10. Binomial Distribution

- **Binomial distribution:** For binomial distribution B(n, p), the probability of x successes is denoted by P($X = x$) or P(X) and is given by $P(X = x) = {}^{n}C_{x}q^{n-x}p^{x}, x = 0, 1, 2, ..n, q = 1 - p$
  Here, P(X) is called the probability function of the binomial distribution.

Example:
An unbiased coin is tossed 5 times. Find the probability of getting atleast 4 heads.

Solution:
Let the random variable X denotes the number of heads.

Here, n = 5 and P (getting a head) $= \frac{1}{2}$

$\therefore p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X = r) = {}^{n}C_{r}p^{r}q^{n-r} = {}^{5}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{5-r} = {}^{5}C_{r}\left(\frac{1}{2}\right)^{5}$

P (getting at-least 4 heads)

$= P(X \geq 4)$

$= P(X = 4) + P(X = 5)$

$= {}^{5}C_{4}\left(\frac{1}{2}\right)^{5} + {}^{5}C_{4}\left(\frac{1}{2}\right)^{5}$

$= (5 + 1)\left(\frac{1}{2}\right)^{5}$

$= 6 \times \frac{1}{32}$

$= \frac{3}{16}$

Normal Distribution

A continuos random variable X with parameter μ and σ2 is said to follow normal distribution if its probability density function is given as

fx=1σ2πe-x-μ22σ2; -∞<x<∞, -∞<μ<∞,σ>00   ; Otherwise

A random variable X that follows normal distribution with parameters μ and σ2 is represented as X~Nμ, σ2. The normal probability distribution or the normal curve is bell-shaped. The curve is symmetric about the mean, μ and the flatness of the curve is determined by its standard deviation, σ.

If $f(x)$ is a probability density function, then Px1<X<x2=∫x1x2fxdx. This is same as the area bounded by the probability density curve $f(x)$, x-axis, $x = x_1$ and $x = x_2$.

The total area under the normal distribution curve is 1. Since the normal distribution curve is symmetric about the mean μ, therefore, PX<μ=PX>μ=12.

Standard Normal Variable

Let *Z* be a random variable defined as $Z=X-\mu\sigma$. The distribution of *Z* is normal distribution with parameters $\mu=0$ and $\sigma2=1$ and is represented as $Z\sim N0, 1$. The distribution with mean, $\mu=0$ and standard deviation $\sigma=1$ is called the standard normal distribution and the random variable *Z* is called the standard normal variable.
A continuos random variable *Z* is said to be standard normal variable if its probability density function is

$fz=12\pi e-z22$; $-\infty<z<\infty 0$; Otherwise