

Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If $\cot \theta = \frac{1}{2}$ and $\sec \phi = \frac{-5}{3}$, where θ lies in quadrant III and ϕ lies in quadrant II, then $\tan (\theta + \phi) = ?$ [1]
a) $\frac{-6}{11}$ b) $\frac{5}{11}$
c) $\frac{2}{11}$ d) $\frac{10}{11}$
2. Number of relations that can be defined on the set $A = \{a, b, c, d\}$ is [1]
a) 24 b) 4^4
c) 16 d) 2^{16}
3. The number of telephone calls received in 245 successive, one-minute intervals at an exchange is given below in the following frequency distribution. [1]

Number	0	1	2	3	4	5	6	7
Frequency	14	21	25	43	51	40	39	12

What is the median of the distribution?

- a) 3.5 b) 5
c) 4 d) 4.5
4. $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$ is equal to [1]
a) 1 b) -1
c) 2 d) -2
5. A line cutting off intercept – 3 from the y-axis and the tangent at angle to the x-axis is $\frac{3}{5}$, its equation is [1]

- a) $5y - 3x + 15 = 0$ b) $5y - 3x - 15 = 0$
c) None of these d) $3y - 5x + 15 = 0$
6. The distance of point P(3, 4, 5) from the yz-plane is [1]
a) 550 b) 5 units
c) 3 units d) 4 units
7. The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$ is [1]
a) -1 b) -4
c) -3 d) -2
8. 4 boys and 4 girls are to be seated in a row. The number of ways in which this can be done, if the boys and girls sit alternately, is [1]
a) $4! \times 4!$ b) $P(8, 8)$
c) none of these d) $2 \times 4! \times 4!$
9. If $f(x) = \frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is equal to: [1]
a) 1 b) $\frac{4}{5}$
c) $\frac{5}{4}$ d) 0
10. $\sqrt{\frac{1+\sin x}{1-\sin x}} = ?$ [1]
a) $\cot \frac{x}{2}$ b) $\tan \frac{x}{2}$
c) $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ d) $\cot\left(\frac{\pi}{4} + \frac{x}{2}\right)$
11. Let $A = \{a, b, c\}$, $B = \{a, b\}$, $C = \{a, b, d\}$, $D = \{c, d\}$ and $E = \{d\}$. Then which of the following statement is not correct? [1]
a) $D \supseteq E$ b) $C - B = E$
c) $B \cup E = C$ d) $C - D = E$
12. The integral part of $(\sqrt{2} + 1)^6$ is [1]
a) 98 b) 96
c) 99 d) 100
13. If C_r denotes nC_r in the expansion of $(1 + x)^n$, then $C_0 + C_1 + C_2 + \dots + C_n = ?$ [1]
a) $2n$ b) 2^n
c) $\frac{1}{3} n(2n + 1)$ d) 2^n
14. solution set of the inequations $x \geq 2$, $x \leq -3$ is [1]
a) $\{ \}$ b) $[-3, 2]$
c) $(-3, 2)$ d) $[2, -3]$
15. If $Q = \{x : x = \frac{1}{y}, \text{ where } y \in \mathbb{N}\}$, then [1]
a) $1 \in Q$ b) $\frac{1}{2} \notin Q$
c) $2 \in Q$ d) $0 \in Q$

16. $(4 \cos^3 15^\circ - 3 \cos 15^\circ) = ?$ [1]
 a) 0 b) 1
 c) -1 d) $\frac{1}{\sqrt{2}}$
17. If $f(x) = x^{100} + x^{99} \dots + x + 1$, then $f'(1)$ is equal to: [1]
 a) 5049 b) 50051
 c) 5050 d) 5051
18. How many 3-digit even numbers can be formed with no digit repeated by using the digits 0, 1, 2, 3, 4 and 5? [1]
 a) 56 b) 52
 c) 50 d) 54
19. **Assertion (A):** Let $A = \{a, b\}$ and $B = \{a, b, c\}$. Then, $A \not\subset B$. [1]
Reason (R): If $A \subset B$, then $A \cup B = B$.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** If the numbers $\frac{-2}{7}$, K , $\frac{-7}{2}$ are in GP, then $k = \pm 1$. [1]
Reason (R): If a_1, a_2, a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.

Section B

21. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$. Are the following true? [2]
 (i) $(a, a) \in R$ for all $a \in N$
 (ii) $(a, b) \in R$ implies $(b, a) \in R$
 (iii) $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$
 Justify your answer in each case.
- OR
- Find the range of the function given by $f(x) = \frac{3}{2-x^2}$.
22. Evaluate: $\lim_{x \rightarrow 0} \frac{\sec 5x - \sec 3x}{\sec 3x - \sec x}$. [2]
23. If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, find $P(A \cap B)$. [2]

OR

- A die is tossed once. What is the probability of getting an even number?
24. Using properties of set, show that: $A \cup (A \cap B) = A$ [2]
25. Find the locus of a point, so that the join of $(-5, 1)$ and $(3, 2)$ subtends a right angle at the moving point. [2]

Section C

26. Determine n if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$ [3]
27. Show that the points $A(4, 6, -3)$, $B(0, 2, 3)$ and $C(-4, -4, -1)$ form the vertices of an isosceles triangle. [3]
28. Find an approximation of $(0.99)^5$ using the first three terms of its expansion. [3]

OR

Using binomial theorem, expand: $(x^2 - \frac{2}{x})^7$.

29. Evaluate $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$ [3]

OR

If $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{when } x \neq \frac{\pi}{2}, \\ \text{and if } \lim_{x \rightarrow \frac{\pi}{2}} f(x) = 3 \end{cases}$

Find the value of k.

30. Each side of an equilateral triangle is 18 cm. The midpoints of its sides are joined to form another triangle whose midpoints, in turn, are joined to form still another triangle. The process is continued indefinitely. Find the sum of the areas of all the triangles. [3]

OR

If a, b, c are the pth, qth and rth terms of a GP, show that $(q - r)\log a + (r - p)\log b + (p - q)\log c = 0$.

31. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics and Mathematics. If 6 students offered all the three subjects, find: [3]
- The total number of students.
 - How many took Maths but not Chemistry.
 - How many took exactly one of the three subjects.

Section D

32. In a survey of 44 villages of a state, about the use of LPG as a cooking mode, the following information about the families using LPG was obtained. [5]

Number of families	0-10	10-20	20-30	30-40	40-50	50-60
Number of villages	6	8	16	8	4	2

- Find the mean deviation about median for the following data.
 - Do you think more awareness was needed for the villagers to use LPG as a mode of cooking?
33. Find the equation of the ellipse whose foci are (4, 0) and (-4, 0), eccentricity = 1/3. [5]

OR

Find the (i) lengths of major and minor axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipses: $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

34. Solve for x, $|x + 1| + |x| > 3$ [5]
35. Prove that $\cos 12^\circ + \cos 60^\circ + \cos 84^\circ = \cos 24^\circ + \cos 48^\circ$ [5]

OR

Prove that: $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$.

Section E

36. Read the text carefully and answer the questions: [4]

Function as a Relation A relation f from a non-empty set A to a non-empty set B is said to be a function, if every element of set A has one and only one image in set B.

In other words, we can say that a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component.

If f is a function from a set A to a set B, then we write

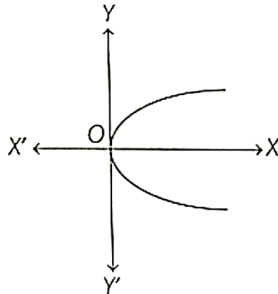
$f : A \rightarrow B$ or $A \xrightarrow{f} B$.

and it is read as f is a function from A to B or f maps A to B .

- (i) If $f(x) = \frac{1}{2 - \sin 3x}$, then find the range (f).
- (ii) If $f(1 + x) = x^2 + 1$, then find the $f(2 - h)$.
- (iii) If $f(x) = x^2 + 2x + 3$, then find the value of among $f(1)$, $f(2)$ and $f(3)$.

OR

What is the equation of a given figure?



37. **Read the text carefully and answer the questions:**

[4]

In a hostel 60% of the students read Hindi newspapers, 40% read English newspapers and 20% read both Hindi and English newspapers.



- (i) A student is selected at random. She reads Hindi or English newspaper?
- (ii) A student is selected at random. Did she read neither Hindi nor English newspapers?
- (iii) A student is selected at random. She reads Hindi but not English Newspaper?

OR

A student is selected at random. She reads English but not Hindi Newspaper?

38. **Read the text carefully and answer the questions:**

[4]

The conjugate of a complex number z , is the complex number, obtained by changing the sign of imaginary part of z . It is denoted by $\bar{z}$.

The modulus (or absolute value) of a complex number, $z = a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $|z|$. i.e.

$$|z| = \sqrt{a^2 + b^2}$$

Multiplicative inverse of z is $\frac{\bar{z}}{|z|^2}$. It is also called reciprocal of z .

$$z\bar{z} = |z|^2$$

- (i) If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then find $|f(z)|$.
- (ii) Find the value of $(z + 3)(\bar{z} + 3)$.

Solution

Section A

1.

(c) $\frac{2}{11}$

Explanation: In quadrant III, $\sin \theta < 0$, $\cos \theta < 0$ and $\tan \theta > 0$

In quadrant II, $\sin \phi > 0$, $\cos \phi < 0$ and $\tan \theta < 0$

Now, $\cot \theta = \frac{1}{2} \Rightarrow \tan \theta = 2$

$\sec \phi = \frac{-5}{3} \Rightarrow \cos \phi = \frac{-3}{5}$

$\therefore \sin^2 \theta = (1 - \cos^2 \phi) = \left(1 - \frac{9}{25}\right) = \frac{16}{25} \Rightarrow \sin \phi = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan \phi = \left(\frac{4}{5} \times \frac{5}{-3}\right) = \frac{-4}{3}$

$\therefore \tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{\left(2 - \frac{4}{3}\right)}{\left\{1 - \left(2 \times \frac{-4}{3}\right)\right\}} = \frac{\left(\frac{2}{3}\right)}{\left(1 + \frac{8}{3}\right)} = \left(\frac{2}{3} \times \frac{3}{11}\right) = \frac{2}{11}$

2.

(d) 2^{16}

Explanation: No. of elements in the set $A = 4$. Therefore, the no. of elements in $A \times A = 4 \times 4 = 16$. As, the no. of relations in $A \times A =$ no. of subsets of $A \times A = 2^{16}$.

3.

(c) 4

Explanation: Given frequency distribution is

Number of calls (x_i)	0	1	2	3	4	5	6	7
Frequency (f_i)	14	21	25	43	51	40	39	12
Number of calls (x_i)	Frequency (f_i)					Cumulative Frequency (cf)		
0	14					14		
1	21					35		
2	25					60		
3	43					103		
4	51					154		
5	40					194		
6	39					233		
7	12					245		
Total	$N = \sum f_i = 245$							

Here, $\frac{N}{2} = \frac{245}{2} = 122.5$

The cumulative frequency 154 which is equal or just greater than $\frac{N}{2}$.

$\therefore$ Required median = Value of the variable corresponding to the cumulative frequency 154 = 4

4.

(b) -1

Explanation: Given, $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} = \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{-(\pi - x)}$

$= -1 \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \pi - x \rightarrow 0 \Rightarrow x \rightarrow \pi \right]$

5. (a) $5y - 3x + 15 = 0$

Explanation: Here, it is given that

$$\tan \theta = \frac{3}{5}$$

We know that,

Slope of a line, $m = \tan \theta$

$$\Rightarrow \text{Slope of line, } m = \frac{3}{5}$$

Since, the lines cut off intercepts -3 on y -axis then the line is passing through the point $(0, -3)$.

Therefore, the equation of line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - (-3) = \frac{3}{5}(x - 0)$$

$$\Rightarrow y + 3 = \frac{3}{5}x$$

$$\Rightarrow 5y + 15 = 3x$$

$$\Rightarrow 5y - 3x + 15 = 0$$

6.

(c) 3 units

Explanation: Given point is $P(3, 4, 5)$

$\therefore$ Distance of P from yz -plane

$$= \sqrt{(0 - 3)^2 + (4 - 4)^2 + (5 - 5)^2} \text{ [using distance formula]}$$

$$= \sqrt{9} = 3 \text{ units}$$

7.

(d) -2

Explanation: -2

$$\begin{aligned} & \frac{i^{502} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1 \\ &= \frac{i^{4 \times 148} + i^{4 \times 147 + 2} + i^{4 \times 147} + i^{4 \times 146 + 2} + i^{4 \times 146}}{i^{4 \times 145 + 2} + i^{4 \times 145} + i^{4 \times 144 + 2} + i^{4 \times 144} + i^{4 \times 143 + 2}} - 1 \quad [\because i^4 = 1 \text{ and } i^2 = -1] \\ &= \frac{1 + i^2 + 1 + i^2 + 1}{i^2 + 1 + i^2 + 1 + i^2} - 1 \\ &= \frac{1}{-1} - 1 \\ &= -2 \end{aligned}$$

8.

(d) $2 \times 4! \times 4!$

Explanation: There are 4 boys and 4 girls and the row can start either with a boy or girl, therefore the number of ways are $4!$

$$\times 4! \times 2$$

9.

(c) $\frac{5}{4}$

Explanation: Given that, $f(x) = \frac{x-4}{2\sqrt{x}}$

$$f'(x) = \frac{1}{2} \left[\frac{\sqrt{x} \cdot 1 - (x-4) \cdot \frac{1}{2\sqrt{x}}}{x} \right] = \frac{1}{2} \left[\frac{2x - x + 4}{2\sqrt{x} \cdot x} \right] = \frac{1}{2} \left[\frac{x+4}{2(x)^{3/2}} \right]$$

$$\therefore f'(x) \text{ at } x = 1 = \frac{1}{2} \left[\frac{1+4}{2 \times 1} \right] = \frac{5}{4}$$

10.

(c) $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

$$\text{Explanation: } \sqrt{\frac{1+\sin x}{1-\sin x}} = \sqrt{\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}} = \left\{ \frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)} \right\}^{\frac{1}{2}} = \left\{ \tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right) \right\}^{\frac{1}{2}} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

11.

(d) $C - D = E$

Explanation: $C - D = \{a, b, c\} - \{c, d\} = \{a, b\}$

But $E = \{d\}$

Hence $C - D \neq E$

12.

(c) 99

Explanation: We have $(1+x)^n = 1 + {}^nC_1(x) + {}^nC_2(x)^2 + \dots + (x)^n$

$$\text{Hence } (\sqrt{2} + 1)^6 = 1 + {}^6C_1(\sqrt{2}) + {}^6C_2(\sqrt{2})^2 + {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^4 + {}^6C_5(\sqrt{2})^5 + (\sqrt{2})^6$$

$$\Rightarrow (\sqrt{2} + 1)^6 = 1 + 6(\sqrt{2}) + 15 \times 2 + 20 \times 2(\sqrt{2}) + 15 \times 4 + 6 \times 4(\sqrt{2}) + 8$$

$$= 99 + 70\sqrt{2}$$

Hence integral part of $(\sqrt{2} + 1)^6 = 99$

13.

(d) 2^n

Explanation: Here, we know that $C_0 + C_1 + C_2 + \dots + C_n = 2^n$

14. (a) $\{ \}$

Explanation: $x \geq 2 \Rightarrow x \in [2, \infty)$

$$x \leq -3 \Rightarrow x \in (-\infty, -3]$$

Hence solution set of $x \geq 2$ and $x \leq -3$ is $[2, \infty) \cap (-\infty, -3] = \Phi$

15. (a) $1 \in \mathbb{Q}$

Explanation: $\mathbb{N}$ is set of natural number, so

$$x = \frac{1}{y}$$

When $y = 1$ then $x = 1$

So, $1 \in \mathbb{Q}$

16.

(d) $\frac{1}{\sqrt{2}}$

Explanation: We know that $(4 \cos^3 \theta - 3 \cos \theta) = \cos 3\theta$

$$\therefore (4 \cos^3 15^\circ - 3 \cos 15^\circ) = \cos(3 \times 15^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

17.

(c) 5050

Explanation: Given, $f(x) = x^{100} + x^{99} \dots + x + 1$

$$\therefore f'(x) = 100x^{99} + 99x^{98} \dots + x + 1$$

$$\text{So, } f'(1) = 100 + 99 + 98 + \dots + 1$$

$$= \frac{100}{2} [2 \times 100 + (100 - 1)(-1)]$$

$$= 50[200 - 99] = 50 \times 101$$

$$= 5050$$

18.

(b) 52

Explanation: Numbers with 0 at units place $= (5 \times 4 \times 1) = 20$

$$\text{Numbers with 2 at units place} = (4 \times 4 \times 1) = 16$$

$$\text{Numbers with 4 at units place} = (4 \times 4 \times 1) = 16$$

$$\text{Total numbers} = (20 + 16 + 16) = 52$$

19.

(d) A is false but R is true.

Explanation: Assertion $A = \{a, b\}$, $B = \{a, b, c\}$

Since, all the elements of A are in B. So,

$$A \subset B$$

Reason $\therefore A \subset B$

$$\Rightarrow A \cup B = B$$

Hence, Assertion is false and Reason is true.

20.

(a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: If $\frac{-2}{7}$, K , $\frac{-7}{2}$ are in G.P.

$$\text{Then, } \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$[\therefore \text{common ratio } (r) = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots]$$

$$\begin{aligned}\therefore \frac{k}{\frac{-7}{2}} &= \frac{\frac{-7}{2}}{k} \\ \Rightarrow \frac{7}{-2}k &= \frac{-7}{2} \times \frac{1}{k} \\ \Rightarrow 7k \times 2k &= -7 \times (-2) \\ \Rightarrow 14k^2 &= 14 \\ \Rightarrow k^2 &= 1 \Rightarrow k = \pm 1\end{aligned}$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. Here $R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$

(i) No $(3, 3) \in R$ because $3 \neq 3^2$

(ii) No. $(9, 3) \in R$ but $(3, 9) \in R$

(iii) No. $(81, 9) \in R$ $(9, 3) \in R$ but $(81, 3) \notin R$

OR

$$\begin{aligned}\text{Let } f(x) &= \frac{3}{2-x^2} = y \\ \Rightarrow 2-x^2 &= \frac{3}{y} \Rightarrow x^2 = 2 - \frac{3}{y} \\ \text{Since } x^2 &\geq 0, 2 - \frac{3}{y} \geq 0 \\ \Rightarrow \frac{2y-3}{y} &\geq 0 \\ \Rightarrow 2y-3 &\geq 0 \\ \Rightarrow 2y-3 &\geq 0 \text{ and } y > 0 \text{ or } 2y-3 \leq 0 \text{ and } y < 0 \\ \Rightarrow y &\geq \frac{3}{2} \text{ or } y < 0 \\ \Rightarrow y &\in (-\infty, 0) \cup [3/2, \infty) \\ \therefore \text{Range of } f &\text{ is } (-\infty, 0) \cup [3/2, \infty)\end{aligned}$$

$$\begin{aligned}22. \text{ We have: } \lim_{x \rightarrow 0} &\left[\frac{\sec 5x - \sec 3x}{\sec 3x - \sec x} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{\frac{1}{\cos 5x} - \frac{1}{\cos 3x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{\cos 3x - \cos 5x}{\cos 5x \cos 3x \left\{ \frac{\cos x - \cos 3x}{\cos x \cos 3x} \right\}} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{(\cos 3x - \cos 5x) \cos x}{\cos 5x \{ \cos x - \cos 3x \}} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{-2 \sin \left(\frac{3x+5x}{2} \right) \sin \left(\frac{3x-5x}{2} \right) \times \cos x}{\cos(5x) \left[-2 \sin \left(\frac{x+3x}{2} \right) \sin \left(\frac{x-3x}{2} \right) \right]} \right] \quad \left[\because \cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{\sin(4x) \times \sin(-x) \times \cos x}{\cos(5x) \times \sin(2x) \times \sin(-x)} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{\sin(4x)}{4x} \times \frac{4x}{\frac{\sin(2x)}{2x} \times 2x} \times \frac{\cos x}{\cos 5x} \right] \\ &= \frac{4}{2} \frac{\cos 0}{\cos 0} \\ &= 2\end{aligned}$$

23. We have to find $P(A \cap B)$.

Given:

$$P(A) = 0.3, P(B) = 0.4 \text{ and } P(A \cup B) = 0.5$$

By addition theorem, we have:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.5 = 0.3 + 0.4 - P(A \cap B)$$

$$\Rightarrow 0.5 = 0.7 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 0.7 - 0.5$$

$$= 0.2$$

$$\text{Hence, } P(A \cap B) = 0.2$$

OR

In tossing a die once, then the sample space of the event is given by

$S = \{1, 2, 3, 4, 5, 6\}$ and, therefore, $n(S) = 6$.

Let E_2 = event of getting an even number. Then,

$E_2 = \{2, 4, 6\}$ and, therefore, $n(E_2) = 3$.

$$\therefore P(\text{getting an even number}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

24. We know that if $A \subset B$ then

$$A \cap B = A$$

$$\text{Also } A \cap B \subset A$$

$$\therefore A \cup (A \cap B) = A$$

25. Suppose P (h, k) be a moving point and let A (- 5,1) and B (3, 2) be given points.

By the given condition

$$\angle APB = 90^\circ$$

$\therefore \triangle APB$ is a right angle triangle

$$\Rightarrow AB^2 = AP^2 + PB^2$$

$$\Rightarrow (3 + 5)^2 + (2 - 1)^2 = (h + 5)^2 + (k - 1)^2 + (h - 3)^2 + (k - 2)^2$$

$$\Rightarrow 65 = 2(h^2 + k^2 + 2h - 3k) + 39$$

$$\Rightarrow h^2 + k^2 + 2h - 3k - 13 = 0$$

Therefore, the locus of (h, k) is $x^2 + y^2 + 2x - 3y - 13 = 0$.

Section C

26. Here, we have ${}^{2n}C_3 : {}^nC_3 = 12 : 1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{3!(2n-3)!}{2n \times (2n-1) \times (2n-2) \times (2n-3)!}}{\frac{3!(n-3)!}{n \times (n-1) \times (n-2) \times (n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{3!}{2n \times (2n-1) \times (2n-2)}}{\frac{3!}{n \times (n-1) \times (n-2)}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times 2 \times (n-1)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

$$\Rightarrow 4 \times (2n - 1) = 12 \times (n - 2)$$

$$\Rightarrow 8n - 4 = 12n - 24$$

$$\Rightarrow 12n - 8n = 24 - 4$$

$$\Rightarrow 4n = 20$$

$$\therefore n = 5$$

27. To prove: Points A, B, C form an isosceles triangle.

Formula: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Here,

$$(x_1, y_1, z_1) = (4, 6, -3)$$

$$(x_2, y_2, z_2) = (0, 2, 3)$$

$$(x_3, y_3, z_3) = (-4, -4, -1)$$

$$\text{Length AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(0 - 4)^2 + (2 - 6)^2 + (3 - (-3))^2}$$

$$= \sqrt{(-4)^2 + (-4)^2 + (6)^2}$$

$$= \sqrt{16 + 16 + 36}$$

$$\text{Length AB} = \sqrt{68} = 2\sqrt{17}$$

$$\text{Length BC} = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$$

$$\begin{aligned}
&= \sqrt{(-4-0)^2 + (-4-2)^2 + (-1-3)^2} \\
&= \sqrt{(-4)^2 + (-6)^2 + (-4)^2} \\
&= \sqrt{16 + 36 + 16} \\
\text{Length BC} &= \sqrt{68} = 2\sqrt{17} \\
\text{Length AC} &= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2} \\
&= \sqrt{(-4-4)^2 + (-4-6)^2 + (-1-(-5))^2} \\
&= \sqrt{(-8)^2 + (-10)^2 + (2)^2} \\
&= \sqrt{64 + 100 + 4}
\end{aligned}$$

$$\text{Length AC} = \sqrt{168}$$

Here, AB = BC

∴ vertices A, B, C forms an isosceles triangle.

$$\begin{aligned}
28. \text{ We have } (0.99)^5 &= (1 - 0.01)^5 \\
&= {}^5C_0 - {}^5C_1(0.01) + {}^5C_2(0.01)^2 - \dots \\
&= 1 - 0.05 + 0.001 - \dots \\
&= 0.951
\end{aligned}$$

OR

To find: Expansion of $\left(x^2 - \frac{3x}{7}\right)^7$

$$\text{Formula used: } {}^nC_r = \frac{n!}{(n-r)!r!}$$

We know that, $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

Here We have, $\left(x^2 - \frac{3x}{7}\right)^7$

$$\begin{aligned}
&\Rightarrow \left[{}^7C_0(x^2)^{7-0} \right] + \left[{}^7C_1(x^2)^{7-1} \left(-\frac{3x}{7}\right)^1 \right] + \left[{}^7C_2(x^2)^{7-2} \left(-\frac{3x}{7}\right)^2 \right] + \left[{}^7C_3(x^2)^{7-3} \left(-\frac{3x}{7}\right)^3 \right] + \left[{}^7C_4(x^2)^{7-4} \left(-\frac{3x}{7}\right)^4 \right] \\
&+ \left[{}^7C_5(x^2)^{7-5} \left(-\frac{3x}{7}\right)^5 \right] + \left[{}^7C_6(x^2)^{7-6} \left(-\frac{3x}{7}\right)^6 \right] + \left[{}^7C_7 \left(-\frac{3x}{7}\right)^7 \right] \\
&\Rightarrow \left[\frac{7!}{0!(7-0)!} (x^2)^7 \right] - \left[\frac{7!}{1!(7-1)!} (x^2)^6 \left(\frac{3x}{7}\right) \right] + \left[\frac{7!}{2!(7-2)!} (x^2)^5 \left(\frac{9x^2}{49}\right) \right] - \left[\frac{7!}{3!(7-3)!} (x^2)^4 \left(\frac{27x^3}{343}\right) \right] \\
&+ \left[\frac{7!}{4!(7-4)!} (x^2)^3 \left(\frac{81x^4}{2401}\right) \right] - \left[\frac{7!}{5!(7-5)!} (x^2)^2 \left(\frac{243x^5}{16807}\right) \right] + \left[\frac{7!}{6!(7-6)!} (x^2)^1 \left(\frac{729x^6}{117649}\right) \right] - \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543}\right) \right] \\
&- \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543}\right) \right] + \left[21(x^{10}) \left(\frac{9x^2}{49}\right) \right] - \left[35(x^8) \left(\frac{27x^3}{343}\right) \right] \\
&+ \left[35(x^6) \left(\frac{81x^4}{2401}\right) \right] - \left[21(x^4) \left(\frac{243x^5}{16807}\right) \right] + \left[7(x^2) \left(\frac{729x^6}{117649}\right) \right] - \left[1 \left(\frac{2187x^7}{823543}\right) \right] \\
&\Rightarrow x^{24} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7 \\
&x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7
\end{aligned}$$

$$\begin{aligned}
29. \text{ Here } \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} \\
&= \lim_{x \rightarrow -2} \frac{\frac{x+2}{2x}}{x+2} \\
&= \lim_{x \rightarrow -2} \frac{x+2}{2x} \times \frac{1}{x+2} \\
&= \lim_{x \rightarrow -2} \frac{1}{2x} = \frac{1}{2 \times -2} = \frac{-1}{4}
\end{aligned}$$

OR

We have,

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{when } x \neq \frac{\pi}{2}, \text{ and if } \lim_{x \rightarrow \frac{\pi}{2}} f(x) = 3 \end{cases}$$

$$\text{LHL } f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x} = \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{k \sin h}{\pi - \pi + 2h} = \lim_{h \rightarrow 0} \frac{k \sin h}{2h} \\
&= \frac{k}{2} \cdot 1 = \frac{k}{2} \left[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right]
\end{aligned}$$

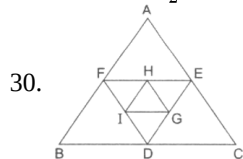
$$\text{RHL } f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{k \cos x}{\pi - 2x} = \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-k \sin h}{\pi - \pi - 2h}$$

$$= \lim_{h \rightarrow 0} \frac{-k \sin h}{-2h} = \frac{k}{2} \left[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right]$$

Since, $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 3$

Therefore, $\frac{k}{2} = 3 \Rightarrow k = 6$



Suppose $\triangle ABC$ be the given triangle having each side 18 cm. Suppose D, E, F be the midpoints of BC, CA, AB respectively to form $\triangle DEF$. Suppose G, H, I be the midpoints of DE, EF and FD respectively to form $\triangle GHI$.

We continue this process indefinitely. Then, we have

The sides of these triangle are 18 cm, 9 cm, $\frac{9}{2}$ cm, ..., and so on.

Sum of the areas of all triangles so formed

$$= \frac{\sqrt{3}}{4} \{ (18)^2 + (9)^2 + \left(\frac{9}{2}\right)^2 + \left(\frac{9}{4}\right)^2 + \dots \infty \} \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} \{ 324 + 81 + \frac{81}{4} + \frac{81}{16} + \dots \infty \} \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} \cdot \frac{a}{(1-r)} \text{ cm}^2, \text{ where } a = 324 \text{ and } r = \frac{81}{324} = \frac{1}{4}$$

$$= \left\{ \frac{\sqrt{3}}{4} \times \frac{324}{\left(1 - \frac{1}{4}\right)} \right\} \text{ cm}^2 = 108\sqrt{3} \text{ cm}^2$$

OR

We know that a, b and c are the pth, qth and rth term of GP.

Let us assume the required GP as A, AR, AR₂, AR₃ ...

Now, the nth term in the GP, $a_n = AR^{n-1}$

pth term $a_p = AR^{p-1} = a \dots (1)$

qth term, $a_q = AR^{q-1} = b \dots (2)$

rth term, $a_r = AR^{r-1} = c \dots (3)$

$$\frac{(1)}{(2)} \rightarrow \frac{AR^{p-1}}{AR^{q-1}} = R^{p-q} = \frac{a}{b} \dots (i)$$

$$\frac{(2)}{(3)} \rightarrow \frac{AR^{q-1}}{AR^{r-1}} = R^{q-r} = \frac{b}{c} \dots (ii)$$

$$\frac{(3)}{(1)} \rightarrow \frac{AR^{r-1}}{AR^{p-1}} = R^{1-p} = \frac{c}{a} \dots (iii)$$

Taking logarithm on both sides of equation (i), (ii) and (iii), we get $(p - q)\log R = \log a - \log b$

$$\therefore (p - q) = \frac{\log a - \log b}{\log R} \dots (4)$$

$$(q - r)\log R = \log b - \log c$$

$$\therefore (q - r) = \frac{\log b - \log c}{\log R} \dots (5)$$

$$(r - p)\log R = \log c - \log a$$

$$\therefore (r - p) = \frac{\log c - \log a}{\log R} \dots (6)$$

Now multiply log c with (4), log a with (5), log b with (6) then add all. We get.

$$(p - q)\log c + (q - r)\log a + (r - p)\log b$$

$$= \left(\frac{\log a - \log b}{\log R} \right) \log c + \left(\frac{\log b - \log c}{\log R} \right) \log a + \left(\frac{\log c - \log a}{\log R} \right) \log b$$

On solving the above equation, we will get,

$$(p - q)\log x + (q - r)\log a + (r - p)\log b = 0$$

Hence proved.

31. Given, $n(P) = 18$, $n(C) = 23$, $n(M) = 24$, $n(C \cap M) = 13$,

$n(P \cap C) = 12$, $n(P \cap M) = 11$ and $n(P \cap C \cap M) = 6$

i. Total no. of students in the class

$$= n(P \cup C \cup M)$$

$$= n(P) + n(C) + n(M) - n(P \cap C) - n(P \cap M) - n(C \cap M) + n(P \cap C \cap M)$$

$$= 18 + 23 + 24 - 12 - 11 - 13 + 6 = 35$$

ii. No. of students who took Mathematics but not Chemistry

$$= n(M - C)$$

$$= n(M) - n(M \cap C)$$

$$= 24 - 13 = 11$$

iii. No. of students who took exactly one of the three subjects

$$= n(P) + n(C) + n(M) - 2n(M \cap P) - 2n(P \cap C) - 2n(M \cap C) + 3n(P \cap C \cap M)$$

$$= 18 + 23 + 24 - 2 \times 11 - 2 \times 12 - 2 \times 13 + 3 \times 6$$

$$= 65 - 22 - 24 - 26 + 18$$

$$= 83 - 72 = 11$$

Section D

32. i.

Number of families	Mid value (x_i)	Number of villages (f_i)	cf	$ x_i - M $	$f_i x_i - M $
0 – 10	5	6	6	20	120
10 – 20	15	8	14	10	80
20 – 30	25	16	30	0	0
30 – 40	35	8	38	10	80
40 – 50	45	4	42	20	80
50 – 60	55	2	44	30	60
		44			420

Here, $N = 44$

Now, $\frac{N}{2} = \frac{44}{2} = 22$, which, lies in the cumulative frequency of 30, therefore median class is 20-30.

$\therefore l = 20, f = 16, cf = 14$ and $h = 10$

$$\therefore \text{Median (M)} = l + \frac{\frac{N}{2} - cf}{f} \times b$$

$$= 20 + \frac{22 - 14}{16} \times 10$$

$$= 20 + \frac{8}{16} \times 10 = 20 + 5 = 25$$

$$\therefore \text{Mean deviation about median} = \frac{\sum_{i=1}^6 f_i |x_i - M|}{\sum f_i} = \frac{420}{44} = 9.55$$

ii. There is a need for awareness among villagers for using LPG as a mode of cooking. Because it will help in keeping the environment clean and will also help in saving of forests.

33. Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (i)$$

The coordinate of foci are $(+ae, 0)$ and $(-ae, 0)$.

$$\therefore ae = 4 [\because \text{foci} : (\pm 4, 0)]$$

$$\Rightarrow a \times \frac{1}{3} = 4 \left[\because e = \frac{1}{3} \right]$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

$$\text{Now, } b^2 = a^2 (1 - e^2)$$

$$\Rightarrow b^2 = 144 \left[1 - \left(\frac{1}{3} \right)^2 \right]$$

$$\Rightarrow b^2 = 144 \left[1 - \frac{1}{9} \right]$$

$$\Rightarrow b^2 = 144 \times \frac{8}{9}$$

$$\Rightarrow b^2 = 16 \times 8 = 128$$

Substituting $a^2 = 144$ and $b^2 = 128$ in equation (i), we get

$$= \frac{x^2}{144} + \frac{y^2}{128} = 1$$

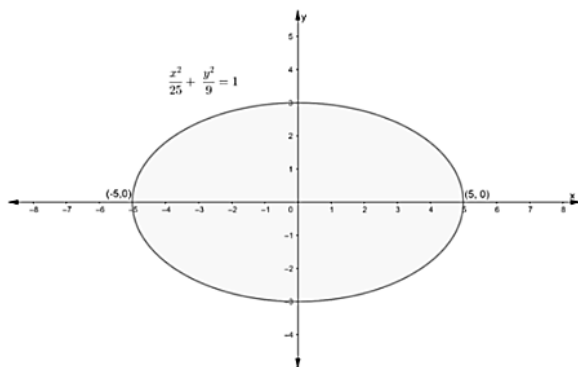
$$\Rightarrow \frac{1}{16} \left[\frac{x^2}{9} + \frac{y^2}{8} \right] = 1$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{8} = 16$$

This is the required equation of the ellipse.

OR

Given: $\frac{x^2}{25} + \frac{y^2}{9} = 1 \dots(i)$



Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, $25 > 9$

So, above equation is of the form.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Comparing eq. (i) and (ii), we get

$$a^2 = 25 \text{ and } b^2 = 9$$

$$\Rightarrow a = 5 \text{ and } b = 3$$

i. To find: Length of major axes

Clearly, $a > b$, therefore the major axes of the ellipse are along x-axes.

$\therefore$ Length of major axes = 2a

Length of major axes = 2×5

ii. To find: Coordinates of the Vertices

Clearly, $a > b$

$\therefore$ Coordinate of vertices = (a, 0) and (-a, 0)

Coordinate of vertices = (5, 0) and (-5, 0)

iii. To find: Coordinates of the foci

We know that,

Coordinates of foci = $(\pm c, 0)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$$c^2 = a^2 - b^2$$

$$= 25 - 9$$

$$c^2 = 16$$

$$c = \sqrt{16}$$

$$c = 4 \dots(ii)$$

$\therefore$ Coordinates of foci = $(\pm a, 0) = (\pm 4, 0)$

iv. To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{4}{5} [\text{from (ii)}]$$

v. To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (3)^2}{5}$$

$$\text{Length of Latus Rectum} = \frac{18}{5}$$

34. We have, $|x + 1| + |x| > 3$

Put $x + 1 = 0$ and $x = 0 \Rightarrow x = -1$ and $x = 0$

$\therefore x = -1, 0$ are critical point.

So, we will consider three intervals $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$

Case I: When $-\infty < x < -1$, then $|x + 1| = -(x + 1)$ and $|x| = -x$

$$\therefore |x + 1| + |x| > 3$$

$$\Rightarrow -x - 1 - x > 3$$

$$\Rightarrow -2x - 1 > 3$$

$$\Rightarrow -2x > 4$$

$$\Rightarrow x < -2$$

Case II: When $-1 \leq x < 0$, then $|x + 1| = x + 1$ and $|x| = -x$

$$\therefore |x + 1| + |x| > 3$$

$$\Rightarrow x + 1 - x > 3 \Rightarrow 1 > 3 \text{ [not possible]}$$

Case III: When $0 \leq x < \infty$, then $|x + 1| = x + 1$ and $|x| = x$

$$\therefore |x + 1| + |x| > 3$$

$$\Rightarrow x + 1 + x > 3$$

$$\Rightarrow 2x + 1 > 3 \Rightarrow 2x > 2$$

$$\therefore x > 1$$

On combining the results of cases I, II and III, we get

$$x < -2 \text{ and } x > 1$$

$$\therefore x \in (-\infty, -2) \cup (1, \infty)$$

$$35. \text{LHS} = \cos 12^\circ + \cos 60^\circ + \cos 84^\circ$$

$$= \cos 12^\circ + (\cos 84^\circ + \cos 60^\circ)$$

$$= \cos 12^\circ + \left[2 \cos \left(\frac{84^\circ + 60^\circ}{2} \right) \times \cos \left(\frac{84^\circ - 60^\circ}{2} \right) \right]$$

$$[\because \cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)]$$

$$= \cos 12^\circ + \left[2 \cos \frac{144^\circ}{2} \times \cos \frac{24^\circ}{2} \right]$$

$$= \cos 12^\circ + [2 \cos 72^\circ \times \cos 12^\circ] = \cos 12^\circ [1 + 2 \cos 72^\circ]$$

$$= \cos 12^\circ [1 + 2 \cos(90^\circ - 18^\circ)]$$

$$= \cos 12^\circ [1 + 2 \sin 18^\circ] [\because \cos(90^\circ - \theta) = \sin \theta]$$

$$= \cos 12^\circ \left[1 + 2 \left(\frac{\sqrt{5}-1}{4} \right) \right] [\because \sin 18^\circ = \frac{\sqrt{5}-1}{4}]$$

$$= \left(1 + \frac{\sqrt{5}-1}{2} \right) \cos 12^\circ = \left(\frac{\sqrt{5}+1}{2} \right) \cos 12^\circ$$

$$\text{RHS} = \cos 24^\circ + \cos 48^\circ$$

$$= 2 \cos \left(\frac{24^\circ + 48^\circ}{2} \right) \cos \left(\frac{24^\circ - 48^\circ}{2} \right) [\because \cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)]$$

$$= 2 \cos 36^\circ \cos(-12^\circ)$$

$$= 2 \cos 36^\circ \times \cos 12^\circ [\because \cos(-\theta) = \cos \theta]$$

$$= 2 \times \frac{\sqrt{5}+1}{4} \times \cos 12^\circ = \frac{\sqrt{5}+1}{2} \times \cos 12^\circ [\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}]$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

OR

We have,

$$\text{LHS} = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$$

$$\Rightarrow \text{LHS} = \cos 60^\circ (\cos 20^\circ \cos 40^\circ) \cos 80^\circ$$

$$\Rightarrow \text{LHS} = \frac{1}{2} \times \frac{1}{2} (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ \left[\because \cos \frac{\pi}{3} = \frac{1}{2} \right]$$

$$\Rightarrow \text{LHS} = \frac{1}{4} \{ [\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)] \cos 80^\circ \} [\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B)]$$

$$\Rightarrow \text{LHS} = \frac{1}{4} \{ (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ \}$$

$$\Rightarrow \text{LHS} = \frac{1}{4} \left\{ \left(\frac{1}{2} + \cos 20^\circ \right) \cos 80^\circ \right\}$$

$$\begin{aligned}
&\Rightarrow \text{LHS} = \frac{1}{4} \left\{ \frac{1}{2} \cos 80^\circ + \cos 80^\circ \cos 20^\circ \right\} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{ \cos 80^\circ + 2 \cos 80^\circ \cos 20^\circ \} \\
&\Rightarrow \text{LHS} = \frac{1}{8} [\cos 80^\circ + \{ \cos (80^\circ + 20^\circ) + \cos (80^\circ - 20^\circ) \}] \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{ \cos 80^\circ + \cos 100^\circ + \cos 60^\circ \} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{ \cos 80^\circ + \cos (180^\circ - 80^\circ) + \cos 60^\circ \} \\
&\Rightarrow \text{LHS} = \frac{1}{8} \{ \cos 80^\circ - \cos 80^\circ + \cos 60^\circ \} [\because \cos (180^\circ - x) = -\cos x] \\
&\Rightarrow \text{LHS} = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = \text{RHS}
\end{aligned}$$

Section E

36. Read the text carefully and answer the questions:

Function as a Relation A relation f from a non-empty set A to a non-empty set B is said to be a function, if every element of set A has one and only one image in set B .

In other words, we can say that a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component.

If f is a function from a set A to a set B , then we write

$$f: A \rightarrow B \text{ or } A \xrightarrow{f} B.$$

and it is read as f is a function from A to B or f maps A to B .

$$(i) f(x) = \frac{1}{2 - \sin 3x}$$

Here, $2 - \sin 3x$ can never be zero as $\sin 3x$ will always less than 2.

$\therefore$ Domain of $f(x)$ will be $x \in \mathbb{R}$.

Now, $f(x)$ will be maximum when $2 - \sin 3x$ is minimum.

$2 - \sin 3x$ will be minimum when $\sin 3x = 1$.

$$\therefore f(x)_{\max} = \frac{1}{2-1} = 1$$

$f(x)$ will be minimum when $2 - \sin 3x$ is maximum.

$2 - \sin 3x$ will be maximum when $\sin 3x = -1$.

$$\therefore f(x)_{\min} = \frac{1}{2-(-1)} = \frac{1}{3}$$

So, range of $f(x)$ will be $\left[\frac{1}{3}, 1 \right]$.

$$(ii) \text{ We have, } f(1+x) = x^2 + 1 \dots (i)$$

On substituting $x = (1 - h)$ in eq. (i), we get

$$f(1 + 1 - h) = (1 - h)^2 + 1$$

$$f(2 - h) = 1 + h^2 - 2h + 1$$

$$= h^2 - 2h + 2$$

$$(iii) f(x) = x^2 + 2x + 3$$

at $f(1)$, $f(2)$, $f(3)$

$$f(1) = (1)^2 + 2(1) + 3 = 1 + 2 + 3 = 6$$

$$f(2) = (2)^2 + 2(2) + 3 = 4 + 4 + 3 = 11$$

$$f(3) = (3)^2 + 2(3) + 3 = 9 + 6 + 3 = 18$$

OR

$$x = y^2$$

37. Read the text carefully and answer the questions:

In a hostel 60% of the students read Hindi newspapers, 40% read English newspapers and 20% read both Hindi and English newspapers.



(i) H: Student read Hindi newspaper

E: Student read English newspaper

$$P(H) = \frac{60}{100}, P(E) = \frac{40}{100}, P(H \cap E) = \frac{20}{100}$$

$$P(E \cup H) = P(E) + P(H) - P(E \cap H)$$

$$\Rightarrow P(E \cup H) = \frac{40}{100} + \frac{60}{100} - \frac{20}{100} = \frac{80}{100}$$

$$\Rightarrow P(E \cup H) = \frac{4}{5} = 80\%$$

$\Rightarrow$ 80% of students read English or Hindi newspaper.

(ii) H: Student read Hindi newspaper

E: Student read English newspaper

$$P(H) = \frac{60}{100}, P(E) = \frac{40}{100}, P(H \cap E) = \frac{20}{100}$$

$$P(E \cup H)' = 1 - P(E \cup H)$$

$$\Rightarrow P(E \cup H)' = 1 - \frac{4}{5} = \frac{1}{5} = 20\%$$

$\Rightarrow$ 20% of students read neither English nor Hindi newspapers.

(iii) H: Student read Hindi newspaper

E: Student read English newspaper

$$P(H) = \frac{60}{100}, P(E) = \frac{40}{100}, P(H \cap E) = \frac{20}{100}$$

$$P(E' \cap H) = P(H) - P(E \cap H)$$

$$\Rightarrow P(E' \cap H) = \frac{60}{100} - \frac{20}{100} = \frac{40}{100} = 40\%$$

$\Rightarrow$ 40% of students read only Hindi newspapers.

OR

H: Student read Hindi newspaper

E: Student read English newspaper

$$P(H) = \frac{60}{100}, P(E) = \frac{40}{100}, P(H \cap E) = \frac{20}{100}$$

$$P(E \cap H') = P(E) - P(E \cap H)$$

$$\Rightarrow P(E' \cap H) = \frac{40}{100} - \frac{20}{100} = \frac{20}{100} = 20\%$$

$\Rightarrow$ 20% of students read only English newspaper.

38. Read the text carefully and answer the questions:

The conjugate of a complex number z , is the complex number, obtained by changing the sign of imaginary part of z . It is denoted by $\bar{z}$.

The modulus (or absolute value) of a complex number, $z = a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $|z|$, i.e.

$$|z| = \sqrt{a^2 + b^2}$$

Multiplicative inverse of z is $\frac{\bar{z}}{|z|^2}$. It is also called reciprocal of z .

$$z\bar{z} = |z|^2$$

(i) Let $z = 1 + 2i$

$$\Rightarrow |z| = \sqrt{1^2 + 4} = \sqrt{5}$$

$$\text{Now, } f(z) = \frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$$

$$= \frac{6-2i}{1-1-4i-4} = \frac{6-2i}{-4-4i}$$

$$= \frac{(2-2i)(2+2i)}{(3-i)(2+2i)}$$

$$= \frac{(2-2i)(2+2i)}{(2-2i)(2+2i)} = \frac{6+4i+2}{4+4}$$

$$= \frac{8+4i}{8} = 1 + \frac{1}{2}i$$

$$f(z) = 1 + \frac{1}{2}i$$

$$\therefore |f(z)| = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{4+1}{4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$$

(ii) Given that: $(z + 3)(\bar{z} + 3)$

Let $z = x + yi$

$$\text{So } (z + 3)(\bar{z} + 3) = (x + yi + 3)(x - yi + 3)$$

$$= [(x + 3) + yi][(x + 3) - yi]$$

$$= (x + 3)^2 - y^2i^2$$

$$= (x + 3)^2 + y^2$$

$$= |x + 3 + iy|^2$$

$$= |z + 3|^2$$