4.3 ELASTIC WAVES. ACOUSTICS

4.150 Since the temperature varies linearly we can write the temperature as a function of x, which is, the distance from the point A towards B.

i.e., $$T = T_1 + \frac{T_2 - T_1}{l} x, \; [0 < x < l]$$

hence, $$dT = \left(\frac{T_2 - T_1}{l}\right) dx \qquad (1)$$

In order to travel an elemental distance of dx which is at a distance of x from A it will take a time

$$dt = \frac{dx}{\alpha \sqrt{T}} \qquad (2)$$

From Eqns (1) and (2), expressing dx in terms of dT, we get

$$dt = \frac{l}{\alpha \sqrt{T}} \left(\frac{l\, dT}{T_2 - T_1}\right)$$

Which on integration gives

$$\int_0^t dt = \frac{l}{\alpha (T_2 - T_1)} \int_{T_1}^{T_2} \frac{dT}{\sqrt{T}}$$

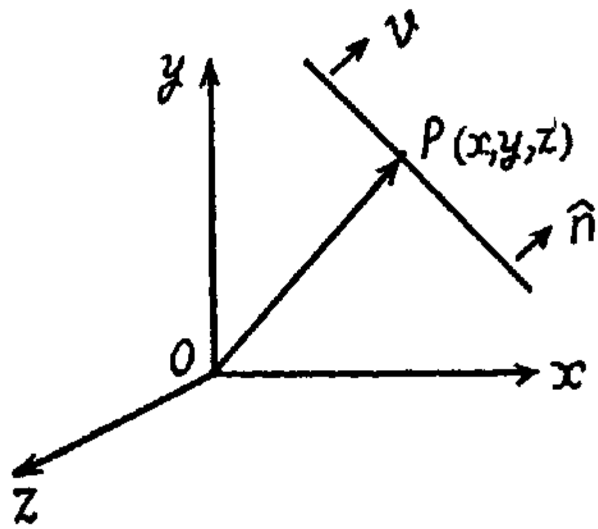
or, $$t = \frac{2l}{(T_2 - T_1)} \left(\sqrt{T_2} - \sqrt{T_1}\right)$$

Hence the sought time $t = \dfrac{2l}{\alpha\left(\sqrt{T_1} + \sqrt{T_2}\right)}$

4.151 Equation of plane wave is given by

$\xi(r, t) = a \cos(\omega t - \vec{k} \cdot \vec{r})$, where $\vec{k} = \frac{\omega}{v} \hat{n}$ called the wave vector

and $\hat{n}$ is the unit vector normal to the wave surface in the direction of the propagation of wave.



or, $$\xi(x, y, z) = a \cos(\omega t - k_x x - k_y y - k_z z)$$
$$= a \cos(\omega t - k x \cos\alpha - k y \cos\beta - k z \cos\gamma)$$

Thus $$\xi(x_1, y_1, z_1, t) = a \cos(\omega t - k x_1 \cos\alpha - k y_1 \cos\beta - k z_1 \cos\gamma)$$

and $$\xi(x_2, y_2, z_2, t) = a \cos(\omega t - k x_2 \cos\alpha - k y_2 \cos\beta - k z_2 \cos\gamma)$$

Hence the sought wave phase difference

$$\varphi_2 - \varphi_1 = k\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]$$

or $$\Delta\varphi = |\varphi_2 - \varphi_1| = k\left|\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]\right|$$
$$= \frac{\omega}{v}\left|\left[(x_1 - x_2)\cos\alpha + (y_1 - y_2)\cos\beta + (z_1 - z_2)\cos\gamma\right]\right|$$

4.152 The phase of the oscillation can be written as

$$\Phi = \omega t - \vec{k}\cdot\vec{r}$$

When the wave moves along the x–axis

$$\Phi = \omega t - k_x x \quad \text{(On putting } k_y = k_z = 0\text{)}.$$

Since the velocity associated with this wave is v_1

We have $$k_x = \frac{\omega}{v_1}$$

Similarly $$k_y = \frac{\omega}{v_2} \text{ and } k_z = \frac{\omega}{v_3}$$

Thus $$\vec{k} = \frac{\omega}{v_1}\hat{e}_x + \frac{\omega}{v_2}\hat{e}_y + \frac{\omega}{v_3}\hat{e}_3.$$

4.153 The wave equation propagating in the direction of +ve x axis in medium K is give as

$$\xi = a \cos(\omega t - k x)$$

So, $\xi = a \cos k(v t - x)$, where $k = \frac{\omega}{v}$ and v is the wave velocity

In the refrence frame K', the wave velocity will be $(v - V)$ propagating in the direction of +ve x axis and x will be x'. Thus the sought wave equation.

$$\xi = a \cos k[(v - V)t - x']$$

or, $$\xi = a \cos\left[\left(\omega - \frac{\omega}{v}V\right)t - k x'\right] = a \cos\left[\omega t\left(1 - \frac{V}{v}\right) - k x'\right]$$

4.154 This follows on actually putting

$$\xi = f(t + \alpha x)$$

in the wave equation $$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}$$

(We have written the one dimensional form of the wave equation.) Then

$$\frac{1}{v^2} f''(t + \alpha x) = \alpha^2 f''(t + \alpha x)$$

so the wave equation is satisfied if

$$\alpha = \pm \frac{1}{v'}$$

That is the physical meaning of the constant α .

4.155 The given wave equation

$$\xi = 60 \cos (1800\, t - 5{\cdot}3\, x)$$

is of the type

$$\xi = a \cos (\omega t - k x), \quad \text{where } a = 60 \times 10^{-6}\,\text{m}$$

$$\omega = 1800 \text{ per sec and } k = 5{\cdot}3 \text{ per metre}$$

As $$k = \frac{2\pi}{\lambda}, \quad \text{so } \lambda = \frac{2\pi}{k}$$

and also $$k = \frac{\omega}{v}, \quad \text{so } v = \frac{\omega}{k} = 340 \text{ m/s}$$

(a) Sought ratio $= \dfrac{a}{\lambda} = \dfrac{a k}{2\pi} = 5{\cdot}1 \times 10^{-5}$

(b) Since $\xi = a \cos (\omega t - k x)$

$$\frac{\partial \xi}{\partial t} = - a \omega \sin (\omega t - k x)$$

So velocity oscillation amplitude

$$\left(\frac{\partial \xi}{\partial t} \right)_m \text{ or } v_m = a \omega = 0{\cdot}11 \text{ m/s} \tag{1}$$

and the sought ratio of velocity oscillation amplitude to the wave propagation velocity

$$= \frac{v_m}{v} = \frac{0{\cdot}11}{340} = 3{\cdot}2 \times 10^{-4}$$

(c) Relative deformation $= \dfrac{\partial \xi}{\partial x} = a k \sin (\omega t - k x)$

So, relative deformation amplitude

$$= \left(\frac{\partial \xi}{\partial x} \right)_m = a k = (60 \times 10^{-6} \times 5{\cdot}3)\,\text{m} = 3{\cdot}2 \times 10^{-4} \text{ m} \tag{2}$$

From Eqns (1) and (2)

$$\left(\frac{\partial \xi}{\partial x} \right)_m = a k = \frac{a \omega}{v} = \frac{1}{v} \left(\frac{\partial \xi}{\partial t} \right)_m$$

Thus $\left(\dfrac{\partial \xi}{\partial x} \right)_m = \dfrac{1}{v} \left(\dfrac{\partial \xi}{\partial t} \right)_m$, where $v = 340$ m/s is the wave velocity.

4.156 (a) The given equation is,

$$\xi = a \cos (\omega t - k x)$$

So at $t = o,$

$$\xi = a \cos kx$$

Now, $$\frac{d\xi}{dt} = -a\omega \sin(\omega t - kx)$$

and $$\frac{d\xi}{dt} = a\omega \sin kx, \text{ at } t = 0.$$

Also, $$\frac{d\xi}{dx} = +ak\sin(\omega t - kx)$$

and at $t = 0,$

$$\frac{d\xi}{dx} = -ak\sin kx.$$

Hence all the graphs are similar having different amplitudes, as shown in the answer-sheet of the problem book.

(b) At the points, where $\xi = 0$, the velocity direction is positive, i.e., along +ve $x-$ axis in the case of longitudinal and + ve y- axis in the case of transverse waves, where $\frac{d\xi}{dt}$ is positive and vice versa.

For sought plots see the answer-sheet of the problem book.

4.157 In the given wave equation the particle's displacement amplitude $= a e^{-\gamma x}$
Let two points x_1 and x_2, between which the displacement amplitude differ by $\eta = 1\%$

So, $$a e^{-\gamma x_1} - a e^{-\gamma x_2} = \eta a e^{-\gamma x_1}$$

or $$e^{-\gamma x_1}(1-\eta) = e^{-\gamma x_2}$$

or $$\ln(1-\eta) - \gamma x_1 = -\gamma x_2$$

or, $$x_2 - x_1 = -\frac{\ln(1-\eta)}{\gamma}$$

So path difference $= -\frac{\ln(1-\eta)}{\gamma}$

and phase difference $= \frac{2\pi}{\lambda} \times$ path difference

$$= -\frac{2\pi}{\lambda}\frac{\ln(1-\eta)}{\gamma} \approx \frac{2\pi\eta}{\lambda\cdot\gamma} = 0{\cdot}3 \text{ rad}$$

4.158 Let S be the source whose position vector relative to the reference point O is $\vec{r}$.
Since intensities are inversely proportional to the square of distances,

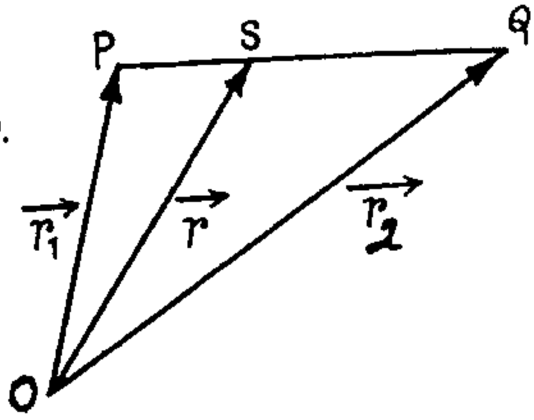
$$\frac{\text{Intensity at } P\,(I_1)}{\text{Intensity at } Q\,(I_2)} = \frac{d_2^2}{d_1^2}$$

where $d_1 = PS$ and $d_2 = QS$.

But intensity is proportional to the square of amplitude.

So, $\frac{a_1^2}{a_2^2} = \frac{d_2^2}{d_1^2}$ or $a_1 d_1 = a_2 d_2 = k$ (say)

Thus $d_1 = \frac{k}{a_1}$ and $d_2 = \frac{k}{a_2}$



Let $\hat{n}$ be the unit vector along PQ directed from P to Q.

Then $$\vec{PS} = d_1 \hat{n} = \frac{k}{a_1}\hat{n}$$

and $$\vec{SQ} = d_2 \hat{n} = \frac{k}{a_2}\hat{n}$$

From the triangle law of vector addition.

$$\vec{OP} + \vec{PS} = \vec{OS} \quad \text{or} \quad \vec{r_1} + \frac{k}{a_1}\hat{n} = \vec{r}$$

or $$a_1 \vec{r_1} + k\hat{n} = a_1 \vec{r} \qquad (1)$$

Similarly $$\vec{r} + \frac{k}{a_2}\hat{n} = \vec{r_2} \quad \text{or} \quad a_2 \vec{r_2} - k\hat{n} = a_2 \vec{r} \qquad (2)$$

Adding (1) and (2),

$$a_1 \vec{r_1} + a_2 \vec{r_2} = (a_1 - a_2)\vec{r}$$

Hence $$\vec{r} = \frac{a_1 \vec{r_1} + a_2 \vec{r_2}}{a_1 + a_2}$$

4.159 (a) We know that the equation of a spherical wave in a homogeneous absorbing medium of wave damping coefficient γ is :

$$\xi = \frac{a'_0 e^{-\gamma r}}{r}\cos(\omega t - kr)$$

Thus particle's displacement amplitude equals

$$\frac{a'_0 e^{-\gamma r}}{r}$$

According to the conditions of the problem,

at $$r = r_0,\; a_0 = \frac{a'_0 e^{-\gamma r_0}}{r_0} \qquad (1)$$

and when $$r = r,\quad \frac{a_0}{\eta} = \frac{a'^{-\gamma r}_0}{r} \qquad (2)$$

Thus from Eqns (1) and (2)

$$e^{\gamma(r-r_0)} = \eta \frac{r_0}{r}$$

or, $$\gamma(r-r_0) = \ln(\eta r_0) - \ln r$$

or, $$\gamma = \frac{\ln\eta + \ln r_0 - \ln r}{r - r_0} = \frac{\ln 3 + \ln 5 - \ln 10}{5} = 0{\cdot}08\ \text{m}^{-1}$$

(b) As $\xi = \frac{a'_0 e^{-\gamma r}}{r} \cos(\omega t - k r)$

So, $$\frac{\partial \xi}{\partial t} = -\frac{a'_0 e^{-\gamma r}}{r} \omega \sin(\omega t - k r)$$

$$\left(\frac{\partial \xi}{\partial t}\right)_m = \frac{a'_0 e^{-\gamma r}}{r} \omega$$

But at point A, $\frac{a'_0 e^{-\gamma r}}{r} = \frac{a_0}{\eta}$

So, $\left(\frac{\partial \xi}{\partial t}\right)_m = \frac{a_0 \omega}{\eta} = \frac{a_0 2\pi}{\eta} = \frac{50\times10^{-6}}{3} \times 2 \times \frac{22}{7} \times 1{\cdot}45 \times 10^3 = 15\ \text{m/s}$

4.160 (a) Equation of the resultant wave,

$$\xi = \xi_1 + \xi_2 = 2a \cos k\left(\frac{y-x}{2}\right) \cos\left\{\omega t - \frac{k(x+y)}{2}\right\},$$

$$= a' \cos\left\{\omega t - \frac{k(x+y)}{2}\right\}, \text{ where } a' = 2a\cos k'\left(\frac{y-x}{2}\right)$$

Now, the equation of wave pattern is,

$$x + y = k, \text{ (a Const.)}$$

For sought plots see the answer-sheet of the problem book.

For antinodes, i.e. maximum intensity

$$\cos\frac{k(y-x)}{2} = \pm 1 = \cos n\pi$$

or, $$\pm(x-y) = \frac{2n\pi}{k} = n\lambda$$

or, $$y = x \pm n\lambda,\ n = 0, 1, 2, \ldots$$

Hence, the particles of the medium at the points, lying on the solid straight lines $(y = x \pm n\lambda)$, oscillate with maximum amplitude.

For nodes, i.e. minimum intensity,

$$\cos\frac{k(y-x)}{2} = 0$$

or, $$\pm\frac{k(y-x)}{2} = (2n+1)\frac{\pi}{2}$$

or, $$y = x \pm (2n+1)\lambda/2,$$

and hence the particles at the points, lying on dotted lines do not oscillate.

(b) When the waves are longitudinal,

For sought plots see the answer-sheet of the problem book.

$$k(y-x) = \cos^{-1}\frac{\xi_1}{a} - \cos^{-1}\frac{\xi_2}{a}$$

or, $$\frac{\xi_1}{a} = \cos\left\{k(y-x) + \cos^{-1}\frac{\xi_2}{a}\right\}$$

$$= \frac{\xi_2}{a}\cos k(y-x) - \sin k(y-x)\sin\left(\cos^{-1}\frac{\xi_2}{a}\right)$$

$$= \frac{\xi_2}{a}\cos k(y-x) - \sin k(y-x)\sqrt{1-\frac{\xi_2^2}{a^2}} \qquad (1)$$

from (1),

if $$\sin k(y-x) = 0 \sin(n\pi)$$

$$\xi_1 = \xi_2 (-1)^n$$

thus, the particles of the medium at the points lying on the straight lines, $y = x \pm \frac{n\lambda}{2}$ will oscillate along those lines (even n), or at right angles to them (odd n).

Also from (1),

if $$\cos k(y-x) = 0 = \cos(2n+1)\frac{\pi}{2}$$

$$\frac{\xi_1^2}{a^2} = 1 - \xi_2^2/a^2, \text{ a circle}.$$

Thus the particles, at the points, where $y = x \pm (n \pm 1/4)\lambda$, will oscillate along circles. In general, all other particles will move along ellipses.

4.161 The displacement of oscillations is given by $\xi = a\cos(\omega t - kx)$

Without loss of generality, we confine ourselves to $x = 0$. Then the displacement maxima occur at $\omega t = n\pi$. Concentrate at $\omega t = 0$. Now the energy density is given by

$$w = \rho a^2 \omega^2 \sin^2 \omega t \qquad \text{at } x = 0$$

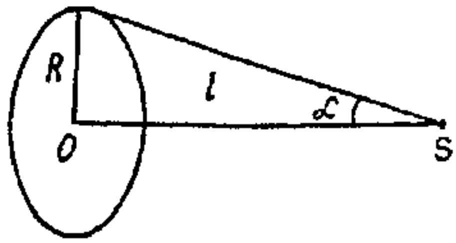
$T/6$ time later (where $T = \frac{2\pi}{\omega}$ is the time period) than $t = 0$.

$$w = \rho a^2 \omega^2 \sin^2\frac{\pi}{3} = \frac{3}{4}\rho a^2\omega^2 = w_0$$

Thus $$\langle w \rangle = \frac{1}{2}\rho a^2 \omega^2 = \frac{2w_0}{3}.$$

4.162 The power output of the source much be

$$4\pi l^2 I_0 = Q \text{ Watt.}$$



The required flux of accoustic power is then : $Q = \frac{\Omega}{4\pi}$

Where Ω is the solid angle subtended by the disc enclosed by the ring at S. This solid angle is

$$\Omega = 2\pi\,(1 - \cos\alpha)$$

So flux
$$\Phi = I_0\, I_0 \left(1 - \frac{l}{\sqrt{r^2 + R^2}}\right) 2\pi l^2$$

Substitution gives $\Phi = 2\Pi \times 30 \left(1 - \frac{l}{\sqrt{1 + \frac{1}{4}}}\right) \mu\text{W} = 1.99\ \mu\,\text{W}.$

Eqn. (1) is a well known result stich is derived as follows; Let SO be the polar axis. Then the required solid angle is the area of that part of the surface a sphere of much radius whose colatitude is $\le \alpha$.

Thus
$$\Omega = \int_0^{\alpha} 2\pi\,\sin\theta\, d\theta = 2\pi\,(1 - \cos\alpha).$$

4.163 From the result of 4.162 power flowing out through anyone of the opening

$$= \frac{P}{2}\left(1 - \frac{h/2}{\sqrt{R^2 + (h/2)^2}}\right)$$

$$= \frac{P}{2}\left(1 - \frac{h}{\sqrt{4R^2 + h^2}}\right)$$

As total power output equals P, so the power reaching the lateral surface must be.

$$= P - 2\cdot\frac{P}{2}\left(1 - \frac{h}{\sqrt{4R^2 + h^2}}\right) = \frac{ph}{\sqrt{4R^2 + h^2}} = 0\cdot 07\text{W}$$

4.164 We are given

$$\xi = a\cos kx\,\omega t$$

so
$$\frac{\partial\xi}{\partial x} = -a k \sin kx \cos\omega t \quad\text{and}\quad \frac{\partial\xi}{\partial t} = -a\omega\cos kx\sin\omega t$$

Thus
$$(\xi)_{t=0} = a\cos kx,\ (\xi)_{t=T/2} = -a\cos kx$$

$$\left(\frac{\partial\xi}{\partial x}\right)_{t=0} = a k\sin kx,\ \left(\frac{\partial\xi}{\partial x}\right)_{t=T/2} = -a k\sin kx$$

(a) The graphs of (ξ) and $\left(\frac{\partial \xi}{\partial x}\right)$ are as shown in Fig. (35) of the book (p.332).

(b) We can calculate the density as follows :

Take a parallelopiped of cross section unity and length dx with its edges at x and $x+dx$.

After the oscillation the edge at x goes to $x+\xi(x)$ and the edge at $x+dx$ goes to $x+dx+\xi(x+dx)$
$= x+dx+\xi(x)+\frac{\partial \xi}{\partial x}dx$. Thus the volume of the element (originally dx) becomes

$$\left(1+\frac{\partial \xi}{\partial x}\right)dx$$

and hence the density becomes $\rho = \dfrac{\rho_0}{1+\dfrac{\partial \xi}{\partial x}}$.

On substituting we get for the density $\rho(x)$ the curves shown in Fig.(35). referred to above.

(c) The velocity $v(x)$ at time $t = T/4$ is

$$\left(\frac{\partial \xi}{\partial t}\right)_{t=T/4} = -a\,\omega\cos kx$$

On plotting we get the figure (36).

4.165 Given $\xi = a\cos kx\cos\omega t$

(a) The potential energy density (per unit volume) is the energy of longitudinal strain. This is

$$w_p = \left(\frac{1}{2}\text{stress} \times \text{strain}\right) = \frac{1}{2}E\left(\frac{\partial \xi}{\partial x}\right)^2, \qquad \left(\frac{\partial \xi}{\partial x}\text{ is the longitudinal strain}\right)$$

$$w_p = \frac{1}{2}E\,a^2k^2\sin^2 kx\cos^2\omega t$$

But $\qquad \dfrac{\omega^2}{k^2} = \dfrac{E}{\rho}\quad$ or $\quad E k^2 = \rho\,\omega^2$

Thus $\qquad w_p = \dfrac{1}{2}\rho\,a^2\omega^2\sin^2 kx\cos^2\omega t$

(b) The kinetic energy density is

$$= \frac{1}{2}\rho\left(\frac{\partial \xi}{\partial t}\right)^2 = \frac{1}{2}\rho\,a^2\omega^2\cos^2 kx\sin^2\omega t.$$

On plotting we get Fig. 37 given in the book (p. 332). For example at $t = 0$

$$w = w_p + w_k = \frac{1}{2}\rho\, a^2 \omega^2 \sin^2 k\,x$$

and the displacement nodes are at $x = \pm \frac{\pi}{2k}$ so we do get the figure.

4.166 Let us denote the displacement of the elements of the string by

$$\xi = a \sin k\,x \, \cos \omega\, t$$

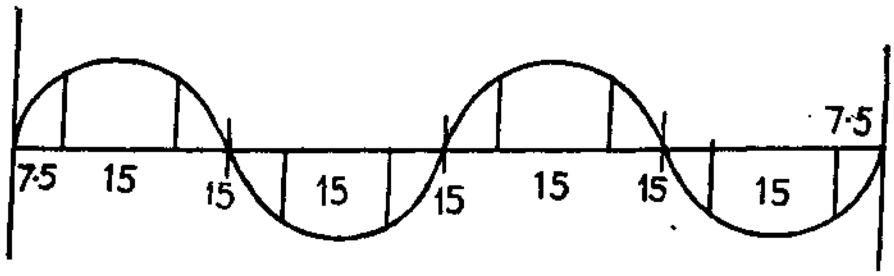
since the string is 120 cm long we must have $k \cdot 120 = n\,\pi$

If x_1 is the distance at which the displacement amplitude first equals 3·5 mm then

$$a \sin k\,x_1 = 3{\cdot}5 = a \sin(k\,x_1 + 15\,k)$$

Then $$k\,x_1 + 15\,k = \pi - k\,x_1 \quad \text{or} \quad k\,x_1 = \frac{\pi - 15\,k}{2}$$

One can convince ourself that the string has the form shown below



It shows that $$k \times 120 = 4\,\pi, \quad \text{so} \quad k = \frac{\pi}{30}\,\text{cm}^{-1}$$

Thus we are dealing with the third overtone

Also $$k\,x_1 = \frac{\pi}{4} \quad \text{so} \quad a = 3{\cdot}5\sqrt{2}\ \text{mm} = 4{\cdot}949\ \text{mm}.$$

4.167 We have $n = \frac{1}{2\,l}\sqrt{\frac{T}{m}} = \frac{1}{2\,l}\sqrt{\frac{T\,l}{M}}$ Where M = total mass of the wire. When the wire is stretched, total mass of the wire remains constant. For the first wire the new length $= l + \eta_1\, l$ and for the record wire, the length is $l + \eta_2 l$. Also $T_1 = \alpha(\eta_1\, l)$ where α is a constant and $T_2 = \alpha(\eta_2\, l)$. Substituting in the above formula.

$$\nu_1 = \frac{1}{2(l + \eta_1\, l)}\sqrt{\frac{(\alpha\,\eta_1\, l)(l + \eta_1\, l)}{M}}$$

$$\nu_2 = \frac{1}{2(l + \eta_2\, l)}\sqrt{\frac{(\alpha\,\eta_2\, l)(l + \eta_2\, l)}{M}}$$

$$\therefore \quad \frac{\nu_2}{\nu_1} = \frac{1 + \eta_1}{1 + \eta_2}\sqrt{\frac{\eta_2}{\eta_1} \cdot \frac{1 + \eta_2}{1 + \eta_1}}$$

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{\eta_2(1 + \eta_1)}{\eta_2(1 + \eta_2)}} = \sqrt{\frac{0{\cdot}04(1 + 0{\cdot}02)}{0{\cdot}02(1 + 0{\cdot}04)}} = 1{\cdot}4$$

4.168 Let initial length and tension be l and T respectively.

So, $$\nu_1 = \frac{1}{2l}\sqrt{\frac{T}{\rho_1}}$$

In accordance with the problem, the new length

$$l' = l - \frac{l \times 35}{100} = 0\cdot65\,l$$

and new tension, $T' = T + \frac{T \times 70}{100} = 1\cdot7\,T$

Thus the new frequency

$$\nu_2 = \frac{1}{2l'}\sqrt{\frac{T'}{\rho_1}} = \frac{1}{2 \times 0\cdot65\,l}\sqrt{\frac{1\cdot7\,T}{\rho_1}}$$

Hence $$\frac{\nu_2}{\nu_1} = \frac{\sqrt{1\cdot7}}{0\cdot65} = \frac{1\cdot3}{0\cdot65} = 2$$

4.169 Obviously in this case the velocityof sound propagation

$$v = 2\nu\,(l_2 - l_1)$$

where l_2 and l_1 are consecutive lengths at which resonance occur

In our problem, $(l_2 - l_1) = l$

So $$v = 2\nu l = 2 \times 2000 \times 8\cdot5 \text{ cm/s} = 0\cdot34 \text{ km/s}.$$

4.170 (a) When the tube is closed at one end

$$\nu = \frac{v}{4l}(2n+1), \quad \text{where} \quad n = 0, 1, 2, \ldots$$

$$= \frac{340}{4 \times 0\cdot85}(2n+1) = 100\,(2n+1)$$

Thus for $n = 0, 1, 2, 3, 4, 5, 6, \ldots,$ we get

$$n_1 = 100\,1\,H_z,\; n_2 = 300\,1\,H_z,\; n_3 = 500\,1\,H_z,\; n_4 = 700\,1\,H_z,$$

$$n_5 = 900\,1\,H_z,\; n_6 = 1100\,1\,H_z,\; n_7 = 1300\,1\,H_z$$

Since ν should be $< \nu_0 = 1250\,1\,H_z$, we need not go beyond n_6.

Thus 6 natural oscillations are possible.

(b) Organ pipe opened from both ends vibrates with all harmonics of the fundamental frequency. Now, the fundamental mode frequency is given as

$$\nu = v/\lambda$$

or, $$\nu = v/2\,l$$

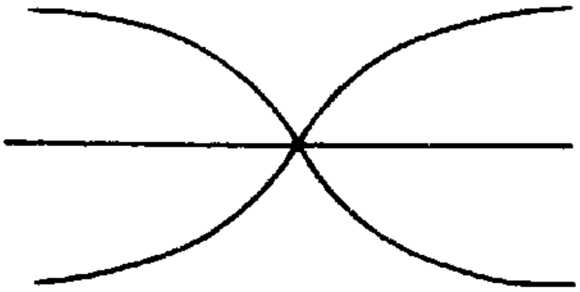
Here, also, end correction has been neglected. So, the frequencies of higher modes of vibrations are given by

$$\nu = n\,(v/2\,l) \qquad (1)$$

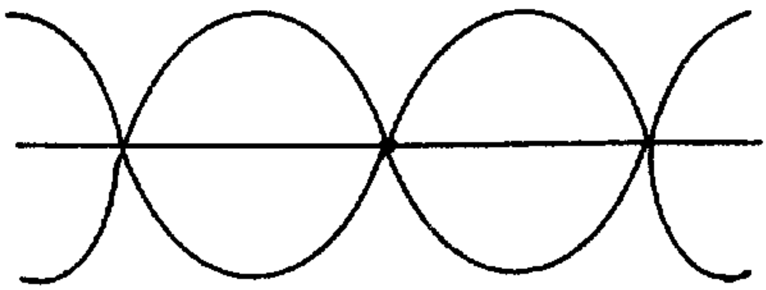
or, $\nu_1 = v/2l$, $\nu_2 = 2(v/2l)$, $\nu_3 = 3(v/2l)$

It may be checked by putting the values of n in the equation (1) that below 1285 Hz, there are a total of six possible natural oscillation frequencies of air column in the open pipe.

4.171 Since the copper rod is clamped at mid point, it becomes a mode and the two free ends will be anitinodes. Thus the fundamental mode formed in the rod is as shown in the Fig. (a).



4.171 (a)



4.171 (b)

In this case, $$l = \frac{\lambda}{2}$$

So, $$\nu_0 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{E}{\rho}}\sqrt{\frac{E}{e}}$$

where E = Young's modules and ρ is the density of the copper

Similarly the second mode or the first overtone in the rod is as shown above in Fig. (b).

Here $$l = \frac{3\lambda}{2}$$

Hence $$\nu_1 = \frac{3v}{2l} = \frac{3}{2l}\sqrt{\frac{E}{\rho}}$$

$$\nu = \frac{2n+1}{2l}\sqrt{\frac{E}{\rho}} \text{ where } n = 0, 1, 2 \ldots$$

Putting the given values of E and ρ in the general equation

$$\nu = 3{\cdot}8\,(2n+1)\text{ k Hz}$$

Hence $\nu_0 = 3\cdot 8$ k.Hz, $\nu_1 = (3{\cdot}8 \times 3)$ k Hz, $\nu_2 = (3{\cdot}8) \times 5 = 19$ k Hz,
$\nu_3 = (3{\cdot}8 \times 7) = 26{\cdot}6$ k Hz, $\nu_4 = (3{\cdot}8 \times 9) = 34{\cdot}2$ k Hz,
$\nu_5 = (3{\cdot}8 \times 11) = 41{\cdot}8$ k Hz, $\nu_6 = (3{\cdot}8) \times 13$ k Hz $= 49{\cdot}4$ k Hz and
$\nu_7 = (3{\cdot}8) \times 14$ k Hz > 50 k Hz.

Hence the sought number of frequencies between 20 to 50 k Hz equals 4.

4.172 Let two waves $\xi_1 = a\cos(\omega t - kx)$ and $\xi_2 = a\cos(\omega t + kx)$, superpose and as a result, we have a standing wave (the resultant wave) in the string of the form $\xi = 2a\cos kx \cos\omega t$.

According to the problem $2a = a_m$.

Hence the standing wave excited in the string is

$$\xi = a_m \cos kx \cos \omega t \tag{1}$$

or,

$$\frac{\partial \xi}{\partial t} = -\omega a_m \cos kx \sin \omega t \tag{2}$$

So the kinetic energy confined in the string element of length dx, is given by :

$$dT = \frac{1}{2}\left(\frac{m}{l}dx\right)\left(\frac{\partial \xi}{\partial t}\right)^2$$

or,

$$dT = \frac{1}{2}\left(\frac{m}{l}dx\right) a_m^2 \omega^2 \cos^2 kx \sin^2 \omega t$$

or,

$$dT = \frac{m a_m^2 \omega^2}{2l} \sin^2 \omega t \cos^2 \frac{2\pi}{\lambda} x\, dx$$

Hence the kinetic energy confined in the string corresponding to the fundamental tone

$$T = \int dT = \frac{m a_m^2 \omega^2}{2l} \sin^2 \omega t \int_0^{\lambda/2} \cos^2 \frac{2\pi}{\lambda} x\, dx$$

Because, for the fundamental tone, length of the string $l = \frac{\lambda}{2}$

Integrating we get, $T = \frac{1}{4} m a_m^2 \omega^2 \sin^2 \omega t$

Hence the sought maximum kinetic energy equals, $T_{max} = \frac{1}{4} m a_m^2 \omega^2$,

because for T_{max} , $\sin^2 \omega t = 1$

(ii) Mean kinetic energy averaged over one oscillation period

$$\langle T \rangle = \frac{\int T\, dt}{\int dt} = \frac{1}{4} m a_m^2 \omega^2 \frac{\int_0^{2\pi/\omega} \sin^2 \omega t\, dt}{\int_0^{2\pi/\omega} dt}$$

or,

$$\langle T \rangle = \frac{1}{8} m a_m^2 \omega^2 .$$

4.173 We have a standing wave given by the equation

$$\xi = a \sin kx \cos \omega t$$

So,

$$\frac{\partial \xi}{\partial t} = -a \omega \sin kx \sin \omega t \tag{1}$$

and

$$\frac{\partial \xi}{\partial t} = a k \cos kx \cos \omega t \tag{2}$$

The kinetic energy confined in an element of length dx of the rod

$$dT = \frac{1}{2}(\rho S\,dx)\left(\frac{\partial \xi}{\partial t}\right)^2 = \frac{1}{2}\rho S a^2 \omega^2 \sin^2 \omega t \, \sin^2 kx \, dx$$

So total kinetic energy confined into rod

$$T = \int dT = \frac{1}{2}\rho S a^2 \omega^2 \sin^2 \omega t \int_0^{\lambda/2} \sin^2 \frac{2\pi}{\lambda} x\,dx$$

or, $$T = \frac{\pi S a^2 \omega^2 \rho \sin^2 \omega t}{4k} \qquad (3)$$

The potential energy in the above rod element

$$dU = \int \partial U = -\int_0^{\xi} F_\xi \, d\xi, \text{ where } F_\xi = (\rho S\,dx)\frac{\partial^2 \xi}{\partial t^2}$$

or, $$F_\xi = -(\rho S\,dx)\,\omega^2 \xi$$

so, $$dU = \omega^2 \rho S\,dx \int_0^{\xi} \xi\,d\xi$$

or, $$dU = \frac{\rho\,\omega^2 S \xi^2}{2} dx = \frac{\rho\,\omega^2 S a^2 \cos^2 \omega t \, \sin^2 kx \, dx}{2}$$

Thus the total potential energy stored in the rod $U = \int dU$

or, $$U = \rho\,\omega^2 S a^2 \cos^2 \omega t \int_0^{\lambda/2} \sin^2 \frac{2\pi}{\lambda} x\,dx$$

So, $$U = \frac{\pi \rho S a^2 \omega^2 \cos^2 \omega t}{4k}$$

To find the potential energy stored in the rod element we may adopt an easier way. We know that the potential energy density confined in a rod under elastic force equals :

$$U_D = \frac{1}{2}(\text{stress} \times \text{strain}) = \frac{1}{2}\sigma\,\varepsilon = \frac{1}{2} Y \varepsilon^2$$

$$= \frac{1}{2}\rho v^2 \varepsilon^2 = \frac{1}{2}\frac{\rho\,\omega^2}{k^2}\varepsilon^2$$

$$= \frac{1}{2}\frac{\rho\,\omega^2}{k^2}\left(\frac{\partial \xi}{\partial x}\right)^2 = \frac{1}{2}\rho a^2 \omega^2 \cos^2 \omega t \cos^2 kx$$

Hence the total potential energy stored in the rod

$$U = \int U_D \, dV = \int_0^{\lambda/2} \frac{1}{2} \rho a^2 \omega^2 \cos^2 \omega t \cos^2 kx \, S \, dx$$

$$= \frac{\pi \rho S a^2 \omega^2 \cos^2 \omega t}{4k} \qquad (4)$$

Hence the sought mechanical energy confined in the rod between the two adjacent nodes

$$E = T + U = \frac{\pi \rho \omega^2 a^2 S}{4k}.$$

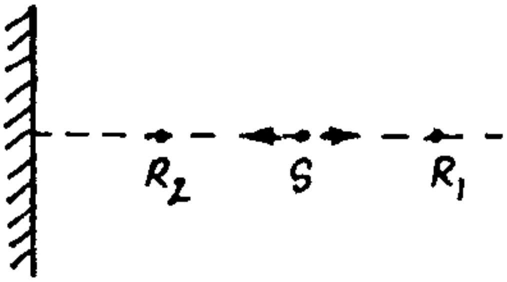
4.174 Receiver R_1 registers the beating, due to the sound waves reaching directly to it from source and the other due to the reflection from the wall.

Frequency of sound reaching directly from S to R_1

$$\nu_{S \to R_1} = \nu_0 \frac{v}{v - u} \text{ when } S \text{ moves towards } R_1$$

and $$\nu'_{S \to R_1} = \nu_0 \frac{v}{v + u} \text{ when } S \text{ moves towards the wall}$$

Now frequency reaching to R_1 after reflection from wall



$$\nu_{W \to R_1} = \nu_0 \frac{v}{v + u}, \text{ when } S \text{ moves towards } R_1$$

and $$\nu'_{W \to R_1} = \nu_0 \frac{v}{v - u}, \text{ when } S \text{ moves towards the wall}$$

Thus the sought beat frequency

$$\Delta \nu = \left(\nu_{S \to R_1} - \nu_{W \to R_1} \right) \text{ or } \left(\nu'_{W \to R_1} - \nu_{S \to R_1} \right)$$

$$= \nu_0 \frac{v}{v - u} - \nu_0 \frac{v}{v + u} = \frac{2 \nu_0 v u}{v^2 - u^2} \approx \frac{2 u \nu_0}{v} = 1 \text{ Hz}$$

4.175 Let the velocity of tuning fork is u. Thus frequency reaching to the observer due to the tuning fork that approaches the observer

$$\nu' = \nu_0 \frac{v}{v - u} \; [v = \text{ velocity of sound }]$$

Frequency reaching the observer due to the tunning fork that recedes from the observer

$$\nu'' = \nu_0 \frac{v}{v + u}$$

So, Beat frequency $\nu - \nu'' = \nu = \nu_0 v \left(\frac{1}{v - u} - \frac{1}{v + u} \right)$

or, $$\nu = \frac{2 \nu_0 v u}{v^2 - u^2}$$

So, $$\nu u^2 + (2 v \nu_0) u - v^2 \nu = 0$$

Hence $$u = \frac{-2\,v\,\nu_0 \pm \sqrt{4\nu_0^2 v^2 + 4 \nu^2 v^2}}{2\,\nu}$$

Hence the sought value of u, on simplifying and noting that $u > 0$

$$u = \frac{v\,\nu_0}{\nu}\left(\sqrt{1 + \left(\frac{\nu}{\nu_0}\right)^2} - 1\right)$$

4.176 Obviously the maximum frequency will be heard when the source is moving with maximum velocity towards the receiver and minimum frequency will be heard when the source recedes with maximum velocity. As the source swing harmonically its maximum velocity equals $a\,\omega$. Hence

$$\nu_{max} = \nu_0 \frac{v}{v - a\,\omega} \text{ and } \nu_{min} = \nu_0 \frac{v}{v + a\,\omega}$$

So the frequency band width $\Delta\nu = \nu_{max} - \nu_{min} = \nu_0\, v\left(\frac{2\,a\,\omega}{v^2 - a^2\omega^2}\right)$

or, $$(\Delta\nu\, a^2)\,\omega^2 + (2\,\nu_0\, v\, a)\,\omega - \Delta\nu\, v^2 = 0$$

So, $$\omega = \frac{-2\,\nu_0\, v\, a \pm \sqrt{4\nu_0^2 v^2 a^2 + \Delta\nu^2 a^2 v^2}}{2\,\Delta\nu\, a^2}$$

On simplifying (and taking + sign as $\omega \to 0$ if $\Delta\nu \to 0$)

$$\omega = \frac{v\,\nu_0}{\Delta\nu\, a}\left(\sqrt{1 + \left(\frac{\Delta\nu}{\nu_0}\right)^2} - 1\right)$$

4.177 It should be noted that the frequency emitted by the source at time t could not be received at the same moment by the receiver, becouse till that time the source will cover the distance $\frac{1}{2} w\, t^2$ and the sound wave will take the further time $\frac{1}{2} w\, t^2 / v$ to reach the receiver. Therefore the frequency noted by the receiver at time t should be emitted by the source at the time $t_1 < t$. Therefore

$$t_1 + \left(\frac{1}{2} w\, t_1^2 / v\right) = t \tag{1}$$

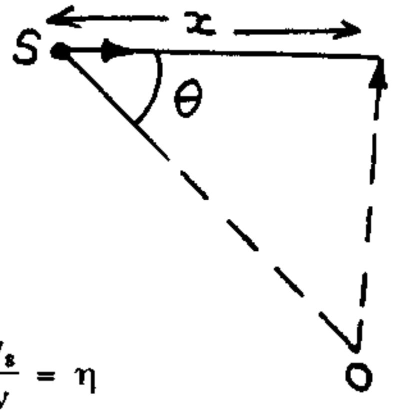
and the frequency noted by the receiver

$$\nu = \nu_0 \frac{v}{v + w\,t_1} \tag{2}$$

Solving Eqns (1) and (2), we get

$$\nu = \frac{\nu_0}{\sqrt{1 + \frac{2\,w\,t}{v}}} = 1\cdot 35 \text{ kHz.}$$

4.178 (a) When the observer receives the sound, the source is closest to him. It means, that frequency is emitted by the source sometimes before (Fig.) Figure shows that the source approaches the stationary observer with velocity $v_s \cos\theta$.



Hence the frequency noted by the observer

$$\nu = \nu_0 \left(\frac{v}{v - v_s \cos\theta} \right)$$

$$= \nu_0 \left(\frac{v}{v - \eta\, v \cos\theta} \right) = \frac{\nu_0}{1 - \eta \cos\theta} \quad (1)$$

But $\frac{x}{v_s} = \frac{\sqrt{l^2 + x^2}}{v}$, So, $\frac{x}{\sqrt{l^2 + x^2}} = \frac{v_s}{v} = \eta$

or, $\cos\theta = \eta$ (2)

Hence from Eqns. (1) and (2) the sought frequency

$$\nu = \frac{\nu_0}{1 - \eta^2} = 5 \text{ kHz}$$

(b) When the source is right in front of O, the sound emitted by it will not be Doppler shifted because $\theta = 90°$. This sound will be received at O at time $t = \frac{l}{v}$ after the source has passed it. The source will by then have moved ahead by a distance $v_s t = l\eta$. The distance between the source and the observer at this time will be $l\sqrt{1 + \eta^2} = 0.32$ km.

4.179 Frequency of sound when it reaches the wall

$$\nu' = \nu \frac{v + u}{v}$$

wall will reflect the sound with same frequency ν'. Thus frequency noticed by a stationary observer after reflection from wall

$\nu'' = \nu' \frac{v}{v - u}$, since wall behaves as a source of frequency ν'.

Thus, $$\nu'' = \nu \frac{v + u}{v} \cdot \frac{v}{v - u} = \nu \frac{v + u}{v - u}$$

or, $$\lambda'' = \lambda \frac{v - u}{v + u} \quad \text{or} \quad \frac{\lambda''}{\lambda} = \frac{v - u}{v + u}$$

So, $$1 - \frac{\lambda''}{\lambda} = 1 - \frac{v - u}{v + u} = \frac{2u}{v + u}$$

Hence the sought percentage change in wavelength

$$= \frac{\lambda - \lambda''}{\lambda} \times 100 = \frac{2u}{v + u} \times 100\,\% = 0.2\% \text{ decrease.}$$

4.180 Frequency of sound reaching the wall.

$$\nu = \nu_0 \left(\frac{v - u}{v}\right) \tag{1}$$

Now for the observer the wall becomes a source of frequency ν receding from it with velocity u

Thus, the frequency reaching the observer

$$\nu' = \nu \left(\frac{v}{v + u}\right) = \nu_0 \left(\frac{v - u}{v + u}\right) \quad \text{[Using (1)]}$$

Hence the beat frequency registered by the receiver (observer)

$$\Delta \nu = \nu_0 - \nu' = \frac{2 u \nu_0}{v + u} = 0.6 \text{ Hz.}$$

4.181 Intensity of a spherical sound wave emitted from a point source in a homogeneous absorbing medium of wave damping coefficient γ is given by

$$I = \frac{1}{2} \rho a^2 e^{-2\gamma r} \omega^2 v$$

So, Intensity of sound at a distance r_1 from the source

$$= \frac{I_1}{r_1^2} = \frac{1/2 \, \rho a^2 e^{-2\gamma r_1} \omega^2 v}{r_1^2}$$

and intensity of sound at a distance r_2 from the source

$$= I_2/r_2^2 = \frac{1/2 \, \rho a^2 e^{-2\gamma r_2} \omega^2 v}{r_2^2}$$

But according to the problem $\quad \frac{1}{\eta} \frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}$

So, $\quad \frac{\eta r_1^2}{r_2^2} = e^{2\gamma(r_2 - r_1)} \quad \text{or} \quad \ln \frac{\eta r_2^2}{r_1^2} = 2\gamma (r_2 - r_1)$

or, $\quad \gamma = \frac{\ln(\eta r_2^2/r_1^2)}{2(r_2 - r_1)} = 6 \times 10^{-3} \text{ m}^{-1}$

4.182 (a)Loudness level in bells $= \log \frac{I}{I_0}$. (I_0 is the theshold of audibility.)

So, loudness level in decibells, $L = 10 \log \frac{I}{I_0}$

Thus loudness level at $x = x_1 = L_{x_1} = 10 \log \frac{I_{x_1}}{I_0}$

Similarly $\quad L_{x_2} = 10 \log \frac{I_{x_2}}{I_0}$

Thus $\quad L_{x_2} - L_{x_1} = 10 \log \frac{I_{x_2}}{I_{x_1}}$

or, $$L_{x_2} = L_{x_1} + 10\log\frac{1/2\,\rho\,a^2\,\omega^2\,v\,e^{-2\gamma x_2}}{1/2\,\rho\,a^2\,\omega^2\,v\,e^{-2\gamma x_1}} = L_{x_1} + 10\log\, e^{-2\gamma(x_2 - x_1)}$$

$$L_{x_2} = L_{x_1} - 20\,\gamma\,(x_2 - x_1)\,\log e$$

Hence $$L' = L - 20\,\gamma\,x\log e \;\;[\text{ since } (x_2 - x_1) = x\,]$$

$$= 20\,dB - 20\times 0{\cdot}23\times 50\times 0{\cdot}4343\;\,dB$$

$$= 60\,dB - 10\,dB = 50\,dB$$

(b) The point at which the sound is not heard any more, the loudness level should be zero. Thus

$$0 = L - 20\,\gamma\,x\log e \quad \text{or} \quad x = \frac{L}{20\,\gamma\log e} = \frac{60}{20\times 0{\cdot}23\times 0{\cdot}4343} = 300 \text{ m}$$

4.183 (a) As there is no damping, so

$$L_{r_0} = 10\log\frac{I}{I_0} = 10\;\log\frac{1/2\,\rho\,a^2\,\omega^2\,v/r_0^2}{1/2\,\rho\,a^2\,\omega^2\,v} = -\;20\;\log r_0$$

Similarly $L_r = -\;20\log r$

So, $$L_r - L_{r_0} = 20\;\log(\,r_0/r\,)$$

or, $$L_r = L_{r_0} + 20\log\left(\frac{r_0}{r}\right) = 30 + 20\times\log\frac{20}{10} = 36\,dB$$

(b) Let r be the sought distance at which the sound is not heard.

So, $$L_r = L_{r_0} + 20\log\frac{r_0}{r} = 0 \;\;\text{or,}\;\; L_{r_0} = 20\log\frac{r}{r_0} \quad\text{or}\quad 30 = 20\log\frac{r}{20}$$

So, $$\log_{10}\frac{r}{20} = 3/2 \quad\text{or}\quad 10^{(3/2)} = r/20$$

Thus $$r = 200\sqrt{10} = 0{\cdot}63\;\text{Km.}$$

Thus for $r > 0{\cdot}63$ km no sound will be heard.

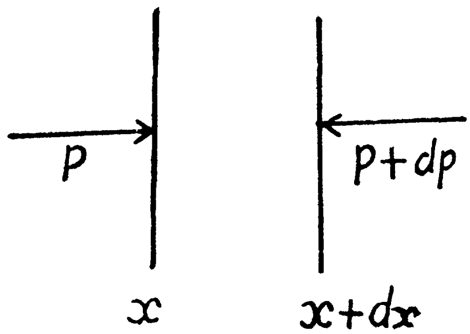
4.184 We treat the fork as a point source. In the absence of damping the oscillation has the form

$$\frac{\text{Const.}}{r}\cos(\,\omega\,t - k\,r\,)$$

Because of the damping of the fork the amplitude of oscillation decreases exponentially with the retarded time (i.e. the time at which the wave started from the source.). Thus we write for the wave amplitude.

$$\xi = \frac{\text{Const.}}{r}\;e^{-\beta\left(t - \frac{r}{v}\right)}$$

This means that $$\frac{e^{-\beta\left(t+\tau-\frac{r_A}{v}\right)}}{r_A} = \frac{e^{-\beta\left(t+\tau-\frac{r_B}{v}\right)}}{r_B}$$



Thus $$e^{-\beta\left(\tau+\frac{r_B-r_A}{v}\right)} = \frac{r_A}{r_B} \quad \text{or } \beta = \frac{\ln\frac{r_B}{r_A}}{\tau+\frac{r_B-r_A}{v}} = 0.12\ s^{-1}$$

4.185 (a) Let us consider the motion of an element of the medium of thickness dx and unit area of cross-section. Let ξ = displacement of the particles of the medium at location x. Then by the equation of motion

$$\rho\, dx\, \ddot{\xi} = -dp$$

where dp is the pressure increment over the length dx

Recalling the wave equation

$$\ddot{\xi} = v^2 \frac{\partial^2 \xi}{\partial x^2}$$

we can write the foregoing equation as

$$\rho v^2 \frac{\partial^2 \xi}{\partial x^2} dx = -dp$$

Integrating this equation, we get

$$\Delta p = \text{surplus pressure} = -\rho v^2 \frac{\partial \xi}{\partial x} + \text{Const.}$$

In the absence of a deformation (a wave), the surplus pressure is $\Delta p = 0$. So 'Const' = 0 and

$$\Delta p = -\rho v^2 \frac{\partial \xi}{\partial x}.$$

(b) We have found earlier that

$$w = w_k + w_p = \text{total energy density}$$

$$w_k = \frac{1}{2}\rho\left(\frac{\partial \xi}{\partial t}\right)^2,\ w_p = \frac{1}{2}E\left(\frac{\partial \xi}{\partial x}\right)^2 = \frac{1}{2}\rho v^2\left(\frac{\partial \xi}{\partial x}\right)^2$$

It is easy to see that the space-time average of both densities is the same and the space time average of total energy density is then

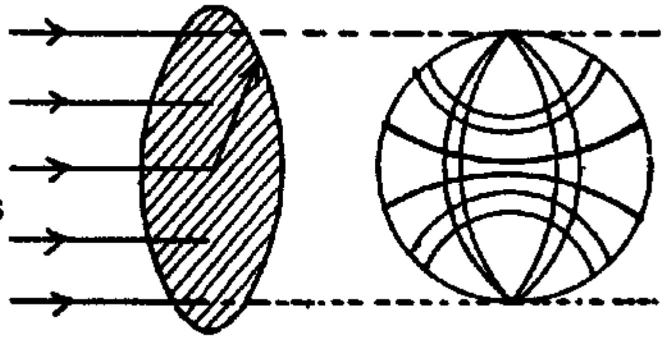
$$\langle w \rangle = \left\langle \rho v^2 \left(\frac{\partial \xi}{\partial x}\right)^2 \right\rangle$$

The intensity of the wave is

$$I = v\langle w \rangle = \left\langle \frac{(\Delta p)^2}{\rho v} \right\rangle$$

Using $$\langle (\Delta p)^2 \rangle = \frac{1}{2}(\Delta p)_m^2 \quad \text{we get} \quad I = \frac{(\Delta p)_m^2}{2\rho v}.$$

4.186 The intensity of the sound wave is



$$I = \frac{(\Delta p)_m^2}{2\rho v} = \frac{(\Delta p)_m^2}{2\rho\nu\lambda}$$

Using $v = \nu\lambda$, ρ is the density of air.

Thus the mean energy flow reaching the ball is

$$\pi R^2 I = \pi R^2 \frac{(\Delta p)_m^2}{2\rho\nu\lambda}$$

πR^2 being the effective area (area of cross section) of the ball.

Substitution gives 10.9 mW.

4.187 We have $\dfrac{P}{4\pi r^2}$ = intensity $= \dfrac{(\Delta p)_m^2}{2\rho v}$

or $$(\Delta p)_m = \sqrt{\frac{\rho v P}{2\pi r^2}}$$

$$= \sqrt{\frac{1{\cdot}293\ \text{kg/m}^3 \times 340\ \text{m/s} \times 0{\cdot}80\ \text{w}}{2\pi \times 1{\cdot}5 \times 1{\cdot}5\ \text{m}^2}} = \sqrt{\frac{1{\cdot}293 \times 340 \times {\cdot}8}{2\pi \times 1{\cdot}5 \times 1{\cdot}5}} \left(\frac{\text{kg kg m}^2\,\text{s}^{-3}\,\text{m s}^{-1}}{m^5}\right)^{\frac{1}{2}}$$

$$= 4{\cdot}9877\ \left(\text{kg m}^{-1}\,\text{s}^{-2}\right) = 5\ \text{Pa}.$$

$$\frac{(\Delta p)_m}{p} = 5 \times 10^{-5}$$

(b) We have $$\Delta p = -\rho v^2 \frac{\partial \xi}{\partial x}$$

$$(\Delta p)_m = \rho v^2 k \xi_m = \rho v\, 2\pi\nu\, \xi_m$$

$$\xi_m = a = \frac{(\Delta p)_m}{2\pi\rho v\nu} = \frac{5}{2\pi \times 1{\cdot}293 \times 340 \times 600} = 3\ \mu\text{m}$$

$$\frac{\xi_m}{\lambda} = \frac{3 \times 10^{-6}}{340/600} = \frac{1800}{340} \times 10^{-6} = 5 \times 10^{-6}$$

4.188 Express L in bels. (i.e. $L = 5$ bels).

Then the intensity at the relevant point (at a distance r from the source) is : $I_0{\cdot}10^L$

Had there been no damping the intensity would have been : $e^{2\gamma r}\, I_0{\cdot}10^L$

Now this must equal the quantity

$\dfrac{P}{4\pi r^2}$, where P = sonic power of the source.

Thus $$\frac{P}{4\pi r^2} = e^{2\gamma r}\, I_0{\cdot}10^L$$

or $$P = 4\pi r^2 e^{2\gamma r} I_0{\cdot}10^L = 1.39\ \text{W}.$$