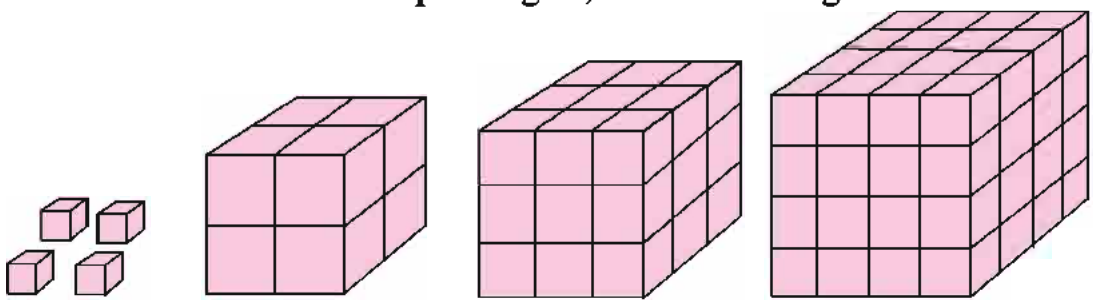
Chapter 2

Cube and Cube root

2.1 Cube and Cube root

Take some cubes from your mathematics kit and pay attention, you will find that all the cubes have equal length, breadth and height. By adding or arranging these cubic blocks, bigger cubes can also be made. But remember this; these cubes should also have equal lengths, breadth and height.



Unit cube | Cube of double units | Cube of triple units | Cube of four units

Figure 2.1

Sr. No.	Number of unit cubes in one side of large cube	Number of unit cubes in making of large cubes
1	1	1
2	2	8
3	3	27
4	4	----------
5	5	----------

Table 2.1

Now consider the numbers 1,8,27..... These numbers are called perfect cube or cube numbers. Can you tell, why this name is given to these numbers? These numbers are obtained only when this number is multiplied three times by the same number. Let us see

$$1 = 1 \times 1 \times 1 = 1^3$$
$$8 = 2 \times 2 \times 2 = 2^3$$
$$27 = 3 \times 3 \times 3 = 3^3$$

Since $5^3 = 5 \times 5 \times 5 = 125$, that is why 125 is a perfect cube. 9 is not a cube number because $9 \neq 3 \times 3 \times 3$ and there is no such natural number by which we can obtain 9 by multiplying itself three times. We know that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$. This clarifies that 9 is not a perfect cube. Cubes of 1 to 10 numbers are given below. Also, find the cube and fill in the blanks:-

Cube numbers	Numbers
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 =$ .....
6	$6^3 = 6 \times 6 \times 6 =$ .....
7	$7^3 = 7 \times 7 \times 7 =$ .....
8	$8^3 = 8 \times 8 \times 8 =$ .....
9	$9^3 = 9 \times 9 \times 9 =$ .....
10	$10^3 = 10 \times 10 \times 10 =$ .....

Table 2.2

We know that $2^2 = 4$, where $4 = 2 \times 2$ or $2 + 2$. Similarly, $2^3 = 8$, where $8 = 2 \times 2 \times 2$. Is it equal to $(2 + 2 + 2)$?

2.1.1 Cube of even and odd numbers

It is clear that there is only 10 cube between 1 to 1000. Look at the cube of even numbers and odd numbers in table 2.2. You will find that cube of even number is always even number and cube of odd number is always odd number.

2.1.2 Unit digit of cube numbers

Consider the unit digits of cubes in the above table. How many numbers have unit digit 1 in their cubes? Which numbers have same unit digit of cube that contain in base numbers. You will find that when any number has digit 0,1,4,5,6 at their unit place then the unit of cube also contain the same digit .

Do and Learn

1. Find the unit digit of cubes given below:

(i) 1331 (ii) 4444 (iii) 159 (iv) 1005

2. The cube of 46 will be even or odd ?

2.2 Some patterns related to cube Numbers

2.2.1 To add the successive odd numbers:

Look at the format of an addition of odd numbers

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

By seeing this pattern, tell that how many successive odd numbers will be required to obtain the 10^3 by addition?

Do and Learn

•According to the above pattern, find the below in form of addition of odd numbers.

(i) 7^3 (ii) 8^3

2.2.2 Cube and their prime factors

Consider on the prime factors of some numbers and their cubes.

Number	Cube Number
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$10 = 2 \times 5$	$10^3 = 1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 =$.....

On the basis of the above pattern, it is clear that all the prime factors of cube numbers can be presented in the group of three similar factors.

Example 1 Is 729 a perfect cube number? Think.

Solution By prime factorisation of 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

You can see that in prime factor two groups of triples can be made. Therefore 729 is a perfect cube.

Example 2 Is 432 a perfect cube?

Solution Resolving 432 into prime factor, we have

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Clearly, in grouping the same factor in triples factor 2 is left over. Thus, 432 is not a perfect cube.

Example 3 Is 5400 a perfect cube? If not, then find the smallest natural number by which 5400 must be multiplied so that the product is a perfect cube.

Solution Resolving 5400 into prime factor, we get

$5400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

We find that the prime factor of 5400 can be grouped into triples of 2 and 3 but 5 occurs as a prime factor only twice. Therefore, 5400 is not a perfect cube.

If we multiply 5400 by 5, 5 will also occur as a prime factor thrice and the product will be perfect cube. Thus, 5 is the smallest natural number.

Example 4 Is 1188 a perfect cube? If not, then find the smallest natural number by which 1188 must be divided so that the quotient is a perfect cube.

Solution Resolving 1188 into prime factor, we get

$$1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$$

Prime factor 2 and 11 are not in a group of triples. Therefore, 1188 is not a perfect cube. In the above factorization of 1188, 2 occur twice and 11occurs single. So, if we divide 1188 by 2 x 2 x 11 then 2 and 11 will not occur in prime factor of quotient. Thus, 44 is the smallest natural number by which 1188 must be divided for the perfect cube number.

Hence, Resultant perfect cube = $1188 \div 44 = 27 = 3^3$

i.e., after grouping the prime factors in triple, product of remaining prime factors are divided to obtain the perfect cube number.

Do and Learn

- Check the perfect cube in following numbers

(i) 2700	(ii) 16000	(iii) 64000	(iv) 900
(v) 125000	(vi) 36000	(vii) 21600	(viii) 10000
(ix) 27000000	(x) 11000		

Exercise 2.1

1. Which of the following numbers are not perfect cube?

   (i) 512 (ii) 243 (iii) 1000 (iv) 100 (v) 2700

2. Find out the smallest number multiplied by the following numbers to get the perfect cube?

   (i) 108 (ii) 500 (iii) 5400 (iv) 10584

3. Find out the smallest number by which following numbers must be divided to get the perfect cube?

   (i) 108 (ii) 500 (iii) 5400 (iv) 10584

4. Rehan works in soap factory. He is playing with arranging cubic soap by making cubes. If he has to arrange 216 soap then how many soaps will occur in first line of cube.

2.3 Cube Root

In the beginning of this chapter, we made large cubes by arranging cubic blocks of mathematic kit. Recall this action and tell that how many blocks are there on one side of cube made of 125 blocks? As we can find the side length by making cubes but, this work can also be done by using cube root.

To determine the square root is the reverse process of finding square similarly, to find out cube root is also a reverse process of finding cube.

As we know that $2^3 = 8$ that is why we can say that 2 is the cube root of 8. It is represented by $\sqrt[3]{8}$ Symbol of cube root is $\sqrt[3]{\ }$

Consider the following table:

Cube	Cube root	Cube	Cube root
$1^3 = 1$	$\sqrt[3]{1}$	$6^3 = 216$	$\sqrt[3]{216}$
$2^3 = 8$	$\sqrt[3]{8}$	$7^3 = 343$	$\sqrt[3]{343}$
$3^3 = 27$	$\sqrt[3]{27}$	$8^3 = 512$	$\sqrt[3]{512}$
$4^3 = 64$	$\sqrt[3]{64}$	$9^3 = 729$	$\sqrt[3]{729}$
$5^3 = 125$	$\sqrt[3]{125}$	$10^3 = 1000$	$\sqrt[3]{1000}$

Table 2.3

2.3.1 Determine cube root by prime factor method:

To determine the cube root of 1728 by prime factor method

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Resolving the given number into prime factor

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping the factors in triples of equal factors, we get

$1728 = 2^3 \times 2^3 \times 3^3 = (2 \times 2 \times 3)^3$

By taking cube root on both side, we get

$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

Example 5 Find the cube root of 17576.

Solution Prime factors of 17576

$17576 = 2 \times 2 \times 2 \times 13 \times 13 \times 13$

$\sqrt[3]{17576} = 2 \times 13 = 26$

Example 6 Find the cube root of 9261 by prime factor method.

Solution Prime factors of 9261 $= \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

So $\sqrt[3]{9261} = 3 \times 7 = 21$

2.3.2 Cube root of a perfect cube(by observation method)

If you know that the given number is a perfect cube then the following steps are used to find the cube root of this number:

Step-1 Suppose 175616 is a cube, starting from the right side make, a group of 3-3 digits. We get.

$$\underset{\text{Second group}}{175} \quad \underset{\text{First group}}{616}$$

Cube roots of any given cube can be estimated step by step process. Here we get two groups 616 and 175 of three digits.

Step -2 By first group 616, we obtain 6 as the unit digit of cube root because the last digit of 616 is 6. We know that 6 occur at unit place when cube root has 6 as an unit digit ($6^3 = 216$).

Step-3 Now consider the second group 175. 175 occurs between $5^3 = 125$ and $6^3 = 216$.

i.e., $5^3 < 175 < 6^3$

So the digit on second place will be 5

Thus, $\sqrt[3]{175616} = 56$

Example 7 Find the cube root of 13824 by estimation method.

Solution Given 13824

Step-1 Starting from the right create groups of 3-3 digits we get.

$\underline{13} \quad \underline{824}$

Step-2 First group is 824 and its first digit is 4 which can only obtain by cube of unit digit 4 ($4^3 = 64$). Therefore, 4 will be the unit digit.

Step-3 Consider the second group 13 which occurs between $2^3 = 8$ and $3^3 = 27$.
i.e., $2^3 < 13 < 3^3$
So the digit on second place will be 2.
Thus, the cube root of given number will be 24.

$\sqrt[3]{13824} = 24$

Exercise 2.2

1. Determine True/ False in the given statements.
   (i) Every even number has even cube.
   (ii) A perfect cube does not end with double zero(00).
   (iii) No one perfect cube end with 8.
   (iv) If square of any number is ending with 5 then its cube is end with 25.
   (v) Cube of single digit is also a single digits number.
   (vi) Cube of double digit number is of 4 to 6 digits.
2. Find the cube roots of the following numbers by estimate and prime factor method. Verify your answer-

(i) 64	(ii) 343	(iii) 5832	(iv) 74088
(v) 3375	(vi) 10648	(vii) 46656	(viii) 91125

We Learnt

1. Cube of a number is that number raised to the power 3., That means when a number multiplies thrice to itself.
2. Power 3 of any number is equal to the cube of that number. E.g., $2^3 = 2 \times 2 \times 2 = 8$
3. Cube of even number is always even number and cube of odd number is always odd number.
4. Numbers which ends with 0,1,4,5,6, cube of that number also has same unit digit.
5. In Cube number some prime factor occurs in triples, so the group of factors in triples can be created.
6. To find out cube root is an inverse operation of finding out cube.
7. Cube root of any perfect cube can be determined by the prime factor method.
8. Cube root of any large number can be calculated by estimate method by making group of triples from the right side.