Combinatorics.

Sum Rule →

Let $E_1, E_2, \ldots, E_n$ are mutually exclusive events which can happen in $e_1, e_2, \ldots, e_n$ ways respectively, then no. of ways in which "E_1 or $E_2 \ldots$ or E_n can happen is "$e_1 + e_2 + \cdots + e_n$"

i.e. $n(E_1 \text{ or } E_2 \text{ or } \cdots \text{ or } E_n) = (e_1 + e_2 + \cdots + e_n)$

no. of ways.

Product Rule →

Let $E_1, E_2, \ldots, E_n$ are independent events which can happen $e_1, e_2, \ldots e_n$ ways respectively, then no. of ways in which "E_1 and E_2 and $\cdots$ and E_n" can happen is "$e_1 \cdot e_2 \cdot \ldots \cdot e_n$".

i.e. $n(E_1 \text{ and } E_2 \ldots \text{ and } E_n) = (e_1 \cdot e_2 \cdot \ldots \cdot e_n)$.

Q.1. A pair of distinct dice were tossed. No. of ways we get a sum of 7 or 8 is what?

→ $7 \rightarrow \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$8 \rightarrow \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$.

total no. of ways to get 7 or 8

$= 6 + 5 = \boxed{11}$

(*) i.e. if a same die is flipped twice. Then it won't matter if 1 comes first or last in $(\underline{6,1})$

* In the above example, if the two dice are identical, then the reqd. no. of ways is

$3+3 = \boxed{6}$.

Q.2. How many ternary sequences of length 5 are possible? (a ternary sequence can have 3 digits, 0, 1 and 2).

→ length = 5

3 ways	3 ways	3 ways	3 ways	3 ways
×	×	×	×	×

In the sequence, each digit we can chose in 3 ways.

By product rule, the reqd. no. of sequences are

$3^5 = 243$

Q.3. How many integers betn 10^5 and 10^6 have no digits other than 0, 2, 5, 8?

→ 6 digit integer.

	3 ways	4 ways	4 ways	4 ways	4 ways	4 ways
first digit cant be '0' →	×	×	×	×	×	×
Smallest →	2	0	0	0	0	0
biggest →	8	8	8	8	8	8

∴ By product rule ⇒

no. of digits = $3 \times (4)^5$

$= \boxed{3072}$

Q.4. Let we have 6 different English movies, 8 different Telugu movies and 10 different Hindi movies. In How many ways we can chose 2 movies of different languages?

→ E.T. or T.H. or H.E.
(6×8) + (8×10) + (10×6)

= 48 + 80 + 60

$= \boxed{188}$ ways.

Q.5. How many integers in the set {1, 2, 3, ..., 1000} have distinct digits?

→ $\times \to 9$

$\times \; \times \to \quad = 81$
9 ways (no '0')   9 ways (can't be the one in the first place)

$\times \; \times \; \times \to 9 \cdot (9) \cdot (8)$.

Required no. of integers = $9 + 9\cdot(9) + 9\cdot(9)(8)$

$= 738.$

Q.5. How many integers are there in the set with at least one digit repeating?

set → $\{1, 2, 3, \ldots, 1000\}$

→

at least one digit remaining

$= 1000 - (\text{No digit repeated})$

$= 1000 - 738$

$= \boxed{262}$

Q.6. How many 4-digit integers are there with digit '6' appearing exactly once?

→ case-I → If the first digit is '6' →

$\underset{}{6} \quad \underset{9 \text{ ways}}{\times} \quad \underset{9 \text{ ways}}{\times} \quad \underset{9 \text{ ways}}{\times}$

then each of the remaining digits we can chose in 9 ways.

$\therefore \quad 9 \times 9 \times 9 = 729$ ways.

case - II → If the first digit is not 6

× × × ×
8
ways

then first digit we can chose in 8 ways and digit '6' can appear in 3 ways. and each of the remaining two digits, we can chose in 9 ways

∴ by product rule,

$$8 \times 3 \times 9 \times 9 = 1944$$

∴ By sum rule, the required no. of digits

$$= 729 + 1944 = \boxed{2673}$$

Q.7. Suppose 4 dice were tossed/rolled, then no. of possible outcomes with atleast one dice shows a 2 ?

→ we will find out no. dice in which no die shows 2 →

5ways 5ways 5ways 5ways.
× × × ×

$$= 5^4 \text{ ways.}$$

And total no. of outcomes with 4 dice are

$$= 6^4$$

∴ Reqd. no. of outcomes $= 6^4 - 5^4$

$= \boxed{671}$

Q.8. Suppose n players are enrolled in a singles elimination tennis tournament. How many matches need to be conducted to decide the winner?

→ Each match eliminates a player.

∴ we have a one-one correspondence b/w no. of matches and no. of losers, since we have to eliminate (n-1) players, we have to organize $\boxed{(n-1)}$ matches

Q.9. A set A has n elements. How many ways we can choose subsets P and Q of A so that $P \cap Q$ is empty set.

→ Each element of A can appear like:

i) the element may appear in P()

ii) the element may appear in Q()

iii) the element may appear in none.

∴ $\boxed{3^n}$

Q10. A fruit salad can be made using at least one of the fruits → mango, Apple, Pineapple, watermelon and banana ?

→ Required no. of varieties $= 2^5 - 1$

$= \boxed{31}$

Permutations →

An arrangement of (or) ordered selectⁿ of objects is called a "permutation".

for {a, b, c},

ab	bc	ac	} w/o repetition
ba	cb	ca	
aa	bb	cc	} with repitition

Permutations w/o repititions :

$p(n, k) = {}^{n}p_k$

= no. of permutations of 'n' distinct objects taking/taken k at a time w/o repetitions.

$${}^{n}p_k = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(k-1))$$

n	$(n-1)$	$(n-2)$	...	$(n-(k-1))$
×	×	×	...	×
1st	2nd	3rd		kth object

$$\therefore {}^{n}p_k = \frac{n!}{(n-k)!}$$

${}^{n}p_1 = n$ ${}^{n}p_3 = n(n-1)\cdot(n-2)$

${}^{n}p_2 = n \cdot (n-1)$ and so on.

$^{n}P_{n} = n!$

Q.11. How many ways 6 different persons can sit in a room w (row) ?

→ I II III IV V VI.

$6P_6 = 6! = \boxed{720.}$

∴ Required no. of ways = $6P_6 = 6! = 720.$

Q.12. How many ways 10 different books can be distributed among 15 persons so that no person can take more than one book and max. no. of books are to be distributed. ?

→ $15P_{10} = 15 \times 14 \times 13 \times 12 \times 11$

=

Q.13. How many ways 5 boys and 5 girls can sit in a row so that boys and girls should sit alternately ?

→

case I →

1 2 3 4 5 6 7 8 9 1

Boys → 5!

Girls → 5!

case - II →

odd numbered places ← Girls

$5!$ ways.

even numbered places ← Boys

$5!$ ways.

These two cases are mutually exclusive and exhaustive,

∴ By sum rule,

req. no. of ways = $(5! \cdot 5!) + (5! 5!)$

$= 2(5! 5!) = 2(120 \times 120)$

$= \boxed{28800}$

Q.14. How many ways 5 boys and 5 girls can sit in a row so that no 2 boys can sit side by side ?

→ $- G_1 - G_2 - G_3 - G_4 - G_5 -$

Now, we have 6 places among the girls for the 5 boys to sit.

∴ Boys can sit in $^6P_5$ ways.

Girls can sit in $^5P_5$ ways.

$\therefore$ total read ways $= {}^5P_5 \times {}^6P_6$

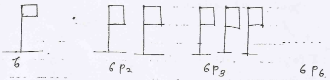
$= 5! \times 6!$

$= \boxed{86,400}$

30/9/2013

Q.15. No. of | How many signals can be generated using 6 different colored flags ? if any no. of the flags can be hoised at a time in a row ?

$\rightarrow$



$\Rightarrow$ the required no. of signals

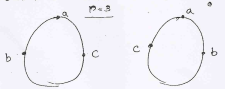
$= 6 + {}^6P_2 + \cdots + {}^6P_6$

$= 6 + 30 + 120 + 360 + 720 + 720$

$= \boxed{1956}$

Permutation in a circular manner →

Number of permutations of 'n' distinct objects (taken all at a time) around a circle is $(n-1)!$

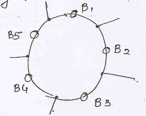


so $(n-1)! = 2! = 2$ ways.

Q.16. How many ways 5 boys and 5 girls can sit around a circular table so that boys and girls should sit alternately ?

→

5 Boys can sit in a circle in $(5-1)! = 4!$ ways.



Girls can sit in $5!$ ways.

∴ Required no. of ways $= 4! \times 5!$

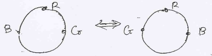
$= 24 \times 120$

$= \boxed{2880}$

a time)

Note : Number of garlands (necklaces) possible with n different colored beads is

$$\frac{(n-1)!}{2}$$



In case of necklaces, the above two combinations are same.

Permutations with repetitions :

$U(n,k) \rightarrow$ no. of permutations of 'n' distinct objects, taken 'k' at a time with unlimited repetitions.

$$\therefore \boxed{U(n,k) = n^k}$$

$$\underbrace{\underset{1}{\overset{n \text{ ways}}{\times}} \quad \underset{2}{\overset{n \text{ ways}}{\times}} \quad \underset{3}{\times} \quad \cdots \quad \underset{k}{\overset{n \text{ ways}}{\times}}}_{n^k}$$

Q.17. How many 5-letter permutations are possible with letters a, b, and c ?

→ Required no. of permutations

$= n^k$

a a a a a b
a a a a a c
12 3!

c a b
a b a

$= 3^5$

$= \boxed{243}$

Q.18 How many ways we can distribute 15 different books among 10 persons?

$\rightarrow$ $\underset{B_1}{\overset{10}{\uparrow}}$ $\underset{B_2}{\overset{10}{\uparrow}}$ $\underset{B_3}{\overset{10}{\uparrow}}$ $\cdots$ $\underset{B_{15}}{\overset{10}{\uparrow}}$

$= \boxed{10^{15}}$

Q.19. How many ways 6 children can be admitted to 10 different schools?

$\rightarrow$ $\underset{C_1}{\overset{10}{\uparrow}}$ $\underset{C_2}{\overset{10}{\uparrow}}$ $\underset{C_3}{\overset{10}{\uparrow}}$ $\underset{C_4}{\overset{10}{\uparrow}}$ $\underset{C_5}{\overset{10}{\uparrow}}$ $\underset{C_6}{\overset{10}{\uparrow}}$.

$= \boxed{10^6.}$

Note → Suppose we have 'n' objects of which 'n_1' objects are alike, 'n_2' objects are alike and 'n_k' objects are alike, then

no. of permutations of n objects taken all at a time

$$= \frac{n!}{n_1! \cdot n_2! \cdots n_k!}$$

Q.20. How many 10-letter permutations are possible with the letters {a, a, b, b, b, c, c, c, c, d} if all the letters of the set are used at a time?

→ the required no of permutations

$$= \frac{10!}{2!\cdot 3!\cdot 4!\cdot 1!} = \frac{10\times 9\times 8\times 7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{2!}\cdot \cancel{3!}\cdot \cancel{4!}}$$

$$= 90\times 4\times 7\times 5 = \boxed{12600}$$

Q.21. How many different messages of length 5 can be sent with 3 dashes and two dots if all the 5 symbols are used at a time?

→ Required no. of permutations

$$= \frac{5!}{3!\times 2!} = \frac{5\times 4\times \cancel{3!}}{\cancel{3!}\cdot 2!} =$$

$$= \boxed{10}$$

Q.22. How many binary sequences of length 10 are possible with six 1's and four 0's? if all the 10 bits are used at a time?

$$\rightarrow \frac{10!}{6!\times 4!} = \frac{10\times 9\times 8\times 7\times \cancel{6!}}{\cancel{6!}\times 4!}$$

$$= \boxed{210}$$

TTTTT HHHHH

Q.23. A coin is tossed 10 times. No. of outcomes possible with 5 heads and 5 tails is what?

$$\rightarrow \frac{10!}{5!\,5!} = \boxed{252}$$

Q.24. How many ways 10 office buildings can be painted so that 3 are in blue, 2 are in brown, and 5 are in white color?

$$\rightarrow \frac{10!}{3!\times 2!\times 5!} = \boxed{2520}$$

(ordered partitions)

Q.25. How many ways 10 persons can be divided into 3 teams so that first team consists 3 members, 2nd team contains 2 members and 3rd team contains 5 members?

$\rightarrow$ Required no. of ways

$$= \frac{10!}{3!\cdot 2!\cdot 5!} = \boxed{2520}$$

Q.26. (Unordered partitions) (No first, second team)

or — "How many ways 10 persons can be divided into 5 teams of 2 each?"

$\rightarrow$ Required no. of ways

$$= \frac{10!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!\cdot 5!}$$

5 teams.

$$= \boxed{945}$$

(unordered)

Combinations →

An unordered selection of objects is called a "combination".

ex → $\{a, b, c\}$

2-letter combinations → $\{a,b\}, \{b,c\}, \{a,c\}$.

Combinations without repetitions →

$^nC_k = C(n,k)$ ⇒ No. of combinations of 'n' distinct objects taken 'k' at a time without repetitions.

$$^nC_k = \frac{n\cdot(n-1)\cdot(n-2)\ldots\ldots(n-(k-1))}{1\cdot 2\cdot 3\cdot\ldots\cdot k}$$

$$= \boxed{\frac{^nP_k}{k!}} = \boxed{\frac{n!}{(n-k)!\,(k!)}}$$

$$\boxed{^nC_k = {^nC_{(n-k)}}}$$

ex. $^{10}C_8 = {^{10}C_2} = \frac{10\cdot 9}{1\cdot 2} = 45$

1) $^nC_0 = 1$

2) $^nC_1 = n$

3) $^nC_2 = \frac{n\cdot(n-1)}{1\cdot 2}$

4) $^nC_3 = \dfrac{n\cdot(n-1)(n-2)}{1\cdot 2\cdot 3}$

5) $^nC_n = 1.$

(*) $^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n.$

Q.1. How many ways we can distribute 10 similar books among 15 persons so that no person can take more than one book?

→ Here, only 10 persons will get books. The 10 persons we can choose in $^{15}C_{10}$ ways.

∴ Required no. of ways = $^{15}C_{10}$

$\boxed{1}$.

Here, we can select the books in one way and can also distribute it in 1 way.

Q.2. How many binary sequences of length 10 are possible with exactly 3 zeros?

→ 7 1's and 3 0's.

X X X ... X

1 2 3 ... 10

* In the sequence, we have to select 3 positions for placing 3 zero's.

→ $^{10}C_3$ ways.

1010101111

273

$\therefore$ Required no. of binary sequences

$$= {}^{10}C_3 = {}^{10}C_7$$

$$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$$

$$= \boxed{120}$$

Q.3. How many binary sequences of length 10 with exactly 4 0's and no two 0's are consecutive?

→ 111111 0000

' 1 ' 1 ' 1 ' 1 ' 1 ' 1 '

zeros can appear.

$\therefore$ No. binary sequences $= {}^7C_4$.

$$= \frac{7!}{3!\,4!} = \frac{7 \times \cancel{6}^{3} \times 5 \times 4!}{2 \times 1 \times 3!}$$

$$= \boxed{\cancel{105}} = \boxed{35}$$

(9)

Q.4. How many 5-digit integers are possible so that in each of these integers, every digit is greater than the digit on its right?

→ To meet the given condⁿ, we have to chose 5 distinct decimal digits and then we have to arrange them in descending order

∴ Required no. digits = $^{10}C_5 \times 1$

$= \boxed{252}$.

Q.5. Suppose n couples are in a party. If each person shakes hands with every other person except his/her spouse, then no. of different handshakes possible in the party is?

→ 2n persons

No. of handshakes possible with 2n persons

$= {}^{2n}C_2$.

Every person does not shake hands with his/her spouse

total no. of handshakes = $^{2n}C_2 - n$

$= \frac{2n \cdot (2n-1)}{1 \cdot 2} - n$

ba ${}^5C_5$ ba ba

$$= \boxed{2n(n-1).}$$

Q.6. How many ways six persons {a,b,c,d,e,f} can speak at a convention with b speaking after a?

→ b speaking after a ⇒ b should not speak before a.

1 2 3 4 5 6.

The speeches of a and b can be arranged in $6C_2$ ways. because we can allots 2 slots in $6C_2$ ways. and the remaining 4 speeches can be arranged in 4! ways.

∴ Required no of ways $= 6C_2 \times 4!$

$$= 6C_3 \times 3!$$

$$= \boxed{120}$$

Q.7. How many ways 10 MCQ's which 4 choices for each question can be answered so that exactly 3 answers are correct?

→ 3 questions can be correctly answered in $10C_3$ ways.

$$\frac{10!}{7!\,3!} = \frac{10 \times 9 \times 8}{3 \times 2} = \boxed{120}$$

and also remaining 7 questions must be wrongly answered. This can be in 3^7 ways.

$\therefore$ By product rule $\Rightarrow$

$10C_3 \times 3^7 =$

Q.8. How many ways 5 out of 10 persons can sit around a circular table ?

$\rightarrow$ We can select 5 persons from 10 in $10C_5$ ways.

And those 5 persons can sit around a circular table in $4!$ ways.

$\therefore$ Required no of ways $= 10C_5 \times 4!$

$= 252 \times 24$

$= \boxed{6048}$

Q.9. How many ways can we select a comittee of 5 members out of 5 men and 5 women so that at least 1 women is included in the comittee ?

$\rightarrow$ $5C_4 \cdot 5C_1 + 5C_3 \cdot 5C_2 + 5C_2 \cdot 5C_3 + 5C_1 \cdot 5C_4 + 5C_5$

at least 1 woman $\rightarrow$ complement

$\downarrow$

no woman selected.

10 C 8
5 0 4 +
6 0 4 8

84

$\therefore$ No. women selected = all men selected

$= {}^5C_5 = 1$

and total are $= {}^{10}C_5$.

$\therefore$ at least one women selected

$= {}^{10}C_5 - 1$

$= \boxed{251}$

Q.10. How many rectangles are there in a chess-board which are not squares?

→ 2×2 chessboard ?

A rectangle can be formed by any two horizontal lines and any two vertical lines.

The chessboard consists of 9 horizontal and 9 vertical lines.

$\therefore$ No. of rectangles (including squares) in the chessboard $= {}^9C_2 \times {}^9C_2$

$= 36 \times 36 = 1296.$

No. of squares in a chessboard are →

$\underset{(8\times8)}{1^2} + \underset{(7\times7)}{2^2} + \underset{(6\times6)}{3^2} + \cdots + \underset{(1\times1)}{64\,(8^2)}$

$$= \frac{n \cdot (n+1)(2n+1)}{6}$$

$n = 8$

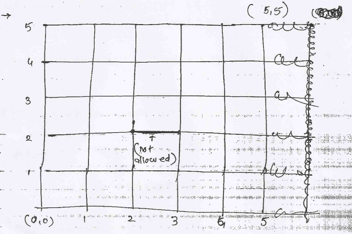
$$= \frac{8 \times 9 \times 17}{6} = \boxed{204} \text{ squares}$$

$\therefore$ total no. of rectangles that are not squares

$= 1296 - 204$

$= \boxed{1092}$

Q.11. A robot is placed at O(0,0). At/In each operation, the robot can move 1 unit to the right (+ve x-axis) or 1 unit up (+ve y-axis). How many different paths are there from (0,0) to (5,5) if the robot is not allowed to use the line segment joining (2,2) and (3,2)?



Each path consists of 5 moves along x-axis and 5 moves along y-axis.

$\therefore$ the no. of paths from $(0,0)$ to $(5,5)$

= no. of binary sequences possible with 5 x's and 5 y's.

$= {}^{10}C_5$

No. of paths from $(0,0)$ to $(5,5)$ via the line segment joining $(2,2)$ to $(3,2)$ =

${}^4C_2 \times {}^5C_2$.

= 60 paths.

$\therefore$ Reqd. no. of paths = total no of possible paths − paths via $\{(2,2)$ to $(3,2)\}$.

$= 252 - 60 = \boxed{192}$

Q.12. There are 12 stations on rail-line. How many ways a special train can stop at 4 of these stations so that no two stops are consecutive stations?

→ Let us denote the stations where train stops by 0's and the other stations by 1's.

then the required no. of ways

= no. of binary seq. possible with 8 1's and 4 0's so that no two zeros are consecutive.

$\Rightarrow \underbrace{-1-1-1-1-1-1-1-1-}_{9 \text{ places}} \mid 0\,0\,0\,0$

$= {}^9C_4 = \boxed{126}$ ways

Combinations with Repetitions →

$V(n,k) = {}^nV_k$

= no. of combinations of n distinct objects taken k at a time with unlimited repetitions.

(k can be > n).

the no. of ways = $C(n-1+k, k)$

= no. of ways we can distribute 'k' similar balls into 'n' numbered boxes.

Q.13. No. of nonnegative integers solutions to the equation $x_1+x_2+\cdots+x_n = k$ &

→ let $k=10$, $n=5$.

then $x_1+x_2+x_3+x_4+x_5 = 10$.

⇒ no. of binary sequences / bit strings possible with $(n-1)$ 1's and k 0's,

$C(n-1+k, k)$

Q.14. How many ways we can distribute 10 similar books among 5 persons?

→ Required no. of ways = $V(5, 10)$

$V(n, k) = C(n-1+k, k)$

$V(5, 10) = C(14, 10) = {}^{14}C_{10}$

$= {}^{14}C_4 = \dfrac{\overset{7}{\cancel{14}} \times 13 \times 12 \times 11}{4 \times 3 \times 2} = \boxed{1001}$

(***) Q.15. How many ways we can distribute 16 similar balls in 4 numbered boxes? (distinguishable) so that each box contains at least one ball.

→ To meet the condition, let us put one ball in each box in only one way.

B_1	B_2	B_3	B_4
1	1	1	1

Remaining 12 balls, we can distribute into 4 boxes in any way ⇒

$V(n, k) = V(4, 12)$ ways.

$= C(4-1+12, 12) = C(15, 12)$ ways.

$= \boxed{455}$

possible. $\to \textcircled{3}$ $\frac{3}{3^{10}}$

Same as previous example now.
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 9$

$\therefore V C(6,9) = C(6-1+9, 9) = C(14, 9)$

$= \boxed{2002}$

Q.19. No. of ternary sequences possible with 6 1's, 6 2's, and 4 0's if so that each '1' is followed by '2'

$\to$ Consider 6 pairs of '12'. How we have 7 places among these pairs for placing the 4 0's

$\therefore$ the reqd. no. of ternary sequences

$= VC(7,4) = C(10,4)$

$= \frac{10!}{6! \times 4!} = \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}\ 4 \times 3 \times 2}$

$= \boxed{210}$.

Q.20. Suppose A question paper has 10 questions for 30 marks. The first question should carry 5 marks and each of the rem. quest^{ns} carry at least 2 marks. How many ways the marks can be distributed among the 10 questions subjected to above cond^{ns}. ?

$\to$ 9. $\to$ boxes. 25 balls.

9 $\to$ 7 balls.

$V(9,7)$ $C(15,7)$. $^{15}C_7$.

${}^{10}C_4$ $\quad$ 12 (2) (12) (2) (19) (12)! 0 (120) 0 0 $\quad$ $V(7,4)$. $C(10,4)$ $\quad {}^{10}C_4$ $\quad \frac{10!}{6!\,4!}$

${}^5P_5 =$ (56)

To meet the given conditions, let us all of 5 marks to first questⁿ and (2×9) to 9 questⁿˢ.

So, we are left with 7 marks and to dist. among last 9 questⁿˢ.

∴ Required no of ways.

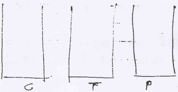
→ $V(9,7) = C(15,7)$

$= \boxed{6435}$

Q. 21. Suppose 10 persons are in a canteen which offers coffee, tean and pepsi. How many ways they can order their drinks as a group if each person can select or wants only one of the 3 drinks?

→

Required no. of orders.



$= V(3,10)$

$= C(12,10) = \frac{12\cdot 11}{1\cdot 2} = 66.$

Pigeonhole Principle →

If A is the average number of pigeons per a pigeonhole, then

a) some pigeonhole contains at least $\lceil A \rceil$ pigeons.

b) some pigeonhole contains at most $\lfloor A \rfloor$ pigeons.

So, let 10 pigeons and 4 pigeonhols.

$$\therefore A = \frac{10}{4} = 2.5$$

a) some pigeonhole contains at least $\lceil A \rceil = 3$ pigeons.

b) some pigeonhole contains at most $\lfloor A \rfloor = 2$ pigeon

* If $n+1$ pigeons are distributed/kept in n pigeonholes then $A = \frac{n+1}{n} = 1 + \frac{1}{n}$.

$\therefore \lceil A \rceil = 2$. $\lfloor A \rfloor = 1$.

pigeon-hole princ.

$\therefore$ By P.H.P, some pigeonhole contains at least 2 pigeons.

and some other pigeonhole contains at most 1 pigeon.

* If $(2n+1)$ pigeons are kept in n pigeonholes, then

$$A = \frac{2n+1}{n} = 2 + \frac{1}{n}$$

$\lceil A \rceil = 3$ $\lfloor A \rfloor = 2$

a) Some pigeonhole contains at least 3 pigeons.

b) Some pigeonhole contains at most 2 pigeons.

* If k is any positive integer, and if $kn+1$ pigeons are kept in n pigeonholes, then

$$A = \frac{kn+1}{n} = k + \frac{1}{n}$$

$\lceil A \rceil = k+1$

$\lfloor A \rfloor = k.$

$\therefore$ a) Some pigeonhole contains at least '$k+1$' pigeons.

b) Some pigeonhole contains at most 'k' pigeons.

Note → Suppose we have 'n' pigeonholes, the min. no. of pigeons~~holes~~ reqd. to ensure that some pigeonhole contains at least $(k+1)$ pigeons

$= (kn+1).$

108. 108, 20, ...

$\frac{12}{n} = 108.12C$ ... 12 ... $\frac{12}{132}$ 94

Q.1. what is the min. no. of persons we have to choose randomly to ensure that at least 10 persons were born in the same month.

→ n = 12. months. ← pigeonholes.

K + 1 = 10.

k = 9. n = 12.

$\therefore (9 \times 12) + 1 = \boxed{109}$

(OR)

$A = \left\lceil \frac{x}{12} \right\rceil = 10.$

$= \left\lceil \frac{109}{12} \right\rceil = 10$

$\therefore \boxed{109}$

Q.2. If 410 letters were distributed in 50 apartments which of the foll. st. are necessarily true.?

Sol) At least 9 per

✓ S1) Some apartment received at least 9 letters

✓ S2) Some apartment received at most 8 letters.

✗ S3) Some apartment received at least 10 letters

✗ S4) Some apartment received at most 7 letters

✓ S5) Some apt. received at least 7 letters.

$\frac{410}{50}$ 50 8/6 ... 1 2 3 4 5

56) Some apt. received at most 10 letter.

→ $A = \frac{410}{50} = 8.2$

$\lceil A \rceil = 9$ $\lfloor A \rfloor = 8$

By P.H.P., S_1 & S_2 follow.

S_3 is also true coz there are all apts receiving letters ≥ 7.

S_4) no. letters ≤ 10. always true.

Q.3. A bag contains 6 red balls, 8 blue balls, 10 green balls and 15 white balls and 20 yellow balls. what is the min. no. of balls we have to choose randomly from the bag to ensure that we get at least 6 balls of same color.

→ 6 - R 8 - B 10 - G 15 - W 20 - Y

$\underbrace{\hspace{8em}}$

5 pigeon holes. $n = 5$.

$k + 1 = 6$. balls (pigeons).

$\therefore k = 5$

Min. no. of balls reqd $= kn + 1$

$= (5 \times 5) + 1 = 26$

Q.4. In the above example, what is the min. no. of balls we have to choose from the bag to ensure that we get at least 9 balls of same color?

→ to find the min. no. of balls reqd., include

$$\underbrace{\overset{\text{all Red}}{6} + \overset{\text{all blue}}{8}}_{\uparrow} + \underset{G}{8} + \underset{W}{8} + \underset{Y}{8}$$

(we have to include them). 14 + 24

= 38 + 1

∴ Min no of balls reqd. to be drawn = $\boxed{39}$

Q.5. In the above ex, what is the min. no. of balls we have to choose from bag to ensure that we get at least 12 balls of same color?

→ $(\underbrace{\underset{R}{6} + \underset{B}{8} + \underset{G}{10}} + \underbrace{\underset{W}{11} + \underset{Y}{11}}) + 1$

= (24 + 22) + 1

= 46 + 1

= $\boxed{47}$

(***)

Q. 6. A C.S. dept. offer 4 yr. B.Tech programme, ~~To form~~ a with an intake of 60 students each year. A student's club can be formed in the dept with any

a) any 10 second year students.

(or)

b) any 8 third year students.

(or)

c) any 6 final year students.

what is the min. no of students you have to pick randomly from dept. to ensure that the student's club is formed ?

→ $\underbrace{(60 + 9 + 7 + 5)}_{\text{I} \quad \text{II} \quad \text{III} \quad \text{IV}} + 1$

with these 81 students, we cannot form a club.

$= (81) + 1$

$= \boxed{82}$

constructing a pigeonhole

Q. 7.

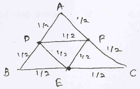
(*****)

5 points were randomly selected in an equilateral Δ with each side 1 unit. There will be at least 2 points (of those 5 points) such that the distance betⁿ the two points cannot exceed

a) 1/2 b) 1/3 c) 1/4 d) 1/5

* Apply pigeon hole principle by dividing the Δ into 4 parts.

→ * Divide the Δ into 4 equal parts.



By pigeon hole princ., the avg. no. of points in a small Δ is $5/4 = 1.25$

Now $\therefore A = 1.25$ $\therefore \lceil A \rceil = \underline{\underline{2}}$

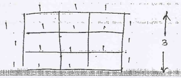
$\therefore$ By P.H.P., at least two of those 5 points lie in one of the small Δs.

(**) the distance betⁿ any two points in a small Δ can not exceed $1/2$.

Q.8. 10 points were randomly selected in a square with each side 3 units. There will be at least 2 points (of those 10 pts.) such that the distance betⁿ them cannot exceed →

a) 1 ✓b) $\sqrt{2}$ c) $\sqrt{2}-1$ d) $2-\sqrt{2}$

→ * Divide the $\square$ in 9 equal parts.



for a small square

The avg. no. of points in a small square

$A = \frac{10}{9} = 1.111$ $\lceil A \rceil = 2.$

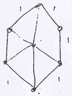
By P.H.P, at least two points belong to same pigeonhole.

$\therefore$ The max distance betn any 2 points in a square is diagonal $= \sqrt{2}$.

$\therefore$ ans. $\rightarrow$ $\boxed{\sqrt{2}}$

Q.9. 7 points were randomly selected in a regular hexagon with each side 1 unit. There will be at least 2 points such that dist betn them is?

a) $\rightarrow$



$\therefore$ There exist at least 2 points whose dist. does not exceed 1.

Euler Function → (of n).

If 'n' is a positive integer, then Euler function of 'n' denoted by $\phi(n)$ is defined as,

(***) $\phi(n)$ = no. of positive integers which are $\leq n$ and relatively prime (or coprime) to n.

® Relatively Prime / Co-prime →
Two +ve integers a and b are said to co-prime if

$$G.C.D.\ (a,b) = 1$$

Q.1. what is Euler function 6.

$$\rightarrow \phi(6) = \{1, 2, 3, 4, 5, 6\}$$

$$= \boxed{2}$$

Q.2. $\phi(7) = \{1, 2, 3, 4, 5, 6, 7\}$

$$= \boxed{6}$$

Note → If 'n' is a prime no., then $\boxed{\phi(n) = n-1}$

Q.3. $\phi(8) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$= \boxed{4}$$

18×10
$9 \times 2 \times 10$
$3 \times 3 \times 2 \times 2 \times 5$

$$\phi(n) = \left\{ \frac{n \times (P_1 - 1) \cdot (P_2 - 1) \cdot \cdots \cdot (P_k - 1)}{P_1 \cdot P_2 \cdot P_3 \cdots P_k} \right\}$$

where, $P_1, P_2, \cdots, P_k \rightarrow$ distinct prime factors of 'n'.

Q-4. No. of +ve integers which are less than 110 and relatively prime to 110 is ?

$\rightarrow$ $\cancel{100}$ $110 = 2 \times 5 \times 11$
$\quad P_3 \quad P_2 \quad P_1$

$$\begin{array}{r|l} 2 & 110 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$$

$$\therefore \phi(n) = \phi(110) = \frac{\cancel{110}^{10} \times (10 \times 4 \times 1)}{\cancel{11} \times 5 \times 2}$$

$$= \boxed{40}$$

Q-5. No. of +ve int. less than 180 and <180.

$180 =$

$$\rightarrow \phi(180) = \frac{\cancel{180} \times (4)(2)(2)(1)(1)}{\cancel{3} \times \cancel{3} \times 2 \times 2 \times \cancel{5}}$$

$= 16.$

we have to take distinct prime nos

$$= \frac{\cancel{180} \times 4 \times 2}{\cancel{3} \times \cancel{5} \times 2} = \boxed{48}$$

Q. 6. $\phi(323)$

$\rightarrow$ $323 = 19 \times 17$. $17 \overline{)323}$ 19.

$\therefore$ $\phi(323) = \dfrac{\cancel{323} \times (16 \times 18)}{19 \times 17}$

$= \boxed{288}$.

Q. 7. $\phi(113)$

$\rightarrow$ 113 is prime.

$\therefore$ $\phi(n) = 113 - 1 = \boxed{112}$.

Q. 8. Let $n = p^2 \cdot q$ where p and q are prime nos. No. of +ve integers m such that $1 \leq m \leq n$ and G.C.D. of $\{m, n\} = 1$ is ?

$\rightarrow$ Reqd. no. of +ve integers

$$\phi(n) = n \cdot \frac{(p-1)(q-1)}{p \cdot q}.$$

$$= p^2 \cdot q \cdot \frac{(p-1)(q-1)}{p \cdot q} = \boxed{p(p-1)(q-1)}.$$

* if suppose a no. n is given.
then we have to check if its prime or not
upto $\sqrt{n}$ only....

Derangements (kind of permutations) :

A permutation of 'n' distinct objects, in which no object appears at its original / correct place is called a "derangement of objects".

No. of derangements of 'n' distinct objects taken all at a time

$$D_n = n! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots + \left(\frac{(-1)^n}{n!} \right) \right\}$$

ex :

suppose $n=2$, $D_2 = 2! \times \frac{1}{2!} = 1$

$n=3$, $D_3 = 3! \times \left\{ \frac{1}{2!} - \frac{1}{3!} \right\}$

$= \frac{3!}{2!} - \frac{3!}{3!} = 3 - 1 = 2$

$n=4$, $D_4 = 4! \times \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right\}$

$= \frac{4!}{2!} - \frac{4!}{3!} + \frac{4!}{4!} = 12 - 4 + 1$

$= 9$

$n=5$, $D_5 = 5! \times \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right\}$

$= 60 - 20 + 5 - 1 = 44.$

1, 2, 3.
2 2 2,

$n = 6$, $D_6 = 265$

Q.1. How many 1-1 functions are possible with 6 elements so that no element is mapped to itself?

$$\begin{array}{ccc} A & \to & A \\ 1 & \leftrightarrow & 1 \\ 2 & \leftrightarrow & 2 \\ 3 & \leftrightarrow & 3 \\ 4 & \leftrightarrow & 4 \\ 5 & \leftrightarrow & 5 \\ 6 & \leftrightarrow & 6. \end{array}$$

Required no. of one-one functions is

$D_6 = \boxed{265}$

Q.2. How many ways we can put 5 letters in 5 envelopes (@ 1 letter per envelope) so that

a) no letter is correctly placed. (i.e. letter l_i is not in envelope e_i ($i = 1$ to 5).

b) at least one letter is correctly placed

c) exactly two letters are correctly placed.

d) atmost one letter is correctly placed.

e) at least one letter is wrongly placed.

f) exactly one letter is wrongly placed.

→ a) l_1 l_2 l_3 l_4 l_5

e_1 e_2 e_3 e_4 e_5.

Required no. of ways

$$= D_5 = 5! \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right\}$$

$$= \boxed{44}$$

b) total No. of ways 5 lettee in 5 envelopes

$= 5! = 120.$

No. of ways in which no letter is correctly placed $= D_5 = 44.$

∴ No. of ways in which at least one letter is correctly placed

$= 120 - 44 = \boxed{76.}$

c) No. of ways we can select 2 letters out of 5

$$= 5C_2 = \frac{5!}{3! \cdot 2!} = \frac{5 \times 4}{2} = 10$$

And these two letters can be placed in correct envelope in one way.

$\therefore$ No. of ways 2 out of 5 letters can be correctly placed $= 10 \times 1 = \underline{10}$.

And also, remaining 3 letters should be placed in wrong envelopes, in D_3 ways.

$\therefore D_3 = 2$.

$\therefore$ Required no of ways $= 10 \times 2 = \boxed{20}$

d) atmost one ≤ 1

$= 0$ (or) $= 1$

Now we have two cases where no letter is correctly placed or only one letter is correctly placed.

No. of ways in which no letter is correctly placed is $D_5 = 44$.

and no. of ways in which only one letter is correctly placed is $5C_1 \cdot D_4$

$\therefore$ $5C_1 \cdot D_4 = \frac{5!}{4! \times 1} \times 9 = 5 \times 9 = 45$.

By sum rule,

$\therefore D_5 + 5C_1 \cdot D_4 = 44 + 45 = \boxed{89}$

e) $\rightarrow$ there is only one way, we can put all 5 letters correctly. In all the remaining cases, at least one letter will be wrongly placed.

$\therefore$ Required no. of ways $= 5! - 1$

$= \boxed{119}$

f) $l_1 \quad l_2 \quad l_3 \quad l_4 \quad l_5$

$e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5$

* It is not possible to put only one letter in wrong envelope because if we put one letter in wrong envelope, then the letter corresponding to that envelope also goes to wrong envelope.

$\therefore$ Required no. of ways $= \boxed{0}$

Q.3. No. of derangements possible for the sequence $\{a, b, c, d, e, f, g, h, i, j\}$ so that

a) the first five letters of this sequence are in first five places.

b) None of the first five letters of the sequence are in first 5 places.

$\rightarrow$ a) $\rightarrow$ $\{\underbrace{a, b, c, d, e}_{D_5}, \underbrace{f, g, h, i, j}_{D_5}\}$

$D_5 \times D_5$

* The first 5 letters of the sequence can be deranged in first 5 places in D_5 ways.

6 156

* IIIly, the last 5 letters of the seq. can be deranged in last 5 places in D_5 ways.

$\therefore$ Required no. of derangements

$= D_5 \times D_5$

$= 44 \times 44 = 1936.$

b) → Any permutation of the given sequence in which the first 5 letters are in last 5 places & last 5 letters are in first 5 places is a derangement.

$\therefore$ Reqd. no. of derangements

$= 5! \times 5! = \boxed{14400}$

$\{\underbrace{a, b, c, d, e}, \underbrace{f, g, h, i, j}\}.$

5! ways

5! ways

Q.4. Suppose 5 diff. books are distributed among 5 students @ one book per student. Further suppose that the books were returned by the students and again distributed among the students later on.
How many ways this can be done so that no student can take the same book twice?

→

First time, the books can be issued to the students in $5!$ ways.

Second time,

$B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5$

$S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5$

(Second time)

the books can be distributed in D_5 ways.

$\therefore$ Required no. of ways

$= 5! \times D_5 = 44 \times 120$

$= \boxed{5280}$

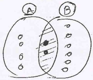
Q.

Mutual

Principle of inclusion and exclusion →

* Let A and B are any two sets.

then no.of elements (A or B)



$= n(A) + n(B) - n(A \text{ and } B)$

i.e. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$= 6 + 8 - 2 = \boxed{12}$

* Let A, B and C are any 3 sets.

then no. of ele. (A or B or C)

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$

$+ n(A \cap B \cap C)$.

* $n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D)$

$- n(A \cap B) - n(A \cap C) - n(A \cap D) - n(B \cap C) - n(B \cap D)$

$- n(C \cap D) + n(A \cap B \cap C) + n(A \cap C \cap D) + n(B \cap C \cap D)$

$+ n(A \cap B \cap D) - n(A \cap B \cap C \cap D)$

54
32
86

$- 172 + 32 - 4$
$= 130 - 76 + 32$

Q.1. Let A, B, C, D are 4 sets such that

$n(A) = 42$, $n(B) = 36$, $n(C) = 28$, $n(D) = 24$.

Intersect^n of any two of these four sets contains 12 elements, and

Intersect^n of any 3 of these 4 sets contains 8 elements, and

$n(A \cap B \cap C \cap D) = 4$. then $n(A \cup B \cup C \cup D) = ?$

→

$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) -$

$n(A \cap B) - n(A \cap C) - n(A \cap D) - n(B \cap C) - n(B \cap D)$

$- n(C \cap D) + n(A \cap B \cap C) + n(A \cap C \cap D) + n(A \cap B \cap D)$

$+ n(B \cap C \cap D) - n(A \cap B \cap C \cap D)$

$= 42 + 36 + 28 + 24 - (6 \times 12) + (4 \times 8) - 4$

$= \boxed{86}$

Q.2. In a class of 100 students,

48 students can speak French,

39 students can Speak German,

and 24 students can speak both lang.

8 4 24 39 24 15 | 81 24 63 | 39 8.7 24 15 24. | 48 39 | 248 39 24 111 | 111 24

How many students can speak

a) at least one of the two languages ?

b) None of the two lang. ?

c) only one of the two lang ?

→ a) $n(F \cup G) = n(F) + n(G) - n(F \cap G)$

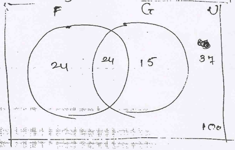
$= 48 + 39 - 24$

$= \boxed{63.}$

b) $n(\overline{F \cup G}) = 100 - 63$

$= \boxed{37}$

c) the given data can be represented as



No. of students who can speak only one of the two lang. $= 24 + 15$

$= \boxed{39}.$

Q.3. In the set $\{1, 2, 3, \ldots, 1000\}$, how many integers are not divisible by 2, 3 or 5 ?

→ Let $n(2)$ → no. of integers in the set div. by 2.

$n(3)$ → no. of integers in the set div. by 3.

$n(5)$ → no. of integers in the set div. by 5.

$$n(2 \cup 3 \cup 5) = n(2) + n(3) + n(5) - n(2 \cap 3) - n(3 \cap 5) - n(2 \cap 5) + n(2 \cap 3 \cap 5)$$

$$= 500 + 333 + 200 - 166 - 66 - 100 + 33.$$

$$= 1033 - 332 + 33$$

$$= \boxed{734}$$

∴ Reqd. no. of integers $= 1000 - 734$

$$= \boxed{266.}$$

Q.4. In a class of 100 students,

40 students failed in maths,

30 students failed in Physics,

25 students failed in Chemistry,

20 students failed in maths & Physics,

$$\begin{array}{r} 166 \\ 66 \\ \hline 232 \end{array} \qquad \begin{array}{r} 1066 \\ 332 \\ \hline 0734 \end{array}$$

100

15 students failed in physics and chem,

10 students failed in maths and chem., and

6 students failed in all the 6 subjects.

a) How many students failed in at least one of the three subjects.

b) " — " none of the three subjects ?

c) " — " only one of the three subjects ?

d) " — " exactly two of the three subjects ?

e) " — " at least two of the three subjects ?

→ Let $n(M)$ = no. of students failed in Maths.

a) n (failed in at least one subject)

$$= n(M \cup P \cup C) = n(P) + n(M) + n(C) - n(M \cap P)$$

$$- n(P \cap C) - n(M \cap C) + n(P \cap M \cap C)$$

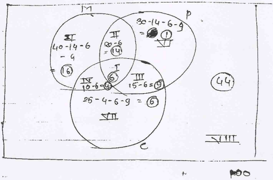
$$= 30 + 40 + 25 - 20 - 15 - 10 + 6$$

$$= \boxed{56}$$

b) $n(\overline{M \cup P \cup C}) = 100 - 56$

$= \boxed{44.}$

The given data can be represented by Venn diagram as follows,



c) → No. of students failed in only one sub

$= 16 + 1 + 6 = \boxed{23}$

d) No. of students failed in exactly 2 sub.

$= 14 + 4 + 9 = \boxed{27}$

e) No. of students failed in at least 2 subjects

$= 14 + 4 + 9 + 6 = \boxed{33}$

100 − 33.
67 67 − 44.

20 − 9.

117

Q.5. In a competition, 34 students received awards.

13 students received awards in biology.

17 students received awards in chemistry,

21 students received awards in Physics.

3 students received awards in all 3 subjects.

How many students received awards

a) in exactly two subjects.

b) only one of the three subjects.

a) →

$n(B \cup P \cup C) = n(B) + n(P) + n(C) - n(B \cap P) - n(P \cap C) - n(B \cap C) + n(B \cap P \cap C)$.

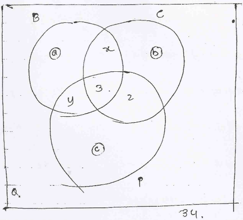
$34 = 13 + 17 + 21 - n(B \cap C) - n(P \cap C) - n(B \cap P) + 3$

$\therefore n(B \cap C) + n(P \cap C) + n(B \cap P) = \boxed{20}$

$(x+3) + (y+3) + (z+3) = 20$

$x + y + z = 20 - 9 = \boxed{11}$

Venn diagram representation :



No. of students received awards in exactly two subjects = $\boxed{11}$

b) No. of students receiving awards in exactly one subjects

$$= 34 - \underbrace{14}_{x+y+2+3} = \boxed{20}$$

Q. 6. Y

$3^4 - 14$
$= 0$

Recurrence Relations →

Let $a_0, a_1, a_2, \ldots, a_n, \ldots\ldots$ be a sequence of real numbers.

A formula which relates 'a_n' with one or more of the preceding terms $a_{n-1}, a_{n-2}, \ldots$ is called a " Recurrence Relation "

$$a_n = f\{ a_{n-1}, a_{n-2}, \ldots \}. \leftarrow \text{Recurrence Relat}^n.$$

ex. → For the arithmatic sequence,

$$\{ a, a+d, a+2d, \ldots\ldots, \}$$

the recurrence relation is

$$a_n = a_{n-1} + d. \quad \text{where } a_0 = a.$$
$$(n \geq 1)$$

ex. → For the geometric sequence,

$$\{ a, a.t, a.t^2, a.t^3, \ldots\ldots \}$$

the recurrence relation is

$$a_n = a_{n-1}.t. \quad (n \geq 1) \quad a_0 = a.$$

ex. For the Fibonacci sequence,

$$\{ 1, 1, 2, 3, 5, 8, 13, 21, \ldots \}.$$

the recurrence relation is,

$a_n = a_{n-1} + a_{n-2}$ $(n \geq 2)$ where $a_0 = a_1 = 1$.

* Recurrence Relations

Linear — Non-linear

A) Linear Recurrence Relations →

A recurrence relation of the form

$$C_0 \cdot a_n + C_1 \cdot a_{n-1} + \cdots + C_k \cdot a_{n-k} = f(n) \quad -(1)$$

is a "linear Recurrence Relation of order k".

* Note → if $f(n) = 0$, then eqn (1) is said to be "homogeneous linear recurrence relation".

if $f(n) \neq 0$, then eqn (1) is said to be "inhomogeneous linear rec. relation".

(*) Formation of Recurrence Relations →

ex. →

Q.1. If a_n = no. of binary sequences of length with no consecutive 0's, then recurrence relation for a_n is what ?

→

x x x x
1 2 3 .... n

case 1) → If the first bit of the sequence is '1', then the remaining 'n-1' bits we can choose in 'a_{n-1}' ways.

∴ No. of binary seq. possible = a_{n-1} (if we take first bit '1').

Case 2) → If the first bit is '0', then second bit should be '1' and the remaining bits we can choose in 'a_{n-2}' ways.

∴ No. of binary seq. possible = a_{n-2} (if we take first bit '0').

∴ By sum rule, the recurrence relation for 'a_n' is

$$a_n = a_{n-1} + a_{n-2} \quad (n>2).$$

Q.2. Using the above recurrence relatⁿ, find the value of a_5.

how to find initial values

→ For the above recurrence relatⁿ, the initial values are

$a_1 = 2$ (binary seq. of length one with no consec. 0's).

$a_2 = \begin{matrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{matrix} = \underline{\underline{3}}$ (binary seq. of length 2 with no consec. 0's).

$\therefore$ <u>initial values</u>

$a_1 = 2$

$a_2 = 3.$

for

$\therefore n = 3, \quad a_3 = a_2 + a_1 = 5$

$n = 4, \quad a_4 = a_3 + a_2 = 5 + 3 = 8.$

$n = 5, \quad a_5 = a_4 + a_3 = 8 + 5 = \boxed{13.}$

Q.3. If a_n = no. of ternary sequences of length 'n' with even no. of 0's then recurrence relation for 'a_n' is ?

$\rightarrow$ case 1) 2 ways

$$\underset{1}{\overset{\times}{=}} \quad \overbrace{\underset{2}{\times} \quad \underset{3}{\times} \quad \cdots \cdots \quad \underset{n}{\times}}^{a_{n-1}}$$

case 1) If the first digit of the sequence is <u>not zero</u> then we can choose the first digit in 'two ways' and the remaining $n-1$ digits we can choose in 'a_{n-1}' ways.

$\therefore$ The no. of ternary seq. $= 2(a_{n-1})$ (ternary seq. of length n with first digit not 0.)

a_2 a_{n-2} $0.\ a_{n-1}$ $(4 \cdot a_{n-2})$ a_{n-1} base (5) (2)

Case 2) → If the first digit is zero, then the remaining sequence should contain odd no of 0's.

∴ The no. of ternary sequences

$$= \underset{\substack{\uparrow \\ \text{total no.} \\ \text{of sequences.}}}{3^{n-1}} - \underset{\substack{\uparrow \\ \text{no of ternary seq.} \\ \text{with even no. of seq. 0's.}}}{(a_{n-1})}$$

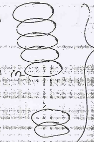
∴ Recurrence relation for a_n is

$$a_n = 2(a_{n-1}) + (3^{(n-1)} - a_{n-1})$$

$$\boxed{\therefore a_n = (a_{n-1}) + 3^{(n-1)}} \quad (n>1) \text{ and } a_1 = 2,\ a_2 = 5.$$

Q.9. If a_n = no of ways we can arrange a pile (or) stack of 'n' chips using Red, white, Green, Gold and Blue chips so that no two gold chips are together. then find the recurrence relat^n for a_n.

→ case 1) → Let the first chip is not gold.



So, the first chip we can select in 4 ways.

and the remaining (n-1) chips we can arrange in a_{n-1} ways.

$\therefore$ By product rule,

No of ways $= 4 \cdot (a_{n-1})$

(if first chip is not Gold).

Case 2) $\rightarrow$ Let the first chip is Gold, then second chip we can choose in 4 ways.

And remaining $n-2$ chips we can arrange in 'a_{n-2}' ways.

$\therefore$ By product Rule,

No. of ways $= 4 \cdot (a_{n-2})$

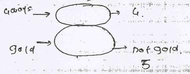
(if first chip is Gold).

$\therefore$ Now, by sum Rule,

$a_n =$ case 1) + case 2)

$= 4 \cdot a_{n-1} + 4 \cdot a_{n-2}$

$\boxed{\therefore a_n = 4\,(a_{n-1} + a_{n-2})}$ $(n>2)$ $a_1 = 5$, $a_2 = 24$



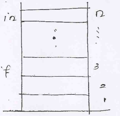
$= 20+4 = 24$

a_1 G (a_{n-1}) + a_{n-2} + a_{n-3} ways

Q-5. If a_n = no of ways a person can climb a flight of 'n' steps, if he/she can skip at most two steps at a time.

Then recurrence relatⁿ for a_n = ?

→ The person has 3 options in first move →



case 1) → In his first move, if the person covers one step. (skips no step), then the remaining steps he can climb in a_{n-1} ways.

case 2) → In his first move, if the person covers two steps by (skipping first), then the rem. steps he can climb in a_{n-2} ways.

case 3) → In his " - ", if the person covers 3 steps (skips 2), then the rem. steps " - " in a_{n-3} ways.

$\therefore a_n = a_{n-1} + a_{n-2} + a_{n-3}$. $n > 3$.

$a_1 = 1$ $a_2 = 2$

and $a_3 = 4$.