Sample Paper 16

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. The class interval of a given observation is 15 to 20, then the class mark for this interval will be:

(a) 11.5 (b) 17.5

(c) 12 (d) 14 1

2. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag. The probability of getting neither a green ball nor a red ball is:

(a) $\frac{5}{20}$ (b) $\frac{3}{20}$

(c) $\frac{7}{20}$ (d) $\frac{1}{20}$ 1

3. The value of sin 30° cos 60° + sin 60° cos 30° is:

(a) 0 (b) 1

(c) 2 (d) 4 1

4. The outer and inner diameters of a circular ring are 34 cm and 32 cm respectively. The area of the ring is:

(a) 27π cm^2 (b) 30π cm^2

(c) 33π cm^2 (d) 31π cm^2 1

5. If $2 \sin 2\theta = \sqrt{3}$, then find the value of θ.

(a) 30° (b) 60°

(c) 90° (d) 45° 1

6. The degree of the polynomial, $x^5 - x^4 + 2$ is:

(a) 2 (b) 5

(c) 1 (d) 0 1

7. If $\sin A = \frac{1}{2}$, $\cos B = 1$, $0 < A, B \leq \frac{\pi}{2}$, then the value of cot (A + B) is:

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) $\sqrt{3}$ 1

8. If $r = 3$ is a root of quadratic equation $kr^2 - kr - 3 = 0$, then the value of k is:

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{5}{2}$ 1

9. The degree of the polynomial $(x + 1)(x^2 - x + x^4 - 1)$ is:

(a) 5 (b) 4

(c) 3 (d) 2 1

10. Write the exponent of 3 in the prime factorisation of 1944.

(a) 3 (b) 1

(c) 5 (d) 4 1

11. When placed along a wall, a ladder forms a 60° angle with the ground. If the ladder's foot is 10 metres from the wall, the length of ladder is:

(a) 4 m (b) 8 m

(c) $8\sqrt{3}$ m (d) 20 m 1

12. The value of k for which the pair of linear equations $kx - 3y = k - 2$, $12x + ky = k$ has no solution.

(a) 2 (b) 5

(c) 4 (d) None of these 1

13. How many multiples of 4 lie between 10 and 205?

(a) 49 (b) 80

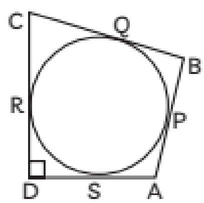
(c) 45 (d) 46 1

14. The zeros of the polynomial $x^2 - 3x - m(m + 3)$ is:

(a) $-2, (m + 1)$ (b) $(m + 3), 4$

(c) $(m + 1), 2$ (d) $-m, (m + 3)$ 1

15. In the figure $\angle ADC = 90°$, BC = 38 cm, CD = 28 cm and BP = 25 cm

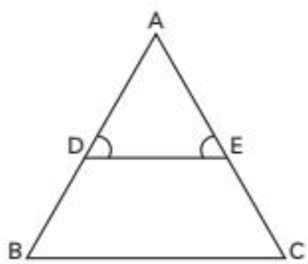


The radius of the circle is:

(a) 20 cm (b) 15 cm

(c) 16 cm (d) 18 cm 1

16. In the figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, that $\triangle BAC$ is a/an:



(a) scalene (b) equilateral

(c) isosceles (d) right-angled 1

17. If a line segment AB of length 6 cm is divided internally by a point C in the ratio 3 : 2, then the length of AC is:

(a) 4 cm (b) 2.5 cm

(c) 2.6 cm (d) 3.6 cm 1

18. Find the class marks of the class 45 – 60.

(a) 36.4 (b) 7.5

(c) 52.5 (d) 50 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** $(\cos^4 A - \sin^4 A)$ is equal to $2\cos^2 A - 1$.

Statement R (Reason): The value of $\cos\theta$ decreases as θ increases. 1

20. **Statement A (Assertion):** If the value of mode and mean is 60 and 66 respectively, then the value of median is 68.

Statement R (Reason):

Median = $\frac{1}{3}$(mode + 2 mean) 1

SECTION - B 10 marks

(Section B consists of 5 questions of 2 marks each.)

21. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q (2, –5) and R (–3, 6), find the coordinates of P.

OR

Prove that the points (2, –2), (–2, 1) and (5, 2) are the vertices of a right-angled triangle. 2

22. If the HCF (210, 55) is expressible in the form $210 \times 5 - 55y$, then find y.

OR

Prove that the number 4^n, n being a natural number, can never end with the digit 0. 2

23. α, β are the zeros of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$. Find the value of k, if $\alpha + \beta = \frac{1}{3}\alpha\beta$. 2

24. Evaluate :

$$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$ 2

25. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. How much canvas cloth is required to just cover the heap? 2

SECTION - C

18 marks

(Section C consists of 6 questions of 3 marks each.)

26. Find the greatest number of 6-digits exactly divisible by 15, 24 and 36.

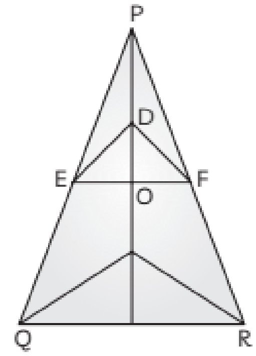
OR

All the black face cards are removed from a pack of 52 playing cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting a: (A) face card (B) red card. 3

27. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. 3

28. Points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of $\triangle ABC$. Find the coordinates of the Q and R on median BE and CF respectively, such that BQ : QE = 2 : 1 3

29. In the given figure, DE || OQ and DF || OR, Prove that EF || OQ.



OR

Let s denote the semi-perimeter of a triangle ABC in which BC = a, CA = b, AB = c. If a circle touches the side BC, CA, AB at D, E, F respectively, prove that BD = $s - b$. 3

30. The rain water from a roof of dimensions 22 m × 20 m drains into a cylindrical vessel having the base of diameter 2 m and height 3.5 m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm. 3

31. The median of the following frequency distribution is 24. Find the missing frequency f_1 and f_2.

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	Total
Number of Students	4	6	f_1	10	25	f_2	18	5	100

3

SECTION - D

20 marks

(Section D consists of 4 questions of 5 marks each.)

32. The tangent at a point C of a circle with centre O and a diameter AB when extended intersect at P. If $\angle PCA = 110^\circ$., find the measure of $\angle CBA$.

OR

The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m above the lower window. At certain instant, the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon from the ground. 5

33. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, then find the length of AD. 5

34. Ajay had some bananas and he divided them into two lots A and B. He sold lot A at the rate of ₹ 2 for 3 bananas and lot B at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold lot A at the rate of ₹ 1 per banana and lot B at the rate of ₹ 4 for 5 bananas his total collection would have been ₹ 460. Find the total number of bananas he had. 5

35. Agam received an overall of 30 marks in math and English on a class quiz. The total of his marks in Mathematics and English would have been 210 if he had got 2 more in Mathematics and 3 less in English. Find out what he scored in the two subjects.

OR

There are 12 balls in a box, and x of them are black. What is the probability that one ball will be picked at random from the box , will be a black ball? If there are now six additional black balls in the box, the probability of drawing a black ball is now double of what it was before. Find x. 5

SECTION - E
12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *The diagram shows a grid of squares. A button is placed on one of the squares. A fair dice is thrown. If 1,2,3 or 4 is thrown, the button is moved one square to the left.*

If 5 or 6 is thrown, the button is moved one square to the right.

	X				Q	Y		P

On the basis of the above information, answer the following questions:

(A) The button is placed on square X. the dice is thrown once. Then find, the probability that the button is moved to the right. 1

(B) On the other occasion, the button is placed on square Y. The dice is thrown once and the button is moved. The die is thrown a second time and the button is moved again. Find the probability that the button is moved at P and button is moved at Y.

OR

Find the probability that the button is moved at P, Q or Y. 2

(C) Find the probability that the button is moved at Q. 1

37. *Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 5 m and length 450 m.*

A level surface is laid inside the tunnel to carry the railway lines. The Diagram 1 shows the circular cross – section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $\angle AOB = 90^\circ$. The space below AB is filled with rubble (debris from the demolition buildings).

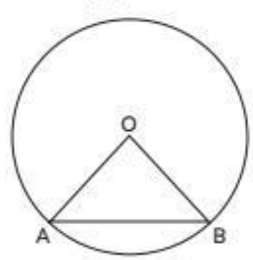


Diagram 1

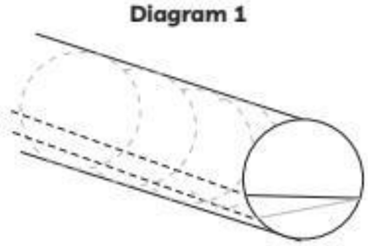


Diagram 2

Steel girders are erected above the tracks to strengthen the tunnel. Some of these are shown in Diagram 2. The girders are erected at 6 m intervals along the length of the tunnel, with one at each end.

On the basis of the above information, answer the following questions:

(A) Find the volume of earth removed to make the tunnel. 1

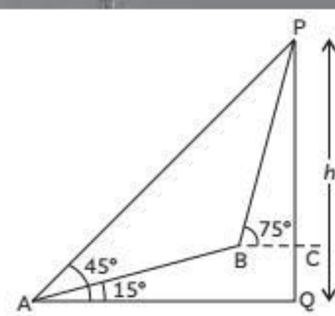
(B) Find the area of ΔAOB shown in Diagram. 1

(C) Find the length of each girder.

OR

Find total length of steel required in the 450 m length of tunnel and how many girders are errected? 2

38. *A radio mast PQ , of height 'h' metres, is standing vertically on the horizontal ground. From A, the angle of elevation of the top of the mast is found to be 45°. On moving 50 m up a slope of 15°, the angle of elevation of P is found to be 75° from B, The horizontal through B is BC.*



On the basis of the above information, answer the following questions:

(A) Find the measure of $\angle APC$. 1

(B) Find the measure of $\angle BPC$. 1

(C) Find the length BP.

OR

Find the length AP and also find the approximate value of *h*. 2

SOLUTION

SECTION - A

1. *(b) 17.5*

Explanation:

$$\text{Class mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$
$$= \frac{20+15}{2}$$
$$= \frac{35}{2}$$
$$= 17.5$$

2. *(c)* $\frac{7}{20}$

Explanation: A ball which is neither a green ball nor a red ball is necessary a white ball.

So, required probability $= \frac{7}{5+8+7} = \frac{7}{20}$

3. *(b) 1*

Explanation: $\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$,

$\cos 60° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Therefore,
sin30°cos60° + sin60°cos30°

$$\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{\sqrt{3}}{2}\right)$$
$$= \left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)$$
$$= \frac{4}{4}$$
$$= 1$$

Caution

➥ *Learn the table of trigonometric ratios for specific angles properly for solving such types of questions.*

4. *(c)* $33\pi\ cm^2$

Explanation: Area of the circular ring,

$$A = \pi\,[17^2 - 16^2]\ \text{cm}^2 = 33\pi\ \text{cm}^2$$

5. *(a) 30°*

Explanation: $2 \sin 2\theta = \sqrt{3}$

$$\Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2}$$
$$= \sin 60°$$
$$\Rightarrow \quad 2\theta = 60°$$
$$\Rightarrow \quad \theta = 30°$$

6. *(b) 5*

Explanation: Degree is the highest power of the variable in any polynomial.

7. *(d)* $\sqrt{3}$

Explanation:

Given, $\sin A = \frac{1}{2}$, $\cos B = 1$

and $0 \le A, B \le \frac{\pi}{2}$

Now, $\sin A = \frac{1}{2}$

i.e., $\sin A = \sin 30°$

∴ $A = 30°$

and $\cos B = 1$

i.e., $\cos B = \cos 0°$

∴ $B = 0°$

Now, $\cot (A + B) = \cot (30° + 0°)$

$= \cot 30°$

$= \sqrt{3}$

8. *(b)* $\frac{1}{2}$

Explanation: As $r = 3$ is a root of $kr^2 - kr - 3 = 0$, we have:

$9k - 3k - 3 = 0 \Rightarrow 6k - 3 = 0$

or $k = \frac{1}{2}$

9. *(a)* 5

Explanation: Given polynomial can be written as,

$x^5 + x^4 + x^3 - 2x - 1$

This is a polynomial of degree 5.

Caution

➥ *While finding the degree of a polynomial, arrange the terms in descending powers.*

10. *(c)* 5

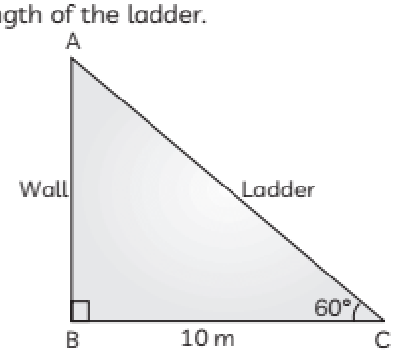
Explanation: Prime factorisation of 1944

$$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$$
$$= 2^3 \times 3^5$$

Hence, the power of 3 is 5.

11. *(d)* 20 m

Explanation: Let AB be the wall, AC be the length of the ladder.



In right angled triangle ABC,

$$\cos 60° = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{10}{AC}$$

$$AC = 10 \times 2 = 20$$

Therefore, the length of the ladder is 20 m.

12. *(d)* None of these

Explanation: Given, pair of equations are

$$kx - 3y = k - 2$$

and $12x + ky = k$

Here, $a_1 = k, b_1 = -3, c_1 = -(k-2)$

and $a_2 = 12, b_2 = k, c_2 = -k$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \uparrow \frac{c_1}{c_2}$$

i.e., $$\frac{k}{12} = \frac{-3}{k} \neq \frac{-(k-2)}{-k}$$

Then, $k^2 = -36$

(which is not possible)

$$-3k \neq k^2 - 2k$$
$$k^2 \neq -k$$
$$k \neq -1$$

Then, $k \neq -1$

Caution

➥ *While comparing the given equations with standard equation, we should also consider the signs of constants.*

13. *(a)* 49

Explanation: Multiples of 4 between 10 and 205 are 12, 16, ... 204

The above series is an A.P. with $a = 12$, $d = 4$ and $l = 204$

Let, the number of terms in this A.P. be 'n'.

Then, $a_n = a + (n-1)d$

$$204 = 12 + (n-1) \times 4$$
$$204 = 12 + 4n - 4$$
$$4n = 204 - 8$$
$$4n = 196$$
$$n = 49$$

Hence, number of multiples of 4 between 10 and 205 are 49.

14. *(d)* $-m, (m + 3)$

Explanation: Let, $P(x) = x^2 - 3x - m(m+3)$

$x^2 - 3x - m(m+3)$

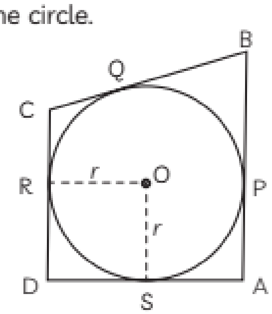
$$= x^2 - [(m+3) - m]\,x - m(m+3)$$
$$= x^2 - (m+3)\,x + mx - m(m+3)$$
$$= x[x - (m+3)] + m[x - (m+3)]$$
$$= (x+m)[x - (m+3)]$$

Thus, zeroes of given polynomial are $-m$, $(m + 3)$.

15. *(b)* 15 cm

Explanation: Join OR and OS.

In quadrilateral RDSO, all the angles are right angles and adjacent sides are equal as OR = OS. So, RDSO is a square of side equal to the radius of the circle.



$\therefore$ DS = DR = OR = OS

Now, BQ = BP gives CQ = BC – BQ

= (38 – 25) cm

= 13 cm

$\Rightarrow$ CR = 13 cm

Also, DR = CD – CR

= (28 – 13) cm

= 15 cm.

Thus, the radius of the circle is 15 cm.

16. *(c) isosceles*

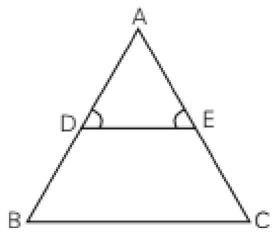
Explanation: Here, ∠E = ∠D

∴ AD = AE ...(i)

(sides opposite to equal sides are equal)

and $\frac{AD}{BD} = \frac{AE}{EC}$

∴ BD = EC ...(ii) [from (i)]



Adding (i) and (ii), we get

AD + DB = AE + EC

AB = AC

∴ ΔABC is an isosoceles triangle.

Caution

While writing the proportional sides of the similar triangles, remember that the sides opposite to equal sides must be equal.

17. *(d) 3.6 cm*

Explanation: Length of AC = $\frac{3}{3+2}\times 6$

$= \frac{18}{5} = 3.6$ cm

18. *(c) 52.5*

Explanation: Class mark of 45-60 $= \frac{45+60}{2}$

$= \frac{105}{2} = 52.5$

19. *(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)*

Explanation:

$\cos^4 A - \sin^4 A = (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= (\cos^2 A - \sin^2 A).1$

$= \cos^2 A - (1 - \cos^2 A)$

20. *(d) Assertion (A) is false but reason (R) is true.*

Explanation:

Median = $\frac{1}{3}$(mode + 2 mean)

$= \frac{1}{3}(60 + 2 \times 66)$

= 64

SECTION - B

21. Let the coordinates of point P be $(2y, y)$. As P is equidistant from Q and R,

PQ = PR or $PQ^2 = PR^2$

i.e. $(2 - 2y)^2 + (-5 - y)^2 = (-3 - 2y)^2 + (6 - y)^2$

$\Rightarrow 4y^2 - 8y + 4 + 25 + 10y + y^2$

$= 9 + 4y^2 + 12y + 36 - 12y + y^2$

$\Rightarrow 2y = 16$

$\Rightarrow y = 8$

Thus, the coordinates of point P are (16, 8).

OR

Let the three given points be A (2, –2), B (–2, 1) and C (5, 2). Then, using distance formula, we have

AB = $\sqrt{(-2 - 2)^2 + (1 + 2)^2} = \sqrt{16 + 9} = 5$

BC = $\sqrt{(5 + 2)^2 + (2 - 1)^2} = \sqrt{49 + 1}$

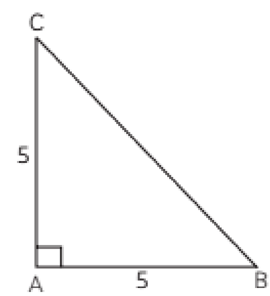
$= \sqrt{50} = 5\sqrt{2}$

CA = $\sqrt{(2 - 5)^2 + (-2 - 2)^2} = \sqrt{9 + 16}$

$= \sqrt{25} = 5$

∴ $AB^2 + CA^2 = BC^2$

Hence, ABC is a right-triangle, right-angled at A.



Caution

The distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

It gives the same answer.

22. Here,

210 = 2 × 3 × 5 × 7

55 = 5 × 11

So, HCF (210, 55) = 5

Thus, 5 = 210 × 5 – 55y

gives $55y = 209 \times 5$

$\Rightarrow$ $55y = 1045$

$\Rightarrow$ $y = 19$

OR

If 4^n ends with digit 0, then it must have 5, 2 as its factors.

But, $(4)^n = (2^2)^n = 2^{2n}$, *i.e.*, the only prime factor of 4^n is 2.

Also, we know from the Fundamental Theorem of Arithmetic that the prime factorisation of each number is unique.

$\therefore$ 4^n can never end with digit 0.

Caution

➥ *Only numbers having 2 and 5 as factors, can end with digit 0.*

23. Here,

$$\alpha + \beta = \frac{k+6}{1} = k + 6$$

and $$\alpha \times \beta = \frac{2(2k-1)}{1} = 2(2k-1)$$

Since $$\alpha + \beta = \frac{1}{3}\alpha\beta,$$

$$\Rightarrow \quad k + 6 = \frac{1}{3}[2(2k-1)]$$

$$\Rightarrow \quad 3k + 18 = 4k - 2$$

$$\Rightarrow \quad k = 20$$

24. $$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{67}{12}$$

25. Base diameter of cone = 24 m

Base radius of cone, r = 12 m

Height of cone, h = 3.5 m

Now, slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{12^2 + 3.5^2}$

$= \sqrt{144 + 12.25}$

$= \sqrt{156.25}$

$= 12.5$ m

Area of canvas cloth = C.S.A. of cone

$= \pi rl$

$= 3.14 \times 12 \times 12.5$

$= 471$

Hence, area of canvas cloth required is 471 m^2.

SECTION - C

26. The required greatest 6-digit number which is a multiple of 24, 15 , 36 is a multiple of LCM (24, 15, 36).

Now, $24 = 2^3 \times 3$; $15 = 3 \times 5$; $36 = 2^2 \times 3^2$

So, LCM (24, 15, 36) = $2^3 \times 3^2 \times 5 = 360$

So, the required number is 2777×360, i.e. 999720, because 2778×360 is a 7-digit number.

OR

In a pack of 52 cards, black face cards are six.

So, the number of remaining cards in the pack is 46.

Thus, 46 cards consists of:

26 red cards + 20 black cards (from 1 to 10 each)

Hence, total number of possible outcomes = 46.

Now,

(A) P (face card) = $\frac{6}{46} = \frac{3}{23}$

[6 face cards of red colour]

(B) P (red card) = $\frac{26}{46} = \frac{13}{23}$

27. Here, $a = 5$, $l = 45$ and S = 400.

Let the AP contains 'n' terms. Then, n^{th} term is the last term.

$\Rightarrow$ $a + (n-1)d = 45$

$\Rightarrow$ $5 + (n-1)d = 45$

$\Rightarrow$ $(n-1)d = 40$ ...(i)

Also, $S_n = \frac{n}{2}[2a + (n-1)d] = 400$

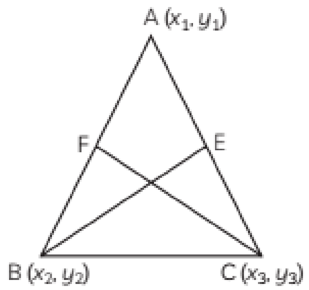
$\frac{n}{2}[10 + 40] = 400$ [using (i)]

$\Rightarrow$ $50n = 800$

$\Rightarrow$ $n = 16$

From (i), $d = \frac{40}{15}$ *i.e.* $\frac{8}{3}$

28.



The points A (x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC.

Median from B meets AC at E.

Coordinates of E are

$$\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (x_4, y_4)$$

Let, the coordinates of Q be (x, y)

$$\therefore \quad x = \frac{mx_4+nx_2}{m+n}$$

$$= \frac{\frac{x_1+x_3}{2}\times 2+x_2\times 1}{2+1}$$

$$= \frac{x_1+x_2+x_3}{3}$$

and
$$y = \frac{my_4+ny_2}{m+n}$$

$$= \frac{\left(\frac{y_1+y_3}{2}\right)\times 2+y_2\times 1}{2+1}$$

$$= \frac{y_1+y_2+y_3}{3}$$

$$\therefore Q\,(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly, finding the coordinates of R dividing CF in the ratio of 2 : 1, we get

$$\therefore R(p, q) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

So, Q and R are the same points and their coordinates are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

29. In ΔOQP, DE || OQ

$$\frac{PE}{EQ} = \frac{PD}{DO} \qquad \text{...(i)}$$

In ΔOPR, DF || OR

$$\frac{PD}{DO} = \frac{PF}{FR} \qquad \text{...(ii)}$$

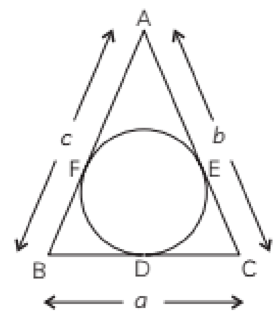
From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

∴ From ΔPQR,

EF || QR

OR



We know, lengths of tangents drawn from an external point to a circle are equal.

∴ BD = BF, CD = CE, AE = AF.

Now, from the figure, we have

$$\begin{aligned} BD &= a - CD \\ &= a - CE && (\because CD = CE) \\ &= a - (b - AE) \\ &= a\,(b - AF) && (\because AE = AF) \\ &= a - b + (c - BF) \\ &= a - b + c - BF \\ &= a - b + c - BD && (\because BD = BF) \end{aligned}$$

Thus, $\quad 2BD = a - b + c \qquad$ ...(i)

Now, $\quad s = \dfrac{a+b+c}{2} \qquad$ ...(ii)

From (i) and (ii), we have :

$$s - b = \frac{a+b+c}{2} - b$$

$$= \frac{a-b+c}{2} = BD.$$

Hence Proved.

30. Let, the height of rainfall be 'h' m.

Then, volume of roof = Volume of cylindrical vessel

$$l \times b \times h = \pi r^2 h$$

$$22 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times 3.5$$

$$h = \frac{22\times 3.5}{22\times 7\times 20}$$

$$= \frac{0.5}{20} = \frac{5}{200} = \frac{1}{40}$$

$$= 0.025 \text{ m}$$

$$= 2.5 \text{ cm.}$$

31.

Marks	f	$c.f.$
0-5	4	4
5-10	6	10
10-15	f_1	$10 + f_1$
15-20	10	$20 + f_1$
20-25	25	$45 + f_1$
25-30	f_2	$45 + f_1 + f_2$
30-35	18	$63 + f_1 + f_2$
35-40	5	$68 + f_1 + f_2$
Total	100	

Now, $68 + f_1 + f_2 = 100$

$f_1 + f_2 = 100 - 68 = 32$

Here, median is 24, so median class is 20-25

Then, $l = 20$, $h = 5$, $c.f. = 20 + f_1$

and $f = 25$, $\frac{N}{2} = 50$, Median = 24

Then,
$$M_e = l + \frac{\left(\frac{N}{2} - c.f.\right)}{f} \times h$$

$$24 = 20 + \left(\frac{50 - 20 - f_1}{25}\right) \times 5$$

$$\Rightarrow \quad 4 = \left(\frac{30 - f_1}{5}\right)$$

$$\Rightarrow \quad 20 = 30 - f_1$$

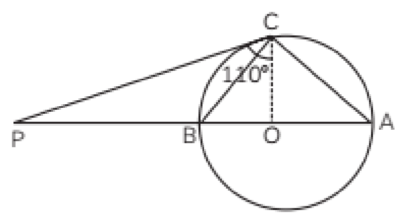
$$\Rightarrow \quad f_1 = 10$$

Now, $f_2 = 32 - 10 = 22$

Hence, the values of f_1 and f_2 are 10 and 22 respectively.

SECTION - D

32.



Given, $\angle PCA = 110°$

A, O, B, P all are on the same line and P and C are points on the tangent.

AB is a diameter of a circle.

$\therefore \quad \angle BCA = 90°$ ($\because$ Angle inscribed in a semi-circle)

and $\quad \angle OCP = 90°$ ($\because$ Tangent at any point on a circle is perpendicular to the radius)

Now, $\angle PCA = \angle PCO + \angle OCA$

$\Rightarrow \quad 110° = 90° + \angle OCA$

$\Rightarrow \quad \angle OCA = 20°$

Now, in ΔAOC

$\Rightarrow \quad AO = OC$ (Radi of circle)

$\Rightarrow \quad \angle OCA = \angle CAO = 20°$

In ΔABC,

$\Rightarrow \quad \angle CAB + \angle CBA + \angle BCA = 180°$

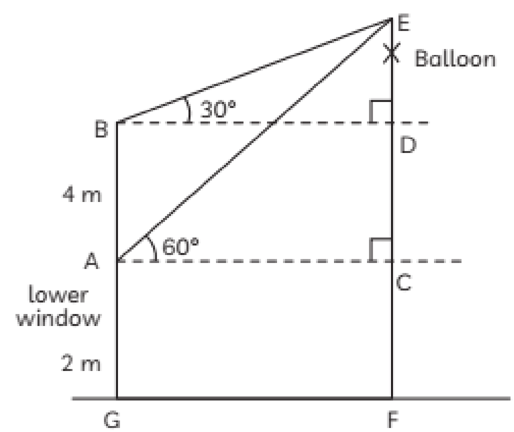
$\Rightarrow \quad 20° + \angle CBA + 90° = 180°$

$\Rightarrow \quad \angle CBA = 70°$

OR

In the figure, A and B represent two windows and E is the position of the balloon. If balloon is 'h' metres above the ground, then

in ΔACE,



$$\tan 60° = \frac{CE}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h - 2}{AC}$$

or $\quad AC = \frac{h-2}{\sqrt{3}}$ .....(i)

Also, in ΔBDE,

$$\tan 30° = \frac{DE}{BD}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-6}{AC}$ ($\because$ BD = AC)

$\Rightarrow \quad AC = \sqrt{3}\,(h - 6)$

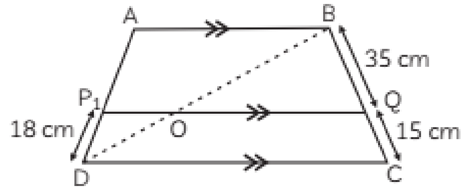
$\Rightarrow \quad \frac{h-2}{\sqrt{3}} = \sqrt{3}\,(h - 6)$ [Using (i)]

$\Rightarrow \quad h - 2 = 3(h - 6)$

$\Rightarrow \quad h = 8$

Hence, the height of the balloon above the ground is 8 metres.

33.



Join BD, so that it intersects PQ at O.

In ΔABD, PO || AB [$\because$ PQ || AB]

By basic proportionality theorem,

$$\frac{DP}{AP} = \frac{DO}{OB} \quad ...(i)$$

In ΔBDC, OQ || DC [$\because$ PQ || DC]

By basic proportionality theorem,

$$\frac{BQ}{QC} = \frac{OB}{OD}$$

$$\Rightarrow \frac{QC}{BQ} = \frac{OD}{OB} \quad ...(ii)$$

From equations (i) and (ii)

$$\frac{DP}{AP} = \frac{QC}{BQ}$$

$$\Rightarrow \frac{18}{AP} = \frac{15}{35}$$

34. Let, the number of bananas in first lot be x and that in second lot be y.

$\therefore$ Total number of bananas = $x + y$

In the first case, price of x bananas at the rate ₹ 2 per 3 bananas = $\frac{2x}{3}$ and price of y bananas at the rate of ₹ 1 per banana = y.

According to the given condition,

$$\frac{2x}{3} + y = 400$$

$$\Rightarrow 2x + 3y = 1200 \quad ...(i)$$

In second case, price of x bananas at the rate of ₹ 1 per bananas = x and price of y bananas at the rate of ₹ 4 per 5 bananas = $\frac{4}{5}y$.

According to given condition,

$$x + \frac{4}{5}y = 460$$

$$\Rightarrow 5x + 4y = 2300 \quad ...(ii)$$

Multiplying (i) by 5 are (ii) by 2, and then subtracting them.

$$10x + 15y = 6000$$
$$10x + 8y = 4600$$
$$- \quad - \quad -$$

$$7y = 1400$$

$$\Rightarrow y = 200$$

Put $y = 200$, in equation (i) we get

$$2x + 3 \times 200 = 1200$$

$$\Rightarrow x = 300$$

$$\therefore x + y = 300 + 200 = 500$$

Thus, Ajay had total of 500 bananas.

35. Let Agam's marks in Mathematics = x

Let Agam's marks in English = $30 - x$

If, he had got 2 marks more in Mathematics, his marks would be = $x + 2$

If, he had got 3 marks less in English, his marks in English would be = $30 - x - 3$

$= 27 - x$

According to given condition:

$$(x + 2)(27 - x) = 210$$

$$\Rightarrow 27x - x^2 + 54 - 2x = 210$$

$$\Rightarrow x^2 - 25x + 156 = 0$$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$, We get $a = 1$, $b = -25$ and $c = 156$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{25 \pm \sqrt{(25)^2 - 4(1)(156)}}{2 \times 1}$$

$$\Rightarrow x = \frac{25 \pm \sqrt{625 - 624}}{2}$$

$$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$$

$$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$$

$$\Rightarrow x = 13, 12$$

Therefore, Agam's marks in Mathematics = 13 or 12

Agam's marks in English = $30 - x$

$= 30 - 13$

$= 17$

Or

Agam's marks in English = $30 - x$

$= 30 - 12$

$= 18$

Therefore, his marks in Mathematics and English are (13,17) or (12,18).

OR

There are 12 balls in the box.

Therefore, total number of favourable outcomes = 12

The number of favourable outcomes = x

Therefore, P_1 = P (getting a black ball) = $\frac{x}{12}$

If 6 more balls put in the box, then

Total number of favourable outcomes = 12 + 6 = 18

And number of favourable outcomes = $x+6$

$\therefore$ P_2 = P (getting a black ball) = $\frac{x+6}{18}$

According to question,

$$P_2 = 2P_1$$

$$\Rightarrow \quad \frac{x+6}{18} = 2\times\frac{x}{12}$$

$$\Rightarrow \quad \frac{x+6}{18} = \frac{x}{6}$$

$$\Rightarrow \quad 6(x+6) = 18x$$

$$\Rightarrow \quad 18x - 6x = 36$$

$$\Rightarrow \quad 12x = 36$$

$$\Rightarrow \quad x = \frac{36}{12}$$

$$\Rightarrow \quad x = 3$$

SECTION - E

36. (A) P(right) = $\frac{2}{6}=\frac{1}{3}$

(B) getting (1, 2, 3 or 4 twice) = $\frac{2}{6}\times\frac{2}{6}$

$= \frac{1}{3}\times\frac{1}{3}$

$= \frac{1}{9}$

As per the situation given, if the button is at Y and it is moved twice, then it will reach to Y again if it is first move to left then to right or vice-versa.

$\therefore$ P(Y) = $\frac{2}{6}\times\frac{4}{6}+\frac{4}{6}\times\frac{2}{6}=\frac{16}{36}=\frac{4}{9}$

OR

By adding the probabilities obtained in (B), we get

Probability (P, Q or Y) = $\frac{1}{9}+0+\frac{4}{9}$

$= \frac{5}{9}$

(C) As per the situation given in part (B), if the button is at Y and it is moved twice, then there is no possibility that it reach to Q.

37. (A) Volume of tunnel= $\pi r^2 h$

$= \pi \times 5 \times 5 \times 450$

$= 11250\pi$ cu cm

(B) Area of $\Delta AOB = \frac{1}{2} \times AO \times OB$

$= \frac{1}{2} \times 5 \times 5$

= 12.5 sq. m

(C) As steel girders are in the shape of a circle or a sector of the circle with centra/angle 270°.

So, circumference of the circle with central angle 270°

$= \frac{270°}{360°}\times 2\pi r$

$= \frac{3}{4}\times 2\pi r$

$= 1.5\pi \times 5$

$= 7.5\pi$ cm

OR

Total length $= 7.5\,\pi \times 76$

$= 570\,\pi$ m

No. of girders = $\frac{450}{6}$

= 75

$\therefore$ Total girder = 75 + 1 (one at last)

= 76

38. (A) In ΔAPQ

$\angle A + \angle P + \angle Q = 180°$

$45° + \angle P + 90° = 180°$

$\Rightarrow \quad \angle P = 180° - 90° - 45°$

$= 180° - 135°$

$= 45°$

(B) In ΔBPC

$\angle B + \angle BPC + \angle C = 180°$

$\Rightarrow 75° + \angle BPC + 90° = 180°$

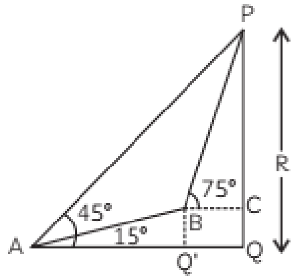
$\Rightarrow \quad \angle BPC = 180° - 165°$

$= 15°$

(C) In ΔAPQ

$$\tan 45° = \frac{PQ}{AQ}$$

$$PQ = AQ = h$$



In ΔABQ',

$$\sin 15° = \frac{BQ'}{AB}$$

$$BQ' = 50 \sin 15°$$

$$= 13 \text{ m}$$

(Taking sin 15° = 0.26)

Then, $BQ' = CQ = 13$ m

$$PC = h - 13$$

In ΔABQ'

$$\cos 15° = \frac{AQ'}{AB}$$

$$AQ' = 50 \cos 15°$$

$$= 48.25 \simeq 49$$

(Taking cos 15° = 0.965)

$$QQ' = AQ - AQ'$$

$$= h - 49$$

$$BC = QQ' = h - 49$$

Now, in ΔPBC

$$\tan 75° = \frac{PC}{BC}$$

$$3.73 = \frac{h-13}{h-49}$$

$$\Rightarrow \quad 3.73h - 182.77 = h - 13$$

$$\Rightarrow \quad 2.73h = 169.77$$

$$\Rightarrow \quad h = 62.186$$

$$\sin 75° = \frac{PC}{PB}$$

$$1 = \frac{62.2-13}{PB}$$

$$PB = 49.2 = 50 \text{ m}$$

OR

In ΔAPQ

$$\sin 45° = \frac{h}{AP}$$

$$AP = \frac{h}{\frac{1}{\sqrt{2}}} = \sqrt{2}h$$

The approximate height h, as calculated in part (C) is 50 m.