16. Mensuration

Exercise 16.1

1. Question

Find the total surface area of the cuboid with l =4 m, b =3 m, and h = 1.5 m.

Answer

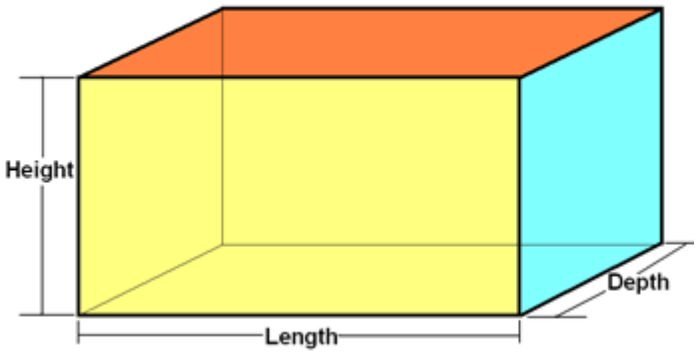
Given: Length (l) = 4m

Breadth (b) = 3m

Height (h) =1.5m

To Find: Total Surface Area of a Cuboid

As we know, the total surface area includes area of all 6 faces of a cuboid



So, Total Surface Area of a Cuboid = **2 {(l × b) + (b × h) + (h × l)}**

= 2 (4 × 3 + 3 × 1.5 + 1.5 × 4)

= 2 (12 + 4.5 + 6)

= 2 (22.5)

Total Surface Area of a Cuboid = 45 m^2

2. Question

Find the area of four walls of a room whose length 3.5 m, breadth 2.5m, and height 3 m.

Answer

Given: Length (l) = 3.5m

Breadth (b) = 2.5m

Height (h) = 3m

As we know, the room is in cuboid shape.

So, Area of Four walls = Lateral Surface Area of Cuboid = **2h (l + b)**

= 2 × 3 (3.5 + 2.5)

= 6 (6)

= $36m^2$

3. Question

The dimensions of a room are l =8 m, b =5 m, h =4 m. Find the cost of distempering its four walls at the rate of ₹ 40/ m^2.

Answer

Given: Length (l) = 8m

Breadth(b) = 5m

Height (h) = 4m

Firstly, we find the Lateral Surface Area of Cuboid i.e.

Area of Four Walls = **2h (l + b)**

= 2 × 4 (8+5)

= 8 (13)

= 104 m^2

Cost of distempering of 1 m^2 area = ₹ 40

So, Cost of distempering of 104 m^2 = ₹ 40 × 104 = ₹ 4160

4. Question

A room is 4.8 m long, 3.6 m broad and 2 m high. Find the cost of laying tiles on its floor and its four walls at the rate of ₹ 100/ m^2.

Answer

Given: Length (l) = 4.8m

Breadth (b) = 3.6m

Height (h) = 2m

Total Surface Area of the room = **2 {(l × b) + (b × h) + (h × l)}**

But here we should **not** consider the ceiling of the room.

So, the required area of the room = **lb + 2 (lh + bh)**

= (4.8 × 3.6) + 2(4.8 × 2 + 3.6 × 2)

= 17.28 + 2 (9.6 + 7.2)

= 17.28 + 33.6

= 50.88 m^2

Cost of laying tiles per square meter = ₹ 100

Cost of laying tiles for 50.88 m^2= ₹100 × 50.88

= ₹5088

5. Question

A closed box is 40 cm long, 50 cm wide and 60 cm deep. Find the area of the foil needed for covering it.

Answer

Given: Length (l) = 40m

Breadth (b) = 50m

Height (h) = 60m

Total Surface Area of a cuboid = **2{(l × b) + (b × h) + (h × l)}**

= 2 (40 × 50 + 50 × 60 + 60 × 40)

= 2 (2000 + 3000 +2400)

= 2 (7400) = 14800cm^2

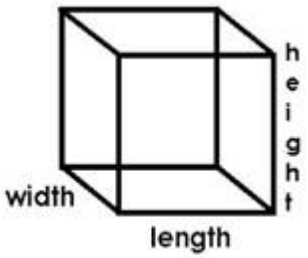
6. Question

The total surface area of a cube is 384 cm^2. Calculate the side of the cube.

Answer

Given the Total surface area of a cube = 384 cm^2

As we know, the Total surface area of a cube = **6 (side of the cube)^2** = 6a^2



So, according to the question:

Total Surface Area of a Cube = 384 = $6a^2$

$\Rightarrow a2 = \frac{384}{6} = 64$

$\Rightarrow a2 = 64$

$\Rightarrow a = \sqrt{64}$

= ±8

a = 8cm (side can't be negative)

7. Question

The L.S.A of a cube is 64 cm^2. Calculate the side of the cube.

Answer

Given: Lateral Surface Area of a Cube = $64cm^2$

Lateral Surface Area of a Cube = $\mathbf{4a^2}$

So, According to the question

Lateral Surface Area of a Cube = 64 = $4a^2$

$\Rightarrow 4a^2 = 64$

$\Rightarrow a^2 = \frac{64}{4} = 16$

$\Rightarrow a = \sqrt{16} = \pm 4 = 4cm$ (side can't be negative)

8. Question

Find the cost of whitewashing the four walls of a cubical room of side 4 m at the rate of ₹ 20/ m^2.

Answer

Given Side of a cubical room = 4m

So, Lateral Surface Area of a Cube = **4(side of a cube)^2**

= 4 $(4)^2$

= 64m^2

Cost of white washing per square meter = ₹20

Cost of white washing for 64 m^2 = ₹20 × 64 = ₹1280

Cost of white washing the four walls of a cubical room is ₹1280

9. Question

A cubical box has edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has a smaller total surface area?

(ii) If each edge of the cube is doubled, how many times will its T.S.A increase?

Answer

Given: Edge of a cubical box = 10cm

Dimensions of cuboidal box:

Length(l) = 12.5cm

Breadth(b) = 10cm

Height(h) = 8cm

(i) Total Surface Area of a cube = 6 $(\text{side of cube})^2$

= $6(10)^2$

= 6 × 10 × 10

= 600 cm^2

Total Surface Area of a Cuboidal box = 2{(l × b) + (b × h) + (h × l)}

= 2 (12.5 × 10 + 10 × 8 + 8 × 12.5)

= 2 (125 + 80 + 100)

= 2 (305)

= 610 cm^2

(ii) Given: If each edge of a cube is doubled then a'= 2a = 2 × 10 = 20cm

New Total Surface Area of a cube = $6(a')^2$

$= 6(20)^2$

$=2400\ cm^2$

So, new total surface area is 4 times the old total surface area.

Answer :If edge is doubled than area is increased by 4 times

Exercise 16.2

1. Question

Find the total surface area and volume of a cube whose length is 12 cm.

Answer

Given: Length of a cube = 12cm

Total Surface Area of a cube = $6(\text{side of cube})^2$

$= 6(12)^2$

$= 6(144)$

$= 864\ cm^2$

Volume of a cube = $(\text{side of cube})^3$

$= (12)^3$

$= 12 \times 12 \times 12$

$= 1728\ cm^3$

2. Question

Find the volume of a cube whose surface area is 486 cm^2.

Answer

Given: Surface Area of a Cube = 486 cm^2

So, firstly we find the side of a cube

Total Surface Area of a Cube = $486 = 6a^2$

$$\Rightarrow a2 = \frac{486}{6} = 81$$

$\Rightarrow a^2 = 81$

$\Rightarrow$ a = $\sqrt{81}$ = ± 9 = 9cm (side can't be negative)

Therefore, Side of a cube = 9 cm

Now, Volume of a Cube = $a^3 = (9)^3 = 9 \times 9 \times 9 = 729\ cm^3$

3. Question

A tank, which is cuboidal in shape, has volume 6.4 m^3. The length and breadth of the base are 2 m and 1.6 m respectively. Find the depth of the tank.

Answer

Given: Volume of a cuboidal tank = 6.4 m^3

Length (l) = 2 m

Breadth (b) = 1.6 m

Depth = ?

Volume of a cuboidal tank = lbh

$\Rightarrow$ 6.4 = 2 × 1.6 × h

$\Rightarrow$ 6.4 = 3.2 × h

$\Rightarrow h = \frac{6.4}{3.2}$

= 2 m

Therefore, the depth of a tank is 2 m

4. Question

How many m^3 of soil has to be excavated from a rectangular well 28 m deep and whose base dimensions are 10 m and 8 m. Also, find the cost of plastering its vertical walls at the rate of ₹ 15/m^2.

Answer

Given: Depth of a well = 28 m

Length = 10 m

Width = 8 m

Volume of a well = l × b × d

= 10 × 8 × 28

= 80 × 28

= 2240 m^3

2240 m^3 of soil has to be excavated from a rectangular well.

So, to calculate the cost of plastering.

Firstly we have to find the lateral surface of a rectangular well

Lateral Surface Area = 2h (l+b)

= 2 × 28 (10+8)

= 56 × 18

= 1008 m^2

Cost of plastering per square metre = ₹ 15

Cost of plastering for 1008 m^2 = ₹ 15 × 1008 = ₹15120

So, the cost of plastering the vertical walls of a rectangular well is ₹ 15120

5. Question

A solid cubical box of fine wood costs ₹ 256 at the rate ₹ 500/m^3. Find its volume and length of each side.

Answer

Let the side of a cubical box = 'a' m

Therefore, Volume of a cubical box = a^3 m^3

Now, Cost of 1 m^3 = ₹500

Therefore, Cost of a^3 m^3 = ₹256

$$\Rightarrow a3 = \frac{256}{500}$$

$$\Rightarrow a3 = 0.512$$

$$\Rightarrow a = \sqrt[3]{0.512}$$

$$\Rightarrow a = \sqrt[3]{\frac{512}{1000}}$$

$$\Rightarrow a = \sqrt[3]{\frac{8 \times 8 \times 8}{10 \times 10 \times 10}}$$

$$\Rightarrow a = \frac{8}{10}$$

$\Rightarrow$ a = 0.8 m or 80 cm

Length of each side = 0.8 m

volume of a cubical box = $a^3 = (0.8)^3 = 0.512\ m^3$

Additional Problems 16

1. Question

Three metal cubes whose edges measure 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find (i) side-length (ii) total surface area of the new cube. What is the difference between the total surface area of the new cube and the sum of the total surface areas of the original three cubes?

Answer

Given, three metal cubes, V1, V2 and V3 with sides 3 cm, 4 cm and 5 cm respectively.

(i) They are melted to form a single cube and let its length be x.

We know, **Volume of a cube = $(side)^3$.**

Thus,

Volume of new cube = Volume of V1+Volume of V2+Volume of V3

$x^3 = 3\times3\times3 + 4\times4\times4 + 5\times5\times5$

$x^3 = 27+64+125$

$x^3 = 216$

x= 6 cm

(ii) We know, **Surface area of a cube = $6\times(side)^2$.**

Surface Area of new cube = $6\times(side)^2$

$= 6 \times (6)^2$

$= 6\times36$

$=216\ cm^2$

Sum of total surface areas of the original three cubes = Surface Area of V1+Surface Area of V2+Surface Area of V3

Sum of total surface areas of the original three cubes = $6 \times (3)^2 + 6 \times (4)^2 + 6 \times (5)^2$

$= 6 \times 9 + 6 \times 16 + 6 \times 25$

= 54 + 96+ 150

=300

∴ Difference between the total surface area of the new cube and the sum of total surface areas of the original three cubes= 300-216

= 84 cm^2

2. Question

Two cubes, each of volume 512 cm^3 are joined end to end. Find the lateral and total surface areas of the resulting cuboid.

Answer

Given, Two cubes of volume 512 cm^3each and they are joined end-to-end.

The side of each cube=?

We know, **Volume of a cube = $(side)^3$.**

So, Volume = $(side)^3$

512 = $(side)^3$

Side = 8 cm

We know, **Lateral Surface Area of cuboid = 2h(l+b)**

Lateral Surface Area = 2h(l+b)

= 2×8×(16+8)

= 16×24

= 384 cm^2

We know, **Surface area of a cuboid= 2 × (lb + bh+ hl).**

Total surface areas of the resulting cuboid= 2 × (lb + bh+ hl)

= 2×(16×8 + 8×8 + 8×16)

= 2×(128 + 64 + 128)

= 2× 300

=600 cm^2

3. Question

The length, breadth, and height of a cuboid are in the ratio 6:5:3. If the total surface area is 504 cm^2, find its dimension. Also, find the volume of the cuboid.

Answer

Given, length, breadth, and height of a cuboid are in the ratio 6:5:3 and the total surface area is 504 cm^2.

Let length, breadth, and height of a cuboid be 6x, 5x and 3x.

We know, **Total Surface area of a cuboid= 2 × (lb + bh+ hl).**

Total Surface area of a cuboid= 504

2 × (6x× 5x + 5x × 3x + 3x × 6x) = 504

$30x^2 + 15x^2 + 18x^2 = 252$

$63x^2=252$

$$x^2 = \frac{252}{63}$$

$X^2 =4$

X=2

∴ Length = 6x = 6 × 2 =12 cm

Breadth = 5x = 5 × 2 =10 cm

Height = 3x = 3 × 2 =6 cm

We know, **Volume of a cuboid= l×b×h**

∴ Volume = 12×10×6

= 720 cm^3

4. Question

Find the area of four walls of a room having length, breadth, and height as 8 m, 5 m, and 3 m respectively. Find the cost of whitewashing the walls at the rate of Rs. 15/m^2.

Answer

Given, a room with length, breadth, and height as 8 m, 5 m, and 3 m respectively and the cost of

whitewashing the walls per m^2 is Rs. 15.

We know, **Area of four walls = 2×(l+b)×h**

∴ Area of four walls = 2×(l+b)×h

= 2×(8+5)×3

= 2×13×3

= 78 m^2

∴ Cost of white washing the walls at the rate of Rs. 15/m^2= 15×78

= Rs. 1170

5. Question

A room is 6 m long, 4 m broad and 3 m high. Find the cost of laying tiles on its floor and four walls at the cost of Rs. 80/m^2.

Answer

Given, a room with length, breadth, and height as 6 m, 4 m, and 3 m respectively and the cost of

laying tiles on its floor and four walls per m^2 is Rs. 80.

We know, **Area of four walls = 2×(l+b)×h**

∴ Area of four walls = 2×(l+b)×h

= 2×(6+4)×3

= 2×10×3

= 60 m^2

Area of floor = length × breadth

= 6 × 4

= 24 m^2

∴ Cost of laying tiles on its floor and four walls at the rate of Rs. 80/m^2= 80×(60+24)

= Rs. 80×84

= Rs. 6720

6. Question

The length, breadth, and height of a cuboid are in the ratio 5:3:2. If its volume is 35.937 m^3, find its dimension. Also, find the total surface area of the cuboid.

Answer

Given, length, breadth, and height of a cuboid are in the ratio 5:3:2 and its volume is 35.937 m^3.

Let length, breadth, and height be 5x, 3x and 2x.

We know, **Volume of a cuboid= l×b×h**

∴ Volume = l×b×h

35.937 = 5x × 3x × 2x

$35.937 = 30x^3$

$$\mathbf{x^3 = \frac{35.937}{30}}$$

X^3=1.1979

X= 1.06 m

∴ Length = 5x = 5 × 1.06 = 5.3 m

Breadth = 3x = 3 × 1.06 =3.18 m

Height = 2x = 2 × 1.06 = 2.12 m

We know, **Total Surface area of a cuboid= 2 × (lb + bh+ hl).**

∴ Total Surface area of a cuboid= 2 × (lb + bh+ hl)

= 2 × (5.3×3.18 + 3.18×2.12 + 2.12×5.3)

= 2 × (16.854 + 6.7416 + 11.236)

= 2 × 34.8316

= 69.66 m^2

7. Question

Suppose the perimeter of one face of a cube is 24 cm. What is its volume?

Answer

Given, the perimeter of one face of a cube is 24 cm.

We know, **Perimeter of one face of a cube = 4 × side**

∴ Perimeter of one face of a cube = 4 × side

24 = 4 × side

$$\text{side} = \frac{24}{4}$$

Side= 6 cm

We know, **Volume of a cube = $(side)^3$**

∴ Volume= $(6)^3$

= 6×6×6

= 216 cm^2

8. Question

A wooden box has inner dimensions l =6 m, b =8 m and h =9 m and it has a uniform thickness of 10 cm. The lateral surface of the outer side has to be painted at the rate of Rs. 50/ m^2. What is the cost of painting?

Answer

Given, a wooden box has inner dimensions l =6 m, b =8 m and h =9 m and a uniform thickness of

10 cm.

∴ Length = 6 + 0.20

= 6.2 m

∴ Breadth = 8 + 0.20

= 8.2 m

∴ Height = 9 + 0.20

= 9.2 m

We know, **Lateral Surface Area of cuboid = 2h(l+b)**

Lateral Surface Area = 2h(l+b)

= 2×9.2×(6.2+8.2)

= 18.4×14.4

= 264.96 m^2

The rate of painting lateral surface of the outer side = Rs. 50/m^2

∴ Cost of painting = Rs. 50 × 264.96

= Rs. 13248

9. Question

Each edge of a cube is increased by 20%. What is the percentage increase in the volume of the cube?

Answer

Let the length of a side of a cube be x.

We know, **Volume of a cube = $(side)^3$**

∴ Volume of a cube = x^3

Each edge of a cube is increased by 20%.

∴ New Side= x + 20% of x

$$= x + \frac{20}{100} \times x$$

$$= x + \frac{1}{5} \times x$$

$$= \frac{6}{5} x$$

= 1.2x

∴ New Volume of cube = $(1.2x)^3$

= $1.728\ x^3$

Percentage increase in the volume = $\frac{\text{New volume} - \text{Old volume}}{\text{Old volume}} \times 100$

$$= \frac{1.728\ x^3 - x^3}{x^3} \times 100$$

$$= \frac{0.728\ x^3}{x^3} \times 100$$

=0.728×100

=72.8%

10. Question

Suppose the length of a cube is increased by 10% and its breadth is decreased by 10%. Will the volume of the new cuboid be the same as that of the cube? What about the total surface areas? If they change, what would be the percentage change in both the cases?

Answer

Let side-length of the cube be x.

Then,

Volume of cube = $(side)^3$

= x^3

Total Surface Area of cube = $6(side)^2$

= $6x^2$

Given, the length of a cube is increased by 10% and its breadth is decreased by 10%.

∴ New length = x + 10% of x

$$= x + \frac{10}{100} \times x$$

$$= x + \frac{1}{10} \times x$$

$$= \frac{11}{10} x$$

= 1.1x

And, New Breadth = x - 10% of x

$$= x - \frac{10}{100} \times x$$

$$= x - \frac{1}{10} \times x$$

$$= \frac{9}{10} x$$

= 0.9x

∴ Volume of the new cuboid = l×b×h

= 1.1x× 0.9x × x

= 0.99 x^3

∴ Surface Area of the new cuboid = 2 × (lb + bh+ hl)

= 2 × (1.1x × 0.9x + 0.9x× x + x × 1.1x)

= 2 × (0.99x^2 + 0.9x^2 + 1.1x^2)

= 2 × 2.99x^2

= 5.98x^2 m^2

Percentage change in the volume = $\frac{\text{New volume} - \text{Old volume}}{\text{Old volume}} \times 100$

$$= \frac{0.99\,x^3 - x^3}{x^3} \times 100$$

$$= \frac{-0.01\,x^3}{x^3} \times 100$$

= -0.01×100

= -1% (decrease)

Percentage change in the Surface Area

$$= \frac{\text{New Surface Area} - \text{Old Surface Area}}{\text{Old Surface Area}} \times 100$$

$$= \frac{5.98x^2 - 6x^2}{6x^2} \times 100$$

$$= \frac{-0.02\,x^2}{2} \times 100$$

= -0.02×100

= -2% (decrease)

Hence, Volume decreases by 1% and Surface Area decreases by 2%.