CBSE Sample Paper -01 (solved)
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions. These are MCQs. Choose the correct option.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. If two zeros of the polynomial $f(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

2. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ΔABC is a right triangle.

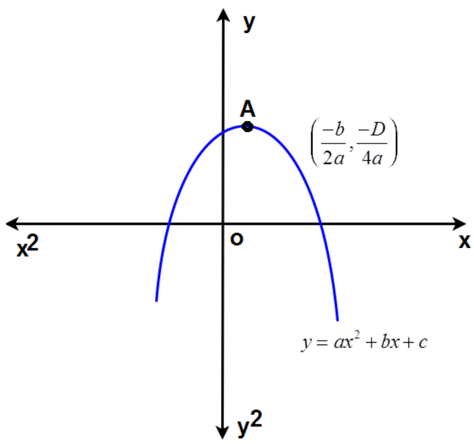
3. Evaluate cos60°cos30° + sin60°sin30°.

4. Prove that $\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

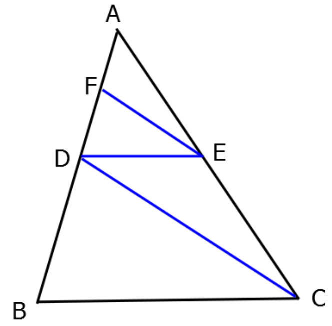
5. Find the median of the daily wages of ten workers from the following data:
22, 25, 18, 20, 28, 15, 27, 10, 9, 16

SECTION – B

6. The graph of $y = ax^2 + bx + c$ is given in the following figure. Identify the signs of a, b and c.



7. In the given figure, DE || BC and CD || EF. Prove that $AD^2 = AB \times AF$.



8. If $\sin\theta + \sin^2\theta = 1$, find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$

9. For the following grouped frequency distribution, find the mode.

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

10. ABC is a right triangle, right angled at C. If A = 30° and AB = 40 units, find the remaining two sides and ∠B of ΔABC.

SECTION – C

11. Prove that $3\sqrt{2}$ is irrational.

12. Solve: $\frac{x}{a}+\frac{y}{b}=2$; $ax - by = a^2 - b^2$

13. The mean of the following frequency distribution is 1.46. Find the missing frequencies.

Number of accidents (x)	0	1	2	3	4	5	Total
Frequency (f)	46	f_1	f_2	25	10	5	200

14. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at same point, the ladder is turned to other side of the street to reach a window 12 m high. Find the width of the street.

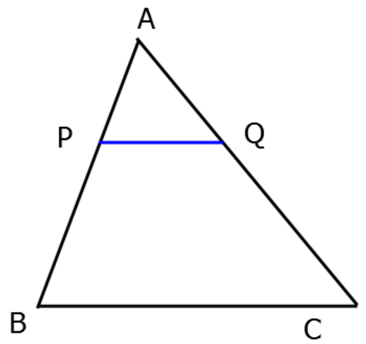
15. If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B \leq 90°$, $A > B$ then find A and B.

16. Prove $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

17. Find the values of x and y if the total frequency and the median of the following data is 100 and 525, respectively.

Class interval	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	x	12	17	20	y	9	7	4

18. P and Q are points on sides AB and AC, respectively of ΔABC. If AP = 3 cm, PB = 6 cm, AQ = 5 cm and QC = 10 cm, show that BC = 3PQ.



19. If α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find the polynomial whose zeros are 2α + 3β and 3α + 2β.

20. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

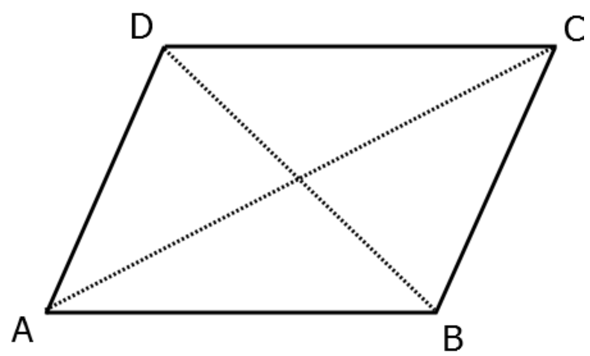
SECTION – D

21. Find all the zeros of the polynomial $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

22. Show graphically that the system of equations $2x + 4y = 10$; $3x + 6y = 12$ has no solution.

23. Prove that if the corresponding sides of two triangles are proportional, then they are similar.

24. ABCD is a rhombus. Prove that $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$



25. If $\cot B = \frac{12}{5}$, prove that $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.

26. If (secA + tanA)(secB + tanB)(secC + tanC) = (secA – tanA)(secB – tanB)(secC – tanC), prove that each of the side is equal to ±1.

27. Apply step-deviation method to find the arithmetic mean of the following frequency distribution.

Variate (x)	5	10	15	20	25	30	35	40	45	50
Frequency (f)	20	43	75	67	72	45	39	9	8	6

28. If cosecA = $\sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - \cos^2 A}$.

29. Draw a cumulative frequency curve and cumulative frequency polygon for the following frequency distribution by less than method.

Age (in years)	0-9	10-19	20-9	30-39	40-49	50-59	60-69
Number of persons	5	15	20	23	17	11	9

30. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

31. The percentage of salary that 10 households donate to an orphanage is given below:

5, 3, 10, 5, 2, 4, 7, 8, 1, 5

Find the mean, median and mode of the data. Also tell the values depicted by the persons of these households.

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SECTION – A

1. **Solution:**

Let $\alpha = \sqrt{3}$ and $\beta = -\sqrt{3}$ be the given zeros and γ be the third zero. Then,

$$\alpha + \beta + \gamma = -\left(\frac{-4}{1}\right) \qquad \left[\text{Using } \alpha + \beta + \gamma = \frac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}\right]$$

$\Rightarrow \quad \sqrt{3} - \sqrt{3} + \gamma = 4$

$\Rightarrow \quad \gamma = 4$

Hence, third zero is 4.

2. **Solution:**

We have AC = BC and $AB^2 = 2AC^2$

Now, $AB^2 = 2AC^2$

$\Rightarrow \quad AB^2 = AC^2 + AC^2$

$\Rightarrow \quad AB^2 = AC^2 + BC^2 \qquad [\because AC = BC \text{ (Given)}]$

$\Rightarrow \quad \Delta ABC$ is a right triangle right angled at C.

3. **Solution:**

We have cos60°cos30° + sin60°sin30°

$$= \frac{1}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\times\frac{1}{2}=\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}=2\left(\frac{\sqrt{3}}{4}\right)=\frac{\sqrt{3}}{2}$$

4. **Solution:**

We have LHS = $\cot^2\theta - \frac{1}{\sin^2\theta}$

$$= \cot^2\theta - \text{cosec}^2\theta \qquad \left[\because \frac{1}{\sin\theta} = \text{cosec}\theta\right]$$

$$= -1 = \text{RHS} \qquad \left[\because 1+\cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta - \text{cosec}^2\theta = -1\right]$$

5. **Solution:**

Arranging the wages in ascending order of magnitude, we have

9, 10, 15, 16, 18, 20, 22, 25, 27, 28

Since there are 10 observations, therefore, median is the arithmetic mean of

$\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$ observations.

Thus, median = $\frac{18+20}{2}=19$

SECTION – B

6. **Solution:**

We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a < 0$.

We also observe that the vertex of the parabola is in first quadrant.

$\therefore \quad -\frac{b}{2a} > 0 \Rightarrow -b < 0 \Rightarrow b > 0$

Parabola $y = ax^2 + bx + c$ cuts Y-axis at P. On Y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of P are $(0, c)$. As P lies on the positive direction of Y-axis, therefore, $c > 0$.

Hence, $a < 0$, $b > 0$ and $c > 0$.

7. **Solution:**

In ΔABC, we have DE || BC

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$ [By basic proportionality theorem] ...(i)

In ΔADC, we have FE || DC

$\Rightarrow \frac{AD}{AF} = \frac{AC}{AE}$ [By basic proportionality theorem] ...(ii)

From (i) and (ii), we get

$\Rightarrow \frac{AB}{AD} = \frac{AD}{AF} \quad \Rightarrow \quad AD^2 = AB \times AF$

8. **Solution:**

We have $\sin\theta + \sin^2\theta = 1 \quad \Rightarrow \quad \sin\theta = 1 - \sin^2\theta \quad \Rightarrow \quad \sin\theta = \cos^2\theta$

Now, $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$

$= (\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta) + 2(\cos^4\theta + \cos^2\theta - 1)$

$= (\cos^4\theta + \cos^2\theta)^3 + 2(\cos^4\theta + \cos^2\theta - 1)$

= $(\sin^2\theta + \cos^2\theta)^3 + 2(\sin^2\theta + \cos^2\theta - 1)$ [$\because \cos^2\theta = \sin\theta, \therefore \cos^4\theta = \sin^2\theta$]

= $1 + 2(1 - 1) = 1$

9. **Solution:**

We observe that the class 12-15 has maximum frequency. Therefore, this is the modal class.

We have,

$l = 12, h = 3, f = 23, f_1 = 10$ and $f_2 = 21$

$\therefore$ Mode = $l + \dfrac{f - f_1}{2f - f_1 - f_2} \times h$

= $12 + \dfrac{23 - 10}{46 - 10 - 21} \times 3$

= $12 + \dfrac{13}{15} \times 3 = 12 + \dfrac{13}{5} = 14.6$

10. **Solution:**

In ΔABC, we have,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $30° + \angle B + 90° = 180°$ [$\because \angle A = 30°$ and $\angle C = 90°$]

$\Rightarrow$ $\angle B = 180° - 120° = 60°$

Now, $\cos A = \dfrac{AC}{AB}$

$\Rightarrow$ $\cos 30° = \dfrac{AC}{40}$

$\Rightarrow$ $\dfrac{\sqrt{3}}{2} = \dfrac{AC}{40}$

$\Rightarrow$ $AC = \dfrac{\sqrt{3}}{2} \times 40$ $\Rightarrow$ $AC = 20\sqrt{3}$ units

And , $\sin A = \dfrac{BC}{AB}$

$\Rightarrow$ $\sin 30° = \dfrac{BC}{40}$

$\Rightarrow$ $\dfrac{1}{2} = \dfrac{BC}{40}$

$\Rightarrow$ $BC = \dfrac{1}{2} \times 40$ $\Rightarrow$ BC = 20 units

Hence, AC = $20\sqrt{3}$ units and BC = 20 units

SECTION – C

11. **Solution:**

Let us assume, to the contrary, that $3\sqrt{2}$ is rational. Then, there exist co-prime positive integers a and b such that

$$3\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \quad \sqrt{2}=\frac{a}{3b}$$

$\Rightarrow \quad \sqrt{2}$ is rational [$\because$ 3, a and b are integers, $\therefore \frac{a}{3b}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational. So, out assumption is not correct.

Hence, $3\sqrt{2}$ is an irrational number.

12. **Solution:**

The given system of equations may be written as

$bx + ay - 2ab = 0$

$ax - by - (a^2 - b^2) = 0$

By cross multiplication, we have

$$\Rightarrow \quad \frac{x}{-a(a^2-b^2)-(-b)(-2ab)}=\frac{-y}{-b(a^2-b^2)-a(-2ab)}=\frac{1}{b(-b)-a(a)}$$

$$\Rightarrow \quad \frac{x}{-a(a^2-b^2)-2ab^2}=\frac{-y}{-b(a^2-b^2)+2a^2b}=\frac{1}{-b^2-a^2}$$

$$\Rightarrow \quad \frac{x}{-a(a^2-b^2+2b^2)}=\frac{-y}{-b(a^2-b^2-2a^2)}=\frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-a(a^2+b^2)}=\frac{-y}{-b(-a^2-b^2)}=\frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad x=\frac{-a(a^2+b^2)}{-(a^2+b^2)}=a \text{ and } y=\frac{-b(a^2+b^2)}{-(a^2+b^2)}=b$$

Hence, solution of the given system of equations is $x = a$ and $y = b$.

13. **Solution:**

Calculation of mean

x_i	f_i	f_ix_i
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$N = 86 + f_1 + f_2$	$\sum f_ix_i = 140 + f_1 + 2f_2$

We have, $N = 200$

$\Rightarrow \quad 200 = 86 + f_1 + f_2$

$\Rightarrow \quad f_1 + f_2 = 114 \quad$ ...(i)

Also, mean = 1.46

$$\Rightarrow \quad 1.46 = \frac{\sum f_ix_i}{N}$$

$$\Rightarrow \quad 1.46 = \frac{140 + f_1 + 2f_2}{200}$$

$\Rightarrow \quad 292 = 140 + f_1 + 2f_2$

$\Rightarrow \quad f_1 + 2f_2 = 152 \quad$ ...(ii)

From (i), $f_1 = 114 - f_2$

Putting the value of f_1 in (ii), we have

$114 - f_2 + 2f_2 = 152$

$\Rightarrow \quad f_2 = 152 - 114 = 38$

Putting the value of f_2 in (i), we have

$f_1 + 38 = 114$

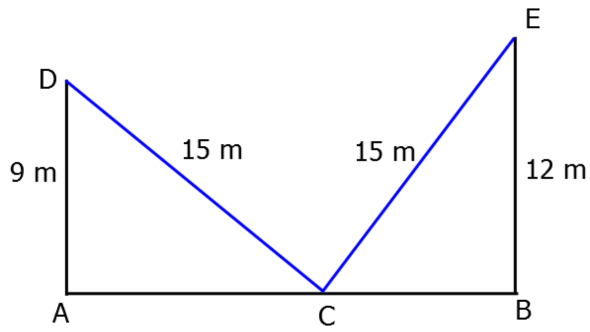
$\Rightarrow \quad f_1 = 114 - 38 = 76$

Thus, we have $f_1 = 76$ and $f_2 = 38$.

14. **Solution:**

Let AB be the width of the street and C be the foot of the ladder. Let D and E be the windows at heights of 9 m and 12 m, respectively from the ground. Then, CD and EF are the two positions of the ladder.

Clearly, AD = 9 m, BE = 12 m, CD = CE = 15 m.



In ΔACD, we have

$CD^2 = AC^2 + AD^2$

$\Rightarrow \quad 15^2 = AC^2 + 9^2$

$\Rightarrow \quad AC^2 = 225 - 81 = 144$

$\Rightarrow \quad AC = 12$ m

In ΔBCE, we have

$CE^2 = BC^2 + BE^2$

$\Rightarrow \quad 15^2 = BC^2 + 12^2$

$\Rightarrow \quad BC^2 = 225 - 144 = 81$

$\Rightarrow \quad BC = 9$ m

Hence, width of the street = AB = AC + CB = (12 + 9) m = 21 m

15. **Solution:**

We have sin(A + B) = 1

$\Rightarrow \quad \sin(A + B) = \sin 90°$

$\Rightarrow \quad A + B = 90°$ ...(i)

And, $\cos(A - B) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \cos(A - B) = \cos 30°$

$\Rightarrow \quad A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$(A + B) + (A - B) = 90° + 30°$

$\Rightarrow \quad 2A = 120° \quad \Rightarrow \quad A = 60°$

Putting A = 60° in (i), we get

$60° + B = 90° \quad \Rightarrow \quad B = 90° - 60° = 30°$

Thus, A = 60° and B = 30°

16. **Solution:**

We have

LHS $= (\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\text{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$

$$= \left(\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\frac{1}{\sin\theta}\right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta\frac{1}{\cos\theta}\right)$$

$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$

$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 4$

$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$ $\quad$ [$\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta$]

$= 7 + \cot^2\theta + \tan^2\theta$

= RHS

17. **Solution:**

Calculation of median

Class intervals	**Frequency (*f*)**	**Cumulative frequency (*cf*)**
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
		Total = 100

We have, $N = \sum fi = 100$

$\Rightarrow \quad 76 + x + y = 100$

$\Rightarrow \quad x + y = 24$

It is given that the median is 525. Clearly, it lies in the class 500-600.

$\therefore \quad l = 500, h = 100, f = 20, F = 36 + x$ and $N = 100$

Now, median = $l + \dfrac{\frac{N}{2} - F}{f} \times h$

$$\Rightarrow \quad 525 = 500 + \frac{\frac{100}{2} - (36 + x)}{20} \times 100$$

$\Rightarrow$ $525 - 500 = \frac{50 - 36 - x}{20} \times 100$

$\Rightarrow$ $25 = (14 - x) \times 5$

$\Rightarrow$ $25 = 70 - 5x$

$\Rightarrow$ $5x = 45$

$\Rightarrow$ $x = \frac{45}{5}$ $\Rightarrow$ $x = 9$

Putting $x = 9$ in $x + y = 24$, we get

$9 + y = 24$

$\Rightarrow$ $y = 24 - 9 = 15$

Thus, $x = 9$ and $y = 15$.

18. **Solution:**

We have,

AB = AP + PB = 3 + 6 = 9 cm

And, AC = AQ + QC = 5 + 10 = 15 cm

$\therefore$ $\frac{AP}{AB} = \frac{3}{9} = \frac{1}{3}$ and $\frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow$ $\frac{AP}{AB} = \frac{AQ}{QC}$

Thus, in triangles APQ and ABC, we have

$\frac{AP}{AB} = \frac{AQ}{QC}$ and $\angle A = \angle A$

Therefore, by SAS criterion of similarity, we have

$\Delta APQ \sim \Delta ABC$

$\Rightarrow$ $\frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow$ $\frac{PQ}{BC} = \frac{AQ}{AC}$

$\Rightarrow$ $\frac{PQ}{BC} = \frac{5}{15} = \frac{1}{3}$

$\Rightarrow$ BC = 3PQ

19. **Solution:**

Since α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$

∴ $\alpha + \beta = -\left(-\frac{5}{2}\right) = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Let S and P denote respectively the sum and product of zeros of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5\times\frac{5}{2}=\frac{25}{2}$

And, $P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$

$= 6(\alpha + \beta)^2 + \alpha\beta$

$= 6\times\left(\frac{5}{2}\right)^2+\frac{7}{2}=6\times\frac{25}{4}+\frac{7}{2}=\frac{75}{2}+\frac{7}{2}=41$

Hence, the required polynomial is given by

$g(x) = k(x^2 - Sx + P)$

$= \left(x^2-\frac{25}{2}x+41\right)$, where k is any non-zero real number.

20. **Solution:**

We have

LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta + \cos^2\theta)\{(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta + \cos^2\theta\}] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\{(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\} - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta\cos^2\theta - 3\sin^4\theta - 3\cos^4\theta + 1$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta)^2 + 1$

$= -1 + 1 = 0 =$ RHS

SECTION – D

21. **Solution:**

We know that, if $x = \alpha$ is a zero of a polynomial, then $x - \alpha$ is a factor of $f(x)$. Since $\sqrt{2}$ and $-\sqrt{2}$ are zeros of $f(x)$, therefore, $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of $f(x)$.

Now, we divide $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ by $g(x) = x^2 - 2$ to find the other zeros of $f(x)$.

We have,

$$
\begin{array}{r|l}
 & 2x^2-3x+1 \\
\hline
x^2-2 & 2x^4-3x^3-3x^2+6x-2 \\
 & 2x^4 \qquad\quad -4x^2 \\
 & - \qquad\qquad + \\
\hline
 & \quad -3x^3+x^2+6x-2 \\
 & \quad -3x^3 \qquad +6x \\
 & \quad + \qquad\quad - \\
\hline
 & \qquad\quad x^2 \qquad -2 \\
 & \qquad\quad x^2 \qquad -2 \\
 & \qquad\quad - \qquad + \\
\hline
 & \qquad\qquad 0
\end{array}
$$

By division algorithm, we have

$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(2x^2 - 2x - x + 1)$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})\{2x(x - 1) - (x - 1)\}$

$\Rightarrow \quad 2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x - \sqrt{2})(x + \sqrt{2})(x - 1)(2x - 1)$

Hence, the zeros of the given polynomial are $\sqrt{2}, -\sqrt{2}$, 1 and $\frac{1}{2}$.

22. **Solution:**

Graph of $2x + 4y = 10$:

We have,

$2x + 4y = 10 \quad \Rightarrow \quad 4y = 10 - 2x \quad \Rightarrow \quad y = \frac{10-2x}{4} \quad \Rightarrow \quad y = \frac{5-x}{2}$

When $x = 1$, we have $y = \frac{5-x}{2} = \frac{5-1}{2} = \frac{4}{2} = 2$

When $x = 3$, we have $y = \frac{5-x}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$

Thus, we have the following table:

x	1	3
y	2	1

Graph of $3x + 6y = 12$:

We have,

$3x + 6y = 12 \quad \Rightarrow \quad 6y = 12 - 3x \quad \Rightarrow \quad y = \frac{12-3x}{6} \quad \Rightarrow \quad y = \frac{4-x}{2}$

When $x = 2$, we have $y = \frac{4-x}{2} = \frac{4-2}{2} = \frac{2}{2} = 1$

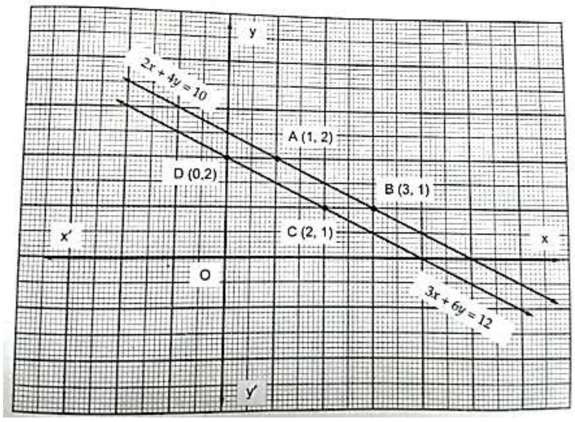
When $x = 0$, we have $y = \frac{4-x}{2} = \frac{4-0}{2} = \frac{4}{2} = 2$

Thus, we have the following table:

x	2	0
y	1	2

Plot the points A (1, 2) and B (3, 1) of $2x + 4y = 10$ on a graph paper. Join A and B and extend it on both sides as shown in the figure.

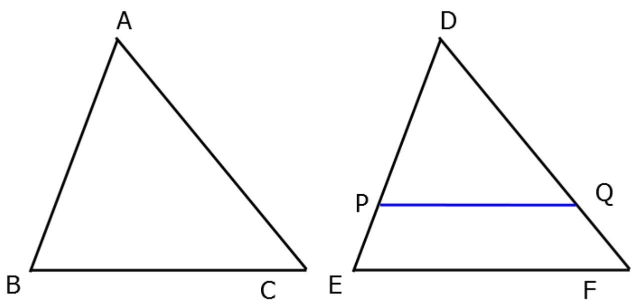
Also, plot the points C (2, 1) and D (0, 2) of $3x + 6y = 12$ on the same graph paper. Join C and D and extend it on both sides as shown in the figure.



We find that the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

23. **Solution:**

Given: Two triangles ABC and DEF such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



To prove: $\Delta ABC \sim \Delta DEF$

Construction: Let P and Q be points on DE and DF respectively such that DP = AB and DQ = AC. Join PQ,

Proof: We have $\frac{AB}{DE}=\frac{AC}{DF}$

$\Rightarrow \quad \frac{DP}{DE}=\frac{DQ}{DF}$ [$\because$ AB = DP and AC = DQ]

$\Rightarrow \quad PQ \parallel EF$ [By the converse of Thale's theorem]

$\Rightarrow \quad \angle DPQ = \angle E$ and $\angle DQP = \angle F$ [Corresponding angles]

Thus, in triangles DPQ and DEF, we have $\angle DPQ = \angle E$ and $\angle DQP = \angle F$.

Therefore, by AA-criterion of similarity, we have

$\Delta DPQ \sim \Delta DEF$ ...(i)

$\Rightarrow \quad \frac{DP}{DE}=\frac{PQ}{EF}$ [By definition of similarity]

$\Rightarrow \quad \frac{AB}{DE}=\frac{PQ}{EF}$ [$\because$ DP = AB]

But, $\frac{AB}{DE}=\frac{BC}{EF}$

$\therefore \quad \frac{PQ}{EF}=\frac{BC}{EF}$

$\Rightarrow \quad PQ = BC$

Thus, in triangles ABC and DPQ, we have

AB = DP, AC = DQ and BC = PQ

Therefore, by SSS criterion of congruence, we have

$\Delta ABC \cong \Delta DPQ$ ...(ii)

From (i) and (ii), we have

$\Delta ABC \cong \Delta DPQ$ and $\Delta DPQ \sim \Delta DEF$

$\Rightarrow \quad \Delta ABC \sim \Delta DPQ$ and $\Delta DPQ \sim \Delta DEF$ [$\because \Delta ABC \cong \Delta DPQ \Leftrightarrow \Delta ABC \sim \Delta DPQ$]

$\Rightarrow \quad \Delta ABC \sim \Delta DEF$

24. **Solution:**

Let the diagonals AC and BD of rhombus ABCD intersect at O.

Since the diagonals of a rhombus bisect each other at right angles.

∴ $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$ and AO = CO, BO = OD.

Since ΔAOB is a right triangle right angled at O,

∴ $AB^2 = OA^2 + OB^2$

⇒ $AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$ [∵ OA = OC and OB = OD]

⇒ $4AB^2 = AC^2 + BD^2$ ...(i)

Similarly, we have

$4BC^2 = AC^2 + BD^2$ ...(ii)

$4CD^2 = AC^2 + BD^2$ ...(iii)

$4AD^2 = AC^2 + BD^2$ ...(iv)

Adding (i), (ii), (iii) and (iv), we get

$4(AB^2 + BC^2 + CD^2 + AD^2) = 4(AC^2 + BD^2)$

⇒ $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

25. **Solution:**

We have,

$\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$

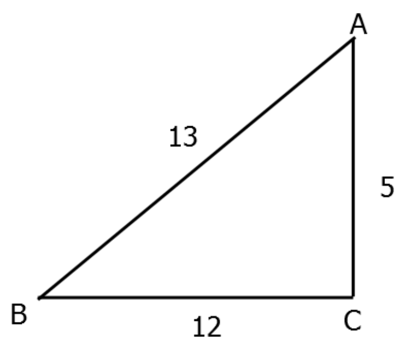
So, we draw a right triangle ABC, right angled at C such that

Base = BC = 12 units and Perpendicular = AC = 5 units

By Pythagoras theorem, we have

$AB^2 = BC^2 + AC^2$

$= 12^2 + 5^2 = 169$



⇒ $AB = \sqrt{169} = 13$

∴ $\sin B = \frac{AC}{AB} = \frac{5}{13}$, $\tan B = \frac{AC}{BC} = \frac{5}{12}$ and $\sec B = \frac{AB}{BC} = \frac{13}{12}$

Now, LHS $= \tan^2 B - \sin^2 B$

$= (\tan B)^2 - (\sin B)^2$

$$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$= \frac{25}{144} - \frac{25}{169}$$

$$= 25\left(\frac{1}{144} - \frac{1}{169}\right)$$

$$= 25\left(\frac{169-144}{144\times169}\right)$$

$$= 25\times\frac{25}{144\times169} = \frac{25\times25}{144\times169} = \frac{5^2\times5^2}{12^2\times13^2} \quad ...(i)$$

And, RHS $= \sin^4 B \sec^2 B$

$= (\sin B)^4 (\sec B)^2$

$$= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 = \frac{5^4\times13^2}{13^4\times12^2}$$

$$= \frac{5^4}{13^2\times12^2} = \frac{5^2\times5^2}{13^2\times12^2} \quad ...(ii)$$

From (i) and (ii), we have $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.

26. **Solution:**

We have,

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiplying both sides by $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, we get

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$
$$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$$

$$\Rightarrow \quad (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$$
$$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$$

$$\Rightarrow \quad 1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$$

$$\Rightarrow \quad (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm1$$

Similarly, multiplying both sides by $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$, we get

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm1$$

27. **Solution:**

Let the assumed mean be $A = 25$ and $h = 5$.

Calculation of mean

Variate **x_i**	**Frequency** **f_i**	**Deviations** **$d_i = x_i - 25$**	$u_i = \frac{x_i - 25}{5}$	**f_iu_i**
5	20	–20	–4	–80
10	43	–15	–3	–129
15	75	–10	–2	–150
20	67	–5	–1	–67
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
$N = \sum f_i = 384$				$\sum f_iu_i = -214$

We have,

$N = 384$, $A = 25$, $h = 5$ and $\sum f_iu_i = -214$

$\therefore$ Mean $= \overline{X} = A + h\left\{\frac{1}{N}\sum f_iu_i\right\}$

$= 25 + 5 \times \left(\frac{-214}{384}\right)$

$= 25 - 2.786 = 22.214$

28. **Solution:**

We have,

$\text{cosecA} = \sqrt{2} \Rightarrow \frac{1}{\sin A} = \sqrt{2} \Rightarrow \sin A = \frac{1}{\sqrt{2}}$

Now, $\cos A = \sqrt{1 - \sin^2 A}$

$= \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}$

$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

And, cotA = $\frac{1}{\tan A}=\frac{1}{1}=1$

Hence, $\frac{2\sin^2 A+3\cot^2 A}{4\tan^2 A-\cos^2 A}$ = $\frac{2\times\left(\frac{1}{\sqrt{2}}\right)^2+3(1)^2}{4(1)^2-\left(\frac{1}{\sqrt{2}}\right)^2}$

= $\frac{2\times\frac{1}{2}+3}{4-\frac{1}{2}}=\frac{1+3}{\frac{7}{2}}=\frac{4}{\frac{7}{2}}=\frac{8}{7}$

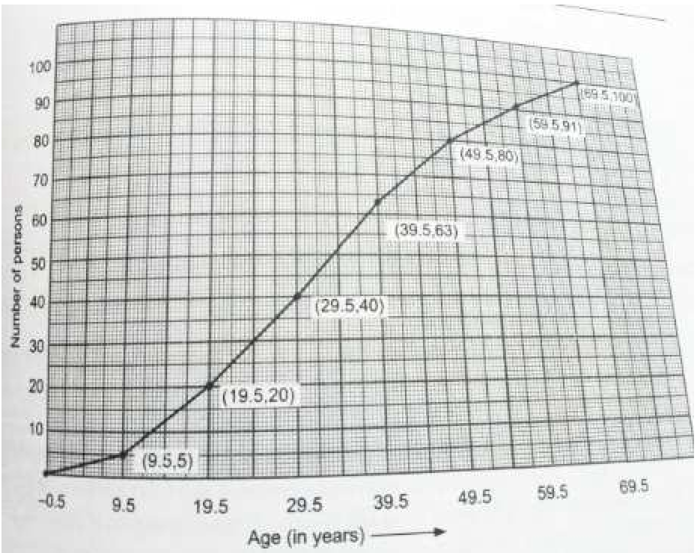
29. **Solution:**

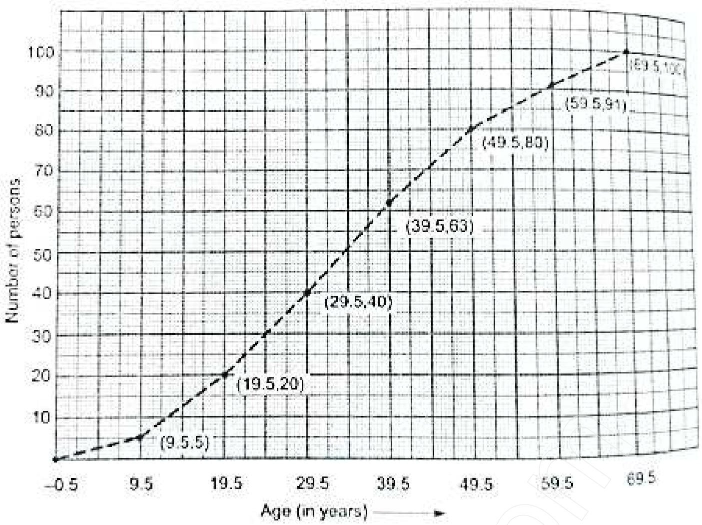
The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under.

Age (in years)	Frequency	Age less than	Cumulative frequency
–0.5-9.5	5	9.5	5
9.5-19.5	15	19.5	20
19.5-29.5	20	29.5	40
29.5-39.5	23	39.5	63
39.5-49.5	17	49.5	80
49.5-59.5	11	59.5	91
59.5-69.5	9	69.5	100

Now, we plot points (9.5, 5), (19.5, 20), (29.5, 40), (39.5, 63), (49.5, 80), (59.5, 91) and (69.5, 100) and join them by a free hand smooth curve to obtain the required ogive as shown in the figure.

The cumulative frequency polygon is obtained by joining these points by line segments as shown below.





30. **Solution:**

Let the actual speed of the train be x km/hr and the actual time taken by y hours. Then,

Distance covered = (xy) km ...(i) [$\because$ Distance = Speed × Time]

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours, i.e., when speed is $(x + 6)$ km/hr, time of journey is $(y - 4)$ hours.

$\therefore$ Distance covered = $(x + 6)(y - 4)$

$\Rightarrow$ $xy = (x + 6)(y - 4)$ [using (i)]

$\Rightarrow$ $-4x + 6y - 24 = 0$

$\Rightarrow$ $-2x + 3y - 12 = 0$ ...(ii)

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours, i.e., when speed is $(x - 6)$ km/hr, time of journey is $(y + 6)$ hours.

$\therefore$ Distance covered = $(x - 6)(y + 6)$

$\Rightarrow$ $xy = (x - 6)(y + 6)$ [using (i)]

$\Rightarrow$ $6x - 6y - 36 = 0$

$\Rightarrow$ $x - y - 6 = 0$ ...(iii)

Thus, we obtain the following system of equations:

$-2x + 3y - 12 = 0$

$x - y - 6 = 0$

By using cross-multiplication, we have,

$$\frac{x}{3\times-6-(-1)\times12} = \frac{-y}{-2\times-6-1\times-12} = \frac{1}{-2\times-1-1\times3}$$

$$\Rightarrow \quad \frac{x}{-30} = \frac{-y}{24} = \frac{1}{-1}$$

$\Rightarrow \quad x = 30$ and $y = 24$.

Putting the values of x and y in equation (i), we get

Distance = 30 × 24 = 720 km

Hence, the length of the journey is 720 km.

31. **Solution:**

Mean of the given data = $\frac{5+3+10+5+2+4+7+8+1+5}{10} = \frac{50}{10} = 5$

Arranging the given data in ascending order, we get

1, 2, 3, 4, 5, 5, 5, 7, 8, 10

We observe that maximum occurring observation is 5. Thus, mode of the given data is 5.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$= \frac{5^{\text{th}} \text{observation} + 6^{\text{th}} \text{observation}}{2} = \frac{5+5}{2} = \frac{10}{2} = 5$$

The values depicted by the persons of these households are social service and caring.