Series **WX1YZ/1**

SET~3

रोल नं.
Roll No.

प्रश्न-पत्र कोड
Q.P. Code **430/1/3**

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित (बुनियादी)
MATHEMATICS (BASIC)

निर्धारित समय: **3 घण्टे**
*Time allowed : **3 hours***

अधिकतम अंक : **80**
*Maximum Marks : **80***

नोट / NOTE :

(i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।

*Please check that this question paper contains **23** printed pages.*

(ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

*Please check that this question paper contains **38** questions.*

(iv) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**

Please write down the serial number of the question in the answer-book before attempting it.

(v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को सावधानी से पढ़िए और उनका पालन कीजिए :

(i)   इस प्रश्न-पत्र में कुल *38* प्रश्न हैं । **सभी** प्रश्न अनिवार्य हैं ।

(ii)   प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **खण्ड-क, ख, ग, घ** तथा **ङ** ।

(iii)   खण्ड – **क** में प्रश्न संख्या *1 से 18* तक बहुविकल्पीय तथा प्रश्न संख्या *19* एवं *20* अभिकथन एवं तर्क आधारित **एक-एक** अंक के प्रश्न हैं ।

(iv)   खण्ड – **ख** में प्रश्न संख्या *21 से 25* तक लघु उत्तरीय-I (*SA-I*) प्रकार के **दो-दो** अंकों के प्रश्न हैं ।

(v)   खण्ड – **ग** में प्रश्न संख्या *26 से 31* तक लघु उत्तरीय-II (*SA-II*) प्रकार के **तीन-तीन** अंकों के प्रश्न हैं ।

(vi)   खण्ड – **घ** में प्रश्न संख्या *32 से 35* तक दीर्घ उत्तरीय प्रकार के **पाँच-पाँच** अंकों के प्रश्न हैं ।

(vii)   खण्ड – **ङ** में प्रश्न संख्या *36 से 38* स्रोत/प्रकरण इकाई आधारित **चार-चार** अंकों के प्रश्न हैं । आंतरिक विकल्प **दो-दो** अंकों के प्रश्न में दिया गया है ।

(viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड – **ख** के *2* प्रश्नों में, खण्ड – **ग** के *2* प्रश्नों में, खण्ड – **घ** के *2* प्रश्नों में तथा खण्ड-**ङ** के *3* प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।

(ix)   जहाँ आवश्यक हो स्वच्छ आकृतियाँ बनाएँ । यदि आवश्यक हो तो $\pi = \dfrac{22}{7}$ लें ।

(x)   कैल्कुलेटर का उपयोग वर्जित है ।

General Instructions :

**Read the following instructions carefully and follow them :**

(i)   *This question paper contains **38** questions. **All** questions are compulsory.*

(ii)   *Question Paper is divided into **5** Sections – **Section A, B, C, D** and **E**.*

(iii)   *In Section–**A** question number **1** to **18** are Multiple Choice Questions (MCQs) and question number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv)   *In Section–**B** question number **21** to **25** are Short Answer-I (SA-I) type questions of **2** marks each.*

(v)   *In Section–**C** question number **26** to **31** are Short Answer-II (SA-II) type questions carrying **3** marks each.*

(vi)   *In Section–**D** question number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii)   *In Section–**E** question number **36** to **38** are Case Based integrated units of Assessment questions carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section B, **2** questions in Section C, **2** questions in Section D and **3** questions in Section E.*

(ix)   *Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.*

(x)   *Use of Calculator is NOT allowed.*

खण्ड – क
(बहुविकल्पीय प्रश्न)

खण्ड – क में **20** प्रश्न हैं और प्रत्येक प्रश्न **1** अंक का है ।

1. यदि $\sqrt{3} \tan\theta = 1$ है, तो θ का मान है :   1
    (a)   30°   (b)   45°
    (c)   60°   (d)   90°

2. संख्या 1728 का अभाज्य गुणनखंड है :   1
    (a) $2^5 \times 3^3$ (b) $2^5 \times 3^4$
    (c) $2^6 \times 3^3$ (d) $2^6 \times 3^2$

3. यदि एक A.P. में, $d = -4$, $n = 7$ और $a_n = 4$ हों, तो a का मान होगा :   1
    (a)   6   (b)   7
    (c)   20   (d)   28

4. दी गई आकृति में, समरूपता की कसौटी, जिससे $\Delta ABC \sim \Delta PQR$ है, है :   1
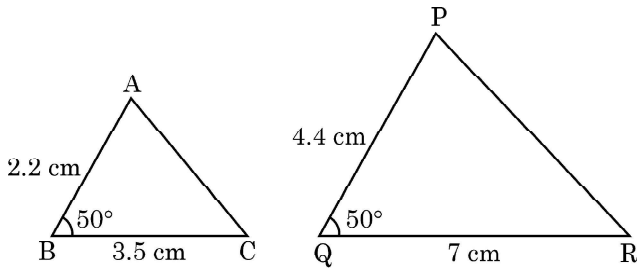
    (a)   SSA (भुजा – भुजा – कोण) समरूपता

    (b)   ASA (कोण – भुजा – कोण) समरूपता

    (c)   SAS (भुजा – कोण – भुजा) समरूपता

    (d)   AA (कोण – कोण) समरूपता

5. त्रिज्या 'r' और ऊँचाई '3r' के एक शंकु का आयतन होगा :   1

    (a) $\frac{1}{3} \pi r^3$ (b) $3 \pi r^3$

    (c) $9 \pi r^3$ (d) πr^3

Section – A

(Multiple Choice Questions)

Section – A consists of 20 questions of **1** mark each.

1. If $\sqrt{3}\tan\theta = 1$, then the value of θ is   **1**

   (a) 30°   (b) 45°

   (c) 60°   (d) 90°

2. The prime factorisation of 1728 is   **1**

   (a) $2^5 \times 3^3$ (b) $2^5 \times 3^4$

   (c) $2^6 \times 3^3$ (d) $2^6 \times 3^2$

3. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then the value of a is   **1**

   (a) 6   (b) 7

   (c) 20   (d) 28

4.   **1**

   In the above figure, the criterion of similarity by which $\triangle ABC \sim \triangle PQR$ is :
   (a) SSA (Side – Side – Angle) Similarity
   (b) ASA (Angle – Side – Angle) Similarity
   (c) SAS (Side – Angle – Side) Similarity
   (d) AA (Angle – Angle) Similarity

5. The volume of a cone of radius 'r' and height '3r' is :   **1**

   (a) $\dfrac{1}{3}\pi r^3$ (b) $3\pi r^3$

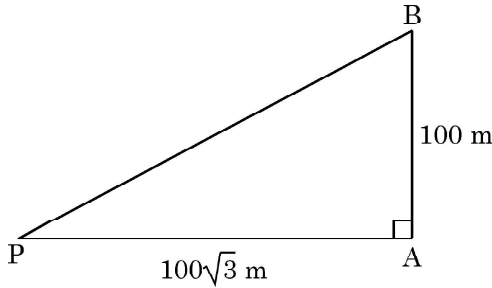   (c) $9\pi r^3$ (d) πr^3

6. दी गई आकृति में, BC और BD केंद्र O वाले वृत्त पर दो स्पर्श-रेखाएँ हैं । वृत्त की त्रिज्या 9 cm है । यदि OB = 15 cm है तो (BC + BD) की लम्बाई है : **1**
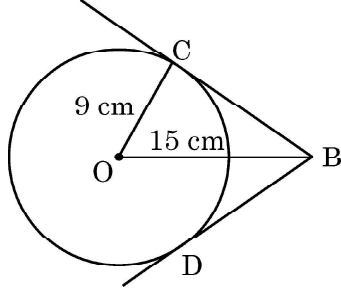
(a) 18 cm (b) 12 cm
(c) 24 cm (d) 36 cm

7. बिंदुओं (3, 0) और (0, –3) के बीच की दूरी है : **1**

(a) $2\sqrt{3}$ इकाई (b) 6 इकाई
(c) 3 इकाई (d) $3\sqrt{2}$ इकाई

8. 100 m ऊँचा एक ऊर्ध्वाधर खंभा, भूमि के एक बिंदु से जो खंभे के पाद से $100\sqrt{3}$ m की दूरी पर है, जो कोण बनाता है उसकी माप है **1**
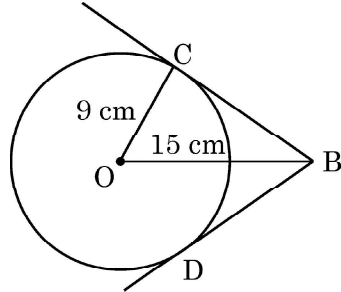

(a) 90° (b) 60°
(c) 45° (d) 30°

9. दो संपूरक कोणों में बड़ा कोण छोटे कोण से 18 डिग्री अधिक है । बड़े कोण की माप है : **1**

(a) 81° (b) 99°
(c) 36° (d) 54°

10. द्विघात समीकरण $2x^2 - 5x - 3 = 0$ का विविक्तकर है : **1**

(a) 1 (b) 49
(c) 7 (d) 19

6. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is : 1

(a) 18 cm (b) 12 cm
(c) 24 cm (d) 36 cm

7. The distance between the points (3, 0) and (0, –3) is 1
(a) $2\sqrt{3}$ units (b) 6 units
(c) 3 units (d) $3\sqrt{2}$ units

8. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of 1
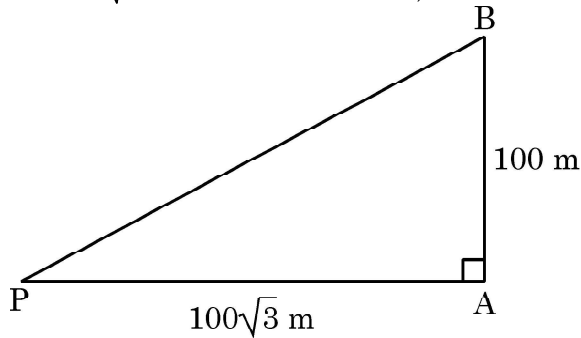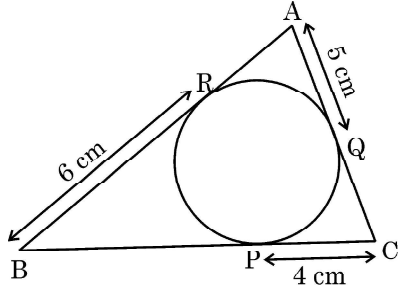

(a) 90° (b) 60°
(c) 45° (d) 30°

9. The larger of two supplementary angles exceeds the smaller by 18 degrees. What is the measure of larger angle ? 1
(a) 81° (b) 99°
(c) 36° (d) 54°

10. The discriminant of the quadratic equation $2x^2 - 5x - 3 = 0$ is 1
(a) 1 (b) 49
(c) 7 (d) 19

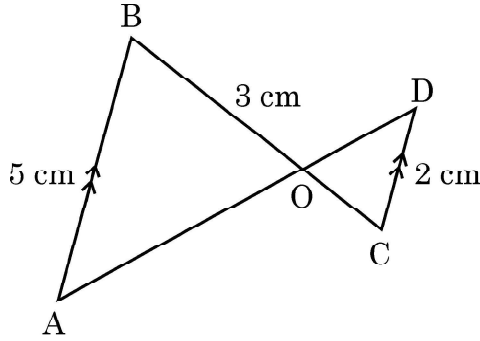11. दी गई आकृति में, त्रिभुज ABC का परिमाप है : **1**

(a)   30 cm   (b)   15 cm

(c)   45 cm   (d)   60 cm

12. दी गई आकृति में, AB || CD । यदि AB = 5 cm, CD = 2 cm और OB = 3 cm हो, तो OC की लम्बाई होगी : **1**
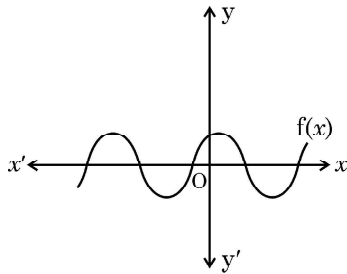

(a) $\dfrac{15}{2}$ cm   (b) $\dfrac{10}{3}$ cm

(c) $\dfrac{6}{5}$ cm   (d) $\dfrac{3}{5}$ cm

13. आकृति में, किसी बहुपद f(x) के लिए y = f(x) का ग्राफ दिखाया गया है । f(x) के शून्यकों की संख्या है : **1**
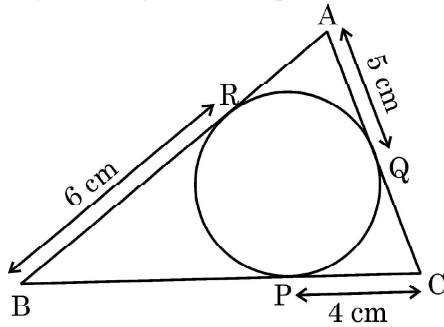

(a)   5   (b)   6

(c)   4   (d)   8

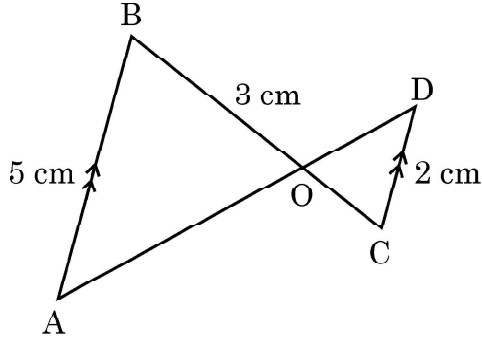11. In the given figure, the perimeter of ∆ABC is : 1

(a) 30 cm   (b) 15 cm
(c) 45 cm   (d) 60 cm

12. In the given figure, AB || CD. If AB = 5 cm, CD = 2 cm and OB = 3 cm, then the length of OC is 1
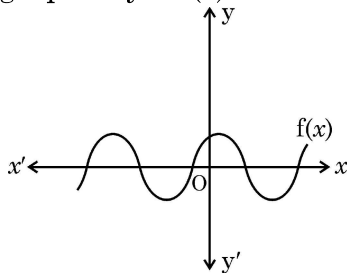

(a) $\frac{15}{2}$ cm   (b) $\frac{10}{3}$ cm

(c) $\frac{6}{5}$ cm   (d) $\frac{3}{5}$ cm

13. The graph of y = f(x) is shown in the figure for some polynomial f(x). 1

The number of zeroes of f(x) is
(a) 5   (b) 6
(c) 4   (d) 8

14. बहुपद $p(x) = 3x^2 - 5x + 2$ के शून्यकों का योग और गुणनफल क्रमशः हैं :   **1**

   (a) $\dfrac{5}{3}, \dfrac{2}{3}$ (b) $\dfrac{-5}{3}, \dfrac{2}{3}$

   (c) $1, \dfrac{2}{3}$ (d) $\dfrac{-5}{3}, \dfrac{-2}{3}$

15. एक पासा एक बार फेंका जाता है । विषम अभाज्य संख्या प्राप्त होने की प्रायिकता है :   **1**

   (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{6}$

   (c) $\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

16. दो समरूप त्रिभुजों की भुजाएँ 4 : 7 के अनुपात में हैं, तो उनके परिमापों का अनुपात होगा   **1**

   (a) 4 : 7   (b) 12 : 21

   (c) 16 : 49   (d) 7 : 4

17. 7 cm व्यास के वृत्त पर दो समांतर स्पर्श-रेखाओं के बीच की दूरी है :   **1**

   (a) 7 cm   (b) 14 cm

   (c) $\dfrac{7}{2}$ cm   (d) 28 cm

18. 52 ताश के पत्तों की एक अच्छी प्रकार से फेंटी गई गड्डी में से एक पत्ता यादृच्छया निकाला जाता है । इस पत्ते के लाल रंग के होने की प्रायिकता होगी :   **1**

   (a) $\dfrac{1}{26}$ (b) $\dfrac{1}{13}$

   (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$

<div align="center">

(अभिकथन – तर्क प्रकार के प्रश्न)

</div>

प्रश्न **19** तथा **20** में एक अभिकथन (A) के बाद एक तर्क-कथन (R) दिया है । निम्न में से सही उत्तर चुनिए :

   (a) (A) तथा (R) दोनों सत्य हैं । (R), कथन (A) की सही व्याख्या करता है ।

   (b) (A) तथा (R) दोनों सत्य हैं, परन्तु (R), कथन (A) की सही व्याख्या नहीं करता ।

   (c) (A) सत्य है, परन्तु (R) असत्य है ।

   (d) (A) असत्य है, परन्तु (R) सत्य है ।

14. The sum and product of zeroes of the polynomial $p(x) = 3x^2 - 5x + 2$ are   1

(a) $\dfrac{5}{3}, \dfrac{2}{3}$

(b) $\dfrac{-5}{3}, \dfrac{2}{3}$

(c) $1, \dfrac{2}{3}$

(d) $\dfrac{-5}{3}, \dfrac{-2}{3}$

15. A die is thrown once. The probability of getting an odd prime number is   1

(a) $\dfrac{1}{2}$

(b) $\dfrac{1}{6}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{2}{3}$

16. The sides of two similar triangles are in the ratio 4 : 7. The ratio of their perimeters is   1

(a) $4 : 7$

(b) $12 : 21$

(c) $16 : 49$

(d) $7 : 4$

17. The distance between two parallel tangents of a circle of diameter 7 cm is :   1

(a) 7 cm

(b) 14 cm

(c) $\dfrac{7}{2}$ cm

(d) 28 cm

18. A card is drawn at random from a well-shuffled deck of 52 cards. The probability of getting a red card is :   1

(a) $\dfrac{1}{26}$

(b) $\dfrac{1}{13}$

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{2}$

(Assertion – Reason based questions)

Directions for Q.19 & Q.20 : In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option :

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **अभिकथन (A) :** रैखिक समीकरण निकाय $3x + 5y - 4 = 0$ और $15x + 25y - 25 = 0$ असंगत है ।   1

तर्क (R) : रैखिक समीकरणों $a_1x + b_1y + c_1 = 0$ और $a_2x + b_2y + c_2 = 0$ का युग्म असंगत होगा, यदि $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ ।

20. **अभिकथन (A) :** वृत्त के किसी बिन्दु पर स्पर्श-रेखा स्पर्श बिंदु से जाने वाली त्रिज्या पर लंब होती है ।   1

तर्क (R) : एक वृत्त के बाहर स्थित एक बिन्दु से उस पर खींची गई स्पर्श-रेखाओं की लम्बाइयाँ समान होती हैं ।

खण्ड – ख

इस खण्ड में लघु उत्तरीय (SA-I) प्रकार के प्रश्न हैं । प्रत्येक प्रश्न के **2** अंक हैं ।

21. मान ज्ञात कीजिए : $\dfrac{3}{2} \tan^2 30° - 2 \cos^2 90° - \dfrac{1}{2} \, \text{cosec}^2 30°$ 2

22. दी गई आकृति में, दो समकोण त्रिभुज ABC तथा AMP हैं, जिनके क्रमशः ∠B और ∠M समकोण हैं । सिद्ध कीजिए कि △ ABC ~ △ AMP.   2
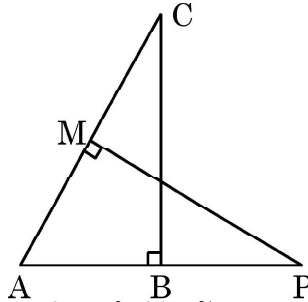
23. (a) उस बिंदु के निर्देशांक ज्ञात कीजिए जो बिंदुओं $(7, -1)$ तथा $(-3, 4)$ को जोड़ने वाले रेखाखंड को आंतरिक रूप से 2 : 3 के अनुपात में विभाजित करता है ।   2

अथवा

(b) y का/के मान ज्ञात कीजिए, जिसके लिए बिन्दुओं A(3, −1) और B(11, y) के बीच की दूरी 10 इकाई है ।

24. अभाज्य गुणनखण्डन का प्रयोग करके 92 और 510 का LCM और HCF ज्ञात कीजिए ।   2

25. (a) x और y के लिए हल कीजिए : $x + y = 6$, $2x - 3y = 4$.   2

अथवा

(b) ज्ञात कीजिए कि निम्न रैखिक समीकरण के युग्म संगत हैं या असंगत :

$5x - 3y = 11$, $-10x + 6y = 22$

19. **Assertion (A) :** The system of linear equations $3x + 5y - 4 = 0$ and $15x + 25y - 25 = 0$ is inconsistent.   1

   Reason (R) : The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

20. **Assertion (A) :** A tangent to a circle is perpendicular to the radius through the point of contact.   1

   Reason (R) : The lengths of tangents drawn from the external point to a circle are equal.

Section – B

This section comprises of short answer (SA-I) type of questions of **2** marks each.

21. Evaluate : $\dfrac{3}{2} \tan^2 30° - 2 \cos^2 90° - \dfrac{1}{2} \operatorname{cosec}^2 30°$ **2**

22. In the given figure, ABC and AMP are two right triangles, right angled at B and M, respectively. Prove that $\Delta \, ABC \sim \Delta \, AMP$.   **2**
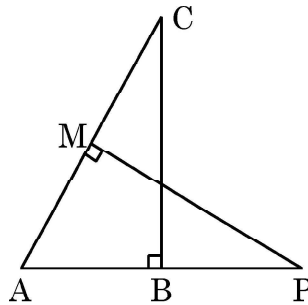
23. (a) Find the coordinates of the point which divides the line segment joining the points $(7, -1)$ and $(-3, 4)$ internally in the ratio $2 : 3$.   **2**

   OR

   (b) Find the value(s) of y for which the distance between the points $A(3, -1)$ and $B(11, y)$ is 10 units.

24. Find the LCM and HCF of 92 and 510, using prime factorisation.   **2**

25. (a) Solve for x and y : $x + y = 6$, $2x - 3y = 4$.   **2**

   OR

   (b) Find out whether the following pair of linear equations are consistent or inconsistent :
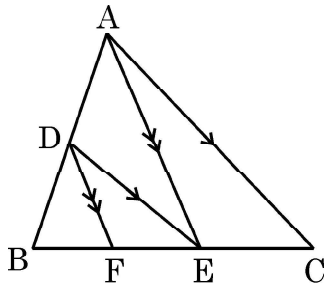 $5x - 3y = 11$, $-10x + 6y = 22$

खण्ड–ग में लघु उत्तरीय (SA-II) प्रकार के प्रश्न हैं और प्रत्येक प्रश्न के **3** अंक हैं ।

26. सिद्ध कीजिए कि $3 + 7\sqrt{2}$ एक अपरिमेय संख्या है, दिया गया है कि $\sqrt{2}$ एक अपरिमेय संख्या है । **3**

27. यदि α, β द्विघात बहुपद $x^2 + 3x + 2$ के शून्यक हों, तो एक द्विघात बहुपद ज्ञात कीजिए जिसके शून्यक $\alpha + 1$, $\beta + 1$ हों । **3**

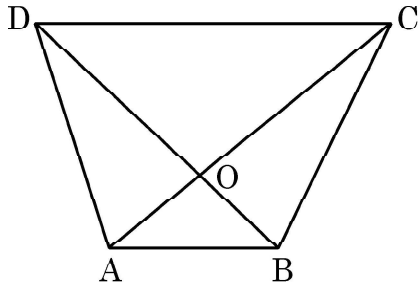28. (a) दी गई आकृति में, DE ‖ AC और DF ‖ AE **3**

सिद्ध कीजिए कि

$$\frac{BF}{FE} = \frac{BE}{EC}$$

अथवा

(b) चतुर्भुज ABCD के विकर्ण परस्पर एक-दूसरे को बिंदु O पर इस प्रकार प्रतिच्छेद करते हैं कि

$$\frac{AO}{BO} = \frac{CO}{OD}$$

दर्शाइए कि चतुर्भुज ABCD एक समलंब है ।

29. सिद्ध कीजिए कि बाह्य बिंदु से वृत्त पर खींची गई स्पर्श-रेखाओं की लम्बाइयाँ बराबर होती हैं । **3**

30. यदि बिंदु Q (0, 1), बिन्दुओं P (5, – 3) तथा R(x, 6) से एकसमान दूरी पर है, तो x का मान ज्ञात कीजिए । दूरी PR भी ज्ञात कीजिए । **3**
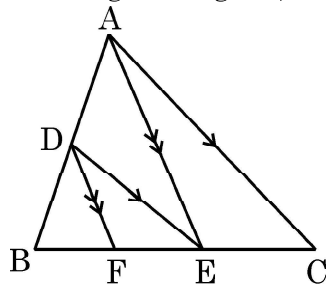
Section – C

This section comprises of Short Answer (SA-II) type questions of **3** marks each.

26. Prove that $3 + 7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. **3**

27. If α, β are zeroes of the quadratic polynomial $x^2 + 3x + 2$, find a quadratic polynomial whose zeroes are $\alpha + 1$, $\beta + 1$. **3**

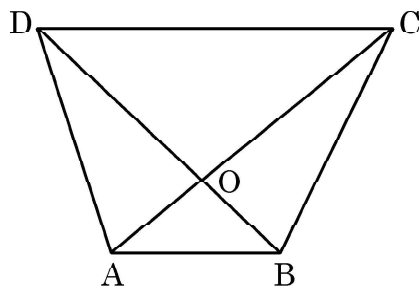28. (a) In the given figure, DE ∥ AC and DF ∥ AE **3**

Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

OR

(b) The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\dfrac{AO}{BO} = \dfrac{CO}{OD}$

Show that quadrilateral ABCD is a trapezium.

29. Prove that the tangents drawn from an external point to a circle are equal in length. **3**

30. If the point Q(0, 1) is equidistant from the points P (5, − 3) and R (x, 6); find the value of x. Also, find the distance PR. **3**

31. (a) सिद्ध कीजिए कि $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2 \sec A$.　　　　3

अथवा

(b) सिद्ध कीजिए कि $(\sin A + \mathrm{cosec}\ A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

खण्ड – घ

खण्ड–घ में दीर्घ उत्तरीय (LA) प्रकार के प्रश्न हैं और प्रत्येक प्रश्न 5 अंक का है ।

32. छात्रों के एक समूह द्वारा एक इलाके में 20 परिवारों पर किए गए एक सर्वेक्षण के परिणामस्वरूप एक परिवार में परिवार के सदस्यों की संख्या के लिए निम्नलिखित आवृत्ति तालिका प्राप्त हुई :　　5

परिवार का आकार	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
परिवारों की संख्या	7	8	2	2	1

उपरोक्त आँकड़ों का माध्य और बहुलक ज्ञात कीजिए ।

33. चावल का एक ढेर एक शंकु के आकार का है जिसका आधार व्यास 24 m और ऊँचाई $\dfrac{7}{2}$ m है । चावल का आयतन ज्ञात कीजिए । ढेर को ठीक से ढँकने के लिए कितने कैन्वस कपड़े की जरूरत है ?　　5

34. (a) एक नहर के तट पर एक टी.वी. टावर ऊर्ध्वाधर रूप से खड़ा है । टावर के ठीक सामने दूसरे तट के एक अन्य बिंदु से टावर के शिखर का उन्नयन कोण 60° है । इसी तट पर इस बिंदु से 20 m दूर और उस बिंदु को टावर के पाद से मिलाने वाली रेखा पर स्थित एक अन्य बिंदु से टावर के शिखर का उन्नयन कोण 30° है । टावर की ऊँचाई ज्ञात कीजिए ।　　5

अथवा

(b) एक हवाई-जहाज जमीन से 4000 m की ऊँचाई पर उड़ते हुए एक पल में दूसरे हवाई-जहाज के ऊपर से ऊर्ध्वाधर रूप से गुजरा, जब जमीन पर एक ही बिंदु से दोनों जहाजों के उन्नयन कोण क्रमशः 60° और 45° हैं । उस पल में दोनों जहाजों के बीच की ऊर्ध्वाधर दूरी ज्ञात कीजिए । ($\sqrt{3} = 1.73$ का प्रयोग करें ।)

35. (a) एक आयताकार खेत का विकर्ण इसकी छोटी भुजा से 60 m अधिक लंबा है । यदि लम्बी भुजा, छोटी भुजा से 80 m अधिक लम्बी है, तो खेत की भुजाओं की लम्बाइयाँ ज्ञात कीजिए ।　　5

अथवा

(b) एक पिता और उसके बेटे की आयु का योग 45 वर्ष है । पाँच वर्ष पहले, उनकी आयु (वर्षों में) का गुणनफल 124 था । उनकी वर्तमान आयु ज्ञात कीजिए ।

31. (a) Prove that $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2 \sec A.$ 3

OR

(b) Prove that $(\sin A + \mathrm{cosec}\, A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A.$

Section – D

This section consists of questions of Long Answer type, of **5** marks each.

32. A survey conducted on 20 families in a locality by a group of students resulted in the following frequency table for the number of family members in a family. 5

Family size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Number of families	7	8	2	2	1

Determine the mean and mode of the above data.

33. A heap of rice is in the form of a cone of base diameter 24 m and height $\dfrac{7}{2}$ m. Find the volume of rice. How much canvas cloth is required to just cover the heap ? 5

34. (a) A TV tower stands vertically on the bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from the point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower. 5

OR

(b) An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

35. (a) The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 80 m more than the shorter side, find the length of the sides of the field. 5

OR

(b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present age.
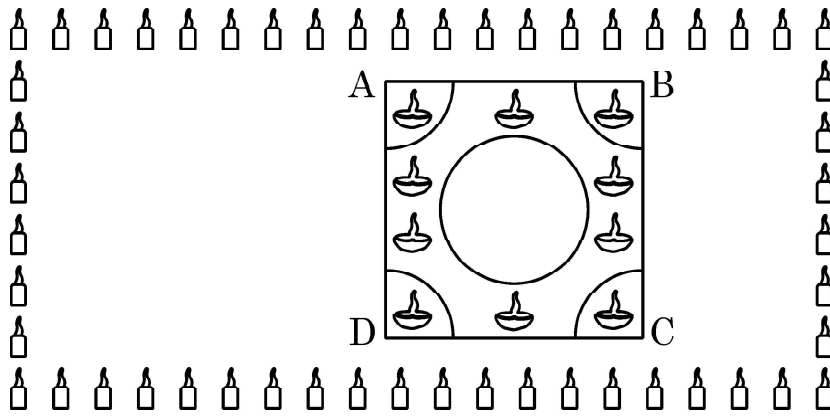
इस खण्ड में **3** प्रकरण अध्ययन प्रकार के प्रश्न हैं । प्रत्येक प्रश्न के **4** अंक हैं ।

36. इंटरस्कूल रंगोली प्रतियोगिता का आयोजन ओडिशा के प्रतिष्ठित स्कूलों में से एक द्वारा किया गया था । रंगोली प्रतियोगिता का विषय दिवाली समारोह था जहाँ छात्रों को गणितीय डिजाइन बनाना था । इस प्रतियोगिता में विभिन्न स्कूलों के विद्यार्थियों ने भाग लिया और रंगोली की सुंदर डिजाइन बनाई । एक ऐसा डिजाइन नीचे बना है :

रंगोली ABCD के रूप में चिह्नित वर्ग के आकार में है । वर्ग की भुजा 40 cm है । इस वर्ग के प्रत्येक कोने पर 10 cm त्रिज्या के चतुर्थांश बनाए जाते हैं (जिनमें दीये रखे जाते हैं) । वर्ग के बीच में 20 cm व्यास का एक वृत्त भी बनाया जाता है ।

उपरोक्त पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

(i)   वर्ग ABCD का क्षेत्रफल कितना है ?   1

(ii) वृत्त का क्षेत्रफल क्या है ?   1

(iii) यदि वर्ग ABCD से वृत्त और चारों चतुर्थांशों को काटकर निकाल लिया जाए, तो शेष बचे वर्ग ABCD का क्षेत्रफल ज्ञात कीजिए ।   2

अथवा

(iii) हटाए गये वृत्त और चार चतुर्थांशों का कुल क्षेत्रफल ज्ञात कीजिए ।   2

Section – E

This section comprises of 3 Case Study questions, each of **4** marks.

36. Interschool Rangoli Competition was organized by one of the reputed schools of Odissa. The theme of the Rangoli Competition was Diwali celebrations where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below.
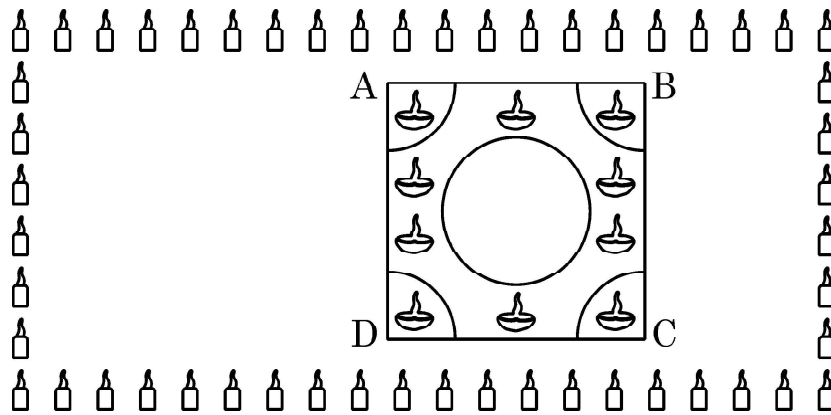
Rangoli is in the shape of square marked as ABCD, side of square being 40 cm. At each corner of a square, a quadrant of circle of radius 10 cm is drawn (in which diyas are kept). Also a circle of diameter 20 cm is drawn inside the square.

(i) What is the area of square ABCD ? **1**

(ii) Find the area of the circle. **1**

(iii) If the circle and the four quadrants are cut off from the square ABCD and removed, then find the area of remaining portion of square ABCD. **2**
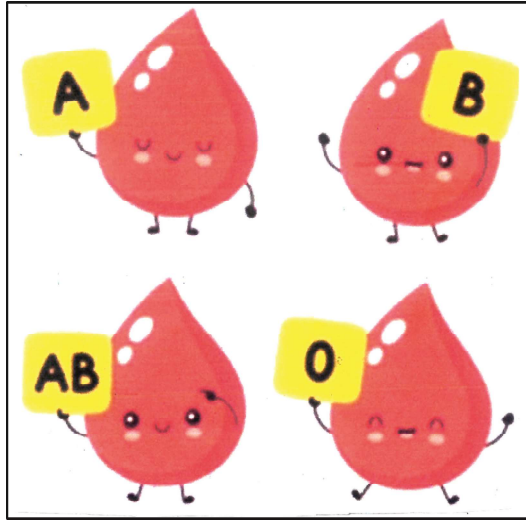
OR

(iii) Find the combined area of 4 quadrants and the circle, removed. **2**

37. ब्लड ग्रुप किसी व्यक्ति के रक्त के प्रकार का वर्णन करता है । यह लाल रक्त कोशिकाओं पर विरासत में मिले एंटीजेनिक पदार्थों की उपस्थिति या अनुपस्थिति के आधार पर रक्त का एक वर्गीकरण है । रक्त के प्रकार भविष्यवाणी करते हैं कि रक्त चढ़ाने में कोई गंभीर प्रतिक्रिया होगी या नहीं ।

50 लोगों के एक नमूने में, 21 लोगों का रक्त टाइप O का, 22 लोगों का रक्त टाइप A का, 5 लोगों का रक्त टाइप B तथा बाकी लोगों का रक्त टाइप AB का है ।

उपरोक्त पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

(i) यादृच्छिक रूप से चुने व्यक्ति का रक्त टाइप O का होने की प्रायिकता क्या है ?   1

(ii) यादृच्छिक रूप से चुने व्यक्ति का रक्त टाइप AB का होने की प्रायिकता क्या है ?   1

(iii) यादृच्छिक रूप से चुने व्यक्ति का रक्त न तो टाइप A का और न ही टाइप B का होने की प्रायिकता क्या है ?   2

अथवा

(iii) यादृच्छिक रूप से चुने व्यक्ति का रक्त या तो टाइप A या टाइप B का या टाइप O का होने की प्रायिकता क्या है ?   2

37. Blood group describes the type of blood a person has. It is a classification of blood based on the presence or absence of inherited antigenic substances on the surface of red blood cells. Blood types predict whether a serious reaction will occur in a blood transfusion.

In a sample of 50 people, 21 had type O blood, 22 had type A, 5 had type B and rest had type AB blood group.
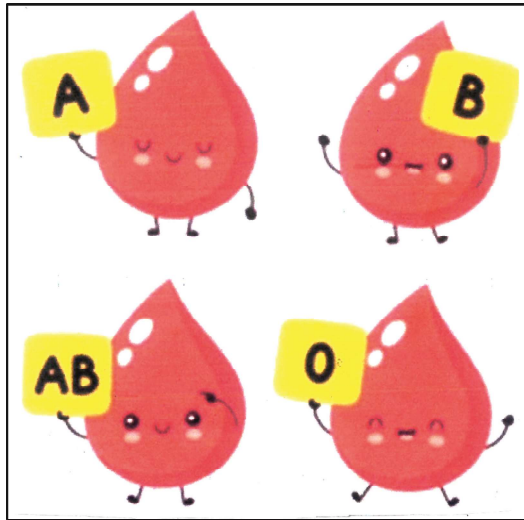
Based on the above, answer the following questions :

(i) What is the probability that a person chosen at random had type O blood ? 1

(ii) What is the probability that a person chosen at random had type AB blood group ? 1

(iii) What is the probability that a person chosen at random had neither type A nor type B blood group ? 2

OR

(iii) What is the probability that person chosen at random had either type A or type B or type O blood group ? 2

38. अहाना एक पौधा प्रेमी होने के कारण अपनी बालकनी को पौधों से भरे एक सुंदर बगीचे में बदलने का फैसला करती है । उसने बालकनी के लिए कुछ पौधे और गमले खरीदे । उसने गमले इस प्रकार रखे कि पहली पंक्ति में गमलों की संख्या 2, दूसरी पंक्ति में गमलों की संख्या 5, तीसरी पंक्ति में 8 और इसी तरह अन्य पंक्तियों में ।
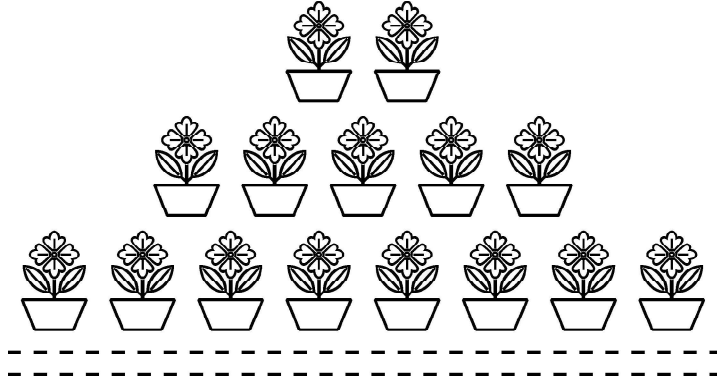
उपरोक्त पर आधारित होकर, निम्न प्रश्नों के उत्तर दीजिए :

(i)   ज्ञात कीजिए कि 10वीं पंक्ति में कितने गमले रखे गए हैं ।   1

(ii) 5वीं और दूसरी पंक्तियों में रखे गमलों की संख्याओं का अन्तर ज्ञात कीजिए ।   1

(iii) यदि अहाना 100 गमले रखना चाहती हो, तो इन सभी गमलों को व्यवस्थित करने में कुल पंक्तियों की संख्या ज्ञात कीजिए ।   2
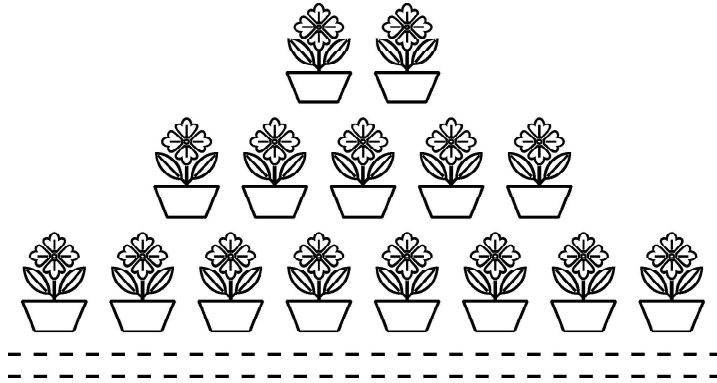
अथवा

(iii) यदि अहाना के पास 12 पंक्तियों का स्थान उपलब्ध हो, तो वह कुल कितने गमले इन पंक्तियों में व्यवस्थित कर सकेगी ?   2

_____

38. Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.

Based on the above information, answer the following questions :

(i)   Find the number of pots placed in the 10th row.   1

(ii) Find the difference in the number of pots placed in 5th row and 2nd row.   1

(iii) If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.   2

OR

(iii) If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement ?   2

———

	Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Secondary School Examination, 2023** **SUBJECT NAME MATHEMATICS (BASIC) (SUBJECT CODE 241) (PAPER CODE 430/1/3)**

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**. However, for MCQs (Q1 to Q20), only first attempt to be evaluated.
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only

	once.
11	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
15	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (BASIC)

SECTION A

1. If $\sqrt{3}\tan\theta = 1$, then the value of θ is

    (a) 30°   (b) 45°

    (c) 60°   (d) 90°

Ans. (a)   30°   *1*

---

2. The prime factorisation of 1728 is

    (a) $2^5 \times 3^3$ (b) $2^5 \times 3^4$

    (c) $2^6 \times 3^3$ (d) $2^6 \times 3^2$

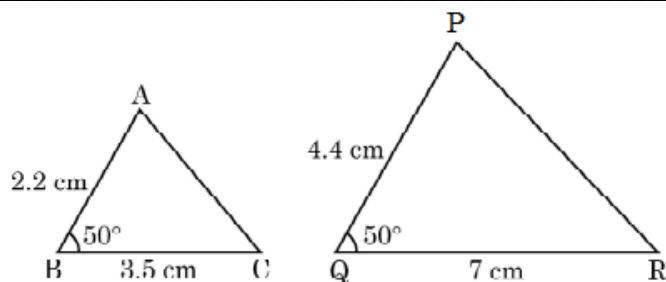**Ans.** (c) $2^6 \times 3^3$ *1*

---

3. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then the value of a is

    (a) 6   (b) 7

    (c) 20   (d) 28

Ans. (d)   28   *1*

4.

In the above figure, the criterion of similarity by which $\triangle ABC \sim \triangle PQR$ is :
(a)   SSA (Side – Side – Angle) Similarity
(b)   ASA (Angle – Side – Angle) Similarity
(c)   SAS (Side – Angle – Side) Similarity
(d)   AA (Angle – Angle) Similarity
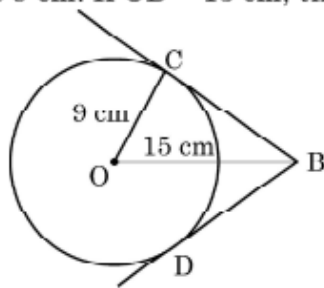
Ans. (c)   SAS (Side – Angle – Side) Similarity   *1*

---

5. The volume of a cone of radius 'r' and height '3r' is :

    (a) $\frac{1}{3}\pi r^3$ (b) $3\pi r^3$

    (c) $9\pi r^3$ (d) πr^3

Ans. (d) πr^3 *1*

---

6. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is :

  (a) 18 cm   (b) 12 cm

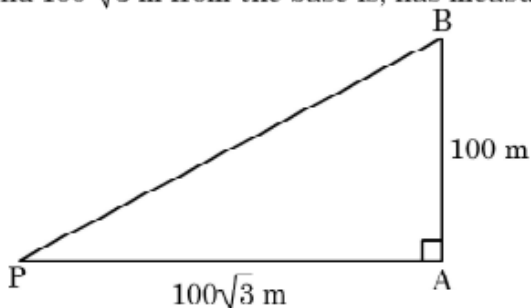 (c) 24 cm   (d) 36 cm

Ans. (c)   24 cm   *1*

---

7. The distance between the points (3, 0) and (0, −3) is

  (a) $2\sqrt{3}$ units   (b) 6 units

  (c) 3 units   (d) $3\sqrt{2}$ units

Ans. (d) $3\sqrt{2}$ units   *1*

---

8. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of

  (a) 90°   (b) 60°

  (c) 45°   (d) 30°

Ans. (d)   30°   *1*

---

9. The larger of two supplementary angles exceeds the smaller by 18 degrees. What is the measure of larger angle ?

  (a) 81°   (b) 99°

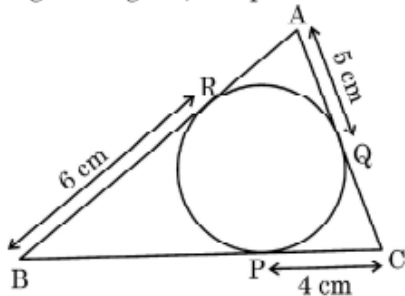 (c) 36°   (d) 54°

Ans. (b)   99°   *1*

---

10. The discriminant of the quadratic equation $2x^2 - 5x - 3 = 0$ is

    (a) 1   (b) 49

    (c) 7   (d) 19

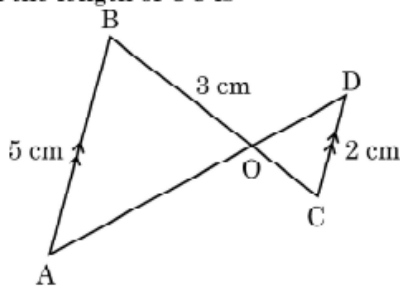**Ans.** (b)   49   *1*

---

11. In the given figure, the perimeter of ΔABC is :

    (a)   30 cm   (b)   15 cm

    (c)   45 cm   (d)   60 cm

Ans. (a)   30 cm   *1*

---

12. In the given figure, AB || CD. If AB = 5 cm, CD = 2 cm and OB = 3 cm, then the length of OC is
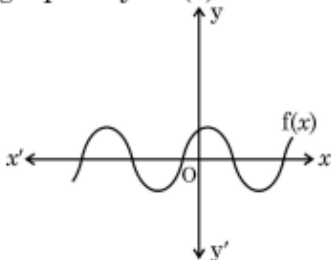

    (a) $\dfrac{15}{2}$ cm   (b) $\dfrac{10}{3}$ cm

    (c) $\dfrac{6}{5}$ cm   (d) $\dfrac{3}{5}$ cm

Ans. (c) $\dfrac{6}{5}$ cm   *1*

---

13. The graph of $y = f(x)$ is shown in the figure for some polynomial $f(x)$.

The number of zeroes of $f(x)$ is

    (a) 5   (b) 6

    (c) 4   (d) 8

Ans. (a)   5   *1*

---

14. The sum and product of zeroes of the polynomial $p(x) = 3x^2 - 5x + 2$ are

   (a) $\dfrac{5}{3}, \dfrac{2}{3}$ (b) $\dfrac{-5}{3}, \dfrac{2}{3}$

   (c) $1, \dfrac{2}{3}$ (d) $\dfrac{-5}{3}, \dfrac{-2}{3}$

Ans. (a) $\dfrac{5}{3}, \dfrac{2}{3}$ *1*

---

15. A die is thrown once. The probability of getting an odd prime number is

   (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{6}$

   (c) $\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

Ans. (c) $\dfrac{1}{3}$ *1*

---

16. The sides of two similar triangles are in the ratio 4 : 7. The ratio of their perimeters is

   (a)   4 : 7   (b)   12 : 21
   (c)   16 : 49   (d)   7 : 4

Ans. (a)   4 : 7   *1*

---

17. The distance between two parallel tangents of a circle of diameter 7 cm is :

   (a)   7 cm   (b)   14 cm
   (c) $\dfrac{7}{2}$ cm   (d)   28 cm

Ans. (a)   7 cm   *1*

---

18. A card is drawn at random from a well-shuffled deck of 52 cards. The probability of getting a red card is :

   (a) $\dfrac{1}{26}$ (b) $\dfrac{1}{13}$

   (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$

Ans. (d) $\dfrac{1}{2}$ *1*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The system of linear equations $3x + 5y - 4 = 0$ and $15x + 25y - 25 = 0$ is inconsistent.

Reason (R) : The pair of linear equations $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ is inconsistent if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

Ans.   (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)   *1*

---

20. **Assertion (A) :** A tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R) : The lengths of tangents drawn from the external point to a circle are equal.
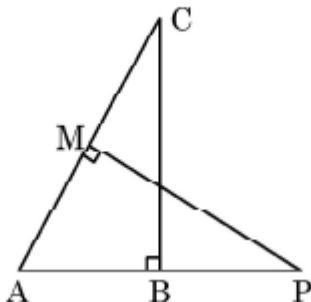
Ans.   (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)   *1*

SECTION B

21. Evaluate : $\dfrac{3}{2} \tan^2 30° - 2 \cos^2 90° - \dfrac{1}{2} \csc^2 30°$

Solution: $\dfrac{3}{2} \left(\dfrac{1}{\sqrt{3}} \right)^2 - 2(0)^2 - \dfrac{1}{2}(2)^2$ $\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2}$

$= -\dfrac{3}{2}$ $\dfrac{1}{2}$

---

22. In the given figure, ABC and AMP are two right triangles, right angled at B and M, respectively. Prove that $\triangle ABC \sim \triangle AMP$.

Solution: In Δ ABC and Δ AMP,

$\angle$ ABC = $\angle$ AMP (90° each)   *1/2*

$\angle$ BAC = $\angle$ MAP (common)   *1*

By AA Similarity

Δ ABC ~ Δ AMP   *1/2*

---

23. (a) Find the coordinates of the point which divides the line segment joining the points (7, −1) and (−3, 4) internally in the ratio 2 : 3.

Solution:

Let P(x, y) divide AB internally in the ratio 2 : 3

$$x = \frac{2 \times -3 + 3 \times 7}{2 + 3} = \frac{15}{5} = 3$$ *1*

$$y = \frac{2 \times 4 + 3 \times -1}{2 + 3} = \frac{5}{5} = 1$$ *1*

Coordinates of the required point are P (3, 1)

OR

(b) Find the value(s) of y for which the distance between the points A(3, −1) and B(11, y) is 10 units.

Solution: AB = 10 units $\Rightarrow AB^2 = 100$

$$\Rightarrow (11 - 3)^2 + (y + 1)^2 = 100$$ *1*

$$\Rightarrow y + 1 = \pm 6$$

$$\Rightarrow y = 5, -7$$ $\frac{1}{2} + \frac{1}{2}$

---

24. Find the LCM and HCF of 92 and 510, using prime factorisation.

Solution: $92 = 2 \times 2 \times 23$ *1/2*

$510 = 2 \times 3 \times 5 \times 17$ *1/2*

HCF = 2   *1/2*

$LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$ *1/2*

---

25. (a) Solve for x and y : $x + y = 6$, $2x - 3y = 4$.

Solution: On solving the given equations and getting

$$x = \frac{22}{5} \text{ and } y = \frac{8}{5}$$

1+1

OR

(b) Find out whether the following pair of linear equations are consistent or inconsistent :

$$5x - 3y = 11, \quad -10x + 6y = 22$$

Solution: $-\dfrac{5}{10} = -\dfrac{3}{6} \neq \dfrac{11}{22}$ or $-\dfrac{1}{2} = -\dfrac{1}{2} \neq \dfrac{1}{2}$

1½

$\Rightarrow$ given pair of linear equations is inconsistent

½

SECTION C

26. Prove that $3 + 7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number.

Solution: Let us assume that $3 + 7\sqrt{2}$ is a rational number.

Let $3 + 7\sqrt{2} = \dfrac{p}{q}$, p, q are integers and $q \neq 0$

1

$$\Rightarrow \sqrt{2} = \frac{p - 3q}{7q}$$

1

RHS is rational but LHS is irrational

$\therefore$ Our assumption is wrong

Hence, $3 + 7\sqrt{2}$ is an irrational number

1

---

27. If α, β are zeroes of the quadratic polynomial $x^2 + 3x + 2$, find a quadratic polynomial whose zeroes are α + 1, β + 1.

Solution: $p(x) = x^2 + 3x + 2$

α, β are its zeroes

$\therefore \alpha + \beta = -3, \alpha\beta = 2$

½ + ½

Now,

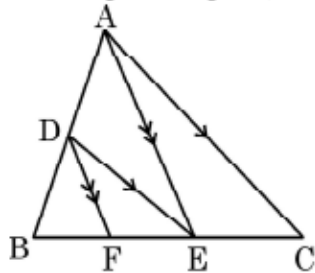$(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = -3 + 2 = -1$

½

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = +2 - 3 + 1 = 0$

1

$\therefore$ Required Polynomial is $k(x^2 + x)$ or $x^2 + x$

½

---

28. (a) In the given figure, DE ∥ AC and DF ∥ AE

Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Solution: In Δ ABE, DF ∥ AE (given), hence by BPT

$$\frac{BD}{DA} = \frac{BF}{FE} \;\rule{2cm}{0.4pt}\; \text{(i)}$$ $1\frac{1}{2}$

In Δ ABC, DE ∥ AC (given), hence by BPT

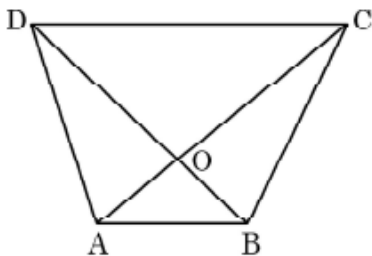$$\frac{BD}{DA} = \frac{BE}{EC} \;\rule{2cm}{0.4pt}\; \text{(ii)}$$ 1

From (i) and (ii)

$$\frac{BF}{FE} = \frac{BE}{EC}$$ $\frac{1}{2}$

OR

(b) The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{OD}$

Show that quadrilateral ABCD is a trapezium.

Solution: In Δ AOB and Δ COD,

$$\frac{AO}{BO} = \frac{CO}{OD} \Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

∠ AOB = ∠ COD (vertically opp. angles)

⇒ Δ AOB ~ Δ COD (SAS Similarity) 2

⇒ ∠ CAB= ∠ACD   (or ∠ DBA= ∠ BDC)

But, these are alternate interior angles

∴ AB ∥ CD ⇒ ▱ABCD is a trapezium 1

29. Prove that the tangents drawn from an external point to a circle are equal in length.

Solution:

(For Fig.) *1*
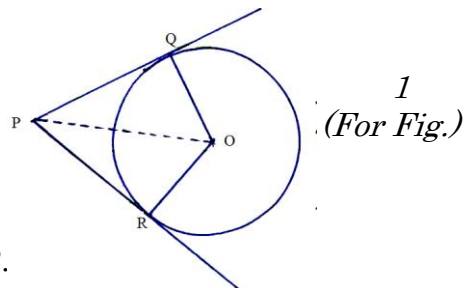
Given: A circle with centre O and PQ, PR are tangents to the circle from an external point P.

To Prove: PQ = PR

Construction: Join OP, OQ, OR $\frac{1}{2}$

Proof : In $\triangle$ OPQ and $\triangle$ OPR

OP = OP (common)

OQ = OR (radii of the same circle)

$\angle$ OQP = $\angle$ ORP (each 90°)

$\Rightarrow \triangle$ POQ $\cong \triangle$ POR (RHS congruence) *1*

$\therefore$ PQ = PR $\frac{1}{2}$

---

30. If the point Q(0, 1) is equidistant from the points P (5, − 3) and R (x, 6); find the value of x. Also, find the distance PR.

Solution: PQ = QR $\Rightarrow$ PQ^2 = QR^2

$(5 - 0)^2 + (- 3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$ *1*

$x^2 = 16$

$x = 4, - 4$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow$ R(4, 6) or (− 4, 6)

If R(4, 6), PR = $\sqrt{(5 - 4)^2 + (- 3 - 6)^2}$ = $\sqrt{82}$ $\frac{1}{2}$

If R(− 4, 6), PR = $\sqrt{(5 + 4)^2 + (- 3 - 6)^2}$ = $\sqrt{162}$ or $9\sqrt{2}$ $\frac{1}{2}$

---

31. (a) Prove that $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2 \sec A$.

Solution: (a) LHS = $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A}$

$= \dfrac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A}$ *1*

$= \dfrac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)\cos A}$ *1*

$= \dfrac{1 + 1 + 2 \sin A}{(1 + \sin A)\cos A} = \dfrac{2(1 + \sin A)}{(1 + \sin A)\cos A}$ $\frac{1}{2}$

$$= \frac{2}{\cos A} = 2 \sec A = RHS$$ $\frac{1}{2}$

<div align="center">OR</div>

(b)   Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

(b) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A$
$+ 2 \cos A \sec A$ 1

$= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2 \sin A \cdot \dfrac{1}{\sin A}$

$+ 2 \cos A \cdot \dfrac{1}{\cos A}$ 1

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 2 + 2$ $\frac{1}{2}$

$= 7 + \tan^2 A + \cot^2 A = RHS$ $\frac{1}{2}$

<div align="center">SECTION D</div>

32.   A survey conducted on 20 families in a locality by a group of students resulted in the following frequency table for the number of family members in a family.

Family size	$1-3$	$3-5$	$5-7$	$7-9$	$9-11$
Number of families	7	8	2	2	1

Determine the mean and mode of the above data.

Solution: For Table: 1

Family size	X_i	F_i	$f_i x_i$
$1-3$	2	7	14
$3-5$	4	8	32
$5-7$	6	2	12
$7-9$	8	2	16
$9-11$	10	1	10
		20	84

Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{84}{20} = 4.2$ $1 + \frac{1}{2}$

Mode: Modal Class $= 3 - 5$ $\frac{1}{2}$

$l = 3,\ f_1 = 8,\ f_0 = 7,\ f_2 = 2,\ h = 2$

Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$= 3 + \left(\frac{8-7}{16-7-2} \right) \times 2 \qquad 1\frac{1}{2}$$

$$= \frac{23}{7} \text{ or } 3.287 \qquad \frac{1}{2}$$

---

33. A heap of rice is in the form of a cone of base diameter 24 m and height $\frac{7}{2}$ m. Find the volume of rice. How much canvas cloth is required to just cover the heap ?

**Solution:** Radius of cone = 12 m, $\quad h = \frac{7}{2}$ m $\qquad \frac{1}{2}$

Volume of cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times \frac{7}{2} \qquad 1\frac{1}{2}$$

$$= 528 \text{ m}^3 \qquad \frac{1}{2}$$

l (slant height) $= \sqrt{h^2 + r^2} = \frac{25}{2}$ m $\qquad 1$

Curved surface area of cone $= \pi r l$

$$= \frac{22}{7} \times 12 \times \frac{25}{2} \qquad 1$$

$$= \frac{3300}{7} \text{ m}^2 \quad \text{or} \quad 471 \cdot 43 \text{ m}^2 \qquad \frac{1}{2}$$

Cloth required to cover the heap $= 471 \cdot 43 \text{ m}^2$

---

34. (a) A TV tower stands vertically on the bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from the point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.
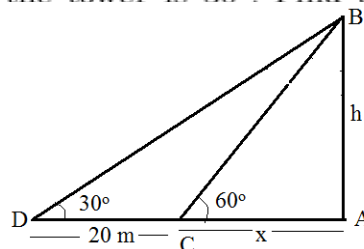
Solution:

For figure *1*

In $\triangle$ ABC, $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3} \, x$ $\qquad 1 + \frac{1}{2}$

In $\triangle$ ABD, $\tan 30° = \frac{h}{20+x} \implies \frac{1}{\sqrt{3}} = \frac{h}{20+x}$ $\qquad 1 + \frac{1}{2}$

430/1/3   13   P.T.O.

$$\sqrt{3}\,h = 20 + x$$

$$\Rightarrow x = 10$$

$\quad\quad\quad$ $\dfrac{1}{2}$

$$\Rightarrow h = \sqrt{3}\,x = 10\sqrt{3}$$

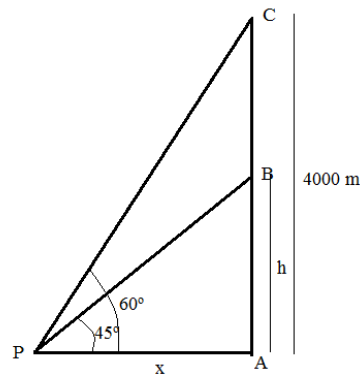$$\Rightarrow \text{Height of tower} = 10\sqrt{3}\ \text{m or } 17.3\ \text{m}$$

$\quad\quad\quad$ $\dfrac{1}{2}$

OR

(b) An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Solution:

For figure *1*

In Δ APB, $\dfrac{h}{x} = \tan 45° \Rightarrow h = x$ $\qquad\qquad$ $1{+}\dfrac{1}{2}$

In Δ APB, $\dfrac{4000}{x} = \tan 60° \Rightarrow x = \dfrac{4000}{\sqrt{3}}$ $\qquad$ $1{+}\dfrac{1}{2}$

$$\Rightarrow h = x = \dfrac{4000}{\sqrt{3}}$$

Distance between the aeroplanes $= 4000 - \dfrac{4000}{\sqrt{3}}$ $\qquad$ $\dfrac{1}{2}$

$$= 4000\left(1 - \dfrac{1}{\sqrt{3}}\right)$$

$$= \dfrac{5080}{3}\ \text{m or } 1693.33\ \text{m (approx.)}$$

$\quad\quad\quad$ $\dfrac{1}{2}$

(**Note:** ½ mark to be deducted for not using $\sqrt{3} = 1.73$)

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35. (a) The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 80 m more than the shorter side, find the length of the sides of the field.

Note: There is an error in the question, so full marks to be awarded to the Candidate, who attempted.

OR

(b) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present age.

Solution: Let age of father = x years
and age of son = 45 − x $\qquad$ *1*
Five years ago, age of father = x − 5
Age of son = 40 − x $\qquad$ *1*
ATQ, (x − 5) (40 − x) = 124 $\qquad$ *1*

$$x^2 - 45x + 324 = 0$$
$$(x - 36)(x - 9) = 0 \qquad \textit{1}$$

x = 36, x = 9 (rejected) $\qquad \frac{1}{2}$

⇒ Father's age = 36 years and son's age = 9 years $\qquad \frac{1}{2}$
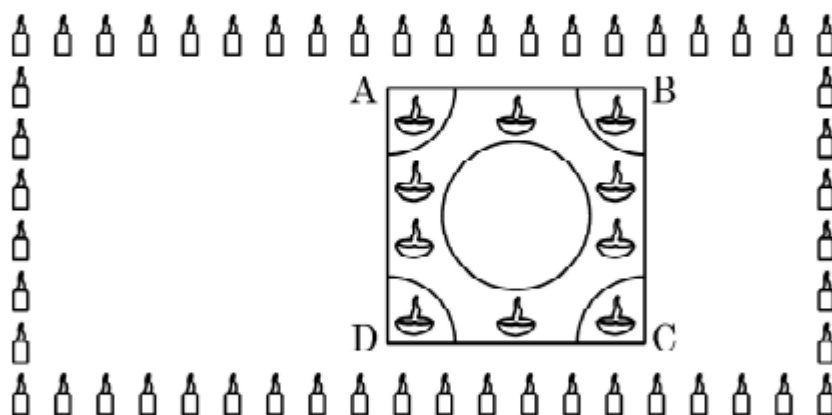

SECTION E

36. Interschool Rangoli Competition was organized by one of the reputed schools of Odissa. The theme of the Rangoli Competition was Diwali celebrations where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below.

Rangoli is in the shape of square marked as ABCD, side of square being 40 cm. At each corner of a square, a quadrant of circle of radius 10 cm is drawn (in which diyas are kept). Also a circle of diameter 20 cm is drawn inside the square.

(i) What is the area of square ABCD ?

(ii) Find the area of the circle.

(iii) If the circle and the four quadrants are cut off from the square ABCD and removed, then find the area of remaining portion of square ABCD.

OR

(iii) Find the combined area of 4 quadrants and the circle, removed.

Solution: (i) Area of square ABCD = $(40)^2$ = 1600 cm^2 *1*

(ii) Area of circle = πr^2 = $\dfrac{22}{7}$ ×10 ×10

$= \dfrac{2200}{7}$ cm^2 or 314.28 cm^2 *1*

(iii) Area of 4 quadrants = $4(\dfrac{1}{4}\pi r^2)$ = $\dfrac{2200}{7}$ cm^2 *1*

Remaining area = $1600 - \left(\dfrac{2200}{7} + \dfrac{2200}{7}\right)$

$= 1600 - \dfrac{4400}{7} = \dfrac{6800}{7}$ cm² or 971.43 cm²   *1*

OR

(iii) Area of 4 quadrants = $4(\dfrac{1}{4}\pi r^2)$ = $\dfrac{2200}{7}$ cm^2 *1*

Combined area of circle + 4 quadrants

$= \dfrac{2200}{7} + \dfrac{2200}{7} = \dfrac{4400}{7}$ cm² or 628.57 cm²   *1*

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37. Blood group describes the type of blood a person has. It is a classification of blood based on the presence or absence of inherited antigenic substances on the surface of red blood cells. Blood types predict whether a serious reaction will occur in a blood transfusion.

In a sample of 50 people, 21 had type O blood, 22 had type A, 5 had type B and rest had type AB blood group.
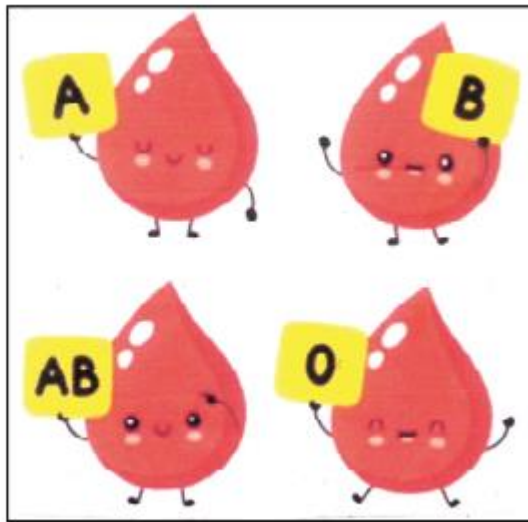
Based on the above, answer the following questions :

(i) What is the probability that a person chosen at random had type O blood ?

(ii) What is the probability that a person chosen at random had type AB blood group ?

(iii) What is the probability that a person chosen at random had neither type A nor type B blood group ?

OR

(iii) What is the probability that person chosen at random had either type A or type B or type O blood group ?

Solution: (i) P(type O) = $\dfrac{21}{50}$ *1*

(ii) No. of people with AB type blood group = 50 – (21 + 22 + 5) = 2 $\dfrac{1}{2}$

P(type AB) = $\dfrac{2}{50}$ or $\dfrac{1}{25}$ $\dfrac{1}{2}$

(iii) P(neither type A nor type B) = $\dfrac{21+2}{50}$ = $\dfrac{23}{50}$ *1+1*

OR

(iii) P(type A or type B or type O) = $\dfrac{21+22+5}{50}$ = $\dfrac{24}{25}$ *1+1*

38. Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.
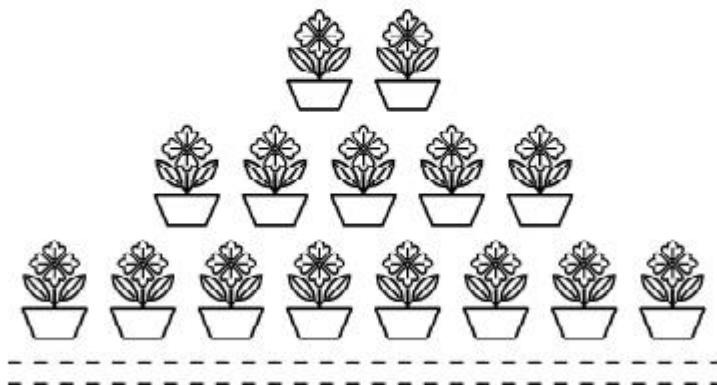
Based on the above information, answer the following questions :

(i)   Find the number of pots placed in the 10th row.

(ii)   Find the difference in the number of pots placed in 5th row and 2nd row.

(iii) If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

OR

(iii) If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement ?

Solution: $a = 2$, $d = 3$

(i)   Number of pots in the 10th row
$$= a_{10} = a + 9d = 29 \qquad 1$$

(ii) $a_5 - a_2 = (a + 4d) - (a + d) = 3d = 9 \qquad 1$

(iii) $S_n = 100 \Rightarrow \dfrac{n}{2}[2(2) + (n - 1)3] = 100 \qquad 1$

$3n^2 + n - 200 = 0 \Rightarrow (3n + 25)(n - 8) = 0$

$\therefore n = 8 \quad (n = -\dfrac{25}{3}$ rejected), $\qquad 1$

OR

(iii) $S_{12} = \dfrac{12}{2}[2(2) + 11(3)] \qquad 1$

$= 222 \qquad 1$