Quantitative Aptitude Test 7

Number of Questions: 35 **Section Marks: 30**

(Quadratic equations and Inequalities)

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Two students independently attempted to solve a quadratic equation in x. One of them copied the constant term wrong and obtained roots as –15 and 16. The other student copied the coefficient of x wrong and obtained his roots as –10 and 21. Find the correct roots of the quadratic equation?

(A) (–15, 14) (B) (–14, 15)
(C) (–25, 7) (D) (–7, 25)

2. $\sqrt{132+\sqrt{132+\sqrt{132+.......}}}$ = ______

(A) 11.11 (B) 12.32
(C) 11 (D) 12

3. The range of k for which the sum as wells as the product of the roots of $6x^2 - kx + 9 - k^2 = 0$ are negative is ___________

(A) (–3, 3) (B) $(-\infty, 3)$
(C) $(-\infty, -3)$ (D) $(3, \infty)$

4. Find the range of k, for which $-x^2 + 4kx + 3k - 1$, is always negative.

(A) $\left(-\frac{1}{4}, 1\right)$ (B) $(1, \infty)$
(C) $\left(-\infty, -\frac{1}{4}\right)$ (D) $\left(-1, \frac{1}{4}\right)$

5. a and b are the roots of the equation $2x^2 - 15x + k = 0$. Find the value of k if $a^2 - b^2 = 45$.

(A) $5\frac{1}{16}$ (B) $10\frac{1}{8}$
(C) $13\frac{1}{2}$ (D) $12\frac{1}{2}$

6. If the sum of the roots of the quadratic equation $3x^2 + (2k + 1)x - k - 5 = 0$ is equal to the product of the roots, which of the following is true?

(A) $k^2 - 4 = 0$ (B) $k^2 - 9 = 0$
(C) $k^2 - 16 = 0$ (D) $k^2 - 25 = 0$

7. If the roots of the equation $x^2 - 7x - 12 = 0$ are diminished by one and then multiplied by two, which of the following equations is formed with those values as the roots?

(A) $x^2 - 10x + 24 = 0$ (B) $x^2 - 6x - 76 = 0$
(C) $x^2 - 2x - 48 = 0$ (D) $x^2 - 10x - 72 = 0$

8. Which of the following statements is true about the roots of the equation $k^2x^2 - kx + (1 + 2x^2) = 0$, where k is a real number?

I. Roots are equal
II. Roots are complex
III. Roots are rational
IV. Roots are real

(A) I and IV (B) III only
(C) I and II (D) II only

9. All the roots of two quadratic equations are positive integers. The sum of the squares of the roots of the first quadratic equation is equal to that of the second quadratic equation. If the sum of the roots of the two equations are 10 and 8 respectively, then what is the greatest possible root of these quadratic equations?

(A) 7 (B) 6
(C) 8 (D) 5

10. If a and b are positive numbers, what is the nature of the roots of the equation $(a + b)x^2 + 2abx + \frac{(a+b)^3}{16} = 0$?

(A) Real and distinct. (B) Real and equal.
(C) Non-real and distinct. (D) Either (B) or (C)

11. If a positive number is increased by three and then squared, the result is 23 more than the original number. Find the original number.

(A) 1 (B) 2
(C) 3 (D) 4

12. Find the value of R, so that one of the roots of $x^2 + 6Rx + 64 = 0$ is the square of the other root.

(A) $\frac{-10}{3}$ (B) $\frac{8}{3}$
(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

13. If the value of p in the equation $x^2 + 2(p + 1)x + 2p = 0$, is real, the roots of the equation are

(A) rational and unequal.
(B) irrational and unequal.
(C) real and unequal.
(D) real and equal.

14. Find the equation whose roots are twice the roots of the equation $3x^2 - 7x + 4 = 0$.

(A) $3x^2 - 14x + 8 = 0$ (B) $3x^2 + 14x + 16 = 0$
(C) $3x^2 + 14x - 16 = 0$ (D) $3x^2 - 14x + 16 = 0$

15. The length of a rectangle is 1 cm more than its breadth. If its diagonal is 29 cm, what is the measure of its breadth? (in cm)

(A) 18 (B) 20
(C) 17 (D) 21

16. A is any single-digit prime number and B is any natural number. How many equations of the form $x^2 - 4\sqrt{A}\,x + 3B = 0$ have both real roots?

(A) 15 (B) 18
(C) 21 (D) 24

17. In a class, eight students play basketball. The remaining students, who represent 7 times the square root of the strength of the class, play football. Find the strength of the class.

(A) 36 (B) 16
(C) 64 (D) 100

18. If the price of a book goes down by ₹20 per dozen, a person can purchase 50 dozen books more for ₹30,000. Find the original price of each book.

(A) ₹10 (B) ₹12
(C) ₹9 (D) ₹8

19. If $-9 \le p \le -5$ and $-17 \le q \le -12$ then which of the following can be concluded?

(A) $\frac{5}{12} \le \frac{p}{q} \le \frac{9}{17}$ (B) $\frac{17}{9} \le \frac{p}{q} \le \frac{12}{5}$

(C) $\frac{5}{17} \le \frac{p}{q} \le \frac{3}{4}$ (D) $\frac{12}{9} \le \frac{p}{q} \le \frac{17}{5}$

20. If $|3x-4| = |5x-12|$, then the sum of the possible values of x is ______.

(A) 4 (B) 6
(C) –4 (D) –6

21. If $|4x-9| = 7$, then the values of $4|x| - |-x^3|$ is ______.

(A) $48, \frac{15}{8}$ (B) $-48, \frac{-15}{8}$

(C) $48, \frac{-15}{8}$ (D) $-48, \frac{15}{8}$

22. Find the range of values of x that satisfy the relation $|2x - 1| - 1 < |x - 2| + 3$.

(A) $-4 < x < 4$ (B) $-6 < x < \frac{1}{2}$

(C) $-6 < x < 4$ (D) $-5 < x < 3$

23. If $E = |x + 4| + |x + 7| + |x - 1|$, then how many integral values of x satisfy the inequality $E \le 14$?

(A) 8 (B) 10
(C) 11 (D) More than 11

24. Which of the following inequalities gives a finite range of values for x?

(A) $6x^3 - x^2 - x < 0$
(B) $x^4 + x^3 - 3x^2 - x + 2 < 0$
(C) $x^3 - x^2 - 5x - 3 < 0$
(D) $x^4 + 3x^3 + 2x^2 > 0$

25. If $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$, then find the range of x.

(A) $\left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ (B) $(-2, -1) \cup (0, 1)$

(C) $(-\infty, -1) \cup \left(-\frac{1}{2}, 1\right)$ (D) $\left(-\frac{1}{2}, \infty\right)$

26. Find the range of x, for which $|x + 2| - 3\,|x - 1| + 4 \ge 0$.

(A) $-2 \le x \le 1$ (B) $-2 \le x \le \frac{9}{2}$

(C) $-\frac{3}{4} \le x \le 4$ (D) None of these

27. a, b, c and d are four positive real numbers whose sum is equal to 4. If $p = \frac{abcd}{(abc + bcd + acd + abd)}$, then find the maximum value of p.

(A) 16 (B) 4

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

28. If $2 < x < 5$ and $10 < y < 30$, then $\frac{y}{x}$ lies between

(A) 5 and 6 (B) 2 and 6
(C) 2 and 15 (D) 6 and 15

29. If $|x| > 6$ and $y > -4$, then which of the following is necessarily true?

(A) $/xy| > 24$ (B) $|xy| < 24$
(C) $|x|\,|y| > 0$ (D) None of these

30. Let $f(x) = \max(3x + 5, 7 - 2x)$, where x is any real number. Then the minimum possible value of $f(x)$ is

(A) $\frac{31}{5}$ (B) $\frac{27}{5}$

(C) $\frac{21}{5}$ (D) $\frac{29}{5}$

31. If $20 \le x \le 35$ and $3y - 2x = 5$, then the minimum value of $\frac{x}{x+y}$ is

(A) 1 (B) $\frac{1}{3}$

(C) $\frac{4}{9}$ (D) $\frac{4}{7}$

32. If a, b and c are positive real numbers. Find the minimum value of $\left(1 + a + \frac{1}{a}\right)\left(1 + b + \frac{1}{b}\right)\left(1 + c + \frac{1}{c}\right)$.

(A) 9 (B) 12
(C) 27 (D) 81

33. If $a \le 25$ and $a + b \ge 10$, then which of the following is always true?

(A) $a - b \ge 40$ (B) $b - a \ge -40$
(C) $a + b \ge 40$ (D) $ab \le 250$

34. If $1 \le x \le 3$, $4 \le y \le 10$ and $2 \le z \le 5$, what is the maximum possible value of $\frac{y}{x+y+z}$?

(A) 5 (B) $\frac{10}{3}$
(C) $\frac{10}{13}$ (D) $\frac{10}{7}$

35. Find the range of values of x for which $\left|\frac{18-2x}{4}\right| < 3$.

(A) $6 < x < 30$ (B) $-3 < x < 15$
(C) $-15 < x < 3$ (D) $3 < x < 15$

Answer Key

1. B	**2.** D	**3.** C	**4.** D	**5.** B	**6.** C	**7.** D	**8.** D	**9.** A	**10.** D
11. B	**12.** A	**13.** C	**14.** D	**15.** B	**16.** C	**17.** C	**18.** A	**19.** C	**20.** B
21. D	**22.** D	**23.** B	**24.** B	**25.** A	**26.** D	**27.** D	**28.** C	**29.** D	**30.** A
31. D	**32.** C	**33.** B	**34.** C	**35.** D					

Hints and Explanations

1. The quadratic equation which has α and β are the roots is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Quadratic equation taken by the first student is

$x^2 - (-15 + 16)x + (-15 \times 16) = 0$

$x^2 - x - 240 = 0$ ————(1)

Quadratic equation taken by the second student is

$x^2 - (-10 + 21)x + (-10 \times 21) = 0$

$x^2 - 11x - 210 = 0$ ————(2)

$\therefore$ Required correct quadratic equation is $x^2 - x - 210 = 0$

$\Rightarrow$ $x^2 - 15x + 14x - 210 = 0$

$\Rightarrow$ $x(x - 15) + 14(x - 15) = 0$

$\Rightarrow$ $(x - 15)(x + 14) = 0$

$\therefore$ Required roots are -14 and 15. Choice (B)

2. Let $X = \sqrt{132+\sqrt{132+\sqrt{132+......}}}$

$\Rightarrow$ $x = \sqrt{132+x}$

Squaring on both sides

$\Rightarrow$ $x^2 = 132 + x \Rightarrow x^2 - x - 132 = 0$

$\Rightarrow$ $x^2 - 12x + 11x - 132 = 0$

$\Rightarrow$ $(x - 12)(x + 11) = 0$

$\therefore$ $x = 12$ ($\therefore$ x is always positive) Choice (D)

3. Given $6x^2 - kx + 9 - k^2 = 0$

The sum of the roots = $\frac{k}{6}$

The product of the roots = $\frac{9-k^2}{6}$

$\frac{k}{6}$ and $\frac{9-k^2}{6}$ are both negative

$\Rightarrow$ $\frac{k}{6} < 0$ and $\frac{9-k^2}{6} < 0$

$\Rightarrow$ $k < 0$ and $9 < k^2$

$\Rightarrow$ $k < 0$ and $k > 3$ or $k < -3$

$\therefore$ $k < -3 \Rightarrow k \in (-\infty, -3)$. Choice (C)

4. It is given that $-x^2 + 4kx + 3k - 1 < 0$

$\Rightarrow$ $-(x^2 - 4kx) + 3k - 1 < 0$

$\Rightarrow$ $-[(x - 2k)^2 - 4k^2] + 3k - 1 < 0$

$\Rightarrow$ $-[(x - 2k)^2] + 4k^2 + 3k - 1 < 0$

Now, for the above expression to be always negative $4k^2 + 3k - 1 < 0 \Rightarrow (4k - 1)(k + 1) < 0$

This is true when $-1 < k < \frac{1}{4}$. Choice (D)

5. The given equation is $2x^2 - 15x + k = 0$

The sum of the roots, $a + b = \frac{15}{2}$ and

the product $ab = \frac{k}{2}$

It is given that $a^2 - b^2 = 45 \Rightarrow a - b = 6$

$a + b = \frac{15}{2}$, $a - b = 6, \Rightarrow a = \frac{27}{4}$, $b = \frac{3}{4}$

$\therefore$ The product of the roots $ab = \left(\frac{27}{4}\right)\left(\frac{3}{4}\right) = \frac{81}{16}$

Now $\frac{k}{2} = \frac{81}{16} \Rightarrow k = \frac{81}{8} = 10\frac{1}{8}$ Choice (B)

6. Sum of the roots = $\frac{-(2k+1)}{3}$

Product of the roots = $\frac{-(k+5)}{3}$

$\frac{-(2k+1)}{3} = \frac{-(k+5)}{3}$

$\Rightarrow$ $2k + 1 = k + 5 \Rightarrow k = 4$

Choice (C)

7. Roots are to be diminished by one and then multiplied by two. i.e., if A, B are roots of given equation, then $2(A-1) = A_1$ and $2(B-1) = B_1$, where A_1 and B_1 are the roots of the new equation. i.e.,

$\Rightarrow \quad A = 1 + \frac{A_1}{2} = \frac{A_1+2}{2}$ and $B = \frac{B_1+2}{2}$

i.e., x of the given equation is to be replaced by $\frac{x+2}{2}$, to obtain the required equation. Given equation is: $x^2 - 7x - 12 = 0$.

Required equation is $\left(\frac{x+2}{2}\right)^2 - \frac{7(x+2)}{2} - 12 = 0$

$(x+2)^2 - 7(2)(x+2) - 4(12) = 0$.

$\Rightarrow \quad x^2 + 4x + 4 - 14x - 28 - 48 = 0$.

$\Rightarrow \quad x^2 - 10x - 72 = 0$. Choice (D)

8. When rewritten, the equation becomes:

$(k^2 + 2)x^2 - kx + 1 = 0$

Discriminant, $D = (k)^2 - 4(1)(k^2 + 2)$

$= -3k^2 - 8 = -(3k^2 + 8)$

$3k^2$ is positive for all real values of k, and hence $(3k^2 + 8)$ is positive; and so $-(3k^2 + 8)$ is negative.

As the discriminant is negative, roots are complex.

Choice (D)

9. Let the roots of the first quadratic equation be α and β and those of the second equation be γ and δ respectively.

Given $\alpha^2 + \beta^2 = \gamma^2 + \delta^2$

Also $\alpha + \beta = 10$ and $\gamma + \delta = 8$.

The possible values of $\alpha^2 + \beta^2$ are 50, 52, 58, 68 and 82 while the possible values $\gamma^2 + \delta^2$ as are 32, 34, 40 and 50. As only 50 is a common value, $\alpha = 5$, $\beta = 5$, $\gamma = 7$ and $\delta = 1$

$\therefore$ The greatest possible root is 7. Choice (A)

10. Dividing both sides of the given equation by $a + b$,

$x^2 + \frac{2abx}{a+b} + \frac{(a+b)^2}{16} = 0$

Discriminant

$= \left(\frac{2ab}{a+b}\right)^2 - \frac{4(a+b)^2}{16} = \left(\frac{2ab}{a+b}\right)^2 - \left(\frac{a+b}{2}\right)^2$

Shown below is the proof that this is always non-positive provided a and b are positive.

$(a-b)^2 \geq 0 \Rightarrow a^2 + b^2 + 2ab \geq 4ab$

dividing both sides by $2(a+b)$ $\frac{a+b}{2} \geq \frac{2ab}{a+b}$

As the expressions on both sides of the inequality are positive, $\left(\frac{a+b}{2}\right)^2 \geq \left(\frac{2ab}{a+b}\right)^2$

$\therefore$ $\Delta < 0$ or $\Delta = 0$

If $\Delta = 0$, the roots are real and equal.

If $\Delta < 0$, the roots are non-real and distinct.

Choice (D)

11. Let the required original number be x.

$(x+3)^2 = 23 + x$.

Hence $x^2 + 6x + 9 = 23 + x$

$\Rightarrow \quad x^2 + 5x - 14 = 0$.

$(x+7)(x-2) = 0$

$\Rightarrow \quad x = -7$ or $x = 2$.

Since the original number is positive, $x = 2$.

Choice (B)

12. If one of the roots is α, the other root is α^2.

Hence the product of the roots $= \alpha(\alpha^2)$.

$\alpha^3 = 64 \Rightarrow \alpha = \sqrt[3]{64} = 4$ and $\alpha^2 = 4^2 = 16$

The sum of roots $= -\left(\frac{6R}{1}\right) = -6R = 4 + 16 = 20$

$R = \left(\frac{20}{-6}\right) = -\frac{10}{3}$ Choice (A)

13. For the equation $x^2 + 2(p+1)x + 2p = 0$

$b^2 - 4ac = [2(p+1)]^2 - 4(2p)] = 4p^2 + 8p + 4 - 8p$

$= 4p^2 + 4$ which is always positive.

Hence the roots of the equation are always real and unequal. Choice (C)

14. For the equation, whose roots are twice the roots of the equation A : $3x^2 - 7x + 4 = 0$, the sum of the roots is twice the sum of the roots of A and the product of the roots is 4 times the product of the roots of A.The required equation is $x^2 - \left(2\left(\frac{7}{3}\right)\right)x + 4\left(\frac{4}{3}\right) = 0$

i.e., $3x^2 - 14x + 16 = 0$ Choice (D)

15. Let ℓ and b be the length and breadth in cm.

Given that $\ell = b + 1$

Also given that diagonal = 29 cm

$\Rightarrow \quad \sqrt{\ell^2 + b^2} = 29$

By squaring on both sides, $(b+1)^2 + b^2 = 29^2$

$\Rightarrow \quad 2b^2 + 2b - 840 = 0$

$\Rightarrow \quad b^2 + b - 420 = 0$

$\Rightarrow \quad (b+21)(b-20) = 0$

$\therefore \quad b = 20$ Choice (B)

16. $(4\sqrt{A})^2 - 4(3B) \geq 0$

$\frac{4}{3} A \geq B$

As A is a single digit prime number, A can be 2, 3, 5 or if $A = 2$, B has 2 possibilities. If $A = 3$, B has 4 possibilities. If $A = 5$, B has 6 possibilities. If $A = 7$, B has 9 possibilities. A total of 21 equations are possible.

Choice (C)

17. Let the strength be x. The number of students who play basketball = 8

The number of students who play football

$= x - 8 = 7\sqrt{x}$

Substituting the choices in place of x in the equation above, only choice (C) satisfies it. Choice (C)

18. Let the initial number of books in dozens = b

Let initial price (in ₹) of books per dozen be p.

$pb = 30,000.$ →(I)

$(50 + b)(p - 20) = 30,000$

$50p - 1000 + pb - 20b = 30,000$

or, $50p - 20b = 1000$

$5p - 2b = 100.$ →(II)

From (I) and (II) $5p - \dfrac{60,000}{p} = 100$

$5p^2 - 100p - 60,000 = 0$

$5p^2 - 600p + 500p - 60,000 = 0$

$5p(p - 120) + 500(p - 120) = 0 \Rightarrow p = 120$

The price of each book = $\dfrac{120}{12} = 10$ Choice (A)

19. $\dfrac{p}{q} = \dfrac{-p}{-q}$

$5 \le -p \le 9$ and $12 \le -q \le 17$

$\dfrac{-p}{-q}$ is maximum when p is maximum and q is minimum

$\therefore$ Max$\left(\dfrac{-p}{-q}\right) = \dfrac{9}{12} = 3/4$

$\dfrac{-p}{-q}$ is minimum when p is minimum and q is maximum.

$\therefore$ Min$\left(\dfrac{-p}{-q}\right) = \dfrac{5}{17}$

$\dfrac{5}{17} \le \dfrac{p}{q} \le \dfrac{3}{4}$ Choice (C)

20. $|3x - 4| = |5x - 12|$

When ever $|p| = |q|$ it follows that $p = \pm q$

$3x - 4 = \pm(5x - 12)$

$\Rightarrow$ $3x - 4 = 5x - 12$ or $3x - 4 = -5x + 12$

$\Rightarrow$ $2x = 8$ or $8x = 16$

$\Rightarrow$ $x = 4$ or $x = 2$

$\therefore$ Required sum of the possible values of x is 6.

Choice (B)

21. $|a| = b \Rightarrow a = \pm b$

$|4x - 9| = 7$

$\Rightarrow$ $4x - 9 = 7$ or $4x - 9 = -7$

$\Rightarrow$ $x = 4$ or $x = 1/2$

$4|x| - |-x|^3$

$= 4(4) - (4)^3$ or $4\left(\dfrac{1}{2}\right) - \left(\dfrac{1}{2}\right)^3$

$= -48$ or $15/8$ Choice (D)

22. $|2x - 1| - 1 < |x - 2| + 3$

$\Rightarrow$ $|2x - 1| - |x - 2| < 4$

We need to consider 3 cases

(1) $x < \dfrac{1}{2}$ (2) $\dfrac{1}{2} \le x < 2$ and (3) $2 \le x$.

For $x < \dfrac{1}{2}$, we get

$-(2x - 1) + (x - 2) < 4$

$\Rightarrow$ $-x < 5 \Rightarrow x > -5$

$\therefore$ $-5 < x < \dfrac{1}{2}$

For $\dfrac{1}{2} \le x < 2$, $2x - 1 + x - 2 < 4$

$\Rightarrow$ $x < \dfrac{7}{3}$ $\therefore \dfrac{1}{2} < x < 2$

For $x \ge 2$, $(2x - 1) - (x - 2) < 4$

$\Rightarrow$ $x < 3$ $\therefore 2 \le x < 3$

Thus the range of x such that the given relation is satisfied is $-5 < x < 3$ Choice (D)

23. For $x = -9$, $E = |-9 + 4| + |-9 + 7| + |-9 - 1| = 17$

For $x = -8$, $E = |-8 + 4| + |-8 + 7| + |-8 - 1| = 14$

For $x = 1$, $E = |1 + 4| + |1 + 7| + |1 - 1| = 13$

For $x = 2$, $E = |2 + 4| + |2 + 7| + |2 - 1| = 16$

Therefore the integral values of x for which the given inequality is satisfied are $-8, -7, -6, -5, -4, -3, -2, -1$, 0 and 1 i.e. a total of 10 values. Choice (B)

24. We need to considering each option separately,

Option A:

$6x^3 - x^2 - x < 0$

$x(2x - 1)(3x + 1) < 0$

The above inequality is satisfied for

$x < -\dfrac{1}{2}$ or $0 < x < \dfrac{1}{2}$

which does not give a finite range of values for x

Option B:

$x^4 + x^3 - 3x^2 - x + 2 < 0$

$(x - 1)^2(x + 1)(x + 2) < 0$ which gives the same solution set as $(x + 1)(x + 2) < 0$ $((x - 1)^2 \ge 0]$

The above inequality is satisfied for $-2 < x < -1$ this is a finite range of values for x.

Option C:

$x^3 - x^2 - 5x - 3 < 0$

$(x + 1)^2(x - 3) < 0$ which gives the same solution set as $x - 3 < 0 \Rightarrow x < 3$

It does not give a finite range of values for x.

Option D:

$x^4 + 3x^3 + 2x^2 > 0$

$x^2(x^2 + 3x + 2) > 0$

$x^2(x + 2)(x + 1) > 0$

The above inequality gives the same solution set as $(x + 2)(x + 1) > 0$ $[x^2 \ge 0]$

The inequality is satisfied for $x < -2$ or $x > -1$ which does not give a finite range of values for x.

Note: A polynomial of odd degree can take values from $-\infty$ to ∞ but a polynomial of even degree has a finite range of values for which it has values of a particular sign. If the coefficient of the leading term (say a) is positive, $f(x) < 0$ for a finite range if $a < 0$, $f(x) > 0$ for a finite range. Choice (B)

25. $\frac{x}{x+1} - \frac{x+2}{x-1} < 0$

$\frac{x(x-1)-(x+2)(x+1)}{(x+1)(x-1)} < 0$

$\frac{x^2 - x - x^2 - 3x - 2}{(x+1)(x-1)} < 0$

$\frac{-4x-2}{(x+1)(x-1)} < 0$

$\frac{-2(2x+1)}{(x+1)(x-1)} < 0$

$\frac{(2x+1)}{(x+1)(x-1)} > 0$

Multiplying both Nr & Dr by $(x + 1)(x - 1)$ we get

$\frac{(2x+1)(x+1)(x-1)}{(x+1)^2(x-1)^2} > 0$

The solution set for the above inequality is the same as that for $(2x + 1)(x + 1)(x - 1) > 0$

Therefore the inequality holds true for $-1 < x < -\frac{1}{2}$ or $x > 1$, i.e., $x \in \left(-1, -\frac{1}{2}\right) \cup (1, \infty)$ Choice (A)

26. $|x + 2| - 3\,|x - 1| + 4 \geq 0$

For $x < -2$, $|x + 2| - 3\,|x - 1| + 4 = -(x + 2) + 3\,(x - 1) + 4$

$\therefore$ $-x - 2 + 3x - 3 + 4 \geq 0 \Rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2}$

But we have taken $x < -2$, thus no solution exists in this range.

For $-2 \leq x < 1$,

$|x + 2| - 3\,|x - 1| + 4 = (x + 2) + 3\,(x - 1) + 4$

$\Rightarrow$ $x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow$ $9 - 2x \geq 0$

$\Rightarrow$ $9 \geq 2x$ $\Rightarrow$ $2x \leq 9$

$\therefore$ $x + 2 + 3x - 3 + 4 \geq 0 \Rightarrow 4x \geq -3 \Rightarrow x \geq -\frac{3}{4}$

Therefore, the range of x satisfying the given condition is $-\frac{3}{4} \leq x < 1$

For $x \geq 1$,

$|x + 2| - 3\,|x - 1| + 4 = (x + 2) - 3\,(x - 1) + 4 - 2x \leq -9$

$\therefore$ $x + 2 - 3x + 3 + 4 \geq 0$

$\Rightarrow$ $9 - 2x \geq 0$

$\Rightarrow$ $2x \leq 9$

$\Rightarrow$ $x \leq \frac{9}{2}$

$\therefore$ $1 \leq x \leq \frac{9}{2}$

Thus, the range of x satisfying the given inequality is $-\frac{3}{4} \leq x \leq \frac{9}{2}$ Choice (D)

27. $AM\,(a, b, c, d) \geq HM\,(a, b, c, d)$

$\frac{a+b+c+d}{4} \geq \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} \geq \frac{16}{a+b+c+d}$

$\frac{bcd+acd+abd+abc}{abcd} \geq \frac{16}{a+b+c+d}$

$\frac{abcd}{abc+bcd+acd+abd} \leq \frac{a+b+c+d}{16}$

$(\because a + b + c + d = 4)$

$\therefore$ $p \leq \frac{1}{4}$. Choice (D)

28. Given $2 < x < 5$ and $10 < y < 30$.

The value of y/x is minimum, for the minimum value of y and the maximum value of x.

$\therefore$ $y/x > \frac{10}{5}$ or $y/x > 2$

The value of y/x is maximum, for the maximum value of y and the minimum value of x.

$\therefore$ $y/x < \frac{30}{2}$ or $y/x < 15$

$\Rightarrow$ $2 < y/x < 15$ Choice (C)

29. Given $|x| > 6$, $y > -4$.

Consider $x = 7$ and $y = 2$; $xy = 14$

$\Rightarrow$ $|xy| = 14|xy| > 24$ is not necessarily true.

Consider $x = 8$ and $y = 5$; $xy = 40$; $|xy| > 40 > 24$

$\therefore$ The second option is not necessarily true.

For $y = 0$; $|x|\,|y| = 0$, hence none of the given options is necessarily true. Choice (D)

30. Given $f(x) = \max(3x + 5, 7 - 2x)$

$f(x)$ has the minimum value when the two expressions are equal.

$\therefore$ $3x + 5 = 7 - 2x$

$5x = 2 \Rightarrow x = \frac{2}{5}$

$\therefore$ The minimum value of $f(x)$ is

$$f\left(\frac{2}{5}\right) = \max\left(\frac{3(2)}{5} + 5, 7 - \frac{2(2)}{5}\right)$$

$$= \max\left(\frac{31}{5}, \frac{31}{5}\right) = \frac{31}{5}$$ Choice (A)

31. $20 \le x \le 35$

$$y = \frac{2x+5}{3}$$

$$\therefore \frac{x}{x+y} = \frac{x}{x + \frac{2x+5}{3}}$$

$$= \frac{3x}{5x+5} = \frac{3}{5 + \frac{5}{x}}$$

This expression is positive for the given range of values of x and it has its minimum value when $5/x$ has its maximum value, i.e. when $x = 20$.

The corresponding value is $\frac{3(4)}{21} = \frac{4}{7}$.

Choice (D)

32. If x is a positive number, the minimum value of $x + \frac{1}{x}$ is 2.

Hence, the minimum value of $1 + x + \frac{1}{x}$ is 3 and for the given expression, it is 3(3)(3) = 27. Choice (C)

33. Given $a \le 25$ and $a + b \ge 10$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - a$

$\Rightarrow$ $a \le 25$ and $b \ge 10 - 25$

$\Rightarrow$ $a \le 25$ and $-b \ge 15$

$\Rightarrow$ $a - b \le 40$

$\Rightarrow$ $b - a \ge -40$ Choice (B)

34. $\frac{y}{x+y+z} = \frac{1}{\frac{x}{y} + 1 + \frac{z}{y}}$; to maximize the given expression, $x + z$ should take minimum and y should take maximum possible value.

$\therefore$ Maximum value $= \frac{1}{\frac{1}{10} + 1 + \frac{2}{10}} = \frac{10}{13}$ Choice (C)

35. Given $\left|\frac{18-2x}{4}\right| < 3$

$\Rightarrow$ $|18 - 2x| < 12$

$\Rightarrow$ $|x - 9| < 6$

The expression $|x - a|$ denotes the distance of the point x from the point a on the number line.

$|x - 9| < 6$ Þ x lies within a distance 6 units from the point 9. i.e., $3 < x < 15$.

Choice (D)