Chapter
Limits and Derivatives

Topic-1: Limit of a Function, Sandwitch Theorem

1 MCQs with One Correct Answer

1. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$. Then $\lim\limits_{a\to 0^+}\alpha(a)$ and $\lim\limits_{a\to 0^+}\beta(a)$ are **[2012]**

(a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and -1
(c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

2. If $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then **[2012]**

(a) $a=1, b=4$ (b) $a=1, b=-4$
(c) $a=2, b=-3$ (d) $a=2, b=3$

3. If $\lim\limits_{x\to 0}\frac{((a-n)nx-\tan x)\sin nx}{x^2}=0$, where n is nonzero real number, then a is equal to **[2003S]**

(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n+\frac{1}{n}$

4. $\lim\limits_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ equals **[2001S]**

(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

5. $\lim\limits_{n\to\infty}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+....+\frac{n}{1-n^2}\right\}$ is equal to **[1984 - 2 Marks]**

(a) 0 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) None

6. If $G(x)=-\sqrt{25-x^2}$ then $\lim\limits_{x\to 1}\frac{G(x)-G(1)}{x-1}$ has the value **[1983 - 1 Mark]**

(a) $\frac{1}{24}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) None

7. If $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim\limits_{x\to\infty}f(x)$ is **[1979]**

(a) 0 (b) ∞
(c) 1 (d) none of these

2 Integer Value Answer/ Non-Negative Integer

8. The value of the limit

$$\lim_{x\to\frac{\pi}{2}}\frac{4\sqrt{2}(\sin 3x+\sin x)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$$

is _____ **[Adv. 2020]**

9. Let m and n be two positive integers greater than 1. If $\lim\limits_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is **[Adv. 2015]**

10. The largest value of non-negative integer a for which $\lim\limits_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is **[Adv. 2014]**

3 Numeric/ New Stem Based Questions

11. Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x\to 0^+}\frac{(1-x)^{\frac{1}{x}}-e^{-1}}{x^a}$$

is equal to a non-zero real number, is _____ **[Adv. 2020]**

4 Fill in the Blanks

12. If $f(9)=9$, $f'(9)=4$, then $\lim\limits_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equals............ **[1988 - 2 Marks]**

13. $\lim_{x\to-\infty}\left[\dfrac{x^4\sin\left(\frac{1}{x}\right)+x^2}{(1+|x|^3)}\right]=$ .... [1987 - 2 Marks]

14. If $f(x)=\begin{cases}\sin x, & x\neq n\pi, n=0,\pm1,\pm2,\pm3,.............\\ 2, & \text{otherwise}\end{cases}$

and $g(x)=\begin{cases}x^2+1, & x\neq 0,2\\ 4, & x=0\\ 5, & x=2\end{cases}$

then $\lim_{x\to0} g[f(x)]$ is .... [1986 - 2 Marks]

5 True / False

15. If $\underset{x\to a}{Lt}[f(x)g(x)]$ exists then both $\underset{x\to a}{Lt}f(x)$ and $\underset{x\to a}{Lt}g(x)$ exist. [1981 - 2 Marks]

6 MCQs with One or More than One Correct Answer

16. Let $f(x)=\dfrac{1-x(1+|1-x|)}{|1-x|}\cos\left(\dfrac{1}{1-x}\right)$ for $x\neq1$. Then [Adv. 2017]

(a) $\lim_{x\to1^-} f(x)=0$

(b) $\lim_{x\to1^-} f(x)$ does not exist

(c) $\lim_{x\to1^+} f(x)=0$

(d) $\lim_{x\to1^+} f(x)$ does not exist

17. For $a\in\mathbb{R}$ (the set of all real numbers), $a\neq-1$, [Adv. 2013]

$$\lim_{n\to\infty}\frac{(1^a+2^a+...+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]}=\frac{1}{60}.$$

Then $a=$

(a) 5 (b) 7 (c) $\dfrac{-15}{2}$ (d) $\dfrac{-17}{2}$

18. $\lim_{x\to1}\dfrac{\sqrt{1-\cos2(x-1)}}{x-1}$ [1998 - 2 Marks]

(a) exists and it equals $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exist because $x-1\to0$

(d) does not exist because the left hand limit is not equal to the right hand limit.

19. The value of $\lim_{x\to0}\dfrac{\sqrt{\frac{1}{2}(1-\cos2x)}}{x}$ [1991 - 2 Marks]

(a) 1 (b) −1
(c) 0 (d) none of these

9 Assertion and Reason/Statement Type Questions

20. Let $f:R\to R$ be a function. We say that f has

PROPERTY 1: If $\lim_{h\to0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2: If $\lim_{h\to0}\dfrac{f(h)-f(0)}{h^2}$ exists and is finite

Then which of the following options is/are correct? [Adv. 2019]

(a) $f(x)=x^{2/3}$ has **PROPERTY 1**

(b) $f(x)=\sin x$ has **PROPERTY 2**

(c) $f(x)=|x|$ has **PROPERTY 1**

(d) $f(x)=x|x|$ has **PROPERTY 2**

10 Subjective Problems

21. $f'(0)=\lim_{n\to\infty} nf\left(\dfrac{1}{n}\right)$ and $f(0)=0$. Using this find $\lim_{n\to\infty}\left((n+1)\dfrac{2}{\pi}\cos^{-1}\left(\dfrac{1}{n}\right)-n\right), \left|\cos^{-1}\dfrac{1}{n}\right|<\dfrac{\pi}{2}$ [2004 - 2 Marks]

22. Use the formula $\lim_{x\to0}\dfrac{a^x-1}{x}=\ln a$ to find $\lim_{x\to0}\dfrac{2^x-1}{(1+x)^{1/2}-1}$ [1982 - 2 Marks]

23. Evaluate : $\lim_{h\to0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ [1980]

24. $f(x)$ is the integral of $\dfrac{2\sin x-\sin2x}{x^3}$, $x\neq0$, find $\lim_{x\to0}f'(x)$ [1979]

25. Evaluate $\lim_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, $(a\neq0)$ [1978]

Topic-2: Limits Using L-Hopital's Rule, Evaluation of Limits of the form 1^∞, Limits by Expansion Method

1 MCQs with One Correct Answer

1. Let $k\in\mathbb{R}$. If $\lim_{x\to0+}(\sin(\sin kx)+\cos x+x)^{\frac{2}{x}}=e^6$, then the value of k is [Adv. 2024]

(a) 1 (b) 2
(c) 3 (d) 4

2. If $\lim_{x\to0}\left[1+x\,\ell n(1+b^2)\right]^{1/x} = 2b\sin^2\theta, b>0$ and $\theta\in(-\pi,\pi]$, then the value of θ is [2011]

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

3. $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2-\frac{\pi^2}{16}}$ equals [2007 - 3 marks]

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$ (c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

4. The value of $\lim_{x\to0}\left((\sin x)^{1/x}+(1/x)^{\sin x}\right)$, where $x>0$ is [2006 - 3M, –1]

(a) 0 (b) –1 (c) 1 (d) 2

5. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is [2004S]

(a) 1 (b) 0 (c) –1 (d) 2

6. $\lim_{h\to0}\frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$, given that $f'(2)=6$ and $f'(1)=4$ [2003S]

(a) does not exist (b) is equal to – 3/2
(c) is equal to 3/2 (d) is equal to 3

7. Let $f:R\to R$ be such that $f(1)=3$ and $f'(1)=6$. Then $\lim_{x\to0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals [2002S]

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

8. The integer n for which $\lim_{x\to0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number is [2002S]

(a) 1 (b) 2 (c) 3 (d) 4

2 Integer Value Answer/ Non-Negative Integer

9. If $\beta=\lim_{x\to0}\frac{e^{x^3}-\left(1-x^3\right)^{\frac{1}{3}}+\left(\left(1-x^2\right)^{\frac{1}{2}}-1\right)\sin x}{x\sin^2 x}$, then the value of 6β is ______. [Adv. 2022]

10. Let $\alpha, \beta\in\mathbb{R}$ be such that $\lim_{x\to0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then $6(\alpha+\beta)$ equals. [Adv. 2016]

3 Numeric/ New Stem Based Questions

11. Let α be a positive real number. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:(\alpha,\infty)\to\mathbb{R}$ be the functions defined by

$$f(x)=\sin\left(\frac{\pi x}{12}\right) \text{ and } g(x)=\frac{2\log_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}.$$

Then the value of $\lim_{x\to\alpha^+}f(g(x))$ is ______. [Adv. 2022]

4 Fill in the Blanks

12. $\lim_{x\to0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}=\ldots$ [1996 - 1 Mark]

13. $\underset{x\to\infty}{Lt}\left(\frac{x+6}{x+1}\right)^{x+4}=$ .... [1990 - 2 Marks]

6 MCQs with One or More than One Correct Answer

14. Let S be the set of all $(\alpha,\beta)\in\mathbb{R}\times\mathbb{R}$ such that

$$\lim_{x\to\infty}\frac{\sin(x^2)(\log_e x)^{\alpha}\sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^{\beta}}=0$$

Then which of the following is (are) correct? [Adv. 2024]

(a) $(-1,3)\in S$ (b) $(-1,1)\in S$
(c) $(1,-1)\in S$ (d) $(1,-2)\in S$

10 Subjective Problems

15. Find $\lim_{x\to0}\{\tan(\pi/4+x)\}^{1/x}$ [1993 - 2 Marks]

Topic-3: Derivatives of Polynomial & Trigonometric Functions, Derivative of Sum, Difference, Product & quotient of two functions

10 Subjective Problems

1. Find the derivative of $\sin(x^2 + 1)$ with respect to x from first principle. **[1978]**

Answer Key

Topic-1 : Limit of a Function, Sandwitch Theorem.

1. (b)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (8)	**9.** (2)	**10.** (2)
11. (1.00)	**12.** (4)	**13.** (–1)	**14.** (1)	**15.** (False)	**16.** (a, d)	**17.** (b, d)	**18.** (d)	**19.** (d)	**20.** (a, c)

Topic-2 : Limits Using L-Hospital's Rule, Evaluation of Limits of the form 1^∞, Limits by Expansion Method

1. (b)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (5)	**10.** (7)
11. (00.50)	**12.** (e^2)	**13.** (e^5)	**14.** (b, c)						

Hints & Solutions

Topic-1: Limit of a Function, Sandwitch Theorem

1. (b) $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$

Let $a+1=y$, then equation reduces to

$(y^{1/3}-1)x^2+(y^{1/2}-1)x+(y^{1/6}-1)=0$

On dividing both sides by $y-1$, we get

$$\left(\frac{y^{1/3}-1}{y-1}\right)x^2+\left(\frac{y^{1/2}-1}{y-1}\right)x+\left(\frac{y^{1/6}-1}{y-1}\right)=0$$

On taking limit as $y\to 1$ i.e. $a\to 0$ on both sides, we get

$$\frac{1}{3}x^2+\frac{1}{2}x+\frac{1}{6}=0 \Rightarrow 2x^2+3x+1=0$$

$\Rightarrow x=-1,\ -\frac{1}{2}$ (roots of the equation)

$$\therefore \lim_{a\to 0^+}\alpha(a)=-1,\ \lim_{a\to 0^+}\beta(a)=-\frac{1}{2}$$

2. (b) Given : $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a)x^2+(1-a-b)x+(1-b)}{x+1}=4$$

For this limit to be finite $1-a=0\Rightarrow a=1$ then given limit reduces to

$$\lim_{x\to\infty}\frac{-bx+(1-b)}{x+1}=4 \Rightarrow \lim_{x\to\infty}\frac{-b+\frac{(1-b)}{x}}{1+\frac{1}{x}}=4$$

$\Rightarrow -b=4$ or $b=-4$, $\therefore a=1,\ b=-4$

3. (d) $\lim_{x\to 0}\frac{[(a-n)nx-\tan x]\sin nx}{x^2}=0$

$$\Rightarrow \lim_{x\to 0} n.\frac{\sin nx}{nx}\left[\left\{(a-n)n-\frac{\tan x}{x}\right\}\right]=0$$

$\Rightarrow n.1\,[(a-n)\,n-1]=0 \Rightarrow a=\frac{1}{n}+n$

[$\because$ n is non zero real number]

4. (b) $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}=\lim_{x\to 0}\frac{\sin(\pi-\pi\sin^2 x)}{x^2}$

$$=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{(\pi\sin^2 x)}{x^2}=\pi$$

5. (b) $\lim_{n\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right)$

$$=\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{1-n^2}=\lim_{n\to\infty}\frac{\Sigma n}{1-n^2}=\lim_{n\to\infty}\frac{n(n+1)}{2(1-n^2)}$$

$$=\lim_{n\to\infty}\frac{1+1/n}{2\left[\frac{1}{n^2}-1\right]}=-1/2$$

6. (d) $\lim_{x\to 1}\frac{-\sqrt{25-x^2}-(-\sqrt{24})}{x-1}$

$$=\lim_{x\to 1}\frac{\sqrt{24}-\sqrt{25-x^2}}{x-1}\times\frac{\sqrt{24}+\sqrt{25-x^2}}{\sqrt{24}+\sqrt{25-x^2}}$$

$$=\lim_{x\to 1}\frac{x^2-1}{(x-1)[\sqrt{24}+\sqrt{25-x^2}]}$$

$$=\lim_{x\to 1}\frac{x+1}{[\sqrt{24}+\sqrt{25-x^2}]}=\frac{2}{2\sqrt{24}}=\frac{1}{2\sqrt{6}}$$

7. (c) $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$;

$$\therefore \lim_{x\to\infty}f(x)=\lim_{x\to\infty}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}}=\sqrt{\frac{1-0}{1+0}}=1$$

8. (8)

$$\lim_{x\to\frac{\pi}{2}}\frac{4\sqrt{2}\cdot 2\sin 2x\cos x}{2\sin 2x\sin\frac{3x}{2}+\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)-\sqrt{2}(1+\cos 2x)}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{8\sqrt{2}\cdot 2\sin x\cos x\cos x}{2\sin 2x\sin\frac{3x}{2}-2\sin 2x\sin\frac{x}{2}-2\sqrt{2}\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{2\sin 2x\left(\sin\frac{3x}{2}-\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{16\sqrt{2}\sin x\cos^2 x}{4\sin x\cos x\left(2\cos x.\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$= \frac{16\sqrt{2}\sin x\cos^2 x}{2\cos^2 x\left(4\sin x\sin\frac{x}{2}-\sqrt{2}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{8\sqrt{2}\sin x}{4\sin x.\sin\frac{x}{2}-\sqrt{2}} = 8$$

9. **(2)** $\lim_{\alpha\to 0}\frac{e^{\cos\alpha^n}-e}{\alpha^m}=\frac{-e}{2}$

$$\Rightarrow \lim_{\alpha\to 0}\frac{e\left[e^{\cos\alpha^n-1}-1\right]}{\cos\alpha^n-1}\times\frac{\cos\alpha^n-1}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow e\lim_{\alpha\to 0}\frac{-2\sin^2\frac{\alpha^n}{2}}{\left(\frac{\alpha^n}{2}\right)^2}\times\frac{\left(\frac{\alpha^n}{2}\right)^2}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow \frac{-e}{2}\alpha^{2n-m}=\frac{-e}{2}\Rightarrow\frac{m}{n}=2$$

10. **(2)** $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{a(1-x)+\sin(x-1)}{(x-1)+\sin(x-1)}\right\}^{1+\sqrt{x}}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{-a+\frac{\sin(x-1)}{x-1}}{1+\frac{\sin(x-1)}{x-1}}\right\}^{1+\sqrt{x}}\Rightarrow\left(\frac{-a+1}{2}\right)^2=\frac{1}{4}$$

$\Rightarrow a=0$ or 2

$\therefore$ Largest value of a is 2.

11. **(1.00)** $\lim_{x\to 0^+}\frac{(1-x)^{1/x}-e^{-1}}{x^a}=\lim_{x\to 0^+}\frac{e^{\left(\frac{\ell n(1-x)}{x}\right)}-\frac{1}{e}}{x^a}$

$$\left[\because (1-x)^{1/x}=e^{1/x\, ln(1-x)}\right]$$

$$\lim_{x\to 0^+}\frac{1}{e}\frac{e^{\left(1+\frac{\ell n(1-x)}{x}\right)}-1}{x^a}=\frac{1}{e}\lim_{x\to 0^+}\frac{\ell n(1-x)+x}{x^{(a+1)}}$$

$$=\frac{1}{e}\lim_{x\to 0^+}\frac{\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-....\right)+x}{x^{a+1}},\ \therefore a=1$$

12. Given : $f(9)=9, f'(9)=4$

$$\therefore \lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$$

$$=\lim_{x\to 9}\frac{(\sqrt{f(x)}-3)(\sqrt{f(x)}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}\cdot\frac{\sqrt{x}+3}{\sqrt{f(x)}+3}$$

$$=\lim_{x\to 9}\frac{\sqrt{x}+3}{\sqrt{f(x)}+3}\times\lim_{x\to 9}\frac{f(x)-9}{x-9}$$

$$=\left[\frac{3+3}{3+3}\right].\ f'(9)=1\times 4=4$$

13. $\lim_{x\to-\infty}\left[\frac{x^4\sin\left(\frac{1}{x}\right)+x^2}{(1+|x|^3)}\right]$

$$=\lim_{x\to-\infty}\frac{x^3}{1+|x|^3}\left[x\sin\left(\frac{1}{x}\right)+\frac{1}{x}\right]$$

$$=\lim_{x\to-\infty}\frac{x^3}{|x|^3}\left[\frac{1}{1+\frac{1}{|x|x^2}}\right]\left[x\sin\left(\frac{1}{x}\right)+\frac{1}{x}\right]$$

$$\left[\because \lim_{x\to 0}\frac{\frac{\sin t}{t}+t}{1+|t|t^2}=1\right]$$

$$=\lim_{x\to-\infty}\frac{x^3}{|x|^3}.1=\lim_{x\to-\infty}\frac{x^3}{-x^3}=-1$$

14. Given :

$$f(x)=\begin{cases}\sin x, x\neq n\pi, n=0,\pm1,\pm2,... \\ 2, \quad \text{otherwise}\end{cases}$$

And $g(x)=\begin{cases}x^2+1, & x\neq 0,2\\ 4, & x=0\\ 5, & x=2\end{cases}$

$$\therefore \lim_{x\to 0}g[f(x)]=\lim_{x\to 0}g(\sin x)\Rightarrow\lim_{x\to 0}(\sin^2 x+1)=1$$

15. **(False)** $f(x)=\frac{|x-a|}{x-a}$ and $g(x)=\frac{x-a}{|x-a|}$ then

$\lim_{x\to a}(f(x)g(x))$ exists but neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists.

16. **(a, d)** Given : $f(x)=\frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x\neq 1$

$$\lim_{x\to 1^-} f(x)=\lim_{h\to 0}\frac{1-(1-h)(1+h)}{h}\cos\left(\frac{1}{h}\right)$$

$$=\lim_{h\to 0}\frac{1-1+h^2}{h}\cos\left(\frac{1}{h}\right)=\lim_{h\to 0} h\cos\left(\frac{1}{h}\right)=0$$

$$\lim_{x\to 1^+} f(x)=\lim_{h\to 0}\frac{1-(1+h)(1+h)}{h}\cos\left(\frac{1}{h}\right)$$

$$=\lim_{h\to 0}\frac{-2h-h^2}{h}\cos\left(\frac{1}{h}\right)=\lim_{h\to 0}(-2-h)\cos\left(\frac{1}{h}\right)$$

$= -2\times$ (Some value oscillating between –1 and 1)

$\therefore\ \lim_{x\to 1^+} f(x)$ does not exist.

17. **(b, d)** Given :

$$\lim_{n\to\infty}\frac{1^a+2^a+---+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+----+(na+n)]}=\frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^a}{(n+1)^{a-1}}\frac{\left(\frac{1}{n}\right)^a+\left(\frac{2}{n}\right)^a+----+\left(\frac{n}{n}\right)^a}{n^2a+\frac{n(n+1)}{2}}=\frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^{a-1}}{(n+1)^{a-1}}\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a}{a+\frac{1}{2}\left(1+\frac{1}{n}\right)}=\frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^{a-1}\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a}{a+\frac{1}{2}\left(1+\frac{1}{n}\right)}=\frac{1}{60}$$

$\because \frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a=\int_0^1 x^a dx$ as $\frac{1}{n}=dx$ and $\frac{r}{n}=x$

when r = 1, $n\to\infty$ then $x\to 0$

when $r=n$ then $x\to 1$

$$\Rightarrow \frac{\int_0^1 x^a dx}{a+\frac{1}{2}}=\frac{1}{60}\Rightarrow \frac{\left[x^{a+1}\right]_0^1}{(a+1)\left(a+\frac{1}{2}\right)}=\frac{1}{60}$$

$$\Rightarrow \frac{1}{(a+1)\left(a+\frac{1}{2}\right)}=\frac{1}{60}$$

$\therefore\ 2a^2+3a-119=0 \Rightarrow (a-7)(2a-17)=0$

$\therefore\ a=7$ or $-\frac{17}{2}$

18. **(d)** $\frac{\sqrt{1-\cos[2(x-1)]}}{x-1}=\frac{\sqrt{2\sin^2(x-1)}}{x-1}$

$$=\sqrt{2}.\frac{\sqrt{\sin^2(x-1)}}{x-1}=\sqrt{2}\frac{|\sin(x-1)|}{x-1}$$

L.H.L. $=\sqrt{2}.\lim_{x\to 1^-}\frac{|\sin(x-1)|}{x-1}=\sqrt{2}.\lim_{h\to 0}\frac{|\sin(-h)|}{-h}$

$$=\sqrt{2}\lim_{h\to 0}\frac{\sin h}{-h}=-\sqrt{2}$$

R.H.L. $=\sqrt{2}\lim_{x\to 1^+}\frac{|\sin(x-1)|}{x-1}$

$$=\sqrt{2}\lim_{h\to 0}\frac{|\sin h|}{h}=\sqrt{2}\lim_{h\to 0}\frac{\sin h}{h}=\sqrt{2}$$

$\because$ L.H.L. $\neq$ R.H.L., $\therefore\ \lim_{x\to 1} f(x)$ does not exist.

19. **(d)** $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$

$$=\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}.2\sin^2 x}}{x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

$\therefore$ L.H.L. $=\lim_{h\to 0}\frac{|\sin(0-h)|}{0-h}=\lim_{h\to 0}\frac{|-\sin h|}{-h}$

$$=\lim_{h\to 0}\frac{\sin h}{-h}=-1$$

R.H.L. $=\lim_{h\to 0}\frac{|\sin(0+h)|}{0+h}=\lim_{h\to 0}\frac{\sin h}{h}=1$

Thus, L.H.L. $\neq$ R.H.L.

Therefore, the given limit does not exist.

20. **(a, c) Property 1:** $\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite

Property 2: $\lim_{h\to 0}\frac{f(h)-f(0)}{h^2}$

(a) $f(x) = x^{2/3}$ for **Property 1**

$$\lim_{h\to 0}\frac{h^{2/3}-0}{\sqrt{|h|}} = \lim_{h\to 0}\frac{|h|^{2/3}}{|h|^{1/2}} = \lim_{h\to 0}|h|^{1/6} = 0$$

∴ option (a) is correct.

(b) $f(x) = \sin x$ for **Property 2**

$$\lim_{h\to 0}\frac{\sin h - \sin 0}{h^2} = \lim_{h\to 0}\frac{\sin h}{h}\times\frac{1}{h}$$

when does not exist.

∴ (b) is incorrect option.

(c) $f(x) = |x|$ for **Property 1**

$$\lim_{h\to 0}\frac{|h|-0}{\sqrt{|h|}} = \lim_{h\to 0}\sqrt{|h|} = 0$$

∴ option (c) is correct

(d) $f(x) = x|x|$ for **Property 2**

$$\lim_{h\to 0}\frac{h|h|-0}{h^2} = \lim_{h\to 0}\frac{|h|}{h}$$

LHL = –1 and RHL = 1

∴ $\lim_{h\to 0}\frac{|h|}{h}$ does not exist

∴ option (d) is incorrect.

21. $$\lim_{n\to\infty}\left[(n+1)\frac{2}{\pi}\cos^{-1}\left(\frac{1}{n}\right)-n\right]$$

$$= \lim_{n\to\infty} n\left[\left(1+\frac{1}{n}\right)\frac{2}{\pi}\cos^{-1}\left(\frac{1}{n}\right)-1\right] = \lim_{n\to\infty} nf\left(\frac{1}{n}\right)$$

where $f(x) = \left[(1+x)\frac{2}{\pi}\cos^{-1}x - 1\right]$ such that

$$f(0) = \left[(1+0)\frac{2}{\pi}\cos^{-1}0-1\right] = \frac{2}{\pi}.\frac{\pi}{2}-1 = 0$$

∴ Using given relation $\lim_{n\to\infty} nf\left(\frac{1}{n}\right) = f'(0)$

given limit becomes

$$= f'(0) = \frac{d}{dx}\left[(1+x)\frac{2}{\pi}\cos^{-1}x-1\right]\Bigg|_{x=0}$$

$$= \frac{2}{\pi}\left[\cos^{-1}x - \frac{1+x}{\sqrt{1-x^2}}\right]\Bigg|_{x=0}$$

$$= \frac{2}{\pi}\left[\frac{\pi}{2}-1\right] = 1-\frac{2}{\pi} = \frac{\pi-2}{\pi}.$$

22. $$\lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1} = \lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1}\times\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

$$= \lim_{x\to 0}\frac{(2^x-1)(\sqrt{1+x}+1)}{1+x-1}$$

$$= \lim_{x\to 0}\frac{2^x-1}{x}.\lim_{x\to 0}(\sqrt{1+x}+1)$$

$$= \ln 2\,.\,(1+1) = 2\ln 2.$$

23. $$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

$$= \lim_{h\to 0}\frac{a^2[\sin(a+h)-\sin a]+2ah\sin(a+h)+h^2\sin(a+h)}{h}$$

$$= \lim_{h\to 0}\frac{a^2\left[2\cos\left(a+\frac{h}{2}\right)\sin\frac{h}{2}\right]}{2\times\frac{h}{2}} + 2a\sin(a+h) + h\sin(a+h)$$

$$= a^2\cos a + 2a\sin a$$

24. Given : $f(x) = \int\frac{2\sin x-\sin 2x}{x^3}dx,\ x\neq 0$

∴ $f'(x) = \frac{2\sin x - \sin 2x}{x^3},\ x\neq 0$

∴ $$\lim_{x\to 0}f'(x) = \lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$$

$$= \lim_{x\to 0}\frac{2\sin x(1-\cos x)(1+\cos x)}{x^3(1+\cos x)}$$

$$= \lim_{x\to 0}2.\frac{\sin^3 x}{x^3}.\frac{1}{1+\cos x} = 2\times(1)^3\times\frac{1}{2} = 1$$

25. $$\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$$

$$= \lim_{x\to a}\frac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{a+2x}+\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{3(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \lim_{x\to a}\frac{(\sqrt{3a+x}+2\sqrt{x})}{3(\sqrt{a+2x}+\sqrt{3x})} = \frac{\sqrt{3a+a}+2\sqrt{a}}{3(\sqrt{a+2a}+\sqrt{3a})}$$

$$= \frac{4\sqrt{a}}{3\times 2\sqrt{3a}} = \frac{2}{3\sqrt{3}}$$

Topic-2: Limits Using L-Hospital's Rule, Evaluation of Limits of the form 1^∞, Limits by Expansion Methoc

1. (b) Let, $\ell = \lim_{x\to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$

Taking log on both sides,

$$\Rightarrow \ln l = \lim_{x\to 0^+} \frac{2}{x}(\sin(\sin kx) + \cos x + x - 1)$$

$$\Rightarrow \ln l = \lim_{x\to 0^+} 2\left(\frac{\sin(\sin kx)}{\sin kx}.\frac{\sin kx}{kx}.\frac{kx}{x} + 1 - \frac{(1-\cos x)}{x^2}.x\right)$$

$$\Rightarrow \ln \ell = 2(k+1) \Rightarrow l = e^{2(k+1)} = e^6$$

$k + 1 = 3 \Rightarrow k = 2$

2. (d) $\lim_{x\to 0}[1 + x\ell n(1+b^2)]^{\frac{1}{x}} = 2b\sin^2\theta$

$$\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}\ell n[1+x\ell n(1+b^2)]} = 2b\sin^2\theta$$

$$\Rightarrow e^{\lim_{x\to 0}\frac{\ell n[1+x\ell n(1+b^2)]}{x\ell n(1+b^2)}\times \ell n(1+b^2)} = 2b\sin^2\theta$$

$$\Rightarrow e^{\ell n(1+b^2)} = 2b\sin^2\theta$$

$$\Rightarrow 1 + b^2 = 2b\sin^2\theta \Rightarrow 2\sin^2\theta = b + \frac{1}{b}$$

We know that $2\sin^2\theta \le 2$ and $b + \frac{1}{b} \ge 2$ for $b > 0$

$\therefore\ 2\sin^2\theta = b + \frac{1}{b} = 2 \Rightarrow \sin^2\theta = 1$

$\because\ \theta \in (-\pi, \pi],\ \therefore \theta = \pm\frac{\pi}{2}$

3. (a) $\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$ $\left[\frac{0}{0}\text{ form}\right]$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\frac{d}{dx}\left[\int_2^{\sec^2 x} f(t)dt\right]}{\frac{d}{dx}\left(x^2 - \frac{\pi^2}{16}\right)} \text{ (using L' Hospital rule)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{f(\sec^2 x).2\sec^2 x\tan x}{2x}$$

$$\left[\because \frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)\,dt\right] = f(h(x))\,h'(x) - f(g(x)).g'(x)\right]$$

$$= \frac{f(2)\times 2\times 2\times 1}{2\times\frac{\pi}{4}} = \frac{8}{\pi}f(2)$$

4. (c) $\lim_{x\to 0} [(\sin x)^{1/x} + (1/x)^{\sin x}]$

$$= \lim_{x\to 0}(\sin x)^{1/x} + \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}$$

$$= 0 + e^{\lim_{x\to 0}\sin x\log\left(\frac{1}{x}\right)}$$

$(\because |\sin x| < 1 \text{ when } x \to 0)$

$$= e^{\lim_{x\to 0}\frac{-\log x}{\operatorname{cosec} x}} = e^{\lim_{x\to 0}\frac{-1/x}{-\operatorname{cosec} x\cot x}}$$

(using L' Hospital rule)

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}.\tan x} = e^0 = 1$$

5. (c) Let $L = \lim_{x\to 0}\frac{f(x^2) - f(x)}{f(x) - f(0)}$ [using L.H. Rule]

$\left[\because f'(a) > 0 \text{ as } f \text{ being strictly increasing}\right]$

$$L = \lim_{x\to 0}\frac{f'(x^2).2x - f'(x)}{f'(x)} = \lim_{x\to 0}\frac{f'(x^2).2x}{f'(x)} - 1 = 0 - 1$$

$= -1$

6. (d) $\lim_{h\to 0}\frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$ $\left[\frac{0}{0}\text{ form}\right]$

$$= \lim_{h\to 0}\frac{f'(2h+2+h^2).(2+2h)}{f'(h-h^2+1).(1-2h)} \text{ [using L.H. rule]}$$

$$= \frac{f'(2).2}{f'(1).1} = \frac{6\times 2}{4\times 1} = 3$$

7. (c) Given $f; R\to R$, $f(1) = 3$ and $f'(1) = 6$

Then $\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{1/x}$

$$= e^{\lim_{x\to 0}\frac{1}{x}[\log f(1+x)-\log f(1)]}$$

$$= e^{\lim_{x\to 0}\frac{\frac{1}{f(1+x)}f'(1+x)}{1}}$$ [using L' Hopital rule]

$$= e^{\frac{f'(1)}{f(1)}} = e^{6/3} = e^2$$

8. (c) $\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$

$$= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)(e^x-\cos x)}{x^n(1+\cos x)}$$

$$= \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)\cdot\left(\frac{e^x-\cos x}{x^{n-2}}\right)\cdot\left(\frac{1}{1+\cos x}\right)$$

$$= 1^2.\frac{1}{2}\lim_{x\to 0}\frac{e^x-\cos x}{x^{n-2}}$$

$$= \frac{1}{2}\lim_{x\to 0}\frac{e^x+\sin x}{(n-2)x^{n-3}}$$ [using L' Hopital's rule]

For this limit to be finite, $n-3=0 \quad \Rightarrow n=3$

9. (5) $\beta = \lim_{x\to 0}\frac{e^{x^3}-(1-x^3)^{\frac{1}{3}}+((1-x^2)^{\frac{1}{2}}-1)\sin x}{x.\frac{\sin^2 x}{x^2}.x^2}$,

Use expansion

$$\beta = \lim_{x\to 0}\left[\frac{\left(1+x^3+\frac{x^6}{2!}+...\right)-\left(1-\frac{1}{3}x^3+\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{1}{2}\right)x^6+...\right)+}{x^3}\right.$$

$$\left.\left(-\frac{1}{2}x^2+\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)x^4+...\right)\left(x-\frac{x^3}{3!}+...\right)\right]$$

$$\beta = \lim_{x\to 0}\frac{x^3\left(1+\frac{1}{3}-\frac{1}{2}\right)}{x^3}$$ (Neglecting higher powers of x)

So, $\beta = \frac{5}{6} \Rightarrow 6\beta = 5$

10. (7) $\lim_{x\to 0}\frac{x^2\sin\beta x}{\alpha x-\sin x} = 1$

$$\Rightarrow \lim_{x\to 0}\frac{x^3\beta}{\alpha x-\sin x} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{x^3\beta}{\alpha x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+....\infty\right)} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{x^3\beta}{(\alpha-1)x+\frac{x^3}{3!}-\frac{x^5}{5!}+....\infty} = 1$$

It is possible, when

$\alpha-1=0$ and $\beta=\frac{1}{3!} \Rightarrow \alpha=1$ and $\beta=\frac{1}{6}$

$$\therefore 6(\alpha+\beta) = 6\left(1+\frac{1}{6}\right) = 7$$

11. (00.50) We have, $g(x) = \frac{2\log_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}$

Now, $\lim_{x\to\alpha^+} g(x) = \lim_{x\to\alpha^+}\frac{\frac{2}{\sqrt{x}-\sqrt{\alpha}}\left(\frac{1}{2\sqrt{x}}\right)}{\frac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\left(\frac{1}{2\sqrt{x}}e^{\sqrt{x}}\right)}$,

$$\left\{\text{when } x\to\alpha^+, f(x)\to\frac{\infty}{\infty}\right\}$$

Apply L.H. Rule

$$= \lim_{x\to\alpha^+}\frac{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}{\sqrt{x}-\sqrt{\alpha}}\cdot\frac{1}{e^{\sqrt{x}}}\cdot 2$$

$$= \lim_{x\to\alpha^+}\frac{e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}}\cdot\frac{2}{e^{\sqrt{x}}} = 2$$

$$= \lim_{x\to\alpha^+} f(g(x)) = f\left(\lim_{x\to\alpha^+} g(x)\right) = \sin\frac{\pi}{6} = \frac{1}{2} = 00.50$$

12. $f(x)^{g(x)} = e^{\log f(x)^{g(x)}} = e^{g(x)\log f(x)}$

$$\Rightarrow \lim_{x\to0}[f(x)]^{g(x)} = e^{\lim_{x\to0} g(x)\log f(x)}$$

$$\therefore \lim_{x\to0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2} = e^{\lim_{x\to0}\frac{1}{x^2}\log\left[\frac{1+5x^2}{1+3x^2}\right]}$$

$$= e^{\lim_{x\to0}\left[5.\frac{\log(1+5x^2)}{5x^2} - 3.\frac{\log(1+3x^2)}{3x^2}\right]} = e^{5-3} = e^2$$

13. $$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left\{\left[1+\frac{5}{x+1}\right]^{\frac{x+1}{5}}\right\}^{5\left(\frac{x+4}{x+1}\right)}$$

$$e^{\lim_{x\to\infty} 5\left(\frac{x+4}{x+1}\right)} = e^{5\lim_{x\to\infty}\left(\frac{1+4/x}{1+1/x}\right)} = e^5 \left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$$

14. **(b, c)** Given, $\lim_{x\to\infty}\frac{\sin(x^2)\sin\left(\frac{1}{x^2}\right)(\ln x)^\alpha}{x^{\alpha\beta}(\ln(1+x))^\beta} = 0$

$$= \lim_{x\to\infty}\frac{(\sin x^2)\sin\left(\frac{1}{x^2}\right)\frac{1}{x^2}(\ln x)^\alpha}{\left(\frac{1}{x^2}\right)x^{\alpha\beta}(\ln(1+x))^\beta} = 0$$

$$= \lim_{x\to\infty}\left(\frac{\ln x}{\ln(1+x)}\right)^\beta . \frac{(\ln x)^{\alpha-\beta}}{x^{\alpha\beta+2}} = 0$$

$$= \lim_{x\to\infty}\frac{(\ln x)^{\alpha-\beta}}{x^{\alpha\beta+2}} = 0$$

It is possible if $\alpha\beta + 2 > 0$ $\alpha\beta > -2$

15. $$\lim_{x\to0}\left\{\tan\left(\frac{\pi}{4}\right)+x\right\}^{\frac{1}{x}} = e^{\lim_{x\to0}\log\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{\frac{1}{x}}}$$

$$= e^{\lim_{x\to0}\frac{\log\tan\left(\frac{\pi}{4}+x\right)}{x}} \qquad \left[\frac{0}{0}\text{form}\right]$$

$$= e^{\lim_{x\to0}\left[\frac{\sec^2\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}\right)+x}\right]} \qquad \text{[using L H rule]}$$

$$= e^{\frac{2}{1}} = e^2$$

Topic-3: Derivatives of Polynomial & Trigonometric Functions, Derivative of Sum, Difference, Product & Quotient of two functions

1. Let $f(x) = \sin(x^2+1)$

$$f'(x) = \lim_{\Delta x\to0}\frac{\sin[(x+\Delta x)^2+1] - \sin[x^2+1]}{\Delta x}$$

$$\Rightarrow f'(x) = \lim_{\Delta x\to0} \frac{2\cos\left(\frac{x^2+(\Delta x)^2+2x\Delta x+1+x^2+1}{2}\right)\sin\left(\frac{x^2+(\Delta x)^2+2x\Delta x+1-x^2-1}{2}\right)}{\Delta x}$$

$$= \lim_{\Delta x\to0}\frac{2\cos\left[x^2+1+x\Delta x+\frac{(\Delta x)^2}{2}\right]\sin\left[\frac{(\Delta x)^2+2x\Delta x}{2}\right]}{\Delta x\left[\frac{\Delta x+2x}{2}\right]} \times \left(\frac{\Delta x+2x}{2}\right)$$

$$= 2\cos(x^2+1)\lim_{\Delta x\to0}\frac{\sin\left[\frac{(\Delta x)^2+2x\Delta x}{2}\right]}{\left[\frac{(\Delta x)^2+2x\Delta x}{2}\right]} \times \left(\frac{\Delta x+2x}{2}\right)$$

$$= 2\cos(x^2+1) \times 1 \times \frac{2x}{2} = 2x\cos(x^2+1)$$