20. Trigonometry: Concept of Measurement of Angle

Let us Work Out 20

1. Question

Let us express the following into degrees, minutes and seconds.

(i) 832’ (ii) 6312’’

(iii) 375’’ (iv) $27\frac{1^\circ}{12}$

(v) 72.04°

Answer

(i) 832’

1 degree = 60 minutes

$\Rightarrow 832' = \frac{832}{60}$ degree

$\Rightarrow 832' = 13\frac{13}{15}$

$\Rightarrow 832' = 13$ degrees 52 minutes

(ii) 6312’’

1 degree = 3600 seconds

$\Rightarrow 6312'' = \frac{6312}{3600}$ degrees

$\Rightarrow 6312'' = \frac{263}{150}$ degree

$\Rightarrow 6312'' = 1\frac{113}{150}$ degree

$\Rightarrow 6312'' = 1$ degree $45\frac{1}{5}$ minutes

⇒ 6312’’ = 1 degree 45 minutes 12 seconds

(iii) 375''

1 degree = 3600 seconds

$$\Rightarrow 375'' = \frac{375}{3600} \text{ degrees}$$

$$\Rightarrow 375'' = \frac{5}{48} \text{ degree}$$

$$\Rightarrow 375'' = 6\frac{1}{4} \text{ minutes}$$

⇒ 375'' = 6 minutes 15 seconds

2. Question

Let us determine the circular values of the followings

(i) 60° (ii) 135°

(ii) -150° (iv) 72°

(v) 22°30' (vi) -62°30'

(vii) 52°52'30''

Answer

(i) 60°

∵ 180° = π

$$\Rightarrow \text{radian} = 60 \times \frac{\pi}{180}$$

$$\Rightarrow \text{radian} = \frac{\pi}{3}$$

(ii) 135°

∵ 180° = π

$$\Rightarrow \text{radian} = 135 \times \frac{\pi}{180}$$

$$\Rightarrow \text{radian} = \frac{3\pi}{4}$$

(ii) -150°

∵ 180° = π

$\Rightarrow \text{radian} = -150 \times \frac{\pi}{180}$

$\Rightarrow \text{radian} = -\frac{5}{6}\pi$

(iv) 72°

$\because 180° = \pi$

$\Rightarrow \text{radian} = 72 \times \frac{\pi}{180}$

$\Rightarrow \text{radian} = \frac{2\pi}{5}$

(v) 22°30’

$\Rightarrow 20° + 30’$

$\Rightarrow 20° + \frac{30}{60}°$

$\Rightarrow 20° + \frac{1}{2}°$

$\Rightarrow \frac{41}{2}°$

$\because 180° = \pi$

$\Rightarrow \text{radian} = \frac{41}{2} \times \frac{\pi}{180}$

$\Rightarrow \frac{41}{360}\pi$

(vi) -62°30’

$\Rightarrow$ -60° - 30’

$\Rightarrow -60° - \frac{1}{2}°$

$\Rightarrow -\frac{121}{2}°$

$\Rightarrow \text{radian} = -\frac{121}{2} \times \frac{\pi}{180}$

$\Rightarrow \text{radian} = -\frac{121}{360}\pi$

(vii) 52°52’30’’

⇒ 52° + 52’ + 30’’

$$\Rightarrow 52° + 52' + \frac{30}{60}'$$

$$\Rightarrow 52° + \frac{105'}{2}$$

$$\Rightarrow 52° + \frac{105}{60 \times 2}°$$

$$\Rightarrow \frac{2503}{48}°$$

∵ 180° = π

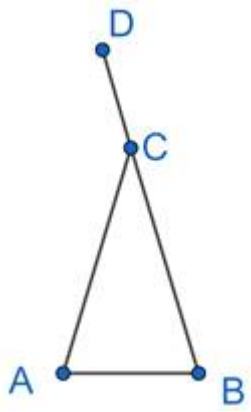
$$\Rightarrow \text{radian} = \frac{2503}{48} \times \frac{\pi}{180}$$

$$\Rightarrow \text{radian} = \frac{2503\pi}{8640}$$

3. Question

In ΔABC, AC=BC and BC is extended upto the point D. If ∠ACD=144°, then let us determine the circular value of each of the angles of ΔABC.

Answer



∠ACD = 144°

∵ DCB is straight line, ∠DCB = 180°

⇒ ∠ACB = 180° - 144° = 36°

∵ AC=BC, by opposite angle property

⇒ ∠CAB = ∠CBA = x

By Angle sum Property

$\Rightarrow \angle CAB + \angle CBA + \angle ACB = 180°$

$\Rightarrow 2x + 36° = 180°$

$\Rightarrow x = 72°$

$\Rightarrow \angle CAB = \angle CBA = 72°$ and $\angle ACB = 36°$

$$\Rightarrow \angle CAB = \angle CBA = 72 \times \frac{\pi}{180}$$

$$\Rightarrow \angle CAB = \angle CBA = \frac{2\pi}{5}$$

And $\angle ACB = 36°$

$$\Rightarrow \angle ACB = 36 \times \frac{\pi}{180}$$

$$\Rightarrow \angle ACB = \frac{\pi}{5}$$

4. Question

If the difference of two acute angles of a right-angled triangle is $\frac{2\pi}{5}$, then let us write the sexagesimal values of two angles.

Answer

let the two angles be x and y.

$\because$ it is a right-angled triangle

$x + y = \frac{\pi}{2}$ ...[1]

Also, by question

$x - y = \frac{2\pi}{5}$ ...[2]

Adding eq. [1] and eq. [2]

$\Rightarrow 2x = \frac{9\pi}{10}$

$\Rightarrow x = \frac{9\pi}{20}$ ...[3]

By eq. [1] and eq. [3]

$$\Rightarrow y = \frac{\pi}{20}$$

Converting x to sexagesimal angle

$1\text{radian} = \frac{180}{\pi}°$

$\Rightarrow \frac{9\pi}{20} = \frac{9\pi}{20} \times \frac{180}{\pi}°$

$\Rightarrow \frac{9\pi}{20} = 81°$

$\Rightarrow x° = 81°$

$\because y° = 90° - x°$

$\Rightarrow y° = 9°$

5. Question

The measure of one angle of a triangle is 65° and other angle is $\frac{\pi}{12}$; let us write the sexagesimal value and circular value of third angle.

Answer

converting $\frac{\pi}{12}$ to sexagesimal

$= \frac{\pi}{12} \times \frac{180}{\pi}$

= 15°

Let c be the third angle

$\because$ sum of angles of a triangle = 180°

$\Rightarrow$ 65° + 15° + c = 180°

$\Rightarrow$ c = 100°

Circular value of c

$100 \times \frac{\pi}{180}$

$= \frac{5\pi}{9}$

6. Question

If the sum of two angles is 135° and their difference is $\frac{\pi}{12}$; then let us determine the sexagesimal value and circular value of two angles.

Answer

converting $\frac{\pi}{12}$ into sexagesimal value

$$\Rightarrow \frac{\pi}{12} \times \frac{180}{\pi}$$

$$\Rightarrow 15°$$

Let the two angles be x and y.

x + y = 135° ...[1]

x - y = 15° ...[2]

adding eq. [1] and eq. [2]

$$\Rightarrow 2x = 150°$$

$$\Rightarrow x = 75° \ldots[3]$$

By [1] and [3]

y = 60°

converting x to circular

$$\Rightarrow 75 \times \frac{\pi}{180}$$

$$\Rightarrow \frac{5\pi}{12}$$

converting y to circular

$$\Rightarrow 60 \times \frac{\pi}{180}$$

$$\Rightarrow \frac{\pi}{3}$$

$$\Rightarrow x = 75° = \frac{5\pi}{12}$$

$$\Rightarrow y = 60° = \frac{\pi}{3}$$

7. Question

If the ratio of three angles of a triangle is 2:3:4, then let us determine the circular value of the greatest angle.

Answer

let the angles be 2x, 3x, 4x

∵ sum of angles of a triangle = 180°

⇒ 2x + 3x + 4x = 180°

⇒ 9x = 180°

⇒ x = 20°

Angles of the triangle

2x = 40°

3x = 60°

4x = 80°

Circular value of 80°

$$\Rightarrow 80 \times \frac{\pi}{180}$$

$$\Rightarrow \frac{4\pi}{9}$$

8. Question

The length of a radius of a circle is 28 cm. Let us determine the circular value of angle subtended by an arc of 5.5 cm length at the centre of this circle.

Answer

let θ be the angle subtended by the arc.

length of arc = rθ

⇒ 28×θ = 5.5

$$\Rightarrow \theta = \frac{11}{56}$$

9. Question

The ratio of two angles subtended by two arcs of unequal lengths at the centre is 5:2 and if the sexagesimal value of the second angle is 30°. Then let us determine the sexagesimal value and the circular value of the first angle.

Answer

let the length of arcs be 5x and 2x

Let r be the radius of the circle

$$\Rightarrow 30 \times \frac{\pi}{180} \times r = 2x \ldots[1]$$

Let θ be the angle subtended by the arc of length 5x.

$\Rightarrow \theta \times r = 5x$ ...[2]

By dividing eq. [1] and eq. [2]

$$\frac{\pi}{6} = \frac{2\theta}{5}$$

$$\Rightarrow \theta = \frac{5\pi}{12}$$

$$\Rightarrow \theta = \frac{5\pi}{12} \times \frac{180}{\pi}$$

$\Rightarrow \theta = 75°$

10. Question

A rotating ray makes an angle $-5\frac{1}{12}\pi$. Let us write by calculating, in which direction the ray has completely rotate and there after what more angle it has produced.

Answer

angle of the ray = $-\frac{63\pi}{12} = -\frac{21\pi}{4}$

The negative sign shows that ray has rotated clockwise.

Adding multiples of 2π

$$-\frac{21\pi}{4} + 3 \times 2\pi$$

$$= \frac{3\pi}{4}$$

$\because$ it is greater than $\frac{\pi}{2}$, so it is in 2nd quadrant.

11. Question

I have drawn an isosceles triangle ABC whose included angle of two equal sides is ∠ABC=45°; the bisector of ∠ABC intersects the side AC at the point D let us determine the circular values of ∠ABD, ∠BAD, ∠CBD and ∠BCD.

Answer

$\angle ABC = 45°$

$$\angle ABC = 45 \times \frac{\pi}{180}$$

$$\Rightarrow \angle ABC = \frac{\pi}{4}$$

$\because$ BD is the angle bisector of $\angle ABC$

$$\Rightarrow \angle ABD = \angle CBD = \frac{\pi}{8}$$

$\angle BAD + \angle ABC + \angle BCD = \pi$

$\because$ ABC is an isosceles triangle

$\Rightarrow \angle BAD = \angle BCD = x$

$$\Rightarrow x + \frac{\pi}{4} + x = \pi$$

$$\Rightarrow 2x = \frac{3\pi}{4}$$

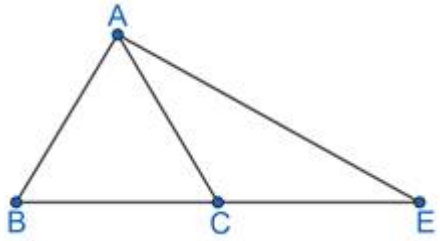
$$\Rightarrow x = \frac{3\pi}{8}$$

$$\Rightarrow \angle BAD = \angle BCD = \frac{3\pi}{8}$$

12. Question

The base BC of the equilateral triangle ABC is extended upto the point E so that CE=BC. By joining A,E, let us determine the circular values of the angles of ΔABC

Answer



∠ABC = ∠BAC = ∠BCA = 60°

$$\Rightarrow \angle ABC = \angle BAC = \angle BCA = 60 \times \frac{\pi}{180}$$

$$\Rightarrow \angle ABC = \angle BAC = \angle BCA = \frac{\pi}{3}$$

∠ACE + ∠ACB = 180°

⇒ ∠ACE = 180° - 60°

⇒ ∠ACE = 120°

∵ BC = CE and BC = AC

⇒ AC = AE

⇒ ∠CAE = ∠AEC = x

∠CAE + ∠AEC + ∠ACE = 180°

⇒ 2x + 120° = 180°

⇒ x = 30°

$$\Rightarrow x = 30 \times \frac{\pi}{180}$$

$$\Rightarrow x = \frac{\pi}{6}$$

$$\Rightarrow \angle CAE = \angle AEC = \frac{\pi}{6}$$

And ∠ACE = 120°

$$\Rightarrow \angle ACE = 120 \times \frac{\pi}{180}$$

$$\Rightarrow \angle ACE = \frac{2\pi}{3}$$

13. Question

If the measures of three angles of quadrilateral are $\frac{\pi}{3}, \frac{5\pi}{6}$ and 90° respectively, then let us determine and write the sexagesimal and circular values of fourth angle.

Answer

sum of angles of quadrilateral = 2π

Let the fourth angle be x

$$\Rightarrow \frac{\pi}{3}+\frac{5\pi}{6}+\frac{\pi}{2}+x=2\pi$$

$$\Rightarrow x=2\pi-\frac{5\pi}{3}$$

$$\Rightarrow x=\frac{\pi}{3}$$

$$\Rightarrow x=\frac{\pi}{3}\times\frac{180}{\pi}^{\circ}$$

$\Rightarrow x = 60°$

14 A1. Question

The end point of the minute hand of a clock rotates in 1 hour

A. $\frac{\pi}{4}$ radian

B. $\frac{\pi}{2}$ radian

C. π radian

D. 2π radian

Answer

angle of complete circle = 2π

Minute hand completes 1 circle in an hour.

14 A2. Question

$\frac{\pi}{6}$ radian equals to

A. 60°

B. 45°

C. 90°

D. 30°

Answer

$\Rightarrow \frac{\pi}{6} \times \frac{180}{\pi}$

$\Rightarrow 30°$

14 A3. Question

The circular value of each internal angle of a regular hexagon is

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{4}$

Answer

Sum of internal angle of a polygon = 180(n-2)

$\Rightarrow$ internal angle of a regular polygon=$\frac{180(n-2)}{n}$

For hexagon n = 6

$\Rightarrow \frac{180(6-2)}{6}$

$\Rightarrow \frac{180 \times 4}{6}$

$\Rightarrow 120°$

$\Rightarrow 120 \times \frac{\pi}{180}$

$\Rightarrow \frac{2\pi}{3}$

14 A4. Question

The measurement of Θ in the relations to S=rΘ is determined by

A. sexagesimal system

B. circular system

C. Those two methods

D. None of these

Answer

Circumference of a circle is 2πr

Where 2π is the angle subtended in circular system and r is the radius.

14 A5. Question

In cyclic quardrilateral ABCD, if ∠a=120°, then the circular of ∠C is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

The sum of opposite angle in a cyclic quadrilateral = π

Converting 120° to cyclic

$$\Rightarrow 120° = 120 \times \frac{\pi}{180}$$

$$\Rightarrow 120° = \frac{2\pi}{3}$$

$$\Rightarrow \frac{2\pi}{3} + \angle c = \pi$$

$$\Rightarrow \angle c = \frac{\pi}{3}$$

14 B. Question

Let us write whether the following statements are true or false:

(i) The angle, formed by rotating a ray centering its end point in anticlockwise direction is positive.

(ii) The angle, formed for completely rotating a ray twice by centering its end point is 720°

Answer

(i) True,

Positive angles are made by rotating anti-clockwise.

(ii) True,

On one rotation angle is 360°

∴ 360° × 2 = 720°

14 C. Question

Let us fill in the blanks:

(i) π radian is a _____angle.

(ii) In sexagesimal system 1 radian equals to _____(approx)

(iii) The circular value of the supplementary angle of the measure $\frac{3\pi}{3}$ is____

Answer

(i) circular

Radian are denotation for circular angles.

(ii) 57.29

$1 \times \frac{180}{\pi}$

=57.29

(iii) 0

Sum of supplementary angles is π

15 A1. Question

If the value of an angle in degree is D and in radian is R; then let us determine the value of $\frac{R}{D}$

Answer

$R = D \times \frac{\pi}{180}$

$$\Rightarrow \frac{R}{D} = \frac{\pi}{180}$$

15 B. Question

Let us write the value of complementary angle of the measure 63°35’15’’

Answer

63° + 35’ + 15’’

$$\Rightarrow 63° + 35' + \frac{15}{60}'$$

$$\Rightarrow 63° + \frac{141}{4}'$$

$$\Rightarrow 63° + \frac{141}{4} \times \frac{1}{60}°$$

$$\Rightarrow 63° + \frac{47}{80}°$$

$$\Rightarrow \frac{5087}{80}°$$

$$\Rightarrow \frac{5087}{80} \times \frac{\pi}{180}$$

$$\Rightarrow \frac{5087\pi}{14400}$$

15 C. Question

If the measures of two angles of a triangle are 65°56’55’’ and 64°3’5’’, then let us determine the circular value of third angle.

Answer

65°56’55’’

$$\Rightarrow 65 + \frac{56}{60} + \frac{55}{60 \times 60}$$

$$\Rightarrow \frac{43883}{720}°$$

In radians

$$\Rightarrow \frac{43883}{720} \times \frac{\pi}{180}$$

= 1.064

Angle 2

64°3’5’’

$$= 64 + \frac{3}{60} + \frac{5}{60 \times 60}$$

$$= \frac{46117}{720}°$$

In radians

$$= \frac{46117}{720} \times \frac{\pi}{180}$$

= 1.130

Third angle

x = π – 1.130 – 1.064

⇒ x = 0.9476

15 D. Question

In a circle, if an arc of 220 cm. length subtends an angle of measure 63° at the centre, then let us determine the radius of the circle.

Answer

converting 63° to radians

$$= 63 \times \frac{\pi}{180}$$

Taking $\pi = \frac{22}{7}$

$$= 63 \times \frac{22}{7 \times 180}$$

$$= \frac{11}{10}$$

Let the radius be r.

$$\Rightarrow r \times \frac{11}{10} = 220$$

⇒ r = 200 cm

15 E. Question

Let us write the circular value of an angle formed by the end point of hour hand of a clock in 1 hour rotation.

Answer

In one complete circle of 12 hours

It completes 2π angle

⇒ In 1 hour, it'll complete

$$= \frac{2\pi}{12}$$

$$= \frac{\pi}{6}$$