Trigonometric Functions

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. A wheel rotates making 20 revolutions per second. If the radius of the wheel is 35 cm, what linear distance does a point of its rim travel in three minutes? (Take $\pi = 22/7$)

(a) 7.92 km (b) 7.70 km
(c) 7.80 km (d) 7.85 km

2. The angles of a triangle are in A.P. and the ratio of angle in degrees of the least to the angle in radians of the greatest is $60 : \pi$, find the angles in degrees.

(a) 30°, 60°, 90° (b) 40°, 60°, 90°
(c) 30°, 30°, 120° (d) 20°, 130°, 30°

3. The large hand of a clock is 42 cm long. How much distance does its extremity move in 20 minutes?

(a) 88 cm (b) 80 cm (c) 75 cm (d) 77 cm

4. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (Use $\pi = \frac{22}{7}$).

(a) 37.5 cm (b) 32.8 cm
(c) 35.7 cm (d) 34.5 cm

5. Find the angle in radian through which a pendulum swings it its length is 75 cm and tip describes an arc of length 21 cm.

(a) 7/25 (b) 6/25 (c) 8/25 (d) 3/25

6. The circular measures of two angles of a triangle are $\frac{1}{2}$ and $\frac{1}{3}$, find the third angle in English system.

(a) 130°15′20″ (b) 132°15′20″
(c) 132°16′22″ (d) 122°16′44″

7. Convert 6 radians into degree measure.

(a) 343°38′11″ (b) 348°33′11″
(c) 433°38′11″ (d) 343°37′12″

8. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, then $x^2 + y^2$ is equal to

(a) 2 (b) 0 (c) 3 (d) 1

9. The value of $\sec^2\theta + \text{cosec}^2\theta$ is equal to

(a) $\tan^2\theta + \cot^2\theta$ (b) $\sec^2\theta \, \text{cosec}^2\theta$
(c) $\sec\theta \, \text{cosec}\theta$ (d) $\sin^2\theta \cos^2\theta$

10. If $x \sin\frac{\pi}{4} \cos^2\frac{\pi}{3} = \frac{\tan^2(\pi/3)\,\text{cosec}\,(\pi/6)}{\sec(\pi/4)\cot^2(\pi/6)}$, then $x =$

(a) 2 (b) 4 (c) 8 (d) 16

11. If $\tan\theta = 3$ and θ lies in III quadrant, then find the value of $\sin\theta$.

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{-5}{\sqrt{10}}$

12. Find x from the equation
$\text{cosec}\,(90° + \theta) + x \cos\theta \cot(90° + \theta) = \sin(90° + \theta)$.

(a) $\cot\theta$ (b) $\tan\theta$ (c) $-\tan\theta$ (d) $-\cot\theta$

13. Find the value of $\tan(\alpha + \beta)$, given that $\cot\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = \frac{-5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$.

(a) 1/11 (b) 2/11 (c) 5/11 (d) 3/11

14. Find the value of 2cos45° sin15°.

(a) $\frac{\sqrt{3}+1}{2}$ (b) 1/2 (c) $\frac{\sqrt{3}-1}{2}$ (d) $\frac{\sqrt{3}}{2}$

15. Find the value of 2sin15°·cos75°.

(a) $\frac{(2+\sqrt{3})}{2}$ (b) 1
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2-\sqrt{3}}{2}$

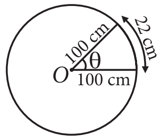
16. The value of $4\sin\alpha \sin\left(\alpha + \frac{\pi}{3}\right)\sin\left(\alpha + \frac{2\pi}{3}\right) =$

(a) $\sin 3\alpha$ (b) $\sin 2\alpha$ (c) $\sin\alpha$ (d) $\sin^2\alpha$

17. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is

(a) 3 (b) 2 (c) –1 (d) 0

18. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm as shown in figure. [Use $\pi = \frac{22}{7}$]



(a) 12°30′ (b) 12°36′ (c) 11°36′ (d) 11°12′

19. Find the radian measure of 520°.

(a) $13\pi/9$ (b) $26\pi/9$ (c) $17\pi/9$ (d) $6\pi/9$

20. If $\frac{\sin A}{\sin B} = m$ and $\frac{\cos A}{\cos B} = n$, then find the value of tan B; $n^2 < 1 < m^2$.

(a) n^2 (b) $\pm\sqrt{\frac{1-n^2}{m^2-1}}$

(c) $n^2/(m^2 - 1)$ (d) m^2

21. If 3 sinθ + 5 cosθ = 4, then find the value of 5 sinθ – 3 cosθ.

(a) $\pm 3\sqrt{2}$ (b) $\pm 5\sqrt{2}$

(c) $\pm\sqrt{2}$ (d) $\pm 7\sqrt{2}$

22. If $\tan\theta = \frac{-4}{3}$, then sinθ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) $\frac{-3}{4}$ or $\frac{3}{4}$

23. If $A + B = \frac{5\pi}{4}$, then value of $(1 + \tan A)(1 + \tan B)$ equals

(a) 1 (b) 2 (c) –2 (d) –1

24. The value of $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ equals

(a) 0 (b) –1

(c) 1 (d) None of these

25. The value of $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$

(a) 11 (b) 12 (c) 13 (d) 14

26. Express 50° 37′ 30″ in radian.

(a) $7\pi/32$ (b) $5\pi/32$ (c) $9\pi/32$ (d) $\pi/32$

27. Find the degree measure corresponding to $\frac{\pi}{32}$ rad.

(a) 5°37′30″ (b) 5°30′20″

(c) 4°30′30″ (d) 4°20′20″

28. Find the degree measure of $\left(\frac{\pi}{9}\right)^c$ and $\left(-\frac{\pi}{3}\right)^c$ respectively.

(a) 20°, –60° (b) 30°, 60°

(c) 20°, 40° (d) 40°, 50°

29. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$ is

(a) 1/6 (b) 1/7 (c) 1/5 (d) 1/2

30. If $\cot x = \frac{4}{3}$ and x lies in third quadrant, then find the value of sec x.

(a) 1/4 (b) 7/4 (c) 2/4 (d) –5/4

31. If $\cos x = \frac{4}{5}$, where $x \in [0, \pi/2]$, then the value of $\cos\left(\frac{x}{2}\right)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{3}{\sqrt{10}}$ (d) $-\frac{2}{5}$

32. The value of $\sqrt{3}$ cosec 20° – sec 20° is

(a) 1 (b) 7 (c) 2 (d) 4

33. Evaluate $2\cos\left(22\frac{1}{2}\right)^\circ \cdot \cos\left(67\frac{1}{2}\right)^\circ$.

(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

34. If $\sin A = \frac{3}{5}$ and A is in first quadrant, then the values of sin2A, cos2A and tan2A are

(a) 24/25, 7/25, 24/7 (b) 1/25, 7/25, 1/7

(c) 24/25, 1/25, 24/7 (d) 1/25, 24/25, 1/24

35. Find the value of

$\sin(40° + \theta)\cos(10° + \theta) - \cos(40° + \theta)\sin(10° + \theta)$.

(a) 1/2 (b) 1/4 (c) 1 (d) 0

36. If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to

(a) 110 (b) 191 (c) 80 (d) 194

37. If $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$, then $\cos A + \sqrt{6}\sin A$ is equal to

(a) $\sqrt{6}\sin A$ (b) $\sqrt{7}\sin A$

(c) $\sqrt{6}\cos A$ (d) $\sqrt{7}\cos A$

38. Find the value of sin20° sin40° sin80°.

(a) $\frac{\sqrt{3}}{8}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\sqrt{3}$

39. The value of $4\sin\alpha \sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha+\frac{2\pi}{3}\right) =$

(a) $\sin 3\alpha$ (b) $\sin 2\alpha$

(c) $\sin\alpha$ (d) $\sin^2\alpha$

40. If α and β lie between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$, given that $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$.

(a) 50/33 (b) 56/33 (c) 52/33 (d) 48/33

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45.

Nitish is playing with Pinwheel toy which he bought from a village fair. He noticed that the pinwheel toy revolves as fast as he blows it.

Consider the Pinwheel toy makes 360 revolutions per minute.

41. The number of revolutions made by Pinwheel toy in 120 seconds is

(a) 720 (b) 120 (c) 240 (d) 360

42. The number of revolutions made by Pinwheel toy in 1 sec is

(a) 7 (b) 9 (c) 6 (d) 8

43. Angle made by Pinwheel toy (in degree) in 6 revolutions is

(a) 2630° (b) 2160° (c) 2360° (d) 2610°

44. The value of 1° in radians is equal to

(a) $\frac{\pi}{170}$ (b) $\frac{\pi}{120}$ (c) $\frac{\pi}{180}$ (d) $\frac{\pi}{60}$

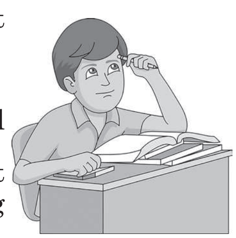
45. Angle made by Pinwheel toy (in radians) in 6 revolutions is

(a) 12π (b) 14π (c) 16π (d) 10π

Case II : Read the following passage and answer the questions from 46 to 50.

Sudhir who is a student of class XI got a Maths assignment from his class teacher.

He did all the questions except a few. If the value of $\sin x = \frac{3}{5}$ and $\cos y = -\frac{12}{13}$, where x and y both lie in second quadrant then help Sudhir in solving these questions.



46. What will be the value of $\cos x$?

(a) $\frac{4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{-4}{5}$ (d) $\frac{3}{5}$

47. What will be the value of $\sin y$?

(a) $\frac{5}{12}$ (b) $\frac{-12}{13}$ (c) $\frac{-5}{13}$ (d) $\frac{5}{13}$

48. Which of the following options is correct?

(a) $\sin(x-y) = \sin x \cos y + \cos x \sin y$

(b) $\sin(x+y) = \cos x \sin y - \sin x \cos y$

(c) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

(d) $\sin(x-y) = \sin x \sin y - \cos x \cos y$

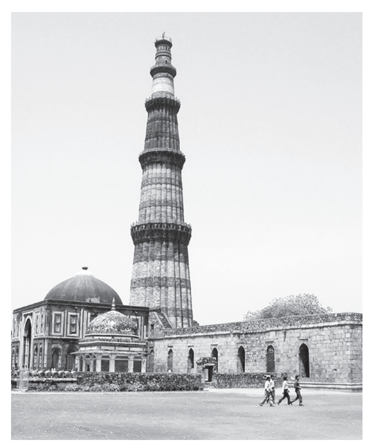
49. The value of $\sin(x+y)$ equals

(a) $\frac{-56}{65}$ (b) $\frac{56}{65}$ (c) $\frac{55}{67}$ (d) $\frac{-55}{67}$

50. Find the value of sin 75°.

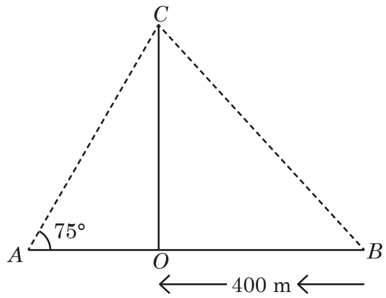
(a) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ (c) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1+\sqrt{3}}{2}$

Case III : Read the following passage and answer the questions from 51 to 55.



Arvind and Murli came to Delhi from Mumbai for an official work. They decided to explore one of the tallest minaret of world *i.e.*, Qutab Minar.

The angle of elevation of Arvind's eyes from a point A to the top of Minaret is 75°. Another point B which is 400 metres far from the Qutab Minar. Given that height of the Minaret is 73 metres.



51 The angle of elevation (in degrees) from point B to the top of the Minaret is

(a) $\angle B = \tan^{-1}\left(\frac{400}{73}\right)$ (b) $\angle B = \sin^{-1}\left(\frac{400}{73}\right)$

(c) $\angle C = \cos^{-1}\left(\frac{400}{73}\right)$ (d) $\angle B = \tan^{-1}\left(\frac{73}{400}\right)$

52. What is the distance of a point A from the foot of Minaret?

(a) $\frac{146}{(\sqrt{3}+1)^2}$ metres (b) $\frac{73}{(\sqrt{3}+1)^2}$ metres

(c) $\frac{73}{(\sqrt{2}+1)^2}$ metres (d) $\frac{73(\sqrt{3}-1)^2}{2}$ metres

53. How long a wire is needed to tied between the top of Minaret and the point A?

(a) $\frac{73}{\sqrt{3}+1}$ metres (b) $\frac{146\sqrt{2}}{(\sqrt{3}+1)}$ metres

(c) $\frac{73\sqrt{2}}{(\sqrt{3}+1)}$ metres (d) $73(2-\sqrt{3})$ metres

54. What is the distance between point A to B?

(a) $\frac{173}{(\sqrt{3}+1)}$ metres (b) $\frac{73\sqrt{2}}{(\sqrt{3}+1)}$ metres

(c) $400-\frac{146}{(\sqrt{3}+1)^2}$ metres

(d) $\frac{800+73(\sqrt{3}-1)^2}{2}$ metres

55. What is the value of angle C?

(a) $105°-\tan^{-1}\left(\frac{73}{400}\right)$ (b) $180°-\tan^{-1}\left(\frac{73}{400}\right)$

(c) $105°+\tan^{-1}\left(\frac{73}{400}\right)$ (d) None of these

Assertion & Reasoning Based MCQs

Directions (Q.-56 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.
(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.
(c) Assertion is correct statement but Reason is wrong statement.
(d) Assertion is wrong statement but Reason is correct statement.

56. Let $\sec\theta + \tan\theta = m$, where $0 < m < 1$.

Assertion : $\sec\theta = \frac{m^2+1}{2m}$ and $\sin\theta = \frac{m^2-1}{m^2+1}$

Reason : θ lies in the third quadrant.

57. Let α be a real number lying between 0 and $\frac{\pi}{2}$ and n be a positive integer.

Assertion : $\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + \ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$.

Reason : $\cot\alpha - \tan\alpha = 2\cot 2\alpha$.

58. Assertion : The value of $\sin(-690°)\cos(-300°) + \cos(-750°)\sin(-240°) = 1$

Reason : The values of sin and cos is negative in third and fourth quadrant respectively.

59. If $A + B + C = 180°$, then

Assertion : $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Reason : $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

60. Assertion : The value of $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$, when θ lies between $(0, 2\pi)$ and $\sin^2\theta = \frac{3}{4}$.

Reason : $\sin\theta$ is positive in the first and second quadrant.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Find the radian measure of 625°.

2. Find the degree measure of $\frac{5\pi}{12}$ radians.

3. Express $-47° 30'$ in radian measure.

4. Find the value of $\cos 240° + \sin\frac{\pi}{3}$.

5. Find the value of $\cos(-1710°)$.

6. What is the value of $\sin\frac{31\pi}{3}$?

7. Convert 4 radians into degree measure.

8. Write $\frac{13\pi}{4}$ radians in the degrees.

9. Find the value of $\tan\left(\frac{31\pi}{3}\right)$.

10. Find the value of $\sin\left(-\frac{11\pi}{3}\right)$.

Short Answer Type Questions (SA-I)

11. If the angles of a triangle are in the ratio 3:4:5, then find the smallest angle in degree and the greatest angle in radians.

12. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, find the ratio of their radii.

13. If $5 \sin x = 3$, x lies in 1st quadrant, then find the value of $\frac{\sec x - \tan x}{\sec x + \tan x}$.

14. If $\sin x = \frac{3}{5}, \frac{\pi}{2} < x < \pi$, then find the value of $\cos x$, $\tan x$, $\sec x$ and $\cot x$.

15. Prove that $\cos^2(45° + x) - \sin^2(45° - x)$ is independent of x.

16. Write the value of $\sqrt{2+\sqrt{2+2\cos\theta}}$ in the simplest form.

17. Express the following as a sum or difference: $2\sin\frac{5\theta}{2}\cos\frac{7\theta}{2}$

18. Find the value of $\cos 55° + \cos 125° + \cos 300°$.

19. Find the value of $\sin(n + 1)x \cdot \sin(n + 2)x + \cos(n + 1)x \cdot \cos(n + 2)x$.

20. Show that : $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Short Answer Type Questions (SA-II)

21. Find the distance from the eye at which a coin of diameter 2 cm should be held so as just to conceal the full moon whose angular diameter is 31′.

22. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 metres when it has traced out 72° at the centre, find the length of the rope.

23. Prove that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$.

24. Prove that $16 \sin^5\theta - 20 \sin^3\theta + 5 \sin \theta = \sin 5\theta$

25. Prove that :

$$\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}.$$

26. Find the value of $\sin\frac{\pi}{18} + \sin\frac{\pi}{9} + \sin\frac{2\pi}{9} + \sin\frac{5\pi}{18}$.

27. Prove that : $\frac{1+\sin x - \cos x}{1 + \sin x + \cos x} = \tan\frac{x}{2}$.

28. Find the value of $m \sin x + n \cos x$, if $\tan\frac{x}{2} = \frac{m}{n}$.

29. Prove that :

$$\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}.$$

30. Prove that :

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}.$$

31. Prove that :

$$\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$$

32. Prove that :

$$\frac{\sin A - \sin 3A + \sin 5A - \sin 7A}{\cos A - \cos 3A - \cos 5A + \cos 7A} = \cot 2A$$

33. Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

34. Prove that :

$(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

35. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$.

Long Answer Type Questions (LA)

36. If A, B, C, D be the angles of a cyclic quadrilateral, taken in order, prove that

(i) $\cos A + \cos B + \cos C + \cos D = 0$

(ii) $\cos(180° + A) + \cos(180° + B) + \cos(180° + C) - \sin(90° + D) = 0$.

37. Prove that :

$$\sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right) = -\frac{3}{4}\sin 3x$$

38. Prove that : $\sin x + \sin y + \sin z - \sin (x + y + z)$

$$= 4\sin\frac{x+y}{2}\sin\frac{y+z}{2}\sin\frac{x+z}{2}$$

39. If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$

prove that $xy + yz + zx = 0$.

40. Find the value of $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(a) :** Radius of the wheel = 35 cm

$\therefore$ Circumference of the wheel = $2\pi \times 35$ cm = 220 cm.

Hence, the linear distance travelled by a point of the rim in one revolution = 220 cm.

Number of revolutions made by the wheel in 3 minutes = $20 \times 3 \times 60 = 3600$.

$\therefore$ The linear distance travelled by a point of the rim in 3 minutes = 220×3600 = 792000 cm = 7.92 km.

2. **(a) :** Let the angles of the triangle be

$(a - d)°, a°, (a + d)°$, where $d > 0$ …(i)

then $(a - d) + a + (a + d) = 180 \Rightarrow a = 60$

$\therefore$ From (i), the angles are $(60 - d)°, 60°, (60 + d)°$

Now, the least angle = $(60 - d)°$

and the greatest angle = $(60 + d)°$

$= (60 + d) \times \frac{\pi}{180}$ radian ($\because 180° = \pi$ radian)

By the given condition, we have

$$\frac{60-d}{\frac{\pi}{180}(60+d)} = \frac{60}{\pi} \Rightarrow \frac{180(60-d)}{(60+d)} = 60$$

$\Rightarrow 180 - 3d = 60 + d \Rightarrow 4d = 120 \Rightarrow d = 30$

$\therefore$ From (i), the angles are $(60 - 30)°, 60°, (60 + 30)°$ *i.e.*, 30°, 60°, 90°.

3. **(a) :** The large hand of the clock makes a complete revolution in 60 minutes.

$\therefore$ Angle traced out by the large hand in 20 minutes (of time) = $\frac{360° \times 20}{60}$ = 120° = $\frac{120\,\pi}{180}$ radian = $\frac{2\pi}{3}$ radian

Hence, the distance moved by the extremity of the large hand = (42) × $\frac{2\pi}{3}$ = 88 cm. $(\because\ l = r\theta)$

4. **(c) :** Here l = 37.4 cm and θ = 60° = $\frac{60\pi}{180}$ radian = $\frac{\pi}{3}$

Hence, by $r = \frac{l}{\theta}$, we have

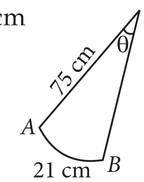
$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22}$ = 35.7 cm

5. **(a) :** Given, length of pendulum = 75 cm

Radius (r) = length of pendulum = 75 cm

Length of arc (l) = 21 cm

Now, $\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$ radian



6. **(c) :** We know that the sum of three angles of a triangle is 180°, *i.e.*, π radian

$\therefore$ The third angle = $\left(\pi - \frac{1}{2} - \frac{1}{3}\right)$ radian

$\simeq \left(\frac{22}{7} - \frac{1}{2} - \frac{1}{3}\right)^c = \left(\frac{97}{42} \times \frac{180}{\pi}\right)^\circ$ $(\because\ \pi^c = 180°)$

$= \left(\frac{97 \times 30}{22}\right)^\circ = \frac{1455}{11}$ degree

$= \left(132\frac{3}{11}\right)^\circ = 132° + \left(\frac{3 \times 60}{11}\right)'$ $(\because\ 1° = 60')$

$= 132° + \left(16\frac{4}{11}\right)' = 132° + 16' + \left(\frac{4}{11} \times 60\right)''$ $(\because\ 1' = 60'')$

= 132° 16′ 22″.

7. **(a) :** We know that π radian = 180°.

Hence, 6 radians = $\frac{180}{\pi}$ × 6 degree = $\frac{1080 \times 7}{22}$ degree

= $343\frac{7}{11}$ degree = 343° + $\frac{7 \times 60}{11}$ minute [as 1° = 60′]

= 343° + 38′ + $\frac{2}{11}$ × 60 seconds [as 1′ = 60″]

= 343° + 38′ + 10.9″ = 343°38′ 11″ approximately.

Hence, 6 radians = 343°38′ 11″ approximately.

8. **(d) :** $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ...(i)

$x\sin\theta = y\cos\theta \Rightarrow y = x\frac{\sin\theta}{\cos\theta}$ ...(ii)

Putting in (i), $x\sin^3\theta + x\frac{\sin\theta}{\cos\theta}\cdot\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow$ $x\sin^3\theta + x\sin\theta\cos^2\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

Putting the value of x in (ii), we get $y = \sin\theta$

$\Rightarrow$ $x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

9. **(b) :** $\sec^2\theta + \text{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{1}{\sin^2\theta\cos^2\theta} = \text{cosec}^2\theta\sec^2\theta$

10. **(c) :** $x\sin 45°\cos^2 60° = \frac{\tan^2 60°\,\text{cosec}\,30°}{\sec 45°\cot^2 30°}$

$\Rightarrow$ $x\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{4} = \frac{3\cdot 2}{\sqrt{2}\cdot 3}$ $\Rightarrow$ x = 8.

11. **(c) :** Given, $\tan\theta = \frac{3}{1}$ and θ lies in III quadrant.

$\sec^2\theta = 1 + \tan^2\theta = 10$ $\Rightarrow$ $\sec\theta = \pm\sqrt{10}$

Since, θ lies in III quadrant, so $\sec\theta = -\sqrt{10}$

$\Rightarrow$ $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\sqrt{10}}$

Now, $\sin^2\theta = 1 - \cos^2\theta = \frac{9}{10}$ $\Rightarrow \sin\theta = \pm\sqrt{\frac{9}{10}}$

Since, θ lies in III quadrant so $\sin\theta = -\sqrt{\frac{9}{10}} = \frac{-3}{\sqrt{10}}$

12. **(b) :** The given equation is

$\text{cosec}(90° + \theta) + x\cos\theta\cot(90° + \theta) = \sin(90° + \theta)$

$\Rightarrow$ $\sec\theta + x\cos\theta\,(-\tan\theta) = \cos\theta$

$\Rightarrow$ $\sec\theta - x\sin\theta = \cos\theta$

$\Rightarrow$ $x\sin\theta = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$

$\Rightarrow$ $x\sin\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$ $\Rightarrow$ $x = \tan\theta$.

13. **(b) :** Given, $\cot\alpha = \frac{1}{2}$ $\Rightarrow$ $\tan\alpha = 2$ and $\sec\beta = \frac{-5}{3}$

Then, $\tan\beta = \sqrt{\sec^2\beta - 1}$ $\Rightarrow$ $\tan\beta = \pm\frac{4}{3}$

But, $\tan\beta = \frac{-4}{3}$ $(\because\ \tan\beta$ is –ve in II quadrant)

$\therefore$ $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = \frac{2 + \left(-\frac{4}{3}\right)}{1 - (2)\left(\frac{-4}{3}\right)} = \frac{2}{11}$

14. **(c) :** We have, 2cos45° sin15°

= sin(45° + 15°) – sin(45° – 15°)

= sin60° – sin30° = $\frac{\sqrt{3} - 1}{2}$

15. **(d) :** We have,

2sin15° cos75° = sin(15° + 75°) + sin(15° – 75°)

= sin90° – sin60° = $\frac{2 - \sqrt{3}}{2}$

16. **(a) :** We have, $4\sin\alpha\sin\left(\alpha + \frac{\pi}{3}\right)\sin\left(\alpha + \frac{2\pi}{3}\right)$

$= 2\sin\alpha\left\{2\sin\left(\alpha + \frac{2\pi}{3}\right)\sin\left(\alpha + \frac{\pi}{3}\right)\right\}$

$= 2\sin\alpha[2\sin(\alpha + 120°)\sin(\alpha + 60°)]$
$= 2\sin\alpha[\cos(\alpha + 120° - \alpha - 60°) - \cos(\alpha + 120° + \alpha + 60°)]$
$= 2\sin\alpha[\cos60° - \cos(180° + 2\alpha)]$
$= 2\sin\alpha\cdot\frac{1}{2} - 2\sin\alpha(-\cos2\alpha)$
$= \sin\alpha + 2\cos2\alpha\sin\alpha = \sin\alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$
$= \sin\alpha + \sin3\alpha - \sin\alpha = \sin3\alpha$

17. (d) : $\sin\theta = 3\sin(\theta + 2\alpha)$
$\Rightarrow \sin(\theta + \alpha - \alpha) = 3\sin(\theta + \alpha + \alpha)$
$\Rightarrow \sin(\theta + \alpha)\cos\alpha - \cos(\theta + \alpha)\sin\alpha$
$= 3\sin(\theta + \alpha)\cos\alpha + 3\cos(\theta + \alpha)\sin\alpha$
$\Rightarrow -2\sin(\theta + \alpha)\cos\alpha = 4\cos(\theta + \alpha)\sin\alpha$
$\Rightarrow \frac{-\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = \frac{2\sin\alpha}{\cos\alpha} \Rightarrow \tan(\theta + \alpha) + 2\tan\alpha = 0$

18. (b) : Given radius, $r = 100$ cm and arc length, $l = 22$ cm
We know that, $l = r\theta$
$\Rightarrow \theta = \frac{l}{r} = \frac{\text{Arc length}}{\text{Radius}} = \frac{22}{100} = 0.22$ radian
$= 0.22 \times \frac{180}{\pi}$ degree $= 0.22 \times \frac{180\times7}{22}$
$= \frac{126°}{10} = \left(12\frac{6}{10}\right)^\circ = 12° + \frac{6}{10}\times 60'$ $[\because 1° = 60']$
$= 12° + 36' = 12°\ 36'$

19. (b) : Required radian measure $= \frac{\pi}{180} \times$ Degree measure
$= \frac{\pi}{180}\times 520 = \frac{26\pi}{9}$

20. (b) : Given, $\frac{\sin A}{\sin B} = m \Rightarrow \sin A = m\sin B$ ...(i)
and $\frac{\cos A}{\cos B} = n \Rightarrow \cos A = n\cos B$ ...(ii)
Squaring (i) and (ii) and then adding, we get
$1 = m^2\sin^2 B + n^2\cos^2 B$
$\Rightarrow \frac{1}{\cos^2 B} = m^2\frac{\sin^2 B}{\cos^2 B} + n^2$ (Dividing by $\cos^2 B$)
$\Rightarrow \sec^2 B = m^2\tan^2 B + n^2$
$\Rightarrow 1 + \tan^2 B = m^2\tan^2 B + n^2 \Rightarrow \tan^2 B = \frac{1-n^2}{m^2-1}$
$\Rightarrow \tan B = \pm\sqrt{\frac{1-n^2}{m^2-1}}$.

21. (a) : Let $x = 5\sin\theta - 3\cos\theta$, then
$x^2 = (5\sin\theta - 3\cos\theta)^2 = 25\sin^2\theta + 9\cos^2\theta - 30\sin\theta\cos\theta$
$= 25(1 - \cos^2\theta) + 9(1 - \sin^2\theta) - 30\sin\theta\cos\theta$
$= 25 + 9 - (25\cos^2\theta + 9\sin^2\theta + 30\sin\theta\cos\theta)$
$= 34 - (3\sin\theta + 5\cos\theta)^2 = 34 - 4^2 = 18$
$\Rightarrow x = \pm\sqrt{18} = \pm3\sqrt{2}$.

22. (b) : Since $\tan\theta = -\frac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant. Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

23. (b) : $\tan(A + B) = \tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$
$\therefore \tan A + \tan B = 1 - \tan A\tan B$
$\Rightarrow \tan A + \tan B + \tan A\tan B = 1$
$\Rightarrow (1 + \tan A) + \tan B(1 + \tan A) = 1 + 1$
(By adding 1 on both sides)
$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

24. (c) : $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ ...(i)
Putting $A = 10°$ in (i), we get
$\therefore \frac{1}{\sqrt{3}}(1 - 3\tan^2 10°) = \tan 10°(3 - \tan^2 10°)$ ...(ii)
Squaring (ii) both sides, we get
$(1 - 3\tan^2 10°)^2 = 3\tan^2 10°(3 - \tan^2 10°)^2$
$\Rightarrow 3\tan^2 10°(9 + \tan^4 10° - 6\tan^2 10°)$
$= 1 + 9\tan^4 10° - 6\tan^2 10°$
$\Rightarrow 3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10° = 1$

25. (c) : $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$
$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)]$
$= 3(1 - 2\sin 2x + \sin^2 2x) + 6 + 6\sin 2x + 4\left[1 - \frac{3}{4}\sin^2 2x\right]$
$= 13 + 3\sin^2 2x - 3\sin^2 2x = 13.$

26. (c) : Here, $30'' = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$
Now, $37'\ 30'' = 37' + \left(\frac{1}{2}\right)' = \left(\frac{75}{2}\right)' = \left(\frac{75}{2\times60}\right)^\circ = \left(\frac{5}{8}\right)^\circ$
$\therefore 50°37'30'' = 50° + \left(\frac{5}{8}\right)^\circ = \left(\frac{405}{8}\right)^\circ$
Hence, $\left(\frac{405}{8}\right)^\circ = \frac{405}{8}\times\frac{\pi}{180}$ rad $= \frac{9\pi}{32}$ rad

27. (a) : Required degree measure $= \frac{180}{\pi} \times$ Radian measure
$= \frac{180}{\pi}\times\frac{\pi}{32} = \left(\frac{45}{8}\right)^\circ = \left(5\frac{5}{8}\right)^\circ = 5° + \left(\frac{5}{8}\times60\right)'$
$= 5° + \left(\frac{75}{2}\right)' = 5° + 37' + \left(\frac{1}{2}\times60\right)'' = 5°\ 37'\ 30''$

28. (a) : (i) $\left(\frac{\pi}{9}\right)^c = \left(\frac{180}{\pi}\times\frac{\pi}{9}\right)^\circ = \left(\frac{180}{9}\right)^\circ = 20°$

(ii) $\left(-\frac{\pi}{3}\right)^c = \left(\frac{180}{\pi}\times\left(-\frac{\pi}{3}\right)\right)^\circ = -60°$.

29. (a) : Given, $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, therefore θ lies in third quadrant. Therefore, in third quadrant $\sin\theta < 0$, $\cos\theta < 0$ and $\tan\theta > 0$.

We have, $\sin^2\theta = 1 - \cos^2\theta = \frac{16}{25}$

$\Rightarrow \quad \sin\theta = \pm\frac{4}{5}$ but $\sin\theta < 0$. $\therefore \sin\theta = -\frac{4}{5}$;

$\sec\theta = \frac{1}{\cos\theta} = \frac{-5}{3}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{-5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{3}$ and $\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$

Now, $\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{-\frac{5}{4}+\frac{3}{4}}{-\frac{5}{3}-\frac{4}{3}} = \frac{-2/4}{-9/3} = \frac{1}{6}$

30. (d) : Since, $\sec^2 x = 1 + \tan^2 x = 1+\left(\frac{3}{4}\right)^2 = \frac{25}{16}$

On taking square root, we get $\sec x = \pm\frac{5}{4}$

But x lies in third quadrant, so $\sec x = -\frac{5}{4}$

31. (c) : We have, $\cos x = \frac{4}{5}, x \in \left[0, \frac{\pi}{2}\right]$

$\cos x = 2\cos^2\frac{x}{2} - 1 \Rightarrow 2\cos^2\frac{x}{2} = \frac{9}{5}$

$\Rightarrow \cos^2\frac{x}{2} = \frac{9}{10} \Rightarrow \cos\frac{x}{2} = \pm\frac{3}{\sqrt{10}}$

$\cos\frac{x}{2}$ is positive in I quadrant $\left(0 < \frac{x}{2} < \frac{\pi}{4}\right)$

$\therefore \quad \cos\frac{x}{2} = \frac{3}{\sqrt{10}}$

32. (d) : $\sqrt{3}\ \operatorname{cosec} 20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°}{2\sin 20°\cos 20°}\right)$

$= 4\left(\frac{\sin 60°\cos 20° - \cos 60°\sin 20°}{\sin 40°}\right)$

$= 4\left(\frac{\sin(60° - 20°)}{\sin 40°}\right) = 4$

33. (c) : We have, $2\cos\left(22\frac{1°}{2}\right)\cdot\cos\left(67\frac{1°}{2}\right)$

$= \cos\left(22\frac{1}{2}+67\frac{1}{2}\right)^\circ + \cos\left(22\frac{1}{2}-67\frac{1}{2}\right)^\circ$

$= \cos 90° + \cos 45° = 0 + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

34. (a) : We have, $\sin A = \frac{3}{5}$

$\Rightarrow \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

and $\tan A = \frac{\sin A}{\cos A} = \frac{3/5}{4/5} = \frac{3}{4}$

Now, $\sin 2A = 2\sin A\cdot\cos A = 2 \times \frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$

$\cos 2A = 1 - 2\sin^2 A = \frac{7}{25}$ and $\tan 2A = \frac{24}{7}$

35. (a) : We have,

$\sin(40° + \theta)\cdot\cos(10° + \theta) - \cos(40° + \theta)\cdot\sin(10° + \theta)$

$= \sin\{(40° + \theta) - (10° + \theta)\} = \sin 30° = \frac{1}{2}$

36. (d) : $\tan A + \cot A = 4$ ...(i)

Squaring (i) both sides, we get

$\tan^2 A + \cot^2 A = 14$ ...(ii)

Squaring (ii) both sides, we get

$\tan^4 A + \cot^4 A = 194$.

37. (b) : $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$

$\Rightarrow \quad \sin A = \left(\sqrt{7}+\sqrt{6}\right)\cos A$

$\Rightarrow \quad \sqrt{7}\sin A - \sqrt{6}\sin A = (7-6)\cos A$

$\Rightarrow \quad \sqrt{7}\sin A = \cos A + \sqrt{6}\sin A$

38. (a) : We have, $\sin 20° \sin 40° \sin 80°$

$= \frac{1}{2}[2\sin 20°\cdot\sin 40°]\sin 80°$

$= \frac{1}{2}[\cos(20° - 40°) - \cos(20° + 40°)]\cdot\sin 80°$

$= \frac{1}{2}[\cos(-20°) - \cos 60°]\sin 80°$

$= \frac{1}{2}\times\frac{1}{2}\left[2\left(\cos 20° - \frac{1}{2}\right)\cdot\sin 80°\right]$

$= \frac{1}{4}[2\cos 20°\times\sin 80° - \sin 80°]$

$= \frac{1}{4}[\sin(20° + 80°) - \sin(20° - 80°) - \sin 80°]$

$= \frac{1}{4}[\sin 100° + \sin 60° - \sin 80°]$

$= \frac{1}{4}[\sin(180° - 80°) + \sin 60° - \sin 80°]$

$= \frac{1}{4}[\sin 80° + \sin 60° - \sin 80°] = \frac{1}{4}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$

39. (a) : We have, $4\sin\alpha \sin\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha+\frac{2\pi}{3}\right)$

$= 2\sin\alpha\left\{2\sin\left(\alpha+\frac{2\pi}{3}\right)\sin\left(\alpha+\frac{\pi}{3}\right)\right\}$

$= 2\sin\alpha[2\sin(\alpha + 120°)\sin(\alpha + 60°)]$

$= 2\sin\alpha[\cos(\alpha + 120° - \alpha - 60°) - \cos(\alpha + 120° + \alpha + 60°)]$

$= 2\sin\alpha[\cos60° - \cos(180° + 2\alpha)]$

$= 2\sin\alpha\cdot\frac{1}{2} - 2\sin\alpha(-\cos2\alpha)$

$= \sin\alpha + 2\cos2\alpha\sin\alpha = \sin\alpha + \sin(2\alpha + \alpha) - \sin(2\alpha - \alpha)$

$= \sin\alpha + \sin3\alpha - \sin\alpha = \sin3\alpha$

40. (b) : Since α and β lies between 0 and $\frac{\pi}{4}$ and $\sin(\alpha - \beta) > 0$, therefore, both $\alpha + \beta$ and $\alpha - \beta$ are positive acute angles.

Now, $\sin(\alpha - \beta) = \frac{5}{13}$

$\Rightarrow \cos(\alpha - \beta) = \sqrt{1-\sin^2(\alpha-\beta)} = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$

and $\cos(\alpha + \beta) = \frac{4}{5}$

$\Rightarrow \sin(\alpha + \beta) = \sqrt{1-\cos^2(\alpha+\beta)} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}$.

Hence, $\tan(\alpha + \beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{3}{4}$

and $\tan(\alpha - \beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5}{12}$.

Now, $\tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{56}{33}$.

41. (a) : Since the number of revolutions made by Pinwheel toy in 1 minute = 360

and 1 min = 60 seconds

So, the number of revolutions made by Pinwheel toy in 60 seconds = 360

$\therefore$ Number of revolutions made by Pinwheel toy in 120 seconds = 720

42. (c) : The number of revolutions made by Pinwheel toy in 1 seconds = $\frac{360}{60} = 6$

43. (b) : Since, angle made made by Pinwheel toy in 1 revolution = 360°. Thus, angle made by Pinwheel toy in 6 revolutions = 360° × 6 = 2160°

44. (c) : $1° = \frac{\pi}{180}$ radians

45. (a) : Angle made by Pinwheel toy in 6 revolutions $= 2160 \times \frac{\pi}{180}$ radians = 12 π radians.

46. (c) : $\because \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

Thus $\cos x = \pm\frac{4}{5}$

Since x lies in second quadrant

$\therefore$ $\cos x$ is negative

$\therefore$ $\cos x = -\frac{4}{5}$

47. (d) : $\sin^2 y = 1 - \cos^2 y$

$= 1 - \frac{144}{169} = \frac{25}{169} = \pm\frac{5}{13}$

Since y lies in second quadrant

$\therefore$ $\sin y$ is positive

$\therefore$ $\sin y = \frac{5}{13}$

48. (c) : $\sin(x + y) = \sin x \cos y + \cos x \sin y$

49. (a) : $\sin(x+y) = \left(\frac{3}{5}\right)\times\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\times\left(\frac{5}{13}\right)$

$= -\frac{36}{65}-\frac{20}{65} = -\frac{56}{65}$

50. (b) : $\sin 75° = \sin(30° + 45°)$

$= \sin 30° \cos 45° + \cos30° \sin 45°$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

51. (d) : Let O be the foot of Minaret. In ΔCOB,

$\tan B = \frac{73}{400}$

$\therefore \angle B = \tan^{-1}\left(\frac{73}{400}\right)$

52. (d) : Distance of point A from the foot of minaret is OA.

In ΔCOA, $\frac{OC}{OA} = \tan 75°$

$\Rightarrow OA = \frac{73}{\tan(45°+30°)} = \frac{73}{\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}}$

$\left[\because (\tan(45°+30°) = \frac{\tan 45°+\tan 30°}{1-\tan 45° \tan 30°} = \frac{\sqrt{3}+1}{\sqrt{3}-1}\right]$

$= \frac{73(\sqrt{3}-1)}{(\sqrt{3}+1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{73(\sqrt{3}-1)^2}{2}$ metres

53 (b) : Wire needed to tied between the top of minaret and point A is equal to the length AC.

In ΔACO, $\frac{OC}{AC} = \sin 75°$

$\Rightarrow \quad AC = \dfrac{73}{\sin 75°} = \dfrac{73}{\sin(45° + 30°)}$

$(\because \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30° = \dfrac{\sqrt{3}+1}{2\sqrt{2}})$

$= \dfrac{73(2\sqrt{2})}{(\sqrt{3}+1)} = \dfrac{146\sqrt{2}}{(\sqrt{3}+1)}$ metres

54. (d) : Since, distance between points A and B is $AO + OB$.

Here, $OA = \dfrac{73(\sqrt{3}-1)^2}{2}$ metres and OB = 400 metres

$\therefore \quad AB = \dfrac{73(\sqrt{3}-1)^2}{2} + 400 = \dfrac{800 + 73(\sqrt{3}-1)^2}{2}$ metres

55. (a) : In D ABC, lC = $180° - \left(75° + \tan^{-1}\left(\dfrac{73}{400}\right)\right)$

$= 180° - (75° + \tan^{-1}\left(\dfrac{73}{400}\right)$

$= 180° - 75° - \tan^{-1}\left(\dfrac{73}{400}\right)$

$= 105° - \tan^{-1}\left(\dfrac{73}{400}\right)$

56. (c) : Given $\sec\theta + \tan\theta = m$, $0 < m < 1$ …(i)

Also, $\sec^2\theta - \tan^2\theta = 1$ ...(ii)

Dividing (ii) by (i), we get $\sec\theta - \tan\theta = \dfrac{1}{m}$ ...(iii)

Note that $\dfrac{1}{m} > 1$ $\quad (\because \ 0 < m < 1)$

Adding (i) and (iii), we get $\sec\theta = \dfrac{m^2+1}{2m} > 0$.

and subtracting (iii) from (i), we get $\tan\theta = \dfrac{m^2-1}{2m} < 0$

As $\sec\theta > 0$ and $\tan\theta < 0$,

$\therefore$ θ lies in the fourth quadrant.

Also, $\sin\theta = \tan\theta \cos\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{m^2-1}{m^2+1}$.

57. (a) : Now, $\cot\alpha - \tan\alpha = \dfrac{1}{\tan\alpha} - \tan\alpha = \dfrac{1-\tan^2\alpha}{\tan\alpha}$

$= 2\left(\dfrac{1-\tan^2\alpha}{2\tan\alpha}\right) = 2\cot 2\alpha$

From here, we get $\tan\alpha = \cot\alpha - 2\cot 2\alpha$

Making repeated use of this identity, we shall obtain

$\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + \ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n \cot 2^n \alpha$

$= (\cot\alpha - 2\cot 2\alpha) + 2(\cot 2\alpha - 2\cot 2^2\alpha) + 2^2(\cot 2^2\alpha - 2\cot 2^3\alpha) + \ldots + 2^{n-1}(\cot 2^{n-1}\alpha - 2\cot 2^n \alpha) + 2^n \cot 2^n \alpha$

$= \cot\alpha$

58. (c) : $\sin(-690°) = -\sin 690° = -\sin(2 \times 360° - 30°)$

$= -(-\sin 30°) = \dfrac{1}{2}$

$\cos(-300°) = \cos 300° = \cos(360° - 60°) = \cos 60° = \dfrac{1}{2}$

$\cos(-750°) = \cos 750° = \cos(2 \times 360° + 30°) = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\sin(-240°) = -\sin 240° = -\sin(180° + 60°)$

$= -(-\sin 60°) = \sin 60° = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \sin(-690°)\cos(-300°) + \cos(-750°)\sin(-240°)$

$= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) + \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{1}{4} + \dfrac{3}{4} = 1.$

59. (a) : L.H.S. $= \cos^2\dfrac{A}{2} + \cos^2\dfrac{B}{2} - \cos^2\dfrac{C}{2}$

$= \dfrac{1+\cos A}{2} + \dfrac{1+\cos B}{2} - \dfrac{1+\cos C}{2}$

$= \dfrac{1 + (\cos A + \cos B - \cos C)}{2}$ ...(i)

Now $\cos A + \cos B - \cos C$

$= 2\cos\dfrac{A+B}{2}\cos\left(\dfrac{A-B}{2}\right) - \cos\left(2\cdot\dfrac{C}{2}\right)$

$= 2\sin\dfrac{C}{2}\cos\left(\dfrac{A-B}{2}\right) - \left(1 - 2\sin^2\dfrac{C}{2}\right)$

$\left(\because \cos\left(\dfrac{A+B}{2}\right) = \cos\left(90° - \dfrac{C}{2}\right) = \sin\left(\dfrac{C}{2}\right)\right)$

$= 2\sin\dfrac{C}{2}\left\{\cos\left(\dfrac{A-B}{2}\right) + \sin\left(\dfrac{C}{2}\right)\right\} - 1$

$= -1 + 2\sin\dfrac{C}{2}\left\{\cos\left(\dfrac{A-B}{2}\right) + \cos\left(\dfrac{A+B}{2}\right)\right\}$

$= -1 + 4\sin\left(\dfrac{C}{2}\right)\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)$ ...(ii)

From (i) and (ii), we get

L.H.S. of the given identity

$= \dfrac{1 + \left(-1 + 4\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right)\right)}{2}$

$= 2\cos\left(\dfrac{A}{2}\right)\cos\left(\dfrac{B}{2}\right)\sin\left(\dfrac{C}{2}\right)$ = R.H.S.

60. (d) : Given $\sin^2\theta = \dfrac{3}{4} \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$ or $-\dfrac{\sqrt{3}}{2}$.

Case I : When $\sin\theta = \dfrac{\sqrt{3}}{2}$, then θ lies either in the first quadrant or second quadrant.

Now, $\sin\theta = \dfrac{\sqrt{3}}{2} = \sin\dfrac{\pi}{3}$

$\Rightarrow \quad \sin\theta = \sin\dfrac{\pi}{3}$ or $\sin\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \dfrac{\pi}{3}$ or $\pi - \dfrac{\pi}{3}$, *i.e.*, $\theta = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$.

Case II : When $\sin\theta = -\frac{\sqrt{3}}{2}$, then θ lies either in the third or fourth quadrant.

Now, $\sin\theta = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$

$= \sin\left(\pi + \frac{\pi}{3}\right)$ or $\sin\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \pi + \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3} \quad \Rightarrow \quad \theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$.

Hence, $\sin^2\theta = \frac{3}{4}, 0 < \theta < 2\pi$

$\Rightarrow \quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

SUBJECTIVE TYPE QUESTIONS

1. $625° = \left(625 \times \frac{\pi}{180}\right)^c = 10.91$ radian

$\left(\because 1° = \frac{\pi}{180} \text{ radian}\right)$

2. $\frac{5\pi}{12}$ radians $= \frac{5\pi}{12} \times \frac{180}{\pi}$ degrees $= 75°$

3. $-47°\ 30' = -\left(47\frac{30}{60}\right)^° = -\left(47\frac{1}{2}\right)^° = -\left(\frac{95}{2}\right)^°$.

Now, $1° = \left(\frac{\pi}{180}\right)^c \Rightarrow -\left(\frac{95}{2}\right)^° = -\left(\frac{\pi}{180} \times \frac{95}{2}\right)^c = -\left(\frac{19\pi}{72}\right)^c$.

4. $\cos 240° + \sin\frac{\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) + \sin\frac{\pi}{3}$

$= -\cos\frac{\pi}{3} + \sin\frac{\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}-1}{2}$

5. $\cos(-1710°) = \cos 1710° = \cos\left(\frac{19\pi}{2}\right) = 0$

6. $\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

7. 4 radians $= \left(4 \times \frac{180}{\pi}\right)^° = \left(\frac{4 \times 180 \times 7}{22}\right)^°$

$= 229\frac{1}{11}$ degree $= 229° + \frac{1 \times 60}{11}$ minute

$= 229° + 5' + \frac{5}{11}$ minute $= 229° + 5' + 27.2''$

Hence, 4 radians = 229°5′27″ approx.

8. $\frac{13\pi}{4}$ radians $= \left(\frac{13\pi}{4} \times \frac{180}{\pi}\right)^° = 585°$

9. $\tan\left(\frac{31\pi}{3}\right) = \tan\left(10\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.

10. $\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

11. Let the three angles be $3x$, $4x$ and $5x$ degrees, then $3x + 4x + 5x = 180° \Rightarrow 12x = 180° \Rightarrow x = 15°$

∴ Smallest angle $3x = 45°$ and greatest angle $= 5x = 75°$

$= 75 \times \frac{\pi}{180}$ radians $= \frac{5\pi}{12}$ radians

12. Let radii of circles be r_1 and r_2.

Given $\theta_1 = 65° \Rightarrow \theta_1 = \left(\frac{65\pi}{180}\right)^c$

and $\theta_2 = 110° \Rightarrow \theta_2 = \left(\frac{110\pi}{180}\right)^c$

Also length of arcs are same

$\therefore \quad \theta_1 r_1 = \theta_2 r_2$

$\Rightarrow \quad \frac{65\pi}{180} r_1 = \frac{110\pi}{180} r_2 \quad \Rightarrow \quad \frac{r_1}{r_2} = \frac{110}{65} = \frac{22}{13}$

$\Rightarrow \quad r_1 : r_2 = 22 : 13$

13. We have, $5\sin x = 3 \Rightarrow \sin x = \frac{3}{5}$

Since, $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \quad \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$.

$\Rightarrow \quad \cos x = \frac{4}{5}$

$\therefore \quad \sec x = \frac{5}{4}$ and $\tan x = \left(\frac{3}{5} \times \frac{5}{4}\right) = \frac{3}{4}$

Hence, $\frac{\sec x - \tan x}{\sec x + \tan x} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}} = \frac{2}{4} \times \frac{4}{8} = \frac{1}{4}$

14. $\sin x = \frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$

$\Rightarrow \quad x$ belongs to second quadrant.

Since, $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

$\Rightarrow \cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}, \sec x = -\frac{5}{4}, \cot x = -\frac{4}{3}$

15. $\cos^2(45° + x) - \sin^2(45° - x)$

$= \cos[(45° + x) + (45° - x)]\cos[(45° + x) - (45° - x)]$

$[\because \cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)]$

$= \cos 90° \cos 2x = (0)\cos 2x = 0$

which does not contain x and hence is independent of x.

16. $\sqrt{2 + \sqrt{2 + 2\cos\theta}} = \sqrt{2 + \sqrt{2(1 + \cos\theta)}}$

$= \sqrt{2 + \sqrt{2 \cdot 2\cos^2\frac{\theta}{2}}} = \sqrt{2 + 2\cos\frac{\theta}{2}}$

$= \sqrt{2\left(1 + \cos\frac{\theta}{2}\right)} = \sqrt{2 \cdot 2\cos^2\frac{\theta}{4}} = 2\cos\frac{\theta}{4}$.

17. $2\sin\frac{5\theta}{2}\cos\frac{7\theta}{2} = \sin\left(\frac{5\theta}{2} + \frac{7\theta}{2}\right) + \sin\left(\frac{5\theta}{2} - \frac{7\theta}{2}\right)$

$[\therefore 2\sin A\cos B = \sin(A + B) + \sin(A - B)]$

$= \sin 6\theta + \sin(-\theta) = \sin 6\theta - \sin\theta \quad [\therefore \sin(-\theta) = -\sin\theta]$

18. $\cos 55° + \cos 125° + \cos 300°$

$= 2\cos 90° \cos 35° + \cos(360° - 60°)$

$= 0 + \cos 60° = \frac{1}{2}$

19. $\sin(n+1)x \cdot \sin(n+2)x + \cos(n+1)x \cdot \cos(n+2)x$

$= \cos[(n+2)x - (n+1)x] = \cos x$

[Applying $\cos(A - B) = \cos A \cos B + \sin A \sin B$]

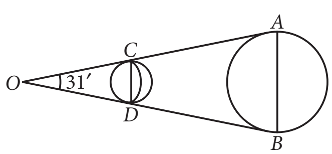
20. $$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$= \left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right) \div \left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right) \div \left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right) \times \left(\frac{1+\tan x}{1-\tan x}\right) = \left(\frac{1+\tan x}{1-\tan x}\right)^2 = \text{R.H.S.}$$

21. Let AB be the diameter of the moon and O, the eye of the observer so that $\angle AOB = 31'$. Let CD be the diameter of the coin. The full moon will be just concealed if the diameter of a coin also subtends the same angle as the diameter of the moon at O, *i.e.*, if $\angle COD = 31'$. As $\angle COD$ is small, CD may be treated as the arc of a circle whose centre is O and radius $= OC$ or OD, the distance of coin from O. Let $OC = OD = r$



Here l = length of arc CD = 2 cm (nearly)

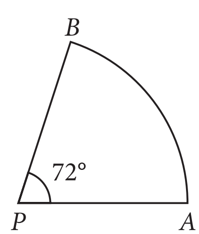
$\theta = 31' = \frac{31}{60} \times \frac{\pi}{180}$ radians

Now, using $r = \frac{l}{\theta}$, we get $[\because l = r\theta]$

$$r = \frac{2 \times 60 \times 180}{31 \times \pi} = \frac{21600 \times 7}{31 \times 22} \qquad \left[\because \pi = \frac{22}{7}\right]$$

$$= \frac{75600}{341}\text{ cm} = \frac{756}{341}\text{ m} = 2.22\text{ m (nearly)}$$

22. Let the post be at point P and let PA be the length of the rope in tight position. Suppose the horse moves along the arc AB so that arc AB = 88 m, and let r be the length of the rope *i.e.*, $PA = r$



Now, $\theta = 72° = \left(72 \times \frac{\pi}{180}\right)$radian

$= \left(\frac{2\pi}{5}\right)$ radian

$$\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{2\pi}{5} = \frac{88}{r}$$

$$\Rightarrow \quad r = 88 \times \frac{5}{2\pi} = 70\text{ m}$$

23. L.H.S. $= \cos 20° \cos 40° \cos 80°$

$= \cos 20° \cos(60° - 20°) \cos(60° + 20°)$

$= \cos 20° (\cos 60° \cos 20° + \sin 60° \sin 20°)$
$(\cos 60° \cos 20° - \sin 60° \sin 20°)$

$$= \cos 20°\left(\frac{\cos 20°}{2} + \frac{\sqrt{3}\sin 20°}{2}\right)\left(\frac{\cos 20°}{2} - \frac{\sqrt{3}}{2}\sin 20°\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} - \frac{3}{4}\sin^2 20°\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} - \frac{3}{4}(1-\cos^2 20°)\right)$$

$$= \cos 20°\left(\frac{\cos^2 20°}{4} + \frac{3\cos^2 20°}{4} - \frac{3}{4}\right)$$

$$= \cos 20°\left(\cos^2 20° - \frac{3}{4}\right)$$

$$= \frac{4\cos^3 20° - 3\cos 20°}{4} = \frac{\cos 60°}{4} = \frac{1}{2 \cdot 4} = \frac{1}{8} = \text{R.H.S.}$$

24. L.H.S. $= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

$= 16\sin^5\theta - 16\sin^3\theta - 4\sin^3\theta + 5\sin\theta + 3\sin\theta - 3\sin\theta$

$= 16\sin^3\theta[\sin^2\theta - 1] + \sin 3\theta + 2\sin\theta$

$= -16\sin^3\theta\cos^2\theta + \sin 3\theta + 2\sin\theta$

$= -(2\sin\theta\cos\theta)^2 \cdot 4\sin\theta + \sin 3\theta + 2\sin\theta$

$= -4\sin^2 2\theta\sin\theta + \sin 3\theta + 2\sin\theta$

$= 2\sin\theta[1 - 2\sin^2 2\theta] + \sin 3\theta$

$= 2\sin\theta\cos 4\theta + \sin 3\theta$

$= \sin 5\theta + \sin(-3\theta) + \sin 3\theta = \sin 5\theta = \text{R.H.S.}$

25. $$\text{L.H.S.} = \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$= 2\left(\left(\frac{1+\cos\frac{\pi}{4}}{2}\right)^2 + \left(\frac{1+\cos\frac{3\pi}{4}}{2}\right)^2\right)$$

$$= \frac{2}{4}\left(\left(1+\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right) = \frac{1}{2}(2+1) = \frac{3}{2} = \text{R.H.S.}$$

26. $\sin\frac{\pi}{18}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{5\pi}{18}$

$= \sin 10° + \sin 20° + \sin 40° + \sin 50°$

$= \sin 50° + \sin 10° + \sin 40° + \sin 20°$

$= \sin(180° - 130°) + \sin 10° + \sin(180° - 140°) + \sin 20°$

$= \sin 130° + \sin 10° + \sin 140° + \sin 20°$

$[\because \sin(\pi - \theta) = \sin\theta]$

$= 2\sin 70° \cos 60° + 2\sin 80° \cos 60°$

$\left[\because \sin x+\sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\right]$

$=2\cdot\frac{1}{2}\sin 70°+2\cdot\frac{1}{2}\sin 80°$

$=\sin 70°+\sin 80° = \sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$

27. L.H.S. $= \frac{1+\sin x-\cos x}{1+\sin x+\cos x}$

$= \frac{2\sin^2\frac{x}{2}+\sin x}{2\cos^2\frac{x}{2}+\sin x} = \frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\cos\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$ = R.H.S.

28. We have, $\tan\frac{x}{2} = \frac{m}{n}$

and, $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\frac{m}{n}}{1+\frac{m^2}{n^2}} = \frac{2mn}{m^2+n^2}$

$\cos x = \sqrt{1-\left(\frac{2mn}{m^2+n^2}\right)^2} = \sqrt{\frac{(m^2+n^2)^2-4m^2n^2}{(m^2+n^2)^2}}$

$= \frac{m^2-n^2}{m^2+n^2}$

$\therefore$ $m\sin x + n\cos x = m\left(\frac{2mn}{m^2+n^2}\right)+n\left(\frac{m^2-n^2}{m^2+n^2}\right)$

$= \frac{2m^2n}{m^2+n^2}+\frac{nm^2}{m^2+n^2}-\frac{n^3}{m^2+n^2} = \frac{3m^2n-n^3}{m^2+n^2}$

29. L.H.S. $= \tan 4x = \tan(2(2x))$

$= \frac{2\tan 2x}{1-\tan^2 2x} = \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$

$= \frac{4\tan x(1-\tan^2 x)^2}{(1-\tan^2 x)[(1-\tan^2 x)^2-4\tan^2 x]}$

$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$ = R.H.S.

30. L.H.S $= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x+\cos^2 y+\sin^2 x+\sin^2 y-2\cos x\cos y-2\sin x\sin y$

$= 2 - 2\cos x\cos y - 2\sin x\sin y$

$= 2 - 2[\cos x\cos y + \sin x\sin y]$

$= 2 - 2\cos(x - y) = 2(1 - \cos(x - y))$

$= 4\sin^2\left(\frac{x-y}{2}\right)$ = R.H.S.

31. L.H.S. $= \cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)$

$= \frac{1}{2}\left[\cos 2x+1+\cos\left(2x+\frac{2\pi}{3}\right)+1 +\cos\left(2x-\frac{2\pi}{3}\right)+1\right]$

$= \frac{1}{2}\left[3+\cos 2x+2\cos 2x\cos\left(\frac{2\pi}{3}\right)\right]$

$[\therefore \cos(A - B) + \cos(A + B) = 2\cos A\cos B]$

$= \frac{1}{2}\left[3+\cos 2x+2\cos 2x\left(\frac{-1}{2}\right)\right]$

$= \frac{1}{2}[3 + \cos 2x - \cos 2x] = \frac{3}{2}$ = R.H.S.

32. Consider,

L.H.S. $= \frac{\sin A-\sin 3A+\sin 5A-\sin 7A}{\cos A-\cos 3A-\cos 5A+\cos 7A}$

$= \frac{(\sin A-\sin 3A)+(\sin 5A-\sin 7A)}{(\cos A-\cos 3A)-(\cos 5A-\cos 7A)}$

$= \frac{2\cdot\cos 2A\sin(-A)+2\cos 6A\cdot\sin(-A)}{2\sin 2A\cdot\sin A-2\sin 6A\cdot\sin A}$

$= \frac{-2\cos 2A\sin A-2\cos 6A\cdot\sin A}{2\sin 2A\cdot\sin A-2\sin 6A\cdot\sin A}$

$= \frac{-2\sin A(\cos 2A+\cos 6A)}{-2\sin A(\sin 6A-\sin 2A)} = \frac{\cos 6A+\cos 2A}{\sin 6A-\sin 2A}$

$= \frac{2\cos 4A\cos 2A}{2\cos 4A\sin 2A} = \cot 2A = \text{R.H.S.}$

33. Consider,

L.H.S. $= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{8\pi}{26}\cos\left(\frac{-2\pi}{26}\right)$

$\left[\text{Using } \cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\right]$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right] = 2\cos\frac{\pi}{13}\left[2\cos\frac{13\pi}{26}\cos\frac{5\pi}{26}\right]$$

$$= 4\cos\frac{\pi}{13}\cdot\cos\frac{\pi}{2}\cdot\cos\frac{5\pi}{26} = 0 = \text{R.H.S.}$$

34. Consider,

L.H.S. = $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x$

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= (\cos 3x \cos x + \sin 3x \sin x) - (\cos^2 x - \sin^2 x)$

$= \cos(3x - x) - \cos 2x$

[Using $\cos(A - B) = \cos A\cos B + \sin A \sin B$ and $\cos^2\theta - \sin^2\theta = \cos 2\theta$]

$= \cos 2x - \cos 2x = 0$ = R.H.S.

35. Given, $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Therefore $\sin\frac{x}{2}$ is positive and $\cos\frac{x}{2}$ is negative.

Now, $\sec^2 x = 1 + \tan^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

Therefore $\cos^2 x = \frac{16}{25} \Rightarrow \cos x = -\frac{4}{5}$

Now, $\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{\frac{5-4}{5}}{2}} = -\sqrt{\frac{1}{10}} - \frac{1}{\sqrt{10}}$

$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{(-1)} = -3$

36. (i) Since A, B, C, D are the angles of a cyclic quadrilateral, we have

$A + C = 180°$ and $B + D = 180°$

$\Rightarrow \quad \cos A = \cos(180° - C)$ and $\cos B = \cos(180° - D)$

$\Rightarrow \quad \cos A = -\cos C$ ...(i)

and $\cos B = -\cos D$ ...(ii)

[$\because \cos(180° - q) = -\cos q$]

Addition. (i) & (ii), we get

$\cos A + \cos B = -(\cos C + \cos D)$

$\Rightarrow \quad \cos A + \cos B + \cos C + \cos D = 0$

(ii) L.H.S. = $-\cos A - \cos B - \cos C - \cos D$

$= -\cos A - \cos B - \cos(180° - A) - \cos(180° - B)$

$= -\cos A - \cos B - (-\cos A) - (-\cos B)$

$= -\cos A - \cos B + \cos A + \cos B = 0$ = R.H.S.

37. L.H.S. $= \sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right)$

$$= \frac{3\sin x - \sin 3x}{4} + \frac{3\sin\left(\frac{2\pi}{3} + x\right) - \sin(2\pi + 3x)}{4} + \frac{3\sin\left(\frac{4\pi}{3} + x\right) - \sin(4\pi + 3x)}{4}$$

$$= \frac{1}{4}\left[3\left\{\sin x + \sin\left(\frac{2\pi}{3} + x\right) + \sin\left(\frac{4\pi}{3} + x\right)\right\} - \{\sin 3x + \sin 3x + \sin 3x\}\right]$$

[$\because \sin(2\pi + x) = \sin x$ and $\sin(4\pi + x) = \sin x$]

$$= \frac{1}{4}\left[3\left\{\sin x + 2\cdot\sin(\pi + x)\cos\left(\frac{\pi}{3}\right)\right\} - 3\sin 3x\right]$$

$$= \frac{1}{4}\left[3\left\{\sin x - 2\cdot\sin x.\left(\frac{1}{2}\right)\right\} - 3\sin 3x\right]$$

[$\therefore \sin(180° + x) = -\sin x$]

$$= \frac{1}{4}[3\{\sin x - \sin x\} - 3\sin 3x] = -\frac{3}{4}\sin 3x = \text{R.H.S.}$$

38. L.H.S. = $(\sin x + \sin y) + (\sin z - \sin(x + y + z))$

$$= \left(2\sin\frac{(x+y)}{2}\cos\frac{(x-y)}{2}\right) + \left(2\cos\frac{(z+x+y+z)}{2}\sin\frac{(z-x-y-z)}{2}\right)$$

$$= 2\sin\frac{(x+y)}{2}\cos\frac{(x-y)}{2} - 2\cos\frac{(x+y+2z)}{2}\sin\frac{(x+y)}{2}$$

$$= 2\sin\frac{(x+y)}{2}\left[\cos\frac{(x-y)}{2} - \cos\frac{(x+y+2z)}{2}\right]$$

$$= 2\sin\frac{(x+y)}{2}\left[\left(-2\sin\frac{(x-y+x+y+2z)}{4}\right)\sin\frac{(x-y-x-y-2z)}{4}\right]$$

$$= 2\sin\left(\frac{x+y}{2}\right)\left[2\sin\frac{(x+z)}{2}\sin\frac{(y+z)}{2}\right]$$

$$= 4\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x+z}{2}\right)\sin\left(\frac{y+z}{2}\right) = \text{R.H.S.}$$

39. Note that $xy + yz + zx = xyz\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$.

If we put $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right)$

$= z\cos\left(\theta + \frac{4\pi}{3}\right) = k$(say)

Then $x=\frac{k}{\cos\theta}, y=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$ and $z=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$

So that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{k}\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right]$

$$=\frac{1}{k}\left[\cos\theta+\cos\theta\cos\frac{2\pi}{3}-\sin\theta\sin\frac{2\pi}{3}+\cos\theta\cos\frac{4\pi}{3}-\sin\theta\sin\frac{4\pi}{3}\right]$$

$$=\frac{1}{k}\left[\cos\theta+\cos\theta\left(\frac{-1}{2}\right)-\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right]$$

$$=\frac{1}{k}\times 0=0$$

Hence, $xy + yz + zx = 0$.

40. $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

$$=\frac{1}{4}\left[\left(2\cos\frac{\pi}{5}\cos\frac{4\pi}{5}\right)\left(2\cos\left(\frac{2\pi}{5}\right)\cos\frac{8\pi}{5}\right)\right]$$

$$=\frac{1}{4}\left[\cos\pi+\cos\frac{3\pi}{5}\right]\left[\cos 2\pi+\cos\frac{6\pi}{5}\right]$$

$$=\frac{1}{4}\left[-1+\cos\left(\pi-\frac{2\pi}{5}\right)\right]\left[1+\cos\left(\pi+\frac{\pi}{5}\right)\right]$$

$$=\frac{1}{4}\left[-1-\cos\frac{2\pi}{5}\right]\left[1-\cos\frac{\pi}{5}\right]$$

$$=\frac{1}{4}\left[-1-\left(\cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right)\right)\right]\left[1-\left(\cos\frac{\pi}{5}\right)\right]$$

$$=\frac{1}{4}\left[-1-\sin\left(\frac{\pi}{10}\right)\right]\left[1-\left(\frac{\sqrt{5}+1}{4}\right)\right]$$

$$=\frac{1}{4}\left[-1-\left(\frac{\sqrt{5}-1}{4}\right)\right]\left[1-\left(\frac{\sqrt{5}+1}{4}\right)\right]$$

$$=\frac{1}{4}\left(\frac{-3-\sqrt{5}}{4}\right)\left(\frac{3-\sqrt{5}}{4}\right)=\frac{1}{64}(-4)=\frac{-1}{16}$$