Chapter – 3
Pair of Linear Equations in Two Variables

Exercise 3.6

Q. 1 Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2, \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2, \frac{4}{\sqrt{x}}+\frac{9}{\sqrt{y}}=-9$

(iii) $\frac{4}{x}+3y=14, \frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2, \frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5, \frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy, 2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4, \frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer: Let $\frac{1}{x}=p \ and \ \frac{1}{y}=q$, then the equations becomes.

$\frac{p}{2}+\frac{q}{3}=2 \Rightarrow 3p+2q-12=0$ … (i)

$\frac{p}{2}+\frac{q}{2}=\frac{13}{6} \Rightarrow 2p+3q-13=0$ … (ii)

Using cross-multiplication method, we obtain,

$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$

$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$

$\frac{p}{10}=\frac{1}{5} \ and \ \frac{q}{15}=\frac{1}{5}$

p = 2 and q = 3

Note: These questions can also be solved by elimination method and the substitution method.

In the elimination method, the coefficient of one variable in both equations is made the same by multiplying the equation and the variable is eliminated. In the substitution method, the value of one variable is calculated in terms of another variable by equation one and then that value is put into another equation.

$\frac{1}{x} = 2 \; and \; \frac{1}{y} = 3$

$x = \frac{1}{2} \; and \; y = \frac{1}{3}$

(ii) Putting $\frac{1}{\sqrt{x}} = p \; and \frac{1}{\sqrt{y}} = q$ in the given equations, we obtain

2p +3q = 2.....(i)

4p - 9q = -1 .....(ii)

Multiplying equation (1) by 3,

we obtain 6p + 9q = 6..... (iii)

Adding equation (ii) and (iii), we obtain

10p = 5

$p = \frac{1}{2}$ … (iv)

$2 \times \frac{1}{2} + 3q = 2$

= 3q = 1

$= q = \frac{1}{3}$

$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2 = x = 4$

$p = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3 = y = 9$

Hence, x = 4 and y = 9.

(iii) Putting $\frac{1}{x} = p$ in given equations, we get

= 4p + 3y = 14

= 4p + 3y - 14 = 0.....(i)

And, 3p - 4y = 23

= 3p - 4y -23 = 0 .....(ii)

By cross- multiplication, we get,

$= \frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$

$= \frac{p}{-125} = \frac{y}{50} = -\frac{1}{25}$

Now,

$= \frac{p}{-125} = -\frac{1}{25}$, so, p = 5

$= \frac{y}{50} = -\frac{1}{25}$, So y = -2

(iv) Putting $\frac{1}{x-1}$ = p and $\frac{1}{y-1}$ = 1, we get

= 5p + q = 2 ……(i)

= 6p – 3q = 1 ……..(ii)

Now, multiplying equation (i) by 3 we get,

= 15p + 3q = 6 ……(iii)

Adding equations (ii) and (iii)

21p = 7

$= p = \frac{7}{21} = \frac{1}{3}$

Putting value of p in equation (iii) we get,

$= 6 \times \frac{1}{3} - 3q = 1$

$= -3q = -1$

$= q = \frac{1}{3}$

we know that,

$p = \frac{1}{x-1} = \frac{1}{3}$

$= 3 = x - 1$

$= x = 4$

and, $q = \frac{1}{y-2} = \frac{1}{3}$

$= 3 = y - 2$

$= y = 5$

Hence $x = 4, y = 5$

v). $\frac{7}{y} - \frac{2}{x} = 5$ ……..(i)

$= \frac{8x+7y}{xy} = 15$

$= \frac{8}{y} + \frac{7}{x} = 15$ …(ii)

Putting $\frac{1}{x} = p \; and \; \frac{1}{y} = q$ in (i) and (ii) to get,

7q - 2p = 5 .....(iii)

8q + 7p = 15 .....(iv)

multiplying equation (iii) by 7 and equation (iv) by 2 . we get,

49q - 14p = 35.....(v)

16q + 14p = 30.....(vi)

After adding equations (v) and (vi) . we get,

65q = 65

= q = 1

Putting value of q in equation (iv) , we get,

8 + 7p = 15

= 7p = 15 - 8 = 7

= p = 1

Now,

p = $\frac{1}{x} = \frac{1}{1} = 1$

q = $\frac{1}{y} = \frac{1}{1} = 1$

Hence , x = 1 and y = 1

(vi) 6x + 3y = 6xy

dividing equation by xy $\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} = \frac{6}{y} + \frac{3}{x} = 6$ .....(i)

2x + 4y = 5xy

dividing equation by xy, $\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy} = \frac{2}{y} + \frac{4}{x} = 5$ .....(i)

Putting $\frac{1}{x}$ = p and $\frac{1}{y}$ = q, we get,

6q + 3p - 6 = 0

2q + 4p - 5 = 0

By cross multiplication method , we get,

= $\frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$

$= \frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$

After comparing we get,

p = 1 and q = $\frac{1}{2}$

Now ,

p = $\frac{1}{x}$ =1 and q = $\frac{1}{y} = \frac{1}{2}$

Hence, x = 1 and y = 2

(vii) Putting $\frac{1}{x+y}$ = p and $\frac{1}{x-y}$ = q, we get

10p + 2q - 4 = 0.....(i)

15p - 5q +2 = 0 .....,,(ii)

By applying cross multiplication method , we get,

$= \frac{p}{4-20} = \frac{q}{-60-(-20)} = \frac{1}{-50-30}$

$= \frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$

After comparing we get,

p = $\frac{1}{5}$ and q =1

Now,

p = $\frac{1}{x+y} = \frac{1}{5}$ So, x + y = 5 …….(iii)

q = $\frac{1}{x-y}$ = 1 = So, x - y = 1 …….(iv)

Adding equations (iii) and (iv) we get,

2x = 6

= x = $\frac{6}{2}$ = 3

Putting value of equation (iii) we get,

y = 2

Hence, x = 3 and y = 2

(viii) Putting $\frac{1}{3x+y}$ = p and $\frac{1}{3x-y}$ = q we get,

p + q = $\frac{3}{4}$ ……..(i)

$\frac{p}{2} - \frac{q}{2} = -\frac{1}{8}$

p – q = - $\frac{1}{4}$ …..(ii)

Addomh (i) and (ii) we get,

2p = $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

= p = $\frac{1}{4}$

Putting value of p in (ii) we get,

= $\frac{1}{4}$ – q = - $\frac{1}{4}$

= q = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Now, p = $\frac{1}{3x+y}$ so, 3x + y = 4 ……..(iii)

q = $\frac{1}{3x-y}$ = 3x - y = 2 ……..(iv)

Adding equations (iii) and (iv) we get,

6x = 6

= x = 1

Putting value of x in equation (iii) we get,

3(1) + y = 4

= y = 1

Hence, x = 1 and y = 1

Q. 2 Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer: (i) Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

While rowing upstream, Ritu's speed slows down and the speed will be her speed minus speed of stream and while rowing downstream her speed will increase and will be equal to sum of her speed and speed of stream. Therefore,

The speed of Ritu while rowing

Upstream = (x - y) km/h

Downstream = (x + y) km/h

According to the question:

Ritu can row downstream 20 km in 2 hours, and

distance = speed × time

⇒ 2 (x+y) = 20

⇒ x+y = 10.....(1)

also, Ritu can row upstream 4 km in 2 hours

⇒ 2 (x - y) = 4

⇒ x-y = 2 .....(2)

Adding equation (1) and (2),

we obtain

⇒ x + y + x - y = 10 + 2

⇒ 2 x = 12

⇒ x = 6

Putting this in equation (1),

6 + y = 10

we obtain y = 4

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii)Let the number of days taken by a woman and a man be x and y respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

and work done by a man in 1 day = $\frac{1}{y}$

According to the question,

2 women and 5 men take 4 days to complete the work

i.e. they take 4 days to complete one work

$\Rightarrow 4\left(\frac{2}{x}+\frac{5}{y}\right)=1$

$\Rightarrow \frac{2}{x}+\frac{5}{y}=\frac{1}{4}$

Also, 3 women and 6 men take 3 days to complete the work i.e. they take 3 days to complete one work

$\Rightarrow 3\left(\frac{3}{x}+\frac{6}{y}\right)=1$

$\Rightarrow \frac{3}{x}+\frac{6}{y}=\frac{1}{3}$

Putting $\frac{1}{x}=p\ and\ \frac{1}{y}=q$ in these equations,

We obtain

$2p+5q=\frac{1}{4}$

$\Rightarrow 8p+20q=1$

And

$3p=6q=\frac{1}{3}$

$\Rightarrow 9p+18q=1$

By cross multiplication, we obtain

$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$

$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$

$\frac{p}{-2}=\frac{-1}{36}\ and\ \frac{q}{-1}=\frac{1}{-36}$

$p=\frac{1}{18}\ and\ q=\frac{1}{36}$

$p=\frac{1}{x}=\frac{1}{18}\ and\ q=\frac{1}{y}=\frac{1}{36}$

x = 18, y = 36

Hence, number of days taken by a woman, x = 18

Number of days taken by a man, y = 36

(iii) Let the speed of train and bus be u km/h and v km/h respectively.

According to the given information,

It takes her 4 hours if she travels 60 km by bus and rest (i.e. 240 km) by train

As $time = \frac{distance}{speed}$

We have

$\frac{60}{u} + \frac{240}{v} = 4$ … (1)

and also, If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer i.e. 4 hours and 10 minutes. Also, 1 hour = 60 minutes.

$\Rightarrow 10\ minutes = \frac{1}{60} \times 10 = \frac{1}{6}\ hour$

And $\Rightarrow 4\ hours\ and\ 10\ minutes = \left(4 + \frac{1}{6}\right) = \frac{25}{6}$ hours

We have

$\frac{100}{u} + \frac{200}{v} = \frac{25}{6}$ ….. (2)

Putting $\frac{1}{u} = p\ and\ \frac{1}{v} = q$ is these equations, we obtain

60p+240q = 4 .....(3)

100p+200q =

600p+1200q = 25 ....(4)

Multiplying equation (3) by 10, we obtain

600p+2400q = 40 ....(5)

Subtracting equation (4) from (5), we obtain

1200q = 15

$q = \frac{15}{1200} = \frac{1}{80}$ … (6)

Substituting in equation (3), we obtain

60p+3 = 4

60p = 1

$p = \frac{1}{60}$

$p = \frac{1}{u} = \frac{1}{60}$ and $q = \frac{1}{v} = \frac{1}{80}$

u = 60km/h and v = 80km/h

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h.