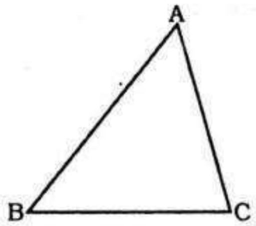
CHAPTER 15

Triangles (Congruence & Similarlity)

We know that a closed figure formed by three intersecting lines is called a triangle. ('Tri' means 'three'). A triangle has three sides, three angles and three vertices. For example, in triangle ABC, denoted as Δ ABC; AB, BC, CA are the three sides, $\angle$A, $\angle$B, $\angle$C are the three angles and A, B, C are three vertices.



Congruence of Triangles

We must have observed that two copies of your photographs of the same size are identical. Similarly, two bangles of the same size, two ATM cards issued by the same bank are identical. We may recall that on placing a one rupee coin on another minted in the same year, they cover each other completely.

They are called congruent figures ('congruent' means equal in all respects or figures whose shapes and sizes are both the same).

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

If ΔPQR is congruent to ΔABC, we write $\Delta PQR \cong \Delta ABC$.

Notice that when Δ PQR $\cong$ Δ ABC, then sides of ΔPQR fall on corresponding equal sides of Δ ABC and so is the case for the angles.

That is, PQ covers AB, QR covers BC and RP covers CA; $\angle$P covers $\angle$A, $\angle$Q covers $\angle$B and $\angle$R covers $\angle$C. Also, there is a one-one correspondence between the vertices. That is, P corresponds to A, Q to B, R to C and so on which is written as

$P \leftrightarrow A$, $Q \leftrightarrow B$, $R \leftrightarrow C$

Note that under this correspondence, $\Delta PQR \cong \Delta ABC$; but it will not be correct to write $\Delta QRP \cong \Delta$ ABC.

Similarly,

FD $\leftrightarrow$ AB, DE $\leftrightarrow$ BC and EF $\leftrightarrow$ CA

and F $\leftrightarrow$ A, D $\leftrightarrow$ B and E $\leftrightarrow$ C

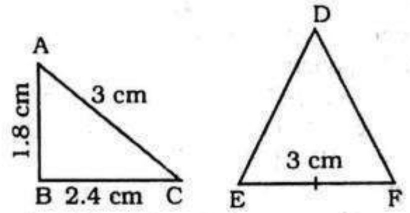
So, $\Delta FDE \cong \Delta ABC$ but writing $\Delta DEF \cong \Delta ABC$ is not correct.

So, it is necessary to write the correspondence of vertices correctly for writing of congruence of triangles in symbolic form.

Note that in congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

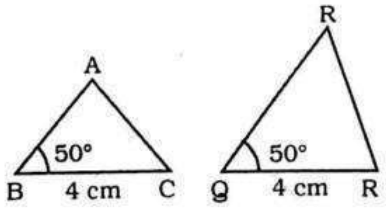
Criteria for Congruence of Triangles

Draw two triangles with one side 3 cm. Are these triangles congruent? Observe that they are not congruent.



(i) (ii)

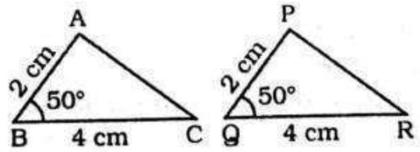
Now, draw two triangles with one side 4 cm and one angle 50°. Are they congruent?



See that these two triangles are not congruent.

So, equality of one pair of sides or one pair of sides and one pair of angles is not sufficient to give us congruent triangles.

The equality of two sides and the included angle is enough for the congruence of triangles.

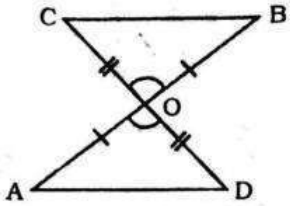


This is the first criterion for congruence of triangles.

Axiom (SAS congruence rule) : Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.

Example 1 : In the figure given below OA = OB and OD = OC. Show that

(i) Δ AOD $\cong$ Δ BOC (ii) AD $\parallel$ BC



Solution : (i) You may observe that in ΔAOD and ΔBOC,

$\left.\begin{array}{l} OA = OB \\ OD = OC \end{array}\right\}$ (Given)

Also, since $\angle AOD$ and $\angle BOC$ form a pair of vertically opposite angles, we have

$\angle AOD = \angle BOC$.

So, $\Delta AOD \cong \Delta BOC$ (by the SAS congruence rule)

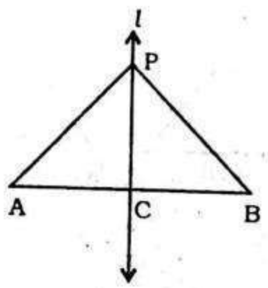
(ii) In congruent triangles AOD and BOC, the other corresponding parts are also equal.

So, $\angle OAD = \angle OBC$ and these form a pair of alternate angles for line segments AD and BC.

Therefore, AD || BC.

Example 2 : AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B.

Solution : Line $l \perp AB$ and passes through C which is the mid-point of AB. You have to show that PA = PB. Consider ΔPCA and ΔPCB.



We have AC = BC (C is the mid-point of AB)

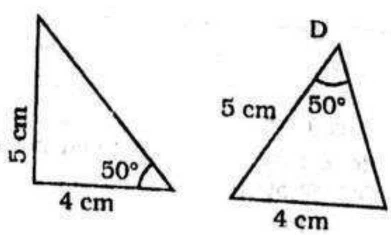
$\angle PCA = \angle PCB = 90°$ (Given)

PC = PC (Common)

So, $\Delta PCA \cong \Delta PCB$ (SAS rule)

and so, PA = PB, as they are corresponding sides of congruent triangles.

Now, let us construct two triangles, whose sides are 4 cm and 5 cm and one of the angles is 50° and this angle is not included in between the equal sides. Are the two triangles congruent?

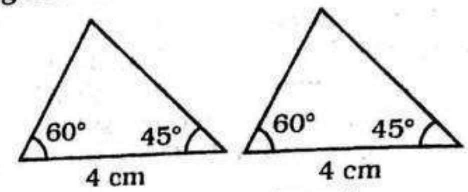


Notice that the two triangles are not congruent.

For triangles to be congruent, it is very important that the equal angles are included between the pairs of equal sides.

So, SAS congruence rule holds but not ASS or SSA rule.

Next, try to construct the two triangles in which two angles are 60° and 45° and the side included between these angles is 4 cm.



The two triangles are congruent.

This result is the Angle-Side-Angle criterion for congruence and is written as ASA criterion. Let us state and prove this result.

Since this result can be proved, it is called a theorem and to prove it, we use the SAS axiom for congruence.

Theorem 1. **(ASA congruence rule) :** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

Proof : We are given two triangles ABC and DEF in which:

$\angle B = \angle E$, $\angle C = \angle F$ and BC = EF

We need to prove that $\Delta ABC \cong \Delta DEF$

For proving the congruence of the two triangles see that three cases arise.

Case (i) : Let AB = DE

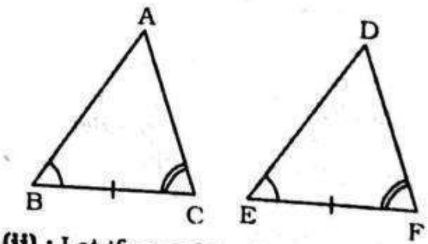
You may observe that

AB = DE (Assumed)

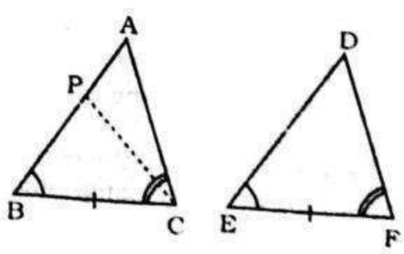
$\angle B = \angle E$ (Given)

BC = EF (Given)

So, $\Delta ABC \cong \Delta DEF$ (By SAS rule)



Case (ii) : Let if possible AB > DE. So, we can take a point P on AB such that PB = DE. Now consider ΔPBC and ΔDEF.



Observe that in ΔPBC and ΔDEF,
PB = DE (By construction)
$\angle B = \angle E$ (Given)
BC = EF (Given)
So, we can conclude that:
$\Delta PBC \cong \Delta DEF$, by the SAS axiom for congruence.

Since the triangles are congruent, their corresponding parts will be equal.

So, $\angle PCB = \angle DFE$

But, we are given that

$\angle ACB = \angle DFE$

So, $\angle ACB = \angle PCB$

Is this possible?

This is possible only if P coincides with A.

or, BA = ED

So, $\Delta ABC \cong \Delta DEF$ (by SAS axiom)

Case (iii) : If AB < DE, we can choose a point M on DE such that ME = AB and repeating the arguments as given in Case (ii), we can conclude that AB = DE and so,

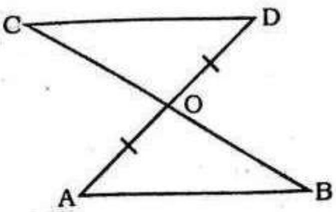
$\Delta ABC \cong \Delta DEF$

Suppose, now in two triangles two pairs of angles and one pair of corresponding sides are equal but the side is not included between the corresponding equal pairs of angles. Are the triangles still congruent? You will observe that they are congruent.

You know that the sum of the three angles of a triangle is 180°. So if two pairs of angles are equal, the third pair is also equal (180° – sum of equal angles).

So, two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal. We may call it as the **AAS Congruence Rule**.

Example 3 : Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD. Show that (i) $\Delta AOB \cong \Delta DOC$ (ii) O is also the mid-point of BC.



Solution : (i) Consider ΔAOB and ΔDOC.

$\angle ABO = \angle DCO$

(Alternate angles as AB || CD and BC is the transversal)

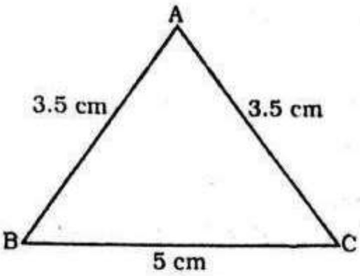
$\angle AOB = \angle DOC$ (Vertically opposite angles)

OA = OD (Given)

Therefore, $\Delta AOB \cong \Delta DOC$ (AAS rule)

(ii) OB = OC (CPCT)

So, O is the mid-point of BC.



Some Properties of a Triangle

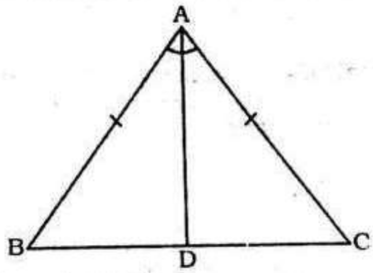
Construct a triangle in which two sides are equal, say each equal to 3.5 cm and the third side equal to 5 cm.

A triangle in which two sides are equal is called an isosceles triangle. So, such triangle is an isosceles triangle.

Theorem 2. Angles opposite to equal sides of an isosceles triangle are equal.

Proof : We are given an isosceles triangle ABC in which AB = AC. We need to prove that $\angle B = \angle C$.

Let us draw the bisector of $\angle A$ and let D be the point of intersection of this bisector of $\angle A$ and BC.



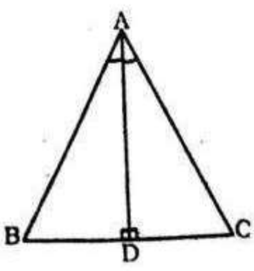
In ΔBAD and ΔCAD,
AB = AC (Given)
$\angle BAD = \angle CAD$ (By construction)
AD = AD (Common)
So, $\Delta BAD \cong \Delta CAD$ (By SAS rule)
So, $\angle ABD = \angle ACD$, since they are corresponding angles of congruent triangles.
So, $\angle B = \angle C$

Theorem 3. The sides opposite to equal angles of a triangle are equal.

This is the converse of Theorem 2.

We can prove this theorem by ASA congruence rule.

Example 4 : In ΔABC, the bisector AD of $\angle A$ is perpendicular to side BC. Show that AB = AC and ΔABC is isosceles.



Solution : In ΔABD and ΔACD,

$\angle BAD = \angle CAD$ (Given)

$AD = AD$ (Common)

$\angle ADB = \angle ADC = 90^\circ$ (Given)

So, $\Delta ABD \cong \Delta ACD$ (ASA rule)

So, $AB = AC$ (CPCT)

or, ΔABC is an isosceles triangle.

Example 5 : E and F are respectively the mid-points of equal sides AB and AC of ΔABC. Show that $BF = CE$.

Solution : In ΔABF and ΔACE,

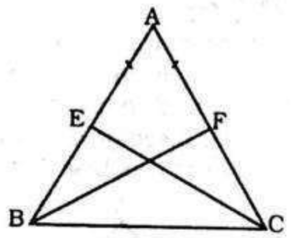
$AB = AC$ (Given)

$\angle A = \angle A$ (Common)

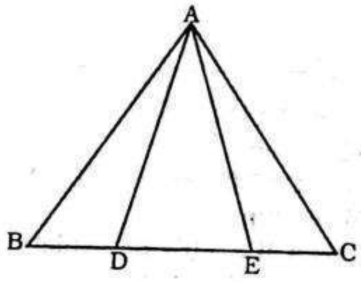
$AF = AE$ (Halves of equal sides)

So, $\Delta ABF \cong \Delta ACE$ (SAS rule)

Therefore, $BF = CE$ (CPCT)



Example 6 : In an isosceles triangle ABC with $AB = AC$, D and E are points on BC such that $BE = CD$. Show that $AD = AE$.



Solution : In ΔABD and ΔACE,

$AB = AC$ (Given)

$AB = AC$ (Given) (1)

$\angle B = \angle C$ (Angles opposite to equal sides) (2)

Also, $BE = CD$

So, $BE - DE = CD - DE$

That is, $BD = CE$ (3)

So, $\Delta ABD \cong \Delta ACE$

(Using (1), (2), (3) and SAS rule).

This gives $AD = AE$ (CPCT)

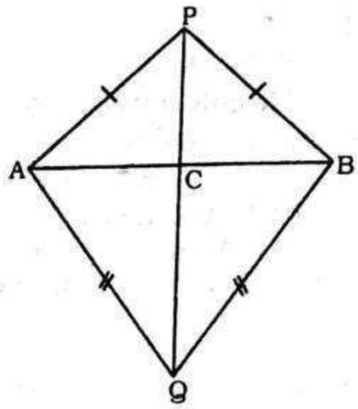
Theorem 4. (SSS congruence rule) : If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

This theorem can be proved using a suitable construction.

Theorem 5. (RHS congruence rule) : If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Note that RHS stands for Right angle Hypotenuse - Side.

Example 7 : AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is the perpendicular bisector of AB.



Solution : We are given that $PA = PB$ and $QA = QB$ and you are to show that $PQ \perp AB$ and PQ bisects AB. Let PQ inersect AB at C.

Let us take ΔPAQ and ΔPBQ.

In these triangles,

$AP = BP$ (Given)

$AQ = BQ$ (Given)

$PQ = PQ$ (Common)

So, $\Delta PAQ \cong \Delta PBQ$ (SSS rule)

Therefore, $\angle APQ = \angle BPQ$ (CPCT).

Now let us consider ΔPAC and ΔPBC.

You have : $AP = BP$ (Given)

$\angle APC = \angle BPC$ ($\angle APQ = \angle BPQ$ proved above)

$PC = PC$ (Common)

So, $\Delta PAC \cong \Delta PBC$ (SAS rule)

Therefore, $AC = BC$ (CPCT) (1)

and $\angle ACP = \angle BCP$ (CPCT)

Also, $\angle ACP + \angle BCP = 180^\circ$ (Linear pair)

So, $2\angle ACP = 180^\circ$

or, $\angle ACP = 90^\circ$ (2)

From (1) and (2), we can easily conclude that PQ is the perpendicular bisector of AB.

Note that, without showing the congruence of ΔPAQ and ΔPBQ, you cannot show that $\Delta PAC \cong \Delta PBC$ even though

$AP = BP$ (Given)

$PC = PC$ (Common)

and $\angle PAC = \angle PBC$ (Angles opposite to equal sides in ΔAPB)

[It is because these results give us SSA rule which is not always valid or true for congruence of triangles. Also the angle is not included between the equal pairs of sides.]

Example 8 : P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.

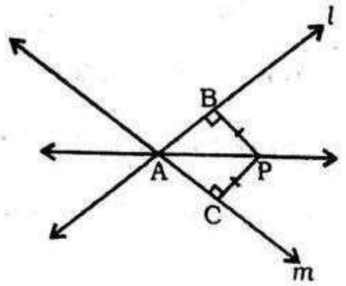
Solution : We are given that lines l and m intersect each other at A. Let PB ⊥ l, PC ⊥ m. It is given that PB = PC.

We are to show that ∠PAB = ∠PAC.

Let us consider ΔPAB and ΔPAC. In these two triangles,

PB = PC (Given)

∠PBA = ∠PCA = 90° (Given)



PA = PA (Common)

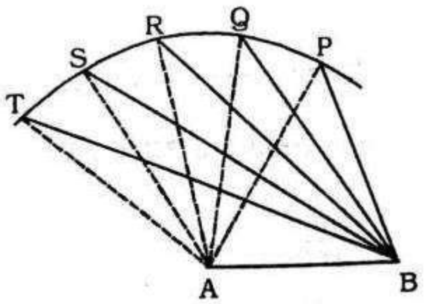
So, ΔPAB ≅ ΔPAC (RHS rule)

So, ∠PAB = ∠PAC (CPCT)

Inequalities in a Triangle

Theorem 6. If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).

We may prove this theorem by taking a point P on BC such that CA = CP in the figure given below.



Theorem 7. In any triangle, the side opposite to the larger (greater) angle is longer.

This theorem can be proved by the method of contradiction.

Now take a triangle ABC and in it, find AB + BC, BC + AC and AC + AB.

We observe that AB + BC > AC,

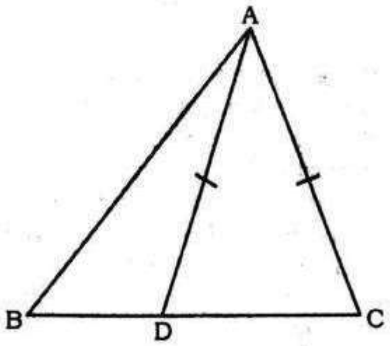
BC + AC > AB and AC + AB > BC.

Theorem 8. The sum of any two sides of a triangle is greater than the third side.

Example 9 : D is a point on side BC of ΔABC such that AD = AC.

Show that AB > AD.

Solution : In ΔDAC,



AD = AC (Given)

So, ∠ADC = ∠ACD

(Angles opposite to equal sides)

Now, ∠ADC is an exterior angle for ΔABD.

So, ∠ADC > ∠ABD

or, ∠ACD > ∠ABD

or, ∠ACB > ∠ABC

So, AB > AC (Side opposite to larger angle in Δ ABC)

or, AB > AD (AD = AC)

SIMILAR FIGURES

The two figures are said to be *congruent*, if they have the same shape and the same size. Two figures having the same shape (and not necessarily the same size) are called *similar figures*.

Similar Figures

We know that all circles with the same radii are congruent, all squares with the same side lengths are congruent and all equilateral triangles with the same side lengths are congruent. Now consider any two (or more) circles. Since all of them do not have the same radius, they are not congruent to each other. Note that some are congruent and some are not, but all of them have the same shape. So they all are, what we call, *similar*. Two similar figures have the same shape but not necessarily the same size. Therefore, all circles are similar. As observed in the case of circles, here also all squares are similar and all equilateral triangles are similar.

From the above, we can say *that all congruent figures are similar but the similar figures need not be congruent.*

Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or *proportion*).

Note that the same ratio of the corresponding sides is referred to as *the scale factor* (or the *Representative Fraction*) for the polygons. You must have heard that world maps (i.e., global maps) and blue prints for the construction of a building are prepared using a suitable scale factor and observing certain conventions.

Similarity of Triangles

Triangle is also a polygon. So, we can state the same conditions for the similarity of two triangles. That is:

Two triangles are similiar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

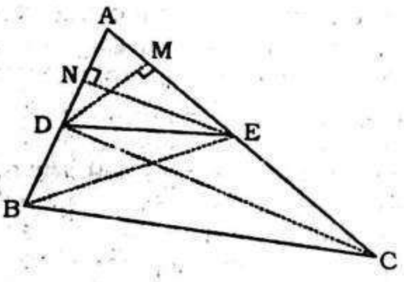
Note that if corresponding angles of two triangles are equal, then they are known as equiangular triangles. A famous Greek mathematician Thales gave an important truth relating to two equiangular triangles which is as follows:

The ratio of any two corresponding sides in two equiangular triangles is always the same.

It is believed that he had used a result called the Basic Proportionality Theorem (now known as the Thales Theorem) for the same.

Theorem 1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof : We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, area of ΔADE

$= \left(= \frac{1}{2}\text{ base} \times \text{height}\right) = \frac{1}{2} AD \times EN.$

So, ar (ADE) = $\frac{1}{2} AD \times EN$

Similarly, ar (BDE) $\frac{1}{2} DB \times EN$,

ar (ADE) = $\frac{1}{2} AE \times DM$ and ar (DEC) = $\frac{1}{2} EC \times DM$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ (1)

and $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ (2)

Now ΔBDE and DEC are on the same base DE and between the same parallels BC and DE.

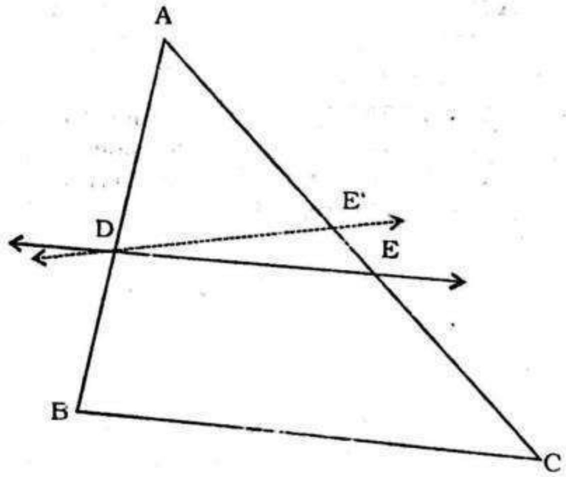
So, ar(BDE) = ar(DEC) (3)

Therefore, from (1), (2) and (3), we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

Theorem 2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

This theorem can be proved by taking a line DE such that $\frac{AD}{DB} = \frac{AE}{EC}$ and assuming that DE is not parallel to BC.



If DE is not parallel to BC, draw a line DE' parallel to BC.

So, $\frac{AD}{DB} = \frac{AE'}{D'C}$

Therefore, $\frac{AE}{EC} = \frac{AE'}{E'C}$

Adding 1 to both sides of above, we see,

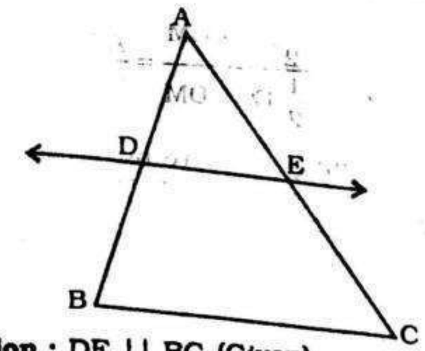
$\frac{AE}{EC} = 1 = \frac{AE'}{E'C} + 1$

$\Rightarrow \frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$

$\Rightarrow \frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow EC = E'C$

Hence, E and E' coincide.

Example 1 : If a line intersects sides AB and AC of a ΔABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$.



Solution : DE || BC (Given)

So, $\frac{AD}{DB} = \frac{AE}{EC}$

or, $\frac{DB}{AD} = \frac{EC}{AE}$

or, $\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

or, $\frac{AB}{AD} = \frac{AC}{AE}$

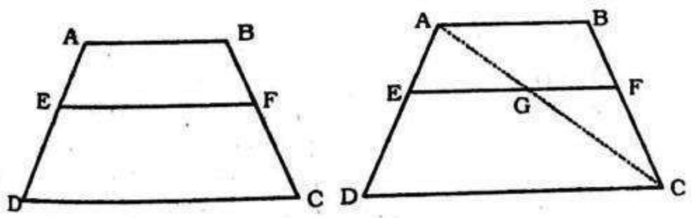
So, $\frac{AD}{AB} = \frac{AC}{AC}$

Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Solution : Let us join AC to intersect EF at G.

AB || DC and EF || AB (Given)

So, EF || DC (Lines parallel to the same line are parallel to each other)



Now, in Δ ADC

EG || DC (As EF || DC)

So, $\frac{AE}{ED} = \frac{AG}{GC}$ (Theorem 1) (1)

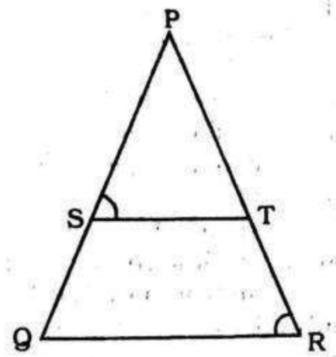
Similarly, from Δ CAB

$\frac{CG}{AG} = \frac{CF}{BF}$ i.e. $\frac{AG}{GC} = \frac{BF}{FC}$ (2)

Therefore, from (1) and (2),

$\frac{AE}{ED} = \frac{BF}{FC}$

Example 3. In the following figure $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



Solution : It is given that $\frac{PS}{SQ} = \frac{PT}{TR}$

So, ST || QR (Theorem 2)

Therefore, $\angle PST = \angle PQR$ (Corresponding angles)(1)

Also, it is given that

$\angle PST = \angle PRQ$ (2)

So, $\angle PRQ = \angle PQR$ [from (1) and (2)]

Therefore, PQ = PR (Sides opposite the equal angles)

i.e. PQR is an isosceles triangle.

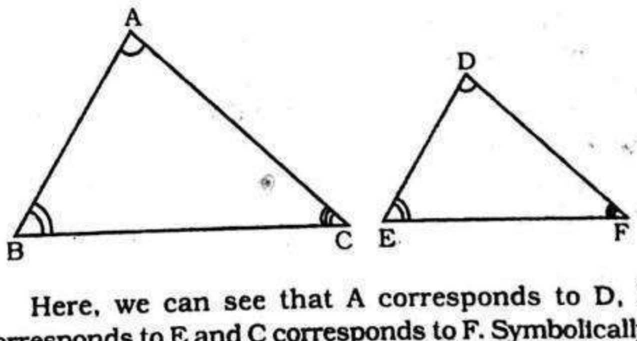
Criteria for Similarity of Triangles

Two triangles are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion).

That is, in Δ ABC and Δ DEF, if

(i) $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then the two triangles are similar.

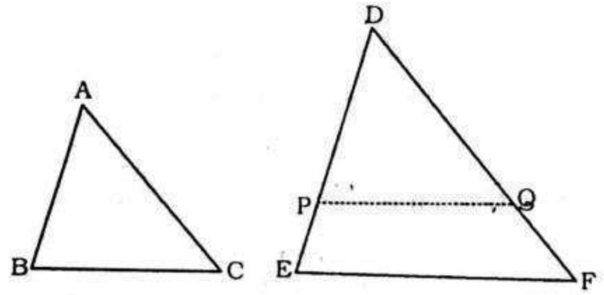


Here, we can see that A corresponds to D, B corresponds to E and C corresponds to F. Symbolically, we write the similarity of these two triangles as 'Δ ABC ~ Δ DEF' and read it as 'triangle ABC is similar to triangle DEF. The symbol '~' stands for 'is similar to'.

It must be noted that as done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF of given similarity, we cannot write Δ ABC ~ Δ EDF or Δ ABC ~ Δ FED. However, we can write Δ BAC ~ Δ EDF.

Theorem 3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles.



This theorem can be proved by taking two triangles ABC and DEF such that $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Cut DP = AB and DQ = AC and join PQ.

So, $\Delta ABC \cong \Delta DPQ$

This gives $\angle B = \angle P = \angle E$ and PQ || EF

Therefore, $\frac{DP}{PE} = \frac{DQ}{QF}$ i.e., $\frac{AB}{DE} = \frac{AC}{DF}$

Similarly, $\frac{AB}{DE} = \frac{BC}{DF}$ and so $\frac{AB}{DE} = \frac{BC}{DF} = \frac{AC}{DF}$.

Remark : If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This may be referred to as the AA similarity criterion for two triangles.

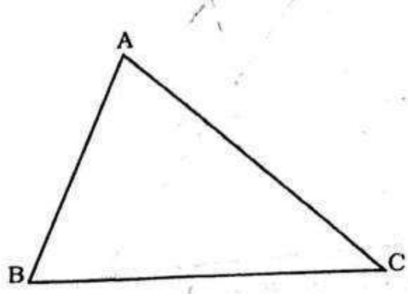
We have seen above that if the three angles of one triangle are respectively equal to the three angles of another triangle, then their corresponding sides are proportional (i.e., in the same ratio).

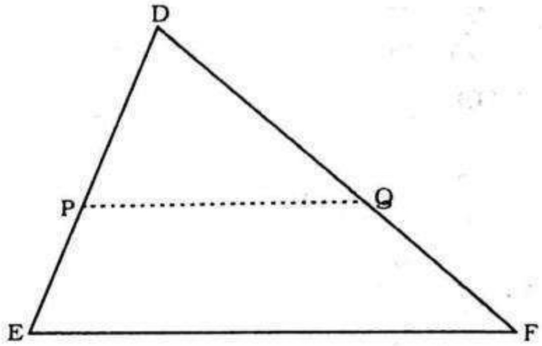
Theorem 4. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.

This criterion is referred to as the SSS (Side–Side–Side) similarity criterion for two triangles.

This theorem can be proved by taking two triangles

ABC and DEF such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ (< 1)





Cut DP = AB and DQ = AC and join PQ.

It can be seen that $\frac{DP}{PE} = \frac{DQ}{QF}$ and PQ || EF

So, $\angle P = \angle E$ and $\angle Q = \angle F$.

Therefore, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

So, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF}$

So, BC = PQ

Thus, $\Delta ABC \cong \Delta DPQ$

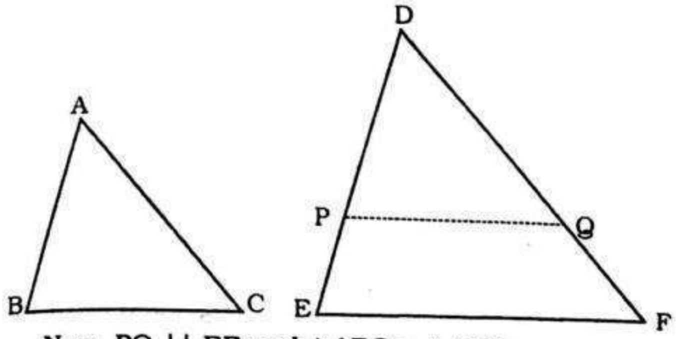
So, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

Remark : We may recall that either of the two conditions namely, (i) corresponding angles are equal and (ii) corresponding sides are in the same ratio is not sufficient for two polygons to be similar. However, on the basis of Theorems 3 and 4, we can now say that in case of similarity of the two triangles, it is not necessary to check both the conditions as one condition implies the other.

Theorem 5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This criterion is referred to as the SAS (Side–Angle–Side) similarity criterion for two triangles.

This theorem can be proved by taking two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{AC}{DF}$ (< 1) and $\angle A = \angle D$. Cut DP = AB, DQ = AC and join PQ.

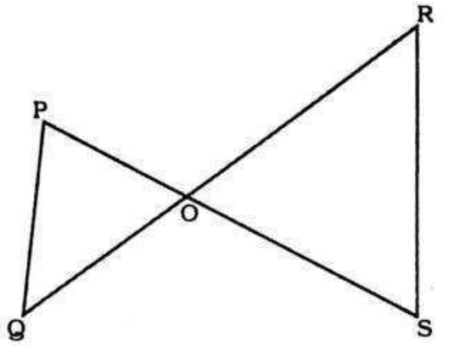


Now, PQ || EF and $\Delta ABC \cong \Delta DPQ$

So, $\angle A = \angle D$, $\angle B = \angle P$ and $\angle C = \angle Q$

Therefore, $\Delta ABC \sim \Delta DEF$

Example 4. If PQ || RS, prove that $\Delta POQ \sim \Delta SOR$.



Solution : PQ || RS (Given)

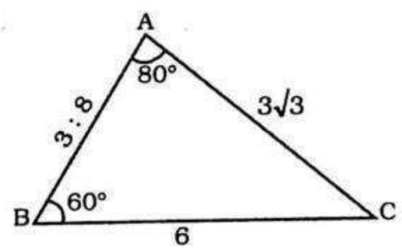
So, $\angle P = \angle S$ (Alternate angles)

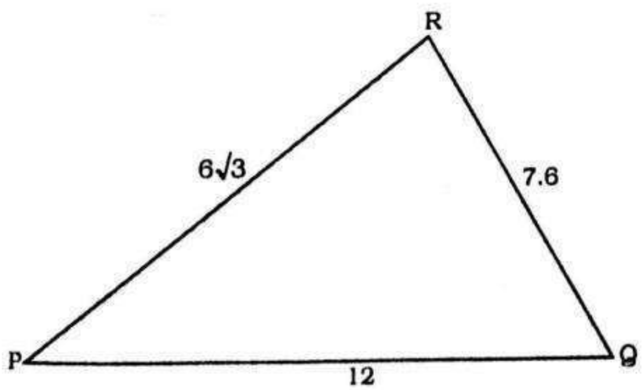
and $\angle Q = \angle R$

Also, $\angle POQ = \angle SOR$ (Vertically opposite angles)

Therefore, $\Delta POQ \sim \Delta SOR$ (AAA similarity criterion)

Example 5 : Observe the figure given below and then find $\angle P$.





Solution : In ΔABC and ΔPQR,

$$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}, \frac{BC}{QP} = \frac{6}{12} = \frac{1}{2} \text{ and } \frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

That is, $\frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$

So, $\Delta ABC \sim \Delta RQP$ (SSS similarity)

Therefore $\angle C = \angle P$

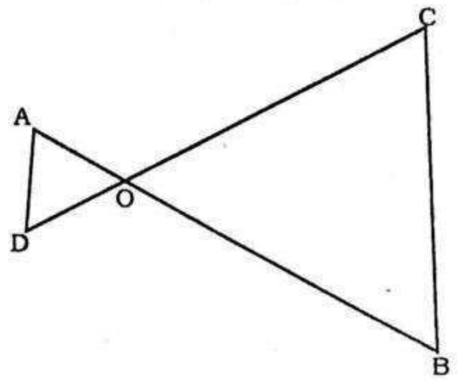
(Corresponding angles of similar triangles)

But, $\angle C = 180° - \angle A - \angle B$ (Angle sum property)

$= 180° - 80° - 60° = 40°$

So, $\angle P = 40°$

Example 6. In the following figure.



OA . OB = OC . OD.

Show that $\angle A = \angle C$ and $\angle B = \angle D$.

Solution : OA . OB = OC . OD (Given)

So, $\frac{OA}{OC} = \frac{OD}{OB}$ (1)

Also, we have $\angle AOD = \angle COB$ (Vertically opposite angles) (2)

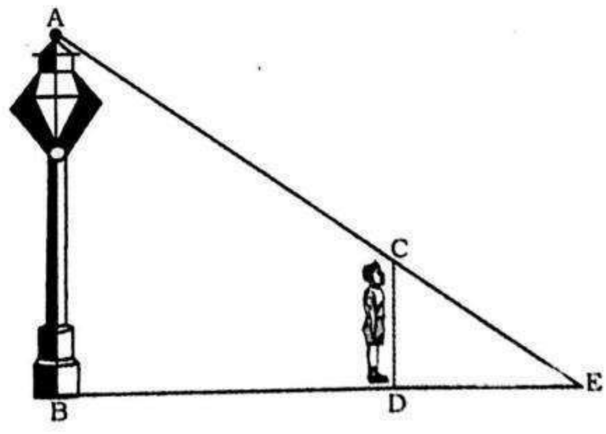
Therefore, from (1) and (2),

$\Delta AOD \sim \Delta COB$ (SAS similarity criterion)

So, $\angle A = \angle C$ and $\angle D = \angle B$

(Corresponding angles of similar triangles)

Example 7. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



Solution : Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post. From the figure, you can see that DE is the shadow of the girl. Let DE be x metres.

Now, BD = 1.2 m × 4 = 4.8m

Note that in Δ ABE and Δ CDE

∠ B = ∠ D (Each is of 90° because lamp-post as well as the girl are standing vertical to the ground)

and ∠ E = ∠ E (Same angle)

So, Δ ABE ~ Δ CDE (AA similarity criterion)

Therefore, $\frac{BE}{DE} = \frac{AB}{CD}$

i.e., $\frac{4.8 + x}{x} = \frac{3.6}{0.9}$ $\left(90cm = \frac{90}{100} m = 0.9m\right)$

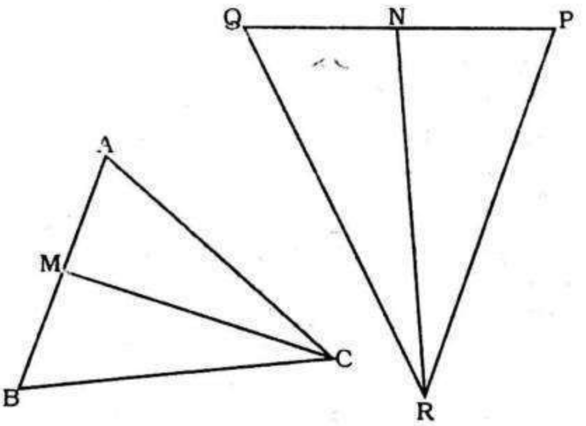
i.e., $4.8 + x = 4x$

i.e., $3x = 4.8$

i.e., $x = 1.6$

So, the shadow of the girl after walking for 4 seconds is 1.6 m long.

Example 8. In the following figure, CM and RN are respectivelly the medians of Δ ABC and Δ PQR. If Δ ABC ~ Δ PQR, prove that :



(i) Δ AMC ~ Δ PNR

(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

(iii) Δ CMB ~ Δ RNQ

Solution : (i) Δ ABC ~ Δ PQR (Given)

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ (1)

and ∠ A = ∠ P, ∠ B = ∠ Q and ∠ C = ∠ R (2)

But, AB = 2 AM and PQ = 2PN

(As CM and RN are medians)

So, from (1) $\frac{2AM}{2\,PN} = \frac{CA}{RP}$

i.e., $\frac{AM}{PN} = \frac{CA}{RP}$ (3)

Also, ∠ MAC = ∠ NPR [From (2)] (4)

So, from (3) and (4),

ΔAMC ~ ΔPNR (SAS similarity) (5)

(ii) From (5) $\frac{CM}{RN} = \frac{CA}{RP}$ (6)

But, $\frac{CA}{RP} = \frac{AB}{PQ}$ [From (1) (7)

Therefore, $\frac{CM}{RN} = \frac{AB}{PQ}$ [From (6) and (7)] (8)

(iii) Again $\frac{AB}{PQ} = \frac{BC}{QR}$ From (1)

Therefore $\frac{CM}{RN} = \frac{BC}{QR}$ [From (8)] (9)

Also, $\frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$

i.e. $\frac{CM}{RN} = \frac{BM}{QN}$ (10)

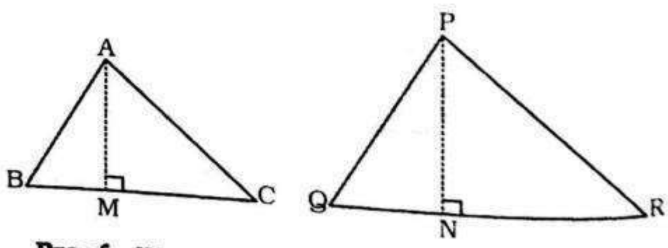
i.e., $\frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$ [From (9) and (10)]

Therefore, Δ CMB ~ Δ RNQ (SSS similarity)

Areas of Similar Triangles

We have learnt that in two similar triangles, the ratio of their corresponding sides is the same. Do you think there is any relationship between the ratio of their areas and the ratio of the corresponding sides? We know that area is measured in square units. So, we may expect that this ratio is the square of the ratio of their corresponding sides. This is indeed true.

Theorem 6. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.



Proof : We are given ΔABC and ΔPQR such that Δ ABC ~ Δ PQR.

We need to prove that

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, ar (ABC) = $\frac{1}{2}$ BC × AM

and ar (PQR) = $\frac{1}{2}$ QR × PN

$$\text{So, } \frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN} \qquad (1)$$

Now, in Δ ABM and Δ PQN,
∠ B = ∠ Q (As Δ ABC ~ Δ PQR)
and ∠ M = ∠ N (Each of 90°)
So, Δ ABM ~ Δ PQN (AA similarity criterion)

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ (2)

Also, Δ ABC ~ Δ PQR (given)

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ (3) (Given)

Therefore $\frac{ar\,(ABC)}{ar\,(PQR)} = \frac{AB}{PQ} = \frac{AM}{PN}$ [From (1) and (3)]

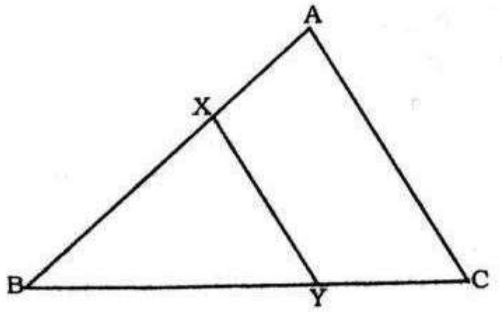
$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (2)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (3), we get

$$\frac{ar\,(ABC)}{ar\,(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Example 9 : In the following figure, the line segment XY is parallel to side AC of Δ ABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$.



Solution : XY || AC Given
So, ∠ BXY = ∠ A and ∠ BYX = ∠ C (Corresponding angles)
Therefore, Δ ABC ~ Δ XBY (AA similarity criterion)

So, $\frac{ar\,(ABC)}{ar\,(XBY)} = \left(\frac{AB}{XB}\right)^2$ (Theorem 6) (1)

Also, ar (ABC) = 2 ar (XBY) (Given)

So, $\frac{ar\,(ABC)}{ar\,(XBY)} = \frac{2}{1}$ (2)

Therefore, from (1) and (2)

$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$, i.e., $\frac{AB}{XB} = \frac{\sqrt{2}}{1}$.

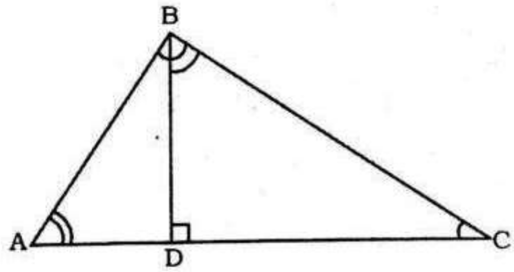
or, $\frac{XB}{AB} = \frac{1}{\sqrt{2}}$ or, $1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$

or, $\frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$, i.e.,

or, $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

Pythagoras Theorem

Now, let us take a right triangle ABC, right angled at B. Let BD be the perpendicular to the hypotenuse AC.



We may note that in Δ ADB and Δ ABC
∠ A = ∠ A
and ∠ ADB = ∠ ABC
So, Δ ADB ~ Δ ABC (1)
Similarly, Δ BDC ~ Δ ABC (2)

So, from (1) and (2), triangles on both sides of the perpendicular BD are similar to the whole triangle ABC.

Also, since Δ ADB ~ Δ ABC
and Δ BDC ~ ∠ ABC
So, Δ ADB ~ Δ BDC

The above discussion leads to the following theorem :

Theorem 7. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

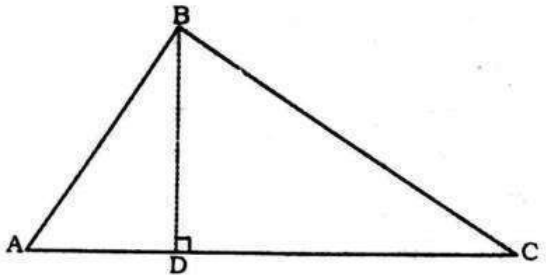
Let us now apply this theorem in proving the Pythagoras Theorem :

Theorem 8. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Proof : We are given a right triangle ABC right angled at B.

We need to prove that $AC^2 = AB^2 + BC^2$

Let us draw $BD \perp AC$



Now, $\Delta ADB \sim \Delta ABC$ (From Theorem)

So $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

or, $AD \cdot AC = AB^2$ (1)

Also, $\Delta BDC \sim \Delta ABC$ (From Theorem)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or, $CD \cdot AC = BC^2$ (2)

Additing (1) and (2),

$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$

or, $AC (AD + CD) = AB^2 + BC^2$

or, $AC \cdot AC = AB^2 + BC^2$

or, $AC^2 = AB^2 + BC^2$

The above theorem was earlier given by an ancient Indian mathematician Baudhayan (about 800 B.C.) in the following form :

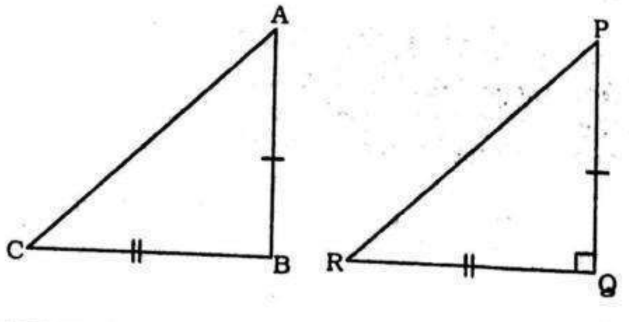
The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e., length and breadth).

For this reason, this theorem is sometimes also referred to as the *Baudhayan Theorem.*

Theorem 9. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Proof : Here, we are given a triangle ABC in which $AC^2 = AB^2 + BC^2$.

We need to prove that $\angle B = 90°$.



To start with, we construct a ΔPQR right angled at Q such that $PQ = AB$ and $QR = BC$.

Now, from ΔPQR, we have :

$PR^2 = PQ^2 + QR^2$

(Pythagoras Theorem, as $\angle Q = 90°$)

or, $PR^2 = AB^2 + BC^2$ (By construction) (1)

But, $AC^2 = AB^2 + BC^2$ (Given) (2)

But, $AC = PR$ [From (1) and (2)]

Now, ΔABC and ΔPQR,

$AB = PQ$ (By construction)

$BC = QR$ (By construction)

$AC = PR$ [Proved in (3) above]

So, $\Delta ABC \cong \Delta PQR$ (SSS congruence)

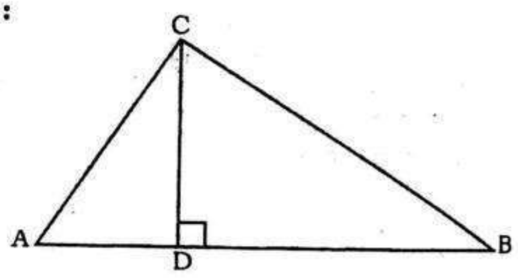
Therefore, $\angle B = \angle Q$ (CPCT)

But $\angle Q = 90°$ (By construction)

So, $\angle B = 90°$

Example 10. In the following figure, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

Solution :



$\Delta ACD \sim \Delta ABC$ (Theorem)

So, $\frac{AC}{AB} = \frac{AD}{AC}$

or, $AC^2 = AB \cdot AD$ (1)

Similarly, $\Delta BCD \sim \Delta BAC$ (Theorem)

So, $\frac{BC}{BA} = \frac{BD}{BC}$

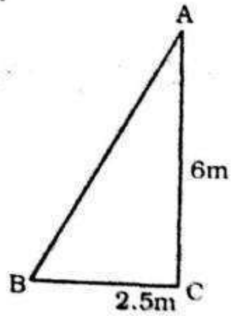
or, $BC^2 = BA \cdot BD$ (2)

Therefore, from (1) and (2)

$\frac{BC^2}{AC^2} = \frac{BA.BD}{AB.AD} = \frac{BD}{AD}$

Example 11 : A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.

Solution : Let AB be the ladder and CA be the wall with the window at A. Also, BC = 2.5 m and CA = 6 m



From Pythagoras Theorem, we have :

$AB^2 = BC^2 + CA^2$

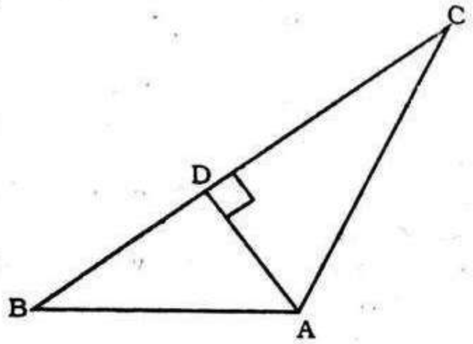
$= (2.5)^2 + (6)^2 = 42.25$

So, AB = 6.5

Thus, length of the ladder is 6.5m.

Example 12. In the following figure, if AD ⊥ BC, prove that $AB^2 + CD^2 = BD^2 + AC^2$.

Solution :



From Δ ADC, we have

$AC^2 = AD^2 + CD^2$ (Pythagoras Theorem) (1)

From Δ ADB, we have

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem) (2)

Subtracting (1) from (2), we have

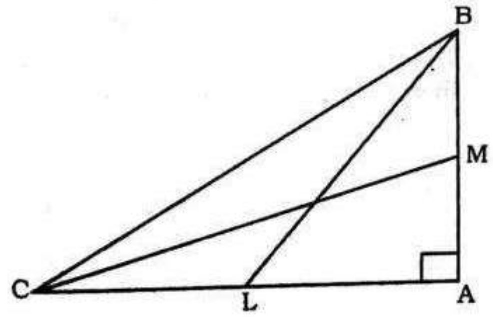
$AB^2 - AC^2 = BD^2 - CD^2$

or, $AB^2 + CD^2 = BD^2 + AC^2$

Example 13. BL and CM are medians of a triangle ABC right angled at A. Prove that

$4(BL^2 + CM^2) = 5\,BC^2$

Solution : BL and CM are medians of a Δ ABC in which ∠A = 90°



From Δ ABC, BC^2

$= AB^2 + AC^2$ (Pythagoras Theorem) (1)

From Δ ABL, $BL^2 = AL^2 + AB^2$

or, $BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$ (L is the mid-point of AC)

or, $BL^2 = \frac{AC^2}{4} + AB^2$

or, $4\,BL^2 = AC^2 + 4\,AB^2$ (2)

From Δ CMA, $CM^2 = AC^2 + AM^2$

or, $CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$ (M is the mid-point of AB)

or, $CM^2 = AC^2 + \frac{AB^2}{4}$

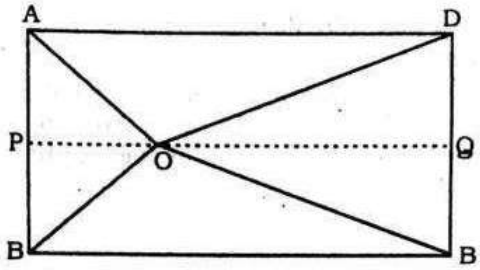
or, $4\,CM^2 = 4\,AC^2 + AB^2$ (3)

Adding (2) and (3), we have

$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$

i.e. $4(BL^2 + CM^2) = 5\,BC^2$ [From (1)]

Example 14. O is any point inside a rectangle ABCD.



Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Solution : Through O, draw PQ || BC so P lies on AB and Q lies on DC.

Now, PQ || BC

Therefore, PQ ⊥ AB and PQ ⊥ DC

(∠ B = 90° and ∠ C = 90°)

So, ∠ BPQ = 90° and ∠ CQP = 90°

Therefore, BPQC and APQD are both rectangles.

Now, From Δ OPB, $OB^2 = BP^2 + OP^2$ (1)

Similarly, from Δ OQD, $OD^2 = OQ^2 + DQ^2$ (2)

From Δ OQC, we have

$OC^2 = OQ^2 + CQ^2$ (3)

and from Δ OAP, we have

$OA^2 = AP^2 + OP^2$ (4)

Adding (1) and (2),

$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$

$= CQ^2 + OP^2 + OQ^2 + AP^2$

(As BP = CQ and DQ = AP)

$= CQ^2 + OQ^2 + OP^2 + AP^2$

$= OC^2 + OA^2$ [From (3) and (4)]

HERON'S FORMULA

Unit of measurement for length or breadth is taken as metre (m) or centimetre (cm) etc.

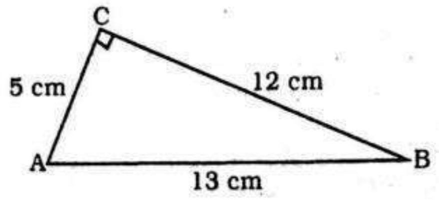
Unit of measurement for area of any plane figure is taken as square metre (m^2) or square centimetre (cm^2) etc.

Suppose that you are sitting in a triangular garden. How would you find its area?

You know that:

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height} \quad (I)$$

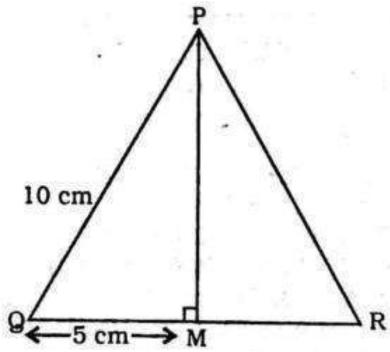
We see that when the triangle is **right angled**, we can directly apply the formula by using two sides containing the right angle as base and height. For example, suppose that the sides of a right triangle ABC are 5 cm, 12 cm and 13 cm; we take base as 12 cm and height as 5 cm. Then the area of ΔABC is given by



$\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 12 × 5 cm^2, i.e., 30 cm^2

Note that we could also take 5 cm as the base and 12 cm as height.

Now suppose we want to find the area of an equilateral triangle PQR with side 10cm. To find its area we need its height. Can you find the height of this triangle?



Take the mid-point of QR as M and join it to P. We know that PMQ is a right triangle. Therefore, by using Pythagoras Theorem, we can find the length PM as shown below:

$PQ^2 = PM^2 + QM^2$

i.e., $(10)^2 = PM^2 + (5)^2$,

since QM = MR.

Therefore, we have $PM^2 = 75$

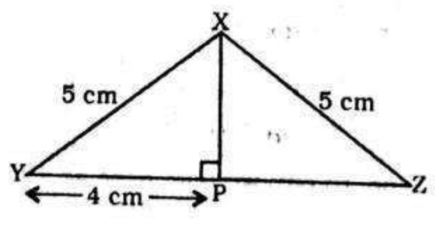
i.e., PM = 75 cm = $5\sqrt{3}$ cm.

Then area of . PQR

$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times 5\sqrt{3}\ cm^2 = 5\sqrt{3}\ cm^2.$

$= 25\sqrt{3}\ cm^2$

Let us see now whether we can calculate the area of an isosceles triangle also with the help of this formula. For example, we take a triangle XYZ with two equal sides XY and XZ as 5 cm each and unequal side YZ as 8 cm.



In this case also, we want to know the height of the triangle. So, from X we draw a perpendicular XP to side YZ. You can see that this perpendicular XP divides the base YZ of the triangle in two equal parts.

Therefore, YP = PZ = $\frac{1}{2}$ YZ = 4 cm

Then, by using Pythagoras theorem, we get

$XP^2 = XY^2 - YP^2$

$= 5^2 - 4^2 = 2^5 - 16 = 9$

So, XP = 3 cm

Now, area of $\Delta XYZ = \frac{1}{2}$ × base YZ × height XP

$= \frac{1}{2} \times 8 \times 3\ cm^2 = 12\ cm^2.$

Area of a Triangle — by Heron's Formula

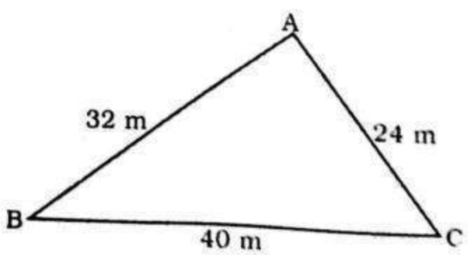
Heron was born in about 10AD possibly in Alexandria in Egypt. He worked in applied mathematics. His works on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of squares, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinders, cones, spheres etc. In this book, Heron has derived the famous formula for the area of a triangle in terms of its three sides.

The formula given by Heron about the area of a triangle, is also known as Hero's formula. It is stated as:

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$	(II)

where a, b and c are the sides of the triangle, and s = semi-perimeter i.e. half the perimeter of the triangle = $\frac{a+b+c}{2}$

This formula is helpful where it is not possible to find the height of the triangle easily. Let us apply it to calculate the area of the triangular park ABC, mentioned above.



Let us take a = 40m, b = 24m, c = 32m, so that we have

$s = \frac{40+24+32}{2} m = 48$ m.

$s - a = (48 - 40)m = 8\ m.$

$s - b = (48 - 24)m = 24m,$

$s - c = (48 - 32)m = 16m.$

Therefore, area of the park ABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48 \times 8 \times 24 \times 16}\ m^2 = 384\ cm^2$

We see that $32^2 + 24^2 = 1024 + 576 = 1600 = 40^2$. Therefore, the sides of the park make a right triangle. The largest side i.e. BC which is 40 m will be the hypotenuse and the angle between the sides AB and AC will be $90°$.

By using Formula I, we can check that the area of the park is

$\frac{1}{2} \times 32 \times 24 m^2 = 384\ m^2$

We find that the area we have got is the same as we found by using Heron's formula.

Now using Heron's formula, you verify this fact by finding the areas of other triangles discussed earlier viz:

(i) equilateral triangle with side 10 cm.

(ii) isosceles triangle with unequal side as 8 cm and each equal side as 5 cm.

You will see that

For (i), we have $s = \frac{10+10+10}{2}$ cm = 15 cm.

Area of triangle = $\sqrt{15(15-10)(15-10)(15-10)}$ cm²

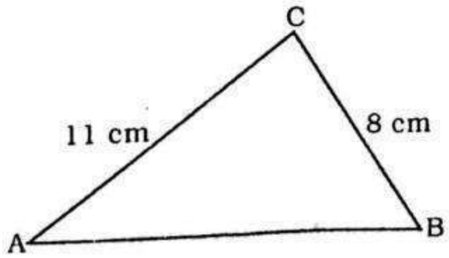
$\sqrt{15 \times 5 \times 5 \times}\ cm^2 = 25\sqrt{3}\ cm^2$

For (ii), we have $s = \frac{8+5+5}{2} cm = 9cm$

Area of triangle = $\sqrt{9(9-8)(9-5)(9-5)}$ cm²

$= \sqrt{9 \times 1 \times 4 \times 4}\ cm^2 = 12\ cm^2$

Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm



Solution : Here we have perimeter of the triangle = 32 cm, $a = 8$ cm and $b = 11$ cm.

Third side $c = 32$ cm − (8 + 11) cm = 13 cm

So, $2s = 32$ i.e. $s = 16$ cm,

$s - a = (16 - 8)$ cm = 8 cm.

$s - b = (16 - 11)$ cm = 5 cm.

$s - c = (16 - 13)$ cm = 3 cm.

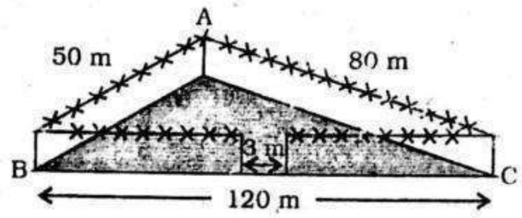
Therefore, area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16 \times 8 \times 5 \times 3} cm^2 = 8\sqrt{30} cm^2$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m. A gardener Kalawati has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of Rs 20 per metre leaving a space 3m wide for a gate on one side.

Solution : For finding area of the park, we have



$2s = 50m + 80m + 120m = 250m.$

i.e., $s = 125m$

Now, $s - a = (125 - 120)m = 5m,$

$s - b = (125 - 80)m = 45m,$

$s - c = (125 - 50)m = 75m.$

Therefore, area of the park

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75} m^2$

$= 375\sqrt{15} m^2$

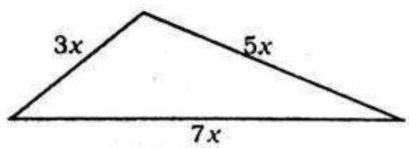
Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m − 3 m (to be left for gate) = $247m$

And so the cost of fencing = Rs 20 × 247 = Rs 4940

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Solution : Suppose that the sides, in metres, are $3x$, $5x$ and $7x$.



Then, we know that $3x + 5x + 7x = 300$ (perimeter of the triangle)

Therefore, $15x = 300$, which gives $x = 20$.

So the sides of the triangle are $3 \times 20m$, $5 \times 20m$ and $7 \times 20m$

i.e., $60m$, $100m$ and $140m$.

We have $s = \frac{60+100+140}{2}\ m = 150m.$

and area will be

$\sqrt{150(150-60)(150-100)(150-140)}m^2$

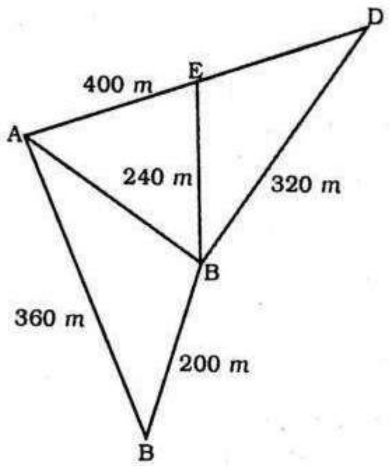
$= \sqrt{150 \times 90 \times 50 \times 10}m$

$= 1500\sqrt{3}m^2$

Application of Heron's Formula in Finding Areas of Quadrilaterals

Example 4 : Dulari has a triangular field with sides 240m, 200m, 360 m, where she grew wheat. In another triangular field with sides 240m, 320m, 400m adjacent to the previous field, she wanted to grow potatoes and onions. She divided the field in two parts by joining the mid-point of the longest side to the opposite verte x and grew patatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m2)

Solution : Let ABC be the field where wheat is grown. Also let ACD be the field which has been divided in two parts by joining C to the mid-point E of AD. For the area of triangle ABC, we have



$a = 200m,\ b = 240m,\ c = 360m$

Therefore, $s = \frac{200+240+360}{2}m = 400m$

So, area for growing wheat

$= \sqrt{400(400-200)(400-240)400-360)}\,m^2$

$= \sqrt{400 \times 200 \times 160 \times 40}m^2$

$= 16000\sqrt{2}m^2 = 1.6 \times \sqrt{2}$ hectares

= 2.26 hectares (nearly)

Let us now calculate the area of triangle ACD.

Here, we have $s = \frac{240+320+400}{2}m = 480m$

So, area of ΔACD

$= \sqrt{480(480-240)(480-320)(480-400)}m^2$

$= \sqrt{480 \times 240 \times 160 \times 80}m^2$

$= 38400m^2 = 3.84$ hectares

We notice that the line segment joining the mid-point E of AD to C divides the triangle ACD in two parts equal in area.

Therefore, area for growing potatoes = area for growing onions

= (3.84 + 2) hectares

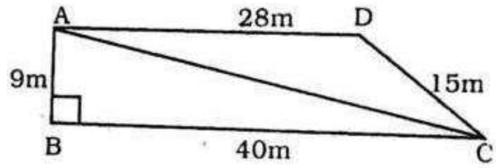
= 1.92 hectares.

Example 5 : Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If AB = 9m, BC = 40m, CD = 15m, DA = 28m and $\angle B = 90°$, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes).

Solution : Since AB = 9 m

and BC = 40 m, $\angle B = 90°$,

we have:



$AC = \sqrt{9^2 + 40^2}m^2$

$= \sqrt{81+1600}m$

$= \sqrt{1681}m = 41m$

Therefore, the first group has to clean the area of triangle ABC, which is right angled.

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 40 \times 9\ m^2 = 180\ m^2$

The second group has to clean the area of triangle ACD, which is scalene having sides

41 m, 15 m and 28 m.

Here, $s = \frac{41+15+28}{2}m = 42m$

Therefore, area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)}m^2$

$= \sqrt{42 \times 1 \times 27 \times 14}m^2 = 126m^2$

So first group cleaned $180m^2$ which is $(180 - 126)$ m^2, i.e., $54m^2$ more than the area cleaned by the second group.

Total area cleaned by all the students

$= (180 + 126)\ m^2 = 306\ m^2$.

Example 6 : Savita has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

Solution : Let ABCD be the field.

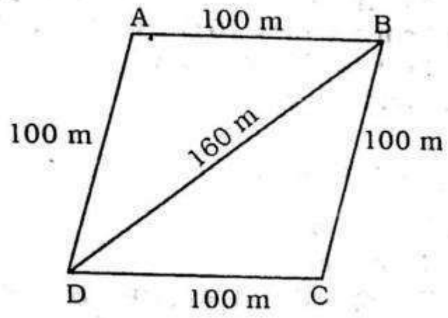
Perimeter = $400m$

So, each side = $400m \div 4 = 100m$.

i.e. AB = AD = $100m$.

Let diagonal BD = $160m$.

Then semi-perimeter s of ΔABD is given by



$$s = \frac{100+100+160}{2} m = 180m$$

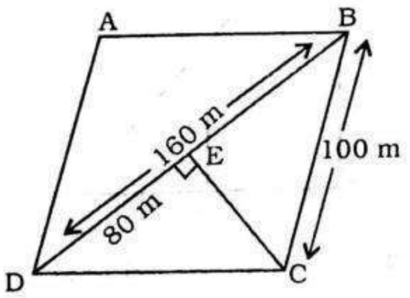
Therefore, area of ΔABD

$$= \sqrt{180(180-100)(180-100)(180-160)} m^2$$

$$= \sqrt{180 \times 80 \times 80 \times 20} m^2 = 4800 m^2$$

Therefore, each of them will get an area of 4800 m^2.

Alternative method : Draw CE $\perp$ BD.



As BD = 160 m, we have

DE = $160m \div 2 = 80m$

And, $DE^2 + CE^2 = DC^2$, which gives

$$CE = \sqrt{DC^2 - DE^2}$$

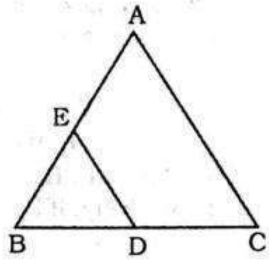
or, $CE = \sqrt{100^2 - 80^2} m = 60m$

Therefore, area of $\Delta BCD = \frac{1}{2} \times 160 \times 60 \; m^2$

$= 4800 m^2$

SOLVED OBJECTIVE QUESTIONS

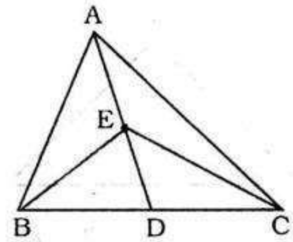
1. ΔABC and ΔBDE are two equilateral triangles such that D is the midpoint of BC. Then, ar(ΔBDE) : ar(ΔABC) = ?



(1) 1 : 2 (2) 1 :4

(3) $\sqrt{3}$: 2 (4) 3 : 4

2. The vertex A of ΔABC is joined to a point D on BC. If E is the midpoint of AD, then ar(ΔBEC) =?



(1) $\frac{1}{2}$ ar(ΔABC) (2) $\frac{1}{3}$ ar(ΔABC)

(3) $\frac{1}{4}$ ar(ΔABC) (4) $\frac{1}{6}$ ar(ΔABC)

3. If the three sides of one triangle are equal to the corresponding sides of the other triangle then the triangles are :

(1) congruent

(2) similar

(3) congruent and similar

(4) None of these

4. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include :

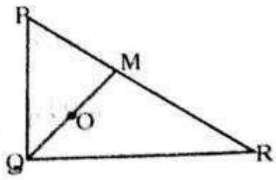
(1) half the sum of the base angles

(2) sum of the base angles

(3) half the difference of the base angles

(4) difference of the base angles

5. If in the figure given $\angle PQR = 90°$, O is the centroid of Δ PQR, PQ = 5 cm and QR = 12 cm, then OQ is equal to :



(1) $3\frac{1}{2}$ cm (2) $4\frac{1}{3}$ cm

(3) $4\frac{1}{2}$ cm (4) $5\frac{1}{3}$ cm

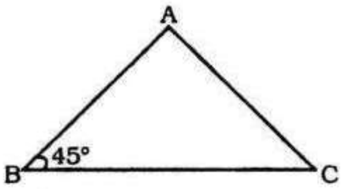
6. If in a triangle ABC, $\angle B = 2\angle C$ and the bisector of $\angle B$ meets CA in D, then the ratio BD : DC would be equal to :

(1) AD : AC (2) AB : AD
(3) AB : AC (4) AC : AB

7. In a triangle ABC, BD and CE are perpendiculars on AC and AB respectively. If BD = CE then the Δ ABC is :

(1) equilateral (2) isosceles
(3) right-angled (4) scalene

8. $\angle ABC$ is equal to 45° as shown in the adjoining figure. If $\frac{AC}{AB} = \sqrt{2}$, then $\angle BAC$ is equal to :



(1) 95° (2) 100°
(3) 105° (4) 110°

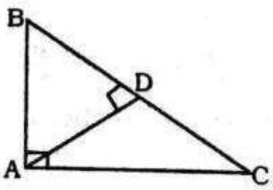
9. If PL, QM and RN are the altitudes of Δ PQR whose orthocentre is O, then P is the orthocentre of :

(1) ΔPQO (2) ΔPQL
(3) ΔQLO (4) ΔQRO

10. ΔABC is such that AB = 3 cm, BC = 2 cm and AC = 2.5 cm. ΔDEF is similar to ΔABC. IF EF = 4 cm, then the perimeter of ΔDEF is :

(1) 5 cm (2) 7.5 cm
(3) 15 cm (4) 18 cm

11. Which of the following is true in the given figure where AD is the altitude to the hypotenuse of a right angled ΔABC?

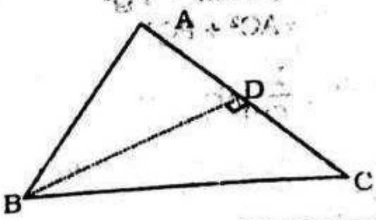


I. $\Delta ABD \sim \Delta CAD$
II. $\Delta ABD \cong \Delta CDA$
III. $\Delta ADB \sim \Delta CAB$

Of these statements the correct ones are combinations of :

(1) I and II (2) I and III
(3) II and III (4) I, II and III

12. Let ABC be an isosceles triangle in which AB = AC and BD $\perp$ AC. Then $BD^2 - CD^2$ is euqal to :



(1) 2 DC.AD (2) 2AD.BC
(3) 3DC.AD (4) $\frac{1}{2}$ AD.DC

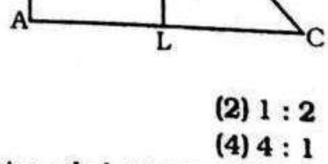
13. If D, E and F are respectively the mid-points of sides BC, AC and AB of a Δ ABC. If EF = 3 cm, FD = 4 cm and AB = 10 cm, then DE, BC and CA respectively will be equal to :

(1) 6, 8 and 20 cm
(2) $\frac{10}{3}$, 9 and 12 cm
(3) 4, 6 and 8 cm
(4) 5, 6 and 8 cm

14. In Δ PQR, $\angle Q = 3a$, $\angle P = a$, $\angle R = b$ and $3b - 5a = 30$, then the triangle is :

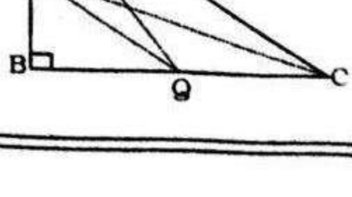
(1) scalene (2) isosceles
(3) equilateral (4) right-angled

15. In a ΔABC shown in the figure M $\angle A = 90°$. Let D be a point on BC such that BD : DC = 1 : 3. If DM and DL respectively are perpendiculars on AB and AC, then DM and LC are in the ratio of :



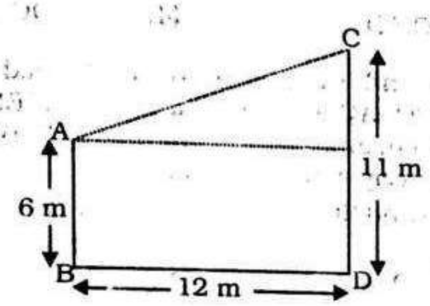
(1) 1 : 3 (2) 1 : 2
(3) 1 : 1 (4) 4 : 1

16. In a right-angled ΔABC, right-angled at B, if P and Q are points on the sides AB and AC respectively, then :



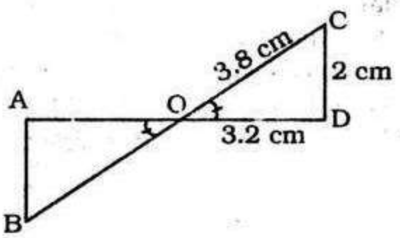
(1) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(2) $2(AQ^2 + CP^2) = AC^2 + PQ^2$
(3) $AQ^2 + CP^2 = AC^2 + PQ^2$
(4) $AQ + CP = \frac{1}{2}(AC + PQ)$

17. In the adjoning figure DC and AB are two poles of lengths 11 m and 6 m respectively. Distance between their feet is 12 m. The distance between their tops is :



(1) 5 m
(2) 6 m
(3) 13 m
(4) 17 m

18. In the adjoining figure, $\Delta ABO \sim \Delta DCO$. If AB = 3 cm, CD = 2cm, OC = 3.8 cm and OD = 3.2 cm, then (OA + OB) is equal to :



(1) 4.8 cm
(2) 5.7 cm
(3) 10.5 cm
(4) 11.5 cm

19. Incentre of a triangle lies in the interior of :
(1) an equilateral triangle only
(2) an isosceles triangle only
(3) right triangle only
(4) any triangle

20. The lengths of the perpendiculars drawn from any point in the interior of an equilateral triangle to the respective sides are p_1, p_2 and p_3. The length of each side of the triangle is

(1) $\frac{2}{\sqrt{3}}(p_1 + p_2 + p_3)$
(2) $\frac{1}{3}(p_1 + p_2 + p_3)$
(3) $\frac{1}{\sqrt{3}}(p_1 + p_2 + p_3)$
(4) $\frac{4}{\sqrt{3}}(p_1 + p_2 + p_3)$

21. In a triangle ABC, the sum of the exterior angles at B and C is equal to :

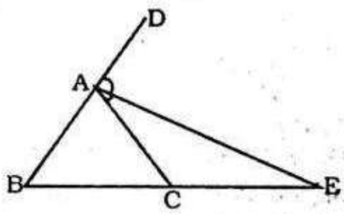
(1) $180° + \angle BAC$
(2) $180° - \frac{1}{2}\angle BAC$
(3) $180° + \frac{1}{2}\angle BAC$
(4) $180° + 2\angle BAC$

22. PQR is a triangle such that PQ = 10cm and PR = 3 cm the side QR is :
(1) equal to 7 cm
(2) greater than 7 cm
(3) less than 7 cm
(4) None of these

23. If D and E are points on the sides AB and AC respectively of a Δ ABC such that DE || BC. If AD = x, DB = $x - 2$, AE = $x + 2$ and EC = $x - 1$. The value of x is :
(1) 2.5
(2) 2
(3) 3
(4) 4

24. Inscribed $\angle ACB$ intercepts AB of circle with centre O. If the bisector of $\angle ACB$ meets arc AB in M then :
(1) m AM > m MB
(2) m AM < m MB
(3) m AM = m MB
(4) None of these

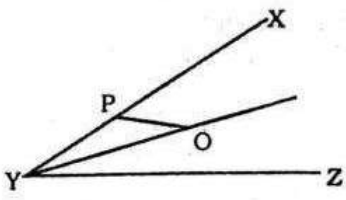
25. In the adjoining figure, AE is the bisector of exterior $\angle CAD$ meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, then CE is equal to :



(1) 6 cm
(2) 12 cm
(3) 18 cm
(4) 20 cm

26. If S is the circumcentre of a Δ ABC, then :
(1) S is equidistant from its sides
(2) S is equidistant from its vertices
(3) SA, SB, SC are the angular bisectors
(4) AS, BS, CS produced are the altitudes on the opposite sides

27. O is any point on the bisector of the acute angle $\angle XYZ$. The line OP is parallel to ZY. Then ΔYPO is :



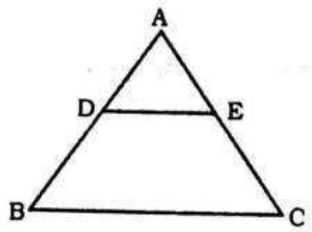
(1) scalene
(2) isosceles but not right-angled
(3) equilateral
(4) right-angled and isosceles

28. Consider the statements :
I. Two of the angles are obtuse.
II. Two of the angles are acute.
III. Each angle is less than 60°.
IV. Each angle is equal to 60°.

In which case/cases is it possible to have a triangle?
(1) All the above cases
(2) I only
(3) I and III only
(4) II and IV only

29. The areas of two equilateral triangles are in the ratio 25 : 36. Their altitudes will be in the ratio :
(1) 36 : 25 (2) 25 : 36
(3) 5 : 6 (4) $\sqrt{5}:\sqrt{6}$

30. ABC is an isosceles triangle in which AB = AC. If D and E are the mid-points of AB and AC respectively. The point B, C, D, E are :



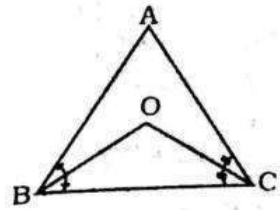
(1) collinear
(2) non-collinear
(3) concyclic
(4) None of these

31. Consider the following statments :
I. If three sides of a triangle are euqal to three sides of another triangle, then the triangles are congruent.
II. If three angles of a triangle are equal to three angles of another triangles respectively, then the two traingels are congruent.
Of these statements :
(1) I is correct and II is false
(2) both I and II are false
(3) both I and II are correct
(4) I is false and II is correct

32. The line segments jonining the mid-points of the sides of a triangle form four triangles each of which is :
(1) similar to the original triangle
(2) congruent to the original triangle
(3) an equilateral triangle
(4) an isosceles triangle

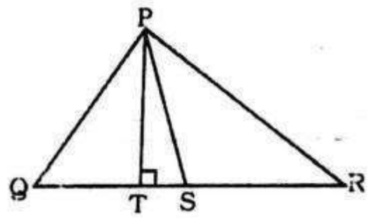
33. The triangle formed by joining the mid-points of the sides of an equilateral triangle is :
(1) a right-angled triangle
(2) an obtuse angled triangle
(3) a scalene triangle
(4) an equilateral triangle

34. OB and OC are respectively the bisectors of ∠ABC and ∠ACB. Then ∠BOC is equal to :



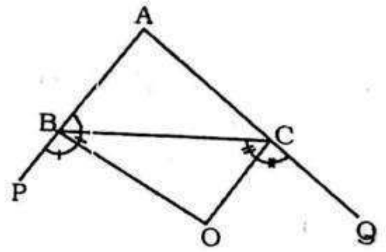
(1) $90° - \frac{1}{2}\angle A$ (2) $90° + \angle A$
(3) $90° + \frac{1}{2}\angle A$ (4) $180° - \frac{1}{2}\angle A$

35. In Δ PQR, PS is the bisector of ∠P and PT ⊥ QR, then ∠TPS is equal to :



(1) $\angle Q + \angle R$ (2) $90° + \frac{1}{2}\angle Q$
(3) $90° - \frac{1}{2}\angle R$ (4) $\frac{1}{2}(\angle Q - \angle R)$

36. In the adjoining figure, sides AB and AC of a Δ ABC are produced to P and Q respectively. The bisectors of ∠PBC and ∠QCB intersect at O. Then ∠BOC is equal to :



(1) $90° - \frac{1}{2}\angle BAC$ (2) $\frac{1}{2}(\angle PBC + \angle QCB)$
(3) $90° + \frac{1}{2}\angle BAC$ (4) None of these

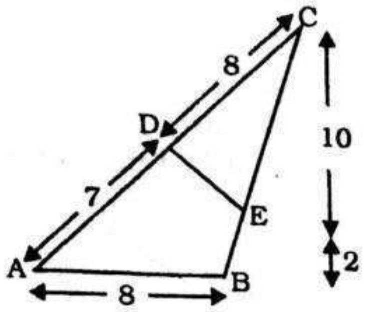
37. If A, B and C are interior angles of triangle ABC, then the value of $\sin^2\frac{B+C}{2} + \sin^2\frac{A}{2}$ is
(1) 1 (2) $\frac{1}{2}$
(3) 2 (4) None of these

38. The sides BC, CA and AB, of a ΔABC are produced in order, forming exterior angles ∠ACD, ∠BAE, and ∠CBF, then

∠ACD + ∠BAE + ∠CBF is equal to :

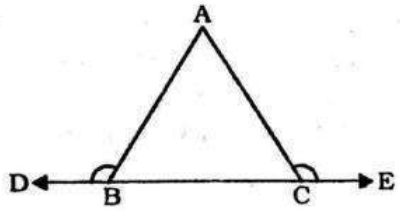
(1) 540°
(2) 360°
(3) 180°
(4) None of these

39. If ∠A = ∠CED and Δ CAB ~ ΔCED then the value of x is :



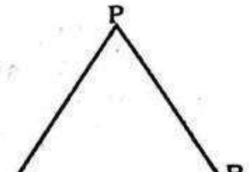
(1) 4 cm
(2) 5 cm
(3) 6 cm
(4) 7 cm

40. In the adjoining figure if m ∠ABC = m ∠ACE, then Δ ABC is :



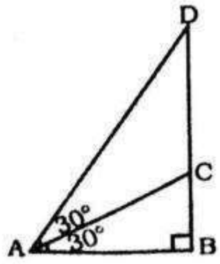
(1) right-angled
(2) isosceles
(3) equilateral
(4) obtuse-angled

41. In triangle PQR length of the side QR is less than twice the length of the side PQ by 2 cm. Length of the side PR exceeds the length of the side PQ by 10 cm. The perimeter is 40 cm. The length of the smallest side of the triangle PQR is :



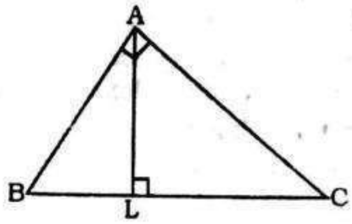
(1) 6 cm.
(2) 8 cm.
(3) 7 cm.
(4) 10 cm.

42. In the adjoning figure which of the following statements is true?



(1) AB = BD
(2) AC = CD
(3) BC = CD
(4) AD < CD

43. In a Δ ABC, ∠A = 90°, AL is drawn perpendicular to BC. Then ∠BAL is equal to :

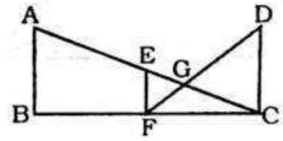


(1) ∠ALC
(2) ∠ACB
(3) ∠BAC
(4) ∠B – ∠BAL

44. In two triangles, the ratio of the areas is 4 : 3 and ratio of their heights is 3 : 4. Find the ratio of their bases.

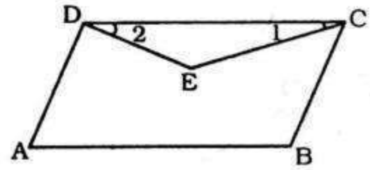
(1) 16 : 9
(2) 9 : 16
(3) 9 : 12
(4) 16 : 12

45. In the adjoining figure AB, EF and CD are parallel lines. Given that EF = 5 cm, GC = 10 cm and DC = 18 cm, then EF is equal to :



(1) 11 cm
(2) 5 cm
(3) 6 cm
(4) 9 cm

46. In the quadrilateral ABCD, the line segments bisecting ∠C and ∠D meet at E. Then the correct statement is :



(1) ∠ A + ∠ B = ∠ CED
(2) ∠ A + ∠ B = 2 ∠ CED
(3) ∠ A + ∠ B = 3 ∠ CED
(4) None of these

47. If AD, BE, CF are the medians of a ΔABC then the correct relation between the sum of the squares of sides to the sum of the squares of median is :

(1) $2(AB^2 + BC^2 + AC^2) = 3(AD^2 + BE^2 + CF^2)$
(2) $4(AB^2 + BC^2 + AC^2) = 3(AD^2 + BE^2 + CF^2)$
(3) $3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$
(4) None of these

48. Match the following :

1. Incentre
2. Circumcentre
3. Centroid
4. Orthocentre

(A) The point of intersection of all the three altitudes of a triangle.

(B) The point of intersection of all the three medians of any triangle.

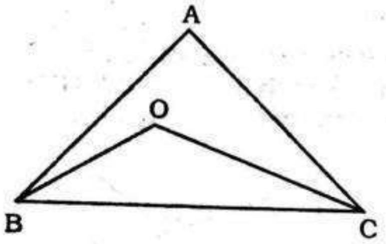
(C) The point of intersection of all the three angle bisectors of a triangle.

(D) The point of intersection of the perpendicular bisectors of the sides of a triangle.

The correct match is :

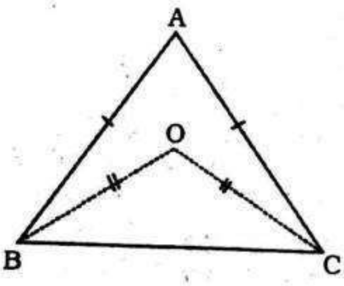
	A	B	C	D
(1)	1	2	3	4
(2)	4	3	1	2
(3)	4	3	2	1
(4)	1	3	4	2

49. In the given figure, $AB > AC$. If BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively, then,



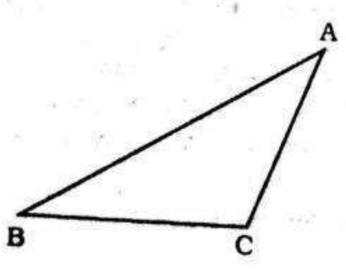
(1) $OB = OC$
(2) $OB > OC$
(3) $OB < OC$
(4) None of these

50. In the given figure, $AB, = AC$ and $OB = OC$. Then, $\angle ABO : \angle ACO =$?



(1) 1 : 1
(2) 2 : 1
(3) 1 : 2
(4) None of these

51. In ΔABC, if $\angle C > \angle B$, then



(1) $BC > AC$
(2) $AB > AC$
(3) $AB < AC$
(4) $BC < AC$

52. O is any point in the interior of ΔABC. Then which of the following is true?

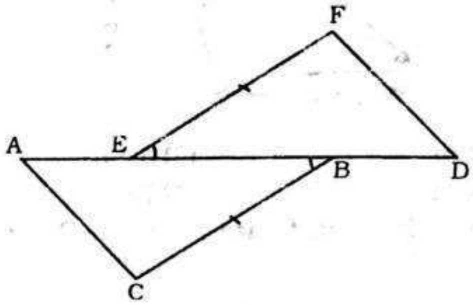
(1) $(OA + OB + OC) > (AB + BC + CA)$

(2) $(OA + OB + OC) > \frac{1}{2}(AB + BC + CA)$

(3) $(OA + OB + OC) < \frac{1}{2}(AB + BC + CA)$

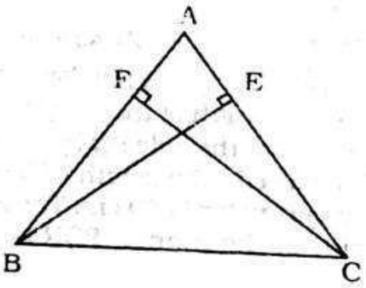
(4) None of these

53. In the given figure, $AE = DB$, $CB = EF$ and $\angle ABC = \angle FED$. Then, which of the following is true?



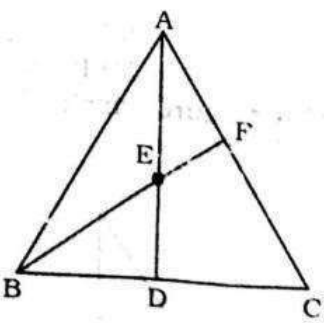
(1) $\Delta ABC \cong DEF$
(2) $\Delta ABC \cong EFD$
(3) $\Delta ABC \cong FED$
(4) $\Delta ABC \cong EDF$

54. In the given figure, $BE \perp CA$ and $CF \perp BA$ such that $BE = CF$. Then, which of the following is true?



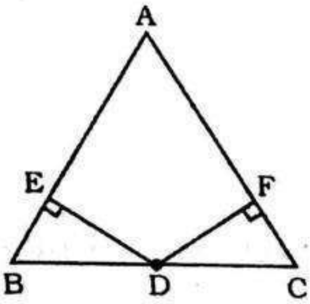
(1) $\Delta ABE \cong \Delta ACF$
(2) $\Delta ABE \cong \Delta AFC$
(3) $\Delta ABE \cong \Delta CAF$
(4) $\Delta ABE \cong \Delta FAC$

55. In the given figure, AD is a median of ΔABC and E is the mid-point of AD. If BE is joined and produced to meet AC in F, then AF = ?



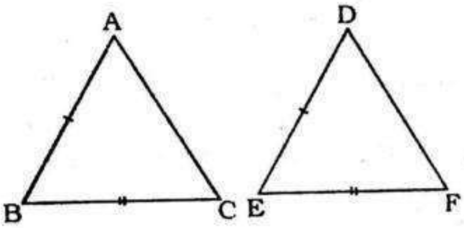
(1) $\frac{1}{2}$ AC
(2) $\frac{1}{3}$ AC
(3) $\frac{2}{3}$ AC
(4) $\frac{3}{4}$ AC

56. In the given figure, D is the midpoint of BC, $DE \perp AB$ and $DF \perp AC$ such that $DE = DF$. Then, which of the following is true?



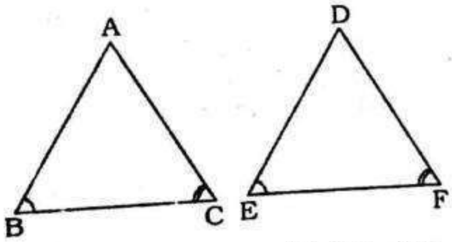
(1) $AB = AC$ (2) $AC = BC$
(3) $AB = BC$ (4) None of these

57. In ΔABC and ΔDEF, it is given that $AB = DE$ and $BC = EF$. In order that $\Delta ABC \cong \Delta DEF$, we must have



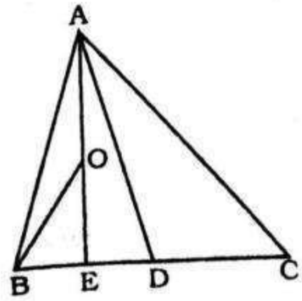
(1) $\angle A = \angle D$ (2) $\angle B = \angle E$
(3) $\angle C = \angle F$ (4) None of these

58. In ΔABC and ΔDEF, it is given that $\angle B = \angle E$ and $\angle C = \angle F$. In order that $\Delta ABC \cong \Delta DEF$, we must have



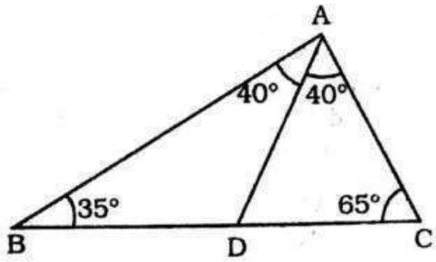
(1) $AB = DF$ (2) $AC = DE$
(3) $BC = EF$ (4) $\angle A = \angle D$

59. In ΔABC, it is given that D is the midpoint of BC ; E is the midpoint of BD and O is the midpoint of AE. Then, ar(ΔBOE) = ?



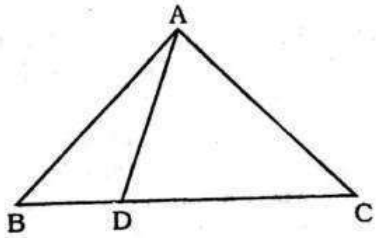
(1) $\frac{1}{3}$ ar(ΔABC) (2) $\frac{1}{4}$ ar(ΔABC)
(3) $\frac{1}{6}$ ar(ΔABC) (4) $\frac{1}{8}$ ar(ΔABC)

60. In ΔABC, $\angle B = 35°$, $\angle C = 65°$ and the bisector AD of $\angle BAC$ meets BC at D. Then, which of the following is true?



(1) $AD > BD > CD$ (2) $BD > AD > CD$
(3) $AD > CD > BD$ (4) None of these

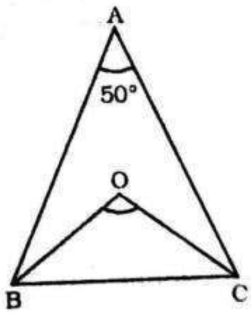
61. In the given figure, $AB > AC$. Then, which of the following is true?



(1) $AB < AD$
(2) $AB = AD$
(3) $AB > AD$
(4) Cannot be determined

62. If the altitudes from two vertices of a triangle to the opposite sides are equal, the the triangle is
(1) equilateral (2) isosceles
(3) scalene (4) right-angled

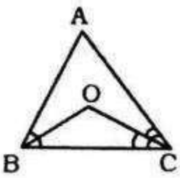
63. In the given figure, BO and CO are the bisectors of $\angle B$ and $\angle C$ respectively. If $\angle A = 50°$, then $\angle BOC$ = ?



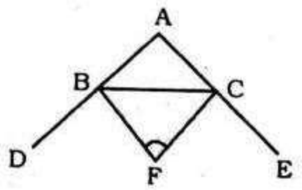
(1) 130° (2) 100°
(3) 115° (4) 120°

64. In the adjoining figure $\angle B = 70°$ and $\angle C = 30°$. BO and CO are the angle bisectors of $\angle ABC$ and $\angle ACB$. Find the value of $\angle BOC$:

(1) 30° (2) 40°
(3) 120° (4) 130°



65. In the given diagram of ΔABC, $\angle B = 80°$, $\angle C = 30°$. BF and CF are the angle bisectors of $\angle CBD$ and $\angle BCE$ respectively. Find the value of $\angle BFC$:



(1) 110° (2) 50°
(3) 125° (4) 55°

66. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm. Triangle DEF is similar to ΔABC. If EF = 12 cm then DE is :

(1) 6 cm (2) 16 cm
(3) 18 cm (4) 15 cm

67. In ΔABC, AB = 5 cm, AC = 7 cm. If AD is the angle bisector of $\angle A$. Then BD : CD is :

(1) 25 : 49 (2) 49 : 25
(3) 6 : 1 (4) 5 : 7

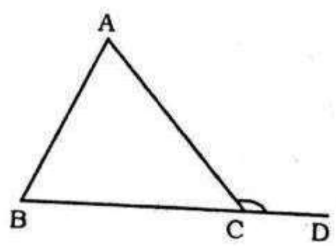
68. In a ΔABC, AB = AC and $AD \perp BC$, then

(1) AB < AD (2) AB > AD
(3) AB = AD (4) $AB \le AD$

69. The difference between altitude and base of a right angled traiangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle ?

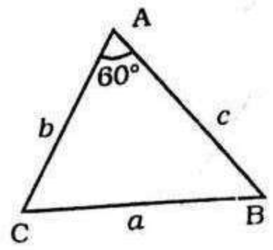
(1) 24 cm (2) 31 cm
(3) 34 cm (4) can't be determined

70. In the triangle ABC, side BC is produced to D. $\angle ACD = 100°$ if BC = AC, then $\angle ABC$ is :



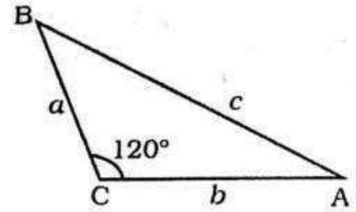
(1) 40° (2) 50°
(3) 80° (4) can't be determined

71. In the adjoining figure $\angle BAC = 60°$ and BC = a, AC = b and AB = c, then, :



(1) $a^2 = b^2 + c^2$ (2) $a^2 = b^2 + c^2 - bc$
(3) $a^2 = b^2 + c^2 - bc$ (4) $a^2 = b^2 + 2bc$

72. In the adjoining figure of ΔABC, $\angle BCA = 120°$ and AB = c, BC = a, AC = b then :



(1) $c^2 = a^2 + b^2 + ba$ (2) $c^2 = a^2 + b^2 - ba$
(3) $c^2 = a^2 + b^2 - 2ba$ (4) $c^2 = a^2 + b^2 + 2ab$

73. What is the ratio of side and height of an equilateral triangle ?

(1) 2 : 1 (2) 1 : 1
(3) $2 : \sqrt{3}$ (4) $\sqrt{3} : 2$

74. The triangle is formed by joining the mid-points of the sides AB, BC and CA of ΔABC and the area of ΔPQR is 6cm², then the area of ΔABC is :

(1) 36 cm² (2) 12 cm²
(3) 18 cm² (4) 24 cm²

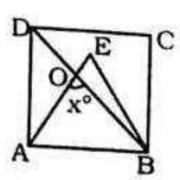
75. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :

(1) 6 cm (2) $6\sqrt{2}$ cm
(3) 4 cm (4) $3\sqrt{2}$ cm

76. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$ then $\angle BOC$ is :

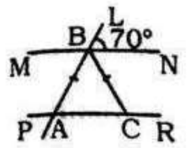
(1) 50° (2) 160°
(3) 100° (4) 130°

77. In the figure ΔABE is an equilateral triangle in a square ABCD. Find the value of angle x in degrees :



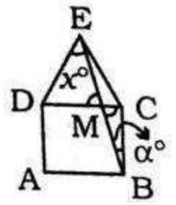
(1) 60° (2) 45°
(3) 75° (4) 90°

78. In the given diagram MN || PR and m ∠LBN = 70°, AB = BC. Find m∠ABC :



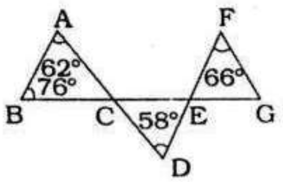
(1) 40° (2) 30°
(3) 35° (4) 55°

79. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find the m∠BED represented by x. Where m ∠EBC = α° :



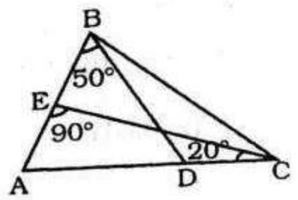
(1) 45° (2) 60°
(3) 30° (4) None of these

80. In the adjoining figure m ∠CAB = 62°, m ∠CBA = 76°, m ∠ADE = 58° and ∠ DFG = 66°
Find m ∠FGE :



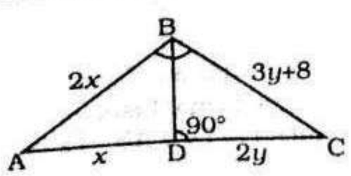
Find m∠FGE :
(1) 44° (2) 34°
(3) 36° (4) None of these

81. In the given figure CE ⊥ AB, m ∠ ACE = 20° and m
∠ ABD = 50°. Find m ∠ BDA :



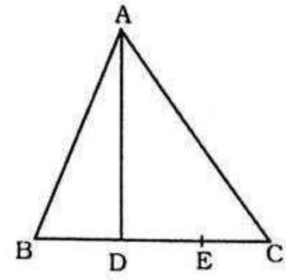
(1) 50° (2) 60°
(3) 70° (4) 80°

82. In the ΔABC, BD bisects ∠ B, and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the value of x and y :



(1) 6, 12 (2) 10, 12
(3) 16, 8 (4) 8, 15

83. In an equilateral triangle ABC, the side BC is trisected at D. Find the value of AD^2.

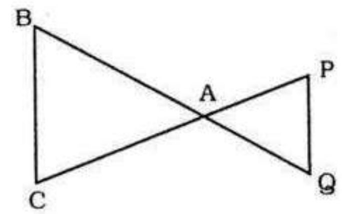


(1) $\frac{9}{7}AB^2$ (2) $\frac{7}{9}AB^2$
(3) $\frac{3}{4}AB^2$ (4) $\frac{4}{5}AB^2$

84. ABC is a triangle in which ∠A = 90°. AN ⊥ BC, AC = 12 cm and AB = 5 cm. Find the ratio of the areas of ΔANC and Δ ANB :
(1) 125 : 44 (2) 25 : 144
(3) 144 : 25 (4) 12 : 5

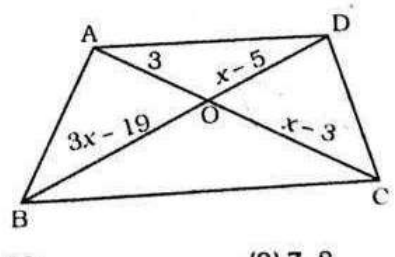
85. A vertical stick 15 cm long casts its shadow 10 cm long on the ground. At the same time a flag pole casts a shadow 60 cm long. Find the height of the flag pole :
(1) 40 cm
(2) 45 cm
(3) 90 cm
(4) None of these

86. In the figure Δ ACB ~ Δ APQ. If BC = 8 cm, PQ = 4 cm, AP = 2.8 cm, find CA :



(1) 8 cm (2) 6.5 cm
(3) 5.6 cm (4) None of these

87. In the figure BC || AD. Find the value of x :



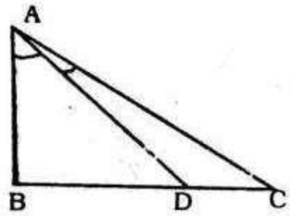
(1) 9, 10 (2) 7, 8
(3) 10, 12 (4) 8, 9

88. In an equilateral triangle of side 2a, calculate the length of its altitude :

(1) $2a\sqrt{3}$ (2) $a\sqrt{3}$

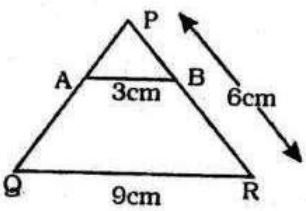
(3) $a\frac{\sqrt{3}}{2}$ (4) None of these

89. In figure AD is the bisector of $\angle BAC$. If BD = 2 cm, CD = 3 cm and AB = 5 cm. Find AC :



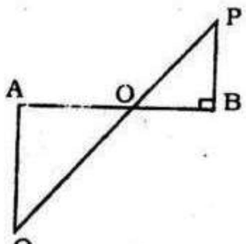
(1) 6 cm (2) 7.5 cm
(3) 10 cm (4) 15 cm

90. In the figure AB | | QR. Find the length of PB :



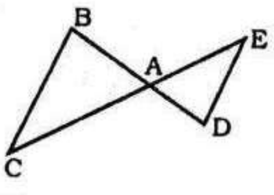
(1) 2 cm (2) 3 cm
(3) 2.5 cm (4) 4 cm

91. In the figure QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ :



(1) 8 cm (2) 9 cm
(3) 15 cm (4) 12 cm

92. In the given figure AB = 12 cm, AC = 15 cm and AD = 6 cm. BC ∥ DE, find the length of AE.

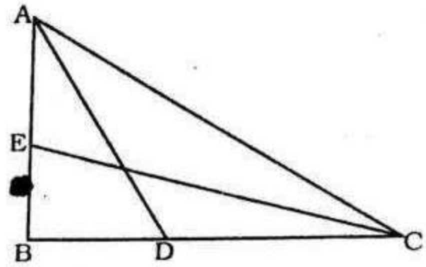


(1) 6 cm (2) 7.5 cm
(3) 9 cm (4) 10 cm

93. AD is the median of a triangle ABC and O is the centroid such that AO = 10 cm. The length of OD in cm is

(1) 4 (2) 5
(3) 6 (4) 8

94. In figure, ABC is right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.



(1) $2\sqrt{5}$ cm (2) 2.5 cm

(3) 5 cm (4) $4\sqrt{2}$ cm

95. In Δ ABC, AB = 10 cm, BC=12 cm, and AC = 14 cm. Find the length of median AD. If G is the centroid, find length of GA:

(1) $\frac{5}{3}\sqrt{7}, \frac{5}{9}\sqrt{7}$

(2) $5\sqrt{7}, 4\sqrt{7}$

(3) $\frac{10}{\sqrt{3}}, \frac{8}{3}\sqrt{7}$

(4) $4\sqrt{7}, \frac{8}{3}\sqrt{7}$

96. $\angle ABC$ is a right angled at A and AD is the altitude to BC. If AB = 7cm and AC = 24 cm. Find the ratio of AD is to AM if M is the mid-point of BC:

(1) 25:41 (2) 32:41

(3) $\frac{336}{625}$ (4) $\frac{625}{336}$

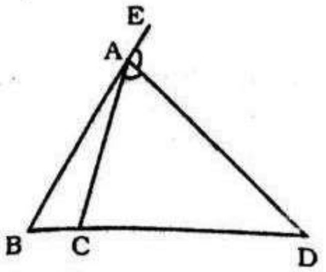
97. Area of $\Delta ABC = 30\ cm^2$. D and E are the mid-points of BC and AB respectively. Find A (ΔBDE):

(1) $10\ cm^2$ (2) $7.5\ cm^2$
(3) $15\ cm^2$ (4) None of these

98. The three sides of a triangles are given which one of the following is not a right angle ?

(1) 20,21,29 (2) 16,63,65
(3) 56,90,106 (4) 36,35,74

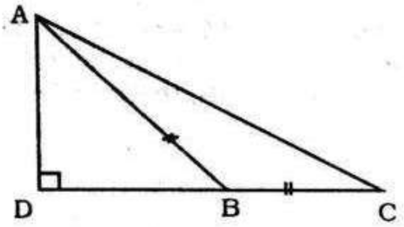
99. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If AB =12 cm, AC = 8 cm and BC = 4 cm, find CD:



(1) 10 cm (2) 6 cm
(3) 8 cm (4) 9 cm

100. In ΔABC, $AB^2 + AC^2 = 2500\ cm^2$ and median AD = 25 cm, Find BC:
(1) 25 cm (2) 40 cm
(3) 50 cm (4) 48 cm

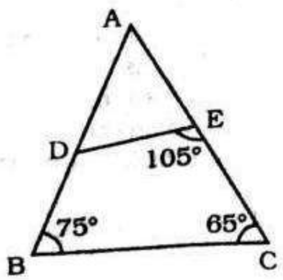
101. In the given figure, AB = BC and $\angle BAC = 15°$, AB = 10 cm. Find the area of ΔABC :



(1) $50\ cm^2$ (2) $40\ cm^2$
(3) $25\ cm^2$ (4) $32\ cm^2$

102. In ABC ,G is the centroid, AB =15 cm, BC = 18 cm and AC = 25 cm, Find GD, where D is the mid-point of BC:
(1) $\frac{1}{3}\sqrt{86}$ cm (2) $\frac{2}{3}\sqrt{86}$ cm
(3) $\frac{8}{3}\sqrt{15}$ cm (4) None of these

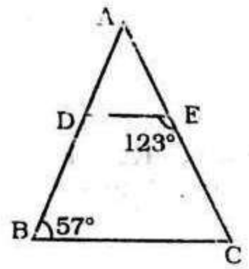
103. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if AE =10 cm, find AB :



(1) 16 cm (2) 12 cm
(3) 15 cm (4) 18 cm

104. Find the maximum area that can be enclosed in a triangle of perimeter 24 cm :
(1) $32\ cm^2$ (2) $16\sqrt{3}\ cm^2$
(3) $16\sqrt{2}\ cm^2$ (4) $27\ cm^2$

105. In the figure, AD =12 cm, AB = 20 cm and AE =10 cm. find EC:



(1) 14 cm (2) 10 cm
(3) 8 cm (4) 15 cm

106. What is the ratio of inradius to the circumradius of a right angled triangle?
(1) 1:2 (2) $1:\sqrt{2}$
(3) 2:5 (4) can't be determined

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

107. In a right-angled triangle ABC, $\angle B$ is the right angle and AC = $2\sqrt{5}$ cm. If AB – BC= 2 cm then the value of $(\cos^2 A - \cos^2 C)$ is :
(1) $\frac{2}{5}$ (2) $\frac{3}{5}$
(3) $\frac{6}{5}$ (4) $\frac{3}{10}$

[SSC Graduate Level Tier-I Exam, 2012]

108. ΔABC and ΔDEF are similar and their areas be respectively 64 cm^2 and 121 cm^2. If EF = 15.4 cm, BC is:
(1) 12.3 cm (2) 11.2 cm
(3) 12.1 cm (4) 11.0 cm

[SSC Graduate Level Tier-I Exam, 2012]

109. If G is the centroid of ΔABC and AG = BC, then $\angle BGC$ is:
(1) 75° (2) 45°
(3) 90° (4) 60°

[SSC Graduate Level Tier-I Exam, 2012]

110. By decreasing 15° of each angle of a triangle, the ratios of their angles are 2:3 : 5, The radian measure of greatest angle is :
(1) $\frac{11\pi}{24}$ (2) $\frac{\pi}{12}$
(3) $\frac{\pi}{24}$ (4) $\frac{5\pi}{24}$

[SSC Graduate Level Tier-I Exam, 2012]

111. D and E are the mid-points of AB and AC of ΔABC; BC is produced to any point P; DE, DP and EP are joined. Then,

(1) $\Delta PED = \frac{1}{4} \Delta ABC$ (2) $\Delta PED = \Delta BEC$

(3) $\Delta ADE = \Delta BEC$ (4) $\Delta BDE = \Delta BEC$

[SSC Graduate Level Tier-I Exam, 2012]

112. The perimeters of two similar triangles ΔABC and ΔPQR are 36cm and 24 cm respectively. If PQ = 10 cm, then AB is:

(1) 25 cm (2) 10 cm
(3) 15 cm (4) 20 cm

[SSC Graduate Level Tier-I Exam, 2012]

113. If G is the centroid and AD be a median with length 12 cm of ΔABC, then the value of AG is

(1) 4 cm (2) 8 cm
(3) 10 cm (4) 6 cm

[SSC Graduate Level Tier-I Exam, 2012]

114. ABC is a right-angled triangle. AD is perpendicular to the hypotenuse BC. If AC = 2 AB, then the value of BD is

(1) $\frac{BC}{2}$ (2) $\frac{BC}{3}$

(3) $\frac{BC}{4}$ (4) $\frac{BC}{5}$

[SSC Graduate Level Tier-I Exam, 2012]

115. In a right-angled triangle ABC, AB = 2.5 cm, cos B = 0.5, $\angle ACB = 90°$. The length of side AC, in cm, is

(1) $5\sqrt{3}$ (2) $\frac{5}{2}\sqrt{3}$

(3) $\frac{5}{4}\sqrt{3}$ (4) $\frac{5}{16}\sqrt{3}$

[SSC Graduate Level Tier-I Exam, 2012]

116. In Δ ABC, AD is drawn perpendicular from A on BC. If AD^2 = BD. CD, then $\angle BAC$ is

(1) 30° (2) 45°
(3) 60° (4) 90°

[SSC Graduate Level Tier-I Exam, 2012]

117. ABC is a triangle. The internal bisector of the angles $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at X, Y and Z respectively. If $\angle A = 50°$, $\angle CZY = 30°$, then $\angle BYZ$, will be

(1) 45° (2) 55°
(3) 35° (4) 30°

[SSC Graduate Level Tier-I Exam, 2012]

118. In Δ ABC, P and Q are the middle points of the sides AB and AC respectively. R is a point on the segment PQ such that PR : RQ = 1 : 2. If PR = 2cm, then BC =

(1) 4 cm (2) 2 cm
(3) 12 cm (4) 6 cm

[SSC Graduate Level Tier-II Exam, 2011]

119. If the ratio of areas of two similar triangles is 9 : 16, then the ratio of their correspon-ding sides is

(1) 3 : 5 (2) 3 : 4
(3) 4 : 5 (4) 4 : 3

[SSC Graduate Level Tier-I Exam, 2012]

120. Let BE and CF be the two medians of a ΔABC and G be their intersection. Also let EF cut AG at O. Then AO : OG is

(1) 1 : 1 (2) 1 : 2
(3) 2 : 1 (4) 3 : 1

[SSC Graduate Level Tier-I Exam, 2012]

121. AC and BC are two equal chords of a circle. BA is produced to any point P and CP, when joined cuts the circle at T. Then

(1) CT : TP = AB : CA (2) CT : TP = CA : AB
(3) CT : CB = CA : CP (4) CT : CB = CP : CA

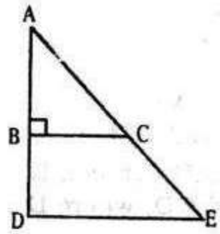
[SSC Graduate Level Tier-I Exam, 2012]

122. The external bisectors of $\angle B$ and $\angle C$ of ΔABC meet at point P. If $\angle BAC = 80°$, then $\angle BPC$ is

(1) 50° (2) 40°
(3) 80° (4) 100°

[SSC Graduate Level Tier-I Exam, 2012]

123. Given that $\angle ABC = 90°$, BC is parallel to DE. If AB = 12, BD = 6 and BC = 10, then the length of DE is



(1) 16 (2) 15
(3) 12 (4) 14

[SSC Graduate Level Tier-I Exam, 2012]

124. ABC is a triangle with $\overline{AC} = \overline{BC}$ and $\angle ABC$ = 50°. The side $\overline{BC}$ is produced to D so that $\overline{BC} = \overline{CD}$. $\angle BAD$ is

(1) 50° (2) 45°
(3) 75° (4) 90°

[SSC Graduate Level Tier-I Exam, 2012]

125. If G be the centroid of ΔABC and the area of ΔGBD is 6 sq. cm, where D is the mid-point of side BC, then the area of ΔABC is

(1) 18 sq. cm (2) 12 sq. cm
(3) 24 sq. cm (4) 36 sq. cm

[SSC Graduate Level Tier-I Exam, 2012]

126. In any triangle ABC, the base angles at B and C are bisected by BO and CO respectively. Then $\angle$ BOC is

(1) $\frac{\pi}{2} + \frac{A}{2}$ (2) $\pi - \frac{A}{2}$

(3) $\frac{(\pi - A)}{2}$ (4) $\frac{\pi}{2} + A$

[SSC Graduate Level Tier-I Exam, 2012]

127. Two sides of a triangle are of length 4 cm and 10 cm. If the length of the third side is 'a' cm, then
(1) $a > 5$
(2) $6 \le a \le 12$
(3) $a < 6$
(4) $6 < a < 14$
[SSC Graduate Level Tier-I Exam, 2012]

128. In ΔABC, AD is the median and AD $= \frac{1}{2}$ BC. If ∠BAD = 30°, then measure of ∠ACB is
(1) 90° (2) 45°
(3) 30° (4) 60°
[SSC Graduate Level Tier-I Exam, 2012]

129. If G is the centroid and AD, BE, CF are three medians of ΔABC with area 72 cm^2, then the area of ΔBDG is :
(1) 12 cm^2 (2) 16 cm^2
(3) 24 cm^2 (4) 8 cm^2
[SSC Graduate Level Tier-I Exam, 2012]

130. The three medians AD, BE and CF of ΔABC intersect at point G. If the area of ΔABC is 60 sq.cm. then the area of the quadrilateral BDGF is :
(1) 10 sq.cm (2) 15 sq.cm
(3) 20 sq.cm (4) 30 sq.cm
[SSC Graduate Level Tier-I Exam, 2012]

131. Consider ΔABD such that ∠ADB = 20° and C is a point on BD such that AB = AC and CD = CA. Then the measure of ∠ABC is :
(1) 40° (2) 45°
(3) 60° (4) 30°
[SSC Graduate Level Tier-I Exam, 2012]

132. In Δ ABC, ∠B = 90°, ∠C = 45° and D is the mid-point of AC. If AC = $4\sqrt{2}$ units, then BD is
(1) $2\sqrt{2}$ units (2) $4\sqrt{2}$ units
(3) $\frac{5}{2}$ units (4) 2 units
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

133. In Δ ABC, ∠B = 60°, ∠C = 40°. If AD bisects ∠BAC and AE ⊥ BC, then ∠EAD is
(1) 10° (2) 20°
(3) 40° (4) 80°
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

134. G is the centroid of ΔABC. If AG = BC, then ∠BGC is
(1) 90° (2) 30°
(3) 60° (4) 120°
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

135. In Δ ABC, O is the centroid and AD, BE, CF are three medians and the area of Δ AOE = 15 cm^2 then area of quadrilateral BDOF is
(1) 20 cm^2 (2) 30 cm^2
(3) 40 cm^2 (4) 25 cm^2
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

136. In ΔABC, AD is the internal bisector of ∠A, meeting the side BC at D. If BD = 5 cm, BC = 7.5 cm, then AB : AC is
(1) 2 : 1 (2) 1 : 2
(3) 4 : 5 (4) 3 : 5
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

137. A straight line parallel to the base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the ΔABE be 36 sq.cm, then the area of the ΔACD is
(1) 18 sq.cm (2) 36 sq.cm
(3) 18 cm (4) 36 cm
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

138. In ΔABC, $\angle BAC = 90°$ and $AB = \frac{1}{2}BC$. Then the measure of ∠ACB is :
(1) 60° (2) 30°
(3) 45° (4) 15°
[SSC FCI Assistant Grade-III Exam, 2012]

139. O is the incentre of ΔABC and ∠A = 30°, then ∠BOC is
(1) 100° (2) 105°
(3) 110° (4) 90°
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

140. The points D and E are taken on the sides AB and AC of ΔABC such that AD = $\frac{1}{3}$ AB, AE = $\frac{1}{3}$ AC. If the length of BC is 15 cm, then the length of DE is :
(1) 10 cm (2) 8 cm
(3) 6 cm (4) 5 cm
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

141. Two medians AD and BE of ΔABC intersect at G at right angles. If AD = 9 cm and BE = 6 cm, then the length of BD, in cm, is
(1) 10 (2) 6
(3) 5 (4) 3
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

142. The in-radius of an equilateral triangle is of length 3 cm. Then the length of each of its medians is

(1) 12 cm (2) $\frac{9}{2}$ cm

(3) 4 cm (4) 9 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

143. In Δ ABC, PQ is parallel to BC. If AP : PB = 1 : 2 and AQ = 3 cm; AC is equal to

(1) 6 cm (2) 9 cm

(3) 12 cm (4) 8 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

144. Let *O* be the in-centre of a triangle *ABC* and *D* be a point on the side *BC* of ΔABC, such that $OD \perp BC$. If $\angle BOD = 15°$, then $\angle ABC =$

(1) 75° (2) 45°

(3) 150° (4) 90°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

145. *D* is any point on side *AC* of ΔABC. If *P, Q, X, Y* are the mid-points of *AB, BC, AD* and *DC* respectively, then the ratio of *PX* and *QY* is

(1) 1 : 2 (2) 1 : 1

(3) 2 : 1 (4) 2 : 3

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

146. ABC is an equilateral triangle. P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ}$ = 5 cm the area of ΔAPQ is :

(1) $\frac{25}{4}$ sq. cm (2) $\frac{25}{\sqrt{3}}$ sq. cm

(3) $\frac{25\sqrt{3}}{4}$ sq. cm (4) $25\sqrt{3}$ sq. cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

147. In a triangle ABC, incentre is O and ∠BOC = 110°, then the measure of ∠BAC is :

(1) 20° (2) 40°

(3) 55° (4) 110°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

148. The ortho centre of a right angled triangle lies

(1) outside the triangle

(2) at the right angular vertex

(3) on its hypotenuse

(4) within the triangle

[SSC FCI Assistant Grade-III Exam, 2012]

149. The angles of a triangle are $(x + 5)°$, $(2x - 3)°$ and $(3x + 4)°$. The value of *x* is

(1) 30 (2) 31

(3) 29 (4) 28

[SSC FCI Assistant Grade-III Exam, 2012]

150. Internal bisectors of angles ∠B and ∠C of a triangle ABC meet at O. If ∠BAC = 80°, then the value of ∠BOC is

(1) 120° (2) 140°

(3) 110° (4) 130°

[SSC Delhi Police S.I. Exam, 19.08.2012]

151. In a triangle ABC, AB + BC = 12 cm, BC + CA = 14 cm and CA + AB = 18 cm. Find the radius of the circle (in cm) which has the same perimeter as the triangle.

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$

(3) $\frac{9}{2}$ (4) $\frac{11}{2}$

152. The lengths of three medians of a triangle are 9 cm, 12 cm and 15 cm. The area (in sq. cm) of the triangle is

(1) 24 (2) 72

(3) 48 (4) 144

153. The sum of all interior angles of a regular polygon is twice the sum of all its exterior angles. The number of sides of the polygon is

(1) 10 (2) 8

(3) 12 (4) 6

154. If the incentre of an equilateral triangle lies inside the triangle and its radius is 3 cm, then the side of the equilateral triangle is

(1) $9\sqrt{3}$ cm (2) $6\sqrt{3}$ cm

(3) $3\sqrt{3}$ cm (4) 6 cm

155. Suppose ΔABC be a right-angled triangle where ∠A = 90° and AD ⊥ BC. If Δ ABC = 40 cm², ΔACD = 10 cm² and $\overline{AC}$ = 9 cm, then the length of BC is

(1) 12 cm (2) 18 cm

(3) 4 cm (4) 6 cm

156. In ΔABC, D and E are points on AB and AC respectively such that DE ∥ BC and DE divides the ΔABC into two parts of equal areas. Then ratio of AD and BD is

(1) 1 : 1 (2) $1 : \sqrt{2} - 1$

(3) $1 : \sqrt{2}$ (4) $1 : \sqrt{2} + 1$

157. I is the incentre of a triangle ABC. If ∠ABC = 65° and ∠ACB = 55°, then the value of ∠BIC is

(1) 130° (2) 120°

(3) 140° (4) 110°

[SSC Graduate Level Tier-II Exam, 16.09.2012]

158. An exterior angle of a regular polygon is 72°. The sum of all the interior angles is

(1) 360° (2) 480°

(3) 520° (4) 540°

159. The angle between the external bisectors of two angles of a triangle is 60°. Then the third angle of the triangle is
(1) 40° (2) 50°
(3) 60° (4) 80°

160. In a right angled ∠ABC, ∠ABC = 90°; BN is perpendicular to AC, AB = 6 cm, AC = 10 cm. Then AN : NC is
(1) 3 : 4 (2) 9 :16
(3) 3 : 16 (4) 1 : 4

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (Ist Sitting)]

161. A circle is inscribed in a square whose length of the diagonal is $12\sqrt{2}$ cm. An equilateral triangle is inscribed in that circle. The length of the side of the triangle is
(1) $4\sqrt{3}$ cm (2) $8\sqrt{3}$ cm
(3) $6\sqrt{3}$ cm (4) $11\sqrt{3}$ cm

162. The area of an equilateral triangle is $4\sqrt{3}$ sq. cm. Its perimeter is
(1) 12 cm (2) 6 cm
(3) 8 cm (4) $3\sqrt{3}$ cm

163. I is the incentre of Δ ABC. If ∠ABC = 60°, ∠BCA = 80°, then the ∠BIC is
(1) 90° (2) 100°
(3) 110° (4) 120°

164. In Δ ABC, draw BE ⊥ AC and CF ⊥ AB and the perpendicular BE and CF intersect at the point O. If ∠ BAC = 70°, then the value of ∠ BOC is
(1) 125° (2) 55°
(3) 150° (4) 110°

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (IInd Sitting)]

165. The ratio of length of each equal side and the third side of an isosceles triangle is 3 : 4. If the area of the triangle is $18\sqrt{5}$ square unit, the third side is
(1) 16 unit (2) $5\sqrt{10}$ unit
(3) $8\sqrt{2}$ unit (4) 12 unit

166. If a, b and c are the sides of a triangle and $a^2 + b^2 + c^2 = ab + bc + ca$, then the triangle is
(1) right-angled (2) obtuse-angled
(3) equilateral (4) isosceles

167. If the perimeter of a right-angled triangle is 56 cm and area of the triangle is 84 sq. cm, then the length of the hypotenuse is (in cm)
(1) 25 (2) 50
(3) 7 (4) 24

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (Ist Sitting)]

168. In a ΔABC, $\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$ and $\overline{BC} = \sqrt{2}\,\overline{AB}$, then ∠ABC is :
(1) 30° (2) 45°
(3) 60° (4) 90°

169. If the orthocentre and the centroid of a triangle are the same, then the triangle is :
(1) Scalene (2) Right angled
(3) Equilateral (4) Obtuse angled

170. In triangle PQR, points A, B and C are taken on PQ, PR and QR respectively such that QC=AC and CR = CB. If ∠QPR = 40°, then ∠ACB is equal to :
(1) 140° (2) 40°
(3) 70° (4) 100°

171. The equidistant point from the vertices of a triangle is called its :
(1) Centroid (2) Incentre
(3) Circumcentre (4) Orthocentre

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

172. The ratio of sides of a triangle is 3 : 4 : 5. If area of the triangle is 72 square unit, then the length of the smallest side is :
(1) $4\sqrt{3}$ unit (2) $5\sqrt{3}$ unit
(3) $6\sqrt{3}$ unit (4) $3\sqrt{3}$ unit

173. If ΔABC is similar to ΔDEF, such that ∠A = 47° and ∠E = 63° then ∠C is equal to :
(1) 40° (2) 70°
(3) 65° (4) 37°

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

174. The area of an isosceles triangle is 4 square unit. If the length of the third side is 2 unit, the length of each equal side is
(1) 4 unit (2) $2\sqrt{3}$ unit
(3) $\sqrt{17}$ unit (4) $3\sqrt{2}$ unit

175. In ΔABC, ∠B = 60° and ∠C = 40°. If AD and AE be respectively the internal bisector of ∠A and perpendicular on BC, then the measure of ∠DAE is
(1) 5° (2) 10°
(3) 40° (4) 60°

176. Internal bisectors of ∠B and ∠C of ΔABC intersect at O. If ∠BOC = 102°, then the value of ∠BAC is
(1) 12° (2) 24°
(3) 48° (4) 60°

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

177. For a triangle, base is $6\sqrt{3}$ cm and two base angles are 30° and 60°. Then height of the triangle is

(1) $3\sqrt{3}$ cm (2) 4.5 cm
(3) $4\sqrt{3}$ cm (4) $2\sqrt{3}$ cm

178. If the length of each side of an equilateral triangle is increased by 2 unit, the area is found to be increased by $3 + \sqrt{3}$ square unit. The length of each side of the triangle is

(1) $\sqrt{3}$ unit (2) 3 unit
(3) $3\sqrt{3}$ unit (4) $1 + 3\sqrt{3}$ unit

179. If in a triangle, the circumcentre, incentre, centroid and orthocentre coincide, then the triangle is

(1) Acute angled (2) Isosceles
(3) Right angled (4) Equilateral

180. The internal bisectors of ∠ABC and ∠ACB of ΔABC meet each other at O. If ∠BOC =110°, then ∠BAC is equal to

(1) 40° (2) 55°
(3) 90° (4) 110°

181. AC is the diameter of a circumcircle of ΔABC. Chord ED is parallel to the diameter AC. If ∠CBE = 50°, then the measure of ∠DEC is

(1) 50° (2) 90°
(3) 60° (4) 40°

182. O is the incentre of ΔABC and ∠BOC = 110°. Find ∠BAC.

(1) 40° (2) 45°
(3) 50° (4) 55°

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

183. ABC is a right angled triangle, right angled at C and p is the length of the perpendicular from C on AB. If a, b and c are the lengths of the sides BC, CA and AB respectively, then

(1) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (2) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

(3) $\frac{1}{p^2} + \frac{1}{a^2} = \frac{1}{b^2}$ (4) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

184. If ΔABC is an isosceles triangle with ∠C = 90° and AC = 5 cm, then AB is :

(1) 5 cm (2) 10 cm
(3) $5\sqrt{2}$ cm (4) 2.5 cm

185. The length of the two sides forming the right angle of a right-angled triangle are 6 cm and 8 cm. The length of its circum-radius is :

(1) 5 cm (2) 7 cm
(3) 6 cm (4) 10 cm

[SSC (10+2) Level Data Entry Operator and LDC Exam, 04.11.2012 (Ist Sitting)]

186. In a triangle ABC, ∠BAC = 90° and AD is perpendicular to BC. If AD = 6 cm and BD = 4 cm, then the length of BC is

(1) 8 cm (2) 10 cm
(3) 9 cm (4) 13 cm

187. Two triangles ABC and DEF are similar to each other in which AB = 10 cm, DE = 8 cm. Then the ratio of the areas of triangles ABC and DEF is

(1) 4 : 5 (2) 25 : 16
(3) 64 : 125 (4) 4 : 7

[SSC (10+2) Level Data Entry Operator and LDC Exam, 04.11.2012 (IInd Sitting)]

1. (1)	**2.** (1)	**3.** (3)	**4.** (1)	**5.** (2)
6. (3)	**7.** (2)	**8.** (3)	**9.** (4)	**10.** (3)
11. (4)	**12.** (1)	**13.** (4)	**14.** (4)	**15.** (1)
16. (3)	**17.** (3)	**18.** (3)	**19.** (4)	**20.** (1)
21. (1)	**22.** (2)	**23.** (4)	**24.** (3)	**25.** (3)
26. (2)	**27.** (2)	**28.** (4)	**29.** (3)	**30.** (3)
31. (1)	**32.** (1)	**33.** (4)	**34.** (3)	**35.** (4)
36. (1)	**37.** (1)	**38.** (2)	**39.** (3)	**40.** (2)
41. (2)	**42.** (1)	**43.** (2)	**44.** (1)	**45.** (4)
46. (2)	**47.** (3)	**48.** (2)	**49.** (2)	**50.** (1)
51. (2)	**52.** (2)	**53.** (1)	**54.** (1)	**55.** (2)
56. (1)	**57.** (2)	**58.** (3)	**59.** (4)	**60.** (2)
61. (3)	**62.** (2)	**63.** (3)	**64.** (4)	**65.** (4)
66. (3)	**67.** (4)	**68.** (2)	**69.** (2)	**70.** (2)
71. (2)	**72.** (1)	**73.** (3)	**74.** (4)	**75.** (4)
76. (4)	**77.** (3)	**78.** (1)	**79.** (1)	**80.** (2)
81. (2)	**82.** (3)	**83.** (2)	**84.** (3)	**85.** (3)
86. (3)	**87.** (4)	**88.** (2)	**89.** (2)	**90.** (1)
91. (3)	**92.** (2)	**93.** (2)	**94.** (1)	**95.** (4)
96. (3)	**97.** (2)	**98.** (4)	**99.** (3)	**100.** (3)
101. (3)	**102.** (2)	**103.** (3)	**104.** (2)	**105.** (1)
106. (4)	**107.** (2)	**108.** (2)	**109.** (3)	**110.** (1)
111. (1)	**112.** (3)	**113.** (2)	**114.** (2)	**115.** (3)
116. (4)	**117.** (3)	**118.** (3)	**119.** (2)	**120.** (3)
121. (3)	**122.** (1)	**123.** (2)	**124.** (4)	**125.** (4)
126. (1)	**127.** (4)	**128.** (4)	**129.** (1)	**130.** (3)
131. (1)	**132.** (1)	**133.** (1)	**134.** (1)	**135.** (2)
136. (1)	**137.** (2)	**138.** (2)	**139.** (2)	**140.** (4)
141. (3)	**142.** (4)	**143.** (2)	**144.** (3)	**145.** (2)

146. (3)	**147.** (2)	**148.** (2)	**149.** (3)	**150.** (4)
151. (2)	**152.** (2)	**153.** (4)	**154.** (2)	**155.** (2)
156. (2)	**157.** (2)	**158.** (4)	**159.** (3)	**160.** (2)
161. (3)	**162.** (1)	**163.** (3)	**164.** (4)	**165.** (4)
166. (3)	**167.** (1)	**168.** (2)	**169.** (3)	**170.** (4)
171. (3)	**172.** (3)	**173.** (2)	**174.** (3)	**175.** (2)
176. (2)	**177.** (2)	**178.** (1)	**179.** (4)	**180.** (1)
181. (4)	**182.** (1)	**183.** (2)	**184.** (3)	**185.** (1)
186. (4)	**187.** (2)			

EXPLANATIONS

1. (1) Let AB = a Then, $BD = \frac{a}{2}$.

$$\therefore \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}\cdot\left(\frac{a}{2}\right)^2}{\frac{\sqrt{3}}{4}\cdot a^2} = \frac{1}{4}$$

So, the required ratio is 1 : 4.

2. (1) Median BE in $\Delta\ ABD$ divides it into two $\triangle$s of equal area.

$\therefore$ ar(ΔBED) $\frac{1}{2}$ ar(ΔABD).

Median CE in ΔACD divides it into two $\triangle$s of equal area.

$\therefore$ ar(ΔCED) = $\frac{1}{2}$ ar(ΔACD).

$\therefore$ ar(ΔBED) + ar(ΔCED)

$= \frac{1}{2}$ {ar(ΔABD) + ar(ΔACD)} = $\frac{1}{2}$ ar (ΔABC)

$\Rightarrow$ ar (ΔBEC) = $\frac{1}{2}$ ar (ΔABC).

3. (3) Here, triangles are congruent. But congruent triangles are always similar.

4. (1) As $\angle BOC = 90° - \frac{1}{2}\angle A$

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \frac{1}{2}\angle A = 90° - \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \frac{1}{2}(\angle B + \angle C) = 90° - \frac{1}{2}\angle A$

Hence, it includes $\frac{1}{2}$ the sum of the base angles.

5. (2) By Pythagoras theorem,

$PR = \sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13$ cm.

$\because$ O is centroid $\Rightarrow$ QM is median and M is mid-point of PR.

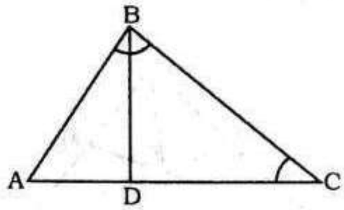
$QM = PM = \frac{13}{2}$

$\because$ Centroid divides median in ratio 2 : 1.

$\therefore OQ = \frac{2}{3}QM = \frac{2}{3} \times \frac{13}{2} = \frac{13}{3}$

$\therefore OQ = 4\frac{1}{3}$ cm

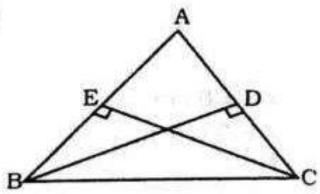
6. (3) As $\angle B = 2\angle C$
$\Rightarrow \angle ABD = \angle BCA$
In Δ ABC and Δ ABD



$\angle A = \angle A$ common,
$\angle ABD = \angle BCA$ (proved)
$\therefore \Delta ABC \sim \Delta ABD$

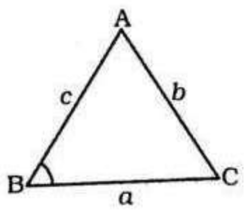
$\Rightarrow \frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$ $\Rightarrow$ BD : BC = AB : AC

7. (2)



As BD = EC, $\angle AEC = \angle BDA = 90°$ each
Also $\angle A = \angle A$ (common)
$\therefore \Delta BDA \cong \Delta AEC$
$\Rightarrow$ AB = AC by concept
$\therefore$ Triangle is an isosceles triangle.

8. (3)



$\frac{AC}{AB} = \sqrt{2}$.

(By Sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$)

$\frac{AC}{\sin B} = \frac{AB}{\sin C}$

$\Rightarrow \frac{AC}{AB} = \frac{\sin B}{\sin C}$ $\because B = 45°$

$\Rightarrow \frac{\sin 45°}{\sin C} = \frac{\sqrt{2}}{1}$ $\Rightarrow \frac{\frac{1}{\sqrt{2}}}{\sin C} = \frac{\sqrt{2}}{1}$

$\Rightarrow \sin C = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$ $\Rightarrow \sin C = \frac{1}{2} = \sin 30$

$\Rightarrow C = 30°$

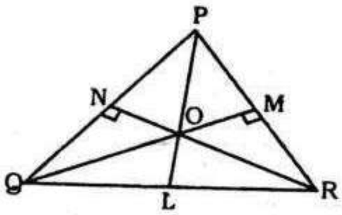
$\therefore \angle BAC = 180° - (\angle B + \angle C)$

$= 180° - (45° + 30°) = 105°$

9. (4) Clearly Δ QRO as

$QP \perp QR$ and $PR \perp QM$

and $OL \perp QR$



$\therefore$ P is point of intersection of altitudes virtually.

10. (3) As $\Delta ABC \sim \Delta DEF$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

But $\frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}$

$\therefore DE = 2AB = 2 \times 3 = 6$ cm

$DF = 2 \times AC = 2 \times 2.5 = 5$cm

$\therefore$ Perimeter of Δ DEF

$= (6 + 5 + 4) = 15$ cm

Quicker Approach

$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$

= Ratio of correspondings sides

$\therefore \frac{(3+2+2.5)}{\text{Perimeter of } \Delta DEF} = \frac{1}{2}$

$\therefore$ Perimeter of $\Delta DEF = 2\ (7.5)$

$= 15$ cm.

11. (4) **I.** : In Δ ABD and Δ CAD

as $\angle ADB = \angle ADC = 90°$ each

$\angle BAD = \angle ACD$ {each $= 90° - \angle DAC$}

$\therefore \Delta ADB \sim \Delta CAD$

II. : In ΔABD and ΔCDA

$\angle ADB = \angle ADC = 90°$ each

$\angle BAD = \angle ACD = 90° - \angle DAC$ (each)

and AD = AD (common)

$\therefore \Delta ABD \cong \Delta CDA$

III. : In Δ ADB and Δ CAB

$\angle ADB = \angle BAC = 90°$ each

$\angle B = \angle B$ (common)

$\therefore \Delta ADB \sim \Delta CAB$

Here I, II and III are correct statements.

12. (1) As ADB is a right angled Δ ,

So, $AB^2 = AD^2 + BD^2$

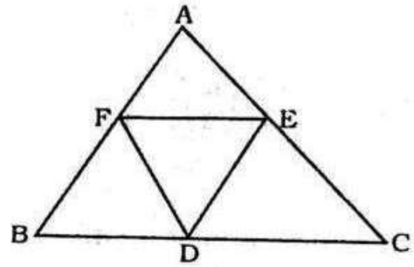
$\Rightarrow AC^2 = AD^2 + BD^2$ $[\because AB = AC]$

$\Rightarrow (AD + DC)^2 = AD^2 + BD^2$

$\Rightarrow AD^2 + DC^2 + 2AD.DC = AD^2 + BD^2$

$\Rightarrow BD^2 - CD^2 = 2CD.AD$

13. (4) As the line joining the mid-points of any two sides of a triangle is parallel to the third side and is half of the third side.



$\therefore DE = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5$ cm.

$EF = \frac{1}{2} BC \Rightarrow BC = 2EF = 2 \times 3 = 6$ cm.

$DF = \frac{1}{2} AC \Rightarrow AC = 2 \times DF = 2 \times 4 = 8$ cm.

14. (4) As $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow a + 3a + b = 180°$

$\Rightarrow 4a + b = 180°$ and $-5a + 3b = 30°$

Solving above equations, $a = 30°$ and $b = 60°$

$\therefore \angle P = 30°, \angle Q = 90°$ and

$\angle R = 60°$.

$\therefore \Delta$ PQR is a right-angled triangle.

15. (1) Consider Δ BMD and Δ DLC

as $\angle BMD = \angle DLC = 90°$ each

Also $\angle BDM = \angle DCL$ corresponding angles

$\therefore \Delta BMD \sim \Delta DLC$

$\therefore \frac{BD}{DC} = \frac{DM}{LC} = \frac{BM}{DL}$

$\Rightarrow \frac{BD}{DC} = \frac{DM}{LC} = \frac{1}{3}$

$\therefore DM : LC = 1 : 3$

16. (3) In ΔABC by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$ .....(i)

and in ΔPBQ

$PQ^2 = PB^2 + BQ^2$ .....(ii)

Adding (i) and (ii),

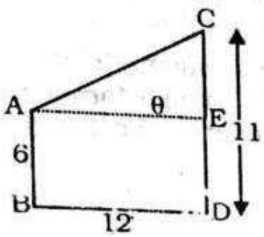
$AC^2 + PQ^2 = (AB^2 + BC^2) + PB^2 + BQ^2$

$= (AB^2 + BQ^2) + (PB^2 + BC^2)$

$\Rightarrow AC^2 + PQ^2 = AQ^2 + PC^2$

$\because$ Δ ABQ and Δ PBC are right triangles.

17. (3) As we can see that distance between tops of poles = AC



Also in ΔAEC,

EC = DC − DE = 11 − AB = 11 − 6 = 5cm

Also, AE = BD

$\Rightarrow$ AE = 12 m $\quad\quad \therefore AC = \sqrt{AE^2 + EC^2}$

$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = 13m$

18. (3) As ΔAOB ~ ΔDOC

$\Rightarrow \frac{AB}{DC} = \frac{AO}{OD} = \frac{BO}{OC} \quad\quad \Rightarrow \frac{AB}{DC} = \frac{AO}{OD} \Leftrightarrow \frac{3}{2} = \frac{AO}{3.2}$

$\Rightarrow AO = \frac{3 \times 3.2}{2} = 4.8$ cm

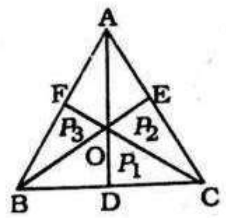
and, $\frac{AB}{DC} = \frac{BO}{OC} \Leftrightarrow \frac{3}{2} = \frac{BO}{3.8}$

$\Rightarrow BO = \frac{3 \times 3.8}{2} = 5.7$ cm

$\Rightarrow$ Required sum = OA + OB = 4.8 + 5.7 = 10.5 cm

19. (4) Incentre is the centre of the circle, so it always lies inside the triangle.

20. (1)



Let the side of ΔABC be x.

O is the point in the interior of ΔABC.

OD, OE, OF are perpendiculars.

$\therefore$ Clearly

ΔOAB + ΔOBC + ΔOAC = ΔABC

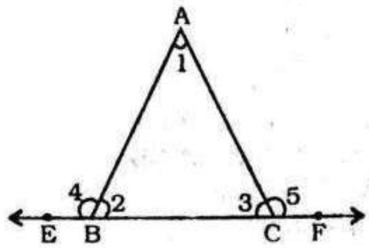
$\Rightarrow \frac{1}{2}x \times p_3 + \frac{1}{2}x \times p_1 + \frac{1}{2}x \times p_2 = \frac{\sqrt{3}}{4}x^2$

$\Rightarrow \frac{1}{2}x(p_3 + p_1 + p_2) = \frac{\sqrt{3}}{4}x^2$

$\Rightarrow p_1 + p_2 + p_3 = \frac{\sqrt{3}}{2}x$

$\Rightarrow x = \frac{2}{\sqrt{3}}(p_1 + p_2 + p_3)$

21. (1) As ∠1 + ∠2 + ∠3 = 180° ....(i)



∠2 + ∠4 = 180° (linear pair)

∠3 + ∠5 = 180° (linear pair)

∴ ∠2 + ∠3 + ∠4 + ∠5 = 360°

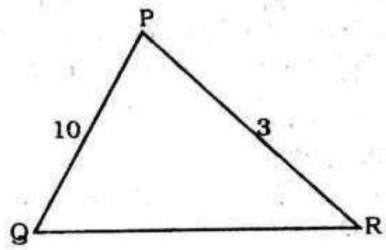
∠4 + ∠5 = 360° − (∠2 + ∠3)

= 360° − (180 − ∠1) from (i)

= 180° + ∠1

⇒ ∠4 + ∠5 = 180° + ∠BAC

22. (2) By the property of sides of a triangle,



PQ + PR > QR ....(i)

QR + PR > PQ ...(ii)

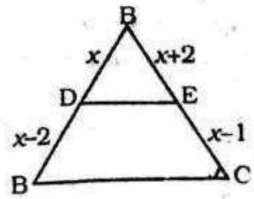
and, PQ + QR > PR ....(iii)

So QR > 7 satisfies case (ii)

∴ QR is greater than 7 cm.

23. (4) As DE || BC, so by basic proportionality theorem.

$\frac{AD}{DB} = \frac{AE}{EC}$



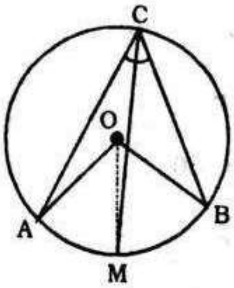
$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x+2)(x-2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4$

24. (3)



$m\,AM = \angle AOM = 2\,\angle ACM$

$m\,MB = \angle MOB = 2\,\angle MCB$

but $\angle ACM = \angle MCB$

$\Rightarrow m\,AM = m\,MB$

25. (3) $\because \frac{BE}{CE} = \frac{AB}{AC}$ as AE is an exterior angle bisector.

Let $CE = x$, $BE = BC + EC = 12 + x$

$\Rightarrow \frac{12 + x}{x} = \frac{10}{6}$ $\qquad \Rightarrow (12 + x)\,6 = 10x$

$\Rightarrow 72 + 6x = 10x$ $\qquad \Rightarrow 4x = 72$

$\Rightarrow x = 18$ cm

26. (2) Circumcentre is the point of intersection of perpendicular bisectors of sides of the triangle. Hence it is equidistant from the vertices of the triangle.

27. (2) As OP || YZ

$\Rightarrow \angle POY = \angle OYZ$

$\Rightarrow \angle PYO = \angle POY \because$ OY is angle bisector of $\angle Y$.

$\therefore PY = PO$

As $\angle XYZ$ is an acute angle

$\therefore \frac{1}{2}\angle XYZ < 45°$

$\therefore \angle POY = \angle PYO \angle 45°$

$\therefore \frac{1}{2}\angle XYZ =< 45°$

$\therefore \angle YPO > 90°$

Hence Δ PYO is isosçeles Δ but not a right-angled triangle.

28. (4) (I) It is not possible to have a triangle in which sum of the two angles is greater than 180°.

(II) In this case, sum of the three angles will be less than 180°.

(II) and (IV) cases : the sum of the three angles will be 180°.

29. (3) The ratio of the areas of two similar triangles is equal to the ratio of squares of the corresponding altitudes.

Ratio of altitudes $= \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$

30. (3) As DE || BC, so $\angle ADE = \angle ABC$

Also, $\angle ABC = \angle ACB$ $\qquad (\because AB = AC)$

$\therefore \angle ADE = \angle ACB$

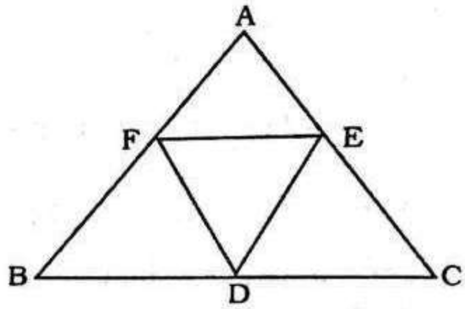
$\therefore \angle ADE + \angle EDB = 180°$

$\Rightarrow \angle ACB + \angle EDB = 180°$

Hence, B, C, D and E are concyclic.

31. (1) As second statement fails in case of equilateral triangles having different lengths of sides. So I is true and II is false.

32. (1) The line segments joining the mid-points of the sides of a triangle form four triangles each of which is similar to the original triangle.



Here, Δ BDF ~ Δ ABC

Also Δ DEC, Δ DEF,

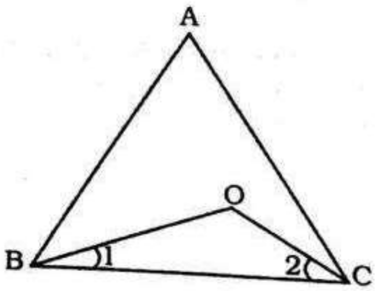
Δ AFE ~ Δ ABC

33. (4) The sides of triangle formed will be half of the sides of the original triangle.

34. (3) In Δ BOC,

$\angle 1 + \angle 2 + \angle BOC = 180°$

$\angle A + \angle B + \angle C = 180°$



$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90°$

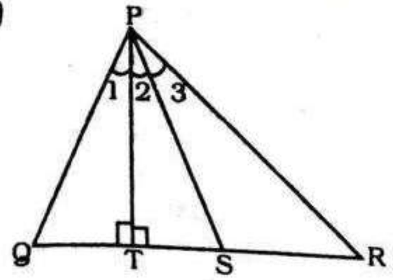
$\Rightarrow \frac{1}{2}(\angle A) + \angle 1 + \angle 2 = 90°$

$\Rightarrow \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$

Put $\angle 1 + \angle 2$ in (i)

$\angle BOC = 180° - \left(90° - \frac{1}{2}\angle A\right) = 90° + \frac{1}{2}\angle A$

35. (4)



$\angle 1 + \angle 2 = \angle 3$

$\angle Q = 90° - \angle 1$

$\angle R = 90° - \angle 2 - \angle 3$

So, $\angle Q - \angle R = (90° - \angle 1) - (90° - \angle 2 - \angle 3)$

$\Rightarrow \angle Q - \angle R = \angle 2 + \angle 3 - \angle 1$ From (i)

$= \angle 2 + (\angle 1 + \angle 2) - \angle 1$

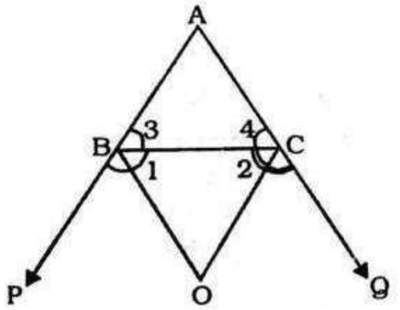
$\Rightarrow \angle Q - \angle R = 2\angle 2 \quad \Rightarrow \frac{1}{2}(\angle Q - \angle R) = \angle TPS$

36. (1) $\angle 1 = 90° - \frac{1}{2}\angle 3$

$\angle 2 = 90° - \frac{1}{2}\angle 4$

Now in ΔBOC

$\angle 1 + \angle 2 + \angle BOC = 180°$



$\angle BOC = 180° - (\angle 2 + \angle 1)$

$= 180° - \left[90° - \frac{1}{2}\angle 4 + 90° - \frac{1}{2}\angle 3\right]$

$\Rightarrow \angle BOC = \frac{1}{2}(\angle 3 + \angle 4)$

$\Rightarrow \angle BOC = \frac{1}{2}(180° - \angle A)$

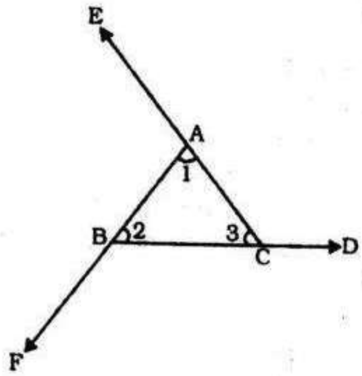
$\because \angle A + \angle 3 + \angle 4 = 180° \quad \Rightarrow \angle BOC = 90° - \frac{1}{2}\angle A$

37. (1) $\because A + B + C = 180° \quad \Rightarrow \frac{B+C}{2} = 90° - \frac{A}{2}$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right) = \cos\frac{A}{2}$

$\therefore \sin^2\frac{B+C}{2} + \sin^2\frac{A}{2} = \cos^2\frac{A}{2} + \sin^2\frac{A}{2} = 1$

38. (2) By exterior angle theorem, we have



$\angle ACD = \angle 1 + \angle 2$

$\angle BAE = \angle 2 + \angle 3$

$\angle CBF = \angle 3 + \angle 1$

$\angle ACD + \angle BAE + \angle CBF$

$= \angle 1 + \angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 1$

$= 2(\angle 1 + \angle 2 + \angle 3) = 2(180°) = 360°$

39. (3) As $\Delta CAB \sim \Delta CED$

$\therefore \frac{CA}{CD} = \frac{CE}{DE} = \frac{CB}{CD}$

So, $\frac{AB}{DE} = \frac{CB}{CD} \quad \therefore \frac{9}{x} = \frac{10+2}{8}$

$\therefore x = \frac{8 \times 9}{12} = 6$ cm.

40. (2) Since $\angle ABD = \angle ACE$

$\therefore 180° - \angle ABC = 180° - \angle ACB$

$\Rightarrow \angle ABC = \angle ACB$

Hence, ΔABC is isosceles triangle.

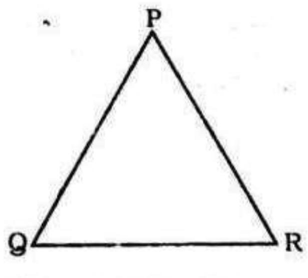
41. (2) In ΔPQR

Here, $QR + 2 = 2PQ$

$QR = 2PQ - 2$ ....(i)

$PR = PQ + 10$ ....(ii)

$\Rightarrow PQ + QR + RP = 40$ ....(iii)



Put (i) and (ii) in (iii)

$PQ + 2PQ - 2 + PQ + 10 = 40$

$\Rightarrow 4PQ = 32$ or $PQ = 8$ cm.

42. (1) Sides opposite to equal angles are equal.

Here, $\angle ADB = \angle BAD = 60°$

So, AB = BD.

43. (2) $\angle BAL + \angle B + 90° = 180°$

$\Rightarrow \angle BAL + \angle B = 90°$

$\Rightarrow \angle BAL = 90° - \angle B$ ....(i)

Now in ΔABC, $\angle ACB + \angle B + \angle A = 180°$

$\Rightarrow \angle ACB + \angle B = 180° - 90°$

$\Rightarrow \angle ACB + \angle B = 90°$

$\Rightarrow \angle ACB = 90° - \angle B$ ....(ii)

From (i) and (ii), $\angle BAL = \angle ACB$

44. (1) Let the height and base of first triangle be h_1 and b_1 respectively and that of second triangle h_2 and b_2 respectively.

Now,

$$\frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \frac{4}{3}$$

$$\Rightarrow \frac{\frac{1}{2}b_1h_1}{\frac{1}{2}b_1h_2} = \frac{4}{3} \qquad \Rightarrow \frac{b_1}{b_2} \times \frac{3}{4} = \frac{4}{3}$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9}$$

45. (4) In Δ GEF and Δ GCD, we have

$\angle EFG = \angle GDC$, (Alternate angles)

$\angle EGF = \angle CGD$ (Vertically opposite angles)

$\Delta GEF \sim \Delta GCD$

$$\therefore \frac{GE}{CG} = \frac{EF}{CD} \Rightarrow \frac{5}{10} = \frac{EF}{18} \Rightarrow EF = \frac{5 \times 18}{10} = 9 \text{ cm}$$

46. (2) $\angle 1 = \frac{1}{2}\angle C$, $\angle 2 = \angle \frac{1}{2}\angle D$

$\angle 1 + \angle 2 + \angle CED = 180°$

$\therefore \angle CED = 180° - (\angle 1 + \angle 2)$

Also $\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A + \angle B + 2(\angle 1 + \angle 2) = 360°$

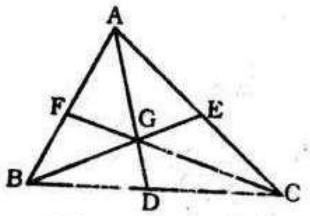
$\angle A + \angle B = 360° - 2(\angle 1 + \angle 2)$

$\angle A + \angle B = 2\angle CED$

47. (3) Let G be the centroid of Δ ABC.

In Δ ABC

[$\because$ The sum of the squares of any two sides is equal to twice the square of half of the third side together with the square of the median bisecting the third side]



$\therefore AB^2 + AC^2$

$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \quad ....(i)$$

$$\Rightarrow AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

$$BC^2 + AB^2 = 2BE^2 + \frac{1}{2}AC^2 \quad ....(ii)$$

$$BC^2 + AC^2 = 2CF^2 + \frac{1}{2}AB^2 \quad ....(iii)$$

Adding (i), (ii) and (iii), we get

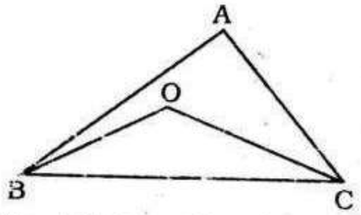
$2(AB^2 + BC^2 + AC^2) = 2(AD^2 + BE^2 + CF^2)$

$+ \frac{1}{2}(AB^2 + BC^2 + AC^2)$

$\therefore 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$

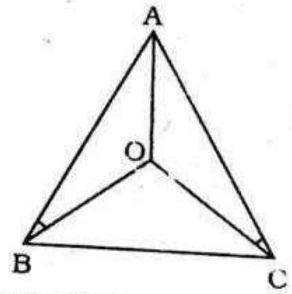
48. (2) Clearly (2) is correct

49. (2) $AB > AC \Rightarrow \angle C > \angle B \Rightarrow \frac{1}{2}\angle C > \frac{1}{2}\angle B$



$\Rightarrow \angle OCB > \angle OBC \Rightarrow OB > OC$.

50. (1) Join OA.



In ΔOAB and ΔOAC, we have:

$AB = AC$ (given), $OB = OC$ (given) and $OA = OA$

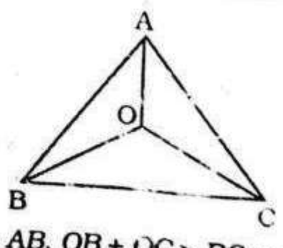
$\therefore \Delta OAB \cong \Delta OAC \Rightarrow \angle ABO = \angle ACO$

$\therefore \angle ABO : \angle ACO = 1:1$

51. (2) Since the side opposite to a greater angle is larger,

$\angle C > \angle B \Rightarrow AB > AC$

52. (2) In ΔOAB, ΔOBC and ΔOCA, we have:



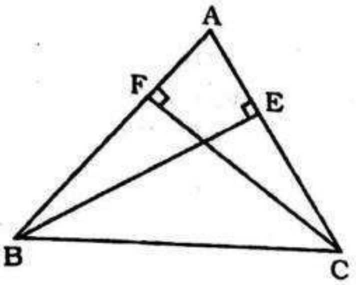
$OA + OB > AB$, $OB + OC > BC$ and $OC + OA > AC$

$\Rightarrow 2(OA + OB + OC) > (AB + BC + CA)$

$\Rightarrow (OA + OB + OC) > \frac{1}{2}(AB + BC + CA)$

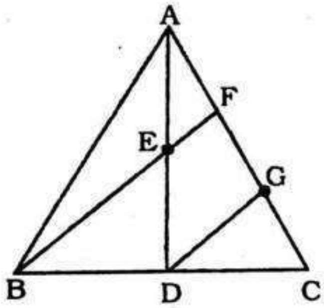
53. (1) $AB = (AD - DB) = (AD - AE)$ and $DE = (AD - AE)$
In ΔABC and ΔDEF, we have;
$AB = DE$ (proved), $CB = EF$ (given) and
$\angle ABC = \angle FED$.
$\therefore\ \Delta ABC \cong \Delta DEF$.

54. (1) In ΔABE and ΔACF, we have:



$BE = CF$ (given), $\angle BEA = \angle CFA = 90°$
and $\angle A = \angle A$
$\therefore\ \Delta ABE \cong \Delta ACF$.

55. (2) Let G be the mid-point of FC. Join DC.
In ΔBCF, D is the mid-point of BC and G is the mid-point of FC.



$\therefore\ DC \parallel BF \Rightarrow DG \parallel EF$.
In $\Delta\ ADG$, E is the mid-point of AD and $EF \parallel DG$.
So, F is the mid-point of AG.
$\therefore\ AF = FG = GC$
[$\because$ G is the mid-point of FC]

$\therefore\ AF = \frac{1}{3} AC$.

56. (1) In right ΔBED and right ΔCFD, we have:
$DE = DF$ (given)
and hyp. BD = hyp.CD
$\therefore\ \Delta BED \cong \Delta CFD$
$\Rightarrow \angle B = \angle C \Rightarrow AC = AB$.

57. (2) For congruence, we must have, $\angle B = \angle E$

58. (3) For congruence, we must have $BC = EF$.

59. (4) Median AD divides ΔABC into two $\triangle$s of equal area.

$\therefore\ ar(\Delta ABD) = \frac{1}{2}\ ar(\Delta ABC)$.

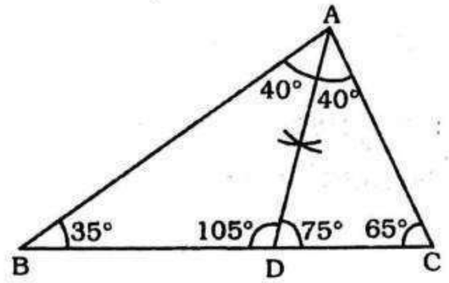
Median AE divides ΔABD into two $\triangle$s of equal area.

$\therefore\ ar(\Delta ABE) = \frac{1}{2}\ ar(\Delta\ ABD) = \frac{1}{4}\ ar(\Delta ABC)$.

Median OB divides ΔABE into two $\triangle$s of equal area.

$\therefore\ ar(\Delta BOE) = \frac{1}{2}\ ar(\Delta\ ABE) = \frac{1}{8}\ ar(\Delta ABC)$.

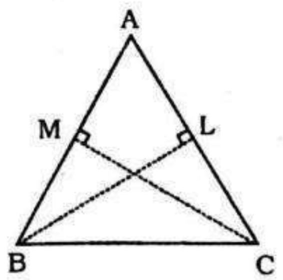
60. (2) In ΔABC, we have:
$\angle B = 35°$, $\angle C = 65°$
$\Rightarrow \angle A = 180° - (35° + 65°) = 80°$



Let AD be the bisector of $\angle BAC$.
Then, $\angle BAD = \angle CAD = 40°$
In ΔABD, $\angle BAD > \angle ABD$
$\Rightarrow BD > AD$.
In ΔACD, $\angle ACD > \angle CAD \Rightarrow AD > CD$.
$\therefore\ BD > AD > CD$.

61. (3) $AB > AC \Rightarrow \angle ACB > \angle ABC$.
Exterior $\angle ADB > \angle ACD$
$\Rightarrow \angle ADB > \angle ACB > \angle ABC$
$\Rightarrow \angle ADB > \angle ABD \Rightarrow AB > AD$

62. (2) A ΔABC is given in which $BL \perp AC$ and $CM \perp AB$ such that $BL = CM$. Then, we have to prove that $AB = AC$.
In ΔABL and ΔACM, we have
$BL = CM$ (given), $\angle BAL = \angle CAM$ (common),



$\angle ALB = \angle AMC$ (each 90°)
$\therefore\ \Delta ABL \cong \angle ACM$ and hence $AB = AC$.
$\therefore\ \Delta ABC$ is isosceles.

63. (3) $\angle A + \angle B + \angle C = 180°$
$\Rightarrow 50° + \angle B + \angle C = 180°$
$\Rightarrow \angle B + \angle C = 130°$
$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 65°$

In ΔOBC,
$\angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180°$

$\Rightarrow 65° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 115°$.

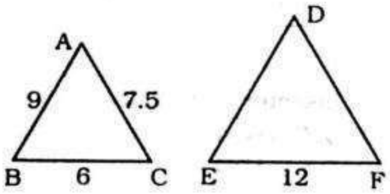
64. (4) $\angle OBC + \angle OCB = \frac{70}{2} + \frac{30}{2} = 50°$

$\therefore \angle BOC = 180° - 50° = 130°$

65. (4) $\angle BFC = 90° - \frac{1}{2}(\angle A)$

$= 90° - \frac{1}{2}[180° - (80° + 30°)] = 55°$

66. (3) The $\Delta ABC \sim \Delta DEF$

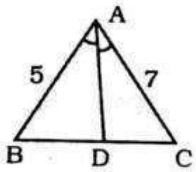


$\frac{AB}{BC} = \frac{DE}{EF} \Rightarrow \frac{9}{6} = \frac{DE}{12}$

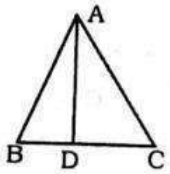
$\Rightarrow$ DE = 18 cm

67. (4) $\frac{AB}{AC} = \frac{BD}{CD} = \frac{5}{7}$

(By angle bisector theorem)



68. (2) AB is hypotenuse and AD is perpendicular in Δ ABD,
AB > AD



69. (2) $x^2 + (x - 17)^2 = 25^2$ (Using Pythagorus theorem)

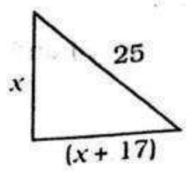
$\Rightarrow x^2 + x^2 - 34x + 289 = 625$

$\Rightarrow 2x^2 - 34x - 336 = 0$

$\Rightarrow x^2 - 17x^2 - 168 = 0$

$\Rightarrow x^2 - 24x + 7x - 168 = 0$

$\Rightarrow x(x - 24) + 17(x - 24) = 0$



$x = 24$ cm
Altitude + Base = 24 + 7 = 31 cm

70. (2) $\angle ACB = 180° - 100° = 80°$
and $\angle BAC + \angle ABC = 180° - 80° = 100°$
$\angle ABC = 50°$ ($\because \angle BAC = \angle ABC$)

71. (2) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (cosine rule)

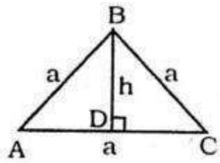
$\Rightarrow \frac{1}{2} = \frac{b^2 + c^2 - a^2}{2bc}$ ($\because \cos 60° = 1/2$)

$\because a^2 = b^2 + c^2 - bc$

72. (1) $\cos C = \frac{a^2 + b^2 - c^2}{2ab} - \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow c^2 = a^2 + b^2 + ab$

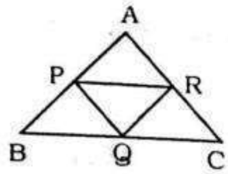
73. (3) $AB^2 = BD^2 + AD^2$ (Pythagorus theorem)



$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\right)^2 \qquad \Rightarrow h^2 = \frac{3}{4}a^2$

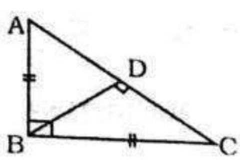
$\Rightarrow h = \frac{\sqrt{3}}{2}a \qquad \Rightarrow \frac{a}{h} = \frac{2}{\sqrt{3}}$

74. (4) There are 4 congruent triangles.



Hence, A (Δ ABC) = 6 × 4 = 24 cm²

75. (4) $BD = \frac{AB \times BC}{AC} = \frac{6 \times 6}{6\sqrt{2}} = 3\sqrt{2}$cm



76. (4) $\angle BOC = 90° + \frac{1}{2}\angle A = 90° + 40° = 130°$

77. (3) $\angle EAB = \angle OAB = 60°$
$\angle ABD = 45°$
$\therefore \angle AOB = 180° - (60° + 45°) = 75°$

78. (1) $\angle LBN = \angle BAC = \angle BCA = 70°$

$\therefore\ \angle ABC = 180° - (70° + 70°) = 40°$

79. (1) $\angle ECB = 60° + 90° = 150°$

$\therefore\ \angle CEB + \angle EBC = 180° - 150° = 30°$

$\therefore\ \angle BEC = 15°\ (\because \angle BEC = \angle EBC)$

$\therefore\ \angle DEC = 60°$

$\therefore\ \angle DEM = 60° - 15° = 45°$

80. (2) $\angle ACB = 42° = \angle ECD$

$\therefore\ \angle CED = 180° - (58° + 42°) = 80° = \angle FEG$

$\therefore \angle FGE = 180° - (66° + 80°) = 34°$

81. (2) $\angle BAD = 70°$

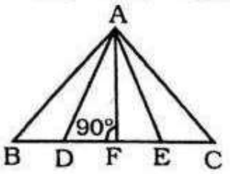
$\therefore\ \angle BDA = 180° - (70° + 50°) = 60°$

82. (3) $\frac{2x}{x} = \frac{3y+8}{2y} \Rightarrow y = 8$

$x = 2y = 16$

83. (2) BD = 2DF

$AB^2 = BF^2 + AF^2$ and $AD^2 = DF^2 + AF^2$

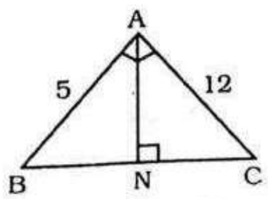


$= \left(\frac{BC}{2} - \frac{BC}{3}\right)^2 + AB^2 - BF^2$

$= \frac{BC^2}{36} + BC^2 - \frac{BC^2}{4} = \frac{BC^2 + 36BC^2 - 9BC^2}{36}$

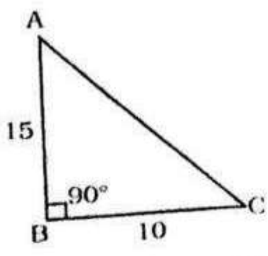
$= \frac{28BC^2}{36} = \frac{7BC^2}{9} = \frac{7}{9}AB^2$

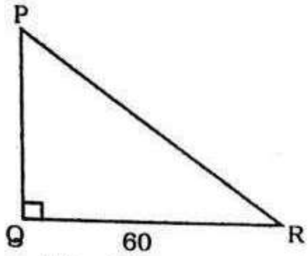
84. (3) $\frac{A(\Delta ANC)}{A(\Delta ANB)} = \frac{\frac{AN \times NC}{2}}{\frac{AN \times BN}{2}}$



$= \frac{NC}{NB} = \frac{\frac{(12 \times 12)}{13}}{\frac{5 \times 5}{13}} = \frac{144}{25}$

85. (3) $\frac{AB}{BC} = \frac{PQ}{QR} \Rightarrow \frac{15}{10} = \frac{PQ}{60}$





$\Rightarrow PQ = 90$ cm

86. (3) $\frac{BC}{PQ} = \frac{AC}{AP} = \frac{AB}{AQ} \Rightarrow \frac{8}{4} = \frac{AC}{2.8} \Rightarrow AC = 5.6$ cm

87. (4) $\frac{3}{x-3} = \frac{x-5}{3x-19}$

$9x - 57 = x^2 - 8x + 15$

$\Rightarrow x^2 - 17x + 72 = 0$

$\Rightarrow x^2 - 8x - 9x + 72 = 0$

$\Rightarrow x(x-8) - 9(x-8) = 0$

$\Rightarrow (x-9)(x-8) = 0$

$\Rightarrow x = 8$ or 9

88. (2) $h = \frac{\sqrt{3}}{2}$ side $= \frac{\sqrt{3}}{2} \times 2a = a\sqrt{3}$

89. (2) $\frac{AB}{AC} = \frac{BD}{CD}$

$\Rightarrow \frac{5}{AC} = \frac{2}{3}$

$\Rightarrow AC = 7.5$ cm

90. (1) $\frac{AB}{QR} = \frac{PB}{PR} \Rightarrow \frac{3}{9} = \frac{PB}{6} \Rightarrow PB = 2$ cm

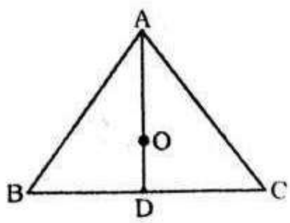
91. (3) $\frac{AO}{BO} = \frac{AQ}{BP} \Rightarrow \frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = 15$ cm

92. (2) $\frac{AB}{AC} = \frac{AD}{AE} \Rightarrow \frac{12}{15} = \frac{6}{AE}$

$\therefore\ AE = 7.5$ cm

93. (2) D, is the mid-point of side BC.

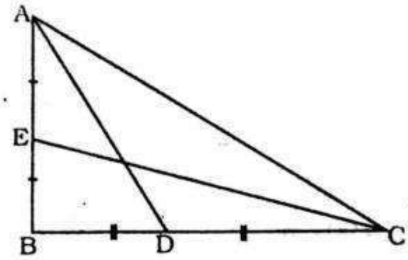
Point O is the centroid that divides AD in the ratio 2 : 1.



$\therefore\ OD = 5$ cm.

94. (1) AC = 5 cm

$AD = \frac{3\sqrt{5}}{2}$ cm



AE = BE and BD = CD

$AB^2 = AC^2 - BC^2 = 25 - BC^2$ ...(i)

and $AB^2 = AD^2 - BD^2 = \left(\frac{3\sqrt{5}}{2}\right)^2 - BD^2$

$= \frac{45}{4} - \frac{BC^4}{4}$ ...(ii)

From equations (i) and (ii)

$BC^2 = \frac{55}{3}$

Now, from equation (i)

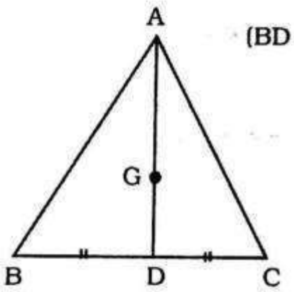
$AB^2 = 25 - \frac{55}{3} = \frac{20}{3}$

Also, $CE^2 = BE^2 + BC^2 = \left(\frac{1}{2}AB\right)^2 + BC^2$

$= \frac{AB^2}{4} + BC^2 = \frac{5}{3} + \frac{55}{3} = \frac{60}{3} = 20$

$\therefore$ CE = $2\sqrt{5}$ cm

95. (4) By apollonius theorem



(BD = CD)

$\Rightarrow AB^2 + AC^2 = 2(AD^2 + BD^2)$

$100 + 196 = 2(AD^2 + 36)$

$\Rightarrow AD^2 = 112$

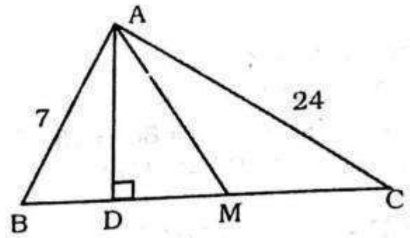
$\therefore$ AD = $4\sqrt{7}$

Now, since $\frac{AG}{GD} = \frac{2}{1}$ $\Rightarrow AG = \frac{2}{3}AD$

$\therefore AG = \frac{2}{3} \times 4\sqrt{7} = \frac{8}{3}\sqrt{7}$ cm

96. (3) $\because$ AM is the median of a right angled triangle.

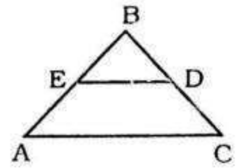
$\therefore AM = \frac{BC}{2} = \frac{25}{2}$



and $AD = \frac{AB \times AC}{BC} = \frac{7 \times 24}{25}$

$\therefore \frac{AD}{AM} = \frac{7 \times 24 \times 2}{25 \times 25} = \frac{336}{625}$

97. (2) ΔBED · ΔBAC



$\therefore \frac{BE}{BA} = \frac{ED}{AC} = \frac{BD}{BC} = \frac{1}{2}$

$\therefore \frac{A(\Delta BED)}{A(\Delta BAC)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore$ A (ΔBDE) = 7.5 cm^2

98. (4) Apply Pythagorus theorem.

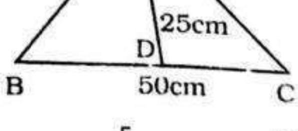
Note : The sides given in option (4) cannot form any type of triangle.

99. (3) $\frac{AB}{BD} = \frac{AC}{CD}$ $\Rightarrow \frac{12}{BC + CD} = \frac{8}{CD}$

$\Rightarrow \frac{12}{4 + CD} = \frac{8}{CD}$ $\Rightarrow$ CD = 8 cm

100. (3) $AB^2 + AC^2 = 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]$

(Appollonius theorem)



$\Rightarrow 2500 = 2\left[625 + \left(\frac{BC}{2}\right)^2\right]$

$\Rightarrow$ BC = 50 cm

101. (3) $\angle BAC = 15°$

$\therefore \angle BCA = 15°$ $(\because AB = BC)$

$\therefore \angle ABC = 180° - (15° + 15°) = 150°$

$\therefore \angle ABD = 30°$ $(180° - 150°)$

$\therefore \sin 30° = \frac{AD}{AB}$

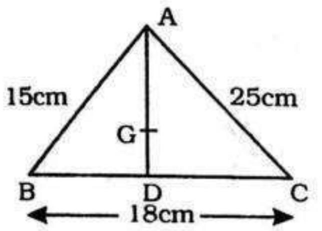
$\Rightarrow \frac{1}{2} = \frac{AD}{AB}$

$\Rightarrow AD = 5$ cm $(\because AB = 10$ cm$)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 10 \times 5 = 25$ cm^2

102. (2) $AB^2 + AC^2 = 2(AD^2 + BD^2)$



$\therefore 225 + 625 = 2(AD^2 + 81)$

$\Rightarrow AD^2 = 344$

$\Rightarrow AD = 2\sqrt{86}$ and $GD = \frac{1}{3} AD$

$\therefore GD = \frac{2}{3}\sqrt{86}$ cm

103. (3) $\angle BDE = 115°$

$\therefore \angle ADE = 65°$ and $\angle AED = 75°$

$\therefore \Delta AED \sim \Delta ABC$

$\therefore \frac{DE}{BC} = \frac{AE}{AB} = \frac{AD}{AC}$

$\therefore \frac{2}{3} = \frac{10}{AB}$

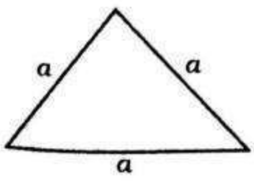
$\Rightarrow AB = 15$ cm

104. (2) For the given perimeter of a triangle the maximum area is enclosed by an equilateral triangle.

$\therefore 3a = 24$ cm

$\Rightarrow a = 8$ cm

$\therefore$ Area $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (8)^2 = 16\sqrt{3}$ cm^2



105. (1) $\Delta ADE \sim \Delta ACB$ (A-A-A property)

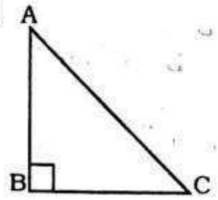
$\therefore \frac{AE}{AB} = \frac{AD}{AC}$ $\Rightarrow \frac{10}{20} = \frac{12}{AC}$

$\Rightarrow AC = 24$ cm

$\therefore EC = AC - AE = 24 - 10 = 14$ cm

106. (4) It is value (side) dependent i.e., it is different for different triangles unlike in equilateral triangle.

107. (2)



Let $BC = x$

$\therefore AB = x + 2$

$\therefore AB^2 + BC^2 = AC^2$

$\Rightarrow (x+2)^2 + x^2 = (2\sqrt{5})^2$

$\Rightarrow x^2 + 4x + 4 + x^2 = 20$

$\Rightarrow 2x^2 + 4x - 16 = 0$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0$

$\Rightarrow x(x+4) - 2(x+4) = 0$

$\Rightarrow (x-2)(x+4) = 0$

$\Rightarrow x = 2 = BC$

$\therefore AB = 2 + 2 = 4$cm

$\therefore \cos^2 A - \cos^2 C = \frac{AB^2}{AC^2} - \frac{BC^2}{AC^2}$

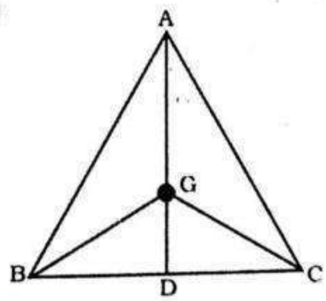
$= \frac{16}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$

108. (2) $\frac{\Delta ABC}{\Delta DEF} = \frac{64}{121} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{8}{11} = \frac{BC}{EF} \Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

$\Rightarrow BC = \frac{8 \times 15.4}{11} = 11.2$ cm

109. (3)



$AG = BC$

$\angle BGC = 90°$

110. (1) $2x + 3x + 5x = 180° - 45° = 135°$

$\Rightarrow 10x = 135°$

$\Rightarrow x = \frac{135}{10} = \frac{27}{2}$

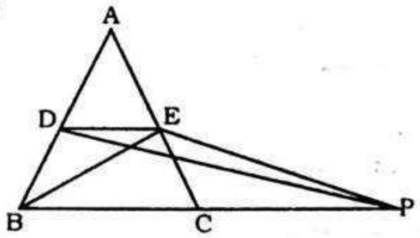
$\therefore$ Largest angle

$= 5x + 15° = \left(5 \times \frac{27}{2}\right)° + 15° = \frac{135 + 30}{2} = \frac{165°}{2}$

$\because$ 180° = π radian

$\therefore \frac{165°}{2} = \frac{\pi}{180} \times \frac{165}{2} = \frac{11\pi}{24}$ radian

111. (1)



DE || BC

$DE = \frac{1}{2} BC$

$\therefore \Delta BDE = \Delta DEP$

$2\Delta BDE = \Delta BEC$

$\therefore \Delta ADE = \Delta BDE$

$\therefore \Delta ABC = 4 \Delta ADE$

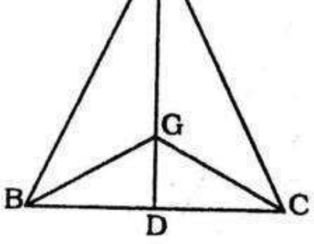
$\therefore PED = \frac{1}{4} \Delta ABC$

112. (3) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AB + BC + CA}{PQ + QR + RP}$

$\Rightarrow \frac{AB}{PQ} = \frac{36}{24}$ $\qquad \Rightarrow \frac{AB}{10} = \frac{36}{24}$

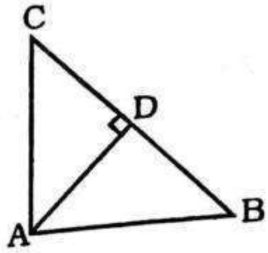
$\Rightarrow AB = \frac{36 \times 10}{24} = 15$ cm

113. (2)



$AG = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8$ cm

114. (2)



In Δ ABD and ΔACD,

$\angle ADC = \angle ADB = 90°$

$\angle CAD = \angle ABD = 90° - \angle DAB$

$\Rightarrow \Delta CAD \sim \Delta ABD$

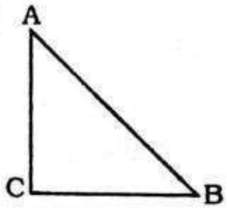
$\therefore \frac{AC}{CD} = \frac{AB}{BD}$

$\Rightarrow \frac{AC}{AB} = \frac{CD}{BD}$

$\Rightarrow \frac{AC + AB}{AB} = \frac{CD + BD}{BD}$

$\Rightarrow \frac{3AB}{AB} = \frac{BC}{BD} \Rightarrow BD = \frac{BC}{3}$

115. (3)



$\angle ACB = 90°$

$AB = 2.5$

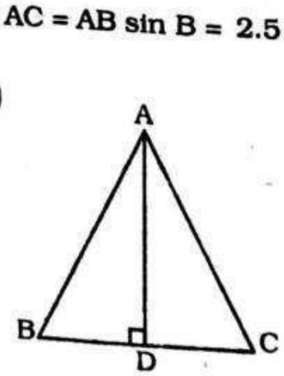
$\cos B = 0.5 = \frac{1}{2}$

$\therefore \sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \sin B = \frac{AC}{AB}$

$\Rightarrow AC = AB \sin B = 2.5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$

116. (4)



In ΔABD and ΔADC

$AB^2 = AD^2 + BD^2$

$AC^2 = AD^2 + DC^2$

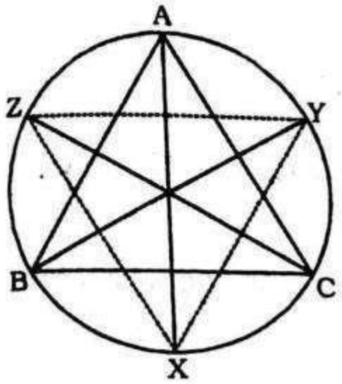
$\therefore AB^2 + AC^2 = 2\ AD^2 + BD^2 + DC^2$

$= 2\ BD \times CD + BD^2 + DC^2$

$\Rightarrow AB^2 + AC^2 = (BD + CD)^2 = BC^2$

$\therefore \angle BAC = 90°$

117. (3)



$\angle BYX = \angle BAX$

$\therefore \angle BYX = \frac{\angle A}{2} = 25°$

$\angle BYZ = \angle BCZ$

$\therefore \angle BYZ = \frac{\angle C}{2}$

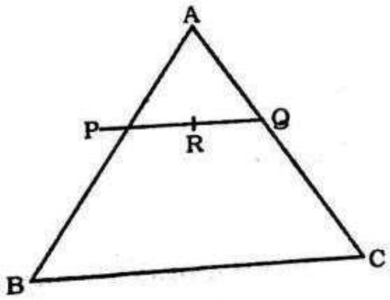
$\angle CBY = \angle CZY = 30°$

$\therefore \angle B = 60°$

$\therefore \angle C = 180° - 50° - 60° = 70°$

$\therefore \angle BYZ = \frac{70}{2} = 35°$

118. (3)



$\frac{PR}{RQ} = \frac{1}{2} \Rightarrow \frac{2}{RQ} = \frac{1}{2}$

$\therefore$ RQ = 4 cm

$\therefore$ PQ = PR + RQ = 2 + 4 = 6 cm

The line joining the mid – points of two sides of a triangle is parallel to and half of the third side.

$\therefore$ BC = 2PQ = 2 × 6 = 12 cm

119. (2) Ratio of corresponding sides $= \sqrt{\frac{9}{16}} = \frac{3}{4}$

120. (3) AF = FB

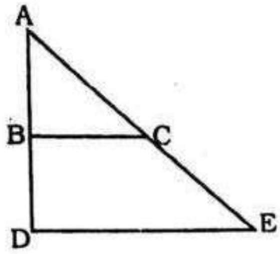
AE = EC $\quad \therefore$ FE || BC

$= \frac{1}{2}BC \quad \therefore$ AO : OG = 2 : 1

121. (3) It is based on fundamental concept.

122. (1) $\angle BPC = 90° - \frac{A}{2} = 90° - 40° = 50°$

123. (2)



BC || DE

$\therefore \angle ABC = \angle ADE = 90°$

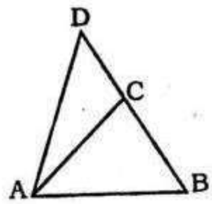
$\angle ACB = \angle AED$

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{BC}{DE} \Rightarrow \frac{12}{18} = \frac{10}{DE}$

$\Rightarrow DE = \frac{18 \times 10}{12} = 15$

124. (4)



AC = BC

$\therefore \Delta$ ABC is isosceles.

$\therefore \angle ABC = \angle CAB = 50°$

$\therefore \angle ACB = 180° - 100° = 80°$

Δ ACD is also isosceles.

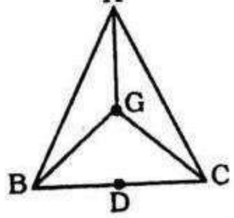
$\therefore \angle CAD = \angle CDA$

$\therefore \angle ACB = 2\angle CAD$

$\therefore \angle CAD = \frac{80}{2} = 40°$

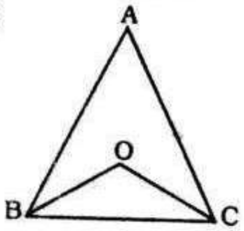
$\therefore \angle BAD = 50° + 40° = 90°$

125. (4)



Area of Δ ABC = 6 × ΔBGD = 6 × 6 = 36 sq. cm.

126. (1)



$\angle A + \angle B + \angle C = \pi$

$\Rightarrow \frac{1}{2}(\angle A + \angle B + \angle C) = \frac{\pi}{2} \quad \Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = \frac{\pi}{2} - \frac{A}{2}$

In Δ OBC,

$\angle OBC + \angle OCB + \angle BOC = \pi$

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} + \angle BOC = \pi$

$\therefore \angle BOC = \pi - \left(\frac{\pi}{2} - \frac{A}{2}\right) = \frac{\pi}{2} + \frac{A}{2}$

127. (4) The sum of any two sides of a triangle is greater than third side and their difference is less than third side.

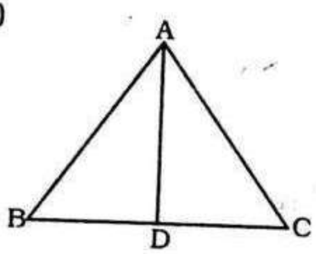
$\therefore a + 4 > 10 \Rightarrow a > 10 - 4$

$\Rightarrow a > 6$

Again, $a - 4 < 10 \Rightarrow a < 14$

$\therefore 6 < a < 14$

128. (4)



BD = DC = AD

$\angle BAD = 30°$

From Δ ABD,

$\angle BAD = 30°$

$\therefore \angle ABD = \angle BAD = 30°$

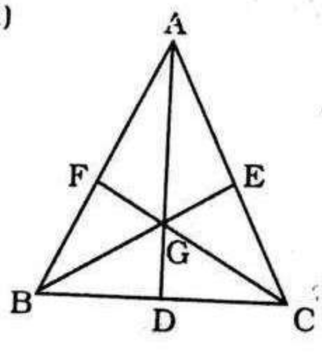
$\therefore \angle ADB = 180° - 2 \times 30° = 120°$

$\therefore \angle ADC = 180° - 120° = 60°$

$\therefore$ AD = DC

$\Rightarrow \angle DAC = \angle ACD = 60°$

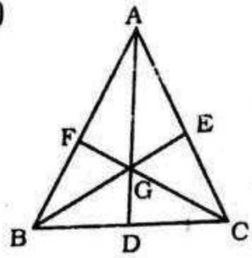
129. (1)



Area of Δ BDG $= \frac{1}{6} \times$ Area ofΔ ABC

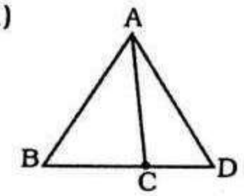
$= \frac{1}{6} \times 72 = 12$ sq. cm

130. (3)



Required area $= \frac{1}{3} \times 60 = 20$ sq. cm

131. (1)



$\angle ADB = 20°$

AB = AC ; CD = CA

$\angle ABC = ?$

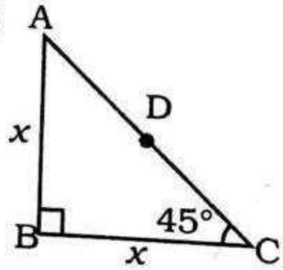
$\therefore \angle CAD = 20°$

$\therefore \angle ACD = 180° - 40° = 140°$

$\therefore \angle ACB = 180° - 140° = 40°$

$\therefore \angle ABC = \quad \angle ACB = 40°$

132. (1)

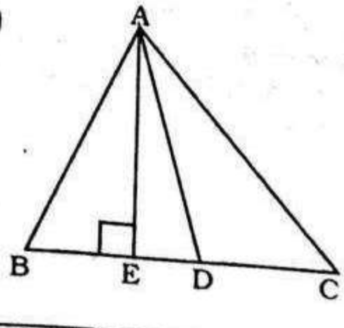


$\Rightarrow 2x^2 = AC^2 \quad \Rightarrow 2x^2 = \left(4\sqrt{2}\right)^2 = 32$

$\Rightarrow x^2 = 16 \quad \Rightarrow x = 4$ units

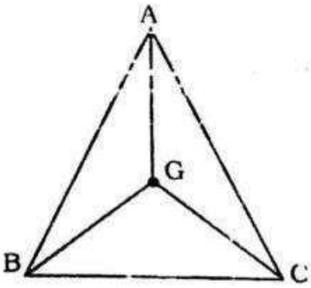
$\therefore BD = \sqrt{AB^2 - AD^2} = \sqrt{16 - 8} = 2\sqrt{2}$ units

133. (1)



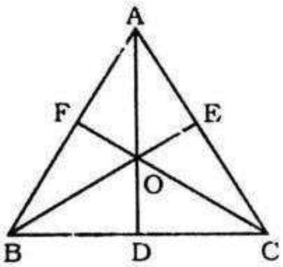
$\angle B = 60°$; $\angle C = 40°$
$\angle A = 180 - 100 = 80°$
$\angle BAD = \angle DAC = 40°$
$\therefore$ From ΔABE,
$\angle BAE = 180° - 60° - 90° = 30°$
$\angle EAD = 40 - 30 = 10°$

134. (1)



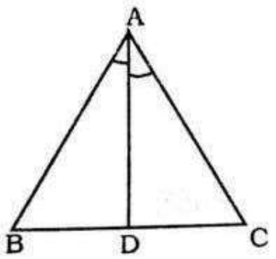
$AG = BC \therefore \angle BGC = 90°$

135. (2)



Area of quadrilateral BDOF = 2 × 15 = 30 sq.cm.

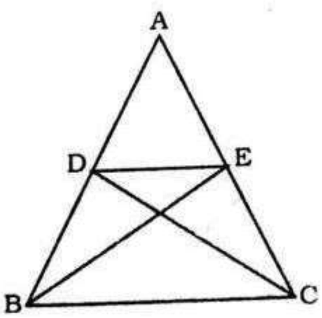
136. (1)



AD is the internal bisector of $\angle A$.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{7.5-2} = \frac{5}{2.5} = 2:1$$

137. (2)



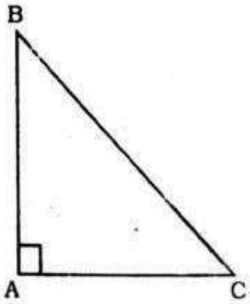
ΔDBC AND ΔEBC lie on the same base and between same parallel lines.

$\therefore \Delta DBC = \Delta BEC$

$\Rightarrow \Delta ABC - \Delta DBC = \Delta ABC - \Delta BEC$

$\Rightarrow \Delta ADE = \Delta ABE = 36$ sq.cm

138. (2)



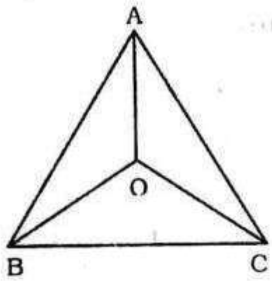
If $AB = x$; $BC = 2x$ units

$\therefore AC = \sqrt{4x^2 - x^2} = \sqrt{3}x$

$\therefore \sin ACB = \frac{AB}{BC} = \frac{1}{2} = \sin 30°$

$\therefore \angle ACB = 30°$

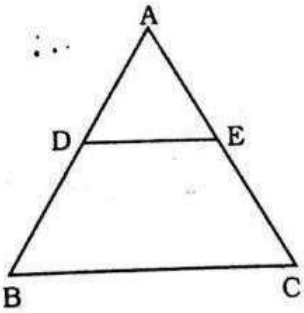
139. (2)



$\angle BOC = 90° + \frac{1}{2}\angle BAC$

$= 90° + 15° = 105°$

140. (4)

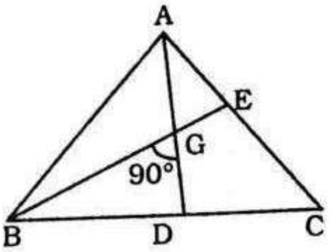


$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$$

$$\therefore \frac{DE}{BC} = \frac{1}{3}$$

$$\Rightarrow DE = \frac{15}{3} = 5 \text{ cm}$$

141. (3)



AD = 9 cm.

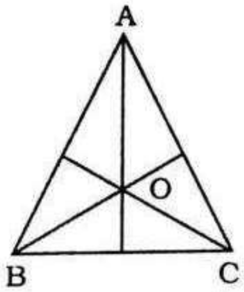
$\Rightarrow GD = \frac{1}{3} \times 9 = 3$ cm

BE = 6 cm

$\Rightarrow BG = \frac{2}{3} \times 6 = 4$ cm

$\therefore BD = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$ cm.

142. (4)

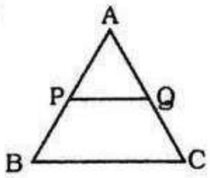


In equilateral triangle centroid, incentre, orthocentre coincide at the same point.

$\therefore \frac{\text{Height}}{3} = \text{in radius}$

$\therefore$ Height = Median = 3 × 3 = 9 cm

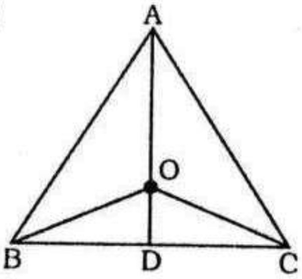
143. (2)



$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$ $\Rightarrow \frac{QC}{AQ} = \frac{2}{1}$

$\Rightarrow \frac{QC + AQ}{AQ} = \frac{3}{1}$ $\Rightarrow AC = 3AQ = 9$ cm

144. (3)



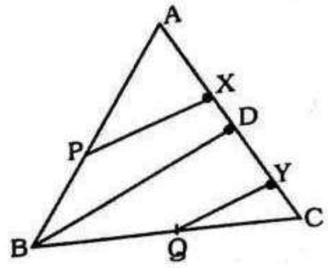
BO is the internal bisector of ∠B

$\angle ODB = 90°$; $\angle BOD = 15°$

$\angle OBD = 180° - 90° - 15° = 75°$

$\angle ABC = 2 \times 75° = 150°$

145. (2)

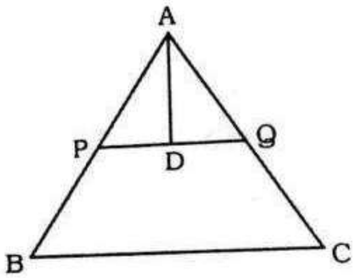


$PX \parallel BD$ and $PX = \frac{1}{2} BD$

$QY \parallel BD$ and $QY = \frac{1}{2} BD$

$\therefore$ PX : QY = 1 : 1

146. (3)



$PQ \parallel BC$

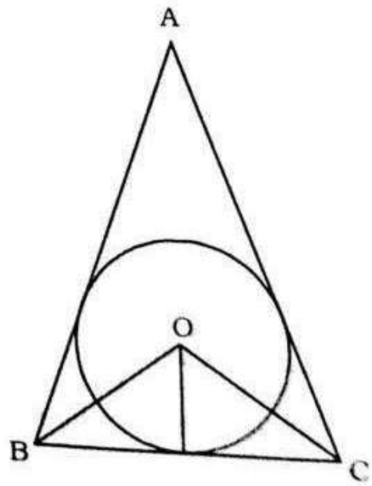
$\angle APQ = \angle ABC = 60°$

$\angle AQP = \angle ACB = 60°$

$\therefore$ Area of $\Delta APQ = \frac{\sqrt{3}}{4} \times (PQ)^2$

$= \frac{\sqrt{3}}{4} \times (5)^2 = \frac{25\sqrt{3}}{4}$ sq.cm.

147. (2)



$\angle BOC = 90^\circ + \frac{A}{2}$

$\Rightarrow 110 = 90^\circ + \frac{A}{2}$

$\Rightarrow A = 2 \times 20 = 40^\circ$

148. (2) at the right angular vertex

149. (3) Sum of angles of a triangle = 180°

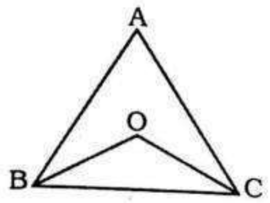
$\therefore x + 5 + 2x - 3 + 3x + 4 = 180^\circ$

$\Rightarrow 6x + 6 = 180^\circ$

$\Rightarrow 6x = 180 - 6 = 174^\circ$

$\Rightarrow x = \frac{174}{6} = 29$

150. (4)



$\angle BAC = 80^\circ$

$\therefore \angle B + \angle C = 180^\circ - 80^\circ = 100^\circ$

$\frac{\angle B}{2} + \frac{\angle C}{2} = 50^\circ$

$\therefore \angle OBC + \angle OCB = 50^\circ$

$\therefore \angle BOC = 180^\circ - 50^\circ = 130^\circ$

151. (2) AB + BC = 12

BC + CA = 14

CA + AB = 18

$\therefore$ 2 (AB + BC + CA)

= 12 + 14 + 18 = 44

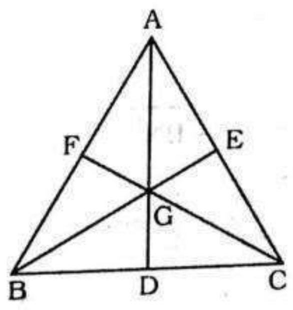
$\Rightarrow$ AB + BC + CA = 22

$\therefore 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2}$ cm

152. (2) AG = 6 cm.



$BG = \frac{2}{3} \times 12 = 8$ cm.

$GC = \frac{2}{3} \times 15 = 10$ cm.

Area of $\Delta ABG = \frac{1}{2} \times 6 \times 8$

= 24 sq. cm.

$\therefore$ Area of $\Delta ABC = 3 \times 24 = 72$ sq. cm.

153. (4) Sum of interior angles

$= (2n - 4) \times 90^\circ$

Sum of exterior angles = 360°

$\therefore (2n - 4) \times 90^\circ = 360^\circ \times 2$

$\Rightarrow 2n - 4 = 2 \times 360^\circ \div 90 = 8$

$\Rightarrow 2n - 4 = 8 \Rightarrow 2n = 12 \Rightarrow n = 6$

154. (2) In radius $= \frac{\text{Side}}{2\sqrt{3}}$

$\Rightarrow 3 = \frac{\text{Side}}{2\sqrt{3}} \Rightarrow \text{Side} = 3 \times 2\sqrt{3}$

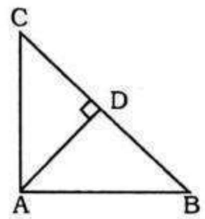
$= 6\sqrt{3}$ cm

155. (2) In Δs ACD and ABC,

$\angle CDA = \angle CAB = 90^\circ$

$\angle C$ is common.

$\therefore \Delta ACD \sim \Delta ABC$



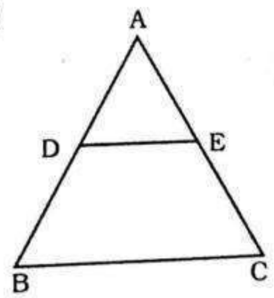
$\therefore \frac{\Delta ACD}{\Delta ABC} = \frac{AC^2}{BC^2}$

$\Rightarrow \frac{10}{40} = \frac{9^2}{BC^2}$

$\Rightarrow BC^2 = 4 \times 9^2$

$\therefore BC = 2 \times 9 = 18$ cm

156. (2)



DE | | BC

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \dfrac{\square BDEC}{\Delta ADE} = \dfrac{1}{1}$

$\Rightarrow \dfrac{\square BDEC}{\Delta ADE} + 1 = 1 + 1$

$\Rightarrow \dfrac{\Delta ABC}{\Delta ADE} = 2 = \dfrac{AB^2}{AD^2}$

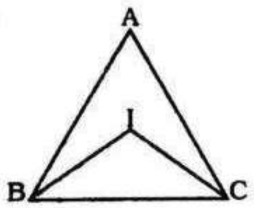
$\Rightarrow \dfrac{AB}{AD} = \sqrt{2}$

$\Rightarrow \dfrac{AB}{AD} - 1 = \sqrt{2} - 1$

$\Rightarrow \dfrac{BD}{AD} = \sqrt{2} - 1$

$\Rightarrow \dfrac{AD}{BD} = \dfrac{1}{\sqrt{2} - 1}$

157. (2)



$\angle IBC = \dfrac{1}{2}\angle ABC = \dfrac{65}{2} = 32.5°$

$\angle ICB = \dfrac{1}{2}\angle ACB = \dfrac{55}{2} = 27.5°$

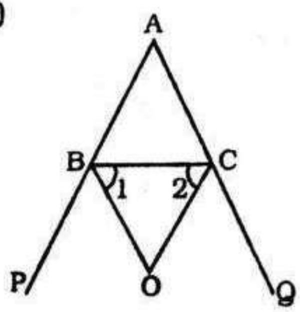
$\therefore \angle BIC = 180° - 32.5° - 27.5° = 120°$

158. (4) Number of sides of polygon= $\dfrac{360}{72} = 5$

$\therefore$ Sum of interior angles

$= (2n - 4) \times 90° = (2 \times 5 - 4) \times 90° = 540°$

159. (3)



$\angle ABC + \angle CBP = 180°$

$\Rightarrow \angle B + 2\angle 1 = 180°$

$\Rightarrow 2\angle 1 = 180° - \angle B$

$\Rightarrow \angle 1 = 90° - \dfrac{1}{2}\angle B$

Again, $\angle ACB + \angle QCB = 180°$

$\Rightarrow \angle 2 = 90° - \dfrac{1}{2}\angle C$

In Δ BOC,

$\angle 1 + \angle 2 + \angle BOC = 180°$

$\Rightarrow 90° - \dfrac{1}{2}\angle B + \angle 90° - \dfrac{1}{2}\angle C + \angle BOC = 180°$

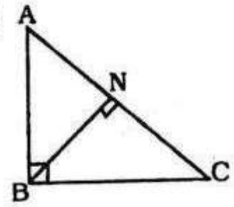
$\Rightarrow \angle BOC = \dfrac{1}{2}(\angle B + \angle C) = \dfrac{1}{2}(180° - \angle A)$

$\Rightarrow \angle BOC = 90° - \dfrac{1}{2}\angle A$

$\Rightarrow 60° = 90° - \dfrac{1}{2}\angle A$

$\Rightarrow \angle A = 60°$

160. (2)



$BC = \sqrt{10^2 - 6^2} = \sqrt{100 - 36}$

$= \sqrt{64} = 8$ cm

Area of Δ ABC,

$= \dfrac{1}{2} \times BC \times AB$

$= \dfrac{1}{2} \times 8 \times 6 = 24$ sq.cm

Again,

$\dfrac{1}{2} AC \times BN = 24$

$\Rightarrow \dfrac{1}{2} \times 10 \times BN = 24$

$\Rightarrow BN = \dfrac{24}{5}$

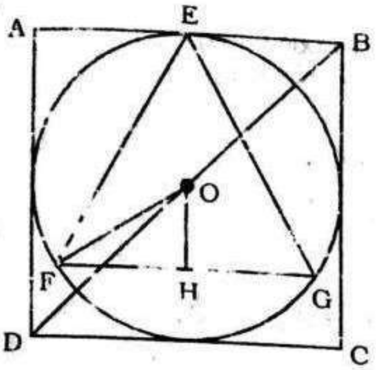
$\therefore NC = \sqrt{BC^2 - BN^2}$

$= \sqrt{64 - \dfrac{576}{25}} = \dfrac{32}{5}$ cm

$AN = 10 - \dfrac{32}{5} = \dfrac{50 - 32}{5} = \dfrac{18}{5}$

$\therefore AN : NC = \dfrac{18}{5} : \dfrac{32}{5} = 9 : 16$

161. (3)



Side of square = $\frac{1}{\sqrt{2}} \times 12\sqrt{2}$ = 12 cm

$\therefore$ Radius of circle = $\frac{12}{2}$ = 6 cm

AB = $2x$ cm

$\therefore$ FH = x cm

$\therefore$ From ΔOFH,

$\cos 30° = \frac{FH}{OF}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{6}$

$\Rightarrow x = \frac{6 \times \sqrt{3}}{2} = 3\sqrt{3}$

Length of side = $6\sqrt{3}$ cm

162. (1) $\frac{\sqrt{3}}{4} x^2 = 4\sqrt{3}$

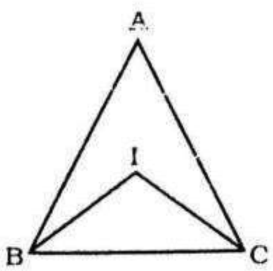
$\Rightarrow x^2 = 4 \times 4$

$\Rightarrow x = 4$ cm

$\therefore$ Perimeter of equilateral triangle

= 3 × 14 = 12 cm

163. (3)



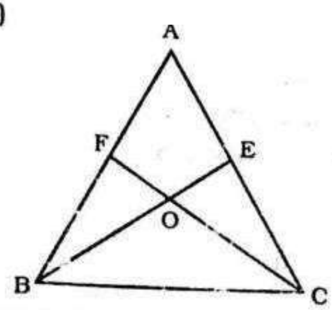
$\angle ABI = \angle IBC = 30°$

$\angle ACI = \angle ICB = 40°$

In ΔBIC,

$\therefore \angle BIC = 180° - 30° - 40° = 110°$

164. (4)



$\angle BAC = 70°$

$\angle ABC + \angle ACB = 110°$ ...(i)

From $\angle$BCF,

$\angle CFB + \angle FBC + \angle FCB = 180°$

$\Rightarrow \angle FBC + \angle FCB = 90°$ ...(ii)

From ΔBCE,

$\angle ECB + \angle EBC = 90°$ ...(iii)

Adding equations (ii) and (iii)

$\angle EBC + \angle FCB = 180° - 110° = 70°$

$\therefore \angle BOC = 180° - 70° = 110°$

165. (4) Sides = $3x$, $3x$ and $4x$

Semi perimeter = $\frac{3x + 3x + 4x}{2} = 5x$

$\therefore \Delta = \sqrt{5x(5x-3x)(5x-3x)(5x-4x)}$

$= \sqrt{5x \times 2x \times 2x \times x}$

$= 2\sqrt{5}x^2$

$\therefore 2\sqrt{5}x^2 = 18\sqrt{5}$

$\Rightarrow x^2 = 9 \Rightarrow x = 3$

$\therefore$ Third side = $4x = 4 \times 3 = 12$ units

166. (3) $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 = 2ab + 2bc + 2ca$

$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

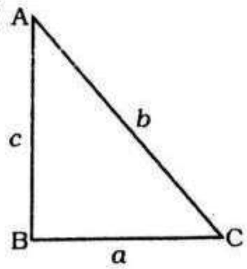
$\Rightarrow a - b = 0 \Rightarrow a = b$

$\Rightarrow b - c = 0 \Rightarrow b = c$

$\Rightarrow c - a = 0 \Rightarrow c = a$

$\therefore a = b = c$

167. (1)



$a + b + c = 56$ ...(i)

$\frac{1}{2} ac = 84$

$\Rightarrow ac = 168$ sq.cm.

$\therefore b^2 = a^2 + c^2$

$\Rightarrow b^2 = (a + c)^2 - 2ac$

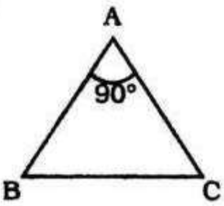
$\Rightarrow b^2 = (56 - b)^2 - 2 \times 168$

$\Rightarrow b^2 = 3136 - 112\,b + b^2 - 336$

$\Rightarrow 112b = 2800$

$\Rightarrow b = \dfrac{2800}{112} = 25$ cm

168. (2)



$AB^2 + AC^2 = BC^2 \Rightarrow \angle BAC = 90°$

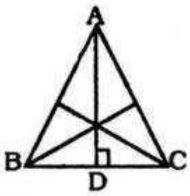
$\Rightarrow AB^2 + AC^2 = 2AB^2$

$\Rightarrow AB^2 = AC^2$

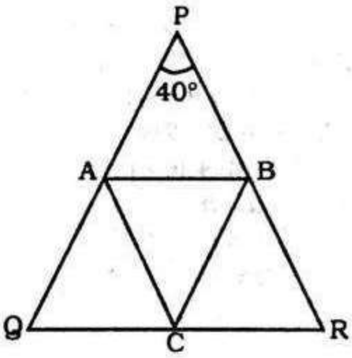
$\Rightarrow AB = AC$

$\therefore \angle ABC = \angle ACB = 45°$

169. (3) In equilateral triangle orthocentre and centroid lie at the same point.



170. (4)



AC= QC

$\therefore \angle QAC = \angle CQA = x$

CR = CB

$\therefore \angle CBR = \angle CRB = y$

$\therefore$ From ΔPQR,

$\angle x + \angle y + 40° = 180$

$\angle x + \angle y = 140°$ ......(i)

Again,

$\angle ACQ + \angle ACB + \angle BCR = 180°$

$\Rightarrow 180° - 2x + \angle ACB + 180° - 2y = 180°$

$\Rightarrow \angle ACB = 2\,(x + y) - 180°$

$= 2 \times 140 - 180° = 100°$

171. (3) The right bisectors of sides meet at a point called circumcentre.

172. (3) Here, $(3x)^2 + (4x)^2 = (5x)^2$

$\therefore$ It is a right angled triangle.

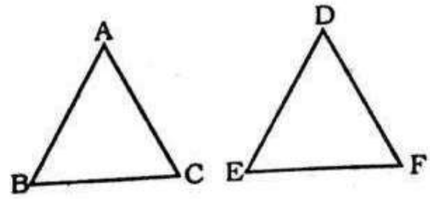
$\therefore$ Area of the triangle $= \dfrac{1}{2} \times 3x \times 4x = 6x^2$

$\therefore 6x^2 = 72 \Rightarrow x^2 = 12$

$\Rightarrow x = 2\sqrt{3}$

$\therefore$ Smallest side $= 3x = 6\sqrt{3}$ units

173. (2)



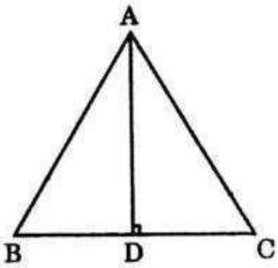
$\Delta ABC \sim \Delta DEF$

$\therefore \angle A = 47° = \angle D$

$\angle B = \angle E = 63°$

$\therefore \angle C = 180° - 47° - 63° = 70°$

174. (3)



AB = AC = x units

BD = DC = 1 unit

$AD = \sqrt{AB^2 - BD^2} = \sqrt{x^2 - 1}$

$\therefore \dfrac{1}{2} \times BC \times AD = 4$

$\Rightarrow \dfrac{1}{2} \times 2 \times \sqrt{x^2 - 1} = 4$

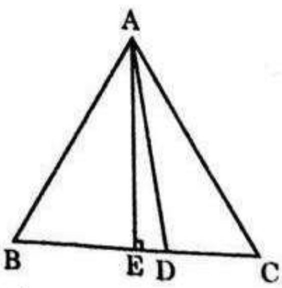
$\Rightarrow \sqrt{x^2 - 1} = 4$

$\Rightarrow x^2 - 1 = 16$

$\Rightarrow x^2 = 17$

$\Rightarrow x = \sqrt{17}$ units

175. (2)



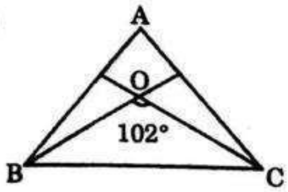
$\angle A = 180° - 60° - 40° = 80°$

$\angle BAD = \frac{80}{2} = 40°$

$\angle BAE = 180° - 60° - 90° = 30°$

$\therefore \angle DAE = 40° - 30° = 10°$

176. (2)



$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$

In ΔBOC,

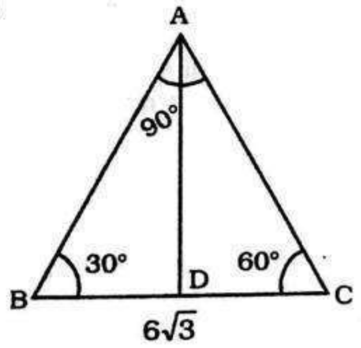
$\angle BOC + \frac{\angle B}{2} + \frac{\angle C}{2} = 180°$

$\Rightarrow 102° + 90° - \frac{\angle A}{2} = 180°$

$\Rightarrow \frac{\angle A}{2} = 102° + 90° - 180° = 12°$

$\Rightarrow \angle A = 24°$

177. (2)



$\sin 30° = \frac{AC}{BC}$

$\Rightarrow \frac{1}{2} = \frac{AC}{6\sqrt{3}} \Rightarrow AC = 3\sqrt{3}$

$\Rightarrow \sin 60° = \frac{AD}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{3\sqrt{3}}$

$\Rightarrow AD = \frac{3\sqrt{3} \times \sqrt{3}}{2} = 4.5$ cm

178. (1) Side of equilateral triangle = x units.

$\therefore \frac{\sqrt{3}}{4}\left((x+2)^2 - x^2\right) = 3 + \sqrt{3}$

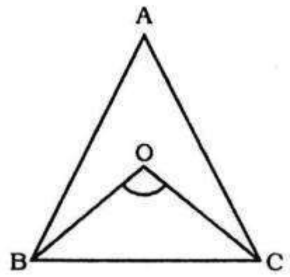
$\Rightarrow \frac{\sqrt{3}}{4}(4x + 4) = 3 + \sqrt{3}$

$\Rightarrow \sqrt{3}x + \sqrt{3} = 3 + \sqrt{3} = \sqrt{3}x = 3$

$\Rightarrow x = \sqrt{3}$ units

179. (4) In an equilateral triangle, centroid, incentre etc lie at the same point.

180. (1)



In ΔABC,

$\angle A + \angle B + \angle C = 180°$ ... (i)

In ΔOBC,

$\angle OBC + \angle BOC + \angle OCB = 180°$

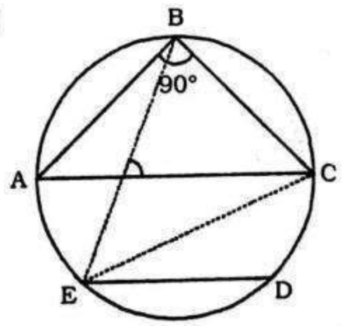
$\Rightarrow \frac{\angle B}{2} + 110° + \frac{\angle C}{2} = 180°$

$\Rightarrow \frac{\angle B + \angle C}{2} = 180° - 110° = 70°$

$\Rightarrow \angle B + \angle C = 140°$

$\therefore \angle A = 180° - 140° = 40°$

181. (4)



$\angle CBE = 50°$

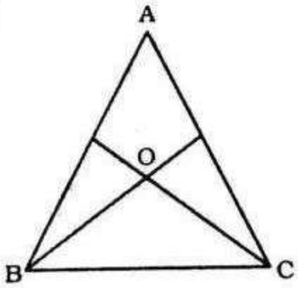
$\angle BAC + \angle BCA = 90°$

$\angle ABE = 90° - 50° = 40°$

$\therefore \angle ABE = \angle ACE = 40°$

$\therefore \angle ACE = \angle DEC = 40°$

182. (1)



In ΔABC,

$\angle A + \angle B + \angle C = 180°$

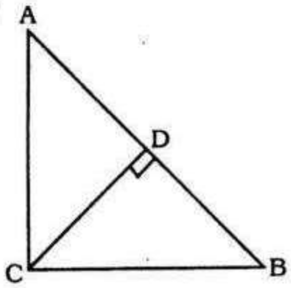
In ΔBOC, $\angle BOC = 110°$

$\therefore \frac{B}{2} + \frac{C}{2} = 180° - 110° = 70°$

$\Rightarrow B + C = 140°$

$\therefore \angle BAC = 180° - 140° = 40°$

183. (2)



$BC = a$; $AC = b$

$\therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{b^2 + a^2}$

Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2}ab$

Again, area of $\Delta ABC = \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times \sqrt{a^2 + b^2} \times p$

$\therefore \frac{1}{2}ab = \frac{1}{2}\sqrt{a^2 + b^2} \times p$

$\Rightarrow ab = \sqrt{a^2 + b^2} \times p$

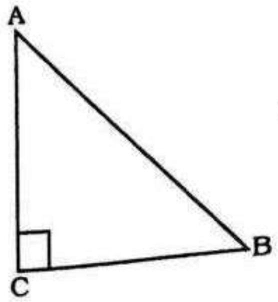
On squaring both sides,

$a^2b^2 = (a^2 + b^2)p^2$

$\Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$ $\Rightarrow \frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{a^2} + \frac{1}{b^2}$

184. (3)

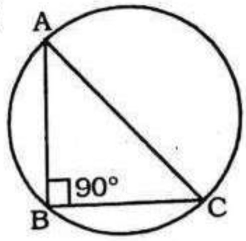


$AC = BC = 5$ cm

$\therefore AB = \sqrt{AC^2 + BC^2}$

$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$ cm

185. (1)



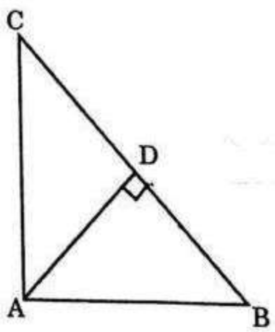
$\angle ABC = 90°$

AC = Diameter of circle

$\therefore AC = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ cm

$\therefore$ Circum-radius = 5 cm

186. (4)



$\angle BAC = 90°$

$AB = \sqrt{AD^2 + BD^2} = \sqrt{36+16} = \sqrt{52}$ cm

ΔABD and ΔABC are similar.

$\therefore \frac{AB}{BC} = \frac{BD}{AB}$

$\Rightarrow AB^2 = BC \times BD$

$\Rightarrow 52 = BC \times 4$

$\Rightarrow BC = \frac{52}{4} = 13$ cm

187. (2) $\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{100}{64} = \frac{25}{16}$