WB JEE

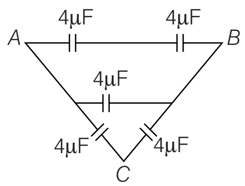
Engineering Entrance Exam

Solved Paper 2016

Physics

Category I (Q.1 to Q.30) *Only one answer is correct. Correct answer will fetch full mark* 1. *Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{4}$ *mark.*

1. Equivalent capacitance between **A** and **B** in the figure is



(a) 20 μF (b) 8 μF
(c) 12 μF (d) 16 μF

2. Two wires of same radius having lengths l_1 and l_2 and resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity will be

(a) $\frac{\rho_1 l_2 + \rho_2 l_1}{\rho_1 + \rho_2}$ (b) $\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$
(c) $\frac{\rho_1 l_1 - \rho_2 l_2}{l_1 - l_2}$ (d) $\frac{\rho_1 l_2 + \rho_2 l_1}{l_1 + l_2}$

3. A hollow metal sphere of radius R is charged with a charge Q. The electric potential and intensity inside the sphere are respectively

(a) $\frac{Q}{4\pi \varepsilon_0 R^2}$ and $\frac{Q}{4\pi \varepsilon_0 R}$
(b) $\frac{Q}{4\pi \varepsilon_0 R}$ and zero
(c) zero and zero
(d) $\frac{4\pi \varepsilon_0 Q}{R}$ and $\frac{Q}{4\pi \varepsilon_0 R^2}$

4. The potential difference V required for accelerating an electron to have the de-Broglie wavelength of 1 Å is

(a) 100 V (b) 125 V
(c) 150 V (d) 200 V

5. The work function of Cesium is 2.27 eV. The cut-off voltage which stops the emission of electrons from a cesium cathode irradiated with light of 600 nm wavelength is

(a) 0.5 V (b) –0.2 V
(c) –0.5 V (d) 0.2 V

6. The number of de-Broglie wavelengths contained in the second Bohr orbit of hydrogen atom is

(a) 1 (b) 2
(c) 3 (d) 4

7. The wavelength of second Balmer line in hydrogen spectrum is 600 nm. The wavelength for its third line in Lyman series is

(a) 800 nm (b) 600 nm
(c) 400 nm (d) 200 nm

8. A ray of light strikes a glass plate at an angle of 60°. If the reflected and refracted rays are perpendicular to each other, the refractive index of glass is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{3}$

9. Light travels through a glass plate of thickness **t** and having refractive index μ. If **c** be the velocity of light in vacuum, time taken by the light to travel through this thickness of glass is

(a) $\frac{t}{\mu c}$ (b) $\frac{tc}{\mu}$ (c) $\frac{\mu t}{c}$ (d) μtc

10. If $\mathbf{x} = \mathbf{at} + \mathbf{bt^2}$, where **x** is in metre **(m)** and **t** is in hour **(h)**, then unit of **b** will be

(a) $\frac{m^2}{h}$ (b) m
(c) $\frac{m}{h}$ (d) $\frac{m}{h^2}$

11. The vectors **A** and **B** are such that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$. The angle between the two vectors will be

(a) 0° (b) 60° (c) 90° (d) 45°

12. At a particular height, the velocity of an ascending body is **u**. The velocity at the same height while the body falls freely is

(a) 2 **u** (b) −**u** (c) **u** (d) −2 **u**

13. Two bodies of masses $\mathbf{m_1}$ and $\mathbf{m_2}$ are separated by a distance R. The distance of the centre of mass of the bodies from the mass $\mathbf{m_1}$ is

(a) $\frac{m_2 R}{m_1 + m_2}$ (b) $\frac{m_1 R}{m_1 + m_2}$
(c) $\frac{m_1 m_2}{m_1 + m_2} R$ (d) $\frac{m_1 + m_2}{m_1} R$

14. The velocity of sound in air at 20°C and 1 atm pressure is 344.2 m/s. At 40°C and 2 atm pressure, the velocity of sound in air is approximately

(a) 350 m/s (b) 356 m/s
(c) 363 m/s (d) 370 m/s

15. The perfect gas equation for 4 g of hydrogen gas is

(a) $pV = RT$ (b) $pV = 2RT$
(c) $pV = \frac{1}{2}RT$ (d) $pV = 4RT$

16. If the temperature of the Sun gets doubled, the rate of energy received on the Earth will increase by a factor of

(a) 2 (b) 4 (c) 8 (d) 16

17. A particle vibrating simple harmonically has an acceleration of 16 cms^{-2} when it is at a distance of 4 cm from the mean position. Its time period is

(a) 1 s (b) 2.572 s
(c) 3.142 s (d) 6.028 s

18. Work done for a certain spring when stretched through 1 mm is 10 joule. The amount of work that must be done on the spring to stretch it further by 1 mm is

(a) 30 J (b) 40 J
(c) 10 J (d) 20 J

19. If the rms velocity of hydrogen gas at a certain temperature is **c**, then the rms velocity of oxygen gas at the same temperature is

(a) $\frac{c}{8}$ (b) $\frac{c}{10}$
(c) $\frac{c}{4}$ (d) $\frac{c}{2}$

20. For air at room temperature, the atmospheric pressure is $\mathbf{1.0 \times 10^5}$ Nm^{-2} and density of air is 1.2 kgm^{-3}. For a tube of length 1.0 m, closed at one end, the lowest frequency generated is 84 Hz. The value of γ (ratio of two specific heats) for air is

(a) 2.1 (b) 1.5 (c) 1.8 (d) 1.4

21. A gas bubble of 2 cm diameter rises through a liquid of density 1.75 g cm^{-3} with a fixed speed of 0.35 cms^{-1}. Neglect the density of the gas. The coefficient of viscosity of the liquid is

(a) 870 poise (b) 1120 poise
(c) 982 poise (d) 1089 poise

22. The temperature of the water of a pond is 0 °C while that of the surrounding atmosphere is −**20** °C. If the density of ice is ρ, coefficient of thermal conductivity is **k** and latent heat of melting is **L**, then the thickness **Z** of ice layer formed increases as a function of time **t** is

(a) $Z^2 = \frac{60k}{\rho L} t$ (b) $Z = \sqrt{\frac{40k}{\rho L}}\, t$
(c) $Z^2 = \frac{40k}{\rho L} \sqrt{t}$ (d) $Z^2 = \frac{40k}{\rho L} t$

23. 1000 droplets of water having 2 mm diameter each coalesce to form a single drop. Given the surface tension of water is $0.072\ Nm^{-1}$. The energy loss in the process is

(a) 8.146×10^{-4} J (b) 4.4×10^{-4} J
(c) 2.108×10^{-5} J (d) 4.7×10^{-1} J

24. A Zener diode having break-down voltage 5.6 V is connected in reverse bias with a battery of emf 10 V and a resistance of **100 Ω** in series. The current flowing through the Zener diode is

(a) 88 mA (b) 0.88 mA
(c) 4.4 mA (d) 44 mA

25. In case of a bipolar transistor $\beta = \mathbf{45}$. The potential drop across the collector resistance of **1 kΩ** is 5V. The base current is approximately

(a) 222 μ A (b) 55 μ A (c) 111 μ A (d) 45 μ A

26. An electron enters an electric field having intensity $\mathbf{E} = \mathbf{3\hat{i} + 6\hat{j} + 2\hat{k}}$ Vm^{-1} and magnetic field having induction $\mathbf{B} = \mathbf{2\hat{i} + 3\hat{j}}$ T with a velocity $\mathbf{v} = \mathbf{2\hat{i} + 3\hat{j}}\ \mathrm{ms}^{-1}$. The magnitude of the force acting on the electron is
(Given, $\mathbf{e = -1.6 \times 10^{-19}}$ C)

(a) 2.02×10^{-18} N (b) 5.16×10^{-16} N
(c) 3.72×10^{-17} N (d) 4.41×10^{-18} N

27. Two coils of self-inductances 6 mH and 8 mH are connected in series and are adjusted for highest coefficient of coupling. Equivalent self-inductance L for the assembly is approximately

(a) 50 mH
(b) 36 mH
(c) 28 mH
(d) 18 mH

28. A 1 μF capacitor C is connected to a battery of 10 V through a resistance 1 MΩ. The voltage across C after 1 s is approximately

(a) 5.6 V (b) 7.8 V
(c) 6.3 V (d) 10 V

29. Two equal resistances, **400 Ω** each, are connected in series with a 8 V battery. If the resistance of first one increases by 0.5%, the change required in the resistance of the second one in order to keep the potential difference across it unaltered is to

(a) increase it by 1 Ω
(b) increase it by 2 Ω
(c) increase it by 4 Ω
(d) decrease it by 4 Ω

30. Angle between an equipotential surface and electric lines of force is

(a) 0° (b) 90°
(c) 180° (d) 270°

Category II (Q. 31 to Q. 35) *Only one answer is correct. Correct answer will fetch full marks* 2. *If correct answer are any combination of more than one answer will fetch* $-\frac{\mathbf{1}}{\mathbf{2}}$ *mark.*

31. A current $\mathbf{I = I_0 e^{-\lambda t}}$ is flowing in a circuit consisting of a parallel combination of resistance **R** and capacitance **C**. The total charge over the entire pulse period is

(a) $\frac{I_0}{\lambda}$ (b) $\frac{2I_0}{\lambda}$
(c) $I_0\lambda$ (d) $e^{I_0\lambda}$

32. For Fraunhoffer diffraction to occur

(a) Light source should be at infinity
(b) Both source and screen should be at infinity
(c) Only the source should be at finite distance
(d) Both source and screen should be at finite distance

33. The temperature of a blackbody radiation enclosed in a container of volume **V** is increased from 100 °C to 1000 °C. The heat required in the process is

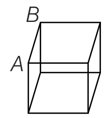
(a) 4.79×10^{-4} cal (b) 9.21×10^{-5} cal
(c) 2.17×10^{-4} cal (d) 7.54×10^{-4} cal

34. A mass of 1 kg is suspended by means of a thread. The system is (i) lifted up with an acceleration of 4.9 ms^{-2}. (ii) lowered with an

acceleration of 4.9 ms^{-2}. The ratio of tension in the first and second case is

(a) 3 : 1
(b) 1 : 2
(c) 1 : 3
(d) 2 : 1

35. The effective resistance between A and B in the figure is $\frac{7}{12}\,\Omega$ if each side of the cube has $1\,\Omega$ resistance. The effective resistance between the same two points, when the link **AB** is removed, is



(a) $\frac{7}{12}\Omega$
(b) $\frac{5}{12}\Omega$
(c) $\frac{7}{5}\Omega$
(d) $\frac{5}{7}\Omega$

Category III (Q. 36 to Q. 40) *One or more answer (s) is (are) correct. Correct answers will fetch marks* 2. *Any combination containing one or more incorrect answer (s) will fetch* 0 *mark. If all correct answers are not marked and also no incorrect answer is marked then score* = **2** × *number of correct answers marked/actual number of correct answers.*

36. A charged particle of mass $\mathbf{m_1}$ and charge $\mathbf{q_1}$ is revolving in a circle of radius **r**. Another charged particle of charge $\mathbf{q_2}$ and mass $\mathbf{m_2}$ is situated at the centre of the circle. If the velocity and time period of the revolving particle be **v** and **T** respectively, then

(a) $v = \sqrt{\frac{q_1 q_2 r}{4\pi\varepsilon_0 m_1}}$

(b) $v = \frac{1}{m_1}\sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 r}}$

(c) $T = \sqrt{\frac{16\pi^3 \varepsilon_0 m_1^2 r^3}{q_1 q_2}}$

(d) $T = \sqrt{\frac{16\pi^3 \varepsilon_0 m_2 r^3}{q_1 q_2}}$

37. The distance between a light source and photoelectric cell is **d**. If the distance is decreased to $\frac{\mathbf{d}}{\mathbf{2}}$, then

(a) the emission of electron per second will be four times
(b) maximum kinetic energy of photoelectrons will be four times
(c) stopping potential will remain same
(d) the emission of electrons per second will be doubled

38. A train moves from rest with acceleration α and in time $\mathbf{t_1}$ covers a distance **x**. It then decelerates to rest at constant retardation β for distance **y** in time $\mathbf{t_2}$. Then,

(a) $\frac{x}{y} = \frac{\beta}{\alpha}$
(b) $\frac{\beta}{\alpha} = \frac{t_1}{t_2}$
(c) $x = y$
(d) $\frac{x}{y} = \frac{\beta t_1}{\alpha t_2}$

39. A drop of water detaches itself from the exit of a tap when (σ = surface tension of water, ρ = density of water, **R** = radius of the tap exit, **r** = radius of the drop)

(a) $r > \left(\frac{2}{3}\frac{R\sigma}{\rho g}\right)^{1/3}$

(b) $r = \frac{2}{3}\frac{\sigma}{\rho g}$

(c) $\frac{2\sigma}{r}$ > atmospheric pressure

(d) $r > \left(\frac{2}{3}\frac{R\sigma}{\rho g}\right)^{2/3}$

40. A rectangular coil carrying-current is placed in a non-uniform magnetic field. On that coil, the total

(a) force is non-zero
(b) force is zero
(c) torque is zero
(d) torque is non-zero

Chemistry

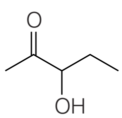
Category I (Q. 1 to Q. 30). *Only one answer is correct. Correct answer will fetch full mark* 1. *Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{4}$ *mark.*

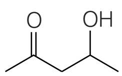
1. Amongst the following compounds, the one that will not respond to Cannizzaro reaction upon treatment with alkali is
   (a) Cl_3CCHO (b) Me_3CCHO
   (c) C_6H_5CHO (d) HCHO

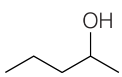
2. Which of the following compounds would not react with Lucas reagent at room temperature?
   (a) $H_2C{=}CHCH_2OH$
   (b) $C_6H_5CH_2OH$
   (c) $CH_3CH_2CH_2OH$
   (d) $(CH_3)_3COH$

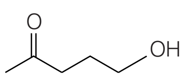
3. Amongst the following compounds, the one which would not respond to iodoform test is
   (a) $CH_3CH(OH)CH_2CH_3$
   (b) $ICH_2COCH_2CH_3$
   (c) CH_3COOH
   (d) CH_3CHO

4. Which of the following will be dehydrated most readily in alkaline medium?

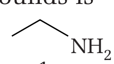
(a) 

(b) 

(c) 

(d) 

5. The correct order of basicity of the following compounds is



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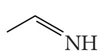

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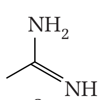

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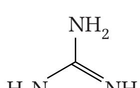


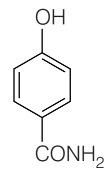
4

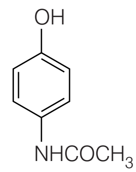
   (a) $1 < 2 < 3 < 4$ (b) $1 < 2 < 4 < 3$
   (c) $2 < 1 < 3 < 4$ (d) $4 < 3 < 2 < 1$

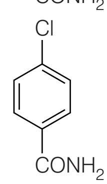
6. Which of the following reactions will not result in the formation of carbon-carbon bonds?
   (a) Cannizzaro reaction
   (b) Wurtz reaction
   (c) Reimer-Tiemann reaction
   (d) Friedel-Crafts acylation

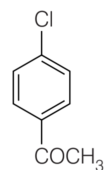
7. Point out the false statement.
   (a) Colloidal sols are homogeneous
   (b) Colloids carry + ve or – ve charges
   (c) Colloids show Tyndall effect
   (d) The size range of colloidal particles is 10-1000 Å

8. The correct structure of the drug paracetamol is

(a) 

(b) 

(c) 

(d) 

9. Which of the following statements regarding Lanthanides is false?
   (a) All lanthanides are solid at room temperature
   (b) Their usual oxidation state is +3
   (c) They can be separated from one another by ion-exchange method
   (d) Ionic radii of trivalent lanthanides steadily increases with increase in atomic number

10. Nitrogen dioxide is not produced on heating
   (a) KNO_3 (b) $Pb(NO_3)_2$
   (c) $Cu(NO_3)_2$ (d) $AgNO_3$

11. The boiling points of HF, **HCl**, HBr and HI follow the order
   (a) HF > HCl > HBr > HI (b) HF > HI > HBr > HCl
   (c) HI > HBr > HCl > H (d) HCl > HF > HBr > HI

12. In the solid state, $\mathbf{PCl_5}$ exists as

(a) $[PCl_4]^-$ and $[PCl_6]^+$ ions
(b) covalent PCl_5 molecules only
(c) $[PCl_4]^+$ and $[PCl_6]^-$ ions
(d) covalent P_2Cl_{10} molecules only

13. Which statement is not correct for *ortho* and *para* hydrogen?

(a) They have different boiling points
(b) *Ortho*-form is more stable than *para*-form
(c) They differ in their nuclear spin
(d) The ratio of *ortho* to *para* hydrogen changes with change in temperature

14. The acid in which **O—O** bonding is present is

(a) $H_2S_2O_3$ (b) $H_2S_2O_6$
(c) $H_2S_2O_8$ (d) $H_2S_4O_6$

15. The metal which can be used to obtain metallic **Cu** from aqueous $\mathbf{CuSO_4}$ solution is

(a) Na (b) Ag
(c) Hg (d) Fe

16. If radium and chlorine combine to form radium chloride, the compound would be

(a) half as radioactive as radium
(b) twice as radioactive
(c) as radioactive as radium
(d) not radioactive

17. Which of the following arrangements is correct in respect of solubility in water?

(a) $CaSO_4 > BaSO_4 > BeSO_4 > MgSO_4 > SrSO_4$
(b) $BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$
(c) $BaSO_4 > SrSO_4 > CaSO_4 > MgSO_4 > BeSO_4$
(d) $BeSO_4 > CaSO_4 > MgSO_4 > SrSO_4 > BaSO_4$

18. The energy required to break one mole of hydrogen-hydrogen bonds in $\mathbf{H_2}$ is 436 kJ. What is the longest wavelength of light required to break a single hydrogen-hydrogen bond?

(a) 68.5 nm (b) 137 nm
(c) 274 nm (d) 548 nm

19. The correct order of **O—O** bond length in $\mathbf{O_2}$, $\mathbf{H_2O_2}$ and $\mathbf{O_3}$ is

(a) $O_2 > O_3 > H_2O_2$
(b) $H_2O_2 > O_3 > O_2$
(c) $O_3 > O_2 > H_2O_2$
(d) $O_3 > H_2O_2 > O_2$

20. The number of σ and π bonds between two carbon atoms in calcium carbide are

(a) one σ, one π (b) one σ, two π
(c) two σ, one π (d) one σ, $1\frac{1}{2}\pi$

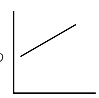
21. An element **E** loses one α and two β particles in three successive stages. The resulting element will be

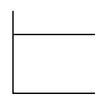
(a) an isobar of *E* (b) an isotone of *E*
(c) an isotope of *E* (d) *E* itself

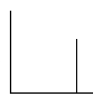
22. An element **X** belongs to fourth period and fifteenth group of the periodic table. Which of the following statements is true?

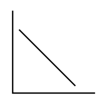
(a) It has completely filled *s*-orbital and a partially filled *d*-orbital
(b) It has completely filled *s*-and *p* -orbitals and a partially filled *d*-orbital
(c) It has completely filled *s*- and *p*-orbitals and a half filled *d*-orbital
(d) It has a half filled *p* -orbital and completely filled *s*-and *d*-orbitals

23. Which of the following plots represent an exothermic reaction?

(a) ln K_p  1/*T*

(b) ln K_p  1/*T*

(c) ln K_p  1/*T*

(d) ln K_p  1/*T*

24. If $\mathbf{p}°$ and $\mathbf{p}$ are the vapour pressure of the pure solvent and solution and $\mathbf{n_1}$ and $\mathbf{n_2}$ are the moles of solute and solvent respectively in the solution then the correct relation between $\mathbf{p}$ and $\mathbf{p}°$ is

(a) $p° = p\left[\frac{n_1}{n_1 + n_2}\right]$ (b) $p° = p\left[\frac{n_2}{n_1 + n_2}\right]$
(c) $p = p°\left[\frac{n_2}{n_1 + n_2}\right]$ (d) $p = p°\left[\frac{n_1}{n_1 + n_2}\right]$

25. Ionic solids with Schottky defect may contain in their structure

(a) cation vacancies only
(b) cation vacancies and interstitial cations
(c) equal number of cation and anion vacancies
(d) anion vacancies and interstitial anions

26. The condition for a reaction to occur spontaneously is

(a) ΔH must be negative
(b) ΔS must be negative
(c) $(\Delta H - T\Delta S)$ must be negative
(d) $(\Delta H + T\Delta S)$ must be negative

27. The order of equivalent conductances at infinite dilution for **LiCl, NaCl** and **KCl** is

(a) LiCl > NaCl > KCl
(b) KCl > NaCl > LiCl
(c) NaCl > KCl > LiCl
(d) LiCl > KCl > NaCl

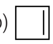
28. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is '**S**'. The corresponding solubility product is K_{sp}. **S** in terms of 'K_{sp}' is given by the relation

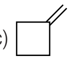
(a) $S = \left(\frac{K_{sp}}{128}\right)^{1/4}$ (b) $S = \left(\frac{K_{sp}}{256}\right)^{1/5}$

(c) $S = (256\,K_{sp})^{1/5}$ (d) $S = (128\,K_{sp})^{1/4}$

29. Ozonolysis of an alkene produces only one dicarbonyl compound. The structure of the alkene is

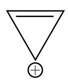
(a) $H_3C—CH{=}CH—CH_3$

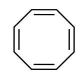
(b) 

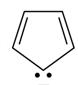
(c) 

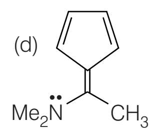
(d) $CH_3—CH{=}CH—CH{=}CH_2$

30. From the following compounds, choose the one which is not aromatic.

(a) 

(b) 

(c) 

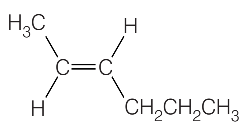
(d) 

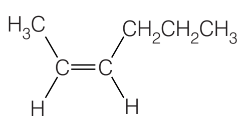
Category II (Q. 31 to Q. 35) *Only one answer is correct. Correct answer will fetch full marks* 2. *Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{2}$ *mark.*

31. Identify **X** in the following sequence of reactions.

$CH_3—CH(Br)—CH(Br)—CH_2—CH_2—CH_3 \xrightarrow[\text{(ii) Na in liquid } NH_3]{\text{(i) } NaNH_2} X$

(a) $CH_3—CH(Br)—CH(NH_2)—CH_2CH_2CH_3$

(b) 

(c) 

(d) $CH_3—CH(NH_2)—CH(NH_2)—CH_2CH_2CH_3$

32. Compound **X** is tested and the results are shown in the table.

Test	Result
Aqueous sodium hydroxide is added, then heated gently.	Gas given off which turns damp red litmus paper blue.
Dilute hydrochloric acid is added.	Effervescence, gas given off which turns lime water milky and acidified $K_2Cr_2O_7$ paper green.

Which ions are present in compound **X**?

(a) Ammonium ions and sulphite ions
(b) Ammonium ions and carbonate ions
(c) Sodium ions and carbonate ions
(d) Ammonium ions and sulphate ions

33. The time taken for an electron to complete one revolution in Bohr orbit of hydrogen atom is

(a) $\frac{4m^2\pi r^2}{n^2h^2}$ (b) $\frac{n^2h^2}{4mr^2}$ (c) $\frac{4\pi^2mr^2}{nh}$ (d) $\frac{nh}{4\pi^2mr^2}$

34. Amongst the following, which should have the highest rms speed at the same temperature?

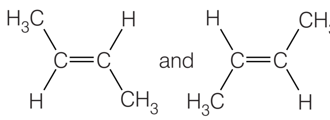
(a) SO_2
(b) CO_2
(c) O_2
(d) H_2

35. The major products obtained during ozonolysis of 2, 3-dimethyl-1-butene and subsequent reductions with **Zn** and $\mathbf{H_2O}$ are

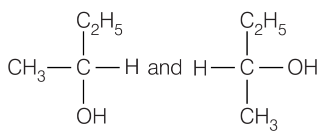
(a) methanoic acid and 2-methyl-2-butanone
(b) methanal and 3-methyl-2-butanone
(c) methanol and 2, 2 -dimethyl-3-butanone
(d) methanoic acid and 2-methyl-3-butanone

Category III (Q. 36 to Q. 40) *One or more answer (s) is (are) correct. Correct answers will fetch marks* 2. *Any combination containing one or more incorrect answer (s) will fetch* 0 *mark. If all correct answers are not marked and also no incorrect answer is marked then score* = **2** × *number of correct answers marked/actual number of correct answers.*

36. Choose the correct statement(s) among the following.

(a)  are enantiomers

(b) CH_3CHO on reaction with HCN gives racemic mixture

(c)  are enantiomers

(d) $CH_3—CH{=}NOH$ shows geometrical isomerism

37. Which of the following statement(s) is (are) correct when a mixture of **NaCl** and $\mathbf{K_2Cr_2O_7}$ is gently warmed with conc. $\mathbf{H_2SO_4}$?

(a) A deep red vapour is evolved
(b) The vapour when passed through NaOH solution, gives a yellow solution
(c) Chlorine gas is also evolved
(d) Chromyl chloride is formed

38. Of the following molecules, which have shape similar to $\mathbf{CO_2}$?

(a) $HgCl_2$ (b) $SnCl_2$ (c) C_2H_2 (d) NO_2

39. In which of the following mixed aqueous solutions, $\mathbf{pH = pK_a}$ at equilibrium?

(1) 100 mL of 0.1 M $\mathbf{CH_3COOH}$ + 100 mL of 0.1 M $\mathbf{CH_3COONa}$
(2) 100 mL of 0.1 M $\mathbf{CH_3COOH}$ + 50 mL of 0.1 M NaOH
(3) 100 mL of 0.1 M $\mathbf{CH_3COOH}$ + 100 mL of 0.1 M NaOH
(4) 100 mL of 0.1 M $\mathbf{CH_3COOH}$ + 100 mL of 0.1 M $\mathbf{NH_3}$

(a) (1) is correct
(b) (2) is correct
(c) (3) is correct
(d) Both (1) and (2) are correct

40. Amongst the following compounds, the one(s) which readily react with ethanolic KCN.

(a) Ethyl chloride (b) Chlorobenzene
(c) Benzaldehyde (d) Salicylic acid

Mathematics

Category I (Q. 1 to Q. 50) *Only one answer is correct. Correct answer will fetch full mark* 1. *Incorrect answer or any combination of more than one answer will fetch* $-1/4$ *mark.*

1. If the solution of the differential equation $x\frac{dy}{dx}+y=xe^{x}$ be $xy=e^{x}\phi(x)+C$, then $\phi(x)$ is equal to

(a) $x+1$ (b) $x-1$
(c) $1-x$ (d) x

2. The order of the differential equation of all parabolas whose axis of symmetry along X-axis is

(a) 2 (b) 3
(c) 1 (d) None of these

3. The line $y=x+\lambda$ is tangent to the ellipse $2x^2+3y^2=1$. Then, λ is

(a) -2 (b) 1
(c) $\sqrt{\frac{5}{6}}$ (d) $\sqrt{\frac{2}{3}}$

4. The area enclosed by $y=\sqrt{5-x^2}$ and $y=|x-1|$ is

(a) $\left(\frac{5\pi}{4}-2\right)$ sq units (b) $\left(\frac{5\pi-2}{2}\right)$ sq units
(c) $\left(\frac{5\pi}{4}-\frac{1}{2}\right)$ sq units (d) $\left(\frac{\pi}{2}-5\right)$ sq units

5. Let S be the set of points, whose abscissae and ordinates are natural numbers. Let $P\in S$, such that the sum of the distance of P from $(8,0)$ and $(0,12)$ is minimum among all elements in S. Then, the number of such points P in S is

(a) 1 (b) 3 (c) 5 (d) 11

6. Time period T of a simple pendulum of length l is given by $T=2\pi\sqrt{\frac{l}{g}}$. If the length is increased by 2%, then an approximate change in the time period is

(a) 2% (b) 1%
(c) $\frac{1}{2}$% (d) None of these

7. The cosine of the angle between any two diagonals of a cube is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

8. If x is a positive real number different from 1 such that $\log_a x, \log_b x, \log_c x$ are in AP, then

(a) $b=\frac{a+c}{2}$ (b) $b=\sqrt{ac}$
(c) $c^2=(ac)^{\log_a b}$ (d) None of these

9. If a, x are real numbers and $|a|<1, |x|<1$, then $1+(1+a)x+(1+a+a^2)x^2+\ldots\infty$ is equal to

(a) $\frac{1}{(1-a)(1-ax)}$ (b) $\frac{1}{(1-a)(1-x)}$
(c) $\frac{1}{(1-x)(1-ax)}$ (d) $\frac{1}{(1+ax)(1-a)}$

10. If $\log_{0.3}(x-1)<\log_{0.09}(x-1)$, then x lies in the interval

(a) $(2,\infty)$ (b) $(1,2)$
(c) $(-2,-1)$ (d) None of these

11. The value of $\sum_{n=1}^{13}(i^n+i^{n+1})$, $i=\sqrt{-1}$ is

(a) i (b) $i-1$
(c) 1 (d) 0

12. If z_1, z_2, z_3 are imaginary numbers such that $|z_1|=|z_2|=|z_3|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=1$, then $|z_1+z_2+z_3|$ is

(a) equal to 1 (b) less than 1
(c) greater than 1 (d) equal to 3

13. If p, q are the roots of the equation $x^2+px+q=0$, then

(a) $p=1, q=-2$ (b) $p=0, q=1$
(c) $p=-2, q=0$ (d) $p=-2, q=1$

14. The number of values of $\mathbf{k}$, for which the equation $\mathbf{x^2 - 3x + k = 0}$ has two distinct roots lying in the interval (0, 1), are

(a) three
(b) two
(c) infinitely many
(d) no value of k satisfies the requirement

15. The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together, is

(a) $\frac{7!}{2!2!}$ (b) $\frac{7!}{2!}$
(c) $\frac{6!}{2!}$ (d) $5! \times 2!$

16. If $\frac{1}{{}^5C_r} + \frac{1}{{}^6C_r} = \frac{1}{{}^4C_r}$, then the value of $\mathbf{r}$ is

(a) 4 (b) 2 (c) 5 (d) 3

17. For positive integer $\mathbf{n}$, $\mathbf{n^3 + 2n}$ is always divisible by

(a) 3 (b) 7 (c) 5 (d) 6

18. In the expansion of $(x-1)(x-2)\ldots(x-18)$, the coefficient of x^{17} is

(a) 684 (b) – 171
(c) 171 (d) – 342

19. $1 + {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta + \ldots + {}^nC_n \cos n\theta$ equals

(a) $\left(2\cos\frac{\theta}{2}\right)^n \cos\frac{n\theta}{2}$ (b) $2\cos^2\frac{n\theta}{2}$
(c) $2\cos^{2n}\frac{\theta}{2}$ (d) $\left(2\cos^2\frac{\theta}{2}\right)^n$

20. If $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are greater than 1, then the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

(a) $\log x \cdot \log y \cdot \log z$
(b) $\log x + \log y + \log z$
(c) 0
(d) $1 - \{(\log x)\cdot(\log y)\cdot(\log z)\}$

21. Let $\mathbf{A}$ be a 3×3 matrix and $\mathbf{B}$ be its adjoint matrix. If $|B| = 64$, then $|A|$ is equal to

(a) ± 2 (b) ± 4
(c) ± 8 (d) ± 12

22. Let $Q = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $x = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, then Q^3x is equal to

(a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

23. Let $\mathbf{R}$ be a relation defined on the set $\mathbf{Z}$ of all integers and $\mathbf{xRy}$, when $\mathbf{x + 2y}$ is divisible by 3, then

(a) R is not transitive
(b) R is symmetric only
(c) R is an equivalence relation
(d) R is not an equivalence relation

24. If $A = \{5^n - 4n - 1 : n \in N\}$ and $B = \{16(n-1) : n \in N\}$, then

(a) $A = B$ (b) $A \cap B = \phi$
(c) $A \subseteq B$ (d) $B \subseteq A$

25. If the function $f : R \to R$ is defined by $f(x) = (x^2 + 1)^{35}$, $\forall\, x \in R$, then $\mathbf{f}$ is

(a) one-one but not onto
(b) onto but not one-one
(c) neither one-one nor onto
(d) both one-one and onto

26. Standard deviation of $\mathbf{n}$ observations $a_1, a_2, a_3, \ldots, a_n$ is σ. Then, the standard deviation of the observations $\lambda a_1, \lambda a_2, \ldots, \lambda a_n$ is

(a) $\lambda\sigma$ (b) $-\lambda\sigma$ (c) $|\lambda|\sigma$ (d) $\lambda^n\sigma$

27. Let $\mathbf{A}$ and $\mathbf{B}$ be two events such that $P(A \cap B) = \frac{1}{6}$, $P(A \cup B) = \frac{31}{45}$ and $P(\overline{B}) = \frac{7}{10}$, then

(a) A and B are independent
(b) A and B are mutually exclusive
(c) $P\left(\frac{A}{B}\right) < \frac{1}{6}$
(d) $P\left(\frac{B}{A}\right) < \frac{1}{6}$

28. The value of $\cos 15^\circ \cos 7\frac{1^\circ}{2} \sin 7\frac{1^\circ}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

29. The smallest positive root of the equation $\tan x - x = 0$ lies in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$
(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(\frac{3\pi}{2}, 2\pi\right)$

30. If in a ΔABC, AD, BE and CF are the altitudes and R is the circumradius, then the radius of the circumcircle of ΔDEF is

(a) $\frac{R}{2}$ (b) $\frac{2R}{3}$
(c) $\frac{R}{3}$ (d) None of these

31. The points $(-a, -b)$, (a, b), $(0, 0)$ and (a^2, ab), $a \neq 0$, $b \neq 0$ are always

(a) collinear
(b) vertices of a parallelogram
(c) vertices of a rectangle
(d) lie on a circle

32. The line AB cuts off equal intercepts $2a$ from the axes. From any point P on the line AB perpendiculars PR and PS are drawn on the axes. Locus of mid-point of RS is

(a) $x - y = \frac{a}{2}$ (b) $x + y = a$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 - y^2 = 2a^2$

33. $x + 8y - 22 = 0$, $5x + 2y - 34 = 0$, $2x - 3y + 13 = 0$ are the three sides of a triangle. The area of the triangle is

(a) 36 sq units
(b) 19 sq units
(c) 42 sq units
(d) 72 sq units

34. The line through the points (a, b) and $(-a, -b)$, passes through the point

(a) $(1, 1)$ (b) $(3a, -2b)$
(c) (a^2, ab) (d) (a, b)

35. The locus of the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = K$ and $\frac{x}{a} - \frac{y}{b} = \frac{1}{K}$, where K is a non-zero real variable, is given by

(a) a straight line (b) an ellipse
(c) a parabola (d) a hyperbola

36. The equation of a line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant, is

(a) $3x + 4y = 15$ (b) $3x + 4y = 45$
(c) $3x + 4y = 9$ (d) $3x + 4y = 27$

37. A line passing through the point of intersection of $x + y = 4$ and $x - y = 2$ makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the X-axis. It intersects the parabola $y^2 = 4(x - 3)$ at points (x_1, y_1) and (x_2, y_2), respectively. Then, $|x_1 - x_2|$ is equal to

(a) $\frac{16}{9}$ (b) $\frac{32}{9}$
(c) $\frac{40}{9}$ (d) $\frac{80}{9}$

38. The equation of auxiliary circle of the ellipse $16x^2 + 25y^2 + 32x - 100y = 284$ is

(a) $x^2 + y^2 + 2x - 4y - 20 = 0$
(b) $x^2 + y^2 + 2x - 4y = 0$
(c) $(x + 1)^2 + (y - 2)^2 = 400$
(d) $(x + 1)^2 + (y - 2)^2 = 225$

39. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOPQ is equilateral. O being the centre. Then, the eccentricity e satisfies

(a) $1 < e < \frac{2}{\sqrt{3}}$ (b) $e = \frac{2}{\sqrt{2}}$
(c) $e = \frac{\sqrt{3}}{2}$ (d) $e > \frac{2}{\sqrt{3}}$

40. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines $x + y = 3$ and $2x + 2y - 1 = 0$, then

(a) $2 < a < 4$ (b) $-\frac{1}{2} < a < 2$
(c) $0 < a < 2$ (d) $-\frac{1}{2} < a < \frac{3}{2}$

41. A straight line joining the points $(1, 1, 1)$ and $(0, 0, 0)$ intersects the plane $2x + 2y + z = 10$ at

(a) $(1, 2, 5)$ (b) $(2, 2, 2)$
(c) $(2, 1, 5)$ (d) $(1, 1, 6)$

42. Angle between the planes $\mathbf{x + y + 2z = 6}$ and $\mathbf{2x - y + z = 9}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

43. If $\mathbf{y = (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n})}$, then the value of $\left(\frac{\mathbf{dy}}{\mathbf{dx}}\right)$ at $\mathbf{x = 0}$ is

(a) 0 (b) −1
(c) 1 (d) 2

44. If $\mathbf{f(x)}$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $\mathbf{f'(3) = 2}$, then $\mathbf{f'(-3)}$ is equal to

(a) 0 (b) 1
(c) 2 (d) 4

45. $\lim_{x \to 1}\left(\frac{1+x}{2+x}\right)^{\frac{(1-\sqrt{x})}{(1-x)}}$ is equal to

(a) 1 (b) does not exist
(c) $\sqrt{\frac{2}{3}}$ (d) $\ln 2$

46. If $\mathbf{f(x)} = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$, then the value of $\mathbf{f''(x)}$ is equal to

(a) x^2 (b) x
(c) 1 (d) 0

47. $\int \frac{\log \sqrt{x}}{3x}\, dx$ is equal to

(a) $\frac{1}{3}(\log\sqrt{x})^2 + C$
(b) $\frac{2}{3}(\log\sqrt{x})^2 + C$
(c) $\frac{2}{3}(\log x)^2 + C$
(d) $\frac{1}{3}(\log x)^2 + C$

48. $\int 2^x[f'(x) + f(x)\log 2]\, dx$ is equal to

(a) $2^x f'(x) + C$
(b) $2^x \log 2 + C$
(c) $2^x f(x) + C$
(d) $2^x + C$

49. $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ is equal to

(a) 1
(b) 0
(c) 2
(d) None of the above

50. The value of
$\lim_{n \to \infty}\left\{\frac{\sqrt{n+1} + \sqrt{n+2} + \ldots + \sqrt{2n-1}}{n^{3/2}}\right\}$ is

(a) $\frac{2}{3}(2\sqrt{2} - 1)$ (b) $\frac{2}{3}(\sqrt{2} - 1)$
(c) $\frac{2}{3}(\sqrt{2} + 1)$ (d) $\frac{2}{3}(2\sqrt{2} + 1)$

Category II (Q. 51 to Q. 65) *Only one answer is correct. Correct answer will fetch full marks* 2. *Incorrect answer or any combination of more than one answer will fetch* $-\frac{1}{2}$ *mark.*

51. The sum of $\mathbf{n}$ terms of the following series $\mathbf{1^3 + 3^3 + 5^3 + 7^3 + \ldots}$ is

(a) $n^2(2n^2 - 1)$ (b) $n^3(n - 1)$
(c) $n^3 + 8n + 4$ (d) $2n^4 + 3n^2$

52. If α and β are roots of $\mathbf{ax^2 + bx + c = 0}$, then the equation whose roots are α^2 and β^2, is

(a) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$
(b) $a^2x^2 + (b^2 - ac)x + c^2 = 0$
(c) $a^2x^2 + (b^2 + ac)x + c^2 = 0$
(d) $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

53. If ω is an imaginary cube root of unity, then the value of $\mathbf{(2-\omega)(2-\omega^2) + 2(3-\omega)(3-\omega^2) + \ldots + (n-1)(n-\omega)(n-\omega^2)}$ is

(a) $\frac{n^2}{4}(n+1)^2 - n$ (b) $\frac{n^2}{4}(n+1)^2 + n$
(c) $\frac{n^2}{4}(n+1)^2$ (d) $\frac{n^2}{4}(n+1) - n$

54. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 126$, then the value of ${}^{n}C_{8}$ is

(a) 10 (b) 7 (c) 9 (d) 8

55. In a group of 14 males and 6 females. 8 and 3 of the males and females, respectively are aged above 40 yr. The probability that a person selected at random from the group is aged above 40 yr given that the selected person is a female, is

(a) $\frac{2}{7}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{5}{6}$

56. The equation $x^3 - yx^2 + x - y = 0$ represents

(a) a hyperbola and two straight lines
(b) a straight line
(c) a parabola and two straight lines
(d) a straight line and a circle

57. The locus of the mid-points of chords of the circle $x^2 + y^2 = 1$, which subtends a right angle at the origin, is

(a) $x^2 + y^2 = \frac{1}{4}$
(b) $x^2 + y^2 = \frac{1}{2}$
(c) $xy = 0$
(d) $x^2 - y^2 = 0$

58. The locus of the mid-points of all chords of the parabola $y^2 = 4ax$ through its vertex is another parabola with directrix

(a) $x = -a$ (b) $x = a$

(c) $x = 0$ (d) $x = -\frac{a}{2}$

59. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2 [x]\, dx$ equals

(a) $\frac{5}{3}$ (b) $\frac{7}{3}$

(c) $\frac{8}{3}$ (d) $\frac{4}{3}$

60. The number of points at which the function $f(x) = \max \{a - x, a + x, b\}$, $-\infty < x < \infty$, $0 < a < b$ cannot be differentiable, is

(a) 0 (b) 1 (c) 2 (d) 3

61. For non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, if $|\mathbf{a} + \mathbf{b}| < |\mathbf{a} - \mathbf{b}|$, then $\mathbf{a}$ and $\mathbf{b}$ are

(a) collinear
(b) perpendicular to each other
(c) inclined at an acute angle
(d) inclined at an obtuse angle

62. General solution of $y\frac{dy}{dx} + by^2 = a\cos x$, $0 < x < 1$ is

(a) $y^2 = 2a\,(2b\sin x + \cos x) + ce^{-2bx}$
(b) $(4b^2 + 1)\, y^2 = 2a\,(\sin x + 2b\cos x) + ce^{-2bx}$
(c) $(4b^2 + 1)\, y^2 = 2a\,(\sin x + 2b\cos x) + ce^{2bx}$
(d) $y^2 = 2a\,(2b\sin x + \cos x) + ce^{-2bx}$

Here, c is an arbitrary constant.

63. The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa increases is/are given by

(a) $\left(3, \frac{16}{3}\right)$ and $\left(-3, -\frac{16}{3}\right)$
(b) $\left(3, -\frac{16}{3}\right)$ and $\left(-3, \frac{16}{3}\right)$
(c) $\left(\frac{1}{16}, \frac{1}{9}\right)$ and $\left(-\frac{1}{16}, -\frac{1}{9}\right)$
(d) $\left(\frac{1}{16}, -\frac{1}{9}\right)$ and $\left(-\frac{1}{16}, \frac{1}{9}\right)$

64. The letters of the word COCHIN are permuted and all permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

(a) 96 (b) 48
(c) 183 (d) 267

65. If the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$, then $A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix}$, $n \in N$, where

(a) $a = 2n$, $b = 2^n$ (b) $a = 2^n$, $b = 2n$
(c) $a = 2^n$, $b = n2^{n-1}$ (d) $a = 2^n$, $b = n2^n$

Category III (Q. 66 to Q. 75) *One or more answer (s) is (are) correct. Correct answer (s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch* 0 *mark. If all correct answer are not marked and also no incorrect answer is marked, then score* = **2** × *number of correct answer marked/actual number of correct answers.*

66. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$, are

(a) $\left(\frac{2}{5}, \frac{1}{5}\right)$

(b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$

(d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

67. If $\phi(t) = \begin{cases} 1, & \text{for } 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$, then

$\int_{-3000}^{3000} \left(\sum_{r'=2014}^{2016} \phi(t - r')\, \phi(t - 2016) \right) dt$ is

(a) a real number
(b) 1
(c) 0
(d) does not exist

68. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = \alpha$ and $y = \beta$, then

(a) $3 \le x \le 7$
(b) $3 \le y \le 7$
(c) $-2 \le y \le 2$
(d) $-2 \le x \le 2$

69. If $z = \sin\theta - i\cos\theta$, then for any integer n,

(a) $z^n + \frac{1}{z^n} = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$

(b) $z^n + \frac{1}{z^n} = 2\sin\left(\frac{n\pi}{2} - n\theta\right)$

(c) $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

(d) $z^n - \frac{1}{z^n} = 2i\cos\left(\frac{n\pi}{2} - n\theta\right)$

70. Let $f : X \to X$ be such that $f[f(x)] = x$, for all $x \in X$ and $X \subseteq R$, then

(a) f is one-to-one
(b) f is onto
(c) f is one-to-one but not onto
(d) f is onto but not one-to-one

71. If A, B are two events such that $P(A \cup B) \ge \frac{3}{4}$ and $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$, then

(a) $P(A) + P(B) \le \frac{11}{8}$

(b) $P(A) \cdot P(B) \le \frac{3}{8}$

(c) $P(A) + P(B) \ge \frac{7}{8}$

(d) None of the above

72. If the first and $(2n-1)$th terms of an AP, GP and HP are equal and their nth terms are respectively a, b, c, then always

(a) $a = b = c$
(b) $a \ge b \ge c$
(c) $a + c = b$
(d) $ac - b^2 = 0$

73. The coordinates of a point on the line $x + y + 1 = 0$, which is at a distance $\frac{1}{5}$ unit from the line $3x + 4y + 2 = 0$, are

(a) (2, − 3)
(b) (−3, 2)
(c) (0, − 1)
(d) (−1, 0)

74. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ units on the line $y - 2x = 1$, then a is equal to

(a) 1
(b) −2
(c) −1
(d) 2

75. If $f(x)$ is a function such that $f'(x) = (x-1)^2(4-x)$, then

(a) $f(0) = 0$
(b) $f(x)$ is increasing in (0, 3)
(c) $x = 4$ is a critical point of $f(x)$
(d) $f(x)$ is decreasing in (3, 5)

Answers

Physics

1. (b)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (*)	**6.** (b)	**7.** (*)	**8.** (d)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (d)
21. (d)	**22.** (d)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (*)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (b)
31. (a)	**32.** (b)	**33.** (*)	**34.** (a)	**35.** (c)	**36.** (*)	**37.** (a,c)	**38.** (a,b)	**39.** (*)	**40.** (a,d)

Chemistry

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (b)
21. (c)	**22.** (d)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (b)
31. (b)	**32.** (d)	**33.** (c)	**34.** (d)	**35.** (b)	**36.** (b,d)	**37.** (a,b,d)	**38.** (a,c)	**39.** (a,b,d)	**40.** (a,c)

Mathematics

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)
11. (b)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (a)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (a)
31. (a)	**32.** (b)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (b)
41. (b)	**42.** (c)	**43.** (c)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (a)	**48.** (c)	**49.** (b)	**50.** (a)
51. (a)	**52.** (a)	**53.** (a)	**54.** (c)	**55.** (b)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (b)	**60.** (c)
61. (d)	**62.** (b)	**63.** (a)	**64.** (a)	**65.** (d)	**66.** (b,d)	**67.** (a,b)	**68.** (a,c)	**69.** (a,c)	**70.** (a,b)
71. (a,c)	**72.** (b,d)	**73.** (b,d)	**74.** (a,b)	**75.** (b,c)					

(*) *No option is correct.*

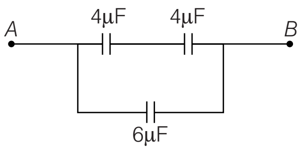
Solutions

Physics

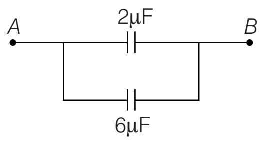
1. $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

$C = 2\mu F$

$2\,\mu F$ will be parallel to $4\mu F$, so the resultant capacitance



$$C' = C_1 + C_2 = 2\mu F + 4\mu F = 6\mu F$$



$$\frac{1}{C''} = \frac{1}{4\mu F} + \frac{1}{4\mu F} = \frac{2}{4}\,\mu F$$

$$C'' = 2\mu F$$

$\therefore$ $C_{AB} = 2\mu F + 6\mu F = 8\mu F$

2. $R = \frac{\rho l}{A}$

where, ρ = resistivity

l = length of the resistance wire

A = area of cross-section

When the wires are connected in series, then

$$R = R_1 + R_2$$

$$R = \frac{\rho_1 l_1}{A_1} + \frac{\rho_1 l_2}{A_2}$$

$$= \frac{\rho_1 l_1}{A} + \frac{\rho_2 l_2}{A} \qquad (\because A_1 = A_2)$$

ρ_{eq} = resistivity of combination

$$l = l_1 + l_2$$

$$A = A$$

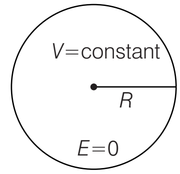
Thus, $\rho_{eq} \cdot \frac{l_1 + l_2}{A} = \frac{\rho_1 l_1 + \rho_2 l_2}{A}$

$$\rho_{eq} = \frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$$

3. Inside a hollow charged sphere, the electric field intensity is always zero.

$\therefore$ $E = 0$

but potential is constant.



Potential on the surface of the hollow sphere

$$V = \frac{KQ}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

This is the potential (V) inside the charged hollow metal sphere.

4. The de-Broglie wavelength associated with electron is given by,

$$\lambda = \frac{h}{\sqrt{2\,meV}}$$

where, h = Planck's constant

m = mass of electron

e = charge on electron

V = potential difference

$\lambda = 1\text{Å} = 10^{-10}$ m (given)

$$\lambda^2 = \frac{h^2}{2\,meV} \Rightarrow V = \frac{h^2}{2\,me\lambda^2}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times (1 \times 10^{-10})^2}$$

$$= \frac{6.6 \times 10^{-34} \times 6.6 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^{-20}}$$

$$= \frac{6.6 \times 6.6 \times 10^2}{2 \times 9.1 \times 1.6} = \frac{4356 \times 10^2}{2912} = 150\text{ V}$$

5. Work function of cesium = 2.27 eV

The wavelength of incident light

$$\lambda = 600\,\text{nm} = 600 \times 10^{-9}\text{ m}$$

The energy of incident photon = $h\nu$

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9} \times 1.6 \times 10^{-19}}\text{ eV}$$

$$= 2.06\text{ eV}$$

$$\frac{hc}{\lambda} < \phi \qquad \text{(work function)}$$

$$2.06\text{ eV} < 2.27\text{ eV}$$

So, no emission of electrons will take place.

Hence, none of the given options is correct.

6. The number of de-Broglie wavelength contained in the second Bohr orbit of Hydrogen atom is 2.

7.
$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} + \frac{1}{n_2^2}\right]$$

For Balmer series,

$$n_1 = 2, n_2 = 3, 4, 5 \ldots.$$

$$\therefore \quad \frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda_B} = R \cdot \frac{3}{16} \quad \ldots\text{(i)}$$

Lyman series is obtained when $n_1 = 1, n_2 = 2, 3, 4, \ldots$

For third line in Lyman series,

$$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = R\left[1 - \frac{1}{16}\right]$$

$$\frac{1}{\lambda_L} = R \cdot \frac{15}{16} \quad \ldots\text{(ii)}$$

$$\frac{1/\lambda_B}{\frac{1}{\lambda_L}} = \frac{R \cdot \frac{3}{16}}{R \cdot \frac{15}{16}}$$

$$\frac{\lambda_L}{\lambda_B} = \frac{3}{15} \quad \ldots\text{(iii)}$$

$$\lambda_L = \frac{3}{15} \times \lambda_B$$

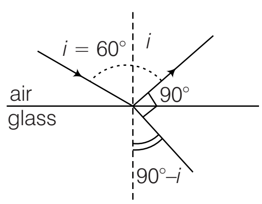
$$\lambda_B = 600\text{nm} = 600 \times 10^{-9}\text{m}$$

$$\lambda_L = \frac{3}{15} \times 600 \times 10^{-9}$$

$$= 120 \times 10^{-9}\text{ m} = 120\text{ nm}.$$

Hence, none of the given options is correct.

8. Here, angle of incidence is 60°



$$\mu = \frac{\sin i}{\sin r}$$

$$\sin i = \mu \sin r$$

here, $\quad r = 90 - i$

$$\sin i = \mu \sin(90° - i) = \mu \cos i$$

$$\mu = \frac{\sin i}{\cos i} = \tan i = \tan 60°$$

$$\mu = \sqrt{3}$$

9. Thickness of glass plate $= t$

Refractive index $= \mu$

Velocity of light $= c$

$$\text{Speed of light} = \frac{\text{distance}}{\text{time}} = \frac{t}{T}$$

$$T = \frac{t}{\text{speed of light}}$$

Speed of light in glass plate

$$v = \frac{c}{\mu} \qquad \left(\because \mu = \frac{c}{v}\right)$$

$$T = \frac{t}{c/\mu} = \frac{\mu t}{c}$$

10. Given, $x = at + bt^2$

$$[x] = [bt^2]$$

$$\text{Unit of} \quad b = \frac{x}{t^2} = \frac{\text{metre}}{(\text{h})^2} = \frac{\text{m}}{\text{h}^2}$$

11. Given, $|A + B| = |A - B|$

$$\therefore \quad (A + B)^2 = (A - B)^2$$

$$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$$

$$2AB\cos\theta + 2AB\cos\theta = 0$$

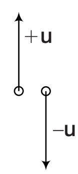
$$4AB\cos\theta = 0$$

Here, $\quad A \neq 0, B \neq 0$

$\therefore \quad \cos\theta = 0$

or $\quad \theta = 90°$

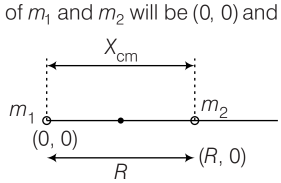
12. Given that velocity of ascending body = u



Let there is no air resistance when body falls freely in downward direction, the direction of falling body will opposite to the ascending body.

∴ The velocity at the same height while the body falls freely = – u

13. The bodies are separated by a distance R, so the coordinates of m_1 and m_2 will be (0, 0) and (R, 0)



From the formula of centre of mass,

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{m_1 \times 0 + m_2 \times R}{m_1 + m_2}$$

$$X_{cm} = \frac{m_2 \cdot R}{m_1 + m_2}$$

14. $T_1 = 273 + 20 = 293\text{ K}$

$T_2 = 273 + 40 = 313\text{ K}$

The velocity of sound wave in gases or air is given by

$$v = \sqrt{\frac{\gamma R T_1}{M}}$$

where, γ = Ratio of C_p / C_V

R = gas constant

$$\therefore \quad v_1 = \sqrt{\frac{\gamma R T_1}{M}}$$

$$v_2 = \sqrt{\frac{\gamma R T_2}{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{293}{313}}$$

Given, $v_1 = 344.2$ m/s

$$\therefore \quad v_2 = v_1 \sqrt{\frac{313}{293}}$$

$$= 344.2 \times \sqrt{1.068}$$

$$= 344.2 \times 1.03 \text{ m/s}$$

$$= 355.75 \text{ m/s}$$

$$\simeq 356 \text{ m/s}$$

15. Mass of gas = 4g (given)

From perfect gas equation

$$pV = nRT$$

where n = number of moles

here, $\left(n = \frac{4}{2} = 2\right)$

$\therefore \quad pV = 2RT$

It will be the perfect gas equation for 4g of hydrogen.

16. Suppose temperature of Sun = T

When it is doubled, $T' = 2T$

From law of radiation (Stefan's law)

$$E \propto T^4 = \sigma T^4$$

σ = Stefan's constant.

$$E_1 \propto T^4$$

$$E_2 \propto (2T)^4$$

$$\frac{E_1}{E_2} = \frac{T^4}{2^4 . T^4}$$

$$E_2 = 16 E_1.$$

17. In SHM, the acceleration of vibrating particle is proportional to displacement

$$|a| = \omega^2 x.$$

Here $a = 16 \text{ cm/s}^2$

$$= 16 \times 10^{-2} \text{ m/s}^2$$

Displacement, $x = 4\text{cm} = 4 \times 10^{-2}$ m

$$\therefore \quad 16 \times 10^{-2} = \omega^2 (4 \times 10^{-2})$$

$$\omega^2 = 4$$

$$\omega = \pm 2 \text{ rad/s}$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.142 \text{ s}$$

18. The work done in stretching the spring, $W = \frac{1}{2} Kx^2$

K = force constant of spring

x = extension in length of the spring

$$W_1 = \frac{1}{2} K x_1^2$$

$$W_2 = \frac{1}{2} K x_2^2$$

$$x_1 = 1$$

$$x_2 = 1 + 1 = 2$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2} K (1)^2}{\frac{1}{2} K (2)^2} = \frac{1}{4}$$

$$W_2 = 4W_1$$

$$W_1 = 10 \text{ J}$$

$$W_2 = 40 \text{ J}$$

$\therefore$ More work required = 40 J – 10 J = 30 J

19. $v_{rms} = \sqrt{\frac{3RT}{M}}$

R = gas constant

T = absolute temperature

M = mass of gas

$$v_{rms} \text{ of } H_2 = \sqrt{\frac{3RT}{M_{H_2}}}$$

$$v_{rms} \text{ of } O_2 = \sqrt{\frac{3RT}{M_{O_2}}}$$

$$\frac{v_{rms}.H_2}{v_{rms}.O_2} = \sqrt{\frac{M_{O_2}}{M_{H_1}}}$$

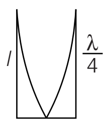
Given $\quad v_{rms}\ H_2 = c$

$$\Rightarrow \quad \frac{c}{v_{rms} O_2} = \sqrt{\frac{M_{O_2}}{M_{H_1}}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

$$\therefore \quad v_{rms} = \frac{c}{4}$$

20. The tube is closed at one end. So, it will act like a closed organ pipe.

The lowest frequency produced in the tube



$$l = \frac{\lambda}{4}, \lambda = 4$$
$$l = 1\text{m}$$
$$f = 84 \text{ Hz}$$
$$v = n\lambda = 84 \times 4$$

When wave propagate in air, the velocity

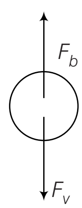
$$v = \sqrt{\frac{\gamma p}{\rho}}$$

Here, $p = 1.0 \times 10^5 \text{N/m}^2$

$\rho = 1.2 \text{ kg/m}^3$

$$v^2 = \frac{\gamma p}{\rho}$$
$$\gamma = \frac{v^2 . \rho}{p}$$
$$= \frac{(84 \times 4)^2 \times 1.2}{1.0 \times 10^5}$$
$$= \frac{135472.2}{10^5} = 1.354$$
$$\approx 1.4$$

21. The gas bubble is rising through the liquid of density 1.75 g/cm^3 with constant speed. The acting forces on bubble are



Buoyancy force of liquid = F_b (in upward direction)

Viscous force due to liquid = F_V (in downward direction)

$\therefore \quad F_V = F_b$

For free falling in a viscous medium

$$F_V = 6\pi\eta r v_T$$

η = viscosity coefficient of the liquid

r = radius of bubble

$$F_b = mg = v \times \rho . g$$
$$= \frac{4}{3} \pi r^3 \rho_e . g$$

$$6\pi\eta r v_t = \frac{4}{3} \pi r^3 \rho_e \cdot g$$
$$\eta = \frac{24\pi \cdot r^3 \rho_e \cdot g}{3 \times 6\pi r v_T} = \frac{2}{9} \cdot \frac{r^2 \rho_e \cdot g}{v_T}$$

here, $D = 2$ cm, $r = 1 \text{ cm} = 1 \times 10^{-2}$ m

$\rho_e = 1.75 \text{ g/cm}^3 = 1.75 \times 10^{+3} \text{ kg/m}^3$

$g = 10 \text{ m/s}^2$

v_T = terminal velocity

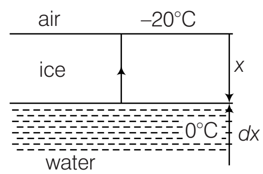
$= 0.35 \times 10^{-2}$ m/s

$$\eta = \frac{2}{9} \times \frac{(10^{-2})^2 \times 1.75 \times 10^{+3} \times 10}{0.35 \times 10^{-2}}$$
$$= \frac{2}{9} \times \frac{10^{-4} \times 1.75 \times 10^3 \times 10}{0.35 \times 10^{-2}}$$
$$= \frac{2 \times 175 \times 10 \times 10}{9 \times 35} \times 10 \text{ poise}$$
$$\simeq 1089 \text{ poise}$$

22. Temperature of water = 0°C

Temperature of surrounding atmosphere = – 20°C

Density of ice = ρ



Coefficient of thermal conductivity = k

latent heat of melting = L

The rate of heat flow $H = \frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{x}$

$$= \frac{kA[(0 - (-20)]}{x}$$
$$= \frac{kA \times 20}{x}$$
$$\frac{dQ}{dt} = \frac{dm}{dt} \cdot L \qquad (\because mL = Q)$$
$$dm = \rho A \cdot dx$$
$$\frac{dQ}{dt} = \rho A \cdot \frac{dx}{dt} L = \frac{kA \cdot 20}{x}$$
$$\int_0^Z x \cdot dx = \frac{20k}{\rho L} \int_0^t dt \Rightarrow Z^2 = \frac{40k}{\rho L} t$$

So, thickness Z increases as a function of time t according to the above equation.

23. Let the radius of single drop = r

Radius of small drop = $\frac{2\text{mm}}{2}$

$= 1$ mm

$= 1 \times 10^{-3}$m

Surface tension of water = 0.072 N / m

The volume of large drop must be equal volume of all small drops.

$$\frac{4}{3}\pi R^3 = 1000\left(\frac{4}{3}\pi r^3\right)$$

$\therefore \quad R = (1000)^{1/3} \cdot r = 10.r$

The initial potential energy of drops system $U_i = s \times n \times 4\pi r^2$

$$= s \times 1000 \times 4\pi r^2$$
$$= 1000\, s \cdot 4\pi r^2$$

When the droplets coalesce, the final potential energy of the system $U_f = s \cdot 4\pi R^2$

$$= s \cdot 4\pi \times 100\, r^2$$

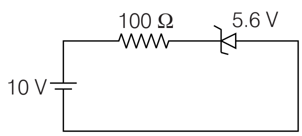
Energy loss in the formation of large drop

$$\Delta E = U_f - U_i$$
$$= s \cdot 4\pi 100 r^2 - 1000 s \cdot 4\pi r^2$$
$$= s \cdot \pi r^2 (400 - 4000)$$
$$= -3600 s \cdot \pi r^2$$
$$= -3600 \times 0.72 \times 3.14 \times (1 \times 10^{-3})^2$$

From catenation

$$= 813.888 \times 10^{-6}$$
$$= 8.1388 \times 10^{-4} \text{ J}$$
$$= 8.146 \times 10^{-4} \text{ J}$$

24.



Breakdown voltage of Zener diode = 5.6 V

Voltage of source = 10 V

Potential difference $\Delta V = (10 - 5.6)$ V = 4.4 V

Resistance of circuit = 100 Ω

The current passing through the Zener diode

$$I = \frac{\Delta V}{R}$$
$$= \frac{4.4}{100}$$
$$= 4.4 \times 10^{-2} \text{ A}$$
$$= 44 \times 10^{-3} \text{ A}$$
$$= 44 \text{ mA}$$

25. Current gain $\beta = 45$

We know that, $\quad \beta = \frac{\Delta I_c}{\Delta I_b}$

$\therefore \quad \frac{\Delta I_c}{\Delta I_b} = 45 \quad \ldots \text{(i)}$

Potential drop across $1\,k\,\Omega$ or $1 \times 10^3\ \Omega$

Collector resistor is 5V.

$$V = I_C R$$
$$I_C = \frac{V}{R} = \frac{5}{1 \times 10^3}$$
$$= 5 \times 10^{-3} \text{ A} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii),

$$\Delta I_B = \frac{\Delta I_C}{45}$$
$$= \frac{5 \times 10^{-3}}{45} = \frac{1}{9} \times 10^{-3}$$
$$= 0.111 \times 10^{-3} \text{ A}$$
$$= 111 \times 10^{-6} \text{ A} = 111\,\mu\text{A}$$

26. Given, $\quad \mathbf{E} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ V/m

$$\mathbf{B} = 2\hat{i} + 3\hat{j}\ \text{T}$$
$$\mathbf{v} = 2\hat{i} + 3\hat{j}\ \text{m/s}$$
$$e = -1.6 \times 10^{-19}\text{C}$$

The magnitude of electric field

$$|\mathbf{E}| = \sqrt{(3)^2 + (6)^2 + (2)^2}$$
$$= \sqrt{9 + 36 + 4}$$
$$= \sqrt{49} = 7$$

The force acting on electron due to electric field

$$\mathbf{F} = q\mathbf{E}$$
$$|\mathbf{F}| = |q|\,|\mathbf{E}|$$
$$= 1.6 \times 10^{-19} \times 7 \text{ N}$$
$$= 11.2 \times 10^{-19} \text{ N}$$
$$= 1.12 \times 10^{-18} \text{ N}$$

Alternate Method

B || v Magnetic field is parallel to velocity of electron.

v || B Magnetic force

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$$
$$= qvB\sin\theta \quad (\because \theta = 0)$$
$$= qvB\sin 0$$
$$= 0$$

$\therefore$ Net force $\quad F = F_e + F_m$

$$= (1.12 \times 10^{-18} + 0) \text{ N}$$
$$= 1.12 \times 10^{-18} \text{ N}$$

Hence, none of the given options is correct.

27. Given, $L_1 = 6$ mH $= 6 \times 10^{-3}$H

$L_2 = 8$ mH $= 8 \times 10^{-3}$ H

If two coils L_1 and L_2 are connected in series, the equivalent self-inductance L for assembly

$$L = L_1 + L_2 + 2\sqrt{L_1 L_2}$$

$= 6 \times 10^{-3} + 8 \times 10^{-3} + 2\sqrt{6 \times 8 \times 10^{-6}}$

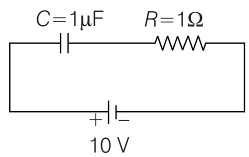
$= 14 \times 10^{-3} + 2\sqrt{48} \times 10^{-3}$

$= 14 \times 10^{-3} + 2 \times 7 \times 10^{-3}$

$(\because \sqrt{48} = 6.92 \simeq 7)$

$= 28 \times 10^{-3}\ \text{H} = 28\text{mH}$

28.



The time constant of the circuit

$\tau = C.R$

Here, $C = 1\,\mu\text{F} = 1 \times 10^{-6}$

$R = 1\,\text{M}\Omega = 1 \times 10^{6}\,\Omega$

$\tau = 1 \times 10^{-6} \times 1 \times 10^{6} = 1\,\text{s}$

We know that in one (1) time constant 63% charging is done.

$\therefore \quad \frac{63}{100} \times q_{\max} \qquad (q = CV)$

$= \frac{63}{100} \times 1 \times 10^{-6} \times 10$

$= \frac{63}{100} \times 1\,\mu\text{F} \times 10 = 6.3\,\mu\text{C}$

$\Rightarrow \quad V = \frac{q}{C} = \frac{6.3 \times 10^{-6} C}{1 \times 10^{-6} \text{F}} = 6.3\ \text{V}$

29. In series combination,

$R = R_1 + R_2$

Same current will pass in R_1 and R_2. According to question, R_1 is increased by 0.5%.

$\therefore$ Increment in the resistance

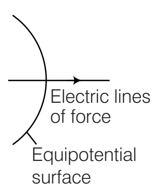
$= \frac{0.5}{100} \times 400$

$= 2.0\ \Omega$

So to keep the potential unchanged in second resistance the change required will be $2.0\ \Omega$ increment.

30. The electric lines of force are always perpendicular to the equpotential surface

$\theta = 90°$



31. The current $I = I_0\, e^{-\lambda t}$ is of exponential nature.

$$I = \frac{dQ}{dt} = I_0\, e^{-\lambda t}$$

$$dQ = I_0 e^{-\lambda t}\, dt$$

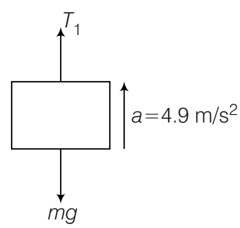
$$\int_0^Q dQ = I_0 \int_{t=0}^{t=\infty} e^{-\lambda t}\, .dt$$

$$Q = I_0 \times \frac{1}{\lambda} = \frac{I_0}{\lambda}$$

32. For Fraunhoffer diffraction to occur, both source and screen should be at infinity, it is essential condition for diffraction pattern.

33. Information in the statement of the questions is insufficient to calculate heat required to raise the temperature from 100°C to 1000°C.

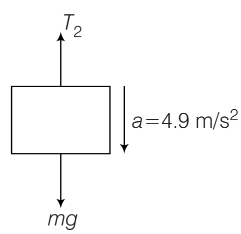
34. (i) When the body of mass m lifted up the forces acting on the body



$$T_1 - mg = \frac{mg}{2}$$

$$T_1 = mg + \frac{mg}{2} = \frac{3mg}{2} \qquad \text{...(i)}$$

(ii) When the body lowered the acting forces



$$(mg - T_2) = \frac{mg}{2}$$

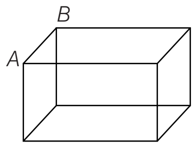
$$T_2 = mg - \frac{mg}{2} = \frac{mg}{2} \qquad \text{...(i)}$$

On dividing Eq. (i) from Eq. (ii) we get

$$\therefore \quad \frac{T_1}{T_2} = \frac{3mg/2}{mg/2} = \frac{3}{1}$$

$$= 3:1$$

35.



Let when link AB is removed, x is an equivalent of remaining part of cube without link.

AB and rest part will be in parallel, so according to question

$$\frac{7}{12} = \frac{R_1 R_2}{R_1 + R} = \frac{1(x)}{1+x} = \frac{x}{1+x}$$

$$\frac{7}{12} = \frac{x}{1+x}$$

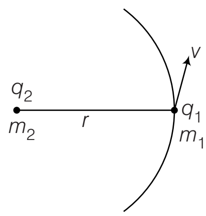
$$12x = 7 + 7x$$

$$12x - 7x = 7$$

$$5x = 7$$

$$x = \frac{7}{5}\,\Omega$$

36.



The necessary centripetal force, to move in a circular path is provided by the Coulomb's force.

$$\frac{m_1 v^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r\, m_1}} \qquad \ldots(i)$$

We know, $v = r\omega$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi \,.\, r}{v} \qquad \ldots(ii)$$

Substitution the value of v from, Eq. (i) we get

$$T = \frac{2\pi r}{\sqrt{\dfrac{q_1 q_2}{4\pi\varepsilon_0\, r \,.\, m_1}}}$$

$$T = \sqrt{\frac{4\pi^2 r^2 \times 4\pi\varepsilon_0 \,.\, r \,.\, m_1}{q_1 q_2}}$$

$$T = \sqrt{\frac{16\pi^3 \varepsilon_0 r^3 .\, m_1}{q_1 q_2}}$$

None of the given options is correct.

Both the charges are positive or both negative. (Since, answers have the product $q_1 q_2$ inside square root. Therefore, circular motion is not possible.

The statement of question is wrong. Either q_1 or q_2 should be negative.

37. The intensity of emitted electrons is

$$I \propto \frac{1}{r^2}$$

$$I_1 \propto \frac{1}{d^2}$$

$$I_2 \propto \frac{1}{(d/2)^2} = 4 \cdot \frac{1}{d^2}$$

and $I \propto$ Number of photons per second $\propto N$

$$\therefore \qquad N \propto \frac{1}{r^2}$$

$\therefore$ Number of emitted electrons will become 4 times.

From Einstein's photo-electric equation

$$KE_{max} = h\nu - \phi$$

where, ϕ = work function of surface

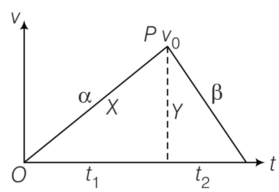
ν = frequency of incident photon

Since, ν remains unchanged, thus KE_{max} as well as stopping potential remains unchanged.

$\because \quad KE_{max} = eV_s \propto \nu$

where, V_s = stopping potential.

38.



Shape of the graph OP shows the acceleration in train

$\therefore$ tan θ = acceleration.

$$\alpha = \frac{v_0}{t_1}$$

Shape of the graph pt shows the deceleration in train.

$\therefore$ Similarly, $\qquad \beta = \dfrac{v_0}{t_2}$

$$\therefore \qquad \frac{\beta}{\alpha} = \frac{v_0/t_2}{v_0/t_1} = \frac{t_1}{t_2}$$

Displacement = area of graph obtained between v-t

$$x = \frac{1}{2} t_1 \,.\, v_0$$

$$y = \frac{1}{2} t_2 \,.\, v_0$$

$$\therefore \qquad \frac{x}{y} = \frac{t_1}{t_2}$$

$$\Rightarrow \qquad \frac{x}{y} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

39. Let mass of the drop = m

Weight of the drop = mg

It will act downward.

The force due to surface tension on the drop = $\sigma 2\pi R$

Where, R = radius of tap

It will act in upward direction.

The drop of water will detach when

$$mg > \sigma 2\pi R$$

But, $$m = v \times \rho = \frac{4}{3}\pi r^3 \rho g$$

$$\therefore \quad \frac{4}{3}\pi r^3 \rho g > \sigma \times 2\pi R$$

$$r > \left[\frac{3}{2}\,\frac{\sigma \,.\, R}{\rho g}\right]^{1/3}$$

Hence, none of the given options is correct.

40. Current-carrying rectangular coil is placed in a non-uniform magnetic field. So, the force on each arm of the coil will different giving the resultant force as non-zero.

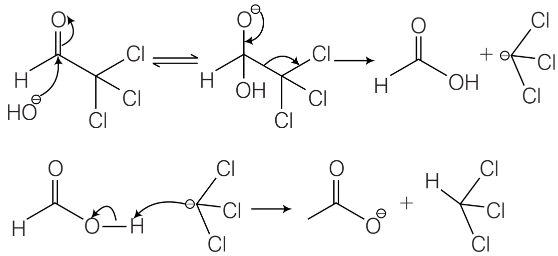
Thus, option (a) is correct.

Also, the torque acting on current-carrying coil in non-uniform magnetic field will be non-zero.

Thus, option (d) is also correct.

Chemistry

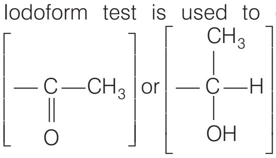
1. For a carbonyl group to give Cannizaro reaction, it should not have an α-hydrogen atom. Among the given compound Ne_3 CCHO, C_6H_5CHO and HCHO respond to Cannizaro reaction but Cl_3CCHO do not due to the following reason.



When hydroxyl group attacks the carbonyl group a tetrahedral intermediate forms. This tetrahedral intermediate will revert to a carbonyl compound by expelling the best leaving group. The $\overset{\ominus}{C}Cl_3$ carbonium is resonance stabilised therefore it is better leaving group than $\overline{O}H$. These initial products exchange a proton to give ion of carboxylic acid and trichloromethane ($CHCl_3$).

2. Primary (1°)-alcohols do not give Lucas reagent test at room temperature. $CH_3CH_2CH_2OH$ does not undergo S_N1 or S_N2 at room temperature.

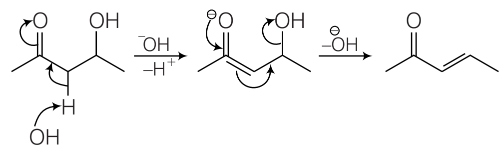
3. Iodoform test is used to detect the presence of



in a molecule.

In the given compounds only CH_3COOH does not fulfill the structural requirement of iodoform test hence, it does not respond to iodoform test.

4.



In alkaline medium the acidic proton present in the molecule is abstracted to give enolate ion. In the next step hydroxide ion if present at β-position leaves giving α-β-unsaturated carbonyl compound. In (a), (c) and (d), dehydration cannot take place due to improper placement of groups.

5. The correct order is 2<1<3<4. Since, more the number of 2° —NH— group, more is the basic nature.

(i) Thus, 3 and 4 are more basic than 1 and 2.

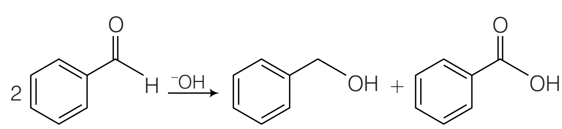
(ii) Between 3 and 4, 4 has one two —NH_2 group while 3 has only one — NH_2 group.

$\therefore$ 4 is more basic than 3.

(iii) Between 1 and 2, 1 has one — NH_2 group with no resonance, thus is more basic than 2.

6. C — C bond is not formed in Cannizaro reaction while other reactions result in the formation of C — C bond.

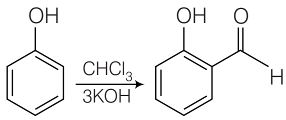
Cannizaro reaction



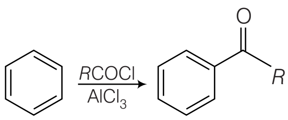
Wurtz reaction

$$2R - X + 2Na \longrightarrow R - R + 2\overset{\oplus}{Na}\overset{\ominus}{X}$$

Reimer-Tiemann reaction



Friedel-Crafts acylation

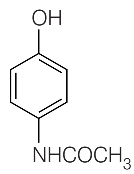


7. Option (a) is false as colloidal sols are not homogeneous.

Colloidal sols are heterogeneous mixture of dispersed phase and dispersion medium.

8. Paracetamol is 4-acetamidophenol, i.e.

N-(4-hydroxy phenyl) ethanamide (IUPAC Name).



9. The option (d) is false as Ionic radii of trivalent lanthanides decreases with increase in atomic number due to lanthanoid contraction.

10. Only KNO_3 on heating do not give NO_2 but all other give

(a) $2KNO_3 \xrightarrow{\Delta} 2KNO_2 + O_2$

(b) $2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2 + O_2$

(c) $2Cu(NO_3)_2 \xrightarrow{\Delta} 2CuO + 4NO_2 + O_2$

(d) $2AgNO_3 \xrightarrow[\text{Red hot}]{\Delta} 2Ag + O_2 + 2NO_2$

Heavy metal nitrates liberate nitrogen dioxide on heating.

11. Due to hydrogen bonding, HF shows highest boiling point.

Thereafter van der Waals' force decide the boiling point.

Larger the size or molecules mass, greater is the van der Waals' forces. Hence, higher is the boiling point.

∵ van der Waals' forces is more for HI than HBr, which is in turn is more than HCl.

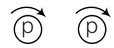
Hence, correct order is HF > HI > HBr > HCl.

12. PCl_5 exists as $[PCl_4]^+$ and $[PCl_6]^-$ in solid state.

$2PCl_5 \longrightarrow [PCl_4]^+ [PCl_6]^-$

13. (i) *Ortho* and *para* hydrogens have different nuclear spin.

In H_2 molecule, two protons in two H atoms with parallel spin are called *ortho*-hydrogen



Nucleus of hydrogen atom with same spin (*ortho*)

and with opposite spin are called *para*-hydrogen.

Nucleus of hydrogen atom with different spin (*para*)

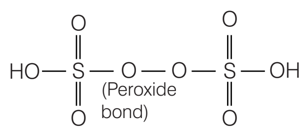
(ii) *Ortho*-hydrogen is more stable than *para* form at and above room temperature only, therefore *para* form always tends to revert in *ortho*-form. Thus, not correct.

(iii) The per cent composition of *ortho* and *para* changes with the temperature. Thus, their ratio also change.

(iv) They have slightly different boiling point.

Hence, (b) is the answer.

14. $H_2S_2O_8$ shows —O—O— (peroxy) bonding as follows:



Peroxodisulphuric acid
or
Marshall's acid

15. Iron (Fe) is used to obtain Cu-metal from $CuSO_4$ (*aq*) because Fe is more reactive than Cu (and also easily available and is cheaper) thus, replace the copper from $CuSO_4$ to give copper (Cu).

$CuSO_4(aq) + Fe(s) \longrightarrow FeSO_4(aq) + Cu(s)$

16. Radium in isolated form and in its chloride (i.e. Ra^+) only differ in the number of orbital electron.

But, radioactivity is independent of chemical environment of an ion or atom, i.e. radioactivity is the phenomenon which does not depend on the orbital electrons but depends only on the composition of nucleus.

17. The correct order of solubility of sulphates in water is

$$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$$

Solubility of 2nd group sulphates decreases as we move down the group due to less release of hydration energy. $Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$ (Ionic Size) As hydration energy decreases more rapidly than latice energy, the solubility decreases down the group.

$$\text{Hydration Energy} \propto \frac{\text{Charge}}{\text{Size}}$$

While lattice energy almost remains constant. Hence, solubility decreases.

18. Given, energy of one mole of H_2 bonds = 436 kJ

$$\because \quad E = \frac{n \cdot hc}{\lambda}$$

(energy for one H—H bond)

(n = number of H—H bonds)

$$\therefore \quad \lambda = \frac{n \cdot hc}{E}$$

$$\lambda = \frac{6.634 \times 10^{-34} \text{ Js} \times 3 \times 10^{8} \text{ ms}^{-1} \times 6.023 \times 10^{23}}{436 \times 10^{3} \text{ J}} \text{ m}$$

$$= 0.2736 \times 10^{-34+8+23-3}$$

$$= 0.2736 \times 10^{-6} \text{m}$$

$$= 273.6 \text{ nm} \approx 274 \text{ nm}$$

19. Correct order of O— O bond length is

$$H_2O_2 > O_3 > O_2$$

(i) Between O_3 and O_2

$\because$ Bond order of $O_2 > O_3$

Thus, bond length of $O_3 > O_2$.

$$\left(\because \text{ Bond order } \propto \frac{1}{\text{Bond length}}\right)$$

(ii) Between O_3 and H_2O_2

$\because O_3$ has π-bonds in resonance so, has shorter bond length than O—O in H_2O_2.

$\therefore$ Hence, (b) is the answer.

20. Calcium carbide is calcium acetylide, having structure

$$Ca^{2+} \cdot \overset{\ominus}{C} \equiv \overset{\ominus}{C}$$

Thus, it has one sigma (σ) and two pi (π) bonds.

21. Let Z = atomic number

M = atomic mass

$$^{M}_{Z}E \xrightarrow{-\alpha} {}_{Z-2}E^{M-4} \xrightarrow{-2\beta} {}_{Z}E^{M-4}$$

Hence, $^{M}_{Z}E$ and ${}_{Z}E^{M-4}$ has same atomic number with different atomic mass thus, they are isotopes.

22. The element belonging to 4th period and 15th group is arsenic (As). The outer electronic configuration of 15th group element is ns^2np^3.

Since arsenic belongs to 4th period, therefore it has filled 'd' orbital too. Thus, the electronic configuration of As is $[Ar]4s^2\,3d^{10}\,4p^3$.

So, it has filled 's' and 'd' orbital and half-filled p-orbital.

23. For exothermic reaction,

$$\because \quad K_p \propto \frac{1}{T}$$

Thus, as $\frac{1}{T}$ increases (i.e. T decreases), K_p increases.

$\because$ ΔH for exothermic reactions is negative.

24. Given,

$p°$ = vapour pressure of pure solvent

p = vapour pressure of solution

n_1 = moles of solute

n_2 = moles of solvent

Thus, according to Raoult's law.

$$\frac{p° - p}{p°} = x_1 = \frac{n_1}{n_1 + n_2}$$

(x_1 = mole fraction of solute)

$$\text{or,} \quad 1 - \frac{p}{p°} = \frac{n_1}{n_1 + n_2}$$

$$\frac{p}{p°} = 1 - \frac{n_1}{n_1 + n_2}$$

$$\Rightarrow \quad \frac{n_1 + n_2 - n_1}{n_1 + n_2}$$

$$\frac{p}{p°} = \frac{n_2}{n_1 + n_2}$$

$$\Rightarrow \quad p = p°\left(\frac{n_2}{n_1 + n_2}\right)$$

25. In Schottky defect, equal number of cations and anions are missing creating equal number of cation and anion vacancies.

26. For a spontaneous reaction, ΔG = negative and ΔS = positive

$$\because \quad \Delta G = \Delta H - T\Delta S = -\text{ve}$$

Hence, $\Delta H - T\Delta S = -$ve only when $\Delta S = +$ ve and $\Delta H = -$ve.

27. $\because$ KCl is more ionic than NaCl while NaCl is more ionic than LiCl.

Also, more the ionic nature, more is the conductivity thus, order of equivalent conductance at infinite dilution is KCl > NaCl > LiCl.

28. $\because$ For a compound $A_x\ B_y$

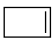
$$K_{sp} = x^x \cdot y^y \cdot S^{x+y}. \text{ (where, } S = \text{ solubility)}$$

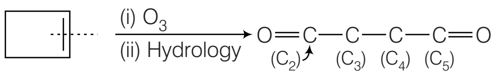
For MX_4

$$K_{sp} = 1 \times (4)^4 \cdot S^{1+4}$$

$$K_{sp} = 256 \times S^5$$

$$\therefore \qquad S = \left(\frac{K_{sp}}{256}\right)^{1/5}$$

29. Only  gives one compound on ozonolysis.



All other options give more than one aldehyde/ketone on ozonolysis.

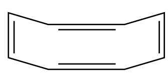
30. For a compound to be aromatic,

(i) It should be planar

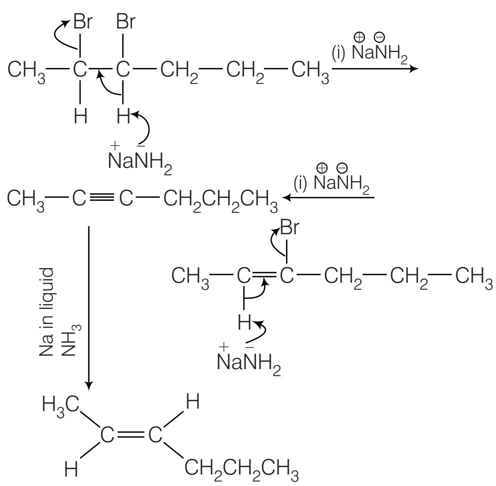
(ii) It should have $(4n+2)\pi$ electrons

(iii) It should be able to delocalise its π electron density.

Among the given compounds,
1, 3, 5, 7–cyclooctatetraene or cyclooctatetraene is not aromatic as its structure is non-planar.



31.



trans-alkene

Na in liquid NH_3 is a Birch reagent. It is used to alkyne into *trans* alkene only.

32. (i) It is a test for ammonia.

$$NH_4Cl + NaOH \xrightarrow{\Delta} \underset{\text{(Basic)}}{NH_3} + NaCl + H_2O$$

$\downarrow$

Turns red litmus blue

(ii) It is a test for SO_4^{2-} ions (Orange)

$$K_2Cr_2O_7 + SO_2 + SO_4^{2-} + HCl \longrightarrow$$

$$\underset{\text{Orange}}{K_2Cr_2O_4} + SO_2 + H_2SO_4$$

$\downarrow \Delta$

$$\underset{\text{(Green)}}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$$

33. Let the distance travelled in T time $= 2\pi r$

(Circumference of orbit)

$$\therefore \qquad \text{Velocity}\,(v) = \frac{2\pi r}{T} \qquad \ldots\text{(i)}$$

$$\text{Also, } v = \frac{n \cdot h}{2\pi m r} \qquad \ldots\text{(ii)}$$

$$\therefore \quad \frac{1}{T\text{ (Time period)}} = \text{frequency } \nu = v \times \frac{1}{2\pi r}$$

From Eq. (ii),

$$\nu = \frac{1}{T} = \frac{nh}{2\pi r \times 2\pi m r}$$

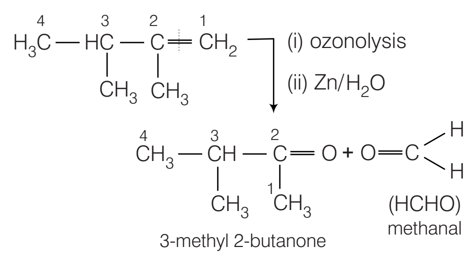
$$\nu = \frac{nh}{4\pi^2 r^2 m}$$

T (time taken in one revolution)

$$= \frac{4\pi^2 r^2 m}{nh}$$

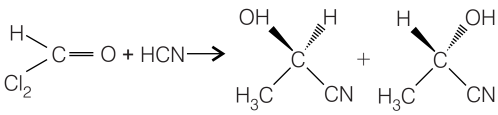
34. $\because$ rms velocity $(v_{rms}) \propto \sqrt{\frac{1}{M}}$ Thus, smaller the mass, more will be the rms speed. H_2 has minimum molar mass, So it has highest value of rms speed.

35.



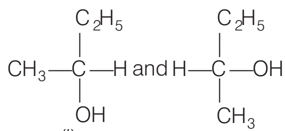
36. (a) These are *cis* and *trans* isomers not enantiomers (Thus, false).

(b) CH_3CHO on reaction with HCN gives racemic mixture of cyanohydrin (Thus, true).



(Thus, true) (cyno-hydrogen)

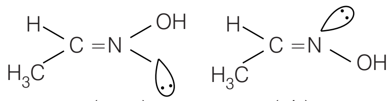
(C)



(i) (ii)

(i) and (ii) both have *R*-configuration thus, are not enantiomers.

(D) $CH_3—CH = NOH$ shows geometrical isomers.



(*trans*) (*cis*)

37. (A) When NaCl and $K_2Cr_2O_7$ warmed with H_2SO_4 (i.e. in acidic medium), they produce deep red vapours of chromyl chloride (CrO_2Cl_2).

(B) When NaOH is passed, the product CrO_3 formed in step (a) will react with NaOH and gives yellow colour solution.

(C) Chlorine gas is not evolved, thus false.

(D) In the given reaction, chromyl chloride (CrO_2Cl_2) is formed. All reactions are as follows:

(i) $K_2Cr_2O_7 + 2H_2SO_4 \longrightarrow 2KHSO_4 + 2CrO_3 + H_2O$

(ii) $NaCl + H_2SO_4 \longrightarrow NaHSO_4 + HCl$

(iii) $\underset{\text{from (i)}}{CrO_3} + \underset{\text{from (ii)}}{2HCl} \longrightarrow \underset{\text{Red vapours}}{CrO_2Cl_2}$

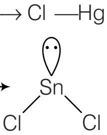
(iv) $CrO_2Cl_2 + 2NaOH \longrightarrow \underset{\text{Yellow solution}}{Na_2CrO_4} + 2HCl$

38. $HgCl_2$ and C_2H_2 both show linear shape as of CO_2.

($\because$ both have zero lone pair of electrons)

(a) $HgCl_2 \longrightarrow Cl—Hg—Cl$ (Linear)

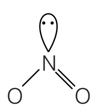
(b) $SnCl_2 \longrightarrow$



(Bent)

(c) $C_2H_2 \longrightarrow H—C\equiv C—H$ (Linear)

(d) $NO_2 \longrightarrow$



(Bent)

39. pH of buffer solutions is given by

$$pH = pK_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

(1) CH_3COOH and CH_3COONa

100 × 0.1   100 × 0.1

No. of milimoles = 10   = 10

$\because$ [salt] = [acid] = 10, $pH = pK_a + \log \frac{10}{10}$

$\therefore$ $pH = pK_a$

(2) $\underset{100\times0.1}{CH_3COOH} + \underset{50\times0.1}{NaOH} \longrightarrow CH_3COONa + H_2O$

	CH_3COOH	NaOH	CH_3COONa	H_2O
Initial	= 10	5	0	0
Final	= 5	0	5	5

$\because$ [salt] = [acid] = 5, $pH = pK_a + \log \frac{5}{5}$

$\therefore$ $pH = pK_a$

(3) $\underset{100\times0.1}{CH_3COOH} + \underset{100\times0.1}{NaOH} \longrightarrow CH_3COONa + H_2O$

	CH_3COOH	NaOH	CH_3COONa	H_2O
Initial	= 10	10	0	0
Final	= 0	0	10	10

[salt] = 10 and [acid] = 0,

$pH = pK_a + \log \frac{10}{0}$

$\therefore$ $pH \neq pK_a$

(4) $CH_3COOH + NH_3 \longrightarrow CH_3COONH_4$

Initial 10 10 $\longrightarrow$ No buffer action

Final Due to weak CH_3COONH_4 salt

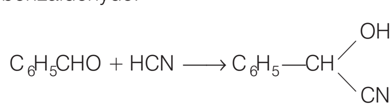
$\therefore$ $pH \neq pK_a$

40. (a) Ethyl chloride gives ethyl cyanide, when it reacts with KCN (ethanolic),

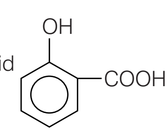
$$C_2H_5Cl + KCN \xrightarrow{\text{Ethanolic}} C_2H_5—CN$$

(b) It does not react with KCN.

(c) Benzaldehyde : KCN gives HCN in alcoholic medium and HCN gives cyanohydrin with benzaldehyde.



(d) Salicylic acid



does not react.

Mathematics

1. Given, $x\dfrac{dy}{dx} + y = xe^x$

$\Rightarrow \quad \dfrac{dy}{dx} + \dfrac{y}{x} = e^x \quad \ldots(i)$

On comparing Eq. (i) by $\dfrac{dy}{dx} + Py = Q$, we get

$$P = \frac{1}{x} \text{ and } Q = e^x$$

$\therefore \quad \text{IF} = e^{\int \frac{1}{x} dx}$

$$= e^{\log x} = x$$

Hence, solution of differential equation,

$$y \cdot x = \int xe^x \, dx + C$$

$\Rightarrow \quad xy = xe^x - \int e^x \, dx + C$

$\Rightarrow \quad xy = xe^x - e^x + C$

$\Rightarrow \quad xy = e^x(x - 1) + C \quad \ldots(ii)$

$\because \quad xy = e^x \, \phi(x) + C \quad$ [given]$\ldots$(iii)

On comparing Eqs. (ii) and (iii), we get

$$\phi(x) = (x - 1)$$

2. The general equation of the parabola is $x = ay^2 + b$, where a and b are arbitrary constants.

So, the differential equation is of order 2.

3. The equation of the line is $y = x + \lambda \quad \ldots(i)$

On comparing it with $y = mx + c$, we get

$$m = 1 \text{ and } c = \lambda.$$

The equation of the ellipse is

$$2x^2 + 3y^2 = 1$$

or $\quad \dfrac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \dfrac{y^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 1$

On comparing it with $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, we get

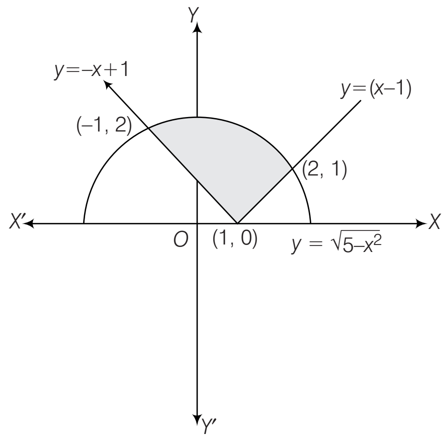
$$a^2 = \frac{1}{2} \text{ and } b^2 = \frac{1}{3}.$$

If the line touches the ellipse, then

$$c^2 = a^2m^2 + b^2 \Rightarrow \lambda^2 = \frac{1}{2} \cdot 1 + \frac{1}{3} = \frac{5}{6}$$

$\Rightarrow \quad \lambda^2 = \dfrac{5}{6} \Rightarrow \lambda = \sqrt{\dfrac{5}{6}}$

4. The graphs of $y = |x - 1|$ and $y = \sqrt{5 - x^2}$ are shown in the figure and the shaded region is the required region bounded by the two curves.



Let A be the area bounded by the given curves. Then,

$$A = \int_{-1}^{1} (\sqrt{5 - x^2} + x - 1)\, dx + \int_{1}^{2} (\sqrt{5 - x^2} - x + 1)\, dx$$

$$= \int_{-1}^{1} \sqrt{5 - x^2}\, dx + \int_{1}^{2} \sqrt{5 - x^2}\, dx + \int_{-1}^{1} (x - 1)\, dx + \int_{1}^{2} (-x + 1)\, dx$$

$$= \int_{-1}^{2} \sqrt{5 - x^2}\, dx + \left[\frac{x^2}{2} - x\right]_{-1}^{1} + \left[-\frac{x^2}{2} + x\right]_{1}^{2}$$

$$= \left[\frac{1}{2} x\sqrt{5 - x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}}\right]_{-1}^{2} - \frac{5}{2}$$

$$= 1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + 1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} - \frac{5}{2}$$

$$= -\frac{1}{2} + \frac{5}{2} \sin^{-1}\left(\frac{2}{\sqrt{5}} \times \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{4}{5}}\right)$$

$$= -\frac{1}{2} + \frac{5}{2} \sin^{-1}(1) = \frac{5\pi}{4} - \frac{1}{2} \text{ sq units}$$

5. Sum of distances of point P from point (8, 0) and (0, 12) will be minimum, if points are collinear.

$\therefore$ Equation of line at point (8, 0) and (0, 12).

$$\frac{x}{8} + \frac{y}{12} = 1$$

$\Rightarrow \quad \dfrac{y}{12} = 1 - \dfrac{x}{8}$

$\Rightarrow \quad y = 12 - \dfrac{x}{8} \times 12$

$\Rightarrow \quad y = 12 - \dfrac{3}{2} x$

$\therefore$ Points $(x, y) \equiv (2, 9), (4, 6)$ and $(6, 3)$.

Hence, the number of such points P in S is 3.

6. Given, $T = 2\pi\sqrt{\frac{l}{g}}$

On differentiating w.r.t. l, we get

$$\frac{dT}{dl} = \frac{2\pi}{\sqrt{g}} \cdot \frac{1}{2\sqrt{l}}$$

$$\therefore \quad \Delta T = \frac{dT}{dl}\Delta l = \frac{\pi}{\sqrt{gl}} \cdot \left(\frac{2l}{100}\right)$$

$$= 2\pi\sqrt{\frac{l}{g}} \cdot \frac{1}{100} = \frac{T}{100}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{100} = 1\%$$

Hence, approximate change in the time period is 1%.

7. Let the direction ratio's of two diagonals of a cube are (1, 1, 1) and (− 1, 1, 1).

$$\therefore \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(1)\times(-1) + (1)\times(1) + (1)\times(1)}{\sqrt{(1)^2 + (1)^2 + (1)^2}\sqrt{(-1)^2 + (1)^2 + (1)^2}}$$

$$= \frac{-1+1+1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

8. Given, $\log_a x, \log_b x$ and $\log_c x$ are in AP.

$$\therefore \quad 2\log_b x = \log_a x + \log_c x$$

$$\Rightarrow \quad 2\log_b x = \frac{1}{\log_x a} + \frac{1}{\log_x c}$$

$$\Rightarrow \quad \frac{2}{\log_x b} = \frac{\log_x ac}{\log_x a \log_x c}$$

$$\Rightarrow \quad 2\log_x c = \frac{\log_x b}{\log_x a}(\log_x ac)$$

$$\Rightarrow \quad 2\log_x c = \log_a b \cdot \log_x ac$$

$$\Rightarrow \quad \log_x c^2 = \log_x (ac)^{\log_a b}$$

$$\therefore \quad c^2 = (ac)^{\log_a b}$$

9. We have,

$$1 + (1+a)x + (1+a+a^2)x^2 + \dots \infty$$

$$= \sum_{n=1}^{\infty}(1 + a + a^2 + \dots + a^{n-1})x^{n-1}$$

$$= \sum_{n=1}^{\infty}\left(\frac{1-a^n}{1-a}\right)x^{n-1}$$

$$= \sum_{n=1}^{\infty}\frac{x^{n-1}}{1-a} - \sum_{n=1}^{\infty}\frac{a^n x^{n-1}}{1-a}$$

$$= \frac{1}{1-a}\sum_{n=1}^{\infty}x^{n-1} - \frac{a}{1-a}\sum_{n=1}^{\infty}(ax)^{n-1}$$

$$= \frac{1}{1-a}[(1 + x + x^2 + \dots \infty)]$$

$$- \frac{a}{1-a}[\{1 + ax + (ax)^2 + \dots \infty\}]$$

$$= \frac{1}{(1-a)} \cdot \frac{1}{(1-x)} - \frac{a}{(1-a)(1-ax)}$$

$$= \frac{1}{(1-ax)(1-x)}$$

10. Given, $\log_{0.3}(x-1) < \log_{0.09}(x-1)$

$$\Rightarrow \quad \log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$$

$$\Rightarrow \quad \log_{0.3}(x-1)^2 < \log_{0.3}(x-1)$$

$$\Rightarrow \quad (x-1)^2 > x-1 \qquad [\because 0.3 < 1]$$

$$\Rightarrow \quad x^2 + 1 - 2x - x + 1 > 0$$

$$\Rightarrow \quad x^2 - 3x + 2 > 0$$

$$\Rightarrow \quad (x-1)(x-2) > 0$$

$$\Rightarrow \quad x < 1, x > 2 \Rightarrow x > 2 \qquad [\because x \nless 1]$$

$$\therefore \quad (2, \infty)$$

Hence, x lies in the interval $(2, \infty)$.

11. We have,

$$\sum_{n=1}^{13}(i^n + i^{n+1})$$

$$= \sum_{n=1}^{13} i^n + \sum_{n=1}^{13} i^{n+1}$$

$$= i\left(\frac{1-i^{13}}{1-i}\right) + i^2\left(\frac{1-i^{13}}{1-i}\right)$$

$$= i\left(\frac{1-i}{1-i}\right) - \left(\frac{1-i}{1-i}\right)$$

$$= i - 1$$

12. We have,

$$|z_1| = |z_2| = |z_3| = 1$$

$$\Rightarrow \quad |z_1|^2 = |z_2|^2 = |z_3|^2 = 1$$

$$\Rightarrow \quad z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$$

$$\Rightarrow \quad \frac{1}{z_1} = \bar{z}_1, \frac{1}{z_2} = \bar{z}_2, \frac{1}{z_3} = \bar{z}_3$$

$$\text{Now,} \quad \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$$\Rightarrow \quad |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$$

$$\Rightarrow \quad |\overline{z_1 + z_2 + z_3}| = 1$$

$$\therefore \quad |z_1 + z_2 + z_3| = 1$$

13. Given, p, q are roots of equation $x^2 + px + q = 0$.

$$\therefore \quad p + q = -p \Rightarrow 2p + q = 0 \qquad \dots \text{(i)}$$

$$\text{and} \quad pq = q$$

$$\Rightarrow \quad p = 1$$

Put $p = 1$ in Eq. (i), we get

$$2 \times 1 + q = 0$$

$$\Rightarrow \quad q + 2 = 0$$

$$\Rightarrow \quad q = -2$$

Hence, $p = 1, q = -2$

14. Given equation is

$$x^2 - 3x + k = 0$$

As, $-\dfrac{b}{2a} = \dfrac{3}{2} \notin (0, 1)$

Since, both roots cannot lie between 0 and 1.

So, no value of k is possible.

15. Given word is 'ARRANGE'.

If R's occur together, number of letters in word = 6

To arrange R's together number of ways $= \dfrac{2!}{2!} = 1$

$\therefore$ Number of ways to permuted 'ARRANGE' $= \dfrac{6!}{2!}$

16. Given, $\dfrac{1}{{}^5C_r} + \dfrac{1}{{}^6C_r} = \dfrac{1}{{}^4C_r}$

$$\Rightarrow \frac{r!\,(5-r)!}{5!} + \frac{r!\,(6-r)!}{6!} = \frac{r!\,(4-r)!}{4!}$$

$$\Rightarrow \frac{(5-r)!}{5} + \frac{(6-r)!}{30} = (4-r)!$$

$$\Rightarrow \frac{(5-r)}{5} + \frac{(6-r)(5-r)}{30} = 1$$

$$\Rightarrow 30 - 6r + 30 - 11r + r^2 = 30$$

$$\Rightarrow r^2 - 17r + 30 = 0$$

$$\Rightarrow r^2 - 15r - 2r + 30 = 0$$

$$\Rightarrow r(r-15) - 2(r-15) = 0$$

$$\Rightarrow (r-2)(r-15) = 0$$

$$\therefore r = 15, 2$$

Hence, the value of r is 2.

17. For $n = 1, 2, 3, \ldots$, we find that $n^3 + 2n$ takes values 3, 12, 33, ..., which are divisible by 3.

18. Coefficient of x^{17} in $(x-1)(x-2)\ldots(x-18)$

$$= -(1 + 2 + 3 + \ldots + 18)$$

$$= -(171) = -171$$

19. Given,

$$1 + {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta + \ldots + {}^nC_n \cos n\theta$$

which is real part of complex number.

$({}^nC_0 + {}^nC_1 e^{i\theta} + \ldots)$,

i.e. $\text{Re}\,({}^nC_0 + {}^nC_1 e^{i\theta} + \ldots)$

$$= \text{Re}\,(1 + e^{i\theta})^n = \text{Re}\,(1 + \cos\theta + i\sin\theta)^n$$

$$= \text{Re}\left(2\cos^2 n\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right)$$

$$\left[\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2} \text{ and } \sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right]$$

$$= \left(2\cos\frac{\theta}{2}\right)^n \text{Re}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^n$$

$$= \left(2\cos\frac{\theta}{2}\right)^n \text{Re}\left(\cos\frac{n\theta}{2} + i\sin\frac{\theta}{2}\right)$$

[by De moivre's theorem]

$$= \left(2\cos\frac{\theta}{2}\right)^n \cdot \cos\frac{n\theta}{2}$$

20. Let $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$$= \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$$

On taking $\dfrac{1}{\log x}, \dfrac{1}{\log y}, \dfrac{1}{\log z}$ common from R_1, R_2 and R_3, we get

$$= \frac{1}{\log x \cdot \log y \cdot \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$\therefore \quad \Delta = 0$ [$\because$ all rows are identical]

21. We have, $|B| = 64$

$$\Rightarrow |\text{adj } A| = 64$$

$$\Rightarrow |A|^2 = 64$$

$$\therefore |A| = \pm 8$$

22. Given,

$$Q = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \text{and } x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Let $Q(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\because \quad Q^3(\theta) = Q(3\theta)$

$$\therefore Q^3\left(\frac{\pi}{4}\right) = \begin{bmatrix} \cos\frac{3\pi}{4} & -\sin\frac{3\pi}{4} \\ \sin\frac{3\pi}{4} & \cos\frac{3\pi}{4} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Now, $Q^3\left(\dfrac{\pi}{4}\right) x = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \end{bmatrix}$

$$\therefore Q^3\left(\frac{\pi}{4}\right) x = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

23. **Reflexivity** For reflexive $(x, x) \in R$.

$$x + 2x = 3x$$

which is divisible by 3.

$\Rightarrow \quad xRx \in R$

Hence, xRy is reflexive.

Symmetric Let $x + 2y = 3\lambda$

$\Rightarrow \quad x = 3\lambda - 2y$

Now, $\quad y + 2x = y + 2(3\lambda - 2y)$

$= y + 6\lambda - 4y$

$= 6\lambda - 3y$

$\Rightarrow \quad y + 2x = 3(2\lambda - y)$

which is divisible by 3.

$\therefore \quad (x, y) \in R \Rightarrow (y, x) \in R$

Hence, xRy is symmetric.

Transitive $\quad x + 2y = 3\lambda \quad \ldots$(i)

and $\quad y + 2z = 3\mu \quad \ldots$(ii)

On adding Eqs. (i) and (ii), we get

$$x + 3y + 2z = 3\lambda + 3\mu$$

$\Rightarrow \quad x + 2z = 3(\lambda + \mu - y)$

which is divisible by 3.

$\because \quad (x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \quad (x, z) \in R$

Hence, xRy is transitive.

$\because$ R is reflexive, symmetric and transitive.

$\therefore$ R is an equivalence relation.

24. We have,

$A = 5^n - 4n - 1 = (1 + 4)^n - 4n - 1$

$= ({}^nC_0 + {}^nC_1 \times 4 + {}^nC_2 \times 4^2 + \ldots + {}^nC_n \times 4^n) - (4n + 1)$

$= 4^2 ({}^nC_2 + {}^nC_3 \times 4 + \ldots + {}^nC_n \times 4^{n-2})$

$\therefore$ A contains some multiples of 16.

Clearly, B contains all multiples of 16 including 0.

$\therefore \quad A \subseteq B$.

25. Given, $f(x) = (x^2 + 1)^{35}, \forall\ x \in R$.

Since, $f(x)$ is even function.

Hence, the function is not one-one.

And $f(x) > 0, \forall\ x \in R$.

Hence, the function is not onto.

$\therefore$ Given, the function is neither one-one nor onto.

26. Let $\quad x_i = a_1, a_2, a_3, \ldots, a_n$;

and $\quad y_i = \lambda a_1, \lambda a_2, \lambda a_3, \ldots, \lambda a_n$.

$\therefore \quad y_i = \lambda x_i$

$\because \quad \bar{y} = \dfrac{\Sigma y_i}{N}$

$= \dfrac{\Sigma \lambda x_i}{n} \quad [\because y_i = \lambda x_i, N = n]$

$= \lambda \dfrac{\Sigma x_i}{n}$

$= \lambda \bar{x}$

$\therefore \quad \sigma_y = \sqrt{\dfrac{\Sigma(y_i - \bar{y})^2}{N}}$

$= \sqrt{\dfrac{\Sigma(\lambda x_i - \lambda \bar{x})^2}{n}}$

$= \sqrt{\dfrac{\lambda^2\ \Sigma(x_i - \bar{x})^2}{n}}$

$= \sqrt{\lambda^2\ \sigma_x^2} = |\lambda|\ \sigma_x = |\lambda|\ \sigma$

27. Given, $\quad P(A \cap B) = \dfrac{1}{6}$

$P(A \cup B) = \dfrac{31}{45}$

and $\quad P(\bar{B}) = \dfrac{7}{10}$

$\therefore \quad P(B) = 1 - \dfrac{7}{10} = \dfrac{3}{10}$

Now, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \dfrac{31}{45} = P(A) + \dfrac{3}{10} - \dfrac{1}{6}$

$\therefore \quad P(A) = \dfrac{31}{45} + \dfrac{1}{6} - \dfrac{3}{10}$

$\therefore \quad = \dfrac{5}{9}$

Then, $\quad P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\frac{1}{6}}{\frac{5}{9}} = \dfrac{3}{10} > \dfrac{1}{6}$

and $\quad P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{1}{6}}{\frac{3}{10}} = \dfrac{5}{9} > \dfrac{1}{6}$

and, $P(A) \times P(B) = \dfrac{5}{9} \times \dfrac{3}{10} = \dfrac{1}{6} = P(A \cap B)$

Hence, A and B are independent.

28. We have, $\cos 15^\circ \cdot \cos 7\dfrac{1^\circ}{2} \cdot \sin 7\dfrac{1^\circ}{2}$

$= \dfrac{1}{2}\cos 15^\circ \left(2\cos 7\dfrac{1^\circ}{2} \cdot \sin 7\dfrac{1^\circ}{2}\right)$

$= \dfrac{1}{2}\cos 15^\circ \left[\sin 2\left(7\dfrac{1^\circ}{2}\right)\right] \quad \left[\because 2\sin\theta \cdot \cos\theta = \sin 2\theta\right]$

$= \dfrac{1}{2}\cos 15^\circ \sin 15^\circ$

$= \dfrac{1}{2} \times \dfrac{1}{2}(2\sin 15^\circ \cos 15^\circ)$

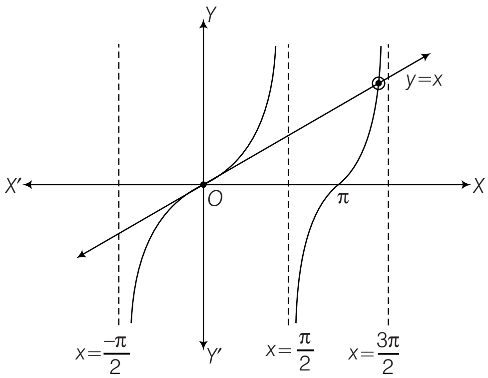
$= \dfrac{1}{4}\sin 30^\circ = \dfrac{1}{4} \times \dfrac{1}{2} = \dfrac{1}{8}$

29. Given equation is

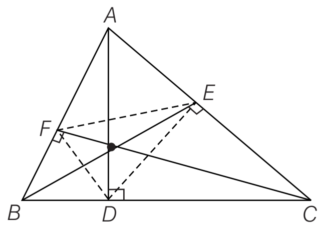
$$\tan x - x = 0$$

$$\Rightarrow \quad \tan x = x$$

Solutions are abscissae of points of intersection of the curves $y = \tan x$ and $y = x$. From the figure, it is clearly visible that solution lies in $\left(\pi, \frac{3\pi}{2}\right)$.



30. Let circumradius of ΔDEF be R'. We know, $\angle FDE = 180° - 2A$ and $FE = R\sin 2A$.



Now, by sine rule in ΔDEF

$$2R' = \frac{EF}{\sin \angle FDE} = \frac{R\sin 2A}{\sin(180° - 2A)}$$

$$\therefore \quad R' = \frac{R}{2}$$

31. Let the four points be $A(-a, -b)$, $B(a, b)$, $C(0, 0)$ and $D(a^2, ab)$.

If A, B and C are collinear.

Then, $$\begin{vmatrix} -a & -b & 1 \\ a & b & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad -a(b-0) + b(a-0) + 1(0) = 0$$

$$\Rightarrow \quad -ab + ab = 0$$

Hence, A, B and C ae collinear.

Again, if B, C and D are collinear.

Then, $$\begin{vmatrix} a & b & 1 \\ 0 & 0 & 1 \\ a^2 & ab & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad a(0 - ab) - b(0 - a^2) + 1(0) = 0$$

$$\Rightarrow \quad -a^2b + a^2b = 0$$

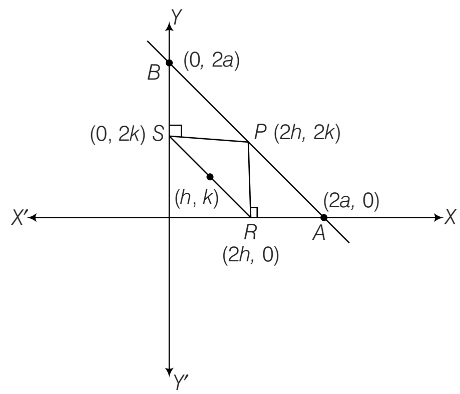
Hence, B, C and D are collinear.

So, the points A, B, C and D are always collinear.

32. Let the equation of line AB is

$$\frac{x}{2a} + \frac{y}{2a} = 1$$

$$\Rightarrow \quad x + y = 2a \qquad \ldots(i)$$



Let the coordinates of the mid-point of RS be (h, k).

So, R and S are $(2h, 0)$ and $(0, 2k)$.

Hence, the mid-point of RS is (h, k).

$\because$ P lies on line AB.

Then, from Eq. (i), we have

$$2h + 2k = 2a$$

$$\Rightarrow \quad h + k = a$$

So, the locus of (h, k) is $x + y = a$.

33. Given three sides of a triangle are

$$x + 8y - 22 = 0 \qquad \ldots(i)$$

$$5x + 2y - 34 = 0 \qquad \ldots(ii)$$

$$2x - 3y + 13 = 0 \qquad \ldots(iii)$$

On solving Eqs. (i) and (ii), we get

$$x_1 = 6, y_1 = 2 \qquad \text{(say)}$$

On solving Eqs. (ii) and (iii), we get

$$x_2 = 4, y_2 = 7 \qquad \text{(say)}$$

On solving Eqs. (i) and (iii), we get

$$x_3 = -2, y_3 = 3 \qquad \text{(say)}$$

If the points of the vertices of the triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of triangle

$$= \frac{1}{2}\begin{vmatrix} 6 & 2 & 1 \\ 4 & 7 & 1 \\ -2 & 3 & 1 \end{vmatrix}$$

$= \frac{1}{2}[6(7-3) - 2(4+2) + 1(12+14)]$

$= \frac{1}{2}(24 - 12 + 26) = \frac{1}{2}(12 + 26)$

$= \frac{1}{2} \times 38$

$= 19$ sq units

34. Let the given points be $A(a, b)$ and $B(-a, -b)$.

The equation of line passing through the points A and B is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - b = \frac{-b-b}{-a-a}(x-a)$$

$$\Rightarrow \quad y - b = \frac{2b}{2a}(x-a)$$

$$\Rightarrow \quad y - b = \frac{b}{a}(x-a)$$

$$\Rightarrow \quad ay - ab = bx - ab$$

$$\Rightarrow \quad bx = ay \qquad \ldots(i)$$

Since, from the given points (a^2, ab) and (a, b) are satisfy the Eq. (i), but (a^2, ab) is the required answer because (a, b) is already given in question.

35. Given equations of straight lines

$$\frac{x}{a} + \frac{y}{b} = K \qquad \ldots(i)$$

and $$\frac{x}{a} - \frac{y}{b} = \frac{1}{K} \qquad \ldots(ii)$$

Let the point of intersection be (α, β).

So, from Eqs. (i) and (ii), we get

$$\frac{\alpha}{a} + \frac{\beta}{b} = K$$

and $$\frac{\alpha}{a} - \frac{\beta}{b} = \frac{1}{K}$$

$$\Rightarrow \quad \left(\frac{\alpha}{a}\right)^2 - \left(\frac{\beta}{b}\right)^2 = 1 \Rightarrow \frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$$

$\therefore$ Locus $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, which is the equation of a hyperbola.

36. Equation of circle is $x^2 + y^2 = 9 \qquad \ldots(i)$

And equation of line is $3x + 4y = 0 \qquad \ldots(ii)$

Equation of line parallel to the line (ii),

$$3x + 4y = k$$

$$\Rightarrow \quad y = \frac{-3x}{4} + \frac{k}{4} \qquad \ldots(iii)$$

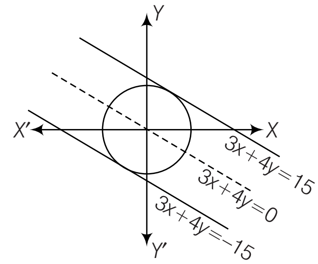
$\because$ Circle touches the line (iii), so by the condition of tangency,

$$c^2 = a^2(1 + m^2)$$

$$\Rightarrow \quad \left(\frac{k}{4}\right)^2 = 9\left[1 + \left(\frac{-3}{4}\right)^2\right]$$

$$\left[\because c = \frac{k}{4}, a = 3, m = \frac{-3}{4}\right]$$

$$\Rightarrow \quad k = \pm 15$$



Hence, $3x + 4y = 15$ touches the circle in the first quadrant.

37. Given equations are

$$x + y = 4 \qquad \ldots(i)$$

and $$x - y = 2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$x = 3 \text{ and } y = 1$$

The line through this point making an angle $\tan^{-1}\frac{3}{4}$ with the X-axis is

$$(y - 1) = \frac{3}{4}(x - 3) \qquad \left[\because m = \frac{3}{4}\right]$$

$$\Rightarrow \quad y = \frac{3x}{4} - \frac{5}{4} = \frac{3x-5}{4} \qquad \ldots(iii)$$

Since, this line intersects the parabola

$y^2 = 4(x-3)$ at points (x_1, y_1) and (x_2, y_2), respectively.

$\therefore$ Putting $y = \frac{3x-5}{4}$ in equation of parabola, we get

$$\left(\frac{3x-5}{4}\right)^2 = 4(x-3)$$

$$\Rightarrow \quad 9x^2 - 94x + 217 = 0$$

$$\Rightarrow \quad x_1 + x_2 = \frac{94}{9} \text{ and } x_1 x_2 = \frac{217}{9}$$

$$\therefore \quad |x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$$

$$= \sqrt{\left(\frac{94}{9}\right)^2 - 4 \times \frac{217}{9}}$$

$$= \frac{32}{9}$$

38. Given equation of ellipse is

$$16x^2 + 25y^2 + 32x - 100y = 284$$

Simplifying the given equation, we have ellipse as

$$\frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

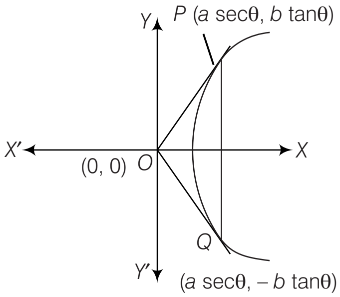
So, the auxiliary equation of circle is

$$(x+1)^2 + (y-2)^2 = 25$$

$$\Rightarrow \quad x^2 + 2x + 1 + y^2 - 4y + 4 = 25$$

$$\therefore \quad x^2 + y^2 + 2x - 4y - 20 = 0$$

39. Given equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and ΔOPQ is an equilateral triangle. PQ is double ordinate of the hyperbola.



Let the coordinates of P and Q be $(a\sec\theta, b\tan\theta)$ and $(a\sec\theta, -b\tan\theta)$, respectively.

In ΔOPQ,

$$OP = PQ$$

$$\Rightarrow \quad a^2\sec^2\theta + b^2\tan^2\theta = (2b\tan\theta)^2$$

$$\Rightarrow \quad a^2\sec^2\theta = 3b^2\tan^2\theta$$

$$\Rightarrow \quad \sin^2\theta = \frac{a^2}{3b^2}$$

Now, $\sin^2\theta < 1$

$$\Rightarrow \quad \frac{a^2}{3b^2} < 1$$

$$\Rightarrow \quad \frac{b^2}{a^2} > \frac{1}{3}$$

$$\Rightarrow \quad 1 + \frac{b^2}{a^2} > \frac{4}{3}$$

$$\therefore \quad e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

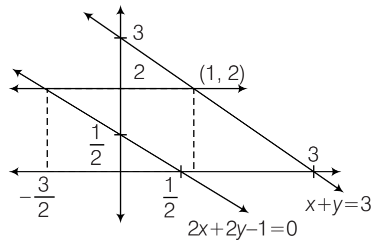
40. Equation of the vertex of conic

$$y^2 - 4y = 4x - 4a$$

$$\Rightarrow \quad y^2 - 4y + 4 = 4x - 4a + 4$$

$$\Rightarrow \quad (y-2)^2 = 4[x - (a-1)]$$

$$\therefore \quad \text{Vertex} = (a-1, 2)$$



From figure,

clearly, $$-\frac{3}{2} < a - 1 < 1$$

$$\therefore \quad -\frac{1}{2} < a < 2$$

41. Equation of line joining the points (1, 1, 1) and (0, 0, 0) is

$$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0} = \lambda \qquad \text{(say)}$$

$$\Rightarrow \quad x = y = z = \lambda$$

So, the point is $(\lambda, \lambda, \lambda)$.

The point intersects the plane $2x + 2y + z = 10$.

$$\therefore \quad 2(\lambda) + 2(\lambda) + \lambda = 10$$

$$\Rightarrow \quad 5\lambda = 10$$

$$\Rightarrow \quad \lambda = 2$$

Hence, the point is (2, 2, 2).

42. Equations of planes are

$$x + y + 2z - 6 = 0 \qquad \ldots\text{(i)}$$

and $$2x - y + z - 9 = 0 \qquad \ldots\text{(ii)}$$

$$\therefore \quad \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{1 \times 2 + 1 \times (-1) + 2 \times 1}{\sqrt{1+1+4}\sqrt{4+1+1}}$$

$$= \frac{2 - 1 + 2}{\sqrt{6} \times \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \quad \theta = \frac{\pi}{3}$$

43. Given,

$$y = (1+x)(1+x^2)(1+x^4) + \ldots + (1+x^{2n})$$

$$\Rightarrow \log y = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \ldots + \log(1+x^{2n})$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \ldots + \frac{2nx^{2n-1}}{1+x^{2n}}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{3x^2}{1+x^3} + \dots + \frac{2nx^{2n-1}}{1+x^{2n}}\right]$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = y\left(\frac{1}{1+0}\right) = 1$$

44. Given that $f(x)$ is an odd differentiable function.

Then, $f(-x) = -f(x)$

$\Rightarrow \quad -f'(-x) = -f'(x)$

$\Rightarrow \quad f'(-x) = f'(x) \quad \dots(i)$

Put $x = 3$ in Eq. (i), we get

$f'(-3) = f'(3)$

$\therefore \quad f'(-3) = 2 \quad [\because f'(3) = 2]$

45. We have, $\lim_{x \to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

$$= \lim_{x \to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})}}$$

$$= \lim_{x \to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1}{(1+\sqrt{x})}}$$

$$= \left(\frac{1+1}{2+1}\right)^{\frac{1}{(1+1)}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}}$$

46. Given, $f(x) = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$

$$= \tan^{-1}\left[\frac{\log e - 2\log x}{\log e + 2\log x}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$$

Let $2\log x = \tan\theta$, we get

$$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] + \tan^{-1}\left[\frac{3+\tan\theta}{1-3\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] + \tan^{-1}[\tan\tan^{-1}(3+\theta)]$$

$$= \frac{\pi}{4} - \theta + \tan^{-1}3 + \theta = \frac{\pi}{4} + \tan^{-1}3$$

$\because \quad f(x) = \text{constant}$

$\therefore \; f''(x) = 0$

47. Let $I = \int \frac{\log\sqrt{x}}{3x}\,dx$

Again, let $\log\sqrt{x} = z \Rightarrow \frac{1}{2x}dx = dz$

$$\therefore \quad I = \int \frac{2z}{3}\,dz = \frac{2}{3}\int z\,dz$$

$$= \frac{2}{3} \cdot \frac{z^2}{2} + C = \frac{1}{3}(\log\sqrt{x})^2 + C$$

48. Let $I = \int 2^x [f'(x) + f(x)\log 2]\,dx$

Consider $g(x) = 2^x f(x)$

$\Rightarrow \quad g'(x) = 2^x f'(x) + 2^x f(x)\log 2$

$\Rightarrow \quad g'(x) = 2^x [f'(x) + f(x)\log 2]$

$\therefore \quad I = \int g'(x)\,dx = g(x) + C = 2^x f(x) + C$

49. Let $I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

$$\Rightarrow \quad I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx = -I$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$\therefore \quad 2I = 0 \;\Rightarrow I = 0$

50. $\because \lim_{n\to\infty}\left[\frac{\sqrt{n+1} + \sqrt{n+2} + \dots + \sqrt{2n-1}}{n^{\frac{3}{2}}}\right]$

$$= \lim_{n\to\infty}\left[\sqrt{1+\frac{1}{n}} + \sqrt{1+\frac{2}{n}} + \dots + \sqrt{1+\frac{n-1}{n}}\right]\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{1+\frac{r}{n}}$$

$$= \int_0^1 \sqrt{1+x}\,dx = \left[\frac{2}{3}(1+x)^{3/2}\right]_0^1 = \frac{2}{3}(2\sqrt{2} - 1)$$

51. We have,

$1^3 + 3^3 + 5^3 + 7^3 + \dots$

Now, $T_n = (2n-1)^3$

$= 8n^3 - 3(2n)^2(1) + 3(2n)(1)^2 - (1)^3$

$= 8n^3 - 12n^2 + 6n - 1$

$\therefore \quad S_n = \Sigma T_n$

$$= \frac{8n^2(n+1)^2}{4} - \frac{12n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} - n$$

$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$

$= n[2n(n+1)^2 - 2(n+1)(2n+1) + 3(n+1) - 1]$

$= n[2n(n^2+1+2n) - 2(2n^2+3n+1) + 3n + 3 - 1]$

$= n[2n^3 + 2n + 4n^2 - 4n^2 - 6n - 2 + 3n + 2]$

$= n[2n^3 - n] = n^2(2n^2 - 1)$

52. Given, α and β are roots of $ax^2 + bx + c = 0$.

$$\therefore \quad \alpha + \beta = \frac{-b}{a}$$

$$\text{and} \quad \alpha\beta = \frac{c}{a}$$

Now, if the roots are α^2 and β^2, then

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} \quad \dots(i)$$

And $\alpha^2\beta^2 = (\alpha\beta)^2 = \left(\frac{c}{a}\right)^2 = \frac{c^2}{a^2}$

The equation whose roots are α^2 and β^2, is

$$x^2 - \left(\frac{b^2}{a^2} - \frac{2c}{a}\right)x + \frac{c^2}{a^2} = 0$$

$\therefore \quad a^2x^2 - (b^2 - 2ac)\,x + c^2 = 0.$

53. Given,

$(2 - \omega)(2 - \omega^2) + 2\,(3 - \omega)(3 - \omega^2) + \dots + (n - 1)(n - \omega)(n - \omega^2)$

Now, $T_n = (n - 1)(n - \omega)(n - \omega^2)$

$= (n - 1)(n^2 - n\omega - n\omega^2 + \omega^3)$

$= (n - 1)(n^2 + n + 1) = n^3 - 1$

$\therefore \quad S_n = \Sigma T_n$

$= \Sigma n^3 - \Sigma 1 = \frac{n^2(n + 1)^2}{4} - n$

54. Given,

$${}^nC_{r-1} = 36 \quad \dots(i)$$

$${}^nC_r = 84 \quad \dots(ii)$$

$${}^nC_{r+1} = 126 \quad \dots(iii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{r}{n - r + 1} = \frac{36}{84}$$

$\Rightarrow \quad 84r = 36n - 36r + 36$

$\Rightarrow \quad 120r = 36n + 36 \quad \dots(iv)$

Also, on dividing Eq. (ii) by Eq. (iii), we get

$$\frac{r + 1}{n - r} = \frac{84}{126}$$

$\Rightarrow \quad 126r + 126 = 84n - 84r$

$\Rightarrow \quad 210r = 84n - 126 \quad \dots(v)$

On solving Eqs. (iv) and (v), we get

$n = 9$ and $r = 3$

$\therefore \quad {}^nC_8 = {}^9C_8 = 9$

55. Number of males = 14

and females = 6

Total = 14 + 6 = 20

Males above 40 yr = 8

and females above 40 yr = 3

Now, P (selected person is female and above 40 yr) $= \frac{3}{6} = \frac{1}{2}$

56. Given equation is

$$x^3 - yx^2 + x - y = 0$$

$\Rightarrow \quad x^2(x - y) + (x - y) = 0$

$\Rightarrow \quad (x^2 + 1)(x - y) = 0$

Now, $\quad x^2 + 1 \neq 0$

and $\quad x - y = 0$

$\therefore \quad x = y$

So, the equation represents a straight line.

57. Let (h, k) be the coordinates of the mid-point of a chord, which subtends a right angle at the origin.

Then, equation of the chord is

$hx + ky - 1 = h^2 + k^2 - 1 \quad$ [using $T = S'$]

$\Rightarrow \quad hx + ky = h^2 + k^2$

The combined equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 1$ and $hx + ky = h^2 + k^2$ is

$$x^2 + y^2 - 1\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

Lines given by the above equation are at right angle.

Therefore, coefficient of x^2 + coefficient of $y^2 = 0$

i.e. $\quad h^2 + k^2 = \frac{1}{2}$

$\therefore \quad x^2 + y^2 = \frac{1}{2}$

58. Let $P(at^2, 2at)$ be a moving point on the parabola $y^2 = 4ax$ and $S(a, 0)$ be its focus. Let $Q(h, k)$ be the mid-point of SP.

Then, $\quad h = \frac{at^2 + a}{2}$

and $\quad k = \frac{2at + 0}{2}$

$\Rightarrow \quad \frac{2h}{a} = t^2 + 1$

and $\quad t = \frac{k}{a}$

$\Rightarrow \quad \frac{2h}{a} = \frac{k^2}{a^2} + 1$

$\Rightarrow \quad k^2 = 2ah - a^2 \quad$ [on eliminating t]

Thus, the locus of (h, k) is $y^2 = 2ax - a^2$

Now, $\quad y^2 = 2ax - a^2$

$\Rightarrow \quad (y - 0)^2 = 2a\left(x - \frac{a}{2}\right)$

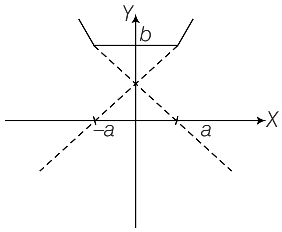
The equation of the directrix of this parabola is

$$x - \frac{a}{2} = -\frac{a}{2}, \text{ i.e. } x = 0.$$

59. Let,

$$I = \int_0^2 x^2 [x]\, dx$$
$$= \int_0^1 x^2 \cdot 0\, dx + \int_1^2 x^2 \times 1\, dx$$
$$= \left[\frac{x^3}{3}\right]_1^2 = \left[\frac{8}{3} - \frac{1}{3}\right] = \frac{7}{3}$$

60. Graph of $f(x)$,



There are two sharp turns. Hence, $f(x)$ cannot be differentiable at two points.

61. Given, $|\mathbf{a} + \mathbf{b}| < |\mathbf{a} - \mathbf{b}|$

Now,

$$|\mathbf{a} + \mathbf{b}|^2 < |\mathbf{a} - \mathbf{b}|^2$$
$$\Rightarrow \quad (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) < (\mathbf{a} - \mathbf{b}) (\mathbf{a} - \mathbf{b})$$
$$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b} < |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b}$$
$$\Rightarrow \quad 4\,\mathbf{a} \cdot \mathbf{b} < 0$$
$$\Rightarrow \quad \mathbf{a} \cdot \mathbf{b} < 0$$
$$\Rightarrow \quad |\mathbf{a}||\mathbf{b}| \cos\theta < 0$$
$$\Rightarrow \quad \cos\theta < 0$$
$$\Rightarrow \quad \theta \in \text{obtuse angle}$$

$\therefore$ $\mathbf{a}$ and $\mathbf{b}$ are inclined at an obtuse angle.

62. Given,

$$y\frac{dy}{dx} + by^2 = a\cos x,\ 0 < x < 1 \qquad \ldots\text{(i)}$$

Let $y^2 = z$

$$\Rightarrow \quad 2y\frac{dy}{dx} = \frac{dz}{dx}$$
$$\Rightarrow \quad y\frac{dy}{dx} = \frac{1}{2}\frac{dz}{dx} \qquad \ldots\text{(ii)}$$
$$\therefore \quad \frac{1}{2}\frac{dz}{dx} + by^2 = a\cos x \qquad \text{[using Eq. (ii)]}$$
$$\Rightarrow \quad \frac{dz}{dx} + 2by^2 = 2a\cos x$$

Now, $\text{IF} = e^{2b\int dx} = e^{2bx}$

$$\therefore \quad z \cdot e^{2bx} = \int 2a\cos x \cdot e^{2bx}\, dx$$
$$\Rightarrow \quad y^2 e^{2bx} = \frac{2a}{4b^2 + 1}(\sin x + 2b\cos x) e^{2bx} + C$$
$$\Rightarrow (4b^2 + 1)\, y^2 = 2a(\sin x + 2b\cos x) + Ce^{-2bx}$$

63. Given that, the ordinate decreases at the same rate at which the abscissa increases, therefore

$$\frac{dy}{dt} = -\frac{dx}{dt} \qquad \ldots\text{(i)}$$

Also, equation of ellipse

$$16x^2 + 9y^2 = 400 \qquad \ldots\text{(ii)}$$

On differentiating w.r.t. t, we get

$$16 \times 2x\frac{dx}{dt} + 9 \times 2y\frac{dy}{dt} = 0$$
$$\Rightarrow \quad 16x\frac{dx}{dt} + 9y\frac{dy}{dt} = 0$$
$$\Rightarrow \quad 9y\frac{dy}{dt} = -16x\frac{dx}{dt}$$
$$\Rightarrow \quad 9y\frac{dy}{dt} = -16x\left(-\frac{dy}{dt}\right) \qquad \text{[using Eq. (i)]}$$
$$\Rightarrow \quad 9y = 16x \ \Rightarrow\ y = \frac{16}{9}x \qquad \ldots\text{(iii)}$$

Using Eq. (iii) in Eq. (ii), we get

$$16x^2 + 9\left(\frac{16}{9}x\right)^2 = 400$$
$$\Rightarrow \quad 16x^2 + \frac{(16)^2}{9}x^2 = 400$$
$$\Rightarrow \quad 16x\left[x + \frac{16}{9}x\right] = 400$$
$$\Rightarrow \quad 16x\left[\frac{25}{9}x\right] = 400 \Rightarrow x^2 = \frac{400 \times 9}{25 \times 16}$$
$$\Rightarrow \quad x^2 = 9 \ \Rightarrow\ x = \pm 3$$

$\therefore$ Required points are $\left(3, \frac{16}{3}\right)$ and $\left(-3, -\frac{16}{3}\right)$.

64. In a dictionary, the words at each stage are arranged in alphabetical order.

In the given problem, we must consider the words beginning with C, C, H, I, N, O in order. So,

Number of words starting with CC = 4!

Number of words starting with CH = 4!

Number of words starting with CI = 4!

Number of words starting with CN = 4!

Next word starting with CO i.e. COCHIN = 1

$\therefore$ Required number of words = $4 \times 4! = 96$

65. We have,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$
$$\therefore \quad A^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 8 & 0 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 2^2 & 0 \\ 2 \cdot 2^2 & 0 & 2^2 \end{bmatrix}$$

Now, $A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ n\cdot 2^n & 0 & 2^n \end{bmatrix}$

But $A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix}$ [given]

$\therefore \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix} = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ n\cdot 2^n & 0 & 2^n \end{bmatrix}$

Hence, $a = 2^n$

and $b = n\,2^n$.

66. Given equation of ellipse is

$$4x^2 + 9y^2 = 1 \quad \text{...(i)}$$

On differentiating, we get

$\Rightarrow \quad 8x + 18yy' = 0$

$\Rightarrow \quad y' = \dfrac{-8x}{18y} = m,$ ...(ii)

Also, $8x = 9y$ [equation of line]

On differentiating, we get

$\Rightarrow \quad 8 = 9y'$

$\Rightarrow \quad y' = \dfrac{8}{9} = m,$ ...(iii)

[$\because$ tangents are parallel to this line]

So, by Eqs. (ii) and (iii), we get

$$\frac{-8x}{18y} = \frac{8}{9}$$

$\Rightarrow \quad -x = 2y$

$\Rightarrow \quad x = -2y$

On substituting $x = -2y$ in Eq. (i), we get

$4(-2y)^2 + 9y^2 = 1$

$\Rightarrow \quad 16y^2 + 9y^2 = 1 \Rightarrow 25y^2 = 1$

$\Rightarrow \quad y = \pm \dfrac{1}{5} \quad \therefore \quad x = \mp \dfrac{2}{5}$

So, required points are $\left(\dfrac{2}{5}, -\dfrac{1}{5}\right)$ and $\left(-\dfrac{2}{5}, \dfrac{1}{5}\right)$.

67. $\displaystyle\int_{-3000}^{3000} \left(\sum_{r'=2014}^{2016} \phi(t - r')\phi(t - 2016) \right) dt$

$= \displaystyle\int_{-3000}^{3000} [\phi(t - 2014)\phi(t - 2016) + \phi(t - 2015)\phi(t - 2016) + \phi(t - 2016)\phi(t - 2016)]dt$

$= \displaystyle\int_{-3000}^{3000} \phi(t - 2016)[\phi(t - 2014) + \phi(t - 2015) + \phi(t - 2016)]dt$

$= \displaystyle\int_{-3000}^{2016} 0\,dt + \int_{2016}^{2017} 1\cdot(0 + 0 + 1)dt + \int_{2017}^{3000} 0\,dt$

$= 0 + 1 + 0 = 1$, which is a real number.

68. Given,

$$x^2 + y^2 - 10x + 21 = 0$$

$\Rightarrow \quad x^2 - 10x + y^2 + 21 = 0$

Now, for real roots of x, $D \ge 0$

$100 - 4(y^2 + 21) \ge 0$

$\Rightarrow \quad y^2 \le 4 \Rightarrow -2 \le y \le 2$

Also,

$$y^2 = -x^2 + 10x - 21$$

Now, for real roots of y,

$-4(x^2 - 10x + 21) \ge 0$

$-x^2 + 10x - 21 \ge 0$

$\Rightarrow \quad (x - 7)(x - 3) \le 0 \quad \therefore \quad 3 \le x \le 7$

69. Given, $z = \sin\theta - i\cos\theta$

$$= \cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right) = e^{i\left(\theta - \frac{\pi}{2}\right)}$$

Now,

$z^n = e^{i\left(n\theta - \frac{n\pi}{2}\right)} = \cos\left(n\theta - \dfrac{n\pi}{2}\right) + i\sin\left(n\theta - \dfrac{n\pi}{2}\right)$

$\Rightarrow \dfrac{1}{z^n} = e^{i\left(\frac{n\pi}{2} - n\theta\right)} = \cos\left(n\theta - \dfrac{n\pi}{2}\right) - i\sin\left(n\theta - \dfrac{n\pi}{2}\right)$

$\therefore \quad z^n + \dfrac{1}{z^n} = 2\cos\left(n\theta - \dfrac{n\pi}{2}\right) = 2\cos\left(\dfrac{n\pi}{2} - n\theta\right)$

and $z^n - \dfrac{1}{z^n} = 2i\sin\left(n\theta - \dfrac{n\pi}{2}\right)$.

70. Given, $f[f(x)] = x$

Now, $f^{-1}(x) = f(x)$

i.e. $f(x)$ is bijective.

Hence, $f(x)$ has to be one-one and onto.

71. We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$ ...(i)

Given, $\dfrac{3}{4} \le P(A \cup B) \le 1$

and $\dfrac{1}{8} \le P(A \cap B) \le \dfrac{3}{8}$

So, $\dfrac{7}{8} \le P(A \cup B) + P(A \cap B) \le \dfrac{11}{8}$

$\therefore \quad P(A) + P(B) \ge \dfrac{7}{8}$ and $P(A) + P(B) \le \dfrac{11}{8}$

72. $\because$ a, b and c are respectively the AM, GM and HM of first and $(2n-1)$th terms.

and also, we know

AM $\ge$ GM $\ge$ HM and AM $\cdot$ HM = GM^2

$\therefore \quad a \ge b \ge c$ and $a\cdot c = b^2$

73. Let (h, k) be the point on line $x + y + 1 = 0$.

So, $h + k + 1 = 0$

$$h = -1 - k \quad \ldots(i)$$

Given, distance between (h, k) and line $3x + 4y + 2 = 0$ is $\frac{1}{5}$.

$\therefore$ $$\frac{1}{5} = \left|\frac{(3h) + (4k) + 2}{\sqrt{3^2 + 4^2}}\right|$$

$\Rightarrow$ $$\frac{1}{5} = \left|\frac{3(-1-k) + 4k + 2}{\sqrt{25}}\right|$$

$\Rightarrow$ $$\frac{1}{5} = \pm\left(\frac{-3 - 3k + 4k + 2}{5}\right)$$

$\Rightarrow$ $$\frac{1}{5} = \pm\left(\frac{k-1}{5}\right)$$

$\Rightarrow$ $\pm(k - 1) = 1$

$\Rightarrow$ $k = 0$ and 2

$\therefore$ $h = -1, -.3$

Hence, the required points are $(-1, 0)$ and $(-3, 2)$.

74. Given, equation of the parabola

$$x^2 = ay$$

i.e. $$y = \frac{x^2}{a} \quad \ldots(i)$$

and the equation of line

$$y - 2x = 1$$

i.e. $$y = 2x + 1 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$\Rightarrow$ $$\frac{x^2}{a} = 2x + 1$$

$\Rightarrow$ $$x^2 = 2ax + a$$

$\Rightarrow$ $$x^2 - 2ax - a = 0 \quad \ldots(iii)$$

$\therefore$ $$x = \frac{2a \pm \sqrt{4a^2 + 4a}}{2}$$

$$= \frac{2(a \pm \sqrt{a^2 + a})}{2} = a \pm \sqrt{a^2 + a}$$

Points of intersection of the parabola and the line are

$$(a + \sqrt{a^2 + a}, 2(a + \sqrt{a^2 + a}) + 1)$$

and $(a - \sqrt{a^2 + a}, 2(a - \sqrt{a^2 + a} + 1)$

$\because$ Distance between the points $= \sqrt{40}$

$\therefore$ $$\sqrt{40} = \sqrt{(2\sqrt{a^2 + a})^2 + (4\sqrt{a^2 + a})^2}$$

$\Rightarrow$ $$40 = 4(a^2 + a) + 16(a^2 + a)$$

$\Rightarrow$ $$2 = a^2 + a \Rightarrow a = 1, -2$$

75. Given,

$$f'(x) = (x - 1)^2 (4 - x)$$

The sign scheme of $f'(x)$

$$\underset{\quad}{+} \;\; \underset{1}{|} \;\; + \;\; \underset{4}{|} \;\; -$$

Clearly, $f(x)$ is increasing, for $x \in (-\infty, 4)$ and decreasing, for $n \in (4, \infty)$.

Since, $f'(x) = 0$ at $x = 4$.

So, $x = 4$ is a critical point.