

13. Complex Numbers

Exercise 13.1

1. Question

Evaluate the following:

(i) i^{457}

(ii) i^{528}

(iii) $\frac{1}{i^{58}}$

(iv) $i^{37} + \frac{1}{i^{67}}$

(v) $\left(i^{41} + \frac{1}{i^{257}}\right)^9$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(vii) $i^{30} + i^{40} + i^{60}$

(viii) $i^{49} + i^{68} + i^{89} + i^{118}$

Answer

i. $i^{457} = i^{(456 + 1)}$

$$= i^{4(114)} \times i$$

$$= (1)^{114} \times i = i \text{ since } i^4 = 1$$

ii. $i^{528} = i^{4(132)}$

$$= (1)^{132} = 1 \text{ since } i^4 = 1$$

iii. $\frac{1}{i^{58}} = \frac{1}{i^{56+2}}$

$$= \frac{1}{i^{56} \times i^2} = \frac{1}{(i^4)^{14} \times i^2} \text{ since } i^4 = 1$$

$$= \frac{1}{i^2} = \frac{1}{-1} = -1 \text{ since } i^2 = -1$$

iv. $i^{37} + \frac{1}{i^{67}} = i^{36+1} + \frac{1}{i^{64+3}} = i + \frac{1}{i^3}$

$$[\text{since } i^4 = 1]$$

$$i^{37} + \frac{1}{i^{67}} = i + \frac{i}{i^4}$$

$$i^{37} + \frac{1}{i^{67}} = i + i = 2i$$

v. $\left(i^{41} + \frac{1}{i^{257}}\right)^9 = \left(i^{40+1} + \frac{1}{i^{256+1}}\right)^9$

$$= \left(i + \frac{1}{i}\right)^9 = (i - i) = 0$$

$$[\text{since } \frac{1}{i} = -i]$$

$$\text{vi. } (i^{77} + i^{70} + i^{87} + i^{414})^3 = (i^{(76+1)} + i^{(68+2)} + i^{(84+3)} + i^{(412+2)})^3$$

$$(i^{77} + i^{70} + i^{87} + i^{414})^3 = (i + i^2 + i^3 + i^2)^3$$

$$[\text{since } i^3 = -i, i^2 = -1]$$

$$= (i + (-1) + (-i) + (-1))^3 = (-2)^3$$

$$(i^{77} + i^{70} + i^{87} + i^{414})^3 = -8$$

$$\text{vii. } i^{30} + i^{40} + i^{60} = i^{(28+2)} + i^{40} + i^{60}$$

$$= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{15}$$

$$= i^2 + 1^{10} + 1^{15} = -1 + 1 + 1 = 1$$

$$\text{viii. } i^{49} + i^{68} + i^{89} + i^{118} = i^{(48+1)} + i^{68} + i^{(88+1)} + i^{(116+2)}$$

$$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{29} \times i^2$$

$$= i + 1 + i - 1 = 2i$$

2. Question

Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number ?

Answer

$$1 + i^{10} + i^{20} + i^{30} = 1 + i^{(8+2)} + i^{20} + i^{(28+2)}$$

$$= 1 + (i^4)^2 \times i^2 + (i^4)^5 + (i^4)^7 \times i^2$$

$$= 1 - 1 + 1 - 1 = 0$$

$$[\text{since } i^4 = 1, i^2 = -1]$$

Hence, $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3 A. Question

Find the value of following expression:

$$i^{49} + i^{68} + i^{89} + i^{110}$$

Answer

$$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48+1)} + i^{68} + i^{(88+1)} + i^{(108+2)}$$

$$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{11} \times i + (i^4)^{27} \times i^2$$

$$= i + 1 + i - 1 = 2i$$

$$[\text{since } i^4 = 1, i^2 = -1]$$

$$i^{49} + i^{68} + i^{89} + i^{110} = 2i$$

3 B. Question

Find the value of following expression:

$$i^{30} + i^{80} + i^{120}$$

Answer

$$i^{30} + i^{80} + i^{120} = i^{(28+2)} + i^{80} + i^{120}$$

$$= (i^4)^7 \times i^2 + (i^4)^{20} + (i^4)^{30}$$

$$= -1 + 1 + 1 = 1$$

$$[\text{since } i^4 = 1, i^2 = -1]$$

$$i^{30} + i^{80} + i^{120} = 1$$

3 C. Question

Find the value of following expression:

$$i + i^2 + i^3 + i^4$$

Answer

$$i + i^2 + i^3 + i^4 = i + i^2 + i^2 \times i + i^4$$

$$= i - 1 + (-1) \times i + 1$$

$$\text{since } i^4 = 1, i^2 = -1$$

$$= i - 1 - i + 1 = 0$$

3 D. Question

Find the value of following expression:

$$i^5 + i^{10} + i^{15}$$

Answer

$$i^5 + i^{10} + i^{15} = i^{(4+1)} + i^{(8+2)} + i^{(12+3)}$$

$$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^3$$

$$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^2 \times i$$

$$= 1 \times i + 1 \times (-1) + 1 \times (-1) \times i$$

$$= i - 1 - i = -1$$

3 E. Question

Find the value of following expression:

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$$

Answer

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = \frac{i^{10}(i^{582} + i^{580} + i^{578} + i^{576} + i^{574})}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = (i^4)^2 i^2 \text{ Since } i^4 = 1, i^2 = -1$$

$$= i^{10}$$

$$= i^2$$

$$= (i^2)^2 (-1)$$

3 F. Question

Find the value of following expression:

$$1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20}$$

Answer

$$1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20} = 1 + (-1) + 1 + (-1) + 1 + \dots + 1$$

$$= 1$$

3 G. Question

Find the value of following expression:

$$(1 + i)^6 + (1 - i)^3$$

Answer

$$\begin{aligned}
(1+i)^6 + (1-i)^3 &= \{(1+i)^2\}^3 + (1-i)^2(1-i) \\
&= \{1+i^2+2i\}^3 + (1+i^2-2i)(1-i) \\
&= \{1-1+2i\}^3 + (1-1-2i)(1-i) \\
&= (2i)^3 + (-2i)(1-i) \\
&= 8i^3 + (-2i) + 2i^2 \\
[\text{since } i^3 = -i, i^2 = -1] \\
&= -8i - 2i - 2 \\
&= -10i - 2 \\
&= -2(1+5i)
\end{aligned}$$

Exercise 13.2**1 A. Question**

Express the following complex numbers in the standard form $a + ib$:

$$(1+i)(1+2i)$$

Answer

Given:

$$\begin{aligned}
\Rightarrow a+ib &= (1+i)(1+2i) \\
\Rightarrow a+ib &= 1(1+2i)+i(1+2i) \\
\Rightarrow a+ib &= 1+2i+i+2i^2 \\
\text{We know that } i^2 &= -1 \\
\Rightarrow a+ib &= 1+3i+2(-1) \\
\Rightarrow a+ib &= 1+3i-2 \\
\Rightarrow a+ib &= -1+3i
\end{aligned}$$

$\therefore$ The values of a, b are $-1, 3$.

1 B. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{3+2i}{-2+i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{3+2i}{-2+i}$$

Multiplying and dividing with $-2-i$

$$\begin{aligned}
\Rightarrow a + ib &= \frac{3+2i}{-2+i} \times \frac{-2-i}{-2-i} \\
\Rightarrow a + ib &= \frac{3(-2-i)+2i(-2-i)}{(-2)^2-(i)^2}
\end{aligned}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{-6-3i-4i-2i^2}{4-i^2}$$

$$\Rightarrow a + ib = \frac{-6-7i-2(-1)}{4-(-1)}$$

$$\Rightarrow a + ib = \frac{-4-7i}{5}$$

$\therefore$ The values of a, b are $-\frac{4}{5}, -\frac{7}{5}$.

1 C. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{1}{(2+i)^2}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{1}{(2+i)^2}$$

$$\Rightarrow a + ib = \frac{1}{2^2+i^2+2(2)(i)}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1}{4-1+4i}$$

$$\Rightarrow a + ib = \frac{1}{3+4i}$$

Multiplying and dividing with 3-4i

$$\Rightarrow a + ib = \frac{1}{3+4i} \times \frac{3-4i}{3-4i}$$

$$\Rightarrow a + ib = \frac{3-4i}{3^2-(4i)^2}$$

$$\Rightarrow a + ib = \frac{3-4i}{9-16i^2}$$

$$\Rightarrow a + ib = \frac{3-4i}{9-16(-1)}$$

$$\Rightarrow a + ib = \frac{3-4i}{25}$$

$\therefore$ The values of a, b is $\frac{3}{25}, \frac{-4}{25}$.

1 D. Question

Express the following complex numbers in the standard form a + i b :

$$\frac{1-i}{1+i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{1-i}{1+i}$$

Multiplying and dividing by 1-i

$$\Rightarrow a + ib = \frac{1-i}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow a + ib = \frac{1^2 + i^2 - 2(1)(i)}{1^2 - (i)^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1+(-1)-2i}{1-(-1)}$$

$$\Rightarrow a + ib = \frac{-2i}{2}$$

$$\Rightarrow a + ib = -i$$

$$\Rightarrow a + ib = 0 - i$$

$\therefore$ The values of a, b is 0, -1.

1 E. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{(2+i)^3}{2+3i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{(2+i)^3}{2+3i}$$

$$\Rightarrow a + ib = \frac{2^3 + i^3 + 3(2)^2(i) + 3(i)^2(2)}{2+3i}$$

$$\Rightarrow a + ib = \frac{8 + (i^2 \cdot i) + 3(4)(i) + 6i^2}{2+3i}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{8 + (-1)i + 12i + 6(-1)}{2+3i}$$

$$\Rightarrow a + ib = \frac{2+11i}{2+3i}$$

Multiplying and dividing with $2-3i$

$$\Rightarrow a + ib = \frac{2+11i}{2+3i} \times \frac{2-3i}{2-3i}$$

$$\Rightarrow a + ib = \frac{2(2-3i) + 11i(2-3i)}{2^2 - (3i)^2}$$

$$\Rightarrow a + ib = \frac{4 - 6i + 22i - 33i^2}{4 - 9i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{4 + 16i - 33(-1)}{4 - 9(-1)}$$

$$\Rightarrow a + ib = \frac{37 + 16i}{13}$$

$\therefore$ The values of a, b are $\frac{37}{13}, \frac{16}{13}$.

1 F. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{(1+i)(1+\sqrt{3}i)}{1-i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{(1+i)(1+\sqrt{3}i)}{1-i}$$

$$\Rightarrow a + ib = \frac{1(1+\sqrt{3}i) + i(1+\sqrt{3}i)}{1-i}$$

$$\Rightarrow a + ib = \frac{1+\sqrt{3}i+i+\sqrt{3}i^2}{1-i}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1+(\sqrt{3}+1)i+\sqrt{3}(-1)}{1-i}$$

$$\Rightarrow a + ib = \frac{(1-\sqrt{3})+(1+\sqrt{3})i}{1-i}$$

Multiplying and dividing with $1+i$

$$\Rightarrow a + ib = \frac{(1-\sqrt{3})+(1+\sqrt{3})i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow a + ib = \frac{(1-\sqrt{3})(1+i) + (1+\sqrt{3})i(1+i)}{1^2 - i^2}$$

$$\Rightarrow a + ib = \frac{1-\sqrt{3}+(1-\sqrt{3})i+(1+\sqrt{3})i+(1+\sqrt{3})i^2}{1-(-1)}$$

$$\Rightarrow a + ib = \frac{(1-\sqrt{3})+(1-\sqrt{3}+1+\sqrt{3})i+(1+\sqrt{3})(-1)}{2}$$

$$\Rightarrow a + ib = \frac{-2\sqrt{3}+2i}{2}$$

$$\Rightarrow a + ib = -\sqrt{3} + i$$

$\therefore$ The values of a, b are $-\sqrt{3}, 1$.

1 G. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{2+3i}{4+5i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{2+3i}{4+5i}$$

Multiplying and dividing with $4-5i$

$$\Rightarrow a + ib = \frac{2+3i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$\Rightarrow a + ib = \frac{2(4-5i)+3i(4-5i)}{4^2-(5i)^2}$$

$$\Rightarrow a + ib = \frac{8-10i+12i-15i^2}{16-25i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{8+2i-15(-1)}{16-25(-1)}$$

$$\Rightarrow a + ib = \frac{23+2i}{41}$$

$\therefore$ The values of a, b are $\frac{23}{41}, \frac{2}{41}$.

1 H. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{(1-i)^3}{1-i^3}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{(1-i)^3}{1-i^3}$$

$$\Rightarrow a + ib = \frac{1^3-3(1)^2(i)+3(1)(i)^2-i^3}{1-i^3}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1-3i+3(-1)-i^3}{1-(-1)i}$$

$$\Rightarrow a + ib = \frac{-2-3i-(-1)i}{1+i}$$

$$\Rightarrow a + ib = \frac{-2-4i}{1+i}$$

Multiplying and dividing with $1-i$

$$\Rightarrow a + ib = \frac{-2-4i}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow a + ib = \frac{-2(1-i)-4i(1-i)}{1^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{-2+2i-4i+4i^2}{1-(-1)}$$

$$\Rightarrow a + ib = \frac{-2-2i+4(-1)}{2}$$

$$\Rightarrow a + ib = \frac{-6-2i}{2}$$

$$\Rightarrow a + ib = -3 - i$$

$\therefore$ The values of a, b are -3, -1.

1 I. Question

Express the following complex numbers in the standard form $a + ib$:

$$(1 + 2i)^{-3}$$

Answer

Given:

$$\Rightarrow a + ib = (1 + 2i)^{-3}$$

$$\Rightarrow a + ib = \frac{1}{(1+2i)^3}$$

$$\Rightarrow a + ib = \frac{1}{1^3 + 3(1)^2(2i) + 2(1)(2i)^2 + (2i)^3}$$

$$\Rightarrow a + ib = \frac{1}{1 + 6i + 4i^2 + 8i^3}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1}{1 + 6i + 4(-1) + 8(i^2)(i)}$$

$$\Rightarrow a + ib = \frac{1}{-3 + 6i + 8(-1)i}$$

$$\Rightarrow a + ib = \frac{1}{-3 - 2i}$$

$$\Rightarrow a + ib = \frac{-1}{3 + 2i}$$

Multiplying and dividing with $3 - 2i$

$$\Rightarrow a + ib = \frac{-1}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$

$$\Rightarrow a + ib = \frac{-3 + 2i}{3^2 - (2i)^2}$$

$$\Rightarrow a + ib = \frac{-3 + 2i}{9 - 4i^2}$$

$$\Rightarrow a + ib = \frac{-3 + 2i}{9 - 4(-1)}$$

$$\Rightarrow a + ib = \frac{-3 + 2i}{13}$$

$\therefore$ the values of a, b are $\frac{-3}{13}, \frac{2}{13}$.

1 J. Question

Express the following complex numbers in the standard form $a + ib$:

$$\frac{3 - 4i}{(4 - 2i)(1 + i)}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{3 - 4i}{(4 - 2i)(1 + i)}$$

$$\Rightarrow a + ib = \frac{3 - 4i}{4(1 + i) - 2i(1 + i)}$$

$$\Rightarrow a + ib = \frac{3 - 4i}{4 + 4i - 2i - 2i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{3 - 4i}{4 + 2i - 2(-1)}$$

$$\Rightarrow a + ib = \frac{3 - 4i}{6 + 2i}$$

Multiplying and dividing with $6 - 2i$

$$\Rightarrow a + ib = \frac{3 - 4i}{6 + 2i} \times \frac{6 - 2i}{6 - 2i}$$

$$\Rightarrow a + ib = \frac{3(6 - 2i) - 4i(6 - 2i)}{6^2 - (2i)^2}$$

$$\Rightarrow a + ib = \frac{18-6i-24i+8i^2}{36-4i^2}$$

$$\Rightarrow a + ib = \frac{18-30i+8(-1)}{36-4(-1)}$$

$$\Rightarrow a + ib = \frac{10-30i}{40}$$

$$\Rightarrow a + ib = \frac{1-3i}{4}$$

$\therefore$ The values of a, b are $\frac{1}{4}, \frac{-3}{4}$.

1 K. Question

Express the following complex numbers in the standard form $a + i b$:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i} \right) \left(\frac{3-4i}{5+i} \right)$$

Answer

Given:

$$\Rightarrow a + ib = \left(\frac{1}{1-4i} - \frac{2}{1+i} \right) \left(\frac{3-4i}{5+i} \right)$$

$$\Rightarrow a + ib = \left(\frac{1+i-2(1-4i)}{(1-4i)(1+i)} \right) \left(\frac{3-4i}{5+i} \right)$$

$$\Rightarrow a + ib = \left(\frac{1+i-2+8i}{1(1+i)-4i(1+i)} \right) \left(\frac{3-4i}{5+i} \right)$$

$$\Rightarrow a + ib = \left(\frac{-1+9i}{1+i-4i-4i^2} \right) \left(\frac{3-4i}{5+i} \right)$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \left(\frac{-1+9i}{1-3i-4(-1)} \right) \left(\frac{3-4i}{5+i} \right)$$

$$\Rightarrow a + ib = \frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}$$

$$\Rightarrow a + ib = \frac{-1(3-4i)+9i(3-i)}{5(5+i)-3i(5+i)}$$

$$\Rightarrow a + ib = \frac{-3+4i+27i-9i^2}{25+5i-15i-3i^2}$$

$$\Rightarrow a + ib = \frac{-3+31i-9(-1)}{25-10i-3(-1)}$$

$$\Rightarrow a + ib = \frac{6+31i}{28-10i}$$

Multiplying and dividing with $28+10i$

$$\Rightarrow a + ib = \frac{6+31i}{28-10i} \times \frac{28+10i}{28+10i}$$

$$\Rightarrow a + ib = \frac{6(28+10i)+31i(28+10i)}{28^2-(10i)^2}$$

$$\Rightarrow a + ib = \frac{168+60i+868i+310i^2}{784-100i^2}$$

$$\Rightarrow a + ib = \frac{168+928i+310(-1)}{784-100(-1)}$$

$$\Rightarrow a + ib = \frac{478+928i}{884}$$

∴ The values of a, b is $\frac{478}{884}, \frac{928}{884}$.

1 L. Question

Express the following complex numbers in the standard form $a + i b$:

$$\frac{5 + \sqrt{2}i}{1 - \sqrt{2}i}$$

Answer

Given:

$$\Rightarrow a + ib = \frac{5 + \sqrt{2}i}{1 - \sqrt{2}i}$$

Multiplying and dividing with $1 + \sqrt{2}i$

$$\Rightarrow a + ib = \frac{5 + \sqrt{2}i}{1 - \sqrt{2}i} \times \frac{1 + \sqrt{2}i}{1 + \sqrt{2}i}$$

$$\Rightarrow a + ib = \frac{5(1 + \sqrt{2}i) + \sqrt{2}i(1 + \sqrt{2}i)}{1^2 - (\sqrt{2}i)^2}$$

$$\Rightarrow a + ib = \frac{5 + 5\sqrt{2}i + \sqrt{2}i + 2i^2}{1 - 2i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{5 + 6\sqrt{2}i + 2(-1)}{1 - 2(-1)}$$

$$\Rightarrow a + ib = \frac{3 + 6\sqrt{2}i}{3}$$

$$\Rightarrow a + ib = 1 + 2\sqrt{2}i$$

∴ The values of a, b are 1, $2\sqrt{2}$.

2 A. Question

Find the real values of x and y, if

$$(x + i y) (2 - 3i) = 4 + i$$

Answer

Given:

$$\Rightarrow (x + iy)(2 - 3i) = 4 + i$$

$$\Rightarrow x(2 - 3i) + iy(2 - 3i) = 4 + i$$

$$\Rightarrow 2x - 3xi + 2yi - 3yi^2 = 4 + i$$

We know that $i^2 = -1$

$$\Rightarrow 2x + (-3x + 2y)i - 3y(-1) = 4 + i$$

$$\Rightarrow (2x + 3y) + (-3x + 2y)i = 4 + i$$

Equating Real and Imaginary parts on both sides, we get

$$\Rightarrow 2x + 3y = 4 \text{ and } -3x + 2y = 1$$

On solving we get,

$$\Rightarrow x = \frac{5}{13}, y = \frac{14}{13}$$

∴ The real values of x and y are $\frac{5}{13}, \frac{14}{13}$.

2 B. Question

Find the real values of x and y, if

$$(3x - 2iy)(2 + i)^2 = 10(1 + i)$$

Answer

Given:

$$\Rightarrow (3x - 2iy)(2 + i)^2 = 10(1 + i)$$

$$\Rightarrow (3x - 2iy)(2^2 + i^2 + 2(2)(i)) = 10 + 10i$$

We know that $i^2 = -1$

$$\Rightarrow (3x - 2iy)(4 + (-1) + 4i) = 10 + 10i$$

$$\Rightarrow (3x - 2iy)(3 + 4i) = 10 + 10i$$

Dividing with $3 + 4i$ on both sides

$$\Rightarrow 3x - 2yi = \frac{10 + 10i}{3 + 4i}$$

Multiplying and dividing with $3 - 4i$

$$\Rightarrow 3x - 2yi = \frac{10 + 10i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i}$$

$$\Rightarrow 3x - 2yi = \frac{10(3 - 4i) + 10i(3 - 4i)}{3^2 - (4i)^2}$$

$$\Rightarrow 3x - 2yi = \frac{30 - 40i + 30i - 40i^2}{9 - 16i^2}$$

$$\Rightarrow 3x - 2yi = \frac{30 - 10i - 40(-1)}{9 - 16(-1)}$$

$$\Rightarrow 3x - 2yi = \frac{70 - 10i}{25}$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow 3x = \frac{70}{25} \text{ and } -2y = -\frac{10}{25}$$

$$\Rightarrow x = \frac{70}{75} \text{ and } y = \frac{1}{5}$$

∴ The values of x and y are $\frac{70}{75}$ and $\frac{1}{5}$.

2 C. Question

Find the real values of x and y, if

$$\frac{(1 + i)x - 2i}{3 + i} + \frac{(2 - 3i)y + i}{3 - i} = i$$

Answer

Given:

$$\Rightarrow \frac{(1 + i)x - 2i}{3 + i} + \frac{(2 - 3i)y + i}{3 - i} = i$$

$$\Rightarrow \frac{((1 + i)x - 2i)(3 - i) + ((2 - 3i)y + i)(3 + i)}{(3 + i)(3 - i)} = i$$

$$\Rightarrow \frac{((1+i)(3-i)x) - (2i)(3-i) + ((2-3i)(3+i)y) + (i)(3+i)}{3^2 - i^2} = i$$

We know that $i^2 = -1$

$$\Rightarrow \frac{(3-i+3i-i^2)x - 6i+2i^2 + (6+2i-9i-3i^2)y + 3i+i^2}{9-(-1)} = i$$

$$\Rightarrow \frac{(3+2i-(-1))x - 6i+2(-1) + (6-7i-3(-1))y + 3i+(-1)}{10} = i$$

$$\Rightarrow (4+2i)x - 3i - 3 + (9-7i)y = 10i$$

$$\Rightarrow (4x+9y-3) + i(2x-7y-3) = 10i$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow 4x+9y-3=0 \text{ and } 2x-7y-3=10$$

$$\Rightarrow 4x+9y=3 \text{ and } 2x-7y=13$$

On solving these equations we get

$$\Rightarrow x=3 \text{ and } y=-1$$

$\therefore$ The real values of x and y are 3 and -1

2 D. Question

Find the real values of x and y, if

$$(1+i)(x+iy) = 2-5i$$

Answer

Given:

$$\Rightarrow (1+i)(x+iy) = 2-5i$$

Dividing with $1+i$ on both sides we get

$$\Rightarrow x+iy = \frac{2-5i}{1+i}$$

Multiplying and dividing with $1-i$

$$\Rightarrow x+iy = \frac{2-5i}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow x+iy = \frac{2(1-i)-5i(1-i)}{1^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow x+iy = \frac{2-2i-5i+5i^2}{1-(-1)}$$

$$\Rightarrow x+iy = \frac{2-7i+5(-1)}{2}$$

$$\Rightarrow x+iy = \frac{-3-7i}{2}$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow x = \frac{-3}{2} \text{ and } y = \frac{-7}{2}$$

$\therefore$ The real values of x and y are $\frac{-3}{2}, \frac{-7}{2}$.

3 A. Question

Find the conjugates of the following complex numbers:

$$4 - 5i$$

Answer

Given complex number is $4-5i$

We know that conjugate of a complex number $a+ib$ is $a-ib$

$\therefore$ The conjugate of $4-5i$ is $4+5i$.

3 B. Question

Find the conjugates of the following complex numbers:

$$\frac{1}{3+5i}$$

Answer

Given

complex number is $\frac{1}{3+5i}$

Let us convert this to standard form $a+ib$,

Multiplying and dividing with $3-5i$

$$\Rightarrow a + ib = \frac{1}{3+5i} \times \frac{3-5i}{3-5i}$$

$$\Rightarrow a + ib = \frac{3-5i}{3^2-(5i)^2}$$

$$\Rightarrow a + ib = \frac{3-5i}{9-25i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{3-5i}{9-25(-1)}$$

$$\Rightarrow a + ib = \frac{3-5i}{34}$$

We know that complex conjugate of a complex number $a+ib$ is $a-ib$.

$$\Rightarrow a - ib = \frac{3+5i}{34}$$

$\therefore$ The conjugate of $\frac{1}{3+5i}$ is $\frac{3+5i}{34}$.

3 C. Question

Find the conjugates of the following complex numbers:

$$\frac{1}{1+i}$$

Answer

Given complex number is $\frac{1}{1+i}$

Let us convert this to the standard form $a+ib$

Multiplying and dividing with $1-i$

$$\Rightarrow a + ib = \frac{1}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow a + ib = \frac{1-i}{1^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{1-i}{1-(-1)}$$

$$\Rightarrow a + ib = \frac{1-i}{2}$$

We know that complex conjugate of a complex number $a+ib$ is $a-ib$.

$$\Rightarrow a - ib = \frac{1+i}{2}$$

$\therefore$ The conjugate of $\frac{1}{1+i}$ is $\frac{1+i}{2}$

3 D. Question

Find the conjugates of the following complex numbers:

$$\frac{(3-i)^2}{2+i}$$

Answer

Given complex number is $\frac{(3-i)^2}{2+i}$

Let us convert this to the standard form $a+ib$

$$\Rightarrow a + ib = \frac{(3-i)^2}{2+i}$$

$$\Rightarrow a + ib = \frac{3^2 + i^2 - 2(3)(i)}{2+i}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{9+(-1)-6i}{2+i}$$

$$\Rightarrow a + ib = \frac{8-6i}{2+i}$$

Multiplying and dividing with $2-i$

$$\Rightarrow a + ib = \frac{8-6i}{2+i} \times \frac{2-i}{2-i}$$

$$\Rightarrow a + ib = \frac{8(2-i)-6i(2-i)}{2^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{16-8i-12i+6i^2}{4-(-1)}$$

$$\Rightarrow a + ib = \frac{16-20i+6(-1)}{5}$$

$$\Rightarrow a + ib = \frac{10-20i}{5}$$

$$\Rightarrow a+ib=2-4i$$

We know that the complex conjugate of a complex number $a+ib$ is $a-ib$

$$\Rightarrow a-ib=2+4i$$

∴ the conjugate of $\frac{(3-i)^2}{2+i}$ is $2+4i$.

3 E. Question

Find the conjugates of the following complex numbers:

$$\frac{(1+i)(2+i)}{3+i}$$

Answer

Given complex number is $\frac{(1+i)(2+i)}{3+i}$

Let us convert this to the standard form $a+ib$

$$\Rightarrow a + ib = \frac{(1+i)(2+i)}{3+i}$$

$$\Rightarrow a + ib = \frac{1(2+i)+i(2+i)}{3+i}$$

$$\Rightarrow a + ib = \frac{2+i+2i+i^2}{3+i}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{2+3i+(-1)}{3+i}$$

$$\Rightarrow a + ib = \frac{1+3i}{3+i}$$

Multiplying and dividing with $3-i$

$$\Rightarrow a + ib = \frac{1+3i}{3+i} \times \frac{3-i}{3-i}$$

$$\Rightarrow a + ib = \frac{1(3-i)+3i(3-i)}{3^2-i^2}$$

$$\Rightarrow a + ib = \frac{3-i+9i-3i^2}{9-(-1)}$$

$$\Rightarrow a + ib = \frac{3+8i-3(-1)}{10}$$

$$\Rightarrow a + ib = \frac{6+8i}{10}$$

We know that complex conjugate of a complex number $a+ib$ is $a-ib$

$$\Rightarrow a - ib = \frac{6-8i}{10}$$

∴ The conjugate of $\frac{(1+i)(2+i)}{3+i}$ is $\frac{6-8i}{10}$.

3 F. Question

Find the conjugates of the following complex numbers:

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$$

Answer

Given complex number is $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Let us convert this into the standard form $a+ib$

$$\Rightarrow a + ib = \frac{3(2+3i)-2i(2+3i)}{1(2-i)+2i(2-i)}$$

$$\Rightarrow a + ib = \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$$

We know that $i^2 = -1$

$$\Rightarrow a + ib = \frac{6+5i-6(-1)}{2+3i-2(-1)}$$

$$\Rightarrow a + ib = \frac{12+5i}{4+3i}$$

Multiplying and dividing with $4-3i$

$$\Rightarrow a + ib = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$\Rightarrow a + ib = \frac{12(4-3i)+5i(4-3i)}{4^2-(3i)^2}$$

$$\Rightarrow a + ib = \frac{48-36i+20i-15i^2}{16-9i^2}$$

$$\Rightarrow a + ib = \frac{48-16i-15(-1)}{16-9(-1)}$$

$$\Rightarrow a + ib = \frac{63-46i}{25}$$

We know that the complex conjugate of a complex number $a+ib$ is $a-ib$

$$\Rightarrow a - ib = \frac{63+46i}{25}$$

$\therefore$ The conjugate of complex number $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\frac{63+46i}{25}$.

4 A. Question

Find the multiplicative inverse of the following complex numbers :

$$1 - i$$

Answer

Given complex number is $Z=1-i$

We know that the multiplicative inverse of a complex number Z is Z^{-1} (or) $\frac{1}{Z}$.

$$\Rightarrow Z^{-1} = \frac{1}{1-i}$$

Multiplying and dividing with $1+i$

$$\Rightarrow Z^{-1} = \frac{1}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow Z^{-1} = \frac{1+i}{1^2-(i)^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z^{-1} = \frac{1+i}{1-(-1)}$$

$$\Rightarrow Z^{-1} = \frac{1+i}{2}$$

$\therefore$ The Multiplicative inverse of $1-i$ is $\frac{1+i}{2}$

4 B. Question

Find the multiplicative inverse of the following complex numbers :

$$(1 + i\sqrt{3})^2$$

Answer

Given complex number is $\mathbf{Z} = (1 + \sqrt{3}i)^2$

$$\Rightarrow \mathbf{Z} = 1^2 + (\sqrt{3}i)^2 + 2(1)(\sqrt{3}i)$$

$$\Rightarrow \mathbf{Z} = 1 + 3i^2 + 2\sqrt{3}i$$

We know that $i^2 = -1$

$$\Rightarrow \mathbf{Z} = 1 + 3(-1) + 2\sqrt{3}i$$

$$\Rightarrow \mathbf{Z} = -2 + 2\sqrt{3}i$$

We know that the multiplicative inverse of a complex number $\mathbf{Z}$ is $\mathbf{Z}^{-1}$ (or) $\frac{1}{\mathbf{Z}}$.

$$\Rightarrow \mathbf{Z}^{-1} = \frac{1}{-2 + 2\sqrt{3}i}$$

Multiplying and dividing with $-2 - 2\sqrt{3}i$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{1}{-2 + 2\sqrt{3}i} \times \frac{-2 - 2\sqrt{3}i}{-2 - 2\sqrt{3}i}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{-2 - 2\sqrt{3}i}{(-2)^2 - (2\sqrt{3}i)^2}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{-2 - 2\sqrt{3}i}{4 - 12i^2}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{-2 - 2\sqrt{3}i}{4 - 12(-1)}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{-2 - 2\sqrt{3}i}{16}$$

$\therefore$ The Multiplicative inverse of $(1 + \sqrt{3}i)^2$ is $\frac{-2 - 2\sqrt{3}i}{16}$.

4 C. Question

Find the multiplicative inverse of the following complex numbers :

$$4 - 3i$$

Answer

Given complex number is $\mathbf{Z} = 4 - 3i$

We know that the multiplicative inverse of a complex number $\mathbf{Z}$ is $\mathbf{Z}^{-1}$ (or) $\frac{1}{\mathbf{Z}}$.

$$\Rightarrow \mathbf{Z}^{-1} = \frac{1}{4 - 3i}$$

Multiplying and dividing with $4 + 3i$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{4 + 3i}{4^2 - (3i)^2}$$

$$\Rightarrow \mathbf{Z}^{-1} = \frac{4 + 3i}{16 - 9i^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z^{-1} = \frac{4+3i}{16-9(-1)}$$

$$\Rightarrow Z^{-1} = \frac{4+3i}{25}$$

$\therefore$ The Multiplicative inverse of $4-3i$ is $\frac{4+3i}{25}$.

4 D. Question

Find the multiplicative inverse of the following complex numbers :

$$\sqrt{5} + 3i$$

Answer

Given complex number is $Z = \sqrt{5} + 3i$

We know that the multiplicative inverse of a complex number Z is Z^{-1} (or) $\frac{1}{Z}$.

$$\Rightarrow Z^{-1} = \frac{1}{\sqrt{5}+3i}$$

Multiplying and dividing with $\sqrt{5}-3i$

$$\Rightarrow Z^{-1} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{(\sqrt{5})^2 - (3i)^2}$$

$$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{5-9i^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{5-9(-1)}$$

$$\Rightarrow Z^{-1} = \frac{\sqrt{5}-3i}{14}$$

$\therefore$ The Multiplicative inverse of $\sqrt{5}+3i$ is $\frac{\sqrt{5}-3i}{14}$.

5. Question

If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$.

Answer

Given:

$$\Rightarrow z_1 = 2-i \text{ and } z_2 = 1+i$$

We have to find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$

We know that $\left| \frac{a}{b} \right|$ is $\frac{|a|}{|b|}$

$$\Rightarrow \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \frac{|z_1 + z_2 + 1|}{|z_1 - z_2 + i|}$$

$$\Rightarrow \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \frac{|2-i+1+i+1|}{|2-i-(1+i)+i|}$$

$$\Rightarrow \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \frac{|4|}{|1-i|}$$

We know that $|a+ib|$ is $\sqrt{a^2 + b^2}$.

$$\Rightarrow \left| \frac{z_1+z_2+1}{z_1-z_2+i} \right| = \frac{\sqrt{4^2+0^2}}{\sqrt{1^2+(-1)^2}}$$

$$\Rightarrow \left| \frac{z_1+z_2+1}{z_1-z_2+i} \right| = \frac{4}{\sqrt{2}}$$

$$\Rightarrow \left| \frac{z_1+z_2+1}{z_1-z_2+i} \right| = 2\sqrt{2}$$

$\therefore$ The value of $\left| \frac{z_1+z_2+1}{z_1-z_2+i} \right|$ is $2\sqrt{2}$.

6. Question

If $z_1 = 2 - i$, $z_2 = -2 + i$, find

i. $\operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right)$

ii. $\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)$

Answer

Given:

$$\Rightarrow z_1 = 2 - i \text{ and } z_2 = -2 + i$$

i. We need to find $\operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right)$

$$\Rightarrow \operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right) = \operatorname{Re}(z_2)$$

$$\Rightarrow \operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right) = \operatorname{Re}(-2 + i)$$

$$\Rightarrow \operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right) = -2$$

$\therefore$ The Real part of $\frac{z_1 z_2}{z_1}$ is -2.

ii. We need to find $\operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right)$

We know that $z\bar{z} = |z|^2$

$$\Rightarrow z_1 \bar{z}_1 = |2 - i|^2$$

We know that for a complex number $\mathbf{Z} = a + ib$ it's magnitude is given by $|z| = \sqrt{a^2 + b^2}$

$$\Rightarrow z_1 \bar{z}_1 = \left(\sqrt{2^2 + (-1)^2} \right)^2$$

$$\Rightarrow z_1 \bar{z}_1 = 5$$

$$\Rightarrow \operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right) = \operatorname{Im} \left(\frac{1}{5} \right)$$

$$\Rightarrow \operatorname{Im} \left(\frac{1}{z_1 \bar{z}_1} \right) = \frac{1}{5}$$

$\therefore$ The Imaginary part of the $\frac{1}{z_1 \bar{z}_1}$ is $\frac{1}{5}$.

7. Question

Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Answer

Given complex number is $Z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$

$$\Rightarrow Z = \frac{(1+i)(1+i) - (1-i)(1-i)}{1^2 - i^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z = \frac{1^2 + i^2 + 2(1)(i) - (1^2 + i^2 - 2(1)(i))}{1 - (-1)}$$

$$\Rightarrow Z = \frac{4i}{2}$$

$$\Rightarrow Z = 2i$$

We know that for a complex number $Z = a + ib$ it's magnitude is given by $|Z| = \sqrt{a^2 + b^2}$

$$\Rightarrow |Z| = \sqrt{0^2 + 2^2}$$

$$\Rightarrow |Z| = 2$$

$\therefore$ The modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ is 2

8. Question

If $x + iy = \frac{a + ib}{a - ib}$, prove that $x^2 + y^2 = 1$

Answer

Given:

$$\Rightarrow x + iy = \frac{a + ib}{a - ib}$$

We know that for a complex number $Z = a + ib$ it's magnitude is given by $|Z| = \sqrt{a^2 + b^2}$

We know that $\left| \frac{a}{b} \right|$ is $\frac{|a|}{|b|}$

Applying Modulus on both sides we get,

$$\Rightarrow |x + iy| = \left| \frac{a + ib}{a - ib} \right|$$

$$\Rightarrow \sqrt{x^2 + y^2} = \frac{|a + ib|}{|a - ib|}$$

$$\Rightarrow \sqrt{x^2 + y^2} = \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + (-b)^2}}$$

$$\Rightarrow \sqrt{x^2 + y^2} = \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \sqrt{x^2 + y^2} = 1$$

Squaring on both sides

$$\Rightarrow (\sqrt{x^2 + y^2})^2 = 1^2$$

$$\Rightarrow x^2 + y^2 = 1$$

$\therefore$ Thus Proved.

9. Question

Find the least positive integral value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

Answer

Let us assume the given complex number be $Z = \left(\frac{1+i}{1-i}\right)$

Multiplying and dividing with $1+i$

$$\Rightarrow Z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow Z = \frac{(1+i)^2}{1^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z = \frac{1^2+i^2+2(1)(i)}{1-(-1)}$$

$$\Rightarrow Z = \frac{1-1+2i}{2}$$

$$\Rightarrow Z = \frac{2i}{2}$$

$$\Rightarrow Z = i$$

We know that i^{2k} is real for $k > 0$.

So, the smallest positive integral 'n' that can make $\left(\frac{1+i}{1-i}\right)^n$ real is 2.

$\therefore$ The smallest positive integral value of 'n' is 2.

10. Question

Find the real values of θ for which the complex number $\frac{1+i \cos \theta}{1-2 i \cos \theta}$ is purely real.

Answer

Let us assume the given complex number as $Z = \frac{1+i \cos \theta}{1-2 i \cos \theta}$

Multiplying and dividing with $1+2i \cos \theta$

$$\Rightarrow Z = \frac{1+i \cos \theta}{1-2 i \cos \theta} \times \frac{1+2 i \cos \theta}{1+2 i \cos \theta}$$

$$\Rightarrow Z = \frac{1(1+2 i \cos \theta)+i \cos \theta(1+2 i \cos \theta)}{1^2-(2 i \cos \theta)^2}$$

$$\Rightarrow Z = \frac{1+2 i \cos \theta+i \cos \theta+2 i^2 \cos^2 \theta}{1-4 i^2 \cos^2 \theta}$$

We know that $i^2 = -1$

$$\Rightarrow Z = \frac{1+3 i \cos \theta+2(-1) \cos^2 \theta}{1-4(-1) \cos^2 \theta}$$

$$\Rightarrow Z = \frac{1-2 \cos^2 \theta+3 i \cos \theta}{1+4 \cos^2 \theta}$$

For a complex number to be purely real, the imaginary part equals to zero.

$$\Rightarrow \frac{3 \cos \theta}{1+4 \cos^2 \theta} = 0$$

$$\Rightarrow 3\cos\theta=0 (\because 1+4\cos^2\theta\geq 1)$$

$$\Rightarrow \cos\theta=0$$

$$\Rightarrow \theta = \frac{(2n+1)\pi}{2}, \text{ for } n \in \mathbb{I}$$

$\therefore$ The values of θ to get the complex number to be purely real is $\frac{(2n+1)\pi}{2}$ for $n \in \mathbb{I}$.

11. Question

Find the smallest positive integer value of n for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number.

Answer

Let us write the given complex number as $Z = \frac{(1+i)^n}{(1-i)^{n-2}}$

Multiplying and dividing with $(1-i)^2$

$$\Rightarrow Z = \frac{(1+i)^n}{(1-i)^{n-2}} \times \frac{(1-i)^2}{(1-i)^2}$$

$$\Rightarrow Z = \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2$$

$$\Rightarrow Z = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n \times (1^2 + i^2 - 2(1)(i))$$

$$\Rightarrow Z = \left(\frac{(1+i)^2}{1^2-i^2}\right)^n \times (1+i^2-2i)$$

We know that $i^2 = -1$

$$\Rightarrow Z = \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^n \times (1+(-1)-2i)$$

$$\Rightarrow Z = \left(\frac{1-1+2i}{2}\right)^n \times (-2i)$$

$$\Rightarrow Z = \left(\frac{2i}{2}\right)^n \times (-2i)$$

$$\Rightarrow Z = i^n \times (-2i)$$

$$\Rightarrow Z = -2i^{n+1}$$

We know that i^{2k} is real for $k \geq 0$.

$\therefore$ The least positive integral of n is 1.

12. Question

If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, find (x, y)

Answer

Given:

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$$

Rationalising denominator

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^3 - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^3 = x + iy$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2-i^2} \right)^3 - \left(\frac{(1-i)^2}{1^2-i^2} \right)^3 = x + iy$$

We know that $i^2 = -1$

$$\Rightarrow \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)} \right)^3 - \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)} \right)^3 = x + iy$$

$$\Rightarrow \left(\frac{1-1+2i}{2} \right)^3 - \left(\frac{1-1-2i}{2} \right)^3 = x + iy$$

$$\Rightarrow \left(\frac{2i}{2} \right)^3 - \left(\frac{-2i}{2} \right)^3 = x + iy$$

$$\Rightarrow i^3 - (-i)^3 = x + iy$$

$$\Rightarrow 2i^3 = x + iy$$

$$\Rightarrow 2i^2 \cdot i = x + iy$$

$$\Rightarrow 2(-1)i = x + iy$$

$$\Rightarrow -2i = x + iy$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow x = 0 \text{ and } y = -2$$

$\therefore$ The values of x and y are 0 and -2.

13. Question

If $\frac{(1+i)^2}{2-i} = x + iy$, find $x + y$.

Answer

Given:

$$\Rightarrow \frac{(1+i)^2}{2-i} = x + iy$$

$$\Rightarrow \frac{1^2+i^2+2(1)(i)}{2-i} = x + iy$$

We know that $i^2 = -1$

$$\Rightarrow \frac{1+(-1)+2i}{2-i} = x + iy$$

$$\Rightarrow \frac{2i}{2-i} = x + iy$$

Multiplying and dividing with $2+i$

$$\Rightarrow \frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$$

$$\Rightarrow \frac{4i+2i^2}{2^2-i^2} = x + iy$$

$$\Rightarrow \frac{2(-1)+4i}{4-(-1)} = x + iy$$

$$\Rightarrow \frac{-2+4i}{5} = x + iy$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow x = \frac{-2}{5} \text{ and } y = \frac{4}{5}$$

$$\Rightarrow x + y = \frac{-2}{5} + \frac{4}{5}$$

$$\Rightarrow x + y = \frac{2}{5}$$

$\therefore$ The value of $x+y$ is $\frac{2}{5}$.

14. Question

If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, find (a, b) .

Answer

Given:

$$\Rightarrow \left(\frac{1-i}{1+i}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{(1-i)^2}{1^2-i^2}\right)^{100} = a + ib$$

We know that $i^2 = -1$

$$\Rightarrow \left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{1-1-2i}{2}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a + ib$$

$$\Rightarrow (-i)^{100} = a + ib$$

$$\Rightarrow i^{100} = a + ib$$

$$\Rightarrow (i^2)^{50} = a + ib$$

$$\Rightarrow (-1)^{50} = a + ib$$

$$\Rightarrow 1 = a + ib$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow a = 1 \text{ and } b = 0$$

$\therefore$ The values of a and b are 1 and 0.

15. Question

If $a = \cos \theta + i \sin \theta$, find the value of $\frac{1+a}{1-a}$.

Answer

Given:

$$\Rightarrow a = \cos \theta + i \sin \theta$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{1+\cos \theta + i \sin \theta}{1-\cos \theta - i \sin \theta}$$

We know that $1+\cos 2\theta = 2\cos^2 \theta$, $1-\cos 2\theta = 2\sin^2 \theta$ and $\sin 2\theta = 2\sin \theta \cos \theta$

$$\Rightarrow \frac{1+a}{1-a} = \frac{2 \cos^2\left(\frac{\theta}{2}\right) + i \left(2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)\right)}{2 \sin^2\left(\frac{\theta}{2}\right) - i \left(2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)\right)}$$

$$\Rightarrow \frac{1+a}{1-a} = \frac{2 \cos\left(\frac{\theta}{2}\right) \left(\cos\left(\frac{\theta}{2}\right) + i \sin\left(\frac{\theta}{2}\right)\right)}{2 \sin\left(\frac{\theta}{2}\right) \left(\sin\left(\frac{\theta}{2}\right) - i \cos\left(\frac{\theta}{2}\right)\right)}$$

$$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right) + i \sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right) - i \cos\left(\frac{\theta}{2}\right)} \times \frac{\sin\left(\frac{\theta}{2}\right) + i \cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right) + i \cos\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right) \left(\sin\left(\frac{\theta}{2}\right) + i \cos\left(\frac{\theta}{2}\right)\right) + i \sin\left(\frac{\theta}{2}\right) \left(\sin\left(\frac{\theta}{2}\right) + i \cos\left(\frac{\theta}{2}\right)\right)}{\sin^2\left(\frac{\theta}{2}\right) - i^2 \cos^2\left(\frac{\theta}{2}\right)}$$

We know that $i^2 = -1$

$$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) + i \cos^2\left(\frac{\theta}{2}\right) + i \sin^2\left(\frac{\theta}{2}\right) + i^2 \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)}{\sin^2\left(\frac{\theta}{2}\right) - (-\cos^2\left(\frac{\theta}{2}\right))}$$

$$\Rightarrow \frac{1+a}{1-a} = \tan\left(\frac{\theta}{2}\right) \times \frac{\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) + i + (-\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right))}{1}$$

$$\Rightarrow \frac{1+a}{1-a} = i \tan\left(\frac{\theta}{2}\right)$$

$\therefore$ The value of $\frac{1+a}{1-a}$ is $i \tan\left(\frac{\theta}{2}\right)$

16 A. Question

Evaluate the following :

$$2x^3 + 2x^2 - 7x + 72, \text{ when } x = \frac{3-5i}{2}$$

Answer

Given:

$$\Rightarrow x = \frac{3-5i}{2}$$

$$\Rightarrow 2x^3 + 2x^2 - 7x + 72$$

$$\Rightarrow 2\left(\frac{3-5i}{2}\right)^3 + 2\left(\frac{3-5i}{2}\right)^2 - 7\left(\frac{3-5i}{2}\right) + 72$$

$$\Rightarrow 2\left(\frac{3^3 - 3(3)^2(5i) + 3(3)(5i)^2 - (5i)^3}{8}\right) + 2\left(\frac{3^2 - 2(3)(5i) + (5i)^2}{4}\right) - \left(\frac{21-35i}{2}\right) + 72$$

We know that $i^2 = -1$

$$\Rightarrow \left(\frac{27-135i+225i^2-125i^3}{4}\right) + \left(\frac{9-30i+25i^2}{2}\right) - \left(\frac{21-35i}{2}\right) + 72$$

$$\begin{aligned}
&\Rightarrow \left(\frac{27-135i+225(-1)-125(-1)(i)}{2} \right) + \\
&\left(\frac{9-30i+25(-1)}{2} \right) - \left(\frac{21-35i}{2} \right) + 72 \\
&\Rightarrow \left(\frac{-198-10i}{4} \right) + \left(\frac{-16-30i}{2} \right) - \left(\frac{21-35i}{2} \right) + 72 \\
&\Rightarrow \left(\frac{-99-5i}{2} \right) + \left(\frac{-37+5i}{2} \right) + 72 \\
&\Rightarrow \left(\frac{-136}{2} \right) + 72 \\
&\Rightarrow -68+72 \\
&\Rightarrow 4
\end{aligned}$$

$$\therefore 2x^3+2x^2-7x+72=4$$

16 B. Question

Evaluate the following :

$$x^4 - 4x^3 + 4x^2 + 8x + 44, \text{ when } x = 3 + 2i$$

Answer

Given:

$$\begin{aligned}
&\Rightarrow x=3+2i \\
&\Rightarrow x^4-4x^3+4x^2+8x+44 \\
&\Rightarrow (3+2i)^4-4(3+2i)^3+4(3+2i)^2+8(3+2i)+44 \\
&\Rightarrow (3^4+4(3)^3(2i)+6(3)^2(2i)^2+4(3)(2i)^3+(2i)^4)-4(3^3+3(3)^2(2i)+3(3)(2i)^2+(2i)^3)+4(3^2+(2i)^2+2(3)(2i))+24+16i+44 \\
&\Rightarrow 81+216i+216i^2+96i^3+16i^4-108-216i-144i^2-32i^3+36+16i^2+48i+24+16i+44
\end{aligned}$$

We know that $i^2=-1$

$$\begin{aligned}
&\Rightarrow 77+64i+88i^2+64i^3+16i^4 \\
&\Rightarrow 77+64i+88(-1)+64(-1)(i)+16(-1)^2 \\
&\Rightarrow 5
\end{aligned}$$

$$\therefore x^4-4x^3+4x^2+8x+44=5$$

16 C. Question

Evaluate the following :

$$x^4 + 4x^3 + 6x^2 + 4x + 9, \text{ when } x = -1 + i\sqrt{2}$$

Answer

Given:

$$\begin{aligned}
&\Rightarrow x=-1+i\sqrt{2} \\
&\Rightarrow x+1=i\sqrt{2} \\
&\Rightarrow (x+1)^4=(i\sqrt{2})^4 \\
&\Rightarrow x^4+4x^3+6x^2+4x+1=2i^4
\end{aligned}$$

We know that $i^2=-1$

$$\Rightarrow x^4 + 4x^3 + 6x^2 + 4x + 1 = 2(-1)^2$$

$$\Rightarrow x^4 + 4x^3 + 6x^2 + 4x + 1 = 2$$

$$\Rightarrow x^4 + 4x^3 + 6x^2 + 4x + 1 + 8 = 2 + 8$$

$$\Rightarrow x^4 + 4x^3 + 6x^2 + 4x + 9 = 10$$

$$\therefore x^4 + 4x^3 + 6x^2 + 4x + 9 = 10$$

16 D. Question

Evaluate the following :

$$x^6 + x^4 + x^2 + 1, \text{ when } x = \frac{1+i}{\sqrt{2}}.$$

Answer

Given:

$$\Rightarrow x = \frac{1+i}{\sqrt{2}}$$

$$\Rightarrow x^6 + x^4 + x^2 + 1$$

$$\Rightarrow x^4(x^2 + 1) + 1(x^2 + 1)$$

$$\Rightarrow (x^4 + 1)(x^2 + 1)$$

$$\Rightarrow \left(\left(\frac{1+i}{\sqrt{2}} \right)^4 + 1 \right) \left(\left(\frac{1+i}{\sqrt{2}} \right)^2 + 1 \right)$$

$$\Rightarrow \left(\frac{1^4 + 4(1)^3(i) + 6(1)^2(i)^2 + 4(1)(i)^3 + i^4}{4} + 1 \right) +$$

$$1 \left(\left(\frac{1^2 + i^2 + 2(i)(1)}{2} \right) + 1 \right)$$

$$\Rightarrow \left(\left(\frac{1 + 4i + 6i^2 + 4i^3 + i^4}{4} \right) + 1 \right) \left(\left(\frac{1 + 2i + i^2}{2} \right) + 1 \right)$$

We know that $i^2 = -1$

$$\Rightarrow \left(\frac{1 + 4i + 6(-1) + 4(-1)i + (-1)^2}{4} \right) +$$

$$1 \left(\left(\frac{1 + 2i + (-1)}{2} \right) + 1 \right)$$

$$\Rightarrow \left(\left(\frac{-4}{4} \right) + 1 \right) \left(\frac{2i}{2} + 1 \right)$$

$$\Rightarrow (-1 + 1)(i + 1)$$

$$\Rightarrow (0)(i + 1)$$

$$\Rightarrow 0$$

$$\therefore x^6 + x^4 + x^2 + 1 = 0$$

16 E. Question

Evaluate the following :

$$2x^4 + 5x^3 + 7x^2 - x + 41, \text{ when } x = -2 - \sqrt{3}i$$

Answer

Given:

$$\Rightarrow x = -2 - \sqrt{3}i$$

$$\Rightarrow 2x^4 + 5x^3 + 7x^2 - x + 41$$

$$\Rightarrow 2(-2 - \sqrt{3}i)^4 + 5(-2 - \sqrt{3}i)^3 + 7(-2 - \sqrt{3}i)^2 - (-2 - \sqrt{3}i) + 41$$

$$\Rightarrow 2(2^4 + 4(2)^3(\sqrt{3}i) + 6(2)^2(\sqrt{3}i)^2 + 4(2)(\sqrt{3}i)^3 + (\sqrt{3}i)^4) - 5(2^3 + 3(2)^2(\sqrt{3}i) + 3(2)(\sqrt{3}i)^2 + (\sqrt{3}i)^3) + 7(2^2 + 2(2)(\sqrt{3}i) + (\sqrt{3}i)^2) + 2 + \sqrt{3}i + 41$$

$$\Rightarrow 16 + 64\sqrt{3}i + 144i^2 + 48\sqrt{3}i^3 + 18i^4 - 40 - 60\sqrt{3}i - 90i^2 - 15\sqrt{3}i^3 + 28 + 28\sqrt{3}i + 21i^2 + \sqrt{3}i + 43$$

We know that $i^2 = -1$

$$\Rightarrow 127 + 33\sqrt{3}i + 75i^2 + 33\sqrt{3}i^3 + 18i^4$$

$$\Rightarrow 127 + 33\sqrt{3}i + 75(-1) + 33\sqrt{3}(-1)(i) + 18(-1)^2$$

$$\Rightarrow 70$$

$$\therefore 2x^4 + 5x^3 + 7x^2 - x + 41 = 70$$

17. Question

For a positive integer n , find the value of $(1-i)^n \left(1 - \frac{1}{i}\right)^n$.

Answer

Given:

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = \left((1-i)\left(1 - \frac{1}{i}\right)\right)^n$$

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = \left(\frac{(1-i)(i-1)}{i}\right)^n$$

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = \left(\frac{i-1-i^2+i}{i}\right)^n$$

We know that $i^2 = -1$

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = \left(\frac{2i-1-(-1)}{i}\right)^n$$

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = \left(\frac{2i}{i}\right)^n$$

$$\Rightarrow (1-i)^n \left(1 - \frac{1}{i}\right)^n = 2^n$$

$$\therefore \text{The values of } (1-i)^n \left(1 - \frac{1}{i}\right)^n = 2^n.$$

18. Question

If $(1+i)Z = (1-i)\bar{Z}$, then show that $Z = -i\bar{Z}$.

Answer

Given:

$$\Rightarrow (1+i)Z = (1-i)\bar{Z}$$

Dividing with $(1+i)$ on both sides we get,

$$\Rightarrow \frac{(1+i)z}{1+i} = \frac{(1-i)\bar{z}}{1+i}$$

$$\Rightarrow z = \frac{1-i}{1+i} \times \frac{1-i}{1-i} \bar{z}$$

$$\Rightarrow z = \frac{(1-i)^2}{1^2-i^2} \bar{z}$$

We know that $i^2 = -1$

$$\Rightarrow z = \frac{1^2+i^2-2(1)(i)}{1-(-1)} \bar{z}$$

$$\Rightarrow z = \frac{1-(-1)-2i}{2} \bar{z}$$

$$\Rightarrow z = \frac{-2i}{2} \bar{z}$$

$$\Rightarrow z = -i\bar{z}$$

$\therefore$ Thus proved

19. Question

Solve the system of equations $\operatorname{Re}(z^2) = 0$, $|z| = 2$.

Answer

Given:

$$\Rightarrow \operatorname{Re}(z^2) = 0 \text{ and } |z| = 2$$

Let us assume $Z = x + iy$

$$\Rightarrow \operatorname{Re}(z^2) = 0$$

$$\Rightarrow \operatorname{Re}((x+iy)^2) = 0$$

$$\Rightarrow \operatorname{Re}(x^2 + (iy)^2 + 2(x)(iy)) = 0$$

$$\Rightarrow \operatorname{Re}(x^2 + i^2y^2 + i(2xy)) = 0$$

We know that $i^2 = -1$

$$\Rightarrow \operatorname{Re}(x^2 - y^2 + i(2xy)) = 0$$

$$\Rightarrow x^2 - y^2 = 0 \text{-----(1)}$$

$$\Rightarrow |z| = 2$$

$$\Rightarrow \sqrt{(x^2 + y^2)} = 2$$

$$\Rightarrow (x^2 + y^2) = 2^2$$

$$\Rightarrow (x^2 + y^2) = 4 \text{-----(2)}$$

Solving (1) and (2) we get

$$\Rightarrow x = \sqrt{2} \text{ and } y = \sqrt{2}.$$

$$\therefore Z = \sqrt{2} + i\sqrt{2}.$$

20. Question

If $\frac{z-1}{z+1}$ is purely imaginary number ($z \neq -1$), find the value of $|z|$.

Answer

Given:

$\Rightarrow \frac{z-1}{z+1}$ is purely imaginary

$\Rightarrow$ Let us assume $\frac{z-1}{z+1} = ki$, where K is any real number

Let us assume $z=x+iy$

$$\Rightarrow \frac{x+iy-1}{x+iy+1} = ki$$

Multiplying and dividing with $(x+1)-iy$

$$\Rightarrow \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = ki$$

$$\Rightarrow \frac{((x-1)(x+1)) - ((x-1)(iy)) + ((x+1)(iy)) - ((iy)(iy))}{(x+1)^2 - (iy)^2} =$$
$$ki$$

$$\Rightarrow \frac{x^2-1-i(xy-y)+i(xy+y)-i^2y^2}{x^2+2x+1-i^2y^2} = ki$$

We know that $i^2=-1$

$$\Rightarrow \frac{x^2-(-y^2)-1+i(2y)}{x^2+2x+1-(-y^2)} = ki$$

$$\Rightarrow \frac{x^2+y^2+1+i(2y)}{x^2+y^2+2x+1} = ki$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow \frac{x^2+y^2-1}{x^2+y^2+2x+1} = 0$$

$$\Rightarrow x^2+y^2-1=0$$

$$\Rightarrow x^2+y^2=1$$

$$\Rightarrow \sqrt{x^2+y^2} = \sqrt{1}$$

$$\Rightarrow |z|=1$$

$$\therefore |z|=1$$

21. Question

If z_1 is a complex number other than -1 such that $|z_1| = 1$ and $z_2 = \frac{z_1-1}{z_1+1}$, then show that the real parts of z_2 is zero.

Answer

Given:

$$\Rightarrow |Z_1| = 1 \text{ and } z_2 = \frac{z_1-1}{z_1+1}$$

Let us assume $z_1=x+iy$

$$\Rightarrow |Z_1|=1$$

$$\Rightarrow \sqrt{x^2+y^2} = 1$$

$$\Rightarrow x^2+y^2=1 \text{-----(1)}$$

$$\Rightarrow z_2 = \frac{z_1-1}{z_1+1}$$

$$\Rightarrow Z_2 = \frac{x+iy-1}{x+iy+1}$$

$$\Rightarrow Z_2 = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$Z_2 = \frac{((x-1)(x+1)) - ((iy)(x-1)) + ((iy)(x+1)) - ((iy)(iy))}{(x+1)^2 - (iy)^2}$$

$$\Rightarrow Z_2 = \frac{x^2 - 1 + i(-xy + y + xy + y) - i^2 y^2}{x^2 + 2x + 1 - i^2 y^2}$$

We know that $i^2 = -1$

$$\Rightarrow Z_2 = \frac{x^2 - (-y^2) - 1 + i(2y)}{x^2 - (-y^2) + 2x + 1}$$

$$\Rightarrow Z_2 = \frac{x^2 + y^2 - 1 + i(2y)}{x^2 + y^2 + 2x + 1}$$

$$\Rightarrow Z_2 = \frac{1 - 1 + i(2y)}{1 + 1 + 2x}$$

$$\Rightarrow Z_2 = \frac{i(2y)}{2 + 2x}$$

$$\Rightarrow Z_2 = \frac{iy}{1 + x}$$

$\therefore z_2$ is an imaginary one.

22. Question

If $|z + 1| = z + 2(1 + i)$, find z .

Answer

Given:

$$\Rightarrow |z + 1| = z + 2(1 + i)$$

Let us assume $z = x + iy$

$$\Rightarrow |x + iy + 1| = x + iy + 2 + 2i$$

$$\Rightarrow \sqrt{(x + 1)^2 + y^2} = (x + 2) + i(y + 2)$$

Equating Real and Imaginary parts on both sides

$$\Rightarrow y + 2 = 0$$

$$\Rightarrow y = -2 \text{-----(1)}$$

$$\Rightarrow \sqrt{(x + 1)^2 + y^2} = x + 2$$

$$\Rightarrow (x + 1)^2 + y^2 = (x + 2)^2$$

$$\Rightarrow x^2 + 2x + 1 + (-2)^2 = x^2 + 4x + 4$$

$$\Rightarrow 2x = 1 + 4 - 4$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\therefore z = \frac{1}{2} - 2i.$$

23. Question

Solve the equation $|z| = z + 1 + 2i$.

Answer

Given:

$$\Rightarrow |z| = z + 1 + 2i$$

Let us assume $z = x + iy$

$$\Rightarrow |x + iy| = x + iy + 1 + 2i$$

$$\Rightarrow \sqrt{x^2 + y^2} = (x + 1) + i(y + 2)$$

Equating Real and Imaginary parts on both sides we get

$$\Rightarrow y + 2 = 0$$

$$\Rightarrow y = -2 \text{-----(1)}$$

$$\Rightarrow \sqrt{x^2 + y^2} = (x + 1)$$

$$\Rightarrow x^2 + (-2)^2 = (x + 1)^2$$

$$\Rightarrow x^2 + 4 = x^2 + 2x + 1$$

$$\Rightarrow 2x = 3$$

$$\Rightarrow x = \frac{3}{2}$$

$$\therefore z = \frac{3}{2} - 2i.$$

24. Question

What is the smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$?

Answer

Given:

$$\Rightarrow (1 + i)^{2n} = (1 - i)^{2n}$$

$$\Rightarrow ((1 + i)^2)^n = ((1 - i)^2)^n$$

$$\Rightarrow (1^2 + i^2 + 2(1)(i))^n = (1^2 + i^2 - 2(1)(i))^n$$

We know that $i^2 = -1$

$$\Rightarrow (1 - 1 + 2i)^n = (1 - 1 - 2i)^n$$

$$\Rightarrow (2i)^n = (-2i)^n$$

We can see that the Relation holds only when n is an even integer.

$\therefore$ The smallest positive integer n is 2.

25. Question

If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$, then find the value of $|z_1 + z_2 + z_3|$.

Answer

Given:

$$\Rightarrow |z_1| = |z_2| = |z_3| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = 1$$

$$\Rightarrow |z_1 + z_2 + z_3| = \left| \frac{z_1 \bar{z}_1}{\bar{z}_1} + \frac{z_2 \bar{z}_2}{\bar{z}_2} + \frac{z_3 \bar{z}_3}{\bar{z}_3} \right|$$

We know that $z\bar{z}=|z|^2$

$$\Rightarrow |z_1 + z_2 + z_3| = \left| \frac{|z_1|^2}{\bar{z}_1} + \frac{|z_2|^2}{\bar{z}_2} + \frac{|z_3|^2}{\bar{z}_3} \right|$$

$$\Rightarrow |z_1 + z_2 + z_3| = \left| \frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3} \right|$$

We know that $|z|=|\bar{z}|$

$$\Rightarrow |z_1 + z_2 + z_3| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right|$$

$$\Rightarrow |z_1 + z_2 + z_3| = 1$$

$$\therefore |z_1 + z_2 + z_3| = 1.$$

26. Question

Find the number of solutions of $z^2 + |z|^2 = 0$.

Answer

Given:

$$\Rightarrow z^2 + |z|^2 = 0$$

Let us assume $z = x + iy$

$$\Rightarrow (x + iy)^2 + (\sqrt{x^2 + y^2})^2 = 0$$

$$\Rightarrow x^2 + (iy)^2 + 2(x)(iy) + x^2 + y^2 = 0$$

$$\Rightarrow 2x^2 + y^2 + i^2 y^2 + i2xy = 0$$

We know that $i^2 = -1$

$$\Rightarrow 2x^2 + y^2 - y^2 + i2xy = 0$$

$$\Rightarrow 2x^2 + i2xy = 0$$

Equating Real and Imaginary parts on both sides we get,

$$\Rightarrow 2x^2 = 0 \text{ and } 2xy = 0$$

$$\Rightarrow x = 0 \text{ and } y \in \mathbb{R}$$

$\therefore z = 0 + iy$ where $y \in \mathbb{R}$. i.e., Infinite solutions.

Exercise 13.3

1 A. Question

Find the square root of the following complex numbers :

$$-5 + 12i$$

Answer

Given:

$$\Rightarrow x + iy = -5 + 12i$$

Here $y > 0$

We know that for a complex number $z = a + ib$

$$\sqrt{a+ib} =$$

$$\Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\sqrt{-5+12i} = \pm \left[\left(\frac{-5+\sqrt{(-5)^2+12^2}}{2} \right)^{\frac{1}{2}} + \right.$$

$$\Rightarrow \left. i \left(\frac{5+\sqrt{(-5)^2+12^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-5+12i} = \pm \left[\left(\frac{-5+\sqrt{25+144}}{2} \right)^{\frac{1}{2}} + \right.$$

$$\Rightarrow \left. i \left(\frac{5+\sqrt{25+144}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-5+12i} = \pm \left[\left(\frac{-5+\sqrt{169}}{2} \right)^{\frac{1}{2}} + \right.$$

$$\Rightarrow \left. i \left(\frac{5+\sqrt{169}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm \left[\left(\frac{-5+13}{2} \right)^{\frac{1}{2}} + i \left(\frac{5+13}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm \left[\left(\frac{8}{2} \right)^{\frac{1}{2}} + i \left(\frac{18}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm \left[4^{\frac{1}{2}} + i 9^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-5+12i} = \pm [2+3i]$$

$$\therefore \sqrt{-5+12i} = \pm [2+3i].$$

1 B. Question

Find the square root of the following complex numbers :

$$-7-24i$$

Answer

Given:

$$\Rightarrow x+iy=-7+24i$$

Here $y < 0$

We know that for a complex number $z=a+ib$

$$\sqrt{a+ib} =$$

$$\Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\sqrt{-7-24i} = \pm \left[\left(\frac{-7 + \sqrt{(-7)^2 + (-24)^2}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow \left. i \left(\frac{7 + \sqrt{(-7)^2 + (-24)^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-7-24i} = \pm \left[\left(\frac{-7 + \sqrt{49+576}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow \left. i \left(\frac{7 + \sqrt{49+576}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-7-24i} = \pm \left[\left(\frac{-7 + \sqrt{625}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow \left. i \left(\frac{7 + \sqrt{625}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm \left[\left(\frac{-7+25}{2} \right)^{\frac{1}{2}} - i \left(\frac{7+25}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm \left[\left(\frac{18}{2} \right)^{\frac{1}{2}} - i \left(\frac{32}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm \left[9^{\frac{1}{2}} - i 16^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-7-24i} = \pm [3 - 4i]$$

$$\therefore \sqrt{-7-24i} = \pm [3 - 4i]$$

1 C. Question

Find the square root of the following complex numbers :

$$1 - i$$

Answer

Given:

$$\Rightarrow x+iy=1-i$$

Here $y < 0$

We know that for a complex number $z=a+ib$

$$\sqrt{a+ib} = \\ \Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\sqrt{1-i} = \pm \left[\left(\frac{1+\sqrt{(1)^2+(-1)^2}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow \left. i \left(\frac{-1+\sqrt{(1)^2+(-1)^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1-i} = \pm \left[\left(\frac{1+\sqrt{1+1}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-1+\sqrt{1+1}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1-i} = \pm \left[\left(\frac{1+\sqrt{2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-1+\sqrt{2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\therefore \sqrt{-5+12i} = \pm \left[\left(\frac{1+\sqrt{2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-1+\sqrt{2}}{2} \right)^{\frac{1}{2}} \right].$$

1 D. Question

Find the square root of the following complex numbers :

$$-8 - 6i$$

Answer

Given:

$$\Rightarrow x+iy=-8-6i$$

Here $y < 0$

We know that for a complex number $z=a+ib$

$$\begin{aligned} \sqrt{a+ib} &= \\ \Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} \sqrt{-8-6i} &= \pm \left[\left(\frac{-8+\sqrt{(-8)^2+(-6)^2}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{8+\sqrt{(-8)^2+(-6)^2}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\begin{aligned} \sqrt{-8-6i} &= \pm \left[\left(\frac{-8+\sqrt{64+36}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{8+\sqrt{64+36}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\begin{aligned} \sqrt{-8-6i} &= \pm \left[\left(\frac{-8+\sqrt{100}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{8+\sqrt{100}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\Rightarrow \sqrt{-8-6i} = \pm \left[\left(\frac{-8+10}{2} \right)^{\frac{1}{2}} - i \left(\frac{8+10}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-8-6i} = \pm \left[\left(\frac{2}{2} \right)^{\frac{1}{2}} - i \left(\frac{18}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-8-6i} = \pm \left[1^{\frac{1}{2}} - i 9^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-8-6i} = \pm [1 - 3i]$$

$$\therefore \sqrt{-8-6i} = \pm [1 - 3i].$$

1 E. Question

Find the square root of the following complex numbers :

$$8 - 15i$$

Answer

Given:

$$\Rightarrow x+iy=8-15i$$

Here $y < 0$

We know that for a complex number $z=a+ib$

$$\begin{aligned} \sqrt{a+ib} &= \\ \Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} \sqrt{8-15i} &= \pm \left[\left(\frac{8+\sqrt{8^2+(-15)^2}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{-8+\sqrt{8^2+(-15)^2}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\begin{aligned} \sqrt{8-15i} &= \pm \left[\left(\frac{8+\sqrt{64+225}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{-8+\sqrt{64+225}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\begin{aligned} \sqrt{8-15i} &= \pm \left[\left(\frac{8+\sqrt{289}}{2} \right)^{\frac{1}{2}} - \right. \\ \Rightarrow & \left. i \left(\frac{-8+\sqrt{289}}{2} \right)^{\frac{1}{2}} \right] \end{aligned}$$

$$\Rightarrow \sqrt{8-15i} = \pm \left[\left(\frac{8+17}{2} \right)^{\frac{1}{2}} - i \left(\frac{-8+17}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{8-15i} = \pm \left[\left(\frac{25}{2} \right)^{\frac{1}{2}} - i \left(\frac{9}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{8-15i} = \pm \left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}} \right]$$

$$\therefore \sqrt{8-15i} = \pm \left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}} \right].$$

1 F. Question

Find the square root of the following complex numbers :

$$-11 - 60\sqrt{-1}$$

Answer

Given:

$$\Rightarrow x+iy = -11 - 60\sqrt{-1}$$

$$\Rightarrow x+iy = -11-60i$$

Here $y < 0$

We know that for a complex number $z = a + ib$

$$\sqrt{a + ib} = \begin{cases} \pm \left[\left(\frac{a + \sqrt{a^2 + b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a + \sqrt{a^2 + b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a + \sqrt{a^2 + b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a + \sqrt{a^2 + b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\sqrt{-11 - 60i} = \pm \left[\left(\frac{-11 + \sqrt{(-11)^2 + (-60)^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{11 + \sqrt{(-11)^2 + (60)^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-11 - 60i} = \pm \left[\left(\frac{-11 + \sqrt{121 + 3600}}{2} \right)^{\frac{1}{2}} - i \left(\frac{11 + \sqrt{121 + 3600}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-11 - 60i} = \pm \left[\left(\frac{-11 + \sqrt{3721}}{2} \right)^{\frac{1}{2}} - i \left(\frac{11 + \sqrt{3721}}{2} \right)^{\frac{1}{2}} \right]$$

$$\sqrt{-11 - 60i} = \pm \left[\left(\frac{-11 + 61}{2} \right)^{\frac{1}{2}} - i \left(\frac{11 + 61}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-11 - 60i} = \pm \left[\left(\frac{50}{2} \right)^{\frac{1}{2}} - i \left(\frac{72}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-11 - 60i} = \pm \left[25^{\frac{1}{2}} - i 36^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-11 - 60i} = \pm [5 - 6i]$$

$$\therefore \sqrt{-11 - 60i} = \pm [5 - 6i]$$

1 G. Question

Find the square root of the following complex numbers :

$$1 + 4\sqrt{-3}$$

Answer

Given:

$$\Rightarrow x + iy = 1 + 4\sqrt{-3}$$

$$\Rightarrow x + iy = 1 + 4(\sqrt{3})(\sqrt{-1})$$

$$\Rightarrow x + iy = 1 + 4\sqrt{3}i$$

Here $y > 0$

We know that for a complex number $z=a+ib$

$$\sqrt{a+ib} = \Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\sqrt{1+4\sqrt{3}i} = \pm \left[\left(\frac{1+\sqrt{1^2+(4\sqrt{3})^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-1+\sqrt{1^2+(4\sqrt{3})^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm \left[\left(\frac{1+\sqrt{1+48}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-1+\sqrt{1+48}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm \left[\left(\frac{1+\sqrt{49}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-1+\sqrt{49}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm \left[\left(\frac{1+7}{2} \right)^{\frac{1}{2}} + i \left(\frac{-1+7}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm \left[\left(\frac{8}{2} \right)^{\frac{1}{2}} + i \left(\frac{6}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm \left[4^{\frac{1}{2}} + i 3^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{1+4\sqrt{3}i} = \pm [2 + \sqrt{3}i]$$

$$\therefore \sqrt{1+4\sqrt{3}i} = \pm [2 + \sqrt{3}i]$$

1 H. Question

Find the square root of the following complex numbers :

$$4i$$

Answer

Given:

$$\Rightarrow x+iy=4i$$

Here $y>0$

We know that for a complex number $z=a+ib$

$$\sqrt{a+ib} =$$

$$\Rightarrow \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\Rightarrow \sqrt{4i} = \pm \left[\left(\frac{0+\sqrt{0^2+4^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{0+\sqrt{0^2+4^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm \left[\left(\frac{0+\sqrt{0+16}}{2} \right)^{\frac{1}{2}} + i \left(\frac{0+\sqrt{0+16}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm \left[\left(\frac{0+\sqrt{16}}{2} \right)^{\frac{1}{2}} + i \left(\frac{0+\sqrt{16}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm \left[\left(\frac{0+4}{2} \right)^{\frac{1}{2}} + i \left(\frac{0+4}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm \left[\left(\frac{4}{2} \right)^{\frac{1}{2}} + i \left(\frac{4}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm \left[2^{\frac{1}{2}} + i 2^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{4i} = \pm [\sqrt{2} + \sqrt{2}i]$$

$$\therefore \sqrt{4i} = \pm [\sqrt{2} + \sqrt{2}i].$$

1 I. Question

Find the square root of the following complex numbers :

$$-i$$

Answer

Given:

$$\Rightarrow x+iy=-i$$

Here $y < 0$

We know that for a complex number $z=a+ib$

$$\Rightarrow \sqrt{a+ib} = \begin{cases} \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} + i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b > 0 \\ \pm \left[\left(\frac{a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{-a+\sqrt{a^2+b^2}}{2} \right)^{\frac{1}{2}} \right], & \text{if } b < 0 \end{cases}$$

$$\Rightarrow \sqrt{-i} = \pm \left[\left(\frac{0+\sqrt{0^2+(-1)^2}}{2} \right)^{\frac{1}{2}} - i \left(\frac{0+\sqrt{0^2+(-1)^2}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-i} = \pm \left[\left(\frac{0+\sqrt{0+1}}{2} \right)^{\frac{1}{2}} - i \left(\frac{0+\sqrt{0+1}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-i} = \pm \left[\left(\frac{0+\sqrt{1}}{2} \right)^{\frac{1}{2}} - i \left(\frac{0+\sqrt{1}}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-i} = \pm \left[\left(\frac{0+1}{2} \right)^{\frac{1}{2}} - i \left(\frac{0+1}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-i} = \pm \left[\left(\frac{1}{2} \right)^{\frac{1}{2}} - i \left(\frac{1}{2} \right)^{\frac{1}{2}} \right]$$

$$\Rightarrow \sqrt{-i} = \pm \left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right]$$

$$\therefore \sqrt{-i} = \pm \left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right].$$

Exercise 13.4

1 A. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$1 + i$$

Answer

Given complex number is $Z=1+i$

We know that the polar form of a complex number $Z=x+iy$ is given by $Z=|Z|(\cos\theta+isin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{1^2 + 1^2}$$

$$\Rightarrow |z| = \sqrt{1+1}$$

$$\Rightarrow |z| = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{1} \right)$$

Since $x>0, y>0$ complex number lies in 1st quadrant and the value of θ will be as follows $0^0 \leq \theta \leq 90^0$.

$$\Rightarrow \theta = \tan^{-1}(1)$$

$$\Rightarrow \theta = \frac{\pi}{4}.$$

$$\Rightarrow Z = \sqrt{2} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)$$

$$\therefore \text{The Polar form of } Z=1+i \text{ is } z = \sqrt{2} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right).$$

1 B. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\sqrt{3} + i$$

Answer

Given Complex number is $Z=\sqrt{3}+i$

We know that the polar form of a complex number $Z=x+iy$ is given by $Z=|Z|(\cos\theta+isin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{(\sqrt{3})^2 + 1^2}$$

$$\Rightarrow |z| = \sqrt{3 + 1}$$

$$\Rightarrow |z| = \sqrt{4}$$

$$\Rightarrow |Z| = 2$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

Since $x > 0, y > 0$ complex number lies in 1st quadrant and the value of θ will be as follows $0^0 \leq \theta \leq 90^0$.

$$\Rightarrow \theta = \frac{\pi}{6}.$$

$$\Rightarrow Z = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right)$$

$$\therefore \text{The Polar form of } Z = \sqrt{3} + i \text{ is } z = 2 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right).$$

1 C. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$1 - i$$

Answer

Given complex number is $z = 1 - i$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{1^2 + (-1)^2}$$

$$\Rightarrow |z| = \sqrt{1 + 1}$$

$$\Rightarrow |z| = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{1} \right)$$

Since $x > 0, y < 0$ complex number lies in 4th quadrant and the value of θ will be as follows $-90^0 \leq \theta \leq 0^0$.

$$\Rightarrow \theta = \tan^{-1}(1)$$

$$\Rightarrow \theta = \frac{-\pi}{4}.$$

$$\Rightarrow Z = \sqrt{2} \left(\cos\left(\frac{-\pi}{4}\right) + i \sin\left(\frac{-\pi}{4}\right) \right)$$

$$\Rightarrow z = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \right)$$

$$\therefore \text{The Polar form of } Z=1+i \text{ is } z = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \right).$$

1 D. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{1-i}{1+i}$$

Answer

$$\text{Given complex number is } z = \frac{1-i}{1+i}$$

$$\Rightarrow z = \frac{1-i}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow z = \frac{(1-i)^2}{1^2 - i^2}$$

We know that $i^2 = -1$

$$\Rightarrow z = \frac{1^2 + i^2 - 2(1)(i)}{1 - (-1)}$$

$$\Rightarrow z = \frac{1 + (-1) - 2i}{2}$$

$$\Rightarrow z = \frac{-2i}{2}$$

$$\Rightarrow z = 0 - i$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{0^2 + (-1)^2}$$

$$\Rightarrow |z| = \sqrt{0 + 1}$$

$$\Rightarrow |z| = \sqrt{1}$$

$$\Rightarrow |z| = 1$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{0} \right)$$

Since $x \geq 0, y < 0$ complex number lies in 4th quadrant and the value of θ will be as follows $-90^\circ \leq \theta \leq 0^\circ$.

$$\Rightarrow \theta = \tan^{-1}(\infty)$$

$$\Rightarrow \theta = \frac{-\pi}{2}.$$

$$\Rightarrow Z = 1 \left(\cos\left(\frac{-\pi}{2}\right) + i \sin\left(\frac{-\pi}{2}\right) \right)$$

$$\Rightarrow z = 1 \left(\cos\left(\frac{\pi}{2}\right) - i \sin\left(\frac{\pi}{2}\right) \right)$$

$$\therefore \text{The Polar form of } Z = \frac{1-i}{1+i} \text{ is } z = 1 \left(\cos\left(\frac{\pi}{2}\right) - i \sin\left(\frac{\pi}{2}\right) \right).$$

1 E. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{1}{1+i}$$

Answer

$$\text{Given complex number is } z = \frac{1}{1+i}.$$

$$\Rightarrow z = \frac{1}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow z = \frac{1-i}{1^2-i^2}$$

We know that $i^2 = -1$

$$\Rightarrow z = \frac{1-i}{1-(-1)}$$

$$\Rightarrow z = \frac{1-i}{2}$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$$

$$\Rightarrow |z| = \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$\Rightarrow |z| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\frac{1}{2}}{\frac{1}{2}} \right)$$

Since $x > 0, y < 0$ complex number lies in 4th quadrant and the value of θ will be as follows $-90^\circ \leq \theta \leq 0^\circ$.

$$\Rightarrow \theta = \tan^{-1}(1)$$

$$\Rightarrow \theta = \frac{-\pi}{4}.$$

$$\Rightarrow Z = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{-\pi}{4}\right) + i \sin\left(\frac{-\pi}{4}\right) \right)$$

$$\Rightarrow Z = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \right)$$

∴ The Polar form of $Z = \frac{1}{1+i}$ is $z = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{\pi}{4}\right) - i \sin\left(\frac{\pi}{4}\right) \right)$.

1 F. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{1+2i}{1-3i}$$

Answer

Given complex number is $z = \frac{1+2i}{1-3i}$.

$$\Rightarrow z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$\Rightarrow z = \frac{1(1+3i)+2i(1+3i)}{1^2-(3i)^2}$$

$$\Rightarrow z = \frac{1+3i+2i+6i^2}{1-9i^2}$$

We know that $i^2 = -1$

$$\Rightarrow z = \frac{1+5i+6(-1)}{1-9(-1)}$$

$$\Rightarrow z = \frac{-5+5i}{10}$$

$$\Rightarrow z = \frac{-1+i}{2}$$

We know that the polar form of a complex number $Z=x+iy$ is given by $Z=|Z|(\cos\theta+i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow |z| = \sqrt{\frac{1}{4} + \frac{1}{4}}$$

$$\Rightarrow |z| = \sqrt{\frac{1}{2}}$$

$$\Rightarrow |z| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\frac{1}{2}}{\frac{1}{2}} \right)$$

Since $x < 0, y > 0$ complex number lies in 2nd quadrant and the value of θ will be as follows $90^\circ \leq \theta \leq 180^\circ$.

$$\Rightarrow \theta = \tan^{-1}(1)$$

$$\Rightarrow \theta = \frac{3\pi}{4}.$$

$$\Rightarrow Z = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$$

$$\therefore \text{The Polar form of } Z = \frac{1+2i}{1-2i} \text{ is } z = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right).$$

1 G. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\sin 120^\circ - i \cos 120^\circ$$

Answer

$$\text{Given complex number is } z = \sin 120^\circ - i \cos 120^\circ$$

$$\Rightarrow z = \frac{\sqrt{3}}{2} - i \left(\frac{-1}{2} \right)$$

$$\Rightarrow z = \frac{\sqrt{3}}{2} + i \left(\frac{1}{2} \right)$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow |z| = \sqrt{\frac{3}{4} + \frac{1}{4}}$$

$$\Rightarrow |z| = \sqrt{1}$$

$$\Rightarrow |z| = 1$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right)$$

Since $x > 0, y > 0$ complex number lies in 1st quadrant and the value of θ will be as follows $0^\circ \leq \theta \leq 90^\circ$.

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\Rightarrow Z = 1 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right)$$

$$\therefore \text{The Polar form of } Z = \sin 120^\circ - i \cos 120^\circ \text{ is } z = 1 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right).$$

1 H. Question

Find the modulus and argument of the following complex numbers and hence express each of them in the polar form :

$$\frac{-16}{1 + i\sqrt{3}}$$

Answer

Given complex number is $z = \frac{-16}{1+i\sqrt{3}}$

$$\Rightarrow z = \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1^2-(i\sqrt{3})^2}$$

$$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1-3i^2}$$

We know that $i^2 = -1$

$$\Rightarrow z = \frac{-16+i16\sqrt{3}}{1-3(-1)}$$

$$\Rightarrow z = \frac{-16+i16\sqrt{3}}{4}$$

$$\Rightarrow z = -4 + i4\sqrt{3}$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{(-4)^2 + (4\sqrt{3})^2}$$

$$\Rightarrow |z| = \sqrt{16 + 48}$$

$$\Rightarrow |z| = \sqrt{64}$$

$$\Rightarrow |z| = 8$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$$

Since $x < 0, y > 0$ complex number lies in 2nd quadrant and the value of θ will be as follows $90^\circ \leq \theta \leq 180^\circ$.

$$\Rightarrow \theta = \tan^{-1}(\sqrt{3})$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$$\Rightarrow Z = 8\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$

$\therefore$ The Polar form of $Z = \frac{-16}{1+i\sqrt{3}}$ is $z = 8\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$.

2. Question

Write $(i^{25})^3$ in polar form.

Answer

Given Complex number is $Z = (i^{25})^3$

$$\Rightarrow Z = i^{75}$$

$$\Rightarrow Z=i^{74}.i$$

$$\Rightarrow Z=(i^2)^{37}.i$$

We know that $i^2=-1$

$$\Rightarrow Z=(-1)^{37}.i$$

$$\Rightarrow Z=(-1).i$$

$$\Rightarrow Z=-i$$

$$\Rightarrow Z=0-i$$

We know that the polar form of a complex number $Z=x+iy$ is given by $Z=|Z|(\cos\theta+isin\theta)$

Where,

$$|Z|=\text{modulus of complex number}=\sqrt{x^2+y^2}$$

$$\theta=\arg(z)=\text{argument of complex number}=\tan^{-1}\left(\frac{|y|}{|x|}\right)$$

Now for the given problem,

$$\Rightarrow |z|=\sqrt{0^2+(-1)^2}$$

$$\Rightarrow |z|=\sqrt{0+1}$$

$$\Rightarrow |z|=\sqrt{1}$$

$$\Rightarrow |z|=1$$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{1}{0}\right)$$

Since $x>0, y<0$ complex number lies in 4th quadrant and the value of θ will be as follows $-90^\circ\leq\theta\leq 0^\circ$.

$$\Rightarrow \theta=\tan^{-1}(\infty)$$

$$\Rightarrow \theta=\frac{-\pi}{2}.$$

$$\Rightarrow Z=1\left(\cos\left(\frac{-\pi}{2}\right)+isin\left(\frac{-\pi}{2}\right)\right)$$

$$\Rightarrow Z=1\left(\cos\left(\frac{\pi}{2}\right)-isin\left(\frac{\pi}{2}\right)\right)$$

$$\therefore \text{The Polar form of } Z=(i^{25})^3 \text{ is } z=1\left(\cos\left(\frac{\pi}{2}\right)-isin\left(\frac{\pi}{2}\right)\right).$$

3 A. Question

Express the following complex numbers in the form $r(\cos\theta+isin\theta)$:

$$1+i\tan\alpha$$

Answer

Given Complex number is $Z=1+i\tan\alpha$

We know that the polar form of a complex number $Z=x+iy$ is given by $Z=|Z|(\cos\theta+isin\theta)$

Where,

$$|Z|=\text{modulus of complex number}=\sqrt{x^2+y^2}$$

$$\theta=\arg(z)=\text{argument of complex number}=\tan^{-1}\left(\frac{|y|}{|x|}\right)$$

We know that $\tan \alpha$ is a periodic function with period π .

We have α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Case1:

$$\Rightarrow \alpha \in \left[0, \frac{\pi}{2}\right)$$

$$\Rightarrow |z| = r = \sqrt{1^2 + \tan^2 \alpha}$$

$$\Rightarrow |z| = r = \sqrt{\sec^2 \alpha}$$

$$\Rightarrow |z| = r = |\sec \alpha|$$

Since $\sec \alpha$ is positive in the interval $\left[0, \frac{\pi}{2}\right)$

$$\Rightarrow |z| = r = \sec \alpha$$

$$\Rightarrow \theta = \arg(z) = \tan^{-1} \left(\frac{\tan \alpha}{1} \right)$$

$$\Rightarrow \theta = \tan^{-1}(\tan \alpha)$$

Since $\tan \alpha$ is positive in the interval $\left[0, \frac{\pi}{2}\right)$

$$\Rightarrow \theta = \alpha$$

$\therefore$ The polar form is $z = \sec \alpha (\cos \alpha + i \sin \alpha)$.

Case2:

$$\Rightarrow \alpha \in \left(\frac{\pi}{2}, \pi\right]$$

$$\Rightarrow |z| = r = \sqrt{1^2 + \tan^2 \alpha}$$

$$\Rightarrow |z| = r = \sqrt{\sec^2 \alpha}$$

$$\Rightarrow |z| = r = |\sec \alpha|$$

Since $\sec \alpha$ is negative in the interval $\left(\frac{\pi}{2}, \pi\right]$.

$$\Rightarrow |z| = r = -\sec \alpha$$

$$\Rightarrow \theta = \arg(z) = \tan^{-1} \left(\frac{\tan \alpha}{1} \right)$$

$$\Rightarrow \theta = \tan^{-1}(\tan \alpha)$$

Since $\tan \alpha$ is negative in the interval $\left(\frac{\pi}{2}, \pi\right]$.

$$\Rightarrow \theta = -\pi + \alpha \quad (\because \theta \text{ lies in 4th quadrant})$$

$$\Rightarrow z = -\sec \alpha (\cos(\alpha - \pi) + i \sin(\alpha - \pi))$$

$$\Rightarrow z = -\sec \alpha (-\cos \alpha - i \sin \alpha)$$

$$\Rightarrow z = \sec \alpha (\cos \alpha + i \sin \alpha)$$

$\therefore$ The polar form is $z = \sec \alpha (\cos \alpha + i \sin \alpha)$

3 B. Question

Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:

$$\tan \alpha - i$$

Answer

Given Complex number is $\tan \alpha - i$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos \theta + i \sin \theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

We know that $\tan \alpha$ is a periodic function with period π .

We have α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Case1:

$$\Rightarrow \alpha \in \left[0, \frac{\pi}{2}\right)$$

$$\Rightarrow |z| = r = \sqrt{\tan^2 \alpha + 1^2}$$

$$\Rightarrow |z| = r = \sqrt{\sec^2 \alpha}$$

$$\Rightarrow |z| = r = |\sec \alpha|$$

Since $\sec \alpha$ is positive in the interval $\left[0, \frac{\pi}{2}\right)$

$$\Rightarrow |z| = r = \sec \alpha$$

$$\Rightarrow \theta = \arg(z) = \tan^{-1} \left(\frac{1}{\tan \alpha} \right)$$

$$\Rightarrow \theta = \tan^{-1}(\cot \alpha)$$

Since $\cot \alpha$ is positive in the interval $\left[0, \frac{\pi}{2}\right)$

$$\Rightarrow \theta = \alpha - \frac{\pi}{2} \quad (\because \theta \text{ lies in 4th quadrant})$$

$$\Rightarrow z = \sec \alpha \left(\cos \left(\frac{-\pi}{2} + \alpha \right) + i \sin \left(\frac{-\pi}{2} + \alpha \right) \right)$$

$$\Rightarrow z = \sec \alpha (\sin \alpha - i \cos \alpha)$$

$\therefore$ The polar form is $z = \sec \alpha (\sin \alpha - i \cos \alpha)$

Case2:

$$\Rightarrow \alpha \in \left(\frac{\pi}{2}, \pi\right]$$

$$\Rightarrow |z| = r = \sqrt{\tan^2 \alpha + 1^2}$$

$$\Rightarrow |z| = r = \sqrt{\sec^2 \alpha}$$

$$\Rightarrow |z| = r = |\sec \alpha|$$

Since $\sec \alpha$ is negative in the interval $\left(\frac{\pi}{2}, \pi\right]$.

$$\Rightarrow |z| = r = -\sec \alpha$$

$$\Rightarrow \theta = \arg(z) = \tan^{-1}\left(\frac{1}{\tan \alpha}\right)$$

$$\Rightarrow \theta = \tan^{-1}(\cot \alpha)$$

Since $\cot \alpha$ is negative in the interval $\left(\frac{\pi}{2}, \pi\right]$.

$$\Rightarrow \theta = \frac{\pi}{2} + \alpha. (\because \theta \text{ lies in 3rd quadrant})$$

$$\Rightarrow z = -\sec \alpha \left(\cos\left(\frac{\pi}{2} + \alpha\right) + i \sin\left(\frac{\pi}{2} + \alpha\right) \right)$$

$$\Rightarrow z = -\sec \alpha (-\sin \alpha + i \cos \alpha)$$

$$\Rightarrow z = \sec \alpha (\sin \alpha - i \cos \alpha)$$

$\therefore$ The polar form is $z = \sec \alpha (\sin \alpha - i \cos \alpha)$.

3 C. Question

Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:

$$1 - \sin \alpha + i \cos \alpha$$

Answer

Given Complex number is $z = 1 - \sin \alpha + i \cos \alpha$

We know that $\sin^2 \theta + \cos^2 \theta = 1$, $\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.

$$\Rightarrow z = \left(\sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right) - 2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \right) + i \left(\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \right)$$

$$\Rightarrow z = \left(\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right) \right)^2 + i \left(\cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \right)$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos \theta + i \sin \theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1}\left(\frac{|y|}{|x|}\right)$$

$$\Rightarrow |z| = \sqrt{(1 - \sin \alpha)^2 + \cos^2 \alpha}$$

$$\Rightarrow |z| = \sqrt{1 + \sin^2 \alpha - 2 \sin \alpha + \cos^2 \alpha}$$

$$\Rightarrow |z| = \sqrt{1 + 1 - 2 \sin \alpha}$$

$$\Rightarrow |z| = \sqrt{(2)(1 - \sin \alpha)}$$

$$\Rightarrow |z| = \sqrt{(2) \left(\sin^2\left(\frac{\alpha}{2}\right) + \cos^2\left(\frac{\alpha}{2}\right) - 2 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right) \right)}$$

$$\Rightarrow |z| = \sqrt{(2) \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right)^2}$$

$$\Rightarrow |z| = \left| \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right) \right|$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)}{\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right)^2} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{(\cos(\frac{\alpha}{2}) - \sin(\frac{\alpha}{2}))(\cos(\frac{\alpha}{2}) + \sin(\frac{\alpha}{2}))}{(\cos(\frac{\alpha}{2}) - \sin(\frac{\alpha}{2}))^2} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\cos(\frac{\alpha}{2}) + \sin(\frac{\alpha}{2})}{\cos(\frac{\alpha}{2}) - \sin(\frac{\alpha}{2})} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\cos(\frac{\alpha}{2})(1 + \tan(\frac{\alpha}{2}))}{\cos(\frac{\alpha}{2})(1 - \tan(\frac{\alpha}{2}))} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{\tan(\frac{\pi}{4}) + \tan(\frac{\alpha}{2})}{1 - \tan(\frac{\pi}{4})\tan(\frac{\alpha}{2})} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right)$$

We know that sine and cosine functions are periodic with period 2π

Here We have 3 intervals as follows:

$$(i) \ 0 \leq \alpha \leq \frac{\pi}{2}$$

$$(ii) \ \frac{\pi}{2} \leq \alpha \leq \frac{3\pi}{2}$$

$$(iii) \ \frac{3\pi}{2} \leq \alpha < 2\pi$$

Case(i):

In the interval $0 \leq \alpha < \frac{\pi}{2}$, $\cos(\frac{\alpha}{2}) > \sin(\frac{\alpha}{2})$ and also $0 < \frac{\pi}{4} + \frac{\alpha}{2} < \frac{\pi}{2}$

so,

$$\Rightarrow |z| = \left| \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right) \right|$$

$$\Rightarrow |z| = \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right)$$

$$\Rightarrow \theta = \frac{\pi}{4} + \frac{\alpha}{2} \quad (\because \theta \text{ lies in } 1^{\text{st}} \text{ quadrant})$$

$$\therefore \text{The polar form is } \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right) \left(\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) + i \sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \right).$$

Case(ii):

In the interval $\frac{\pi}{2} \leq \alpha < \frac{3\pi}{4}$, $\cos(\frac{\alpha}{2}) < \sin(\frac{\alpha}{2})$ and also $\frac{\pi}{2} < \frac{\pi}{4} + \frac{\alpha}{2} < \pi$

so,

$$\Rightarrow |z| = \left| \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right) \right|$$

$$\Rightarrow |z| = -\sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right)$$

$$\Rightarrow |z| = \sqrt{2} \left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right) \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right)$$

$$\Rightarrow \theta = \frac{\pi}{4} + \frac{\alpha}{2} - \pi \quad (\because \theta \text{ lies in } 4^{\text{th}} \text{ quadrant})$$

$$\Rightarrow \theta = \frac{\alpha}{2} - \frac{3\pi}{4}$$

∴ The polar form is $\sqrt{2} \left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right) \right) \left(\cos\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) + i \sin\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) \right)$.

Case(iii):

In the interval $\frac{3\pi}{2} \leq \alpha < 2\pi$, $\cos\left(\frac{\alpha}{2}\right) < \sin\left(\frac{\alpha}{2}\right)$ and also $\pi < \frac{\pi}{4} + \frac{\alpha}{2} < \frac{5\pi}{4}$

so,

$$\Rightarrow |z| = \left| \sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right) \right|$$

$$\Rightarrow |z| = -\sqrt{2} \left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right) \right)$$

$$\Rightarrow |z| = \sqrt{2} \left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right) \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) \right)$$

$$\Rightarrow \theta = \pi - \frac{\pi}{4} - \frac{\alpha}{2} \text{ (since } \theta \text{ presents in first quadrant and tan's period is } \pi \text{)}$$

$$\Rightarrow \theta = \frac{3\pi}{4} - \frac{\alpha}{2}$$

∴ The polar form is $\sqrt{2} \left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right) \right) \left(\cos\left(\frac{3\pi}{4} - \frac{\alpha}{2}\right) + i \sin\left(\frac{3\pi}{4} - \frac{\alpha}{2}\right) \right)$.

3 D. Question

Express the following complex numbers in the form $r(\cos \theta + i \sin \theta)$:

$$\frac{1-i}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$

Answer

Given complex number is $Z = \frac{1-i}{\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)}$

$$\Rightarrow Z = \frac{1-i}{\frac{1}{2} + \frac{i\sqrt{3}}{2}}$$

$$\Rightarrow Z = 2 \times \frac{1-i}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$\Rightarrow Z = 2 \times \frac{1(1-i\sqrt{3}) - i(1-i\sqrt{3})}{1^2 - (i\sqrt{3})^2}$$

$$\Rightarrow Z = 2 \times \frac{1+i^2\sqrt{3} - i(1+i\sqrt{3})}{1-i^2 3}$$

We know that $i^2 = -1$

$$\Rightarrow Z = 2 \times \frac{(1+(-\sqrt{3}) - i(1+i\sqrt{3}))}{1-(-3)}$$

$$\Rightarrow Z = 2 \times \frac{(1-\sqrt{3}) - i(1+i\sqrt{3})}{4}$$

$$\Rightarrow Z = \frac{(1-\sqrt{3}) - i(1+i\sqrt{3})}{2}$$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos \theta + i \sin \theta)$

Where,

$$|Z| = \text{modulus of complex number} = \sqrt{x^2 + y^2}$$

$$\theta = \arg(z) = \text{argument of complex number} = \tan^{-1} \left(\frac{|y|}{|x|} \right)$$

Now for the given problem,

$$\Rightarrow |z| = \sqrt{\left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{-1-\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow |z| = \sqrt{\frac{1+3-2\sqrt{3}+1+2+2\sqrt{3}}{4}}$$

$$\Rightarrow |z| = \sqrt{\frac{8}{4}}$$

$$\Rightarrow |z| = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1} \left(\left| \frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}} \right| \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\left| \frac{1+\sqrt{3}}{1-\sqrt{3}} \right| \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\left| \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} \right| \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\left| \frac{1+3+2\sqrt{3}}{1-3} \right| \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{4+2\sqrt{3}}{2} \right)$$

Since $x < 0, y < 0$ complex number lies in 3rd quadrant and the value of θ will be as follows $-180^\circ \leq \theta \leq -90^\circ$.

$$\Rightarrow \theta = \tan^{-1}(2 + \sqrt{3})$$

$$\Rightarrow \theta = \frac{-7\pi}{12}.$$

$$\Rightarrow Z = \sqrt{2} \left(\cos\left(\frac{-7\pi}{12}\right) + i \sin\left(\frac{-7\pi}{12}\right) \right)$$

$$\Rightarrow z = \sqrt{2} \left(\cos\left(\frac{7\pi}{12}\right) - i \sin\left(\frac{7\pi}{12}\right) \right)$$

$$\therefore \text{The Polar form of } Z = \frac{1-i}{\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)} \text{ is } z = \sqrt{2} \left(\cos\left(\frac{7\pi}{12}\right) - i \sin\left(\frac{7\pi}{12}\right) \right).$$

4. Question

If z_1 and z_2 are two complex number such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.

Answer

Given:

$$\Rightarrow |z_1| = |z_2| \text{ and } \arg(z_1) + \arg(z_2) = \pi$$

Let us assume $\arg(z_1) = \theta$

$$\Rightarrow \arg(z_2) = \pi - \theta$$

We know that $z = |z|(\cos\theta + i\sin\theta)$

$$\Rightarrow z_1 = |z_1|(\cos\theta + i\sin\theta) \text{-----(1)}$$

$$\Rightarrow z_2 = |z_2|(\cos(\pi - \theta) + i\sin(\pi - \theta))$$

$$\Rightarrow z_2 = |z_2|(-\cos\theta + i\sin\theta)$$

$$\Rightarrow z_2 = -|z_2|(\cos\theta - i\sin\theta)$$

Now we find the conjugate of z_2

$$\Rightarrow \bar{z}_2 = -|z_2|(\cos\theta + i\sin\theta) (\because \overline{|z_2|} = |z_2|)$$

Now,

$$\Rightarrow \frac{z_1}{\bar{z}_2} = \frac{|z_1|(\cos\theta + i\sin\theta)}{-|z_2|(\cos\theta + i\sin\theta)}$$

$$\Rightarrow \frac{z_1}{\bar{z}_2} = -1 (\because |z_1| = |z_2|)$$

$$\Rightarrow z_1 = -\bar{z}_2$$

$\therefore$ Thus proved.

5. Question

If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, prove that $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = 0$.

Answer

Given:

$$\Rightarrow z_1 = \bar{z}_2$$

$$\Rightarrow z_3 = \bar{z}_4$$

$$\text{We know that } \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(\bar{z}_2) - \arg(z_4) + \arg(z_2) - \arg(\bar{z}_4)$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = (\arg(z_2) + \arg(\bar{z}_2)) - (\arg(z_4) + \arg(\bar{z}_4))$$

$$\text{We know that } \arg(z) + \arg(\bar{z}) = 0$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = 0 - 0$$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = 0$$

$\therefore$ Thus proved.

6. Question

Express $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ in polar form.

Answer

$$\text{Given Complex number is } z = \sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$$

$$\text{We know that } \sin 2\theta = 2\sin\theta\cos\theta \text{ and } 1 - \cos 2\theta = 2\sin^2\theta$$

$$\Rightarrow z = 2 \sin \frac{\pi}{10} \cos \frac{\pi}{10} + i \left(2 \sin^2 \frac{\pi}{10} \right)$$

$$\Rightarrow z = 2 \sin \frac{\pi}{10} \left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10} \right)$$

$\therefore$ The Polar form of $z = \sin \left(\frac{\pi}{5} \right) + i \left(1 - \cos \frac{\pi}{5} \right)$ is $z = 2 \sin \frac{\pi}{10} \left(\cos \left(\frac{\pi}{10} \right) + i \sin \left(\frac{\pi}{10} \right) \right)$.

Very Short Answer

1. Question

Write the value of the square root of i.

Answer

Let $\sqrt{i} = \sqrt{a + ib}$1

Squaring both sides, we get

$$i^2 = (a^2 - b^2) + 2aib$$

By comparing real and imaginary term, we get

$$2ab = 1 \text{ and } a^2 - b^2 = 0$$

By solving these we get

$$a = b = \frac{1}{\sqrt{2}}$$

By putting value of a and b in 1, we get

$$\sqrt{i} = \pm \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$\sqrt{i} = \pm \frac{1}{\sqrt{2}}(1 + i)$$

2. Question

Write the values of the square root of -i.

Answer

$$\sqrt{-i} = x + iy$$

Squaring both side

$$-i = (x + iy)^2$$

$$= (x^2 - y^2) + 2ixy$$

$$x^2 - y^2 = 0$$

$$2xy = -1$$

As we know all that,

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$(x^2 + y^2)^2 = 0 + 1$$

$$(x^2 + y^2)^2 = 1$$

$$x^2 + y^2 = 1$$

$$x^2 - y^2 = 0 \dots\dots\dots (1)$$

$$x^2 + y^2 = 1 \dots\dots\dots(2)$$

From (1)

$$x^2 = y^2 \dots\dots\dots(3)$$

$$2x^2 = 1 \text{ (because } x^2 = y^2 \text{)}$$

$$2xy = -1$$

it means $xy < 0$

Either $x < 0$, $y > 0$

Or $x > 0$, $y < 0$

X and y have different sign

$$\therefore x + iy = \pm \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right) \text{ or } - \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)$$

3. Question

If $x + iy = \sqrt{\frac{a + ib}{c + id}}$, then write the value of $(x^2 + y^2)^2$.

Answer

$$x + iy = \sqrt{\frac{a + ib}{c + id}}$$

$$x + iy = \sqrt{\left(\frac{a + ib}{c + id} \right) \times \left(\frac{c - id}{c - id} \right)}$$

$$= \sqrt{\left(\frac{(a + ib)(c - id)}{c^2 + d^2} \right)}$$

$$= \sqrt{\left(\frac{(ac + bd) + i(bc - ad)}{c^2 + d^2} \right)}$$

By squaring both sides, we get

$$\Rightarrow (x + iy)^2 = \left(\frac{(ac + bd) + i(bc - ad)}{c^2 + d^2} \right)$$

$$\Rightarrow x^2 - y^2 + 2ixy = \left(\frac{(ac + bd) + i(bc - ad)}{c^2 + d^2} \right)$$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = \left(\frac{ac + bd}{c^2 + d^2} \right), 2xy = \left(\frac{bc - ad}{c^2 + d^2} \right)$$

We know that,

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$\Rightarrow \left(\frac{ac + bd}{c^2 + d^2} \right)^2 + \left(\frac{bc - ad}{c^2 + d^2} \right)^2$$

$$\Rightarrow \left(\frac{a^2c^2 + b^2d^2 + 2abcd + b^2c^2 + a^2d^2 - 2abcd}{(c^2 + d^2)^2} \right)$$

$$\Rightarrow \left(\frac{a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2}{(c^2 + d^2)^2} \right)$$

$$\Rightarrow \left(\frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2} \right)$$

$$\Rightarrow \left(\frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2} \right)$$

$$\Rightarrow \left(\frac{a^2 + b^2}{c^2 + d^2} \right)$$

4. Question

If $\pi < \theta < 2\pi$ and $z = 1 + \cos \theta + i \sin \theta$, then write the value of $|z|$.

Answer

As we all know that,

$$z = 1 + \cos \theta + i \sin \theta$$

$$a = (1 + \cos \theta) \text{ and } b = \sin \theta$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\Rightarrow \sqrt{(1 + \cos \theta)^2 + \sin^2 \theta} \Rightarrow \sqrt{1 + \cos^2 \theta + 2\cos \theta + \sin^2 \theta} \Rightarrow \sqrt{2 + 2\cos \theta}$$

$$\Rightarrow \sqrt{2(1 + \cos \theta)} \Rightarrow \sqrt{4 \cos^2 \frac{\theta}{2}} = 2 \cos^2 \frac{\theta}{2}$$

$\pi < \theta < 2\pi$ it means z lies in second quadrant

$$z = -\theta$$

$$\Rightarrow -2 \cos^2 \frac{\theta}{2}$$

5. Question

If n is any positive integer, write the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$.

Answer

Explanation

As we know that $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$$z = \frac{i^{4n+1} - i^{4n-1}}{2}$$

$$= \frac{((i^4)^n)(i^1 - i^{-1})}{2}$$

$$= \frac{1^n \left(i - \left(\frac{1}{i} \right) \right)}{2}$$

$$= \frac{i^2 - 1}{2i}$$

$$= \frac{-1-1}{2i}$$

$$= -\frac{1}{i} \times \frac{i}{i}$$

$$= -\frac{i}{i^2}$$

$$= -\frac{i}{-1}$$

$$= i$$

6. Question

Write the value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$.

Answer

Explanation

$$\begin{aligned} Z &= \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} \\ &= \frac{(i^2)^{296} + (i^2)^{295} + (i^2)^{294} + (i^2)^{293} + (i^2)^{292}}{(i^2)^{291} + (i^2)^{290} + (i^2)^{289} + (i^2)^{288} + (i^2)^{287}} \\ &= \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1} \\ &= -2 \end{aligned}$$

7. Question

Write $1 - i$ in polar form.

Answer

$$Z = 1 - i = a + ib$$

$$\text{So, } a = 1, b = -1$$

$$|z| = \sqrt{a^2 + b^2} \Rightarrow \sqrt{1^2 + (-1)^2} = \sqrt{2} = r$$

$$\tan \alpha = \left| \frac{b}{a} \right| \Rightarrow \left| \frac{-1}{1} \right| \Rightarrow 1$$

$$\alpha = \frac{\pi}{4} \text{ or } 45^\circ$$

$$\tan \alpha \ a > 0, b < 1$$

$\therefore z$ lies in forth quadrant

$$\arg(z) = \theta$$

$$\Rightarrow -\alpha = -\frac{\pi}{4}$$

Required polar form

$$\begin{aligned} z(\cos \theta + i \sin \theta) &\Rightarrow \sqrt{2} \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right) \Rightarrow \sqrt{2} \left(\cos \left(\frac{\pi}{4} \right) - \right. \\ &= \left. i \sin \left(\frac{\pi}{4} \right) \right) \end{aligned}$$

8. Question

Write $-1 + i\sqrt{3}$ in polar form.

Answer

$$z = -1 + \sqrt{3}i = a + ib$$

So, $a = 1$, $b = -1$

$$|z| = \sqrt{a^2 + b^2} \Rightarrow \sqrt{-1^2 + (\sqrt{3})^2} = 2 = r$$

$$\tan \alpha = \left| \frac{b}{a} \right| \Rightarrow \left| \frac{\sqrt{3}}{-1} \right| \Rightarrow \sqrt{3} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{4} \text{ or } 45^\circ$$

$\tan \alpha$ $a < 0$, $b > 1$

$\therefore z$ lies in second quadrant

$$\arg(z) = \theta \Rightarrow \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Required polar form

$$= z(\cos \theta + i \sin \theta) \Rightarrow 2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

9. Question

Write the argument of $-i$.

Answer

$$z = 0 - i = a + ib$$

So, $a = 0$, $b = -1$

$$\tan \alpha = \left| \frac{b}{a} \right| \Rightarrow \left| \frac{-1}{0} \right| \Rightarrow \text{not defined} = \infty = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4} \text{ or } 45^\circ$$

$a = 0$, $b = -1$

$\therefore z$ lies in forth quadrant

$$\arg(z) = \theta$$

$$\Rightarrow -\alpha = -\frac{\pi}{2}$$

10. Question

Write the least positive integral value of n for which $\left(\frac{1+i}{1-i} \right)^n$ is real.

Answer

$$\begin{aligned} \left(\frac{1+i}{1-i} \right)^n &= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^n \\ &= \left(\frac{(1+i)^2}{(1)^2 - (i)^2} \right)^n \end{aligned}$$

$$= \left(\frac{1 + i^2 + 2i}{2} \right)^n$$

$$= \left(\frac{1 - 1 + 2i}{2} \right)^n$$

$$= \left(\frac{2i}{2} \right)^n$$

$$= i^n$$

As we know that $i^2 = -1$

And value of n is real number so,

$$n = 2$$

11. Question

Find the principal argument of $(1 + i\sqrt{3})^2$.

Answer

As we know that, $z = a + ib$

$$z = (1 + \sqrt{3}i)^2$$

$$= 1^2 + (\sqrt{3}i)^2 + 2 \times 1 \times \sqrt{3}i$$

$$= 1 + i^2 + 2i\sqrt{3}$$

$$= 1 - 3 + 2i\sqrt{3}$$

$$= -2 + 2i\sqrt{3}$$

$$a = -2 \quad b = 2\sqrt{3}$$

$$\tan \alpha = \left| \frac{b}{a} \right|$$

$$= \left| \frac{2\sqrt{3}}{-2} \right|$$

$$= |\sqrt{3}|$$

$$\alpha = \frac{\pi}{3} \text{ or } 60^\circ$$

$$\alpha < 0, \quad b > 1$$

$\therefore z$ lies in second quadrant

$$\arg(z) = \theta$$

$$= \pi - \alpha$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

12. Question

Find z , if $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$.

Answer

$$r = |z| = 4,$$

$$\arg(z) = \frac{5\pi}{6} = \theta$$

$$z = r(\cos \theta + i \sin \theta)$$

$$= 4\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$$

$$= 4\left(\cos\left(\pi - \frac{\pi}{6}\right) + i \sin\left(\pi - \frac{\pi}{6}\right)\right)$$

$$= 4\left(-\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$$

$$= 4\left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right)$$

$$= \frac{4}{2}(-\sqrt{3} + i)$$

$$z = -2\sqrt{3} + 2i$$

13. Question

If $|z - 5i| = |z + 5i|$, then find the locus of z .

Answer

$$z = a + ib$$

$$|a+ib-5i| = |a+ib+5i|$$

$$|a+ib-5i|^2 = |a+ib+5i|^2$$

$$|a + i(b-5)|^2 = |a + i(b+5)|^2$$

$$a^2 + (b-5)^2 = a^2 + (b+5)^2$$

$$a^2 + b^2 + 25 - 10b = a^2 + b^2 + 25 + 10b$$

$$20b = 0$$

$$b = 0$$

b is a imaginary part of z

$$\operatorname{Im}(z) = \frac{z - \bar{z}}{2}$$

$$= y$$

$$= 0$$

$$\operatorname{Im}(z) = 0$$

So, the locus point is real axis

14. Question

If $\frac{(a^2 + 1)^2}{2a - i} = x + iy$, find the value of $x^2 + y^2$.

Answer

$$\begin{aligned}\frac{(a^2 + 1)^2}{2a - i} &= x + iy \\&= \frac{a^4 + 1 + 2a^2}{2a - i} \times \frac{2a + i}{2a + i} \\&= \frac{(2a(a^4 + 1 + 2a^2)) + (i(a^4 + 1 + 2a^2))}{4a^2 + 1} \\x + iy &= \frac{(2a(a^4 + 1 + 2a^2)) + (i(a^4 + 1 + 2a^2))}{4a^2 + 1}\end{aligned}$$

Comparing real and imaginary part, we get

$$x = \frac{(2a(a^4 + 1 + 2a^2))}{4a^2 + 1}, y = \frac{(i(a^4 + 1 + 2a^2))}{4a^2 + 1}$$

So, $x^2 + y^2$

$$\begin{aligned}x^2 + y^2 &= \frac{(2a(a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} + \frac{(i(a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} \\&= \frac{4a^2((a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} + \frac{i^2((a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} \\&= \frac{4a^2((a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} - \frac{1((a^4 + 1 + 2a^2))^2}{(4a^2 + 1)^2} \\&= \frac{(4a^2 - 1)(a^4 + 1 + 2a^2)^2}{(4a^2 + 1)^2} \\&= \frac{(4a^2 - 1)(a^2 + 1)^4}{(4a^2 + 1)^2} \\&= \frac{(a^2 + 1)^4}{(4a^2 + 1)}\end{aligned}$$

15. Question

Write the value of $\sqrt{-25} \times \sqrt{-9}$.

Answer

$$\begin{aligned}\sqrt{-25} \times \sqrt{-9} &= \sqrt{25} \sqrt{-1} \times \sqrt{9} \sqrt{-1} \\&= 5i \times 3i \\&= 15i^2 \\&= -15\end{aligned}$$

16. Question

Write the sum of the series $i + i^2 + i^3 + \dots$ Upto 1000 terms.

Answer

0

Explanation

Here, $a = i$

$$r = \frac{i^2}{i}$$

$$= i$$

$$n = 1000 \text{ terms}$$

$$s_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow s_{1000} = \frac{i(i^{1000} - 1)}{1000 - 1}$$

$$= \frac{i(1 - 1)}{999}$$

$$= 0$$

17. Question

Write the value of $\arg(z) + \arg(\bar{z})$.

Answer

As we all know that,

$$z = r(\cos \theta + i \sin \theta), \theta = \arg(z)$$

$$\bar{z} = r(\cos \theta - i \sin \theta)$$

$$= r(\cos(-\theta) + i \sin(-\theta))$$

$$, -\theta = \arg(\bar{z})$$

$$\text{So, } \arg(z) + \arg(\bar{z}) = \theta - \theta$$

$$= 0$$

18. Question

If $|z + 4| \leq 3$, then find the greatest and least values of $|z + 1|$.

Answer

6 and 0

Explanation

As we all know that,

$$|z_1 + z_2| \leq |z_1| + |z_2| \text{ and } |z_1 + z_2| \geq |z_1| - |z_2|$$

Suppose,

$$Z_1 = z + 4$$

$$Z_2 = -3$$

$$|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

$$|z + 4| - |-3| \leq |z + 4 - 3| \leq |z + 4| + |-3|$$

$$|z + 4| - 3 \leq |z + 1| \leq |z + 4| + 3$$

$$3 - 3 \leq |z + 1| \leq 3 + 3 \text{ (Given } |z + 4| \leq 3)$$

$$0 \leq |z + 1| \leq 6$$

19. Question

for any two complex numbers z_1 and z_2 and any two real numbers a, b find the value of $|az_1 - bz_2|^2 + |az_2 + bz_1|^2$.

Answer

$$\begin{aligned}
& |az_1 - bz_2|^2 + |az_2 + bz_1|^2 \\
&= (az_1 - bz_2)(\overline{az_1 - bz_2}) + (az_2 + bz_1)(\overline{az_2 + bz_1}) \quad (\because \text{by using } z \bar{z} = |z|^2) \\
&= (az_1 - bz_2)(\overline{az_1} - \overline{bz_2}) + (az_2 + bz_1)(\overline{az_2} - \overline{bz_1}) \\
&= a^2 z_1 \overline{z_1} - ab z_1 \overline{z_2} - ab z_2 \overline{z_1} + b^2 z_2 \overline{z_2} + a^2 z_2 \overline{z_2} + ab z_2 \overline{z_1} + ab z_1 \overline{z_2} \\
&\quad + b^2 z_1 \overline{z_1} \\
&= z_1 \overline{z_1} (a^2 + b^2) + z_2 \overline{z_2} (a^2 + b^2) \\
&= |z_1|^2 (a^2 + b^2) + |z_2|^2 (a^2 + b^2) \\
&= (a^2 + b^2) (|z_1|^2 + |z_2|^2)
\end{aligned}$$

20. Question

Write the conjugate of $\frac{2-i}{(1-2i)^2}$.

Answer

$$\begin{aligned}
z &= \frac{2-i}{(1-2i)^2} \\
&= \frac{2-i}{1+4i^2-4i} \\
&= \frac{2-i}{1-4-4i} \\
&= \frac{2-i}{-3-4i} \\
&= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i} \\
&= \left(\frac{(2-i)(-3+4i)}{(-3)^2 - (4i)^2} \right) \\
&= \frac{-6+8i+3i+4}{9+16} \\
&= \frac{-2+11i}{25} \\
&= -\frac{2}{25} + i\frac{11}{25}
\end{aligned}$$

21. Question

If $n \in \mathbb{N}$, then find the value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$.

Answer

As we know that,

$$i = \sqrt{-1}, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

$$z = i^n + i^{n+1} + i^{n+2} + i^{n+3}$$

$$= i^n (1 + i^1 + i^2 + i^3)$$

$$= i^n (1 + i - 1 - i)$$

$$= i^n(0)$$

$$= 0$$

22. Question

Find the real value of a for which $3i^3 - 3ai^2 + (1 - a)i + 5$ is real.

Answer

$$a = 2$$

Explanation

Z is a purely real, it means $\text{Im}(z) = 0$

$$Z = 3i^3 - 3ai^2 + (1-a)i + 5$$

$$= -3i + 3a + (1-a)i + 5$$

$$= (3a+5) + i(-3+1-a)$$

$$= (3a+5) + i(-2-a)$$

$$\text{Re}(z) = 3a+5, \text{Im}(z) = (-2-a)$$

Z is a real so, $\text{Im}(z) = 0$

$$-2-a = 0$$

$$a = -2$$

23. Question

If $|z| = 2$ and $\arg(z) = \frac{\pi}{4}$, find z.

Answer

$$r = |z| = 2, \arg(z) = \frac{\pi}{4} = \theta$$

$$z = r(\cos \theta + i \sin \theta)$$

$$= 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$$

$$= 2\left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} (1 + i)$$

$$z = \sqrt{2} (1 + i)$$

24. Question

Write the argument of $(1 + \sqrt{3}i)(1 + i)(\cos \theta + i \sin \theta)$.

Answer

As we know that,

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) \text{ so,}$$

$$\arg(z_1 z_2 z_3) = \arg(z_1) + \arg(z_2) + \arg(z_3)$$

$$\arg((1 + \sqrt{3}i)(1 + i)(\cos \theta + i \sin \theta))$$

$$= \arg(1 + \sqrt{3}i) + \arg(1 + i) + \arg(\cos \theta + i \sin \theta) \dots \dots \dots (1)$$

$$z_1 = \arg(1 + \sqrt{3}i) = \tan \alpha = \left| \frac{\text{Im}(z_1)}{\text{Re}(z_1)} \right| \Rightarrow \left| \frac{\sqrt{3}}{1} \right| = \frac{\pi}{6}$$

$$\therefore \arg(z_1) = \theta$$

$$\alpha = \frac{\pi}{6}$$

$$z_2 = \arg(1+i)$$

$$\tan \alpha = \left| \frac{\text{Im}(z_1)}{\text{Re}(z_1)} \right|$$

$$\Rightarrow \left| \frac{1}{1} \right| = \frac{\pi}{4}$$

$$\therefore \arg(z_2) = \theta$$

$$\alpha = \frac{\pi}{4}$$

$$z_3 = \arg(\cos \theta + i \sin \theta)$$

$$\Rightarrow 1(\cos \theta + i \sin \theta)$$

$$\arg(z_3) = \theta$$

$$\therefore \text{In } r(\cos \theta + i \sin \theta), \arg(z) = \theta$$

By putting the value of all arg in 1, we get

$$\frac{\pi}{6} + \frac{\pi}{4} + \theta = \frac{5\pi}{12} + \theta$$

MCQ

1. Question

Mark the Correct alternative in the following:

The value of $(1 + i)(1 + i^2)(1 + i^3)(1 + i^4)$ is

- A. 2
- B. 0
- C. 1
- D. i

Answer

We know that

$$i = \sqrt{-1}$$

$$i^2 = i \times i$$

$$= \sqrt{-1} \times \sqrt{-1}$$

$$= -1$$

$$(1+i)(1+i^2)(1+i^3)(1+i^4) = (1+i)(1+(-1))(1+i^3)(1+i^4)$$

$$= (1+i)(0)(1+i^3)(1+i^4)$$

$$= 0$$

2. Question

Mark the Correct alternative in the following:

If $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$ is a real number and $0 < \theta < 2\pi$, then $\theta =$

- A. π
- B. $\pi/2$
- C. $\pi/3$
- D. $\pi/6$

Answer

$$\begin{aligned} \frac{3+2i \sin \theta}{1-2i \sin \theta} &= \frac{3+2i \sin \theta}{1-2i \sin \theta} \times \frac{1+2i \sin \theta}{1+2i \sin \theta} \\ &= \frac{3 - 4 \sin^2 \theta + 8i \sin \theta}{1 + 4 \sin^2 \theta} \\ &= \frac{3 - 4 \sin^2 \theta}{1 + 4 \sin^2 \theta} + i \frac{8 \sin \theta}{1 + 4 \sin^2 \theta} \end{aligned}$$

For real number, imaginary part should be 0

$$\Rightarrow \frac{8 \sin \theta}{1 + 4 \sin^2 \theta} = 0$$

$$\Rightarrow 8 \sin \theta = 0$$

$$\Rightarrow \theta = n \pi$$

As θ belongs to $(0, 2\pi)$ so $\theta = \pi$

3. Question

Mark the Correct alternative in the following:

If $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = a + ib$, then $2 \times 5 \times 10 \times \dots \times (1 + n^2)$ is equal to

- A. $\sqrt{a^2 + b^2}$
- B. $\sqrt{a^2 - b^2}$
- C. $a^2 + b^2$
- D. $a^2 - b^2$
- E. $a + b$

Answer

Given that $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = a + ib \dots (1)$

We can also say that

$(1 - i)(1 - 2i)(1 - 3i) \dots (1 - ni) = a - ib \dots (2)$

Multiply and divide the eq no. 2 with eq no. 1

$$\frac{(1 + i)(1 - i)(1 + 2i)(1 - 2i) \dots (1 + ni)(1 - ni)}{(1 - i)(1 - 2i) \dots (1 - ni)} = \frac{(a + ib)(a - ib)}{a - ib}$$

$$((1)^2 - (i)^2)((1)^2 - (2i)^2) \dots ((1)^2 - (ni)^2) = ((a)^2 - (ib)^2)$$

$$2 \times 5 \times 10 \times \dots \times (1 + n^2) = a^2 + b^2$$

4. Question

Mark the Correct alternative in the following:

If $\sqrt{a+ib} = x+iy$, then possible value of $\sqrt{a-ib}$ is

A. $x^2 + y^2$ B. $\sqrt{x^2 + y^2}$

C. $x + iy$ D. $x - iy$

E. $\sqrt{x^2 - y^2}$

Answer

$$\sqrt{a+ib} = x+iy$$

Square both sides

$$a+ib = (x+iy)^2 = x^2 + i2xy - y^2$$

So, we can say that $a = x^2 - y^2$ and $b = 2xy$

$$a-ib = (x^2 - y^2) - i(2xy)$$

$$= (x)^2 + 2(x)(-iy) + (-iy)^2$$

$$= (x + (-iy))^2$$

$$= (x - iy)^2$$

$$\sqrt{a-ib} = x - iy$$

5. Question

Mark the Correct alternative in the following:

If $z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{6}$, then

A. $|z| = 1, \arg(z) = \frac{\pi}{4}$

B. $|z| = 1, \arg(z) = \frac{\pi}{6}$

C. $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \frac{5\pi}{24}$

D. $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1} \frac{1}{\sqrt{2}}$

Answer

$$z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{6}$$

$$|z| = \sqrt{\cos^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{6}}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\arg(z) = \tan^{-1} \left(\frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{4}} \right)$$

$$= \tan^{-1} \frac{1}{\sqrt{2}}$$

6. Question

Mark the Correct alternative in the following:

The polar form of $(i^{25})^3$ is

A. $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

B. $\cos \pi + i \sin \pi$

C. $\cos \pi - i \sin \pi$

D. $\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$

Answer

$$z = (i^{25})^3 = i^{75} = i^{4 \times 18 + 3}$$

We know that $i^4 = 1$ and $i^3 = -i$

$$z = i^{4 \times 18} \cdot i^3 = 0 - i$$

$$|z| = \sqrt{0^2 + (-1)^2} = 1$$

$$\theta = \tan^{-1} \left(\frac{-1}{0} \right) = \frac{-\pi}{2}$$

$$z = |z|(\cos \theta + i \sin \theta)$$

$$= 1 \left(\cos \frac{-\pi}{2} + i \sin \frac{-\pi}{2} \right)$$

$$= \cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$$

7. Question

Mark the Correct alternative in the following:

If $i^2 = -1$, then the sum $i + i^2 + i^3 + \dots$ upto 1000 terms is equal to

A. 1

B. -1

C. i

D. 0

Answer

We know that

$$i^{4n+1} = i$$

$$i^{4n+2} = i^2 = -1$$

$$i^{4n+3} = i^3 = -i$$

$$i^{4n+4} = i^4 = 1$$

$$i^{4n+1} + i^{4n+2} + i^{4n+3} + i^{4n+4} = i + (-1) + (-i) + 1$$

$$= 0$$

$$S = i + i^2 + i^3 + \dots \text{ upto 1000 terms}$$

We can make the pair of 4 terms because we know that value is repeat after every 4th terms. So, there are total 250 pairs are made and each pair have value equal to 0.

$$S = 0$$

8. Question

Mark the Correct alternative in the following:

If $z = \frac{-2}{1+i\sqrt{3}}$, then the value of $\arg(z)$ is

A. π

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{4}$

Answer

$$z = \frac{-2}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{-2(1-i\sqrt{3})}{4}$$

$$z = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$$

$$\text{Arg}(z) = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right)$$

$$= \tan^{-1}(-\sqrt{3})$$

$$= \frac{2\pi}{3}$$

9. Question

Mark the Correct alternative in the following:

If $a = \cos \theta + i \sin \theta$, then $\frac{1+a}{1-a} =$

A. $\cot \frac{\theta}{2}$

B. $\cot \theta$

C. $i \cot \frac{\theta}{2}$

D. $i \tan \frac{\theta}{2}$

Answer

$$\begin{aligned} \frac{1+a}{1-a} &= \frac{1+(\cos\theta+i\sin\theta)}{1-(\cos\theta+i\sin\theta)} = \frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta} \times \frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta} \\ &= \frac{2i\sin\theta}{2-2\cos\theta} \\ &= 0 + i \frac{\sin\theta}{1-\cos\theta} \\ &= i \cot \frac{\theta}{2} \end{aligned}$$

10. Question

Mark the Correct alternative in the following:

If $(1+i)(1+2i)(1+3i)\dots(1+ni) = a+ib$, then $2 \cdot 5 \cdot 10 \cdot 17 \dots (1+n^2) =$

- A. $a - ib$
- B. $a^2 - b^2$
- C. $a^2 + b^2$
- D. None of these

Answer

Given that $(1+i)(1+2i)(1+3i)\dots(1+ni) = a+ib \dots (1)$

We can also say that

$$(1-i)(1-2i)(1-3i)\dots(1-ni) = a-ib \dots (2)$$

Multiply and divide the eq no. 2 with eq no. 1

$$\frac{(1+i)(1-i)(1+2i)(1-2i)\dots(1+ni)(1-ni)}{(1-i)(1-2i)\dots(1-ni)} = \frac{(a+ib)(a-ib)}{a-ib}$$

$$((1)^2 - (i)^2)((1)^2 - (2i)^2)\dots((1)^2 - (ni)^2) = ((a)^2 - (ib)^2)$$

$$2 \times 5 \times 10 \times \dots \times (1+n^2) = a^2 + b^2$$

11. Question

Mark the Correct alternative in the following:

If $\frac{(a^2+1)^2}{2a-i} = x+iy$, then x^2+y^2 is equal to

- A. $\frac{(a^2+1)^4}{4a^2+1}$
- B. $\frac{(a+1)^2}{4a^2+1}$
- C. $\frac{(a^2-1)^2}{(4a^2-1)^2}$

D. None of these

Answer

$$\frac{(a^2+1)^2}{2a-i} = x + iy$$

$$x + iy = \frac{(a^2+1)^2}{2a-i} \times \frac{2a+i}{2a+i}$$

$$= \frac{(a^2+1)^2(2a+i)}{4a^2+1}$$

$$= \frac{2a(a^2+1)^2}{4a^2+1} + i \frac{(a^2+1)^2}{4a^2+1}$$

$$x = \frac{2a(a^2+1)^2}{4a^2+1} \text{ and } y = \frac{(a^2+1)^2}{4a^2+1}$$

$$x^2 + y^2 = \left(\frac{2a(a^2+1)^2}{4a^2+1} \right)^2 + \left(\frac{(a^2+1)^2}{4a^2+1} \right)^2$$

$$= (4a^2+1) \frac{(a^2+1)^4}{(4a^2+1)^2}$$

$$= \frac{(a^2+1)^4}{4a^2+1}$$

12. Question

Mark the Correct alternative in the following:

The principal value of the amplitude of $(1 + i)$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{12}$

C. $\frac{3\pi}{4}$

D. π

Answer

We know that the principal value of amplitude is value of argument lie between $(-\pi, \pi]$

$$\arg(z) = \tan^{-1}(1) = \frac{\pi}{4}$$

So, $\frac{\pi}{4}$ is called the principal value of the amplitude of $(1 + i)$ because it lies between $(-\pi, \pi]$

13. Question

Mark the Correct alternative in the following:

The least positive integer n such that $\left(\frac{2i}{1+i} \right)^n$ is a positive integer, is

A. 16

B. 8

C. 4

D. 2

Answer

$$\frac{2i}{1+i} = \frac{2i}{(1+i)} \times \frac{(1-i)}{(1-i)} = 1 + i$$

$$\left(\frac{2i}{1+i} \right)^n = (1+i)^n$$

Let check the value of $(1+i)^n$ for different value of n

at $n=1$, $1+i$ (no)

at $n=2$, $(1+i)^2 = 1 + i^2 + 2i = 2i$ (no)

at $n=3$, $(1+i)^2(1+i) = (1+i)(2i) = 2i - 2$ (no)

at $n=4$, $(1+i)^2(1+i)^2 = (2i)^2 = -4$ (no)

at $n=5$, $(1+i)^4(1+i) = -4(1+i)$ (no)

at $n=6$, $(1+i)^4(1+i)^2 = -4(2i)$ (no)

at $n=7$, $(1+i)^6(1+i) = -8i(1+i) = -8i + 8$ (no)

at $n=8$, $(1+i)^4(1+i)^4 = (-4)(-4) = 8$ (yes)

So, we can say that $n = 8$ is the least positive integer for which $\left(\frac{2i}{1+i} \right)^n$ is positive integer.

14. Question

Mark the Correct alternative in the following:

If z is a non-zero complex number, then $\left| \frac{\overline{z}^2}{z\overline{z}} \right|$ is equal to

A. $\left| \frac{\overline{z}}{z} \right|$

B. $|z|$

C. $\left| \overline{z} \right|$

D. None of these

Answer

Let, $z = re^{i\theta}$

$$\overline{z} = re^{-i\theta}$$

$$z\overline{z} = re^{i\theta} \cdot re^{-i\theta} = r^2$$

$$|\overline{z}| = r$$

$$|\overline{z}|^2 = r^2$$

$$\left| \frac{|\overline{z}|^2}{z\overline{z}} \right| = \left| \frac{r^2}{r^2} \right|$$

$$= 1$$

Solve option A

$$\left| \frac{\bar{z}}{z} \right| = \left| \frac{r}{re^{i\theta}} \right|$$

$$= \left| \frac{1}{e^{i\theta}} \right|$$

$$= |e^{-i\theta}|$$

$$= 1$$

15. Question

Mark the Correct alternative in the following:

If $a = 1 + i$, then a^2 equals

A. $1 - i$

B. $2i$

C. $(1 + i)(1 - i)$

D. $i - 1$.

Answer

$$a^2 = (1 + i)(1 + i)$$

$$= 1^2 + 2i + i^2$$

$$= 1 - 1 + 2i$$

$$= 2i$$

16. Question

Mark the Correct alternative in the following:

If $(x + iy)^{1/3} = a + ib$, then $\frac{x}{a} + \frac{y}{b} =$

A. 0

B. 1

C. -1

D. None of these

Answer

$$(x + iy)^{1/3} = a + ib$$

$$x + iy = (a + ib)^3$$

$$= a^3 + (ib)^3 + 3a^2(ib) + 3a(ib)^2$$

$$= a^3 - ib^3 + i3a^2b - 3ab^2$$

$$= (a^3 - 3ab^2) + i(3a^2b - b^3)$$

$$x = a^3 - 3ab^2 \text{ and } y = 3a^2b - b^3$$

$$\frac{x}{a} + \frac{y}{b} = \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$$

$$= a^2 - 3b^2 + 3a^2 - b^2$$

$$= 4(a^2 - b^2)$$

17. Question

Mark the Correct alternative in the following:

$(\sqrt{-2})(\sqrt{-3})$ is equal to

A. $\sqrt{6}$

B. $-\sqrt{6}$

C. $i\sqrt{6}$

D. None of these

Answer

$$\sqrt{-2} = \sqrt{2} i \text{ and } \sqrt{-3} = \sqrt{3} i$$

$$\sqrt{-2}\sqrt{-3} = \sqrt{2}i \times \sqrt{3}i$$

$$= i^2 \sqrt{6}$$

$$= -\sqrt{6}$$

18. Question

Mark the Correct alternative in the following:

The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is

A. 60°

B. 120°

C. 210°

D. 240°

Answer

$$\frac{1-i\sqrt{3}}{1+i\sqrt{3}} = \frac{(1-i\sqrt{3})(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$$

$$= \frac{-2 - 2i\sqrt{3}}{4}$$

$$= \frac{-1}{2} + i \frac{-\sqrt{3}}{2}$$

$$\text{Arg } z = \tan^{-1} \left(\frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} \right)$$

$$= \frac{\pi}{3}$$

$$= 60^\circ$$

But answer is going in 3rd quadrant because $\tan \theta$ is positive but $\sin \theta$ and $\cos \theta$ both are negative and it is possible only in 3rd quadrant.

So, answer is $\pi + 60^\circ = 180^\circ$

19. Question

Mark the Correct alternative in the following:

If $z = \left(\frac{1+i}{1-i} \right)$, then z^4 equals

- A. 1
- B. -1
- C. 0
- D. None of these

Answer

$$\begin{aligned}
 z &= \frac{1+i}{1-i} \\
 &= \frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)} \\
 &= \frac{(1+i)^2}{1^2 - i^2} \\
 &= \frac{1 + i^2 + 2i}{1 + 1} \\
 &= \frac{1 - 1 + 2i}{2} \\
 &= i \\
 z^4 &= i^4 \\
 &= 1
 \end{aligned}$$

20. Question

Mark the Correct alternative in the following:

If $z = \frac{1+2i}{1-(1-i)^2}$, then $\arg(z)$ equals

- A. 0
- B. $\frac{\pi}{2}$
- C. π
- D. None of these

Answer

$$\begin{aligned}
 z &= \frac{1+2i}{1-(1-i)^2} \\
 &= \frac{1+2i}{1+2i} \\
 &= 1+i0 \\
 \text{Arg } z &= \tan^{-1} \frac{0}{1} \\
 &= 0
 \end{aligned}$$

21. Question

Mark the Correct alternative in the following:

If $sz = \frac{1}{(2+3i)^2}$, then $|z| =$

A. $\frac{1}{13}$

B. $\frac{1}{5}$

C. $\frac{1}{12}$

D. None of these

Answer

$$\begin{aligned} z &= \frac{1}{(2+3i)^2} \\ &= \frac{1}{(2+3i)^2} \times \frac{(2-3i)^2}{(2-3i)^2} \\ &= \frac{-5-12i}{169} \\ &= \frac{-5}{169} + i \frac{-12}{169} \\ |z| &= \sqrt{\left(\frac{-5}{169}\right)^2 + \left(\frac{-12}{169}\right)^2} \\ &= \sqrt{\frac{25+144}{(169)^2}} \\ &= \sqrt{\frac{169}{(169)^2}} \\ &= \frac{1}{13} \end{aligned}$$

22. Question

Mark the Correct alternative in the following:

If $z = \frac{1}{(1+i)(2+3i)}$, then $|z| =$

A. 1

B. $1/\sqrt{26}$

C. $5/\sqrt{26}$

D. None of these

Answer

$$z = \frac{1}{(1+i)(2+3i)} = \frac{1}{-1+5i}$$

$$= \frac{1}{(-1+5i)} \times \frac{(-1-5i)}{(-1-5i)}$$

$$= \frac{-1}{26} + i \frac{-5}{26}$$

$$|z| = \sqrt{\left(\frac{-1}{26}\right)^2 + \left(\frac{-5}{26}\right)^2}$$

$$= \sqrt{\frac{1+25}{(26)^2}}$$

$$= \frac{1}{\sqrt{26}}$$

23. Question

Mark the Correct alternative in the following:

If $z = 1 - \cos \theta + i \sin \theta$, then $|z| =$

- A. $2 \sin \frac{\theta}{2}$
- B. $2 \cos \frac{\theta}{2}$
- C. $2 \left| \sin \frac{\theta}{2} \right|$
- D. $2 \left| \cos \frac{\theta}{2} \right|$

Answer

$$|z| = \sqrt{(1 - \cos \theta)^2 + (\sin \theta)^2}$$

$$= \sqrt{2 - 2 \cos \theta}$$

$$= 2 \left| \sin \frac{\theta}{2} \right|$$

24. Question

Mark the Correct alternative in the following:

If $x + i y = (1 + i) (1 + 2 i) (1 + 3i)$, then $x^2 + y^2 =$

- A. 0
- B. 1
- C. 100
- D. None of these

Answer

Given that $(1 + i) (1 + 2i) (1 + 3i) = x + i y \dots(1)$

We can also say that

$$(1 - i)(1 - 2i)(1 - 3i) = x - iy \dots(2)$$

Multiply and divide the eq no. 2 with eq no. 1

$$\frac{(1 + i)(1 - i)(1 + 2i)(1 - 2i)(1 + 3i)(1 - 3i)}{(1 - i)(1 - 2i)(1 - 3i)} = \frac{(x + iy)(x - iy)}{x - iy}$$

$$((1)^2 - (i)^2)((1)^2 - (2i)^2)((1)^2 - (3i)^2) = ((x)^2 - (iy)^2)$$

$$x^2 + y^2 = 2 \times 5 \times 10 = 100$$

25. Question

Mark the Correct alternative in the following:

$$\text{If } z = \frac{1}{1 - \cos \theta - i \sin \theta}, \text{ then Re } (z) =$$

A. 0

B. $\frac{1}{2}$

C. $\cot \frac{\theta}{2}$

D. $\frac{1}{2} \cot \frac{\theta}{2}$

Answer

$$z = \frac{1}{1 - \cos \theta - i \sin \theta} = \frac{1}{(1 - \cos \theta - i \sin \theta)} \times \frac{(1 - \cos \theta + i \sin \theta)}{(1 - \cos \theta + i \sin \theta)}$$

$$= \frac{(1 - \cos \theta) + i \sin \theta}{2 - 2 \cos \theta}$$

$$= \frac{1}{2} + i \frac{\cot \frac{\theta}{2}}{2}$$

$$\text{Re } (z) = \frac{1}{2}$$

26. Question

Mark the Correct alternative in the following:

$$\text{If } x + iy = \frac{3 + 5i}{7 - 6i}, \text{ then } y =$$

A. 9/85

B. -9/85

C. 53/85

D. None of these

Answer

$$x + iy = \frac{3 + 5i}{7 - 6i}$$

$$= \frac{(3 + 5i)}{(7 - 6i)} \times \frac{(7 + 6i)}{(7 + 6i)}$$

$$= \frac{-9 + 53i}{85}$$

$$= \frac{-9}{85} + i \frac{53}{85}$$

$$y = \frac{53}{85}$$

27. Question

Mark the Correct alternative in the following:

If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2 =$

- A. 1
- B. -1
- C. 0
- D. None of these

Answer

$$a + ib = \frac{(1-ix)}{(1+ix)} \times \frac{(1-ix)}{(1-ix)}$$

$$= \frac{1 - x^2 - 2ix}{1 + x^2}$$

$$= \frac{1 - x^2}{1 + x^2} + i \frac{-2x}{1 + x^2}$$

$$a = \frac{1 - x^2}{1 + x^2} \text{ and } b = \frac{-2x}{1 + x^2}$$

$$a^2 + b^2 = \left(\frac{1 - x^2}{1 + x^2} \right)^2 + \left(\frac{-2x}{1 + x^2} \right)^2$$

$$= \frac{(1 - x^2)^2 + (-2x)^2}{(1 + x^2)^2}$$

$$= \frac{1 + x^4 - 2x^2 + 4x^2}{(1 + x^2)^2}$$

$$= \frac{(1 + x^2)^2}{(1 + x^2)^2}$$

$$= 1$$

28. Question

Mark the Correct alternative in the following:

If θ is the amplitude of $\frac{a+ib}{a-ib}$, then $\tan \theta =$

- A. $\frac{2a}{a^2 + b^2}$
- B. $\frac{2ab}{a^2 - b^2}$

C. $\frac{a^2 - b^2}{a^2 + b^2}$

D. None Of these

Answer

$$\begin{aligned}\frac{a+ib}{a-ib} &= \frac{(a+ib)}{(a-ib)} \times \frac{(a+ib)}{(a+ib)} \\ &= \frac{a^2 - b^2 + 2iab}{a^2 + b^2} \\ &= \frac{a^2 - b^2}{a^2 + b^2} + i \frac{2ab}{a^2 + b^2} \\ \tan \theta &= \frac{\left(\frac{2ab}{a^2 + b^2} \right)}{\left(\frac{a^2 - b^2}{a^2 + b^2} \right)} \\ &= \frac{2ab}{a^2 - b^2}\end{aligned}$$

29. Question

Mark the Correct alternative in the following:

If $z = \frac{1+7i}{(2-i)^2}$, then

A. $|z| = 2$

B. $|z| = \frac{1}{2}$

C. $\text{amp}(z) = \frac{\pi}{4}$

D. $\text{amp}(z) = \frac{3\pi}{4}$

Answer

$$\begin{aligned}z &= \frac{1+7i}{3-4i} = \frac{(1+7i)}{(3-4i)} \times \frac{(3+4i)}{(3+4i)} \\ &= \frac{-25 + 25i}{25} \\ &= -1 + i \\ |z| &= \sqrt{(-1)^2 + 1^2} \\ &= \sqrt{2} \\ \text{amp}(z) &= \tan^{-1} \frac{1}{-1} \\ &= \pi - \frac{\pi}{4} \\ &= \frac{3\pi}{4}\end{aligned}$$

30. Question

Mark the Correct alternative in the following:

The amplitude of $\frac{1}{i}$ is equal to

A. 0

B. $\frac{\pi}{2}$

C. $-\frac{\pi}{2}$

D. π

Answer

$$\frac{1}{i} = \frac{1}{i} \times \frac{i}{i}$$

$$= \frac{i}{-1}$$

$$= 0 + i(-1)$$

$$\text{amp} = \tan^{-1} \frac{-1}{0}$$

$$= \frac{-\pi}{2}$$

31. Question

Mark the Correct alternative in the following:

The argument of $\frac{1-i}{1+i}$ is

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. $\frac{5\pi}{2}$

Answer

$$\frac{1-i}{1+i} = \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)}$$

$$= 0 + i(-1)$$

$$\text{arg} = \tan^{-1} \frac{-1}{0}$$

$$= \frac{-\pi}{2}$$

32. Question

Mark the Correct alternative in the following:

The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $-\frac{\pi}{6}$

Answer

$$\frac{1+i\sqrt{3}}{\sqrt{3}+i} = \frac{(1+i\sqrt{3})}{(\sqrt{3}+i)} \times \frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$$

$$= \frac{2\sqrt{3} + 2i}{4}$$

$$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$$

$$\text{amp} = \tan^{-1} \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \tan^{-1} \frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{6}$$

33. Question

Mark the Correct alternative in the following:

The value of $(i^5 + i^6 + i^7 + i^8 + i^9)/(1 + i)$ is

A. $\frac{1}{2}(1+i)$

B. $\frac{1}{2}(1-i)$

C. 1

D. $\frac{1}{2}$

Answer

We know that

$$i^{4n+1} = i$$

$$i^{4n+2} = i^2 = -1$$

$$i^{4n+3} = i^3 = -i$$

$$i^{4n+4} = i^4 = 1$$

$$i^5 + i^6 + i^7 + i^8 + i^9 = i + (-1) + (-i) + 1 + i$$

$$= i$$

$$\frac{(i^5 + i^6 + i^7 + i^8 + i^9)}{(1 + i)} = \frac{i}{1 + i}$$

$$= \frac{i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$= \frac{1}{2}(1 + i)$$

34. Question

Mark the Correct alternative in the following:

$$\frac{1 + 2i + 3i^2}{1 - 2i + 3i^2} \text{ equals}$$

- A. i
- B. -1
- C. -i
- D. 4

Answer

$$\frac{1 + 2i + 3i^2}{1 - 2i + 3i^2} = \frac{1 + i(2 + 3i)}{1 + i(-2 + 3i)}$$

$$= \frac{i\left(\frac{1}{i} + (2 + 3i)\right)}{i\left(\frac{1}{i} + (-2 + 3i)\right)}$$

$$= \frac{-i + (2 + 3i)}{-i + (-2 + 3i)}$$

$$= \frac{2 + 2i}{-2 + 2i}$$

$$= \frac{1 + i}{-1 + i} \times \frac{-1 - i}{-1 - i}$$

$$= -i$$

35. Question

Mark the Correct alternative in the following:

$$\text{The value of } \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1 \text{ is}$$

- A. -1
- B. -2
- C. -3
- D. -4

Answer

We know that

$$i^{4n+1} = i$$

$$i^{4n+2} = i^2$$

$$= -1$$

$$i^{4n+3} = i^3$$

$$= -i$$

$$i^{4n+4} = i^4$$

$$= 1$$

$$i^{592} = i^{4(147)+4}$$

$$= 1$$

$$i^{582} = i^{4(145)+2}$$

$$= -1$$

$$i^{590} = i^{4(147)+2}$$

$$= -1$$

$$i^{580} = i^{4(144)+4}$$

$$= 1$$

$$i^{588} = i^{4(146)+4}$$

$$= 1$$

$$i^{578} = i^{4(144)+2}$$

$$= -1$$

$$i^{586} = i^{4(146)+2}$$

$$= -1$$

$$i^{576} = i^{4(143)+4}$$

$$= 1$$

$$i^{584} = i^{4(145)+4}$$

$$= 1$$

$$i^{574} = i^{4(143)+2}$$

$$= -1$$

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1 = \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1} - 1$$

$$= -2$$

36. Question

Mark the Correct alternative in the following:

The value of $(1 + i)^4 + (1 - i)^4$ is

A. 8

B. 4

C. -8

D. -4

Answer

$$(1 + i)^4 + (1 - i)^4 = ((1 + i)^2)^2 + ((1 - i)^2)^2$$

$$= (2i)^2 + (-2i)^2$$

$$= -4 + -4$$

$$= -8$$

37. Question

Mark the Correct alternative in the following:

If $z = a + ib$ lies in third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant if

A. $a > b > 0$

B. $a < b < 0$

C. $b < a < 0$

D. $b > a > 0$

Answer

If $z = a + ib$ lies in third quadrant then a and b both are less than zero

$$\bar{z} = a - ib$$

$$\frac{\bar{z}}{z} = \frac{a - ib}{a + ib}$$

$$= \frac{a - ib}{a + ib} \times \frac{a - ib}{a - ib}$$

$$= \frac{a^2 - b^2 - 2iab}{a^2 + b^2}$$

$$= \frac{a^2 - b^2}{a^2 + b^2} + i \frac{-2ab}{a^2 + b^2}$$

$$\frac{a^2 - b^2}{a^2 + b^2} < 0 \text{ and } \frac{-2ab}{a^2 + b^2} < 0$$

$a^2 - b^2 < 0$ and $ab > 0$ because $a^2 + b^2$ is always greater than zero

$$(a - b)(a + b) < 0$$

Here a and b both are less than zero that means $(a + b)$ is always less than zero

$$\text{So, } a - b > 0 \Rightarrow a > b$$

Then, final answer is $b < a < 0$

38. Question

Mark the Correct alternative in the following:

If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is

A. $\frac{|z|}{2}$

- B. $|z|$
 C. $2|z|$
 D. None of these

Answer

$$|z| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$$

$$f(z) = \frac{7-z}{1-z^2}$$

$$= \frac{7-(1+2i)}{1-(1+2i)^2}$$

$$= \frac{6-2i}{4-4i}$$

$$= \frac{3-i}{2-2i} \times \frac{2+2i}{2+2i}$$

$$= \frac{8+4i}{8}$$

$$= 1 + i\frac{1}{2}$$

$$|f(z)| = \sqrt{(1)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\sqrt{5}}{2}$$

$$= \frac{|z|}{2}$$

39. Question

Mark the Correct alternative in the following:

A real value of x satisfies the equation $\frac{3-4ix}{3+4ix} = a - ib$ ($a, b \in \mathbb{R}$), if $a^2 + b^2 =$

- A. 1
 B. -1
 C. 2
 D. -2

Answer

$$a - ib = \frac{(3-4ix)}{(3+4ix)} \times \frac{(3-4ix)}{(3-4ix)}$$

$$= \frac{9 - 16x^2 - 24ix}{9 + 16x^2}$$

$$= \frac{9 - 16x^2}{9 + 16x^2} - i \frac{24x}{9 + 16x^2}$$

$$a = \frac{9 - 16x^2}{9 + 16x^2} \text{ and } b = \frac{24x}{9 + 16x^2}$$

$$\begin{aligned}
 a^2 + b^2 &= \left(\frac{9 - 16x^2}{9 + 16x^2} \right)^2 + \left(\frac{24x}{9 + 16x^2} \right)^2 \\
 &= \frac{81 + 256x^4 - 288x^2 + 576x^2}{(9 + 16x^2)^2} \\
 &= \frac{(9 + 16x^2)^2}{(9 + 16x^2)^2} \\
 &= 1
 \end{aligned}$$

40. Question

Mark the Correct alternative in the following:

The complex number z which satisfies the condition $\left| \frac{i+z}{i-z} \right| = 1$ lies on

- A. circle $x^2 + y^2 = 1$
- B. the x-axis
- C. the y-axis
- D. the line $x + y = 1$

Answer

Let, $z = x + iy$

$$\begin{aligned}
 \frac{i+z}{i-z} &= \frac{x + i(y+1)}{-x + i(-y+1)} \\
 &= \frac{x + i(y+1)}{-x + i(-y+1)} \times \frac{-x - i(-y+1)}{-x - i(-y+1)} \\
 &= \frac{-x^2 - y^2 + 1 - 2ix}{x^2 + y^2 - 2y + 1} \\
 &= \frac{-x^2 - y^2 + 1}{x^2 + y^2 - 2y + 1} + i \frac{-2x}{x^2 + y^2 - 2y + 1} \\
 \left| \frac{i+z}{i-z} \right| &= \sqrt{\left(\frac{-x^2 - y^2 + 1}{x^2 + y^2 - 2y + 1} \right)^2 + \left(\frac{-2x}{x^2 + y^2 - 2y + 1} \right)^2} \\
 &= \sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 - 2x^2 - 2y^2) + 4x^2}{(x^2 + y^2 - 2y + 1)^2}} \\
 &= \sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 - 2y^2)}{(x^2 + y^2 - 2y + 1)^2}} \\
 \left| \frac{i+z}{i-z} \right| &= 1 \\
 \sqrt{\frac{(x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 - 2y^2)}{(x^2 + y^2 - 2y + 1)^2}} &= 1
 \end{aligned}$$

$$x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 - 2y^2 = x^4 + y^4 + 1 + 2x^2y^2 + 2x^2 +$$

$$6y^2 - 4y^3 - 2xy(x+y) - 4y$$

$$8y^2 - 4y^3 - 2xy(x + y) - 4y = 0$$

$$y(8y - 4y^2 - 2x(x + y) - 4) = 0$$

$$y = 0 \text{ and } 8y - 4y^2 - 2x(x + y) - 4 = 0$$

So, by $y = 0$ we can say that it lies on x axis

41. Question

Mark the Correct alternative in the following:

If z is a complex number, then

A. $|z|^2 > |\bar{z}|^2$

B. $|z|^2 = |\bar{z}|^2$

C. $|z|^2 < |\bar{z}|^2$

D. $|z|^2 \geq |\bar{z}|^2$

Answer

Let, $z = a + ib$

$$\bar{z} = a - ib = a + i(-b)$$

$$|z| = \sqrt{a^2 + b^2}$$

$$|z|^2 = a^2 + b^2$$

$$|\bar{z}| = \sqrt{a^2 + (-b)^2}$$

$$= \sqrt{a^2 + b^2}$$

$$|\bar{z}|^2 = a^2 + b^2$$

$$|z|^2 = |\bar{z}|^2$$

42. Question

Mark the Correct alternative in the following:

Which of the following is correct for any two complex numbers z_1 and z_2 ?

A. $|z_1 z_2| = |z_1| |z_2|$

B. $\arg(z_1 z_2) = \arg(z_1) \arg(z_2)$

C. $|z_1 + z_2| = |z_1| + |z_2|$

D. $|z_1 + z_2| \geq |z_1| + |z_2|$

Answer

Let, $z_1 = r_1 e^{i\alpha}$ and $z_2 = r_2 e^{i\beta}$

$$|z_1| = r_1 \text{ and } |z_2| = r_2$$

Option A

$$z_1 z_2 = r_1 r_2 e^{i(\alpha+\beta)}$$

$$|z_1 z_2| = r_1 r_2 = |z_1| |z_2|$$

Option A correct

Option B

$$\arg(z_1 z_2) = \alpha + \beta$$

$$= \arg(z_1) + \arg(z_2)$$

Option B not correct

Let, $z_1 = a + ib$ and $z_2 = c + id$

Option C

$$z_1 + z_2 = (a+c) + i(b+d)$$

$$|z_1 + z_2| = \sqrt{(a+c)^2 + (b+d)^2}$$

$$|z_1| = \sqrt{a^2 + b^2} \text{ and } |z_2| = \sqrt{c^2 + d^2}$$

We cannot say anything about option c and option d

43. Question

Mark the Correct alternative in the following:

If the complex number $z = x + iy$ satisfies the condition $|z + 1| = 1$, then z lies on

- A. x-axis
- B. circle with centre $(-1, 0)$ and radius 1
- C. y-axis
- D. None of these

Answer

$$|z + 1| = 1$$

$$|x + iy + 1| = 1$$

$$|(1 + x) + iy| = 1$$

$$\sqrt{(1+x)^2 + y^2} = 1$$

$$(x + 1)^2 + y^2 = 1$$

$$(x - (-1))^2 + (y - 0)^2 = (1)^2$$

So, we can say that it is a circle with centre $(-1, 0)$ and radius 1