Chapter 2

Maxwell Equations and EM-Wave Propogation

CHAPTER HIGHLIGHTS

- *Faraday's Law*
- *Maxwells Equations*
- *Phasor Notation of Fields*
- *Uniform Plane Wave*
- *Wave Propagation in a Loss – Less Medium*
- *Constant Phase Point*
- *Medium*
- *Impedance of Loss Less Media*
- *Wave Equation for a Lossy Medium*
- *Depth of Penetration*
- *Impedance of Lossy Medium*
- *Poynting Vector*
- *Polarization*
- *Reflection and Refraction of Plane Waves*
- *Normal Incidence of Plane Wave on Plane Boundaries*
- *Boundary Conditions*
- *Oblique Incidence*
- *Total Internal Reflection*

INTRODUCTION

In previous chapters, the study of static fields provided a good understanding of basic concepts.

Electrostatic fields are produced by stationary or static electric charge, whereas magneto static fields are produced by charges moving with a constant velocity.

Time varying electromagnetic fields are produced by accelerated charges (or) time-varying currents.

FARADAY'S LAW

According to Faraday's law, induced emf V_{emf} in any closed circuit is equal to the time rate of change of magnetic flux linkage by the circuit.

$$V_{emf} = -N\frac{d\Psi}{dt}$$

N – number of turns

ψ – flux through each turn

The above expression can also be written as follows:

$$V_{emf} = \oint_L \bar{E}.dl = \frac{-d}{dt}\int_s \bar{B}.ds$$

Therefore, the flux can be varied in the following three ways:

1. Variation of magnetic flux density (B)
2. Variation of loop area
3. Variation of Both

(a) Time-varying magnetic field

$$V_{emf} = \oint_L \bar{E}.dl = -\int_s \frac{\partial \bar{B}}{\partial t}.ds$$

Applying stokes theorem

$$\int \left(\nabla \times \bar{E}\right).ds = -\int \frac{\partial \bar{B}}{\partial t}.\partial s$$

$$\therefore\ \nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$$

(b) Time-varying loop area

This is also called motional emf and the electric field called motional electric field.

$$E_m = \frac{\bar{F}}{Q} = \bar{V} \times \bar{B}$$

$$V_{emf} = \oint_L \overline{E_m}.dl = \oint_L (\bar{V} \times \bar{B})\ dl$$

$$\int_s \left(\nabla \times \overline{E_m}\right) ds = \int_s \nabla \times \left(\bar{V} \times \bar{B}\right) ds$$

$$\nabla \times \overline{E_m} = \nabla \times \left(\bar{V} \times \bar{B}\right)$$

(c) Time-varying area and B

$$V_{emf} = \oint_L \bar{E}.dl = -\int_s \frac{\partial \bar{B}}{\partial t}.ds + \oint_L (\bar{V} \times \bar{B}).dl$$

and

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{V} \times \bar{B})$$

Solved Examples

Example 1

In a hundred turn coil, if the flux through each turn is ($t^2 - 2t$) (mWb), the magnitude of induced emf in the coil at a time of 2 sec is

(A) 0.2 volts (B) 2 volts
(C) 20 volts (D) 0.02 volts

Solution

According to Faradays law

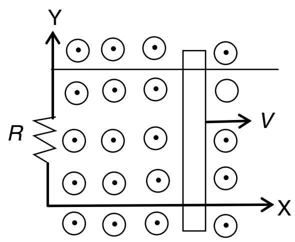
$$(V_{emf}) = \left| N.\frac{d\psi}{dt} \right| \text{m volts } t = 2 \text{ sec}$$

$$= 100.\frac{d}{dt}(t^2 - 2t) \text{ mv} = 100\ (2t - 2)|\text{mv } t = 2 \text{ sec}$$

$$= 100.\ 2 \text{ mv} = 0.\ 2 \text{ volts}$$

Example 2

Consider the loop in the following figure. If $\bar{B} = 0.5\ a_z$ wb/m^2 $R = 20\ \Omega$, l = 0. 1 m and the rod is moving with a velocity of 8 a_x m/s, then the induced emf in the rod.



(A) 2 v (B) 4 v (C) 8 v (D) 1 v

Solution

Since B is constant

$$V_{emf} = \int (\bar{V} \times \bar{B}).\text{dl} = B \text{ vl} = 0.5 \times 8 \times 0.1 = 4 \text{ v}$$

Displacement Current

For magneto static fields, according to Amperes law, $\nabla \times H = J$.

But the divergence of curl of a vector is zero.

Therefore,

$$\nabla.\ (\nabla \times H) = 0 = \nabla.\ J$$

It contradicts the continuity equation. Therefore, the above equation is not compatible for time-varying conditions and is modified by Maxwell.

$$\nabla \times H = J + J_d$$
$$\nabla.\ (\nabla \times H) = \nabla.\ J + \nabla.\ J_d$$

$$0 = \nabla.\ J + \nabla.\ J_d$$

$$\nabla.\ J_d = \frac{\partial \rho_v}{\partial t} \left(\nabla.\ j = \frac{-\partial \rho_v}{\partial t} \right)$$

$$\nabla.\ J_d = \frac{\partial}{\partial t}(\nabla.D)$$

$$J_d = \frac{\partial \bar{D}}{\partial t}$$

J_d is called displacement current density

$$\therefore \nabla \times H = J + J_d$$

$$\nabla \times H = J + \frac{\partial \bar{D}}{\partial t}$$

J is called as conduction current density
$J = \sigma E$ (ohm's law)

$$J_d = \frac{\partial \bar{D}}{\partial t} = \in \frac{\partial \bar{E}}{\partial t}$$

J_d is due to variation of time-varying electric field.

Without the J_d, the electromagnetic wave propagation would be impossible. At low frequencies, J_d is neglected. At radio frequencies, J and J_d are comparable.

Example for this is current through a capacitor when alternating voltage source applied to its plates.

Example 3

A parallel plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage 10 Sin $10^3\ t$ v applied to its plates. $E = 2 \in_0$, then displacement current density is

(A) 0.294 sin $10^3\ t$ (B) 0.588 cos $10^3\ t$
(C) 0.294 cos $10^3\ t$ (D) 0.588 sin $10^3\ t$

Solution

$$J_d = \frac{\partial D}{\partial t}$$

$$D = \in E = \frac{\in}{d}.v$$

$$J_d = \frac{\in}{d}.\frac{\partial v}{\partial t}$$

$$= \frac{2\in_0}{3 \times 10^{-3}} \times 10 \times 10^3 \times \cos 10^3 t$$

$$= 0.588 \times \cos 10^3\ t\ \mu\text{A/m}^2$$

Example 4

A material has conductivity of 10^{-2} ℧ /m and a relative permittivity of 8. The frequency at which the conduction current in the medium is equal to the displacement current is

(A) 225 MHz (B) 2.25 MH
(C) 22.5 MH (D) 22.5 GHz

Solution

$$J_c = J_d$$
$$|\sigma E| = |j\, w\, t\, E|$$
$$\sigma = w\, t$$
$$w = \frac{\sigma}{\in}$$
$$2\,\pi f = \frac{\sigma}{\in}$$
$$f = \frac{\sigma}{2\pi \in}$$
$$f = \frac{10^{-2}}{2\pi 8 \times E_o} = 22.5 \text{ MHz}$$

Generalized Form of Maxwell Equations

	Differential form	Integral form	Phasor form	Remarks
(i)	$\nabla . \bar{D} = \delta v$	$\oint_s \bar{D}.ds = \int_v \rho_v dv$	$\nabla . \bar{D}_p = \rho_v$	Gauss law
(ii)	$\nabla . \bar{B} = 0$	$\oint_s \bar{B}.ds = 0$	$\nabla . \bar{B}_p = 0$	Solenoidal property of magnetic lines
(iii)	$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$	$\oint_L \bar{E}.dl = -\frac{\partial}{\partial t}\int_s \bar{B}.ds$	$\nabla \times \bar{E}_p = -jwB_p$	Faraday's law
(iv)	$\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H}.dl = \int_s \left(J + \frac{\partial \bar{D}}{\partial t} \right).ds$	$\nabla \times \bar{H}_p = \bar{J}_p + j\omega \bar{D}_p$	Modified Amperes law

Suffix 'P' represents the phasor form.

The Lorentz force equation

$$F = Q\,[E + u \times B] \qquad (1)$$
$$\nabla . J = -\frac{\partial \rho_v}{\partial t} \qquad (2)$$
$$D = \varepsilon E = \varepsilon_0 E + P \qquad 3(a)$$
$$B = \mu H = \mu_0 (H + M) \qquad 3(b)$$
$$J = \sigma E + \rho_v\, \mu \qquad 3(c)$$

The boundary conditions for time-varying fields are

$$(E_1 - E_2) \times a_n = 0 \qquad 4(a)$$
$$(H_1 - H_2) \times a_n = k \qquad 4(b)$$
$$(D_1 - D_2).a_n = \rho_s \qquad 4(c)$$
$$(B_2 - B_1).a_n = 0 \qquad 4(d)$$

For a perfect conductor ($\sigma \approx \infty$) in a time-varying field

$$E = 0, H = 0, J = 0 \qquad (5)$$

Hence, $B_n = 0$

For a perfect dielectric ($\sigma \simeq 0$), equation (4) holds except that $k = 0$. But equation (1) to (5) are not Maxwell's equations. They are associated with them.

1. Word Statements of the Field Equations

A word statement of the significance of the field equation is readily obtained from their mathematical statement in the integral form

(a) The magnetomotive force around a closed path is equal to the conduction current plus the time derivative of the electric displacement through any surface bounded by the path.

(b) The electromotive force around a closed path is equal to the Time derivative of the magnetic displacement through any surface bounded by the path.

(c) The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

(d) The net magnetic flux energy through any closed surface is zero.

Example 5

Which one of the following is not a correct Maxwell equation?

(A) $\nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$ (B) $\nabla \times \bar{E} = \frac{d\bar{B}}{dt}$

(C) $\nabla . \bar{D} = \rho$ (D) $\nabla . \bar{B} = 0$

Solution

All the options are correct except (B)

$$\text{i.e., } \nabla \times \bar{E} = -\frac{d\bar{B}}{dt} \text{ (Faraday law)}$$

Example 6

If C is a closed curve enclosing a surface s, then the magnetic field intensity $\bar{H}$ the current density $\bar{J}$ and the electric flux density $\bar{D}$ are related by

(A) $\iint_s \bar{H}.ds = \oint \left(\bar{J} + \frac{d\bar{D}}{dt} \right).d\bar{l}$

(B) $\oint_c \bar{H}.dl = \iint_s \left(\bar{J} + \frac{d\bar{D}}{dt} \right).d\bar{s}$

(C) $\oiint \bar{H}.d\bar{s} = \int_c \left(\bar{J} + \frac{d\bar{D}}{dt} \right).d\bar{l}$

(D) $\int_c \bar{H}.d\bar{l} = \oiint_s \left(\bar{J} + \frac{d\bar{D}}{dt} \right).d\bar{s}$

Solution

According to Amperes law

$$\oint_c \overline{H}.dl = I = \iint_s \left(\overline{J} + \frac{d\overline{D}}{dt}\right).ds$$

Maxwell's Equations

2. For Time-Varying Fields

Point form	Integral form
$\nabla \times \overline{H} = \overline{J_c} + \frac{\partial \overline{D}}{\partial t}$	$\oint \overline{H.dl} = \int_s \left(\overline{J_c} + \frac{\partial \overline{D}}{\partial t}\right)\overline{ds}$ (Ampere's law)
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	$\oint \overline{E.dl} = \int_s \left(-\frac{\partial \overline{B}}{\partial t}\right).\overline{ds}$ (Faraday's law)
$\nabla.\overline{D} = \rho$	$\oint_s \overline{D}.\overline{ds} = \oint_v \rho.dv$ (Gauss law)
$\nabla.\overline{B} = 0$	$\oint_s \overline{B}.\overline{ds} = 0$ (nonexistence of monopole)

3. For Time--Invariant Fields

For free space, $\rho = 0$, $\overline{J}_C = 0$

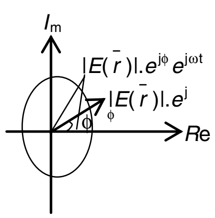
Point form	Integral form
$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$	$\oint \overline{H.dl} = \int_s \left(\frac{\partial \overline{D}}{\partial t}\right)\overline{ds}$
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	$\oint \overline{E.dl} = \int_s \left(-\frac{\partial \overline{B}}{\partial t}\right).\overline{ds}$
$\nabla.\overline{D} = 0$	$\oint_s \overline{D}.\overline{ds} = 0$
$\nabla.\overline{B} = 0$	$\oint_s \overline{B}.\overline{ds} = 0$

Phasor Notation of Fileds

$$\overline{E}(r, t) = \text{Re}\left[E(\overline{r})e^{j\omega t}\right]$$

$E(\overline{r})$- Phasor form

$$E(\overline{r}) = |E(\overline{r})|.e^{j\phi}$$



Time-varying Potentials

V = Electric scalar potential

A = Magnetic vector potential

$$E = -\nabla v - \frac{\partial A}{\partial t}$$

V and A are related as

$$\nabla.\ A = -\mu \in \frac{\partial V}{\partial t}$$

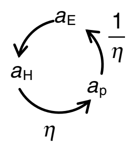
Wave Equations

$$\nabla^2 V - \mu \in \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_v}{\in}$$

$$\nabla^2 A - \mu \in \frac{\partial^2 A}{\partial t^2} = \mu J.$$

Properties of Waves

1. Longitudinal components are absent, that is, there is no field along the direction of propagation.
2. Transverse components are present, that is, fields are present in a plane perpendicular to the direction of propagation.
3. In free space, the intrinsic impendence of the wave is $\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\,\pi \cong 377\,\Omega$
4. $\overline{E}$, $\overline{H}$, and the directions of propagation form a right handed system.



Example 7

Identify which one of the following will not satisfy the wave equation

(A) $50.\ e^{j(\omega t - 2Z)}$ (B) $\cos(\omega(10Z + 2t))$

(C) $\cos x.\cos t$ (D) $\cos(y^3 + 5t)$

Solution

Wave equation is a function of y and t. Option (D) is a function of y^3, t.

∴ Option (D) does not satisfy the wave equation.

Uniform Plane Wave

In a medium, if there present equiphase characteristics, then wave is said to be uniform plane wave.

(or)

If suppose a wave is propagating in z direction, then electric field $E = f(z - v_0 t)$. E is function of z, t and independent of x and y. Such a wave is said to be a uniform plane wave.

$$i.e., \frac{\partial E_z}{\partial z} = 0$$

That is, a uniform plane wave progression in z direction has no z component.

In low loss and loss less medium, electric field or magnetic field exhibit equiphase characteristic because waves progress with uniform velocity.

For uniform plane wave,

$$E_x = \sqrt{\frac{\mu}{\varepsilon}} H_y \text{ and } E_y = \sqrt{\frac{\mu}{\varepsilon}} H_x$$

and $$\frac{E}{H} = \frac{\sqrt{E^2{}_x + E^2{}_Y}}{\sqrt{H^2{}_x + H^2{}_Y}} = \sqrt{\frac{\mu}{\varepsilon}}$$

Wave Propagation In A Loss Less Medium

Wave equation in a source-free region is as follows:

$$\nabla^2 \overline{E} = \mu \in \frac{\partial^2 \overline{E}}{\partial t^2} \text{ (time domain)}$$

$$\nabla^2 \overline{E} = -\omega^2 \text{m} \in \overline{E} \text{ (Frequency domain)}$$

Assuming $\overline{E}$ depends on x and independent of y and z,

$$\therefore \frac{\partial E}{\partial y} = \frac{\partial E}{\partial z} = 0$$

$$\therefore \frac{\partial^2 \overline{E}}{\partial x^2} = -\omega^2 \mu \in \overline{E}$$

$$\frac{\partial^2 \overline{E}}{\partial x^2} = -\beta^2 \overline{E} \text{ (Where } \beta = \omega\sqrt{\mu E}\text{)}$$

If the wave is travelling along x direction $E_x = 0$

Therefore,

$$\frac{\partial^2 E_y}{\partial x^2} + \beta^2 E_y = 0 \text{ and}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \beta^2 E_z = 0$$

General solution for the above equation is

$$E_y(x) = C_1 . e^{-j\beta x} + C_2 . e^{j\beta x}$$

In time domain,

$$E_y(x, t) = \text{Re}[E_y(x). e^{j\omega t}]$$

$$E_y(x, t) = C_1 \cos(\omega t - \beta x) + C_2 \cos(\omega t + \beta x)$$

By assuming $C_1 = 1$ $C_2 = 0$

$$E_y(x, t) = \cos(\omega t - \beta x)$$

Constant Phase Point

It's phase angle with respect to phase reference is always constant irrespective of time 't'.

Constant phase surfaces are planes perpendicular to the direction of propagation of wave and as amplitudes are uniform it is called as uniform plane wave.

1. Phase velocity

$$\beta x - \omega t = \text{constant}$$

$$\beta . \frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{\beta}$$

Velocity with which constant phase point moves is called phase velocity.

$$V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu \in}}$$

If it is free space,
$\mu_r = 1, \in_r = 1$

$$V_0 = \frac{1}{\sqrt{\mu_0 \in_0}}$$

For any other loss less media,

$$V = \frac{V_0}{\sqrt{\mu_r \in_r}}$$

For most of the dielectric $\mu_r = 1$

$$\therefore V = \frac{V_0}{\sqrt{\in_r}}$$

2. Relative phase velocity

$$P = \frac{V}{V_0} = \frac{1}{\sqrt{\in_r}}$$

$\in_r \geq 1$
$\therefore P \leq 1$

Velocity of the wave in any loss less media is always less than its velocity in free space.

3. Wavelength
The distance travelled by a wave in a full cycle of 2π radians.

Example 8

The electric field intensity vector of a plane wave is given by $\overline{E}(r,t) = 10 \sin(3{,}000t + 0.03x + 20a_y)$ where a_y denotes unit vector along y direction. The wave is propagating with a phase velocity.

(A) 1×10^5 m/sec (B) 1×10^6 m/sec
(C) -1×10^5 m/sec (D) -1×10^6 m/sec

Solution

$$V = \frac{\omega}{\beta} = \frac{3000}{0.03} = \frac{3\times10^5}{3}$$

$V = 1 \times 10^5$ m/sec
But the wave is propagation along negative x direction.

Example 9

Refractive index of glass is 1.5. Find the wavelength of a beam of light with a frequency of 10^{12} Hz in glass. Assuming velocity of light is 3×10^8 in vacuum.

(A) 20 μm (B) 200 μm
(C) 2 mm (D) 0.2 μm

Solution

$$\text{Velocity in glass} = \frac{V}{n} = \frac{3\times10^8}{\frac{3}{2}} = 2\times10^8 \text{ m/sec}$$

But $v = f\lambda$

$$\lambda = \frac{v}{f} = \frac{2\times10^8}{10^{12}} = 200\ \mu\text{m}$$

4. Phase Constant (B)

$$B = \frac{2\pi}{\lambda} \text{ (Rad/Meter)}$$

Phase Angle Per Unit Distance

$$V = \frac{\omega}{\beta} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda$$

$$V = f\lambda$$

5. Group Velocity

$$V_g = \frac{\Delta\omega}{\Delta\beta}$$

For Vanishing Small Bandwidths,

$$V_g = \frac{d\omega}{d\beta}$$

Medium

1. Non-Dispersive
2. Dispersive
   1. Non-Dispersive
      Phase Velocity Is Not A Function Of Frequency
      $V_p \neq f(f)$
      $V_p = V_g$ (Group Velocity)
      e.g., Free Space
   2. Dispersive
      Phase velocity is a function of frequency.
      $V_p = f(f)$
      (a) Normally dispersive:

$$\frac{dV_p}{d\lambda} > 0$$

$$V_g < V_p$$

$$V_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta V_p)$$

$$V_g = V_p + \beta.\frac{dV_p}{d\beta}$$

$$V_g = V_p - \beta\frac{dV_p}{d\lambda}$$

(b) Anomalously dispersive

$$\frac{dV_p}{d\lambda} < 0$$

$$V_g > V_p$$

Impedance of Loss Less Media

$$\frac{E_y}{H_z} = \frac{-E_z}{H_y} = \eta = \sqrt{\frac{\mu}{\in}}$$

$$\frac{E}{H} = \frac{\sqrt{E_y^2 + E_z^2}}{\sqrt{H_y^2 + H_2^2}} = \sqrt{\frac{(H_z^2 + H_y^2)\eta^2}{(H_y^2 + H_z^2)}}$$

$$\frac{E}{H} = \eta$$

Wave Equation for a Lossy Medium

$$\nabla^2\ \overline{E} = \mu\,\varepsilon\frac{\partial^2\overline{E}}{\partial t^2} + \mu\sigma\frac{\partial\overline{E}}{\partial t}$$

$$\nabla^2\ \overline{H} = \mu\in\frac{\partial^2\overline{H}}{\partial t^2} + \mu\sigma\frac{\partial\overline{H}}{\partial t}$$

1. Wave propagation in lossy medium

$$\nabla^2\ \overline{E} = -\mu\in\omega^2\overline{E} + \mu\sigma j\omega\overline{E}$$

$$\nabla^2\ \overline{E} + \omega^2\mu\in\overline{E} - j\,\mu\sigma\omega\overline{E} = 0$$

$$\nabla^2\ \overline{E} = \gamma^2\ \overline{E}$$

Where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

γ–propagation constant.

$$\gamma = \alpha + j\,\beta$$

For a wave travelling in x direction $E_x = 0$, $H_x = 0$ and remaining components depends only on x and independent of y and z (Assumption)

$$\frac{d^2E_y}{dx^2} = \gamma^2E_y$$

Phasor value

$E_y(x) = E_0.\ e^{-\gamma x}$

$E_y(x, t) = \text{Re}[E_0.\ e^{-\gamma x}\ e^{j\omega t}]$

$= \text{Re}[E_0.\ e^{-\alpha x}.\ e^{-j\beta x}.\ e^{j\omega t}]$

$E_y(x, t) = E_0.\ e^{-\alpha x}.\ \cos(\omega t - \beta x)$

Due to $e^{-\alpha x}$, the wave is attenuating exponentially

α – Attenuating constant

β – Phase constant $= \dfrac{2\pi}{\lambda}$

$$\alpha = \omega\left[\sqrt{\frac{\mu\in}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\in^2}} - 1\right]}\right]$$

$$\beta = \omega\left[\sqrt{\frac{\mu\varepsilon}{2}\left[1 + \left[\frac{\sigma}{\omega\in}\right]^2 + 1\right]}\right]$$

$$\text{Loss tangent } \tan\theta = \frac{\sigma}{\omega\in} = \frac{|J_c|}{|J_d|}$$

Parameter	General value	Good conductors	Good dielectrics
(1) Attenuation constant	$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}$	$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\in}}$
(2) Propagation constant	$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right]}$	$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$	$\beta = \omega\sqrt{\mu\in}\left(1+\frac{\sigma^2}{8\omega^2\in^2}\right)$
(3) Velocity	$V = \frac{1}{\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right]}}$	$V = \sqrt{\frac{2\omega}{\mu\sigma}}$	$v = \frac{1}{\sqrt{\mu\in}}\left(1-\frac{\sigma^2}{8\omega^2\in^2}\right)$
(4) Characteristic impedance	$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}}$	$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45°$	$\eta = \sqrt{\frac{\mu}{\in}}\left(1+\frac{j\sigma}{2\omega\varepsilon}\right)$

Example 10

The y direction electric field E_y having sinusoidal time variation $e^{j\omega t}$ and space variation in x direction satisfies the equation $\nabla^2 E_y + k^2 E_y = 0$ under source-free region in a loss less medium. What is the solution representing the wave in positive x direction

(A) $E_y = E_0.\ e^{kx}$ (B) $E_y = E_0.\ e^{-kx}$
(C) $E_y = E_0.\ e^{-jkx}$ (D) $E_y = E_0.\ e^{jkx}$

Solution

$$\nabla^2 E_y + k^2 E_y = 0$$

E_y depends only on x

$$\therefore \frac{\partial^2 E_y}{\partial x^2} = -k^2 E_y$$

$$E_y = C_1.\ e^{jkx} + C_2\ e^{-jkx}$$

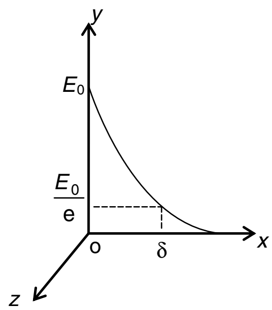
–ve x direction and +ve x direction

$$E_y = E_0.\ e^{-\gamma x}$$

$$E_y = E_0.\ e^{-\alpha}.\ e^{-j\beta x}$$

$$|E_y| = E_0.\ e{-\alpha x}$$

at $x = 0$ $|E_y| = E_0$



at $x = \delta$

$$|E_y| = \frac{E_0}{e} = E_0 . e^{-\alpha\delta}$$

$$\alpha\delta = 1$$

$$\delta = \frac{1}{\alpha}$$

$$\delta = \frac{1}{\omega\times\left[\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma^2}{\omega t}\right]^2}-1\right]}\right]}$$

For a good conductor

$$\delta \approx \frac{1}{\sqrt{\pi f \mu\sigma}}$$

One per cent of depth of penetration:
Wave attenuate to 1% of its initial value,

$$\therefore 0.01\ E_0 = E_0.\ e^{-x_1/\delta}$$

Example 11

Distilled water at 25°C is characterized by $\sigma = 1.7 \times 10^{-4}$ ℧/m and $\in = 78 \in_0$ at a frequency of 6 GHz. Its loss tangent tan θ is

(A) 1.3×10^{-5} (B) 0.65×10^{-5}
(C) 0.65×10^{-6} (D) 1.3×10^{-6}

Solution

$$\tan\theta = \frac{\sigma}{\omega\in} = \frac{1.7\times10^{-4}}{2\pi\times6\times10^9\times78\in_0} = \frac{9\times10^9\times1.7\times10^{-4}}{78\times3\times10^9}$$

$$= \frac{5.1\times10^{-4}}{78} = 6.5384\times10^{-6}$$

Depth of Penetration

At very high frequencies, where σ is large, the wave amplitude attenuates rapidly. Depth of penetration is the distance in which the wave amplitude drop to $\frac{1}{e}$ of its initial value.

$$e^{\frac{x_1}{\delta}} = 100$$

$$\frac{x_1}{\delta} = \ln(100)$$

$x_1 = 4.6\delta$
X_1 – 1% of depth of penetration

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Surface impedance

$$Z_s = \frac{r}{\sigma} = \frac{\varepsilon}{js}$$

Surface Resistance

$$R_s = \sqrt{\frac{\mu\omega}{2\sigma}}$$

Critical Angle

$$\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Skin Effect

The inability of the high-frequency EM waves to penetrate into the conductor, and it is confined to a thin layer and it is called skin effect.

Skin effect is used in wave guides and antennas.

Impedance of Lossy Medium

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

Considering the wave travelling in x direction

$$H_z = \frac{-1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$

But $E_y = E_0 . e^{-\gamma x}$

$$H_z = \frac{\gamma}{j\omega\mu}E_0 . e^{-\gamma x}$$

$$\eta = \frac{E_y}{H_z} = \frac{j\omega\mu}{\gamma}$$

1. **Good dielectric**

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\in)}$$

$$\eta = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\in}}$$

$$\eta = \sqrt{\frac{\mu}{\in}}\left[1 + j\frac{\sigma}{2\omega\in}\right]\left(\frac{\sigma}{\omega\in} << 1\right)$$

2. **Good conductor**

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}.\frac{\sigma}{\omega\varepsilon}}, \text{ when } \frac{\sigma}{\omega\varepsilon} >> 1$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

So, $\gamma = \sqrt{\omega\mu\sigma}\angle 45°$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

Solution for the wave equation is

$E = E_0 e^{-\alpha z}.e^{j(\omega t - \beta z)}a_x$
$H = H_0\, e^{-\alpha z}.e^{j(\omega t - \beta z)}a_y$

When $z = \frac{1}{\alpha}$, $E = \frac{E_0}{e}$ and $H = \frac{H_0}{e}$

i.e., when a wave travels a distance of $\frac{1}{\alpha}$, the amplitude of the wave falls by $\frac{1}{e}$ times

→ This distance $\frac{1}{\alpha}$ is called skin depth or depth of penetration

Skin depth $= \delta = \frac{1}{\alpha}$

$$= \sqrt{\frac{2}{\omega\mu\sigma}} \text{ for good conductors}$$

Summary

(i) $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$; $\gamma = \sqrt{\omega\mu\sigma}\angle 45°$

(ii) $\delta = \frac{1}{\alpha}$ = distance travelled by the wave where the amplitude falls by $\frac{1}{e}$ times initial value

(iii) $\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$. So, $R_s = \sqrt{\frac{\omega\mu}{\sigma}} = \frac{1}{\delta\sigma}$

(iv) Skin depth is very small of the order of a few mm

Example 12

A plane wave propagating through a medium $\varepsilon_r = 27$, $\mu_r = 3$ and $\sigma = 0$ has its electric field strength given by $\bar{E} = 0.5\, e^{2/3}$ sin $(10^8 t - \beta z)$ V/m. The wave impedance is

(A) 125.67 Ω (B) 1.13k Ω
(C) 3.39k Ω (D) 565.5 Ω

Solution

$$\eta = \frac{j\omega\mu}{r}$$

$$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega \in}}$$

given $\sigma = 0$

$$= \sqrt{\frac{\mu}{\in}} = \sqrt{\frac{\mu_0}{\in_0}}.\sqrt{\frac{\mu_r}{\in_r}}$$

$$= 377.\sqrt{\frac{3}{27}} = \frac{377}{3} = 125.67\ \Omega$$

Example 13

For seawater with $\sigma = 50$ ℧/m and $\in_r = 81$, what is the distance for which radio signal can be transmitted with 99% attenuation at 25 KHz

(A) 4.14 m (B) 6.54 m
(C) 2.07 m (D) 3.27 m

Solution

$$\frac{\sigma}{\omega\varepsilon} >> 1$$

99% of attenuation, means 1% of its initial value

$\therefore x_1 = 4.6\delta$

$$= 4.6\ \frac{1}{\sqrt{\pi f\ \mu\sigma}}$$

$$= 4.6\ \frac{1}{\sqrt{\pi \times 4\pi \times 10^{-7} \times 25 \times 10^3 \times 50}}$$

$$x_1 = 2.070 \text{ m}$$

Example 14

A lossy dielectric has an intrinsic impedance of 200 ∠30° at a particular frequency. If at that frequency, the plane wave propagating through the dielectric has the magnetic field component $H = 5\ e^{-\alpha x} \cos\left(\omega t - \frac{1}{2}x\right) a_y$ A/m then skin depth is

(A) 0.288 m (B) 3.464 m
(C) 1.7171 m (D) 0.858 m

Solution

a_{wave} - unit vector along the direction of wave propagation

$a_{wave} = a_x$

$\therefore a_H = a_y$

$\therefore a_E = -a_z$

$$\beta = \frac{1}{2}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2} + 1}\right]^{\frac{1}{2}}$$

But $\frac{\sigma}{w \in} = \tan 2(\theta_n)$

θ_n = angle of intrinsic impedance

$$\frac{\sigma}{\omega \in} = \sqrt{3}$$

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{1}{2\sqrt{3}}$$

$$\beta = \frac{1}{\alpha} = 2\sqrt{3} = 3.464 \text{ m}$$

Example 15

In the above problem, the electric field if the angular frequencies = 0.5 rad/sec

(A) $-e^{\frac{x}{2\sqrt{3}}} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) \alpha_z$ kv/m

(B) $-e^{\frac{-x}{2\sqrt{3}}} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) a_z$ kv/m

(C) $e^{\frac{x}{2\sqrt{3}}} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) a_z$ kv/m

(D) $-e^{\frac{x}{2\sqrt{3}}} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) a_z$ v/m

Solution

$$a_E = -a_z$$

$$\frac{E_0}{H_0} = \eta = 200\angle 30$$

$$E_0 = 10006^{j\pi/6}$$

$$E = \text{Re}(1000\ e^{j\pi/6}.\ e^{-\gamma x}.\ e^{j\omega t}\ a_E)$$

$$E = -\ e^{-\alpha x} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) a_z \text{ kv/m}$$

$$= -e^{\frac{-x}{2\sqrt{3}}} \cos\left(\frac{t-x}{2} + \frac{\pi}{6}\right) a_z \text{ kv/m}$$

Poynting Vector

Poynting's theorem states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within volume minus the conduction losses.

Mathematically,

$$\oint\left(\overline{E}\times\overline{H}\right).ds = -\frac{\partial}{\partial t}\int_v\left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] - \int_v \sigma E^2 dv$$

Poynting vector P is defined as $\overline{E}\times\overline{H}$ in units watts per square meter. It represents the instantaneous power density vector associated with the EM field at a given point. The integration of the poynting vector over any closed surface gives the net power flowing out of that surface.

$\overline{P}$ is normal to both $\overline{E}$ and $\overline{H}$, i.e., along the direction of propagation

$$a_p = a_E \times a_H$$

1. Instantaneous poynting vector

$$\overline{P_i} = \overline{E} \times \overline{H}$$

2. Time average poynting vector

$$\overline{P}_{avg} = \frac{1}{T}\int_0^T \overline{P}_i \, dt$$

3. Total time average power

$$P_{avg} = \int_s P_{avg} .ds$$

4. Behaviour of poynting vector in loss less media:
   [By assuming a uniform plane wave travelling along the x direction]

$\overline{E} = E_y \, a_y$
$\overline{H} = H_z \, a_z$
$\overline{P} = E_y \, H_z \, a_x$
$\overline{P} = P_x \, a_x \quad [P_x = E_y \, H_z]$
$E_y = E_0 \sin(\omega t - \beta x)$

$$H_z = \frac{E_0}{\eta} \sin(\omega t - \beta x)$$

$$P_x = \frac{E_0^2}{\eta} \sin^2(\omega t - \beta x) \ \mathrm{w/m^2}$$

$$\text{Peak value of } P_x = \frac{E_0^2}{\eta}$$

$$\text{Average value of } P_x = \frac{1}{2\pi}\int_0^{2\pi} P_x d(\omega t) = \frac{E_0^2}{2\eta}$$

5. Poynting vector in lossy medium:

$E_y = E_0 . e^{-\gamma x} \, a_y$
$H_z = H_0 . e^{-\alpha x} \, e^{-\beta x} . e^{-j\xi}$

$$\overline{P}_{avg} = \frac{1}{2}\mathrm{Re}\left[\overline{E} \times \overline{H}^*\right]$$

$$= \frac{1}{2} E_0 H_0 e^{-2\alpha x} \, \mathrm{Re}\left[e^{j\xi}\right] a_x$$

$$\overline{P}_{avg} = \frac{1}{2} E_0 H_0 e^{-2\alpha x} \cos \xi$$

For a good conductor $\xi = 45°$

$$\therefore \ \overline{P}_{avg} = \frac{1}{2\sqrt{2}} E_0 H_0 e^{-2\alpha x} \, a_x$$

That is, power absorbed by that loss medium with the distance exponentially decaying manner.

Example 16

The field in A/m of a plane wave propagating in free space is given by $\overline{H} = \frac{3}{\eta_0}\cos(\omega t - \beta.z) + \frac{4}{\eta_0}\sin(\omega t - \beta.z) \, a_y$ then the time average power flow density in watts is

(A) $\frac{25}{\eta_0}$ (B) $25\,\eta_0$
(C) $\frac{12.5}{\eta_0}$ (D) $12.5\,\eta_0$

Solution

$$P = \frac{1}{2}\eta_0 \, |H|^2$$

$$|H_0| = \frac{\sqrt{3^2 + 4^2}}{\eta_0} = \frac{5}{\eta_0}$$

$$P = \frac{1}{2} . \eta_0 . \frac{25}{\eta_0^2}$$

$$P = \frac{12.5}{\eta_0} \text{ watts}$$

Example 17

In free space $\overline{E}(x, t) = 10\cos(\omega t - 2x)\, a_y$ v/m what is the average power crossing a circular area of radius a circular area of radius 4 m in the plane x = constant

(A) 0 watts (B) 3.334 watts
(C) 6.67 watts (D) 13.34 watts

Solution

$\overline{E}(x, t) = 10\cos(\omega t - 2x)\, a_y$ v/m

Comparing with

$$\overline{E} = E_0 \cos(\omega t - \beta x)$$

$$E_0 = 10$$

$$\text{Average power} = \int \overline{P}_{avg} .ds$$

$$= \frac{E_o^2}{2\eta} .\pi(4)^2$$

$$= \frac{10^2}{2 \times 120\pi} \times \pi \times 16$$

$$= \frac{40}{6} \text{ watts}$$

$$= 6.67 \text{ watts}$$

Polarization

The time-varying behaviour of electric field vector at some fixed point in space is referred to as polarization of electromagnetic wave, and the direction in which it is varied is the direction of propagation.

$$\overline{E} = \mathrm{Re}\left[\overline{E}\, e^{j(\omega t - \beta x)}\right]$$

$$\overline{H} = \frac{1}{\eta}\left[a_x \times \overline{E}\right]$$

$$\overline{E} = E_x \, a_x + E_y \, a_y$$

$E_{xp} = E_1$ (Phasor form)
$E_{yp} = E_2 \, e^{j\phi}$ (Phasor form)

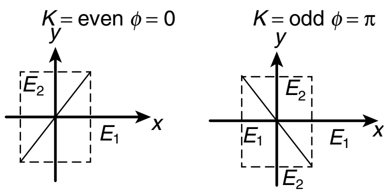
∴ ϕ - Angle which E_y leads E_x

$$\bar{E} = \text{Re}\left[(E_1 a_x + E_2 e^{j\phi} a_y) e^{j(\omega t - \beta x)}\right]$$

$\bar{E} = E_1 \cos(\omega t - \beta x)\, a_x + E_2 \cos(\omega t - \beta x + \phi)\, a_y$
$E_x = E_1 \cos(\omega t - \beta x)$
$E_y = E_2 \cos(\omega t - \beta x + \phi)$

1. $E_x \neq 0$; $E_y = 0$
   - X-polarized wave
2. $E_x = 0$; $E_y \neq 0$
   - Y-polarized wave
3. $\phi = k\pi$, $k = 0, 1, 2, ...$
   - If k is even, E_x and E_y are in phase
   - If k is odd E_x and E_y are out of phase

   This is called as linear polarization



4. Elliptical polarization

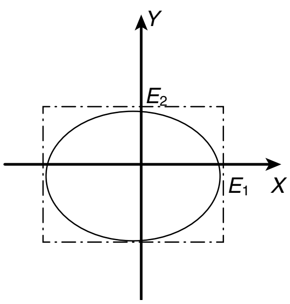
   $\phi = (2k + 1)\dfrac{\pi}{2}$

   $k = 0, 1, 2....$

   E_x and E_y are in phase quadrature.

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1$$

   and $E_1 \neq E_2$.



5. Circular polarization:

   $\phi = (2k + 1)\dfrac{\pi}{2}$

   $k = 0, 1, 2....$

   $E_1 = E_2 = E_0$

   $E_x^2 + E_y^2 = E_0^2$

   Tip of electric field traces a circular path then it is called as circular polarization.

   If k = even

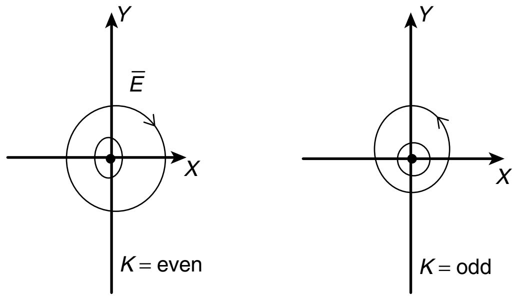
   $E_x = E_0 \cos \omega t$

   $E_y = -E_0 \sin \omega t$

   If k = odd

   $E_x = E_0 \cos \omega t$

   $E_y = -E_0 \sin \omega t$



k = even close wise circular Polarization (Left circular Polarization)

k = odd Counter clockwise circular Polarization (Right circular polarization)

Example 18

A linearly polarized wave can be expressed as a sum of two oppositely

(A) Circular polarized waves
(B) Elliptically polarized waves
(C) Both
(D) Neither of them

Solution

$$\bar{E}_{LCP} = E_0 e^{-j\beta z}(a_x + ja_y)$$

$$\bar{E}_{RCP} = E_0 e^{-j\beta z}[a_x - ja_y]$$

By superposing,

$$\bar{E}_{LCP} + \bar{E}_{RCP} = 2E_0 e^{-j\beta z} a_x$$

Example 19

If $\bar{E} = (a_y + ja_z)e^{-j\beta x}$, then the wave is said to be in which one of the following

(A) Right circular polarized
(B) Right elliptically polarized
(C) Left circular polarized
(D) Left elliptical polarized

Solution

$|E_{0y}| = |E_{0z}|$ and angle is 90° and E_z leads E_y by 90°

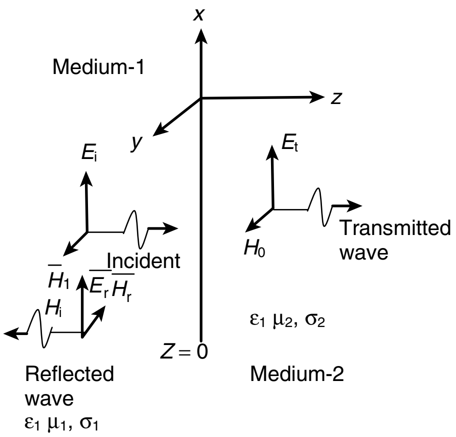
∴ The wave is left circular polarized

1. Left and right notation

- For advancing time, if the $\bar{E}$ field traces anticlockwise rotation
- If the direction of propagation is denoted by right-hand thumb, according to right-hand thumb rule. It is called right elliptical/circular polarization.
- For advancing time, if $\bar{E}$ traces anticlockwise and if left-hand thumb denotes the direction of propagation and fingers the advancing time, the called left circular elliptically polarization.

Reflection and Refraction of Plane Waves

Normal Incidence of Plane Wave on Plane Boundaries



Assuming the electric field does not undergo any variation and the wave is travelling in z direction. Then, the fields are shown in above figure.

Propagation constant $\gamma = \propto + j\,\beta$

$$= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

Incident Wave

$$\overline{E}_i = E_{oi} \cdot e^{-\gamma_1 z}\, a_x$$

$$\overline{H}_i = \frac{E_{0i}}{\eta_1} e^{-\gamma_1} a_y$$

Reflected wave:

$$\overline{E}_r = E_{0r} \cdot e^{\gamma_1 z}\, a_x$$

$$\overline{H}_\gamma = \frac{-E_{0r}}{\eta_1} e^{\gamma_1 z} a_y$$

Transmitted wave:

$$\overline{E}_t = E_{0t} . e^{-\gamma_2 z} a_x$$

$$\overline{H}_t = \frac{E_{0t}}{\eta_2} e^{-\gamma_2 z} a_y$$

Boundary Conditions

Tangential components are continuous

$$\overline{E}_i + \overline{E}_r = \overline{E}_t,\ \overline{H}_i + \overline{H}_r = \overline{H}_t$$

at $z = 0$

$$E_{0i} + E_{0r} = E_{0t}$$

$$\frac{E_{0i}}{\eta_1} - \frac{E_{or}}{\eta_1} = \frac{E_{ot}}{\eta_2}$$

$\therefore\ \rho = \dfrac{E_r}{E_i} = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ (reflection coefficient)

$T = \dfrac{E_t}{E_i} = \dfrac{2\eta_2}{\eta_1 + \eta_2}$ [transmission coefficient]

$1 + \rho = T$

$0 \le |\rho| \le 1$

1. Medium 1 (perfect dielectric)
   Medium 2 (perfect conductor)
   $\sigma_1 = 0 \qquad \sigma_2 = \infty$
   $\overline{E}_i + \overline{E}_r = 0 \qquad$ at $z = 0$
   $E_{or} = -E_{oi}$ at $z = 0$
   $\eta_2 = 0 \qquad \therefore\ \rho = -1$
   $T = 1 - 1 = 0$

$$\overline{E}_1 = \overline{E}_i + \overline{E}_r$$

$$\overline{E}_1 = E_0\left[e^{-\gamma_1 z} - e^{\gamma_1 z}\right] a_x$$

$$\overline{H}_1 = \frac{E_0}{\eta_1}\left[e^{-\gamma_1 z} + e^{\gamma_1 z}\right] a_y$$

   From the above expression, we can say that electric field can undergo change in its direction.

2. Both are perfect dielectric
   $\sigma_1 = 0 \qquad \sigma_2 = 0$
   $\gamma_1 = j\,\beta,\ \gamma_2 = j\,\beta_2$

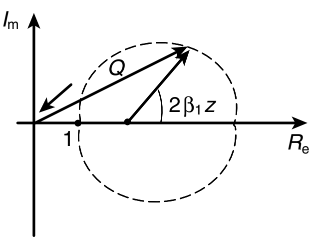
$$\overline{E}_1 = E_0\left[e^{j\beta_1 z} + \rho\, e^{j\beta_1 z}\right] a_x$$

$$\overline{H}_1 = \frac{E_0}{\eta_1}\left[e^{-j\beta z} + \rho e^{-j\beta_1 z}\right] a_y$$

$$\overline{E}_1 = E_0 e^{-j\beta_1 z}\left[1 + \rho e^{j2\beta_1 z}\right] a_x$$

$$|\overline{E}_1| = E_0\,|1 + \rho\, e^{j2\beta_1 z}|$$

   $1 + \rho\, e^{j2\beta_1 z}$ can be expressed as sum of 2 phasors on complex plane.



   Where $Q = \left|1 + P.e^{j2\beta_1 z}\right|$

Example 20

A uniform plane wave in the free space is normally incident on an infinitely thick dielectric slab ($\in_r = 9$). Then, the magnitude of reflection coefficient is

(A) 2 (B) $\dfrac{1}{2}$ (C) 1 (D) 0

Solution

$$\rho = \left|\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right| = \left|\frac{\sqrt{\varepsilon_{r_1}} - \sqrt{\varepsilon_{r_2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}\right|$$

$$= \left|\frac{1-3}{1+3}\right| = \frac{1}{2}$$

Example 21

A plane electromagnetic wave travelling in a perfect dielectric medium of dielectric constant $\in_1$ is incident on its boundary with another dielectric medium of dielectric constant ε_2 (where $\in_2 = \sqrt{3} \in_1$). The angle of incidence $\theta_i = 60°$ with the normal to the boundary surface. Then, the angle of transmitted wave with the boundary surface.

(A) 45° (B) 60°
(C) 30° (D) No transmitted wave

Solution

According to Snell's law

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\in_2}{\in_1}} = \sqrt{3}$$

$$\sin\,\theta_t = \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}} = \frac{1}{2}$$

$$\theta_t = 30°$$

Maxima

$$2\,\beta_1 z = -\,2\,n\,\pi$$

$n = 0, 1, 2.....$

$$z = \frac{-n\pi}{\beta_1}$$

$$z = \frac{-n\lambda_1}{2}$$

$$|E_1|_{max} = E_0(1+\rho)$$

Minima

$$2\beta_1\,z = -(2\,m+1)\,\pi$$

$m = 0, 1, 2...$

$$z = \frac{-(2m+1)\pi}{2\beta} = \frac{-(2m+1)\lambda}{4}$$

Minima occur at odd multiples of quarter wavelength.

$$|\bar{E}_1|_{min} = E_0(1-\rho)$$

$$\frac{|E_1|_{max}}{|E|_{min}} = \text{VSWR (Voltage standing wave ratio)}$$

$$S = \text{VSWR} = \frac{E_0(1+\rho)}{E_0(1-\rho)}$$

$$S = \frac{1+|\rho|}{1-|\rho|}$$

$$|\rho| = \frac{VSWR-1}{VSWR+1}$$

$0 \le |\rho| \le 1$ and VSWR ≥ 1

Plane wave travelling in arbitrary direction:

$$\bar{E}(\bar{r}) = \bar{E}_0 . e^{-j\beta\hat{u}.\bar{r}}$$

u - Unit vector in the direction of propagation of wave.

$$\bar{E} = \bar{E}_0\,e^{-j\beta(x\cos A + y\cos B + z\cos c)}$$

$$\bar{E} = (E_{0x}\,a_x + E_{0y}\,a_y + E_{0z}\,a_z)\,e^{-j\beta(x\cos A + y\cos B + z\cos c)}$$

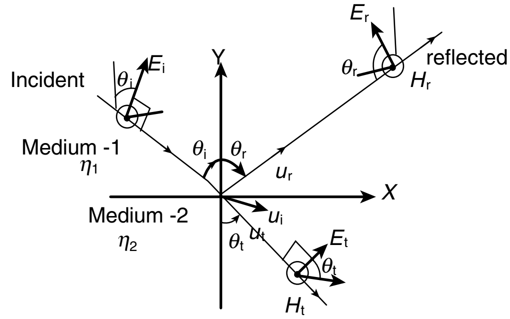
$\beta_x = \beta\cos A$

$\beta_y = \beta\cos B$

$\beta_z = \beta\cos C$

Oblique Incidence

Parallel Polarization



Incident wave

$$\bar{E} = (E_{0x}\,a_x + E_{0y}\,a_y + E_{0z}\,a_z)\,e^{-j\beta_1(x\cos A + y\cos B + z\cos C)}$$

$$\bar{E}_i = (a_x\cos\theta_i + a_y\sin\theta_i)\,E_{0i}\,e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)} \quad (1)$$

$$\bar{H}_i = \frac{E_{0i}}{\eta_1}e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)}\,a_z \quad (2)$$

Reflected wave

$$\bar{E}_r = \rho_{11}E_{0r}\,(-\cos\theta_r\,a_x + \sin\theta_r a_y).e^{-j\beta_1(x\sin\theta_r + y\cos\theta_r)} \quad (3)$$

$$\bar{H}_r = \frac{\rho_{11}E_{0r}}{\eta_1}e^{-j\beta_1(x\sin\theta_r + y\cos\theta_r)}.a_z \quad (4)$$

ρ_{11} - reflection coefficient

Transmitted wave

$$\bar{E}_t = \tau_{11}(\cos\theta_t\,a_x + \sin\theta_t a_y)\,E_{0t}.e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)} \quad (5)$$

$$\bar{H}_t = \frac{T_{11}E_{0t}}{\eta_2}e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)}a_z \quad (6)$$

τ_{11} - Transmission coefficient.

Among (1), (3), (5) tangential components are x terms and y are $\perp^r$ so that $y = 0$

$\therefore$ According to boundary conditions at $x = 0$, $y = 0$

$$E_{0t}\cos\theta_t = E_{0i}\cos\theta_i - E_{0r}\cos\theta_r \text{ and } \theta_r = \theta_i$$

$$\sin\,\theta_t = \frac{n_1}{n_2}\sin\theta_i(snells\,law)$$

$$1 - \rho_{11} = \frac{\cos\theta_t}{\cos\theta_i}\tau_{11}$$

$$1 + \rho_{11} = \frac{n_1}{n_2}\tau_{11}$$

$$\frac{1-\rho_{11}}{1+\rho_{11}} = \frac{\cos\theta_t}{\cos\theta_i}\cdot\frac{n_2}{n_1}$$

$$\rho_{11} = \frac{\frac{\in_2}{\in_1}\cos\theta_i - \sqrt{\frac{\in_2}{\in_1} - \sin^2\theta_i}}{\frac{\in_2}{\in_1}\cos\theta_i + \sqrt{\frac{\in_2}{\in_1} - \sin^2\theta_i}}$$

Brewster angle

The angle of incidence for a parallel polarized wave for which parallel reflection coefficient is zero.

$\therefore\ \theta_i = \theta_B$ (Brewster angle)

$$\therefore\ \frac{\epsilon_2^2}{\epsilon_1^2}\cos^2\theta_B = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_B$$

$$\sin^2\theta_B = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2}$$

$$\cos^2\theta_B = \frac{\epsilon_1}{\epsilon_1 + \epsilon_2}$$

$$\therefore\ \theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

NOTES

1. If the incident wave is a mixed polarized wave, i.e., if it has both parallel and perpendicular component, incident the boundary at $\theta_i = \theta_B$ then $\rho_{11} = 0$, i.e., reflected wave will be purely perpendicular polarized.
2. If it is a circular polarized wave, then reflected wave will be linear polarized.

Example 22

The two dielectric material properties $\epsilon_{r1} = 3$, $\epsilon_{r2} = 9$, $\mu_1 = \mu_0$, $\mu_2 = \mu_0$, then the Brewster angle for the interface of these medium is

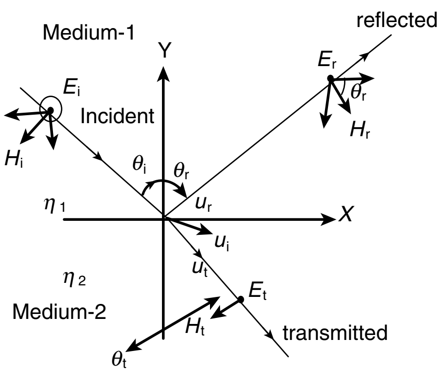
(A) 30° (B) 45° (C) 60° (D) 90°

Solution

$$\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

$$\theta_B = \tan^{-1}(\sqrt{3}) = 60°$$

Perpendicular Polarization



Incident wave

$$\overline{E}_i = a_z.E_0\, e^{-[j\beta_1(x\sin\theta_i - y\cos\theta_i)]}$$

$E_0 = E_{0z}$

$$\overline{H}_i = \frac{E_o}{\eta_1} e^{-j\beta_1(x\sin\theta_i - y\cos\theta_i)}.(-\cos\theta_i a_x - \sin\theta_i a_y)$$

$$\frac{E_0}{\eta_1} = H_0 \text{ (amplitude of } \overline{H}_i)$$

Reflected wave

$$\overline{E}_r = \rho_\perp E_0 e^{-j\beta_1(x\sin\theta_r + y\cos\theta_r)} a_z$$

$$\overline{H}_r = \frac{\rho_\perp E_0}{\eta_1} e^{-j\beta_1(x\sin\theta_r + y\cos\theta_r)}(a_x\cos\theta_r - a_y\sin\theta_r)$$

$\rho^\wedge$ = Perpendicular polarization reflection coefficient

Transmitted wave

$$\overline{E}_t = E_0\, T_\perp e^{-j\beta_2(x\sin\theta_t - y\cos\theta_t)}$$

$$\overline{H}_t = -\frac{E_0 T_\perp}{\eta_2} e^{-j\beta_2(x\sin\theta t - y\cos\theta_t)}(a_x\cos\theta_t - a_y\sin\theta_t)$$

$T^\wedge$ - Transmission coefficient

Applying boundary conditions

$E_i + E_r = E_t$ at $y = 0$ plane (or)

x, z plane

$1 + \rho_\perp^{\ 2} = T_\perp$

$\theta_i = \theta_r$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2}.(Snells\ law)$$

$$-\cos\theta_i + \rho_\perp\cos\theta_i = -T_\perp\frac{\eta_1}{\eta_2}\cos\theta_t$$

$$\rho_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Case (i):

If $\theta_i = 0$

$$\rho^\wedge = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \text{ (normal incidence)}$$

Case (ii):

If medium 2 is perfect conductor, then $\eta_2 = 0$

$\rho_\perp^{\ r} = -1$

Case (iii):

If both are dielectrics

$$\rho_\perp = \frac{\sqrt{\epsilon_1}\cos\theta_i - \sqrt{\epsilon_2}\cos\theta_t}{\sqrt{\epsilon_1}\cos\theta_i + \sqrt{\epsilon_2}\cos\theta_t}$$

by using Snell's law

$$\rho^\wedge{}_r = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

$\rho_{\perp r} \neq 0$ (always)

$\therefore$ Perpendicular polarization have always some reflection and refraction.

$$T_\perp^{\ r} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

Example 23

A plane wave propagating in air with $\overline{E} = (2a_x + 3a_y + 4a_z)\, e^{j(\omega t + 2x + 3y)}$ is incident on a perfectly conducting slab positioned at x £ 0 the $\overline{E}$ of the reflected wave i

(A) $-(2a_x + 3a_y + 4a_z)\, e^{j(\omega t - 2x - 3y)}$ v/m

(B) $(-2a_x - 3a_y + 4a_z)\, e^{j(\omega t - 2x - 3y)}$ v/m

(C) $(2a_x + 3a_y - 4a_z)\, e^{j(\omega t + 2x - +3y)}$ v/m
(D) $-(2a_x + 3a_y + 4a_z)\, e^{j(\omega t + 2x + 3y)}$ v/m

Solution

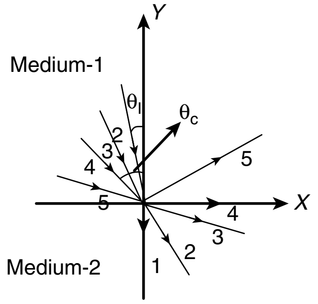
Electric field inside a conductor = 0
$E_t = 0$
$\therefore E_i + E_r = E_t$
$E_i + E_r = 0$
$E_r = -E_i$
$\therefore E_r = -(2a_x + 3a_y + 4a_z)\, e^{j(\omega t - 2x - 3y)}$ v/m

Example 24

A plane EM wave at an oblique incidence at Brewster angle then the reflected wave will be
(A) Purely perpendicular polarized
(B) Purely parallel polarized
(C) Both
(D) Neither of the two

Solution

At Brewster angle, the parallel reflection coefficient = 0
$\therefore \rho_{11} = 0$
The entire parallel polarized wave gets transmitted.
Total internal reflection:



According to Snell's law

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i$$

$\epsilon_1 > \epsilon_2$ (denser to rarer)
As θ_i increases θ_t increases and $\theta_t > \theta_i$
The angle θ_i at which $\theta_t = 90°$ is called critical angle θ_c.
(or)
θ_c is the angle of incidence for which angle of refraction is 90°.

$$\therefore 1 = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_c$$

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\therefore$ If $\theta_i > \theta_c$ then the wave gets total internally reflected.

Example 25

The two medium characteristics with $\epsilon_{r1} = 2, \epsilon_{r2} = 1$
$\mu_1 = \mu_0, \mu_2 = \mu_0$ then the critical angle of incidence for total reflection
(A) 30° (B) 45° (C) 90° (D) 60°

Solution

$$\text{Sin}\,\theta_c = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

$$\sin\theta_c = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$$

Direction for questions 1 to 15: Select the correct alternative from the given choices.

Example 26

Which of the following is true?

(A) $\int \overline{H}.\overline{ds} = \oint\left(J + \frac{\partial D}{\partial t}\right) dl$ (B) $\oint \overline{H}.\overline{ds} = \int\left(J + \frac{\partial D}{\partial t}\right) dl$

(C) $\oint \overline{H}.\overline{dl} = \int\left(J + \frac{\partial D}{\partial t}\right) ds$ (D) $\oint \overline{H}.\overline{dl} = \oint\left(J + \frac{\partial D}{\partial t}\right) ds$

Solution: (C)

Example 27

If a plane wave satisfies the equation $\frac{\partial^2 E_x}{\partial Z^2} = \frac{1}{c^2}\frac{\partial E_x}{\partial t^2}$, the wave propagation in _____
(A) X direction (B) Z direction
(C) Any direction in XZ plane (D) Y direction

Solution: (B)

Example 28

Find a for ferrite at 10GHz, $\varepsilon_r = 9$, $\mu_r = 4$, $\sigma = 10$ ms/m
(A) 10.2 (B) 9.82
(C) 6.68 (D) None of these

Solution

Given
$\varepsilon_r = 9, f = 10 \times 10^9$ Hz
$\mu_r = 4, \sigma = 10$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\,\sqrt{1 - \left(\frac{j\sigma}{\omega\varepsilon}\right)^{\frac{1}{2}}}$$

Example 29

Find the characteristic impedance of the medium whose relative permeability is 1 and $\epsilon_r = 3$
(A) $\eta = 217.66\ \Omega$ (B) $\eta = 218.59\ \Omega$
(C) $\eta = 21.766\ \Omega$ (D) $\eta = 0$

Solution

$\varepsilon_r = 3, \mu_r = 1$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$\eta = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$\eta = 120\pi\sqrt{\frac{1}{3}}$$

$$\eta = 217.66\ \Omega$$

Example 30

Find the skin depth of penetration of a plane wave with a power frequency of 60 Hz and at a microwave frequency 10^{10} Hz given $\sigma = 5.8 \times 10^7$ mole/m

(A) $\delta = 8.53$ m (B) $\delta = 8.53 \times 10^{-3}$ m
(C) $\delta = 8.53 \times 10^{-6}$ m (D) $\delta = 8.53 \times 10^{-9}$ m

Solution

Given $\sigma = 5.8 \times 10^7$

Depth of penetration $\delta = \frac{1}{\alpha}$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$f = 60$ Hz, $\mu_r = 1$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{2\pi\times 60\times 4\pi\times 10^{-7}\times 1\times 5.8\times 10^{7}}}$$

$$\delta = 8.53 \times 10^{-3}\ \text{m}$$

Example 31

Find the velocity of plane wave in a lossless medium having a relative permittivity of 5 and relative permeability of unity.

(A) 134.10×10^8 m/s (B) 136.21×10^8 m/s
(C) 128.26×10^8 m/s (D) 132.10×10^8 m/s

Solution

$$V = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$$

$$V = 134 \times 10^8\ \text{m/s}$$

Example 32

A uniform plane electromagnetic wave is incident normally upon a sheet of dielectric material, which has the following constants. $\epsilon = 4\epsilon_v$, $\mu = \mu_v$, $\sigma = 0$. If the sheet is 2 cm thick and the amplitude of the electric field strength of the incident wave is 100 mV/m, determine the electric field strength of the wave after passing through the sheet

(1) If the frequency is 3,000 MHz
(2) If 30 MHz

(A) $E_t^1 = \frac{8}{9}$ mV/m (B) $E_t^1 = \frac{800}{9}$ mV/m
(C) $E_t^1 = 1$ (D) $E_t^1 = 0$

Solution

$$\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\eta_2 = \sqrt{\frac{\mu_v}{4\,\epsilon_v}} = \frac{120\pi}{2} = 60\pi$$

$$\eta_1 = \sqrt{\frac{\mu_v}{\epsilon_v}} = 120\pi$$

$$\frac{E_t^1}{E_t} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$\frac{E_t^1}{E_i} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} = \frac{4\times 120\times 60\pi}{180\pi\times 180\pi}$$

$$\frac{E_t^1}{E_i} = \frac{8}{9}$$

$$E_t^1 = \frac{8}{9} \times 100 = \frac{800}{9}\ \text{mV/m}$$

Example 33

Given $\overline{E} = E_m \sin(\omega t - \beta x).\overline{a_y}$ in free space, then $\overline{B}$ will be given as

(A) $\frac{E_m\beta}{\omega}\sin(\omega t - \beta z)\overline{a_z}$ (B) $\frac{E_m\beta}{\omega}\cos(\omega t - \beta z)\overline{a_x}$
(C) $\frac{E_m\beta}{\omega}\sin(\omega t - \beta z)\overline{a_y}$ (D) None of these

Solution

As $\nabla \times E = \frac{-\partial B}{\partial t}$

$$\text{(or)} \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m \sin(\omega t - \beta z) & 0 \end{vmatrix} = \frac{\partial \overline{B}}{\partial t}$$

$$\text{(or)}\ \frac{\partial \overline{B}}{\partial t} = \beta E_m \cos(\omega t - \beta z))$$

Integrating we get,

$$\overline{B} = \frac{\beta E_m}{\omega}\sin(\omega t - \beta z)\overline{a_z}$$

Example 34

A plane travelling wave has a peak electric field E_0 of 15 V/m. The medium is lossless with $\mu_r = 1$ and $\varepsilon_r = 12$, find velocity of the wave.

(A) $V = 0.865 \times 10^8$ m/sec (B) $V = 86.5435$ m/sec
(C) $V = 86.5435 \times 10^3$ m/sec (D) None of the above

Solution

Velocity of the wave is given by

$$V = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}}$$

$$V = \frac{1}{\sqrt{(4\pi\times 10^{-7}\times 1)(8.854\times 10^{-12}\times 12)}}$$

$$V = 0.865 \times 10^8\ \text{m/sec}$$

Example 35

A plane travelling wave has a peak electric field E_0 of 15 V/m. The medium is lossless with $\mu_r = 1$ and $\varepsilon_r = 12$, find impedance of the medium

(A) $\eta = 108.83\ \Omega$ (B) $\eta = 106.25\ \Omega$
(C) $\eta = 108.83 \times 10^{-2}\ \Omega$ (D) $\eta = 0\ \Omega$

Solution

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$$

$$\eta = (377)\sqrt{\frac{1}{12}}$$

$$\eta = 108.83\ \Omega$$

Example 36

Maxwell's divergence equation for the magnetic field is given by

(A) $\nabla \times B = 0$ (B) $\nabla . B = 0$
(C) $\nabla \times B = \rho$ (D) $\nabla . B = \rho$

Solution

As the net magnetic flux through any closed surface is always zero.

Example 37

The inconsistency of continuity equation for time varying fields was corrected by Maxwell and the correction applied was

(A) Ampere's Law, $\frac{\partial D}{\partial t}$ (B) Gauss's law, J
(C) Faraday's law, $\frac{\partial B}{\partial t}$ (D) Ampere's law, $\frac{\partial P}{\partial t}$

Solution

Ampere's law, $\frac{\partial D}{\partial t}$

Example 38

Which one of the following sets of equations is independent in Maxwell's equation?

(A) Two curl equations
(B) Two divergence equations
(C) Both the curl and divergence auditions
(D) Two curl equations combined with the continuity equations

Solution: (C)

Example 39

The expression for B, given that in free space $E = 15 \cos(6\pi \times 10^{-8}t - 2\pi z)i_x$ V/m is

(A) $5 \times 10^{-8} \cos(6\pi \times 10^8 t - 2\pi z)i_y$
(B) $90\pi \times 10^8 \sin(6\pi \times 10^8 t - 2\pi z)i_x$
(C) $45 \times 10^8 \sin(6\pi \times 10^8 t - 2\pi z)i_y$
(D) $5 \times 10^{-8} [\cos(6\pi \times 10^8 t - 2\pi z) - \sin 6\pi \times 10^8 t\ 2\pi z)i_z]$

Solution

Given wave is travelling in the $+z$ direction and E is entirely in the x direction. Now, E and H direction gives the direction of travel of power. It is concluded, therefore, that it must be in $+y$ direction

$$i_x \times i_y = i_z$$

Further, $|H| = \frac{|E|}{\eta}$

$$B = \mu_0 H = \frac{\mu_0 E}{\eta}$$

(or) $|B| = \frac{\mu_0 |E|}{\sqrt{\mu_0/\varepsilon_0}} = \sqrt{\mu_0\varepsilon_0}\,|E|$

$$|E| = 1 \times 10^{-8}|E|$$

$$\therefore B = 5 \times 10^{-8} \cos(6\pi \times 10^8 t - 2\pi z)i_y \text{ wb/m}^2$$

Example 40

The depth of penetration of a wave in a lossy dielectric increase with increasing

(A) Conductivity (B) Permeability
(C) Wavelength (D) Permittivity

Solution: (C)

Exercises

Practice Problems I

Direction for questions 1 to 24: Select the correct alternative from the given choices.

1. A varying magnetic flux linking a coil is given by $\phi = \frac{1}{3}\lambda t^3$. If at time $t = 2$ sec the emf induced is 4 v, then the value of λ is
 (A) Zero (B) 1 wb/s^2
 (C) −1 wb/s^2 (D) 4 wb/s^2

2. A parallel plate air-filled capacitor has plate area of 10^{-4} m^2 and a plate separation of 10^{-2}m. It is connected to a 1 v, 3.6 GHz source. Then the magnitude of displacement current is
 (A) 20 mA (B) 200 mA
 (C) 20 A (D) 2 A

3. A material has conductivity of 10^{-4} and a relative permittivity of 4. The frequency at which the conduction and displacement currents are equal

(A) 0.45 MHz (B) 1.35 MHz
(C) 0.9 MHz (D) 1 GHz

4. In free space, $\overline{H} = 0.1 \cos(3 \times 10^6 t - \beta z)\, a_x$ A/m. The expression for $\overline{E}(z, t)$ is
(A) $37.7 \cos(3 \times 10^6 t - \beta z)\, a_z$
(B) $-37.7 \cos(3 \times 10^6\, t - \beta z) a_y$
(C) $37.7 \cos(3 \times 10^6\, t - \beta z) a_y$
(D) $75.4 \cos(3 \times 10^6\, t - \beta z) a_z$

5. The electric field of wave propagating through a loss less medium (μ_0, $4 \in_0$) is $\overline{E} = 10 \cos(9\, \pi \times 10^8\, t - \beta x)\, a_y$. Then, the phase constant β of the wave is
(A) $18\, \pi$ rad/m (B) $6\, \pi$ rad/m
(C) $3\, \pi$ rad/m (D) $\frac{\pi}{3}$ rad/m

6. A wave propagating through a medium [$\varepsilon_r = 8$, $\mu_r = 2$ and $\sigma = 0$] has its electric field given by $\overline{E} = 5 \sin(10^6\, t - \beta z)$ v/m. The wave impedance in ohms is
(A) $120\, \pi$ (B) $30\, \pi$ (C) $60\, \pi$ (D) 180

7. The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity $\in_r$ and relative permeability $\mu_r = 1$ are given by $\overline{E} = E_p\, e^{j(\omega t - 560\pi y)} a_z$ v/m $\overline{H} = 3\, e^{j(\omega t - 560\pi y)}\, a_x$ v/m
Assuming velocity of light in free space $= 3 \times 10^8$ m/s, $\eta_0 = 120\, \pi$, then ε_r and E_p are
(A) 0.6, $120\, \pi$ (B) 0.6, $600\, \pi$
(C) 0.36, $600\, \pi$ (D) 0.36, $120\, \pi$

8. A plane EM wave satisfying the equation $\frac{d^2 E_y}{dz^2} = c^2 \frac{dE_y}{dt^2}$ $\frac{d^2 E_y}{dz^2} = c^2 \frac{dE_y}{dt^2}$, then the wave propagates in the
(A) X direction
(B) Y direction
(C) Z direction
(D) At angle depends on 'c' in the XY plane

9. The intrinsic impedance of Cu of 27 GHz $\mu_0 = 4\, \pi \times 10^{-7}$ H/m
$\in = \frac{10^{-9}}{36\pi}$, $\sigma = 5.8 \times 10^7$ ℧ /m will be
(A) $0.03\, e^{\frac{j\pi}{4}}$ (B) $0.036\, e^{\frac{j\pi}{2}}$
(C) $0.06\, e^{\frac{j\pi}{2}}$ (D) $0.06\, e^{\frac{j\pi}{4}}$

10. If $\overline{E} = (a_y + j a_z)\, e^{jkx - \omega t}$, $\overline{H} = \left(\frac{k}{\omega\mu}\right)(a_z + j a_y) e^{jkx - \omega t}$, then the time averaged poynting vector is
(A) Null vector (B) $\frac{k}{\omega\mu} a_x$
(C) $\frac{2k}{\omega\mu} a_x$ (D) $\frac{k}{2\omega\mu} a_x$

11. When a plane wave travelling in free space is incident normally on a medium having $\varepsilon_r = 9$, then the fraction of power transmitted into the medium is given by
(A) $\frac{1}{4}$ (B) $\frac{1}{9}$ (C) $\frac{8}{9}$ (D) $\frac{3}{4}$

12. The instantaneous electric field of a plane wave propagating in z direction is
$\overline{E}(t) = [a_x E_1 \cos \omega t - a_y E_2 \sin \omega t]\, e^{-jkz}$
Then, the wave is
(A) Linearly polarized
(B) Elliptically polarized
(C) Right circularly polarized
(D) Left circularly polarized

13. A plane EM wave with magnetic field intensity $\overline{H}_i = \cos(10^8 t - \beta z)\, a_x$ mA/m travels in air for $z \le 0$ and is incident normally on a loss less non-magnetic medium of relative permittivity 4 which occupy the region $z \ge 0$. Which one of the following is the expression for the reflected electric field in (mv/m) $\eta_0 = 120\, \pi\, \Omega$
(A) $\overline{E}_r = -40\, \pi \cos(10^8\, t - 3z)\, a_x$
(B) $\overline{E}_r = -40\, \pi \cos(10^8\, t + \frac{z}{3})\, a_x$
(C) $\overline{E}_r = -80\, \pi \cos(10^8\, t - 3z)\, a_x$
(D) $\overline{E}_r = 80\, \pi \cos(10^8\, t + \frac{z}{3})\, a_x$

14. A plane wave from free space with $\overline{E} = 100\, e^{j(0.866y + 0.5z)}\, a_x$ v/m is incident on a dielectric medium $\sigma = 0$ $\in = 4\, \varepsilon_0$, $\mu = \mu_0$ and occupying $z \ge 0$ then Brewster angle
(A) 45° (B) 60°
(C) 27.57° (D) 63.43°

15. A plane wave of wavelength λ is travelling in a direction making an angle 45° with positive x axis and 90° with positive y axis. Then, the $\overline{E}$ field of plane wave can be represented as (E_0 - constant)
(A) $a_y E_0\, e^{j\left(\omega t - \frac{\sqrt{2}\pi}{\lambda} x - \frac{\sqrt{2}\pi}{\lambda} z\right)}$
(B) $a_y E_0\, e^{j\left(\omega t - \frac{\sqrt{2}\pi}{\lambda} x + \frac{\sqrt{2}\pi}{\lambda} z\right)}$
(C) $a_y E_0\, e^{j\left(\omega t + \frac{\sqrt{2}\pi}{\lambda} x - \frac{\sqrt{2}\pi}{\lambda} z\right)}$
(D) $a_y E_0\, e^{j\left(\omega t + \frac{\sqrt{2}\pi}{\lambda} x + \frac{\sqrt{2}\pi}{\lambda} z\right)}$

16. In a certain region, the current density is expressed as $J_c = -10^5\, \nabla V$ A/m² where $V = 10e^{-x}$ siny volts. Find $\sigma = ?$
(A) 10^5 ℧ /m (B) 10^{-5} ℧ /m
(C) -10^5 ℧ /m (D) -10^{-5} ℧ /m

Direction for questions 17 and 18:
The magnetic field intensity of a uniform plane wave in air is 20A/m in the $\hat{a}_y$ direction. The wave is propagating in the $\hat{a}_z$ direction at an angular frequency of 2×10^9 rad/s.

17. Find wave length.
(A) 0.942 m (B) 1.8 m
(C) 1.942 m (D) 0.45 m

18. What is the frequency
(A) 318 MHZ (B) 3.18 MHZ
(C) 31.8 MHZ (D) 0.318 MHZ

19. The electric field component of a wave in free space is given by $E = 10$ $\cos(10^7 t + kz)a_y$ V/m. It implies that
(A) The wave propagates along a_y
(B) The wave length $\lambda = 94.2$ m.
(C) The wave number $K = 0.33$ rad/m
(D) The wave attenuates as it travels.

20. A current sheet of $\overline{K} = 6.5\ \hat{a}_z$ A/m at $x = 0$ separates region1 $x < 0$ where $\overline{H_1} = 10\ \hat{a}_y$ A/m and region2 is characterized by x > 0 if $\mu_{r1} = 3$ and $\mu_{r2} = 4$ Find $\overline{H_2}$
(A) 16.5 $\hat{a}_y$ A/m (B) 16.5 $(-\hat{a}_y)$ A/m
(C) 3.5 $\hat{a}_y$ A/m (D) 3.5 $(-\hat{a}_y)$ A/m

21. A plane slab of dielectric having dielectric constant 5, placed normal to a uniform field with a flux density of 2 C/m^2, is uniformly polarized. The polarization of the slab is
(A) 0.4 C/m^2 (B) 1.6 C/m^2
(C) 2.0 C/m^2 (D) 6.4 C/m^2

22. A plane wave of 10 GHz is incident normally on a dielectric plate of 3 mm thickness. If the phase shift on transmission through the sheet is 90°, then the dielectric constant is
(A) 5.0 (B) 3.25
(C) 4.5 (D) 6.25

23. Find the displacement current density with a parallel plate capacitor where $\varepsilon_0 = 10\varepsilon$, where $A = 0.01$ m^2, $d =$ 5 cm, and $V = 100 \sin 314t$ V
(A) $I_d = 5.25 \times 10^{-3} \cos(314t)$
(B) $I_d = \cos(314t)$
(C) $I_d = 5.55 \cos(314t)$
(D) $I_d = 5.55 \times 10^6 \cos(314t)$

24. Find the skin depth at a frequency of 2MHz is allowed when $\sigma = 38.2$ s/m and $\mu_r = 1$
(A) $\delta = 5.758 \times 10^{-6}$ m
(B) $\delta = 5.758 \times 10^{-5}$ m
(C) $\delta = 0$
(D) $\delta = 5.758$ m

Practice Problems 2

Direction for questions 1 to 21: Select the correct alternative from the given choices.

1. The E field of a uniform plane wave propagating in a dielectric medium is given by $\overline{E}(t, z) = 2\cos(10^8 t - \frac{z}{\sqrt{2}})\, a_x - 2\sin(10^8 t - \frac{z}{\sqrt{2}})\, a_y$.
Then, the dielectric constant of the medium is
(A) 1.25 (B) 2.25 (C) 3.25 (D) 1.717

2. The depth of penetration of EM wave in a medium having conductivity σ at a frequency of 2 MHz is 5 cm. Then, the depth of penetration at a frequency of 32 MHz is
(A) 0.25 cm (B) 4 cm
(C) 2 cm (D) 1.25 cm

3. In a good conductor, the intrinsic impedance is
(A) Real
(B) Imaginary
(C) Have both real and imaginary with phase angle 45°
(D) Null

4. A TEM wave impinges obliquely on a dielectric boundary with $\in_{\gamma 1} = 2$, $\in_{\gamma 2} = 1$. The angle of incident of total reflection is
(A) 30° (B) 60° (C) 45° (D) 90°

5. For an elliptically polarized wave incident on the surface of the interface of a dielectric at the Brewster angle, the reflected wave will be
(A) Elliptically polarized
(B) Linearly polarized
(C) Right circularly polarized
(D) Left circular polarized

6. If $\overline{H} = 0.1 \sin(10^6 \pi t + \beta y)\, a_x$ A/m for a plane wave propagating in free space. Then the time average poynting vector is
(A) $0.6\pi \sin^2(\beta y)\, a_y$ w/m^2
(B) $-0.6\pi\, a_y$ w/m^2
(C) $1.2\pi\, a_x$ /m^2
(D) $-1.2\pi\, a_x$ w/m^2

7. Which one of the following is not correct for a plane wave with
$\overline{H} = 0.5\, e^{-0.1x} \cos(10^8 t - 3x)\, a_y$ A/m
(A) Wave frequency is 10^8 r. p. s
(B) Phase constant is 3 rad/m
(C) Wave travels along +ve x direction
(D) The wave is polarized in the z direction

8. If the $\overline{E}$ field of a plane polarized EM wave travelling in the z - direction is $\overline{E} = a_x E_x + a_y E_y$, then its Hx field is
(A) $\frac{-E_y}{\eta_0} a_x + \frac{E_x}{\eta_0} a_y$ (B) $\frac{E_y}{\eta_0} a_x + \frac{E_x}{\eta_0} a_y$
(C) $\frac{E_x}{\eta_0} a_x + \frac{E_y}{\eta_0} a_y$ (D) $\frac{E_x}{\eta_0} a_x - \frac{E_y}{\eta_0} a_y$

9. Which one of the following statement is wrong about the EM wave
(A) Electric field is perpendicular to the direction of propagation
(B) Magnetic field is perpendicular to the direction of propagation
(C) Transverse components only are absent
(D) Poynting vector is along the direction of propagation

10. According to poynting theorem, the vector product $\bar{E} \times \bar{H}$ is a measure of which of the following?
(A) Stored energy density of electric field
(B) Stored energy density of magnetic field
(C) Power dissipated per unit volume
(D) Rate of energy flow per unit area

11. An EM wave having frequency of gets attenuated by a factor of e^{-2} after propagation a distance d in a good conductor. If the signal frequency is now reduced to $0.25 f_o$ after travelling the same distance d in the same conductor. The signal attenuated by a factor of
(A) e^{-4} (B) $e^{-2\sqrt{2}}$ (C) $e^{-\sqrt{2}}$ (D) e^{-1}

12. The electric field of uniform plane wave in free space, along the positive z direction, is given by $\bar{E} = 10(a_x - j\,a_y)\,e^{-j\,6.28z}$
The frequency and polarization of the wave is
(A) 3 GHz, left circular
(B) 0.3 GHz, right circular
(C) 3 GHz, right circular
(D) 0.3 GHz, left circular

13. A plane wave propagating in air with $\bar{E} = (8\,a_x + 6\,a_y - 3\,a_z)\,e^{j(\omega t+3x-4y)}$ v/m is incident on a perfectly conducting slab positioned at $x \le 0$. The $\bar{E}$ of the reflected wave is
(A) $(-8a_x - 6a_y - a_z)\,e^{j(\omega t+3x-4y)}$ V/m
(B) $(-8a_x + 6a_y - 0.75\,a_z)\,e^{j(\omega t+3x-4y)}$ V/m
(C) $(\frac{8a_x}{3} + \frac{3}{2}a_y - 3\,a_z)\,e^{j(\omega t-3x-4y)}$ V/m
(D) $(-8a_x - 6a_y + 3a_z)\,e^{j(\omega t-3x-4y)}$ V/m

14. The electric field component of time harmonic plane EM wave travelling in a non-magnetic loss less dielectric medium has an amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time average power density vector (in w/m^2) is
(A) $\frac{1}{30\pi}$ (B) $\frac{1}{60\pi}$
(C) $\frac{1}{120\pi}$ (D) $\frac{1}{240\pi}$

15. If electric field is given by
$\bar{E} = 2\cos(\omega t + \beta z)\,a_x$ V/m then the wave travels in
(A) –ve y direction (B) +ve y direction
(C) –ve z direction (D) –ve x direction

16. The unit of $\nabla \times H$ is
(A) Ampere (B) Ampere/meter
(C) Ampere/meter^2 (D) Ampere - meter

17. The time-varying field is given by
$\tilde{E}(0, t) = A\cos\omega t\,\hat{a}_x - B\sin\omega t\,\hat{a}_y$ in which A and B are positive real constants, the wave is said to be
(A) Linearly polarized
(B) Circularly polarized
(C) Elliptically Polarized
(D) Not polarized at all.

18. In a uniform plane wave, E and H are related by
(A) $\frac{E}{H} = 1$ (B) $\frac{E}{H} = \sqrt{\frac{\in}{\mu}}$
(C) $\frac{E}{H} = \sqrt{\frac{\mu}{\in}}$ (D) $\frac{E}{H} = \frac{\mu}{\in}$

19. Maxwell's equation in free space is
(A) $\nabla.B = 0$ (B) $\nabla.B = \rho$
(C) $\nabla.B = J$ (D) $\nabla.B = \sigma J$

20. Identify the wave equation for free space conditions and charge free
(A) $\nabla^2 E = \mu\in\frac{\partial^2 E}{\partial t^2}$ (B) $\nabla^2 E = \mu\in\frac{\partial^2 H}{\partial t^2}$
(C) $\nabla^2 E = \nabla^2 H$ (D) $\nabla^2 H = \mu\in\frac{\partial^2 E}{\partial t^2}$

21. In a lossless medium the intrinsic impedance $\eta = 60\pi$ and $\mu_r = 1$. What is the value of the dielectric constant $\in_r$?
(A) 2 (B) 1
(C) 4 (D) 8

Previous Years' Questions

1. If $\vec{E} = (\hat{a}_x + j\hat{a}_y)\,e^{-jkz-j\omega t}$ v/m and
$\vec{H} = \left(\frac{K}{\omega\mu}\right)(\hat{a}_y + j\hat{a}_x)\,e^{jkz-j\omega t}$ A/m, time average pointing vector is **[2004]**
(A) Null vector (B) $\left(\frac{K}{\omega\mu}\right)\hat{a}_z$
(C) $\left(\frac{2K}{\omega\mu}\right)\hat{a}_z$ (D) $\left(\frac{K}{2\omega\mu}\right)\hat{a}_z$

2. Refractive index of glass is 1.5. Find the wave length of a beam of light with a frequency of 10^{14} Hz in glass. Assume velocity of light is 3×10^8 m/s in vacuum. **[2005]**
(A) 3 μm (B) 3 mm
(C) 2 μm (D) 1 μm

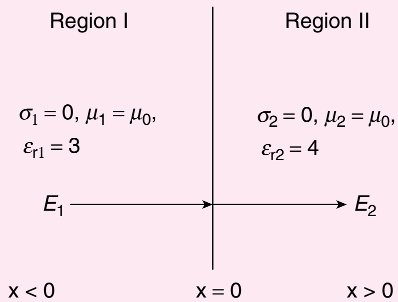
3. The electric field of an electromagnetic wave propagating in the positive Z – direction is given by

$$E = \hat{a}_x \sin(\omega t - \beta z) + \hat{a}_y \sin\left(\omega t - \beta z + \frac{\pi}{2}\right) \text{V/m.}$$

The wave is **[2006]**

(A) linearly polarized in the z direction
(B) elliptically polarized
(C) left-hand circularly polarized
(D) right-hand circularly polarized

4. A medium is divided into regions I and II about $x = 0$ plane, as shown in the figure below. An electromagnetic wave with electric field, $E_1 = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ V/m is incident normally on the interface from region - I. The electric field E_2 in region – II at the interface is **[2006]**



(A) $E_2 = E_1$
(B) $4\hat{a}_x + 0.75\hat{a}_y - 1.25\hat{a}_z$
(C) $3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$
(D) $-3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

5. When a plane wave travelling in free space is incident normally on a medium having $\varepsilon_r = 4.0$, the fraction of power transmitted into the medium is given by **[2006]**

(A) $\frac{8}{9}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{5}{6}$

6. If C is a closed curve enclosing a surface S, then the magnetic field intensity $\vec{H}$, the current density $\vec{J}$ and the electric flux density $\vec{D}$ are related by **[2007]**

(A) $\iint_S \vec{H} \cdot d\vec{s} = \oint_c \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{l}$

(B) $\int_c \vec{H} \cdot d\vec{l} = \oiint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) . d\vec{s}$

(C) $\oiint_s \vec{H} \cdot d\vec{s} = \int_c \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{l}$

(D) $\oint_c \vec{H} \cdot d\vec{l} = \iint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) . d\vec{s}$

7. A plane wave of wavelength l is travelling in a direction making an angle 30° with positive x – axis and 90° with positive y – axis. The $\vec{E}$ field of the plane wave can be represented as (E_0 is constant) **[2007]**

(A) $\vec{E} = \hat{y} E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$

(B) $\vec{E} = \hat{y} E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x - \frac{\sqrt{3}\pi}{\lambda}z\right)}$

(C) $\vec{E} = \hat{y} E_0 e^{j\left(\omega t + \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

(D) $\vec{E} = \hat{y} E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

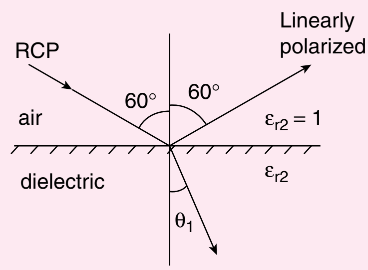
8. The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by

$$\vec{H} = \hat{x}\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{Y}\frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right) \text{A / m,}$$

the time average power flow density in Watts is: **[2007]**

(A) $\frac{\eta_0}{100}$ (B) $\frac{100}{\eta_0}$
(C) $50\eta_0^2$ (D) $\frac{50}{\eta_0}$

9. A right circularly polarized (RCP) plane wave is incident at an angle of 60° to the normal, on an air dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant ε_{r2} is: **[2007]**

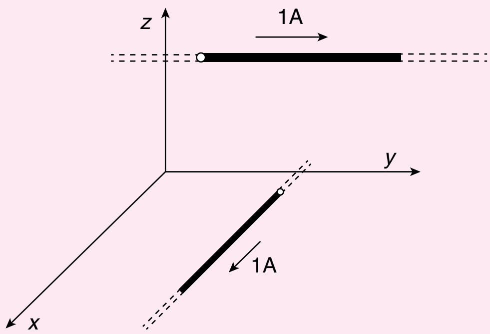


(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

10. A uniform plane wave in the free space is normally incident on an infinitely thick dielectric slab (dielectric constant $\varepsilon_r = 9$). The magnitude of the reflection coefficient is **[2008]**

(A) 0 (B) 0.3
(C) 0.5 (D) 0.8

11. Two infinitely long wires carrying current are as shown in the figure below. One wire is in the y–z plane and parallel to the y-axis. The other wire is in the x–y plane and parallel to the x-axis. Which components of the resulting magnetic field are non-zero at the origin? **[2009]**



(A) x, y, z components (B) x, y components
(C) y, z components (D) x, z components

12. The electric field component of a time harmonic plane EM wave traveling in a nonmagnetic lossless dielectric medium has a amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time-average power density vector (in W/m^2) is **[2010]**

(A) $\frac{1}{30\pi}$ (B) $\frac{1}{60\pi}$

(C) $\frac{1}{120\pi}$ (D) $\frac{1}{240\pi}$

13. A plane wave having the electric field component $\vec{E}_i = 24\cos\left(3\times10^8 t - \beta y\right)\hat{a}_z$ V/m and traveling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\varepsilon = 9\varepsilon_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by **[2010]**

(A) $\frac{1}{10\pi}\cos\left(3\times10^8 t + y\right)\hat{a}_x$ A/m

(B) $\frac{1}{20\pi}\cos\left(3\times10^8 t + y\right)\hat{a}_x$ A/m

(C) $-\frac{1}{20\pi}\cos\left(3\times10^8 t + y\right)\hat{a}_x$ A/m

(D) $-\frac{1}{10\pi}\cos\left(3\times10^8 t + y\right)\hat{a}_x$ A/m

14. Consider the following statements regarding the complex Poynting vector $\bar{P}$ for the power radiated by a point source in an infinite homogeneous and lossless medium. Re($\bar{P}$) denotes the real part of $\bar{P}$, S denotes a spherical surface whose centre is at the point source, and $\hat{n}$ denotes the unit surface normal on S. Which of the following statements is **TRUE**? **[2011]**

(A) Re($\bar{P}$) remains constant at any radial distance from the source.

(B) Re($\bar{P}$) increases with increasing radial distance from the source

(C) $\oiint_S \text{Re}(\bar{P}) . \hat{n}\, dS$ remains constant at any radial distance from the source.

(D) $\oiint_S \text{Re}(\bar{P}) . \hat{n}\, dS$ decreases with increasing radial distance from the source

15. The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity ε_r and relative permeability $\mu_r = 1$ are given by

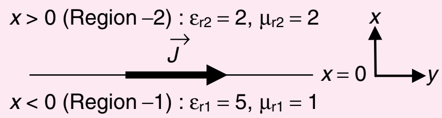
$\bar{E} = E_p e^{j(\omega t - 280\pi y)}\hat{a}_z$ V/m

$\bar{H} = 3e^{j(\omega t - 280\pi y)}\hat{a}_z$ A/m

Assuming the speed of light in free space to be 3×10^8 m/s, the intrinsic impedance of free space to be 120π, the relative permittivity ε_r of the medium and the electric field amplitude E_p are **[2011]**

(A) $\varepsilon_r = 3, E_p = 120\pi$ (B) $\varepsilon_r = 3, E_p = 360\pi$
(C) $\varepsilon_r = 3, E_p = 360\pi$ (D) $\varepsilon_r = 9, E_p = 120\pi$

16. A current sheet $\vec{J} = 10\,\hat{u}_y$ A/m lies on the dielectric interface $x = 0$ between two dielectric media with $\varepsilon_{r1} = 5, \mu_{r1} = 1$ in Region–1 ($x < 0$) and $\varepsilon_{r2} = 2, \mu_{r2} = 2$ in Region–2 ($x > 0$). If the magnetic field in Region–1 at $x = 0^-$ is $\vec{H}_1 = 3\,\hat{u}_x + 30\,\hat{u}_y$ A/m, the magnetic field in Region–2 at $x = 0^+$ is **[2011]**



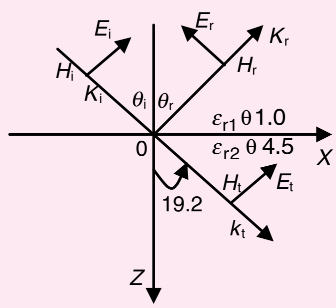
(A) $\bar{H}_2 = 1.5\,\hat{u}_x + 30\,\hat{u}_y - 10\,\hat{u}_z$ A/m
(B) $\bar{H}_2 = 3\,\hat{u}_x + 30\,\hat{u}_y - 10\,\hat{u}_z$ A/m
(C) $\bar{H}_2 = 1.5\,\hat{u}_x + 40\,\hat{u}_y$ A/m
(D) $\bar{H}_2 = 3\,\hat{u}_x + 30\,\hat{u}_y + 10\,\hat{u}_z$ A/m

17. The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction, is given by $\vec{E} = 10\left(\hat{a}_y + j\hat{a}_z\right)e^{-j25x}$. The frequency and polarization of the wave, respectively, are **[2012]**

(A) 1.2GHz and left circular
(B) 4Hz and left circular
(C) 1.2GHz and right circular
(D) 4Hz and right circular

Direction for questions 18 and 19:

A monochromatic plane wave of wavelength $\lambda = 600\mu$m is propagating in the direction as shown in the figure below $\vec{E}_i, \vec{E}_r$ and $\vec{E}_t$ denote incident, reflected, and transmitted electric field vectors associated with wave.



18. The angle of incidence θ_t and the expression for E_t are **[2013]**

(A) 60^0 and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times 10^4 (x+z)}{3\sqrt{2}}}$ V/m

(B) 45^0 and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times 10^4 z}{3}}$ V/m

(C) 45^0 and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times 10^4 (x+z)}{3\sqrt{2}}}$ V/m

(D) 60^0 and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times 10^4}{3}}$ V/m

19. The expression for $\overline{E}_r$ is **[2013]**

(A) $0.23\frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_x)e^{-j\frac{\pi\times 10^4}{3}}$ V/m

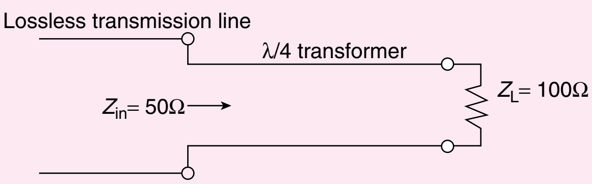
(B) $-\frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_x)e^{j\frac{\pi\times 10^4 z}{3}}$ V/m

(C) $0.44\frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_x)e^{-j\frac{\pi\times 10^4 (x-z)}{3\sqrt{2}}}$ V/m

(D) $\frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times 10^4 (x+z)}{3}}$ V/m

20. In spherical coordinates, let $\hat{a}_\theta, \hat{a}_\varphi$ denote unit vectors along the θ, Φ directions. $E = \frac{100}{r}\sin\theta\cos(\omega t - \beta r)\hat{a}_\theta$ V/m and $H = \frac{0.265}{r}\sin\theta\cos(\omega t - \beta r)\hat{a}_\varphi$ A/m represent the electric and magnetic field components of the EM wave at large distances r from a dipole antenna, in free space. The average power (W) crossing the hemispherical shell located at r = 1 km, $0 \le \theta \le \pi/2$ is __________ **[2014]**

21. To maximize power transfer, a lossless transmission line is to be matched to resistive load impedance via a $\lambda/4$ transformer as shown.

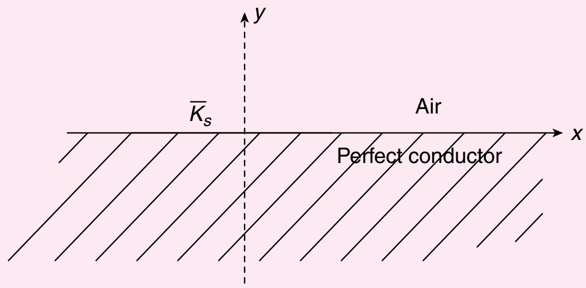


The characteristic impedance (in Ω) of the $\lambda/4$ transformer is _____________ **[2014]**

22. If the electric field of a plane wave is
$\vec{E}(z,t) = \hat{x}\,3x\cos(\omega t - kz + 30^\circ)$ the polarization $-\hat{y}\,4\sin(\omega t - kz + 45^\circ)$ (m V/m),
state of the plane wave is **[2014]**

(A) left elliptical (B) left circular
(C) right elliptical (D) right circular

23. A region shown below contains perfect conducting half-space and air. The surface current $\vec{K}_s$ on the surface of the perfect conductor is $\vec{K}_s = \hat{x}2$ amperes per meter. The tangential $\vec{H}$ field in the air just above the perfect conductor is **[2014]**



(A) $(\hat{x} + \hat{z})\,2$ amperes per meter

(B) $\hat{x}\,2$ amperes per meter

(C) $-\hat{z}\,2$ amperes per meter

(D) $\hat{z}\,2$ amperes per meter

24. Assume that a plane wave in air with an electric field $\vec{E} = 10\cos(\omega t - 3x - \sqrt{3}z)\,\hat{a}_Y$, V/m is incident on a non-magnetic dielectric slab of relative permittivity 3 which covers the region $z > 0$. The angle of transmission in the dielectric slab is _____ degrees. **[2014]**

25. For an antenna radiating in free space, the electric field at a distance of 1 km is found to be 12 mV/m. Given that intrinsic impedance of the free space is 120π Ω, the magnitude of average power density due to this antenna at a distance of 2 km from the antenna (in nW/m^2) is ______. **[2014]**

26. In the electric field component of a plane wave travelling in a lossless dielectric medium is given by $\vec{E}(z, t) = \hat{a}_y\, 2\cos\left(10^8 t - \frac{z}{\sqrt{2}}\right)$ V/m. The wavelength (in m) for the wave is ________. **[2015]**

27. The electric field of a uniform plane electromagnetic wave is

$$\vec{E} = \left(\vec{a}_x + j4\,\vec{a}_y\right)\exp\,[j(2\pi\times 10^7\, t - 0.2z)]$$

The polarization of the wave is **[2015]**

(A) right-handed circular
(B) right-handed elliptical
(C) left-handed circular
(D) left-handed elliptical

28. The electric field of a uniform plane wave travelling along the negative z direction is given by the following equation:

$$\overline{E}_W^i = (\hat{a}_X + j\hat{a}_y) E_0 e^{jkz}.$$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation:

$$E_a = (\hat{a}_X + 2\hat{a}_y) E_1 \frac{1}{r} e^{-jkr}$$

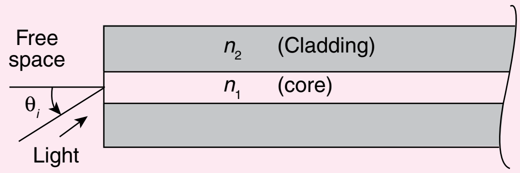
The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively, **[2016]**

(A) Linear, Circular (Clockwise), – 5dB
(B) Circular (clockwise), Linear, –5dB
(C) Circular (clockwise), Linear, –3dB
(D) Circular (anti clockwise), Linear, –3dB

29. Let the electric field vector of a plane electromagnetic wave propagating in a homogeneous medium be expressed as $E = \hat{X} E_x e^{-j(\omega t - \beta z)}$ where the propagation constant β is a function of the angular frequency ω. Assume that $\beta(\omega)$ and E_x are known and are real. From the information available, which one of the following CANNOT be determined? **[2016]**

(A) The type of polarization of the wave.
(B) The group velocity of the wave.
(C) The phase velocity of the wave.
(D) The power flux through the $z = 0$ plane.

30. Light from free space is incident at an angle θ_i to the normal of the facet of a step index large core optical fibre. The core and cladding refractive indices are $n_1 = 1.5$ and $n_2 = 1.4$ respectively. **[2016]**



The maximum value of θ_I (in degrees) for which the incident light will be guided in the core of the fiber is __________.

31. If a right-handed circularly polarized wave is incident normally on a plane perfect conductor, then the reflected wave will be **[2016]**

(A) right-handed circularly polarized.
(B) left-handed circularly polarized.
(C) elliptically polarized with a tilt angle of 45°.
(D) horizontally polarized.

32. Faraday's law of electromagnetic induction is mathematically described by which one of the following equations? **[2016]**

(A) $\nabla \bullet \vec{B} = 0$
(B) $\nabla \bullet \vec{D} = \rho_v$
(C) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
(D) $\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** C	**3.** C	**4.** B	**5.** B	**6.** C	**7.** C	**8.** C	**9.** D	**10.** A
11. D	**12.** B	**13.** B	**14.** D	**15.** A	**16.** A	**17.** A	**18.** A	**19.** A	**20.** A
21. B	**22.** A	**23.** A	**24.** B						

Practice Problems 2

1. B	**2.** D	**3.** C	**4.** C	**5.** B	**6.** B	**7.** D	**8.** A	**9.** C	**10.** D
11. D	**12.** B	**13.** D	**14.** C	**15.** C	**16.** C	**17.** C	**18.** C	**19.** A	**20.** A
21. C									

Previous Years' Questions

1. A	**2.** C	**3.** C	**4.** C	**5.** A	**6.** D	**7.** A	**8.** D	**9.** D	**10.** C
11. D	**12.** C	**13.** A	**14.** D	**15.** D	**16.** A	**17.** A	**18.** C	**19.** A	
20. 55.4 to 55.6		**21.** 70 to 72		**22.** A	**23.** D	**24.** 30°	**25.** 47 to 48		
26. 8.85 to 8.92		**27.** D		**28.** C	**29.** D	**30.** 32 : 33		**31.** B	**32.** C