Electric Charges and Fields

Multiple Choice Questions

Choose and write the correct option(s) in the following questions.

1. Which statement is true for Gauss law? *[CBSE Sample Paper-2022, Term-1)]*

(*a*) All the charges whether inside or outside the Gaussian surface contribute to the electric flux.

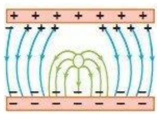
(*b*) Electric flux depends upon the geometry of the Gaussian surface.

(*c*) Gauss theorem can be applied to non-uniform electric field.

(*d*) The electric field over the Gaussian surface remains continuous and uniform at every point.

2. Which of the diagrams correctly represents the electric field between two charged plates if a neutral conductor is placed in between the plates? *[CBSE 2022, (55/2/4), Term-1]*

(*a*)



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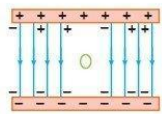

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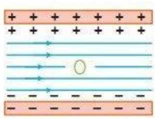

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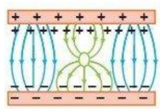
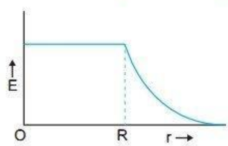


3. The electric field due to a uniformly charged sphere of radius *R* as a function of the distance from its centre is represented graphically by

(*a*)



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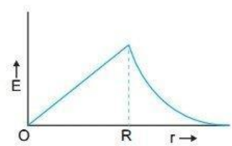
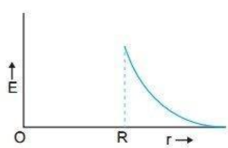
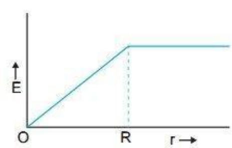


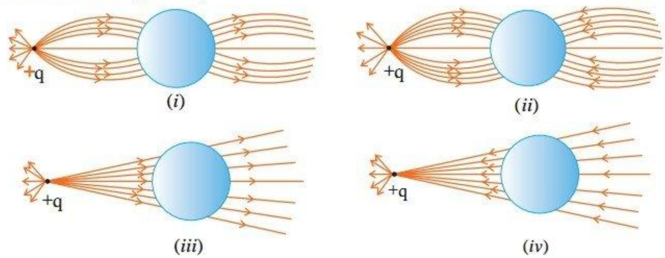
(*c*)



(*d*)

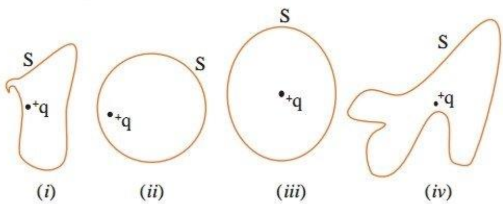


4. **A point positive charge is brought near an isolated conducting sphere (Fig. given below). The electric field is best given by** [*NCERT Exemplar*]



(*a*) Fig (*i*)
(*b*) Fig (*ii*)
(*c*) Fig (*iii*)
(*d*) Fig (*iv*)

5. **The Electric flux through the surface** [*NCERT Exemplar*]



(*a*) in Fig. (*iv*) is the largest.
(*b*) in Fig. (*iii*) is the least.
(*c*) in Fig. (*ii*) is same as Fig. (*iii*) but is smaller than Fig. (*iv*)
(*d*) is the same for all the figures.

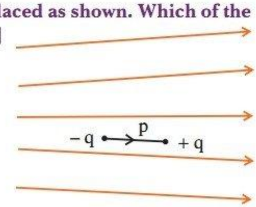
6. **A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed** [*NCERT Exemplar*]

(*a*) perpendicular to the diameter
(*b*) parallel to the diameter
(*c*) at an angle tilted towards the diameter
(*d*) at an angle tilted away from the diameter

7. **An electric dipole placed in a non-uniform electric field can experience** [*CBSE 2020 (55/1/2)*]

(*a*) a force but not a torque.
(*b*) a torque but not a force.
(*c*) always a force and a torque.
(*d*) neither a force nor a torque.

8. **Figure shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct?** [*NCERT Exemplar*]



(*a*) the dipole will not experience any force.
(*b*) the dipole will experience a force towards right.
(*c*) the dipole will experience a force towards left.
(*d*) the dipole will experience a force upwards.

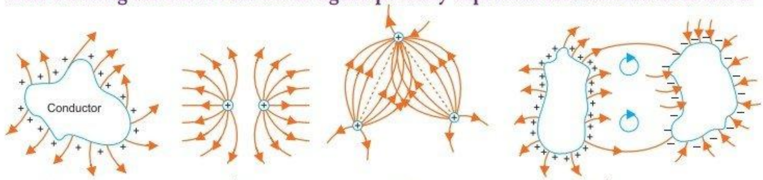
9. **A point charge $+q$, is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is** [*NCERT Exemplar*]

(*a*) directed perpendicular to the plane and away from the plane.
(*b*) directed perpendicular to the plane but towards the plane.
(*c*) directed radially away from the point charge.
(*d*) directed radially towards the point charge.

10. **If $\oint \vec{E}.d\vec{S} = 0$ over a surface, then** *[NCERT Exemplar]*
(*a*) the electric field inside the surface and on it is zero.
(*b*) the electric field inside the surface is necessarily uniform.
(*c*) the number of flux lines entering the surface must be equal to the number of flux lines leaving it.
(*d*) all charges must necessarily be outside the surface.

11. **Two charges are at distance *d* apart in air. Coulomb force between them is *F*. If a dielectric material of dielectric constant *K* is placed between them, the Coulomb force now becomes**
(*a*) F/K (*b*) FK (*c*) F/K^2 (*d*) K^2F

12. **Which among the curves shown in figure possibly represent electrostatic field lines?**



13. **Two point charges *A* and *B*, having charges $+q$ and $-q$ respectively, are placed at certain distance apart and force acting between them is *F*. If 25% charge of *A* is transferred to *B*, then force between the charges becomes**
(*a*) F (*b*) $\frac{9F}{16}$ (*c*) $\frac{16F}{3}$ (*d*) $\frac{4F}{3}$

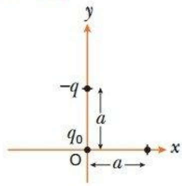
14. **Two large conducting spheres carrying charges Q_1 and Q_2 are kept with their centres *r* distance apart. The magnitude of electrostatic force between them is not exactly $\frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ because** *[CBSE 2020 (55/3/3)]*
(*a*) these are not point charges.
(*b*) charge distribution on the spheres is not uniform.
(*c*) charges on spheres will shift towards the centres of their respective spheres.
(*d*) charges will shift towards the portions of the spheres which are closer and facing towards each other.

15. **A negatively charged object *X* is repelled by another charged object *Y*. However an object *Z* is attracted to object *Y*. Which of the following is the most possibility for the object *Z*?** *[CBSE 2022 (55/2/4), Term-1]*
(*a*) positively charged only (*b*) negatively charged only
(*c*) neutral or positively charged (*d*) neutral or negatively charged

16. **In an experiment three microscopic latex spheres are sprayed into a chamber and became charged with charges +3e, +5e, and – 3e respectively. All the three spheres came in contact simultaneously for a moment and got separated. Which one of the following are possible values for the final charge on the spheres?** *[CBSE 2022 (55/2/4), Term-1]*
(*a*) $+5e, -4e, +5e$ (*b*) $+6e, +6e, -7e$
(*c*) $-4e, +3.5e, +5.5e$ (*d*) $+5e, -8e, +7e$

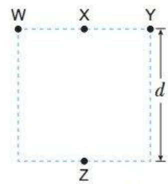
17. **The magnitude of electric field due to a point charge $2q$, at distance r is *E*. Then the magnitude of electric field due to a uniformly charged thin spherical shell of radius *R* with total charge *q* at a distance $\frac{r}{2}(r >> R)$ will be** *[CBSE 2022 (55/2/4), Term-1]*
(*a*) $\frac{E}{4}$ (*b*) 0
(*c*) $2E$ (*d*) $4E$

18. **Three charges q, $-q$ and q_0 are placed as shown in figure. The magnitude of the net force on the charge q_0 at point O is $\left[k = \frac{1}{(4\pi\varepsilon_0)}\right]$** *[CBSE 2022 (55/2/4), Term-1]*



(a) 0 (b) $\frac{2kqq_0}{a^2}$ (c) $\frac{\sqrt{2}\,kqq_0}{a^2}$ (d) $\frac{1}{\sqrt{2}}\frac{kqq_0}{a^2}$

19. **Four objects W, X, Y and Z, each with charge $+q$ are held fixed at four points of a square of side d as shown in the figure. Objects X and Z are on the midpoints of the sides of the square. The electrostatic force exerted by object W on object X is F. Then the magnitude of the force exerted by object W on Z is** *[CBSE 2022 (55/2/4), Term-1]*



(a) $\frac{F}{7}$ (b) $\frac{F}{5}$ (c) $\frac{F}{3}$ (d) $\frac{F}{2}$

20. **A square sheet of side 'a' is lying parallel to XY plane at $z = a$. The electric field in the region is $\vec{E} = cz^2\hat{k}$. The electric flux through the sheet is** *[CBSE 2022 (55/2/4), Term-1]*

(a) a^4c (b) $\frac{1}{3}a^3c$ (c) $\frac{1}{3}a^4c$ (d) 0

21. **Two point charges placed in a medium of dielectric constant 5 are at a distance r between them, experience an electrostatic force 'F'. The electrostatic force between them in vacuum at the same distance r will be** *[CBSE Sample Paper-2022, Term-1]*

(a) $5F$ (b) F (c) $F/2$ (d) $F/5$

22. **Consider an uncharged conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,** *[CBSE Sample Paper-2022, Term-1]*

(a) negative and uniformly distributed over the surface of sphere
(b) positive and uniformly distributed over the surface of sphere
(c) negative and appears at a point on the surface of sphere closest to point charge
(d) zero

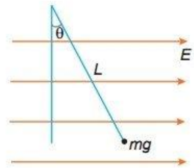
23. **A cylinder of radius r and length l is placed in a uniform electric field parallel to the axis of the cylinder. The total flux for the surface of the cylinder is given by** *[CBSE Sample Paper-2022, Term-1]*

(a) zero (b) πr^2
(c) $E\pi r^2$ (d) $2\,E\pi r^2$

24. **Two parallel large thin metal sheets have equal surface densities 26.4×10^{-12} C/m^2 of opposite signs. The electric field between these sheets is** *[CBSE Sample Paper-2022, Term-1]*

(a) 1.5 N/C (b) 1.5×10^{-16} N/C
(c) 3×10^{-10} N/C (d) 3 N/C

25. A small object with charge q and weight mg is attached to one end of a string of length 'L' attached to a stationary support. The system is placed in a uniform horizontal electric field 'E', as shown in the accompanying figure. In the presence of the field, the string makes a constant angle θ with the vertical. The sign and magnitude of q is [*CBSE Sample Paper-2022, Term-1*]



(*a*) positive with magnitude mg/E

(*b*) positive with magnitude $(mg/E) \tan\theta$

(*c*) negative with magnitude $mg/E \tan\theta$

(*d*) positive with magnitude $E \tan\theta/mg$

26. A point charge situated at a distance 'r' from a short electric dipole on its axis, experiences a force $\vec{F}$. If the distance of the charge is '$2r$', the force on the charge will be [*CBSE 2023 (55/1/1)*]

(*a*) $\frac{\vec{F}}{16}$ (*b*) $\frac{\vec{F}}{8}$

(*c*) $\frac{\vec{F}}{4}$ (*d*) $\frac{\vec{F}}{2}$

27. The magnitude of the electric field due to a point charge object at a distance of 4.0 m is 9 N/C. From the same charged object the electric field of magnitude, $16 NC^{-1}$ will be at a distance of [*CBSE 2023 (55/2/1)*]

(*a*) 1 m (*b*) 2 m (*c*) 3 m (*d*) 6 m

28. An electron experiences n forece $(1 \cdot 6 \times 10^{-16}\text{ N})\,\hat{i}$ in an electric field $\vec{E}$. The electric field is $\vec{E}$ is [*CBSE 2023 (55/3/1)*]

(*a*) $(1.0 \times 10^{3} NC^{-1})\hat{i}$ (*b*) $-(1.0 \times 10^{3} NC^{-1})\hat{i}$

(*c*) $(1.0 \times 10^{-3} NC^{-1})\hat{i}$ (*d*) $-(1.0 \times 10^{-3} NC^{-1})\hat{i}$

29. Two charges q_1 and q_2 are placed at the centres of two spherical counducting shells of radius r_1 and r_2 respectively. The shells are arranged such that their centres are $d\,[> (r_1 + r_2)]$ distance apart. The force on q_2 due to q_1 is: [*CBSE 2023 (55/4/1)*]

(*a*) $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ (*b*) $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(d - r_1)^2}$

(*c*) zero (*d*) $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{[d - (r_1 + r_2)]^2}$

30. An electric dipole of length 2 cm is placed at an angle of 30° with an electric field 2×10^5 N/C. If the dipole experiences a torque of 8×10^{-3} Nm, the magnitude of either charge of the dipole, is [*CBSE 2023 (55/4/1)*]

(*a*) 4 μC (*b*) 7 μC

(*c*) 8 mC (*d*) 2 mC

Answers

1. (*d*)	**2.** (*d*)	**3.** (*b*)	**4.** (*a*)	**5.** (*d*)	**6.** (*a*)	**7.** (*c*)
8. (*c*)	**9.** (*a*)	**10.** (*c*), (*d*)	**11.** (*a*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)
15. (*c*)	**16.** (*b*)	**17.** (*c*)	**18.** (*c*)	**19.** (*b*)	**20.** (*a*)	**21.** (*a*)
22. (*d*)	**23.** (*a*)	**24.** (*d*)	**25.** (*b*)	**26.** (*b*)	**27.** (*c*)	**28.** (*b*)
29. (*a*)	**30.** (*a*)					

Assertion-Reason Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) *Both A and R are true and R is the correct explanation of A.*

(b) *Both A and R are true but R is not the correct explanation of A.*

(c) *A is true but R is false.*

(d) *A is false and R is also false.*

1. **Assertion** (*A*) : A negative charge in an electric field moves along the direction of the electric field.
   Reason (*R*) : On a negative charge a force acts in the direction of the electric field.
   [*CBSE 2022 (55/2/4), Term-1*]
2. **Assertion** (*A*) : Charge is quantized because only integral number of electrons can be transferred.
   Reason (*R*) : There is no possibility of transfer of some fraction of electron.
3. **Assertion** (*A*) : In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.
   Reason (*R*) : In a non-uniform electric field, a dipole experiences a force as well as torque.
   [*CBSE Sample Paper 2021*]
4. **Assertion** (*A*) : Electrostatic field lines start at positive charges and end at negative charges.
   Reason (*R*) : Field lines are continuous curves without any breaks and they form closed loop.
5. **Assertion** (*A*) : Electrons moves away from a region of lower potential to a region of higher potential.
   Reason (*R*) : An electron has a negative charge.
6. **Assertion** (*A*) : All the charge in a conductor gets distributed on whole of its outer surface.
   Reason (*R*) : In a dynamic system, charges try to keep their potential energy minimum.
   [*AIIMS 2018*]
7. **Assertion** (*A*) : When a body acquires negative charge, its mass decreases.
   Reason (*R*) : A body acquires positive charge when it gains electrons.
8. **Assertion** (*A*) : Surface charge density of an irregularly shaped conductor is non-uniform.
   Reason (*R*) : Surface density is defined as charge per unit area.
9. **Assertion** (*A*) : Total flux through a closed surface is zero if no charge is enclosed by the surface.
   Reason (*R*) : Gauss law is true for any closed surface, no matter what its shape or size is.
10. **Assertion** (*A*) : If a proton and an electron are placed in the same uniform electric field, they experience different acceleration.
    Reason (*R*) : Electric force on a test charge is independent of its mass.

Answers

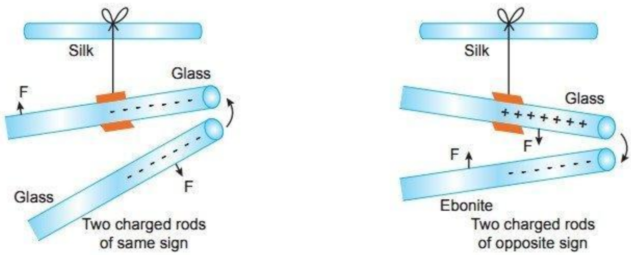
1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*c*) **5.** (*a*) **6.** (*a*) **7.** (*d*)
8. (*b*) **9.** (*a*) **10.** (*b*)

Case-based/Passage-based Questions

Read the paragraph given below and answer the questions that follow:

Frictional Electricity: Induction

The easiest way to experience electric charge is to rub certain solid bodies against each other. Long ago, around 600 BC, the Greeks knew that when amber is rubbed with wool, it acquires the property of attracting light objects such as small pieces of paper. This is because amber becomes electrically charged. If we pass a comb through dry hair, the comb becomes electrically charged and can attract small pieces of paper. An automobile becomes charged when it travels through the air. A paper sheet becomes charged when it passes through a printing machine. A gramophone record becomes charged when cleaned with a dry cloth.

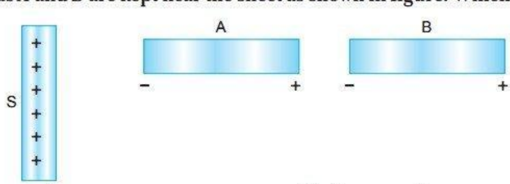


The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body may pass on to the other body. The body that receives the extra electrons becomes negatively charged and the body that donates the electrons becomes positively charged because it has more protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged.

(*i*) Charge Q is distributed to two different metallic spheres having radii R and $2R$ such that both spheres have equal surface charge density. Then charge on larger sphere is

(*a*) $\frac{4Q}{5}$ (*b*) $\frac{3Q}{5}$ (*c*) $\frac{5Q}{4}$ (*d*) $\frac{Q}{5}$

(*ii*) A large non-conducting sheet S is given a uniform charge density. Two uncharged small metal rods A and B are kept near the sheet as shown in figure. Which of the following is true?



(*a*) S attracts A (*b*) S attracts B

(*c*) A attracts B (*d*) All of these

(*iii*) Charge on a body which carries 300 excess electrons is

(*a*) -4.8×10^{-18} C (*b*) 4.8×10^{-18} C

(*c*) -4.8×10^{-17} C (*d*) 4.8×10^{-17} C (*iv*) **Which of the following cannot be true about properties of charge?**

(*a*) Charges can be created or destroyed in equal and unlike pairs only.

(*b*) Proper sign have to be used while adding the charges in a system.

(*c*) Excess of electrons over protons in a body is responsible for positive charge of the body.

(*d*) It is not possible to create or destroy net charge carried by an isolated system.

OR

The cause of charging is

(*a*) actual transfer of neutrons

(*b*) actual transfer of electrons

(*c*) actual transfer of protons

(*d*) none of these

Explanations

(*i*) (*a*) If q, and q' are charges on sphere of radii R and $2R$, then surface charge density will be same.

i.e.,
$$\sigma = \sigma'$$

$$\frac{q}{4\pi R^2} = \frac{q'}{4\pi(2R)^2} \Rightarrow q' = 4q \qquad \left[\because \sigma = \frac{Q}{A}\right]$$

As $q + q' = Q \Rightarrow q + 4q = Q \Rightarrow q = \frac{Q}{5}$

$$\therefore \qquad q' = \frac{4Q}{5}$$

(*ii*) (*d*) If the sheet S is given some positive charge density, then by induction, negative charge develop on ends of A and B, closer to S and an equal positive charge develops on farther ends of A and B as shown in figure. So, S attracts both A and B. Also, A attracts B.

(*iii*) (*c*) According to quantization of charge, $Q = ne$

Hence $n = 300$, $e = -1.6 \times 10^{-19}$ C

So, $Q = 300 \times (-1.6 \times 10^{-19}) = -4.8 \times 10^{-17}$C

(*iv*) (*c*) Excess of electrons over protons in a body is responsible for negative charge of the body.

OR

(*b*) The charging of body is due to transfer of electrons only.

CONCEPTUAL QUESTIONS

Q. 1. Why is the direction of the electric field due to a charged conducting sphere at any point perpendicular to its surface? [*CBSE 2019 (55/2/2)*]

Ans. If electric field is not perpendicular but has a component tangential to the surface of the conductor, it will exert force on charge and make them more. It means electrostatic condition is violated. **1**

[*CBSE Marking Scheme 2019 (55/2/2)*]

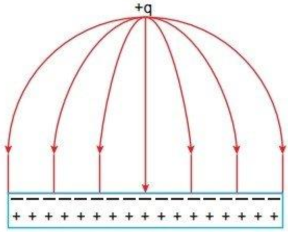
Q. 2. Two electric field lines cannot cross each other. Also, they cannot form closed loops. Give reasons. [*CBSE 2020 (55/2/1)*]

Ans. (*i*) Two electric field lines never cross each other because if they do so there will be two directions of electric field at the point of intersection which is not possible.

(*ii*) Since the electric field lines start from positive charge and terminate at the negative charge hence closed loops are not possible.

Q. 3. Draw the pattern of electric field lines when a point charge $+q$ is kept near an uncharged conducting plate. **[*CBSE 2019 (55/1/3)*]**

Ans.



Q. 4. Does the charge given to a metallic sphere depend on whether it is hollow or solid? Give reason for your answer. **[*CBSE 2017 (55/1/1)*]**

Ans. No, ½

Because the charge resides only on the surface of the conductor. ½

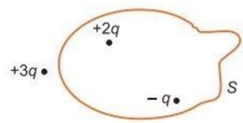
[*CBSE Marking Scheme 2017 (55/1/1)*]

Q. 5. Two identical conducting balls *A* and *B* have charges $-Q$ and $+3Q$ respectively. They are brought in contact with each other and then separated by a distance *d* apart. Find the nature of the Coulomb force between them. **[*CBSE 2019 (55/4/1)*]**

Ans. Final charge on balls A and B $= \dfrac{3Q - Q}{2} = Q$

The nature of the coulomb force between them is **repulsive**.

Q. 6. Fig. shows three point charges $+2q$, $-q$ and $+3q$. The charges $+2q$ and $-q$ are enclosed within a surface '*S*'. What is the electric flux due to this configuration through the surface '*S*'? **[*CBSE Delhi 2010*]**



Ans. Electric flux $= \dfrac{1}{\varepsilon_0} \times$ (Net charge enclosed within the surface)

$$= \frac{1}{\varepsilon_0}(2q - q) = \frac{q}{\varepsilon_0}$$

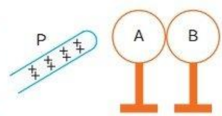
Q. 7. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? **[*CBSE Delhi 2015*]**

Ans. Net electric flux is zero.

Reason : (*i*) Independent to the shape and size.

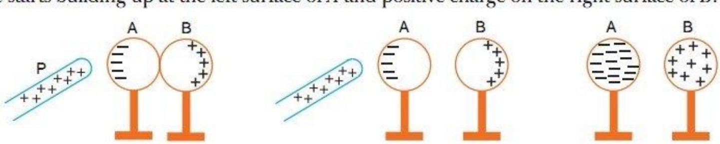
(*ii*) Net charge of the electric dipole is zero.

Q. 8. Two metallic spheres *A* and *B* kept on insulating stands are in contact with each other. A positively charged rod *P* is brought near the sphere *A* as shown in the figure. The two spheres are separated from each other, and the rod *P* is removed. What will be the nature of charges on spheres *A* and *B*? **[*CBSE 2019 (55/3/1)*]**



Ans.
- Sphere *A* will be negatively charged.
- Sphere *B* will be positively charged.

Explanation: If positively charged rod *P* is brought near metallic sphere *A* due to induction negative charge starts building up at the left surface of *A* and positive charge on the right surface of *B*.

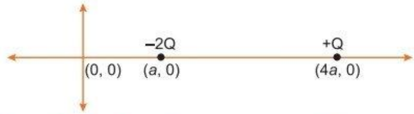


If the two spheres are separated from each other, the two spheres are found to be oppositely charged. If rod P is removed, the charges on spheres rearrange themselves and get uniformly distributed over them.

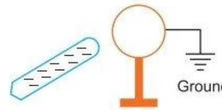
Q. 9. Two charges of magnitudes – $2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '$3a$' with its centre at the origin? **[*CBSE (AI) 2013*]**

Ans. Electric flux, $\phi = \dfrac{-2Q}{\varepsilon_0}$

Concept: Imagine a sphere of radius $3a$ about the origin and observe that only charge $-2Q$ is inside the sphere.

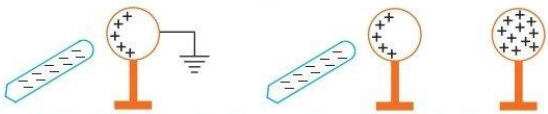


Q. 10. A metal sphere is kept on an insulating stand. A negatively charged rod is brought near it, then the sphere is earthed as shown. On removing the earthing, and taking the negatively charged rod away, what will be the nature of charge on the sphere? Give reason for your answer. **[*CBSE 2019 (55/3/1)*]**



Ans. The sphere will be positively charged due to electrostatic induction.

Explanation: When a negatively charged rod is brought near a metal sphere, the electrons will flow to the ground while the positive charges at the near end will remain held there due to the attractive force of the negative charge on the rod. On disconnecting the sphere from the ground, the positive charge continues to be held at the near end. On removing the electrified rod, the positive charge will spread uniformly over the sphere.

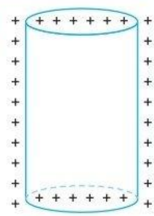


Q. 11. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? **[*CBSE 2016 (55/1/1)*]**

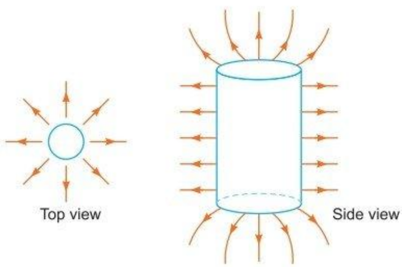
Ans. Electric flux remains unaffected. 1

[*CBSE Marking Scheme 2016 (55/1/1)*]

Q. 12. Sketch the electric field lines for a uniformly charged hollow cylinder shown in figure. **[*NCERT Exemplar*][*HOTS*]**



Ans.



Q. 13. What is the nature of electrostatic force between two point electric charges q_1 and q_2 if
(*a*) $q_1 + q_2 > 0$? **(*b*) $q_1 + q_2 < 0$?**

Ans. (*a*) If both q_1 and q_2 are positive, the electrostatic force between these will be repulsive.

However, if one of these charges is positive and is greater than the other negative charge, the electrostatic force between them will be attractive.

Thus, the nature of force between them can be repulsive or attractive.

(b) If both q_1 and q_2 are –ve, the force between these will be repulsive.

However, if one of them is –ve and it is greater in magnitude than the second +ve charge, the force between them will be attractive.

Thus, the nature of force between them can be repulsive or attractive.

Q. 14. The dimensions of an atom are of the order of an Angstrom. Thus there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero? [*NCERT Exemplar*]

Ans. The electric fields bind the atoms to neutral entity. Fields are caused by excess charges. There can be no excess charge on the inner surface of an isolated conductor. So, the electrostatic field inside a conductor is zero.

Q. 15. An arbitrary surface encloses a dipole. What is the electric flux through this surface? [*NCERT Exemplar*]

Ans. Net charge on a dipole = $-q + q = 0$. According to Gauss's theorem, electric flux through the surface,

$$\phi = \frac{q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

Very Short Answer Questions

Each of the following questions are of 2 marks.

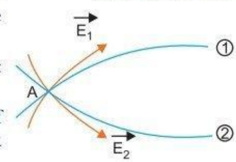
Q. 1. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why is it so?

(b) Explain why two field lines never cross each other at any point. [*CBSE (AI) 2014*]

Ans. (a) An electrostatic field line is the path of movement of a positive test charge ($q_0 \to 0$)

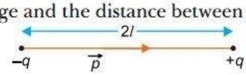
A moving charge experiences a continuous force in an electrostatic field, so an electrostatic field line is always a continuous curve.

(b) Two electric lines of force can never cross each other because if they cross, there will be two directions of electric field at the point of intersection (say A); which is impossible.



Q. 2. Define electric dipole moment. Is it a scalar or a vector quantity? What are its SI unit? [*CBSE Sample Paper 2021, (AI) 2011, 2013, (F) 2009, 2012, 2013*]

Ans. The electric dipole moment is defined as the product of either charge and the distance between the two charges. Its direction is from negative to positive charge.



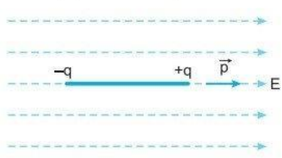
i.e., $|\vec{p}| = q(2l)$

Electric dipole moment is a vector quantity.

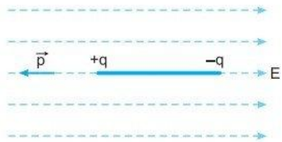
Its SI unit is coulomb-metre.

Q. 3. Depict the orientation of the dipole in (a) stable, (b) unstable equilibrium in a uniform electric field. [*CBSE Delhi 2017*]

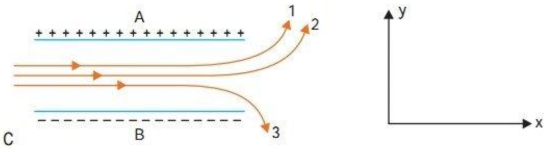
Ans. (a) Stable equilibrium, $\theta = 0°$ $\vec{P}$ is parallel to $\vec{E}$



(*b*) Unstable equilibrium, $\theta = 180°$ $\vec{P}$ is anti parallel to $\vec{E}$



Q. 4. The figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio? [*NCERT*]



Ans. A positively charged particle is deflected towards a negative plate and a negatively charged particle towards a positive plate and shows a parabolic path.

From fig. it is clear that the particles (1) and (2) are deflected towards positive plate; hence, they carry **negative charges**.

Particle (3) is deflected along negative plate, so it carries **positive charge**.

The transverse deflection in a given electric field is

$$y = \frac{1}{2}at^2, \text{ where } a = \frac{qE}{m} \text{ and } t = \left(\frac{x}{u}\right)$$

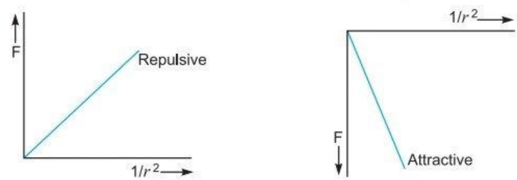
So $$y = \frac{1}{2}\left(\frac{q}{m}\right)\frac{E\,x^2}{u^2} \propto \frac{q}{m}.$$

From fig., it is obvious that the transverse deflection is the maximum for particle (3), hence, particle (3) has the highest charge to mass ratio (q/m).

Q. 5. Plot a graph showing the variation of coulomb force (F) versus $\left(\frac{1}{r^2}\right)$, where r is the distance between the two charges of each pair of charges: (1 μC, 2 μC) and (2 μC, – 3 μC). Interpret the graphs obtained. [*CBSE (AI) 2011*]

Ans. $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}.$$

The graph between F and $\frac{1}{r^2}$ is a straight line of slope $\frac{1}{4\pi\varepsilon_0}q_1 q_2$ passing through origin in both the cases.

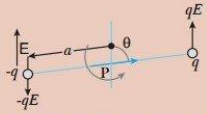


Since, magnitude of the slope is more for attraction, therefore, attractive force is greater than repulsive force.

Q. 6. Derive an expression for the torque acting on an electric dipole of dipole moment $\vec{P}$ placed in a uniform electric field $\vec{E}$. Write the direction along which the torque acts.

[*CBSE 2019 (55/5/1)*]

Ans.



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Force on either charge, $F = qE$

Magnitude of torque = Either of force × ⊥ distance between them. ½

$$\tau = qE\,2a\sin\theta$$

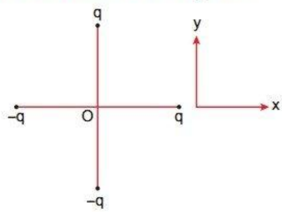
$$\Rightarrow \quad \tau = pE\sin\theta$$

$$\therefore \quad \vec{\tau} = \vec{P} \times \vec{E}$$ ½

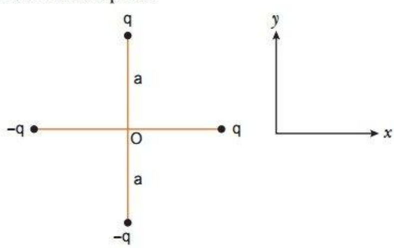
Direction is normal to the paper coming out of it. ½

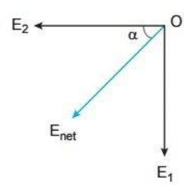
[CBSE Marking Scheme 2019 (55/5/1)]

Q. 7. Two identical dipoles are arranged in x-y plane as shown in the figure. Find the magnitude and the direction of net electric field at the origin O. *[CBSE 2023 (55/4/1)]*



Ans. Electric field at O due to both dipole.





$$\vec{E}_1 = \frac{kq}{a^2}(-\hat{j}-\hat{j}) = -\frac{2kq}{a^2}\hat{j}$$

$$\vec{E}_2 = \frac{kq}{a^2}(-\hat{i}-\hat{i}) = -\frac{2kq}{a^2}\hat{i}$$

So, net electric field at O,

$$|\vec{E}_{net}| = \sqrt{E_1^2+E_2^2} = \sqrt{\left(\frac{2kq}{a^2}\right)^2+\left(\frac{2kq}{a^2}\right)^2}$$

$$= \frac{2\sqrt{2}\,kq}{a^2}\ \text{NC}^{-1}$$

Also, $\tan\alpha = \dfrac{|\vec{E}_1|}{|\vec{E}_2|} = 1 \quad \Rightarrow \quad \alpha = 45°$

Hence, direction of $\vec{E}_{net}$ is 45° to its – x axis or 225° to + x-axis.

Q. 8. **(a) Define electric flux. Write its SI unit.**

(b) A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason. **[*CBSE (F) 2016*]**

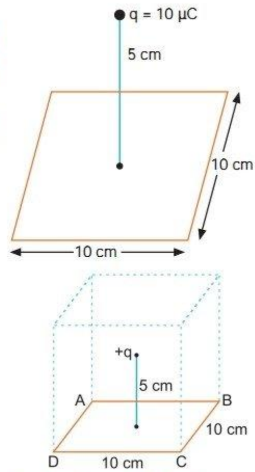
Ans. (a) Total number of electric field lines crossing a surface normally is called electric flux.

Its SI unit is Nm^2C^{-1} or Vm.

(b) Total electric flux through the surface $= \dfrac{q}{\varepsilon_0}$

As charge remains unchanged when size of balloon increases, electric flux through the surface remains unchanged.

Q. 9. **A point charge + 10 μC is at a distance 5 cm directly above the centre of a square of side 10 cm as shown in figure. What is the magnitude of the electric flux through the square? [Hint: Think of the square as one face of a cube with edge 10 cm]** **[*NCERT*] [*HOTS*]**



Ans. Obviously the given square *ABCD* of side 10 cm is one face of a cube of side 10 cm. At the centre of this cube a charge $+q = 10$ μC is placed.

According to Gauss's theorem, the total electric flux through the six faces of cube $= \dfrac{q}{\varepsilon_0}$.

∴ Total electric flux through square,

$$\phi = \frac{1}{6}\frac{q}{\varepsilon_0}$$

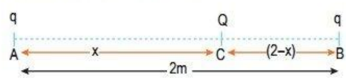
$$= \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= \mathbf{1.88 \times 10^5\ Nm^2C^{-1}}.$$

Q. 10. **Two identical point charges, *q* each, are kept 2 m apart in air. A third point charge *Q* of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of *Q*.** **[*CBSE 2019 (55/1/1)*]**

Ans. System is in equilibrium therefore net force on each charge of system will be zero.

For the total force on 'Q' to be zero



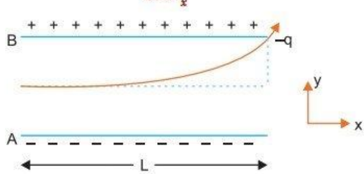
$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(2-x)^2}$$

$\Rightarrow \quad x = 2 - x \quad \Rightarrow \quad 2x = 2$

$\Rightarrow \quad \mathbf{x = 1\ m}$

For the equilibrium of charge "*q*" the nature of charge *Q* must be opposite to the nature of charge *q*.

Q. 11. **A particle of mass *m* and charge (–*q*) enters the region between the two charged plates initially moving along *X*-axis with speed v_x as shown in fig. The length of plate is *L* and an uniform electric field *E* is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $\dfrac{qEL^2}{2mv_x^2}$.** **[*NCERT*] [*HOTS*]**



Ans. Force on particle towards upper plate B, $F_y = qE$ vertical acceleration of particle, $a_y = \frac{qE}{m}$.

Initial vertical velocity $v_y = 0$

Speed of particle along X-axis $= v_x$ (constant)

Time taken by particle between the plates, $t = \frac{L}{v_x}$

From relation, $s = ut + \frac{1}{2}at^2$

Vertical deflection, $y = 0 + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$

$$\therefore \quad y = \frac{qEL^2}{2mv_x^2}$$

Q. 12. Given a uniform electric field $\vec{E} = 5\times10^3\,\hat{i}$ N/C, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the Y-Z plane. What would be the flux through the same square if the plane makes a 30° angle with the X-axis? *[CBSE Delhi 2014]*

Ans. Here, $\vec{E} = 5\times10^3\,\hat{i}$ N/C, *i.e.*, field is along positive direction of X-axis.

Surface area, A = 10 cm × 10 cm = 0.10 m × 0.10 m = 10^{-2} m^2

(*i*) When plane is parallel to Y-Z plane, the normal to plane is along X-axis. Hence

$$\theta = 0^\circ$$

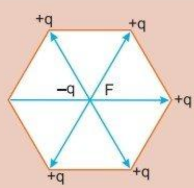
$$\therefore \quad \phi = EA\cos\theta = 5\times10^3\times10^{-2}\cos 0^\circ = \mathbf{50\ NC^{-1}\,m^2}$$

(*ii*) When the plane makes a 30° angle with the X-axis, the normal to its plane makes 60° angle with X-axis. Hence $\theta = 60^\circ$,

$$\therefore \quad \phi = EA\cos\theta = 5\times10^3\times10^{-2}\cos 60^\circ = \mathbf{25\ NC^{-1}\,m^2}$$

Q. 13. Five point charges, each of charge $+q$ are placed on five vertices of a regular hexagon of side 'l'. Find the magnitude of the resultant force on a charge $-q$ placed at the centre of the hexagon. *[CBSE 2019 (55/3/1)]*

Ans.



1

Alternatively:

The forces due to the charges placed diagonally opposite at the vertices of hexagon, on the charge $-q$ cancel in pairs. Hence net force is due to one charge only.

Net Force, $|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}$ 1

[CBSE Marking Scheme 2019 (55/3/1)]

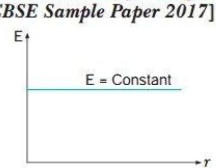
Q. 14. Represent graphically the variation of electric field with distance, for a uniformly charged plane sheet. *[CBSE Sample Paper 2017]*

Ans. Electric field due to a uniformly charged plane sheet.

$$E = \frac{\sigma}{2\varepsilon_0}$$

which is independent of distance.

So, it represents a straight line parallel to distance axis.

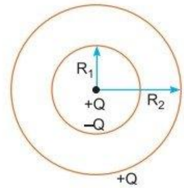


Q. 15. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (*i*) the inner surface, and (*ii*) the outer surface? [*NCERT Exemplar*]

Ans. When a charge $+Q$ is placed at the centre of spherical cavity,

the charge induced on the inner surface $= -Q$

the charge induced on the outer surface $= +Q$

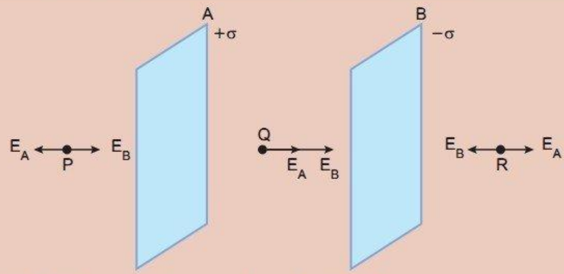


$\therefore$ Surface charge density on the inner surface $= \dfrac{-Q}{4\pi R_1^2}$

Surface charge density on the outer surface $= \dfrac{+Q}{4\pi R_2^2}$

Q. 16. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the electric field (*i*) between the sheets, and (*ii*) outside the sheets. [*CBSE 2019 (55/4/1)*]

Ans.



½

Now Electric field Intensity due to a plane sheet of charge

$$E = \frac{\sigma}{2\varepsilon_0}$$ ½

Here

$$E_A = \frac{+\sigma}{2\varepsilon_0} \text{ and } E_B = \frac{-\sigma}{2\varepsilon_0}$$

(*i*) Electric field at point Q (In between the sheets)

$$\vec{E} = \vec{E}_A + \vec{E}_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$ ½

(*ii*) Field at the point P or R

$$\vec{E} = \vec{E}_A + \vec{E}_B = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$ ½

[*CBSE Marking Scheme 2019 (55/4/1)*]

Q. 17. The given figure shows the electric field lines around three point charges A, B and C.

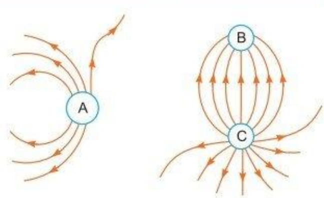
(*a*) Which charges are positive?

(*b*) Which charge has the largest magnitude? Why?

(*c*) In which region or regions of the picture could the electric field be zero? Justify your answer.

(*i*) near A (*ii*) near B (*iii*) near C (*iv*) nowhere.

[*NCERT Exemplar*] [*HOTS*]



Ans. (*a*) Charges A and C are positive since lines of force emanate from them.

(*b*) Charge C has the largest magnitude since maximum number of field lines are associated with it.

(*c*) (*i*) near A.

Justification: There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

Q. 18. Two isolated metal spheres A and B have radii R and $2R$ respectively, and same charge q. Find which of the two spheres have greater energy density just outside the surface of the spheres.
[*CBSE Sample Paper 2016*]

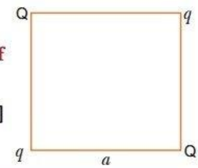
Ans. Energy density,

$$U = \frac{1}{2}\varepsilon_0 E^2$$

But, $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$$\therefore \quad U = \frac{1}{2}\frac{\varepsilon_0 Q^2}{A^2\varepsilon_0^2} \Rightarrow U = \frac{Q^2}{2A^2\varepsilon_0} \Rightarrow \quad U \propto \frac{1}{A^2} \quad \Rightarrow \quad U_A > U_B$$

Q. 19. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure. Find the resultant electric force on a charge Q. [*CBSE 2018 (55/1)*]



Ans. Let us find the force on the charge Q at the point C.

Force due to the other charge Q

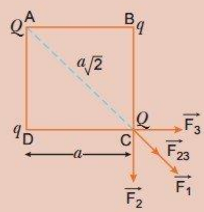
$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2}\right)(\text{along } AC)$$ ½

Force due to the charge q (at B),

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \ (\text{along } BC)$$

Force due to the charge q (at D),

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \ (\text{along } DC)$$ ½



Resultant of these two equal forces

$$F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{qQ(\sqrt{2})}{a^2} \ (\text{along } AC)$$

$\therefore$ Net force on charge Q (at point C) ½

$$F = F_1 + F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$$ ½

This force is directed along AC. (For the charge Q, at the point A, the force will have the same magnitude but will be directed along CA)

[**Note:** Don't deduct marks if the student does not write the direction of the net force, F]

[*CBSE Marking Scheme 2018 (55/1)*]

Q. 20. Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q. [*CBSE 2018 (55/1)*]

Justification: There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

Q. 18. Two isolated metal spheres A and B have radii R and $2R$ respectively, and same charge q. Find which of the two spheres have greater energy density just outside the surface of the spheres. [*CBSE Sample Paper 2016*]

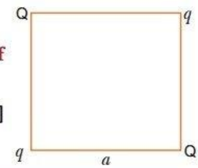
Ans. Energy density,

$$U = \frac{1}{2}\varepsilon_0 E^2$$

But, $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$$\therefore \quad U = \frac{1}{2}\frac{\varepsilon_0 Q^2}{A^2\varepsilon_0^2} \Rightarrow U = \frac{Q^2}{2A^2\varepsilon_0} \Rightarrow \quad U \propto \frac{1}{A^2} \quad \Rightarrow \quad U_A > U_B$$

Q. 19. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure. Find the resultant electric force on a charge Q. [*CBSE 2018 (55/1)*]



Ans. Let us find the force on the charge Q at the point C.

Force due to the other charge Q

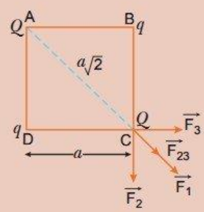
$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2}\right)\text{(along } AC\text{)}$$ ½

Force due to the charge q (at B),

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \text{ (along } BC\text{)}$$

Force due to the charge q (at D),

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2} \text{ (along } DC\text{)}$$ ½



Resultant of these two equal forces

$$F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{qQ(\sqrt{2})}{a^2} \text{ (along } AC\text{)}$$

$\therefore$ Net force on charge Q (at point C) ½

$$F = F_1 + F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$$ ½

This force is directed along AC. (For the charge Q, at the point A, the force will have the same magnitude but will be directed along CA)

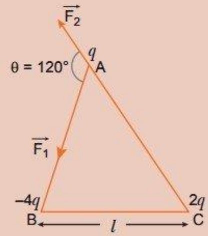
[**Note:** Don't deduct marks if the student does not write the direction of the net force, F]

[*CBSE Marking Scheme 2018 (55/1)*]

Q. 20. Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q. [*CBSE 2018 (55/1)*]

Ans. Force on charge q due to the charge $-4q$

$$F_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{l^2}\right), \text{ along } AB$$ ½



Force on the charge q, due to the charge $2q$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{l^2}\right), \text{ along } CA$$

The forces F_1 and F_2 are inclined to each other at an angle of 120°

Hence, resultant electric force on charge q,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$ ½

$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120^\circ}$$

$$= \sqrt{F_1^2 + F_2^2 - F_1F_2}$$ ½

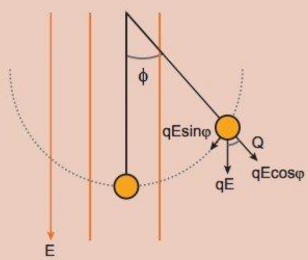
$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)\sqrt{16+4-8}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right)$$ ½

[*CBSE Marking Scheme 2018 (55/1)*]

Q. 21. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically downwards. Find the period of oscillation of the pendulum due to the electrostatic force acting on the sphere, neglecting the effect of the gravitational force.
[*CBSE 2019 (55/3/1)*]

Ans.



½

Restoring force,

$$F_r = -qE\sin\phi$$ ½

$$\Rightarrow \quad ma = -qE\sin\phi$$

When ϕ is small, $\sin\phi \simeq \phi$

$$\Rightarrow \quad ma = -qE\phi$$

$$\Rightarrow \quad m\frac{d^2x}{dt} = -qE\frac{x}{l}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -q\frac{E}{m}\frac{x}{l}$$ ½

Comparing with equation of linear SHM

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \Rightarrow \omega^2 = \frac{qE}{ml}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{qE}{ml}}$$

$$\therefore \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ml}{qE}}$$ ½

Alternatively: The student can use angular SHM intermediate also. Full marks to be awarded for correct answer even without intermediate steps.

[*CBSE Marking Scheme 2019 (55/3/1)*]

Q. 22. An electric dipole of length 2 cm is placed with its axis making an angle of 60° with respect to uniform electric field of 10^5 N/C.

If it experiences a torque of $8\sqrt{3}$ Nm, calculate the magnitude of charge on the dipole, and its potential energy. **[*CBSE Sample Paper 2021*]**

Ans. Here, $2l = 2 \text{ cm} = 2 \times 10^{-2}$ m

$$\theta = 60°, E = 10^5 \text{N/C}, \tau = 8\sqrt{3} \text{ Nm}$$

Using $\tau = pE \sin\theta$

$$\tau = 2\,qlE \sin\theta$$

$$q = \frac{\tau}{2\,l\,E\sin\theta} = \frac{8\sqrt{3}\times 2}{2\times 10^{-2}\times 10^5\times\sqrt{3}} = 8\times 10^{-3}\text{C}$$

Potential energy $= -\,pE\cos\theta = -2qlE\cos\theta$

$$= -2\times 10^{-2}\times 8\times 10^{-3}\times 10^5\times\frac{1}{2} = \mathbf{8\ J}$$

Q. 23. Two point charges $q_1 = +1\ \mu$C and $q_2 = +4\ \mu$C are placed 2 m apart in air. At what distance from q_1 along the line joining the two charges, will the net electric field be zero?

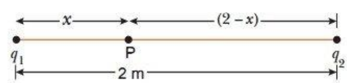
[*CBSE 2020 (55/3/1)*]

Ans. The electric field at point P due to q_1,

$$E_1 = \frac{kq_1}{x^2}$$

The electric field at point P due to q_2,

$$E_2 = \frac{kq_2}{(2-x)^2}$$



At point P, net electric field is zero,

$$\Rightarrow \qquad E_1 = E_2$$

$$\Rightarrow \qquad \frac{kq_1}{x^2} = \frac{kq_2}{(2-x)^2} \qquad \Rightarrow \qquad \frac{1}{x^2} = \frac{4}{(2-x)^2}$$

$$\Rightarrow \qquad \frac{1}{x} = \frac{2}{(2-x)} \qquad \Rightarrow \qquad (2-x) = 2x$$

$$\Rightarrow \qquad 3x = 2$$

$$\therefore \qquad x = \mathbf{\frac{2}{3}\ m}$$

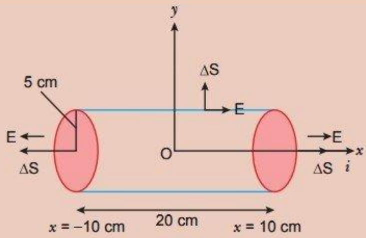
Q. 24. An electric field is uniform and acts along $+x$ direction in the region of positive x. It is also uniform with the same magnitude but acts in $-x$ direction in the region of negative x. The value of the field is $E = 200$ N/C for $x > 0$ and $E = -200$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one flat face is at $x = +10$ cm and the other is at $x = -10$ cm.

Find:

(*i*) **The net outward flux through the cylinder.**

(*ii*) **The net charge present inside the cylinder.** [*CBSE 2020 (55/1/1)*]

Ans. (*i*)



The net outward flux through cylinder,

$\phi = EA + EA = 2EA$ where, $A = \pi r^2$ ½

$\therefore \quad \phi = 2 \times 200 \times 3.14 \times 0.05 \times 0.05$

$= 3.14 \text{ NC}^{-1}\text{m}^2$ ½

(*ii*) The net charge present inside the cylinder,

$q = \varepsilon_0 \phi$

$q = 8.854 \times 3.14 \times 10^{-12}$ ½

$= 2.78 \times 10^{-11}$ C ½

[*CBSE Marking Scheme 2020 (55/1/1)*]

Short Answer Questions

Each of the following questions are of 3 marks.

Q. 1. Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. Find the ratio of the electric fields at their surfaces. [*CBSE 2023 (55/2/1)*]

Ans. Two charged conducting spheres are connected to a wire then their potential on the surfaces are same.

$\left[\text{Since } V = \frac{kq}{r}\right]$

i.e., $V_1 = V_2$

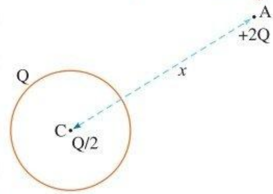
$$\frac{kq_a}{a} = \frac{kq_b}{b} \Rightarrow \frac{q_a}{q_b} = \frac{a}{b}$$

Also, we know, $E = \frac{kq}{r^2}$ (for point change)

then, $E_a = \frac{kq_a}{a^2}$, and $E_b = \frac{kq_b}{b^2}$

Ratio, $$\frac{E_a}{E_b} = \frac{\frac{kq_a}{a^2}}{\frac{kq_b}{b^2}} = \frac{b^2}{a^2} \times \frac{q_a}{q_b} = \frac{\boldsymbol{b}}{\boldsymbol{a}}.$$

Q. 2. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at the centre C and another charge $+2Q$ is placed outside the shell at A at a distance x from the centre as shown in the figure.



(*i*) Find the electric flux through the shell.

(*ii*) State the law used.

(*iii*) Find the force on the charges at the centre C of the shell and at the point A. **[*CBSE East 2016*]**

Ans. (*i*) Electric flux through a Gaussian surface, $\phi = \frac{\text{Total enclosed charge}}{\varepsilon_0}$

Net charge enclosed inside the shell, $q = 0$

$\therefore$ Electric flux through the shell, $\phi = \frac{q}{\varepsilon_0} = 0$

(*ii*) **Gauss's Law:** Electric flux through a Gaussian surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed within it.

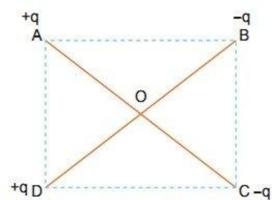
Mathematically, $\oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0} \times q$

(*iii*) We know that electric field or net charge inside the spherical conducting shell is zero. Hence, the force on charge $\frac{Q}{2}$ is zero.

Force on charge at A, $F_A = \frac{1}{4\pi\varepsilon_0} \frac{2Q\left(Q + \frac{Q}{2}\right)}{x^2} = \frac{1}{4\pi\varepsilon_0} \frac{3Q^2}{x^2}$

Q. 3. (*a*) The distance of a far off point on the equatorial plane of an electric dipole is halved. How will the electric field be affected for the dipole?

(*b*) Two identical electric dipoles are placed along the diagonals of a square *ABCD* of side $\sqrt{2}$ m as shown in the figure. Obtain the magnitude and direction of the net electric field at the centre (O) of the square.

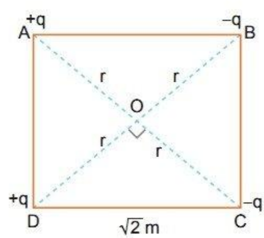


[*CBSE 2023 (55/1/1)*]

Ans. (*a*) If $r' = \frac{r}{2}$

Then, $E'_{eq} = \frac{kp}{(r')^3} = \frac{kp}{\left(\frac{r}{2}\right)^3} = \frac{8kp}{r^3} = 8\,E_{eq}.$

Hence, electric field at new position at equatorial line becomes 8 times of its initial value.



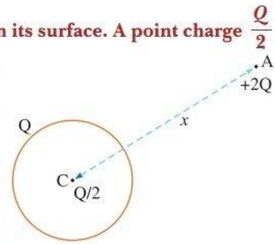
(*b*) Here, $AB = BC = CD = AD = \sqrt{2}$ m and

$q_A = q_D = +q, q_B = q_C = -q$

Electric field due to A and C at O,

$$E_1 = E_A + E_C = \frac{kq}{r^2} + \frac{kq}{r^2} = \frac{kq}{(1)^2} + \frac{kq}{(1)^2} = 2kq \text{ NC}^{-1} \qquad \left[r = \sqrt{2}\sin 45° = \frac{\sqrt{2}}{\sqrt{2}} = 1\text{ m}\right]$$

Q. 2. A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at the centre C and another charge $+2Q$ is placed outside the shell at A at a distance x from the centre as shown in the figure.



(i) Find the electric flux through the shell.

(ii) State the law used.

(iii) Find the force on the charges at the centre C of the shell and at the point A. **[CBSE East 2016]**

Ans. (i) Electric flux through a Gaussian surface, $\phi = \frac{\text{Total enclosed charge}}{\varepsilon_0}$

Net charge enclosed inside the shell, $q = 0$

$\therefore$ Electric flux through the shell, $\phi = \frac{q}{\varepsilon_0} = 0$

(ii) **Gauss's Law:** Electric flux through a Gaussian surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed within it.

Mathematically, $\oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0} \times q$

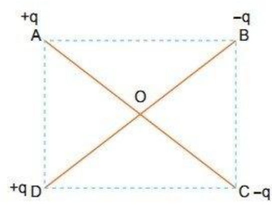
(iii) We know that electric field or net charge inside the spherical conducting shell is zero. Hence, the force on charge $\frac{Q}{2}$ is zero.

$$\text{Force on charge at } A,\ F_A = \frac{1}{4\pi\varepsilon_0}\frac{2Q\left(Q+\frac{Q}{2}\right)}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{x^2}$$

Q. 3. (a) The distance of a far off point on the equatorial plane of an electric dipole is halved. How will the electric field be affected for the dipole?

(b) Two identical electric dipoles are placed along the diagonals of a square $ABCD$ of side $\sqrt{2}$ m as shown in the figure. Obtain the magnitude and direction of the net electric field at the centre (O) of the square.

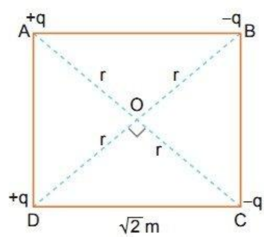
[CBSE 2023 (55/1/1)]



Ans. (a) If $r' = \frac{r}{2}$

Then, $E'_{eq} = \frac{kp}{(r')^3} = \frac{kp}{\left(\frac{r}{2}\right)^3} = \frac{8kp}{r^3} = 8\,E_{eq}.$

Hence, electric field at new position at equatorial line becomes 8 times of its initial value.



(b) Here, $AB = BC = CD = AD = \sqrt{2}$ m and

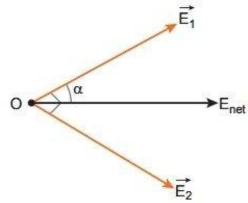
$q_A = q_D = +q,\ q_B = q_C = -q$

Electric field due to A and C at O,

$$E_1 = E_A + E_C = \frac{kq}{r^2} + \frac{kq}{r^2} = \frac{kq}{(1)^2} + \frac{kq}{(1)^2} = 2kq\ \text{NC}^{-1} \qquad \left[r = \sqrt{2}\sin 45^\circ = \frac{\sqrt{2}}{\sqrt{2}} = 1\ \text{m}\right]$$

Again, electric field due to D and B,

$$E_2 = E_D + E_B = \frac{kq}{(r)^2} + \frac{kq}{(r)^2} = \frac{kq}{(1)^2} + \frac{kq}{(1)^2} = 2kq \text{ NC}^{-1}.$$



So, $E_{net} = \sqrt{E_1^{\,2} + E_2^{\,2}}$

$= \sqrt{(2kq)^2 + (2kq)^2}$

$= \mathbf{2\sqrt{2}\ kq\ NC^{-1}}$

For direction, $\alpha = \tan^{-1}\left(\frac{2kq}{2kq}\right)$

$= \tan^{-1}(1) = \mathbf{45°}$

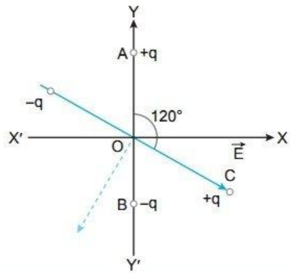
So, E_{net} along 45° from E_1 or E_2.

Q. 4. Two small identical electric dipoles *AB* and *CD*, each of dipole moment $\vec{P}$ are kept at an angle of 120° to each other in an external electric field $\vec{E}$ pointing along the *x*-axis as shown in the figure. Find the

(*a*) dipole moment of the arrangement, and

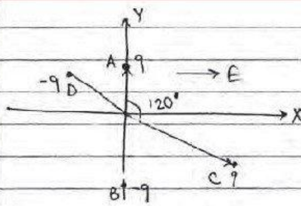
(*b*) magnitude and direction of the net torque acting on it.

[CBSE 2020 (55/2/1)]



Ans.

28



(a) Dipole moment of dipole AB = $\vec{p} = p\hat{j}$.

Dipole moment of dipole CD = $\vec{p} = p\cos 30°\,\hat{i} - p\cos 60°\,\hat{j}$

$= p\frac{\sqrt{3}}{2}\hat{i} - \frac{p}{2}\hat{j}$

∴ net dipole moment $(\vec{P})$ = $p\hat{j} + (p\frac{\sqrt{3}}{2}\hat{i} - \frac{p}{2}\hat{j})$

$= p\frac{\sqrt{3}}{2}\hat{i} + \frac{p}{2}\hat{j}$

∵ $|\vec{P}| = \sqrt{p^2 \times \frac{3}{4} + \frac{p^2}{4}} = \sqrt{p^2} = p.$ $\frac{1}{2}$

∴ magnitude of dipole moment = p.

$\tan\theta = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$.   ∴ angle made by it with +ve x-axis = 30° (Ans i)

Torque acting on a dipole of dipole moment $\vec{p}$ in electric field $\vec{E}$ = $\vec{p} \times \vec{E}$.

For AB, dipole moment = $p\hat{j}$.

field = $E\hat{i}$

∴ torque, $\vec{\tau}_{AB} = (p\hat{j} \times E\hat{i}) = pE(-\hat{k})$.

For CD, dipole moment $= (p\frac{\sqrt{3}}{2}\hat{i} - \frac{p}{2}\hat{j})$

field $= E\hat{i}$

∴ Torque, $\vec{\tau}_{CD} = (p\frac{\sqrt{3}}{2}\hat{i} - \frac{p}{2}\hat{j}) \times E\hat{i}$

$= pE\frac{\sqrt{3}}{2} \times 0 - \frac{p}{2}E.(-\hat{k}) = \frac{pE}{2}\hat{k}$

∴ net torque $= \vec{\tau}_{AB} + \vec{\tau}_{CD} = -\frac{pE}{2}\hat{k} = \frac{pE}{2}(-\hat{k})$

∴ magnitude $= \frac{pE}{2}$,

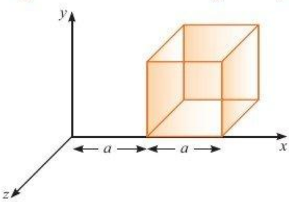
direction = into the plane of paper $(-\hat{k})$.

[*Topper's Answer 2020*]

Q. 5. State Gauss's law in electrostatics. A cube with each side '*a*' is kept in an electric field given by $\vec{E} = C \times \hat{r}$, (as is shown in the figure) where *C* is a positive dimensional constant. Find out [*CBSE (F) 2012*]

(*i*) the electric flux through the cube, and

(*ii*) the net charge inside the cube.



Ans. Gauss's Law in electrostatics states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of that charge.

$$\phi = \oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

(*i*) Net flux, $\phi = \phi_1 + \phi_2$

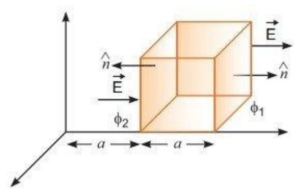
where $\phi_1 = \vec{E}.d\vec{S}$

$= 2aC\, dS \cos 0°$

$= 2aC \times a^2 = 2a^3 C$

$\phi_2 = aC \times a^2 \cos 180° = -a^3C$

∴ $\phi = 2a^3C + (-a^3C) = \mathbf{a^3C\ Nm^2\ C^{-1}}$



(*ii*) Net charge $(q) = \varepsilon_0 \times \phi = a^3C\,\varepsilon_0$ coulomb

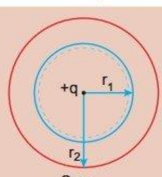
∴ $q = \mathbf{a^3C\,\varepsilon_0}$ **coulomb.**

Q. 6. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge *Q*.

(*a*) A charge *q* is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(*b*) Is the electric field inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not? Explain. [*CBSE 2019 (55/2/1)*]

Ans. (*a*) Diagram ½



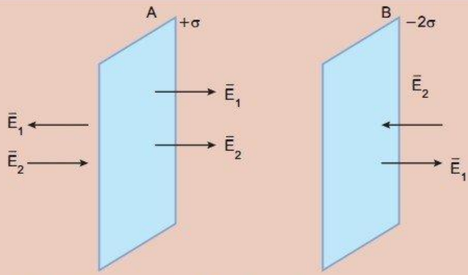
The Surface charge density on inner surface of the shell is $\sigma_1 = -\frac{q}{4\pi r_1^2}$ 1

The surface charge density on outer shell is $\sigma_2 = \frac{Q+q}{4\pi r_2^2}$ ½ (*b*) Consider a Gaussian surface inside the shell, net flux is zero since $q_{net} = 0$. According to Gauss's law it is independent of shape and size of shell. **1**

[*CBSE Marking Scheme 2019 (55/2/1)*]

Q. 7. Two large charged plane sheets of charge densities σ and –2σ C/m² are arranged vertically with a separation of *d* between them. Deduce expressions for the electric field at points (*i*) to the left of the first sheet, (*ii*) to the right of the second sheet, and (*iii*) between the two sheets. [*CBSE 2019 (55/2/1)*]

Ans.



1½

(*i*) Electric field in the region left of first sheet, **½**

$$E_1 = E_1 + E_2$$

$$E_1 = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$E_1 = +\frac{\sigma}{2\varepsilon_0}$$

It is towards right.

(*ii*) Electric field in the region to the right of second sheet **½**

$$E_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$$

$$E_{II} = \frac{\sigma}{2\varepsilon_0}$$

It is towards left.

(*iii*) Electric field between the two sheets, **½**

$$E_{III} = E_1 + E_2$$

$$E_{III} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$E_{III} = \frac{3\sigma}{2\varepsilon_0}$$

Electric field is towards the right.

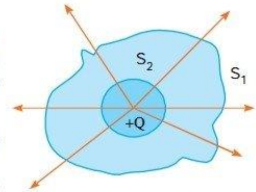
[*CBSE Marking Scheme 2019 (55/2/1)*]

Q.8. (*a*) "The outward electric flux due to charge +*Q* is independent of the shape and size of the surface which encloses it." Give two reasons to justify this statement.

(*b*) Two identical circular loops '1' and '2' of radius *R* each have linear charge densities –λ and +λ C/m respectively. The loops are placed coaxially with their centres $R\sqrt{3}$ distance apart. Find the magnitude and direction of the net electric field at the centre of loop '1'.

[*CBSE Patna 2015*]

Ans. (*a*) In figure, a charge $+Q$ is enclosed inside the surfaces S_1 and S_2.

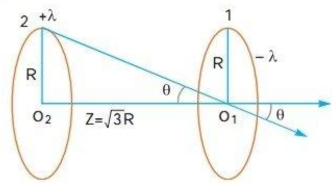


(*i*) For a given charge Q the same number of electric field lines emanating from the surfaces S_1 and S_2 depends on the charge Q and independent to the shape and size of the surfaces of S_1 and S_2.

(*ii*) From Gauss's law the net-outward electric flux through any closed surface of any shape and size is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed within that surface *i.e.*, $\frac{Q}{\varepsilon_0}$

(*b*) Electric field at the centre O_1 due to loop 1 is given by

$$|\vec{E}_1| = 0 \text{ (As Z = 0)}$$



Electric field at a point outside the loop 2 on the axis passing normally through O_2 of loop 2 is

$$|\vec{E}_2| = \frac{\lambda R}{2\varepsilon_0} \frac{Z}{(R^2+Z^2)^{3/2}}$$

Since $\qquad Z = R\sqrt{3}$

$$= \frac{\lambda R}{2\varepsilon_0} \frac{R\sqrt{3}}{(R^2+3R^2)^{3/2}}$$

$$= \frac{\lambda\sqrt{3}}{16\varepsilon_0 R} \text{ towards right (As } \lambda \text{ is positive)}$$

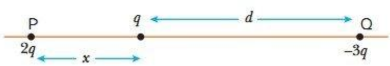
So, net electric field at the centre of loop 1,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= 0 + \frac{\lambda\sqrt{3}}{16\varepsilon_0 R} = \mathbf{\frac{\lambda\sqrt{3}}{16\varepsilon_0 R}}$$

Q. 9. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge $2q$ be placed such that it will not experience any force? [*NCERT Exemplar*]

Ans.



Let the charge $2q$ be placed at point P as shown. The force due to q is to the left and that due to $-3q$ is to the right.

$$\therefore \qquad \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0 (d+x)^2} \Rightarrow (d+x)^2 = 3x^2$$

$$\therefore \qquad 2x^2 - 2dx - d^2 = 0 \quad \Rightarrow \quad x = \frac{d}{2} \pm \frac{\sqrt{3}\,d}{2}$$

(–ve sign shows charge $2q$ at p would lie between q and $-3q$ and hence is unacceptable.)

$$\Rightarrow \qquad x = \frac{d}{2} + \frac{\sqrt{3}\,d}{2} = \mathbf{\frac{d}{2}(1+\sqrt{3})} \text{ to the left of } q.$$

Q. 10. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.

(*a*) What is the surface charge density on the (*i*) inner and (*ii*) outer surface of the sphere?

(*b*) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere. [*CBSE 2020 (55/4/1)*]

Ans. (*a*) Surface charge density on the inner surface $= \dfrac{q}{4\pi r_1^2}$ ½

On the outer surface $= \dfrac{Q-q}{4\pi r_2^2}$ ½

(*b*) For a spherical Gaussian surface $x > r_2$, 1

$$\int \vec{E}.\vec{ds} = \frac{Q-q}{\varepsilon_0}$$

$$E \times 4\pi x^2 = \frac{Q-q}{\varepsilon_0}$$ ½

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q-q}{x^2}$$ ½

[CBSE Marking Scheme 2020 (55/4/1)]

Long Answer Questions

Each of the following questions are of 5 marks.

Q. 1. (*i*) State Coulomb's law in electrostatics and write it in vector form, for two charges.

(*ii*) Gauss's law is based on the inverse-square dependence on distance contained in the Coulomb's law.' Explain.

(*iii*) Two charges *A* (charge q) and *B* (charge $2q$) are located at points (0, 0) and (a, a) respectively. Let $\hat{i}$ and $\hat{j}$ be the unit vectors along x-axis and y-axis respectively. Find the force exerted by *A* on *B*, in terms of $\hat{i}$ and $\hat{j}$. *[CBSE 2023 (55/1/1)]*

Ans. (*i*) **Coulomb's Law:**

It states that the force of attraction or repulsion between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges, *i.e.*,

$$F = k\frac{q_1 q_2}{r^2}$$

where $k = \dfrac{1}{4\pi\varepsilon}$ is constant of proportionality; ε is permittivity of medium between the charges. If ε_0 is permittivity of free space and K the dielectric constant of medium, then $\varepsilon = K\varepsilon_0$

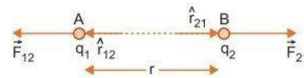
$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$$

For free space $K = 1$, Therefore

$$\therefore \qquad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

In Vector from,

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$



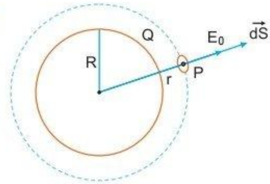
But $$\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{\vec{r}_{21}}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{21}$$

Similarly if $\vec{r}_{12}$ is position vector of q_1 relative to q_2 and $\hat{r}_{12}$ is unit vector from B to A, then

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{12}$$

(*ii*) Let $+q$ charge is placed at a point O and a point P lies at distance r from the point O. Imagine a sphere of radius r and centre O. Thus, point P lies on the surface of the sphere. Now, the surface of the sphere will behave as gaussian surface. Therefore, the intensity of electric field on the surface at all the points will be equal in magnitude and will be directed radially outward.



$\therefore$ The electric flux passing through the spherical surface,

$$\phi_E = E \,.\, S \cos 0°$$

$$\phi_E = E \,.\, S \qquad [S = 4\pi r^2]$$

$$\therefore \quad \phi_E = \pi r^2 E \qquad ...(i)$$

From Gauss's law, $\phi_E = \dfrac{q_{in}}{\varepsilon_0}$ ...(*ii*)

From equation (*i*) and (*ii*), $4\pi r^2 E = \dfrac{q_{in}}{\varepsilon_0}$

or $$E = \frac{q_{in}}{4\pi\varepsilon_0 r^2}$$

Now, imagine a charge q_0 placed at point P.

$\therefore$ Force on q_0, $F = q_0 E$

$$\Rightarrow \quad F = \frac{q_0 q_{in}}{4\pi\varepsilon_0 r^2}$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0 q_{in}}{r^2}$$

Hence it verify coulomb's inverse square law.

(*iii*) Force exerted by A on B,

$$\vec{F}_{BA} = \frac{kq_1 q_2}{r^2}\hat{r}$$

Using distance Formula,

$$r = \sqrt{(a-0)^2 + (a-0)^2} = \sqrt{2}\,a$$

$$\hat{r} = \frac{a\hat{i} + a\hat{j}}{\sqrt{2}\,a} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

Now, $$\vec{F}_{BA} = \frac{kq \times 2q}{(\sqrt{2}a)^2}\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \frac{kq^2}{\sqrt{2}a^2}(\hat{i} + \hat{j}) \text{ N}$$

Q. 2. Find expressions for the force and torque on an electric dipole kept in a uniform electric field. ***[CBSE (AI) 2014; 2019 (55/5/1); 2020 (55/3/1); 2020 (55/5/1); CBSE Sample Paper 2021]***

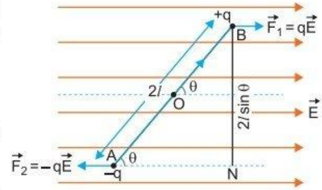
OR

(*i*) Define torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.

(*ii*) What happens if the field is non-uniform?

(*iii*) What would happen if the external field $\vec{E}$ is increasing (*i*) parallel to $\vec{p}$ and (*ii*) anti-parallel to $\vec{p}$? ***[CBSE (F) 2016]***

Ans. Consider an electric dipole placed in a uniform electric field of strength E in such a way that its dipole moment $\vec{p}$ makes an angle θ with the direction of $\vec{E}$. The charges of dipole are $-q$ and $+q$ at separation $2l$ the dipole moment of electric dipole,



$$p = q2l \quad ...(i)$$

Force: The force on charge $+q$ is, $\vec{F}_1 = q\vec{E}$, along the direction of field $\vec{E}$.

The force on charge $-q$ is $\vec{F}_2 = q\vec{E}$, opposite to the direction of field $\vec{E}$.

Obviously forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction; hence net force on electric dipole in uniform electric field is

$$F = F_1 - F_2 = qE - qE = 0 \text{ (zero)}$$

As net force on electric dipole is zero, so dipole does not undergo any translatory motion.

Torque: The forces $\vec{F}_1$ and $\vec{F}_2$ form a couple (or torque) which tends to rotate and align the dipole along the direction of electric field. This couple is called the torque and is denoted by τ.

$\therefore$ Torque τ = magnitude of one force × perpendicular distance between lines of action of forces

$$= qE\,(BN)$$
$$= qE\,(2l \sin\theta)$$
$$= (q2l)\,E \sin\theta$$
$$= pE \sin\theta \quad \text{[using } (i)\text{]} \quad ...(ii)$$

Clearly, the magnitude of torque depends on orientation (θ) of the electric dipole relative to electric field. Torque (τ) is a vector quantity whose direction is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$ given by right hand screw rule.

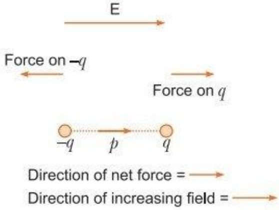
In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$ ...(iii)

Thus, if an electric dipole is placed in an electric field in oblique orientation, it experiences no force but experiences a torque. The torque tends to align the dipole moment along the direction of electric field.

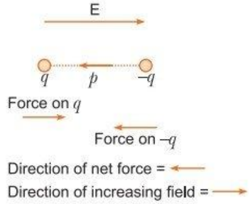
When the field is non-uniform, the net force will evidently be non-zero. There will be translatory motion of the dipole.

When $\vec{E}$ is parallel to $\vec{p}$, the dipole has a net force in the direction of increasing field.

When $\vec{E}$ is anti-parallel to $\vec{p}$, the net force on the dipole is in the direction of decreasing field. In general, force depends on the orientation of $\vec{p}$ with respect to $\vec{E}$.



E parallel to p



E antiparallel to p

Q. 3. Find an expression for the electric field strength at a distant point situated (*i*) on the axis and (*ii*) along the equatorial line of an electric dipole. [*CBSE (AI) 2013; (F) 2015; 2019 (55/5/1)*]

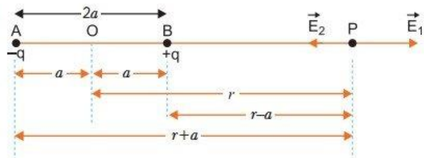
OR

Derive an expression for the electric field intensity at a point on the equatorial line of an electric dipole of dipole moment $\vec{p}$ and length 2*a*. What is the direction of this field?
[*CBSE South 2016; 2019 (55/1/1)*] [*CBSE 2023 (55/1/1)*]

Ans. Consider an electric dipole *AB*. The charges $-q$ and $+q$ of dipole are situated at *A* and *B* respectively as shown in the figure. The separation between the charges is $2a$.

Electric dipole moment, $p = q.2a$

The direction of dipole moment is from $-q$ to $+q$.



(*i*) **At axial or end-on position:** Consider a point *P* on the axis of dipole at a distance *r* from mid-point *O* of electric dipole.

The distance of point *P* from charge $+q$ at *B* is $BP = r - a$

and distance of point *P* from charge $-q$ at *A* is, $AP = r + a$.

Let E_1 and E_2 be the electric field strengths at point *P* due to charges $+q$ and $-q$ respectively. We know that the direction of electric field due to a point charge is away from positive charge and towards the negative charge. Therefore,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ (from } B \text{ to } P) \text{ and } E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2} \text{ (from } P \text{ to } A)$$

Clearly the directions of electric field strengths $\vec{E}_1$ are $\vec{E}_2$ along the same line but opposite to each other and $E_1 > E_2$ because positive charge is nearer.

∴ The resultant electric field due to electric dipole has magnitude equal to the difference of E_1 and E_2 direction from *B* to *P i.e.*

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ra}{(r^2-a^2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{2(q2a)r}{(r^2-a^2)^2}$$

But $q.2\,a = p$ (electric dipole moment)

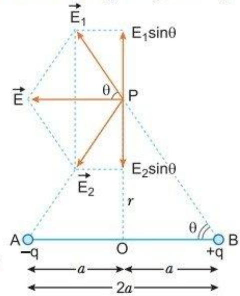
∴ $$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2} \qquad ...(i)$$

If the dipole is infinitely small and point P is far away from the dipole, then $r >> a$, therefore equation (*i*) may be expressed as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{r^4} \text{ or } E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \qquad ..(ii)$$

This is the expression for the electric field strength at axial position due to a short electric dipole.

(*ii*) **At a point of equatorial line:** Consider a point *P* on broad side on the position of dipole formed of charges $+q$ and $-q$ at separation $2a$. The distance of point *P* from mid point (*O*) of electric dipole is r. Let $\vec{E}_1$ and $\vec{E}_2$ be the electric field strengths due to charges $+q$ and $-q$ of electric dipole.



From fig. $AP = BP = \sqrt{r^2 + a^2}$

$\therefore \quad \vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2}$ along B to P

$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2}$ along P to A

Clearly $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude

i.e., $|\vec{E}_1| = |\vec{E}_2|$ or $E_1 = E_2$.

To find the resultant of $\vec{E}_1$ and $\vec{E}_2$, we resolve them into rectangular components.

Component of $\vec{E}_1$ parallel to $AB = E_1 \cos\theta$, in the direction to $\overrightarrow{BA}$

Component of $\vec{E}_1$ perpendicular to $AB = E_1 \sin\theta$ along OP

Component of $\vec{E}_2$ parallel to $AB = E_2 \cos\theta$ in the direction $\overrightarrow{BA}$

Component of $\vec{E}_2$ perpendicular to $AB = E_2 \sin\theta$ along PO

Clearly, components of $\vec{E}_1$ and $\vec{E}_2$ perpendicular to AB: $E_1 \sin\theta$ and $E_2 \sin\theta$ being equal and opposite cancel each other, while the components of $\vec{E}_1$ and $\vec{E}_2$ parallel to AB : $E_1 \cos\theta$ and $E_2 \cos\theta$, being in the same direction add up and give the resultant electric field whose direction is parallel to $\overrightarrow{BA}$.

$\therefore$ Resultant electric field at P is $E = E_1 \cos\theta + E_2 \cos\theta$

But $E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2 + a^2)}$

From the figure, $\cos\theta = \frac{OB}{PB} = \frac{a}{\sqrt{r^2 + a^2}} = \frac{a}{(r^2 + a^2)^{1/2}}$

$$E = 2E_1 \cos\theta = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2 + a^2)} \cdot \frac{a}{(r^2 + a^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0} \frac{2qa}{(r^2 + a^2)^{3/2}}$$

But $q.2a = p =$ electric dipole moment ...(*iii*)

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$$

If dipole is infinitesimal and point P is far away, we have $a << r$, so a^2 may be neglected as compared to r^2 and so equation (*iii*) gives

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$

i.e., electric field strength due to a short dipole at broadside on position

$$E = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3} \text{ in the direction parallel to } \overrightarrow{BA} \quad \text{...(iv)}$$

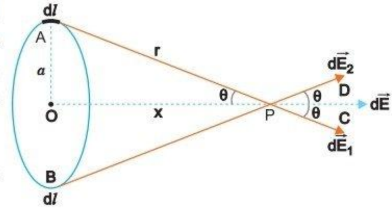
Its direction is parallel to the axis of dipole from positive to negative charge.

It may be noted clearly from equations (ii) and (iv) that electric field strength due to a short dipole at any point is inversely proportional to the cube of its distance from the dipole and the electric field strength at axial position is twice that at broad-side on position for the same distance.

Important: Note the important point that the electric field due to a dipole at large distances falls off as $\frac{1}{r^3}$ and not as $\frac{1}{r^2}$ as in the case of a point charge.

Q. 4. A charge is distributed uniformly over a ring of radius '*a*'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge. [*CBSE Delhi 2016, 2020 (55/5/1)*]

Ans. Consider a point P on the axis of uniformly charged ring at a distance x from its centre O. Point P is at distance $r = \sqrt{a^2 + x^2}$ from each element dl of ring. If q is total charge on ring, then, charge per metre length, $\lambda = \dfrac{q}{2\pi a}$.



The ring may be supposed to be formed of a large number of ring elements.

Consider an element of length dl situated at A.

The charge on element, $dq = \lambda\, dl$

$\therefore$ The electric field at P due to this element

$$d\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}, \text{ along } \overrightarrow{PC}$$

The electric field strength due to opposite symmetrical element of length dl at B is

$$d\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}, \text{ along } \overrightarrow{PD}$$

If we resolve $d\vec{E}_1$ and $d\vec{E}_2$ along the axis and perpendicular to axis, we note that the components perpendicular to axis are oppositely directed and so get cancelled, while those along the axis are added up. Hence, due to symmetry of the ring, the electric field strength is directed along the axis. The electric field strength due to charge element of length dl, situated at A, along the axis will be

$$dE = dE_1 \cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\cos\theta$$

But, $\cos\theta = \dfrac{x}{r}$

$$\therefore \quad dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl x}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{r^3}dl$$

The resultant electric field along the axis will be obtained by adding fields due to all elements of the ring, *i.e.*,

$$\therefore \quad E = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{r^3}dl = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{r^3}\int dl$$

But, $\int dl$ = whole length of ring $= 2\pi a$ and $r = (a^2 + x^2)^{1/2}$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{(a^2 + x^2)^{3/2}}2\pi a$$

As, $\lambda = \dfrac{q}{2\pi a}$, we have $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\left(\dfrac{q}{2\pi a}\right)x}{(a^2 + x^2)^{3/2}}2\pi a$

or, $$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2 + x^2)^{3/2}}, \text{ along the axis}$$

At large distances *i.e.*, $x >> a$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x^2}$,

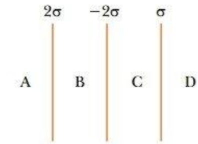
i.e., the electric field due to a point charge at a distance x.

For points on the axis at distances much larger than the radius of ring, the ring behaves like a point charge.

Q. 5. **(a) State Gauss's law in electrostatics. Show with help of suitable figure that outward flux due to a point charge Q, in vacuum within gaussian surface, is independent of its size and shape.**

(b) In the figure there are three infinite long thin sheets having surface charge density $+2\sigma$, -2σ and $+\sigma$ respectively. Give the magnitude and direction of electric field at a point to the left of sheet of charge density $+2\sigma$ and to the right of sheet of charge density $+\sigma$.

[CBSE Sample Paper 2021]



Ans. (a) **Statement:** ***The net-outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times the total charge contained within that surface, i.e.,***

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\Sigma q$$

where $\oint_S$ indicates the surface integral over the whole of the closed surface, $\sum q$ is the algebraic sum of all the charges (*i.e.*, net charge in coulombs) enclosed by surface S and remain unchanged with the size and shape of the surface.

Proof: Let a point charge $+q$ be placed at centre O of a sphere S. Then S is a Gaussian surface.

Electric field at any point on S is given by

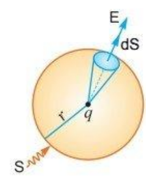
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

The electric field and area element points radially outwards, so $\theta = 0°$.

Flux through area $d\vec{S}$ is

$$d\phi = \vec{E}\,.\,d\vec{S} = E\,dS\cos 0° = E\,dS$$

Total flux through surface S is



$$\phi = \oint_S d\phi = \oint_S EdS = E\oint_S dS = E \times \text{Area of sphere}$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}4\pi r^2 \quad \text{or,} \quad \boldsymbol{\phi = \frac{q}{\varepsilon_0}}$$ which proves Gauss's theorem.

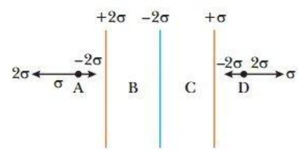
(b) At A, both σ and 2σ will act in left and -2σ will act in right, so, charge density at A can be given as

$$E_A = \frac{2\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boldsymbol{\frac{\sigma}{2\varepsilon_0}}$$

The net electric field at A is towards left.

Similarly at point D, σ and 2σ will act in right and -2σ will act in left.

So, $$E_D = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} = \boldsymbol{\frac{\sigma}{2\varepsilon_0}}$$



The net electric field at D is towards right.

Q. 6. **(i) Using Gauss Theorem show mathematically that for any point outside the shell, the field due to a uniformly charged spherical shell is same as the entire charge on the shell, is concentrated at the centre.** *[CBSE 2019 (55/4/1)]*

(ii) Why do you expect the electric field inside the shell to be zero according to this theorem?

OR *[CBSE Allahabad 2015]*

A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's theorem, derive an expression for the electric field at a point outside the shell. *[CBSE Delhi 2009]*

Draw a graph of electric field $E(r)$ with distance r from the centre of the shell for $0 \le r < \infty$.

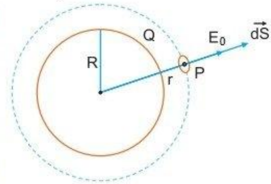
OR

Find the electric field intensity due to a uniformly charged spherical shell at a point (*i*) outside the shell and (*ii*) inside the shell. Plot the graph of electric field with distance from the centre of the shell. **[*CBSE North 2016; 2020 (55/1/1)*]**

OR

Using Gauss's law obtain the expression for the electric field due to a uniformly charged thin spherical shell of radius *R* at a point outside the shell. Draw a graph showing the variation of electric field with *r*, for $r > R$ and $r < R$. **[*CBSE (AI) 2013; 2020 (55/2/1)*]**

Ans. **(*i*) Electric field intensity at a point outside a uniformly charged thin spherical shell:** Consider a uniformly charged thin spherical shell of radius R carrying charge Q. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius r ($>R$), concentric with given shell. If $\vec{E}_0$ is electric field outside the shell, then by symmetry electric field strength has same magnitude E_0 on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_0$ and $d\vec{S}$ is zero at each point. Hence, electric flux through Gaussian surface. $\phi = \oint_S \vec{E}_0 \cdot d\vec{S}$.



$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E_0 dS \cos 0 = E_0 . 4\pi r^2$$

Now, Gaussian surface is outside the given charged shell, so charge enclosed by Gaussian surface is Q.

Hence, by Gauss's theorem

$$\oint_S \vec{E}_0 \cdot d\vec{S} = \frac{1}{\varepsilon_0} \times \text{ charged enclosed}$$

$$\Rightarrow \qquad E_0\, 4\pi r^2 = \frac{1}{\varepsilon_0} \times Q \Rightarrow E_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Thus, ***electric field outside a charged thin spherical shell is the same as if the whole charge Q is concentrated at the centre.***

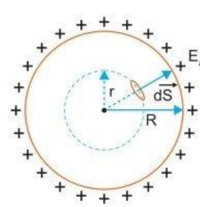
If σ is the surface charge density of the spherical shell, then

$$Q = 4\pi R^2 \sigma \text{ coulomb}$$

$$\therefore \qquad E_0 = \frac{1}{4\pi\varepsilon_0} \frac{4\pi R^2 \sigma}{r^2} = \frac{R^2 \sigma}{\varepsilon_0 r^2}$$

(*ii*) Electric field inside the shell (hollow charged conducting sphere):

The charge resides on the surface of a conductor. Thus a hollow charged conductor is equivalent to a charged spherical shell. To find the electric field inside the shell, we consider a spherical Gaussian surface of radius r ($< R$) concentric with the given shell. If $\vec{E}$ is the electric field inside the shell, then by symmetry electric field strength has the same magnitude E_i on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E}_i$ and $d\vec{S}$ is zero at each point. Hence, electric flux through Gaussian surface



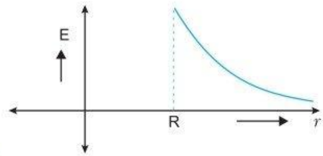
$$= \oint_S \vec{E}_i . d\vec{S} = \oint E_i\, dS \cos 0 = E_i . 4\pi r^2$$

Now, Gaussian surface is inside the given charged shell, so charge enclosed by Gaussian surface is zero.

Hence, by Gauss's theorem

$$\oint_S \vec{E}_i \cdot d\vec{S} = \frac{1}{\varepsilon_0} \times \text{charge enclosed}$$

$$\Rightarrow \quad E_i\, 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 \;\Rightarrow\; E_i = 0$$



Thus, ***electric field at each point inside a charged thin spherical shell is zero. The graph is shown in fig.***

Q. 7. **(*i*) Use Gauss' law to obtain an expression for the electric field due to an infinitely long thin straight wire with uniform linear charge density λ.** ***[CBSE 2020 (55/5/1), 2023 (55/2/1)]***

(*ii*) An infinitely long positively charged straight wire has a linear charge density λ. An electron is revolving in a circle with a constant speed *v* such that the wire passes through the centre, and is perpendicular to the plane, of the circle. Find the kinetic energy of the electron in terms of magnitudes of its charge and linear charge density λ on the wire.

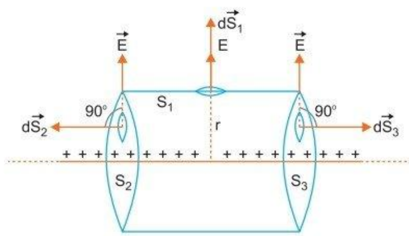
(*iii*) Draw a graph of kinetic energy as a function of linear charge density λ. ***[CBSE 2023 (55/2/1)]***

Ans. (*i*) **Electric field due to infinitely long, thin and uniformly charged straight wire:** Consider an infinitely long line charge having linear charge density λ coulomb metre^{-1} (linear charge density means charge per unit length). To find the electric field strength at a distance *r*, we consider a cylindrical Gaussian surface of radius *r* and length *l* coaxial with line charge. The cylindrical Gaussian surface may be divided into three parts:

(*i*) Curved surface S_1 (*ii*) Flat surface S_2 and (*iii*) Flat surface S_3.

By symmetry, the electric field has the same magnitude *E* at each point of curved surface S_1 and is directed radially outward.

We consider small elements of surfaces S_1, S_2 and S_3 The surface element vector $d\vec{S}_1$ is directed along the direction of electric field (*i.e.*, angle between $\vec{E}$ and $d\vec{S}_1$ is zero); the elements $d\vec{S}_2$ and $d\vec{S}_3$ are directed perpendicular to field vector $\vec{E}$ (*i.e.*, angle between $d\vec{S}_2$ and $\vec{E}$ is 90° and so also angle between $d\vec{S}_3$ and $\vec{E}$).



Electric Flux through the cylindrical surface

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E} \cdot d\vec{S}_1 + \int_{S_2} \vec{E} \cdot d\vec{S}_2 + \int_{S_3} \vec{E} \cdot d\vec{S}_3$$

$$= \int_{S_1} E\, dS_1 \cos 0° + \int_{S_2} E\, dS_2 \cos 90° + \int_{S_3} E\, dS_3 \cos 90°$$

$$= \int E\, dS_1 + 0 + 0$$

$$= E \int dS_1 \qquad \text{(since electric field } E \text{ is the same at each point of curved surface)}$$

$$= E\, 2\pi r l \qquad \text{(since area of curved surface} = 2\,\pi\, r l)$$

As λ is charge per unit length and length of cylinder is *l* therefore, charge enclosed by assumed surface = (λ*l*)

∴ By Gauss's theorem

$$\oint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \times \text{charge enclosed}$$

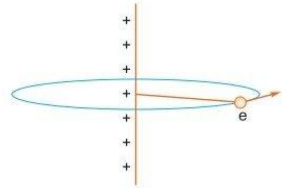
$$\Rightarrow \quad E\, 2\pi r l = \frac{1}{\varepsilon_0}(\lambda l) \quad \Rightarrow \quad \boldsymbol{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

Thus, the electric field strength due to a line charge is inversely proportional to *r*.

(ii) Infinitely long charged wire produces a radical electric field.

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad ...(i)$$

The revolving electron experiences an electrostatic force and provides necessarily centripetal force.

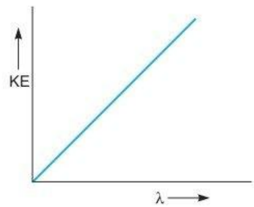


$$eE = \frac{mv^2}{r} \qquad ...(ii)$$

$$\Rightarrow \qquad \frac{e\lambda}{2\pi\varepsilon_0 r} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{e\lambda}{2\pi\varepsilon_0}$$

Kinetic energy of the electron, $K = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\varepsilon_0}$

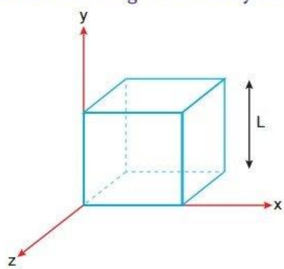
(iii)



Q. 8. (i) Define electric flux and write its SI unit.

(ii) Use Gauss law to obtain the expression for the electric field due to a uniformly charged infinite plane sheet.

(iii) A cube of side L is kept in space, as shown in the figure. An electric field $\vec{E} = (Ax+B)\hat{i}$ NC^{-1} exists in the region. Find the net charge enclosed by the cube. *[CBSE 2023 (55/3/1)]*

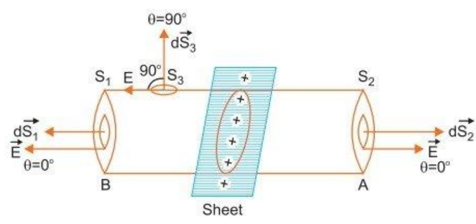


Ans. (i) **Electric flux:** It is defined as the total number of electric field lines passing through an area normal to its surface.

Also, $\phi = \oint \vec{E} \cdot d\vec{S}$

The SI unit is Nm^2/C or volt-metre.

(ii) Let electric charge be uniformly distributed over the surface of a thin, non-conducting infinite sheet. Let the surface charge density (*i.e.*, charge per unit surface area) be σ. We need to calculate the electric field strength at any point distant r from the sheet of charge.



To calculate the electric field strength near the sheet, we now consider a cylindrical Gaussian surface bounded by two plane faces A and B lying on the opposite sides and parallel to the charged sheet and the cylindrical surface perpendicular to the sheet (fig). By symmetry the electric field strength at every point on the flat surface is the same and its direction is normal outwards at the points on the two plane surfaces and parallel to the curved surface.

Total electric flux

or $$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E} \cdot d\vec{S}_1 + \int_{S_2} \vec{E} \cdot d\vec{S}_2 + \int_{S_3} \vec{E} \cdot d\vec{S}_3$$

$$\oint_S \vec{E} \cdot d\vec{S} = \int_{S_1} E\, dS_1 \cos 0° + \int_{S_2} E\, dS_2 \cos 0° + \int_{S_3} E\, dS_3 \cos 90°$$

$$= E\int dS_1 + E\int dS_2 = Ea + Ea = 2Ea$$

$\therefore$ Total electric flux $= 2Ea$

As σ is charge per unit area of sheet and a is the intersecting area, the charge enclosed by Gaussian surface $= \sigma a$

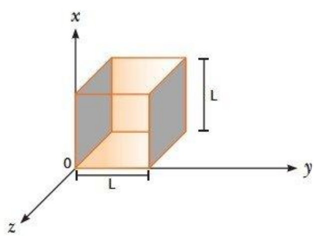
According to Gauss's theorem,

Total electric flux $= \frac{1}{\varepsilon_0} \times$ (total charge enclosed by the surface)

i.e., $$2Ea = \frac{1}{\varepsilon_0}(\sigma a) \qquad \therefore \quad E = \frac{\sigma}{2\varepsilon_0}$$

Thus electric field strength due to an infinite flat sheet of charge is independent of the distance of the point.

(*iii*) Given, $\vec{E} = (Ax + B)\hat{i}\ \text{NC}^{-1}$



From Gauss's Law, $\oint E.ds = \frac{q_{in}}{\varepsilon_0}$

Only flux through shaded portion will contribute.

Flux through face A, $\phi_A = \vec{E}_A . \vec{ds}$ where, at $x = 0$,) $\vec{E}_A = \big(A(0) + B\big)\hat{i}$

$$= B\hat{i} . L^2(-\hat{i}) \quad \phi_A = -BL^2$$

Flux through face B, $\phi_B = \vec{E} . \vec{ds}$ where, at $x = L$, $\vec{E}_B = (AL + B)\hat{i}$

$$= (AL + B)\hat{i} . L^2\,\hat{i} = (AL^3 + BL^2)$$

Hence, Net flux $= \phi_A + \phi_B = -BL^2 + AL^3 + BL^2$

$$= AL^3$$

Again, from Gauss theorem, $\phi_{net} = \frac{q_{in}}{\varepsilon_0}$

$$\therefore \qquad q_{in} = AL^3\,\varepsilon_0\,\mathbf{C}$$

Q. 9. Apply Gauss's Theorem to find the electric field near a charged conductor.

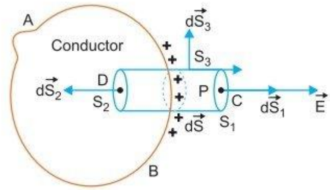
OR

Show that the electric field at the surface of a charged conductor is $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ where σ is surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

[CBSE (AI) 2010]

Ans. Let a charge Q be given to a conductor, this charge under electrostatic equilibrium will redistribute and the electric field inside the conductor is zero (*i.e.*, $E_{in}=0$).

Let us consider a point P at which electric field strength is to be calculated, just outside the surface of the conductor. Let the surface charge density on the surface of the conductor in the neighbourhood of P be σ coulomb/metre^2. Now consider a small cylindrical box CD having one base C passing through P; the other base D lying inside the conductor and the curved surface being perpendicular to the surface of the conductor.



Let the area of each flat base be a. As the surface of the conductor is equipotential surface, the electric field strength **E** at P, just outside the surface of the conductor is perpendicular to the surface of the conductor in the neighbourhood of P.

The flux of electric field through the curved surface of the box is zero, since there is no component of electric field E normal to curved surface. Also the flux of electric field through the base D is zero, as electric field strength inside the conductor is zero. Therefore the resultant flux of electric field through the entire surface of the box is same as the flux through the face C. This may be analytically seen as:

If S_1 and S_2 are flat surfaces at C and D and S_3 is curved surface, then

$$\text{Total electric flux } \oint_S \vec{E}\cdot d\vec{S} = \int_{S_1} \vec{E}\cdot d\vec{S}_1 + \int_{S_2} \vec{E}\cdot d\vec{S}_2 + \int_{S_3} \vec{E}\cdot d\vec{S}_3$$

$$= \int_{S_1} E\, dS_1 \cos 0 + \int_{S_2} \vec{0}\cdot d\vec{S}_2 + \int_{S_3} E\, dS_3 \cos 90^\circ$$

$$\therefore \qquad \oint_S E\, dS_1 = Ea$$

As the charge enclosed by the cylinder is (σa) coulomb, we have, using Gauss's theorem,

$$\text{Total electric flux} = \frac{1}{\varepsilon_0} \times \text{charge enclosed}$$

$$\Rightarrow \qquad Ea = \frac{1}{\varepsilon_0}(\sigma a) \qquad \text{or} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Thus the electric field strength at any point close to the surface of a charged conductor of any shape is equal to $1/\varepsilon_0$ times the surface charge density σ. This is known as Coulomb's law. The electric field strength is directed radially away from the conductor if σ is positive and towards the conductor if σ is negative.

If $\hat{n}$ is unit vector normal to surface in outward direction, then $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$.

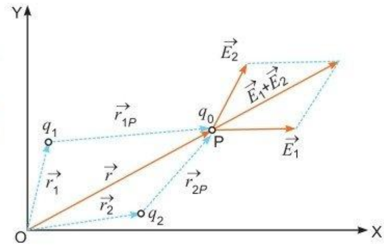
Obviously electric field strength near a plane conductor is twice of the electric field strength near a non-conducting thin sheet of charge.

Q. 10. *(a)* **Consider a system of n charges $q_1, q_2, \ldots q_n$ with position vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots, \vec{r_n}$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r_p}$, due to this system of charges.**

(b) **Three point electric charges $+q$ each are kept at the vertices of an equilateral triangle of side a. Determine the magnitude and sign of the charge to be kept at the centroid of the triangle so that the charges at the vertices remain in equilibrium.** *[CBSE (F) 2015] [HOTS]*

Ans. (*a*) **Electric field due to a system of point charges.**

Consider a system of N point charges q_1, q_2, ..., q_n, having position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ with respect to origin O. We wish to determine the electric field at point P whose position vector is $\vec{r}$. According to Coulomb's law, the force on charge q_0 due to charge q_1 is



$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r_{2p}^2}\hat{r}_{1P}$$

where $\hat{r}_{1P}$ is a unit vector in the direction from q_1 to P and r_{1P} is the distance between q_1 and P. Hence the electric field at point P due to charge q_1 is

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Similarly, electric field at P due to charge q_2,

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$

According to the principle of superposition of electric fields, the electric field at any point due to a group of point charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.

Hence, the electric field at point P due to the system of n charges is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

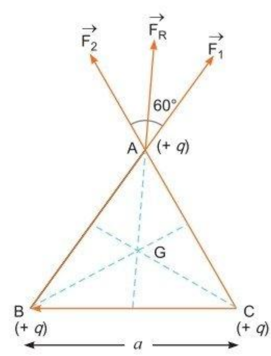
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \ldots + \frac{q_n}{r_{nP}^2}\hat{r}_{nP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

(*b*) The charge at any vertex will remain in equilibrium if the net force experienced by this charge due to all other three charges is zero.

Let Q be the required charge to be kept at the centroid G.

Considering the charge at A,



Force $\vec{F}_1$ on charge at A due to charge at B

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{BA}$$

Force $\vec{F}_2$ on charge at A due to charge at C

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{CA}$$

Since angle between $\vec{F}_1$ and $\vec{F}_2$ is 60°.

$$\vec{F}_1 + \vec{F}_2 = \sqrt{3}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \text{ along } \overrightarrow{GA}$$

Also, the distance of centroid G from any vertex is $\frac{a}{\sqrt{3}}$.

The nature of charge to be kept at G has to be opposite (–ve) so that it exerts a force of attraction on charge $(+q)$ kept at A to balance the force $\vec{F}_1 + \vec{F}_2$.

Force exerted by $(-Q)$ kept at G on charge $(+q)$ at $A = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q.3q}{a^2}$ along $\overrightarrow{AG}$

Equating the two forces, being equal and opposite

$$\sqrt{3}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} = -\frac{1}{4\pi\varepsilon_0}\frac{3Qq}{a^2} \quad \Rightarrow \quad Q = -\frac{q}{\sqrt{3}}$$

Questions for Practice

1. Choose and write the correct option in the following questions.

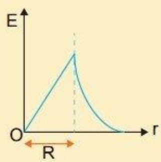
(*i*) Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The point on x axis at which net electric field is zero due to these charges is

[CBSE Sample Paper-2022, Term-1)]

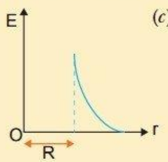
(*a*) $8L$ (*b*) $4L$ (*c*) $2L$ (*d*) L

(*ii*) Which one of the following plots represents the variation of electric field with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell)

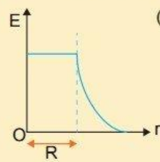
(*a*)



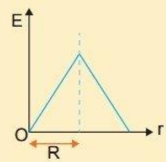
(*b*)



(*c*)



(*d*)



(*iii*) An object has charge of 1 C and gains 5.0×10^{18} electrons. The net charge on the object becomes *[CBSE 2022 (55/2/4), Term-1]*

(*a*) – 0.80 C (*b*) + 0.80 C (*c*) + 1.80 C (*d*) + 0.20 C

(*iv*) The electric flux emerging out from 1C charge is *[CBSE 2020 (55/3/1)]*

(*a*) $\frac{1}{\varepsilon_0}$ (*b*) 4π (*c*) $\frac{4\pi}{\varepsilon_0}$ (*d*) ε_0

(*v*) In which of the following cases the electric field strength is independent of distance?

[CBSE 2020 (55/3/1)]

(*a*) Due to a point charge (*b*) Due to a line charge

(*c*) Due to a spherical charge (*d*) Due to infinite flat sheet of charge

2. ***In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.***

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false and R is also false.***

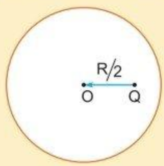
(*i*) **Assertion (*A*) :** If a proton and an electron a replaced in the same uniform electric field, they experience different acceleration.

Reason (*R*) : Electric force on a test charge is independent of its mass.

(*ii*) **Assertion** (*A*) **:** The charge given to a metallic sphere does not depend on whether it is hollow or solid.

Reason (*R*) **:** Since the charge resides only on the surface of the conductor.

3. Two insulated charged copper spheres *A* and *B* of identical size have charges q_A and $-3q_A$ respectively. When they are brought in contact with each other and then separated, what are the new charges on them?
4. Two charges of magnitudes $-3Q$ and $+2Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '$5a$' with its centre at the origin?
5. A charge Q μC is placed at the centre of a cube. What is the electric flux coming out from any one surface? [*CBSE (AI) 2012*]
6. Two identical point charges, q each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.
7. Draw the pattern of electric field lines, when a point charge $-Q$ is kept near an uncharged conducting plate. [*CBSE 2019 (55/1/1)*]
8. A charge q is placed at the centre of a cube of side l. What is the electric flux passing through two opposite faces of the cube? [*CBSE (AI) 2012*]
9. Figure shows a point charge $+Q$, located at a distance $\frac{R}{2}$ from the centre of a spherical metal shell. Draw the electric field lines for the given system. [*CBSE Sample Paper 2016*]



10. An electric dipole is held in a uniform electric field.
    (*i*) Show that the net force acting on it is zero.
    (*ii*) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of 180°. [*CBSE (AI) 2012*]
11. Two concentric metallic spherical shells of radii R and $2R$ are given charges Q_1 and Q_2 respectively. The surface charge densities on the outer surfaces of the shells are equal. Determine the ratio $Q_1 : Q_2$. [*CBSE (F) 2013*]
12. Consider a uniform electric field $\vec{E} = 3\times10^3\hat{i}\ \text{NC}^{-1}$. (*a*) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the *yz* plane? (*b*) What is the flux through the same square if the normal to its plane makes a 60° angle with the *x*-axis? [*NCERT*]
13. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\ \text{C/m}^2$ What is electric field strength *E*: (*a*) in the outer region of the first plate, (*b*) in the outer region of the second plate, and (*c*) between the plates? [*NCERT*]
14. A long charged cylinder of linear charge density $+\lambda_1$ is surrounded by a hollow coaxial conducting cylinder of linear charge density $-\lambda_2$. Use Gauss's law to obtain expressions for the electric field at a point (*i*) in the space between the cylinders, and (*ii*) outside the larger cylinder.
15. Four point charges $q_A = 2\ \mu\text{C}$, $q_B = -5\mu\text{C}$, $q_C = 2\ \mu\text{C}$ and $q_D = -5\ \mu\text{C}$ are located at the corners of a square *ABCD* of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the sphere? [*NCERT*]

16. Two large charged plane sheets of charge densities σ and -2σ C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (*i*) to the left of the first sheet, (*ii*) to the right of the second sheet, and (*iii*) between the two sheets.

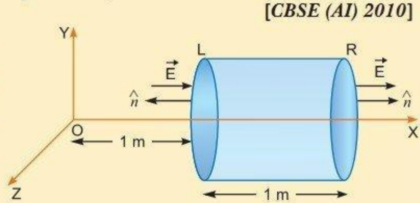
17. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge 'Q'. A charge 'q' is placed at the centre of the shell.

    (*a*) What is the surface charge density on the (*i*) inner surface, (*ii*) outer surface of the shell?

    (*b*) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell.

    [*CBSE (AI) 2010*]

18. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50\,x\,\hat{i}$, where E is in NC^{-1} and x is in metres. Find



    (*i*) net flux through the cylinder.

    (*ii*) charge enclosed by the cylinder. **[*CBSE Delhi 2013*]**

19. Two parallel uniformly charged infinite plane sheets, '1' and '2', have charge densities $+\sigma$ and -2σ respectively. Give the magnitude and direction of the net electric field at a point

    (*i*) in between the two sheets and (*ii*) outside near the sheet '1'. **[*CBSE Ajmer 2015*]**

20. The electric field E due to any point charge near it is defined as $E = \lim_{q \to 0} \frac{F}{q}$ where q is the test charge and F is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression? Draw the electric field lines of point charge Q when (*i*) $Q > 0$ and (*ii*) $Q < 0$.

21. Two point charges of $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.

22. Two charges of value 2 μC and –50 μC are placed 80 cm apart. Calculate the distance of the point from the smaller charge where the intensity is zero.

Answers

1. (*i*) (*c*) (*ii*) (*b*) (*iii*) (*d*) (*iv*) (*a*) (*v*) (*d*)

2. (*i*) (*b*) (*ii*) (*a*)

3. $-q_A, -q_A$ **4.** $\frac{-Q}{\varepsilon_0}$ **5.** $\frac{Q}{6\varepsilon_0}$ μVm **6.** 1 m **8.** $\frac{q}{3\varepsilon_0}$ Vm

10. (*ii*) $2pE$ **11.** 1 : 4 **12.** (*a*) 30 Nm^{2}C^{-1} (*b*) 15 Nm^{2}C^{-1} **13.** (*a*) 0 (*b*) 0 (*c*) 1.92×10^{-10} NC^{-1}