⑪ Algebra

11.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a). Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].
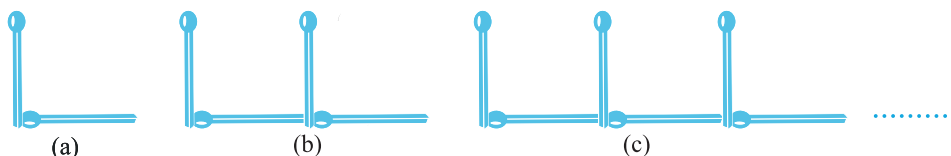

Fig. 11.1

Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.

While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required $= 2 \times$ number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, n $= 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required $= 2 \times n$.

Instead of writing $2 \times n$, we write $2n$.

Note that $2n$ is same as $2 \times n$.

Ameena says to her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required

$= 2 \times 1 = 2$

For $n = 2$, the number of matchsticks required

$= 2 \times 2 = 4$

For $n = 3$, the number of matchsticks required

$= 2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".

Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2n

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,....

Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

n is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable n.

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).
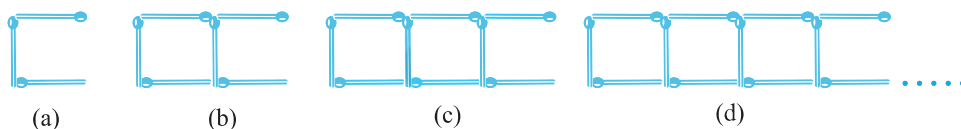

(a)   (b)   (c)   (d)

Fig. 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....

Can you complete the entries left blank in the table?

Sarita comes up with the rule :

Number of matchsticks required $= 3n$

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).
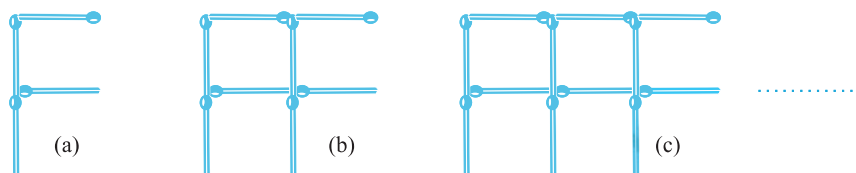

(a)   (b)   (c)

Fig. 11.3

Can you now write the rule for making patterns of F?

In the example on matchstick patterns, we use the variable n to give us the rule for the number of sticks required to make a pattern. An important use of variables in mathematics is in writing general rules like those we have found.

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (∨), triangle (△), square (▢) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter *n* to show a variable. Raju asks, "Why not *m*"? There is nothing special about *n*, any letter can be used.

> **One may use any letter as *m, l, p, x, y, z* etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not variable. But *n* in the examples we have looked at above is a variable. It takes on various values 1, 2, 3, 4, ...**

Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter *m* stands for the number of notebooks a student wants to buy; *m* is a variable, which can take any value 1, 2, 3, 4, ... The total cost of *m* notebooks is given by the rule :

The total cost in rupees $= 5 \times$ number of note books required $= 5m$

If Munnu wants to buy 5 notebooks, then taking m = 5, we say that Munnu should carry ₹ 5 × 5 or ₹ 5 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig11.4). How many children can there be in the drill?
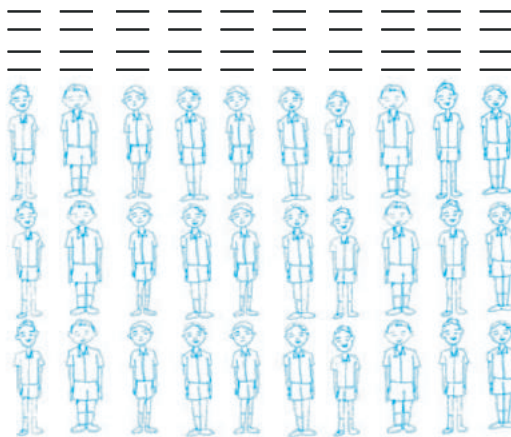

Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2 × 10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, r is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles. But we know that, Sarita's marbles = Ameena's marbles + 10.

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)

7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder as in (Fig 11.5). She has 8 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?

Fig 11.5

8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is ℓ, how many laddus did she make?

10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

<div align="center">

(a)   (b)   (c)   (d)
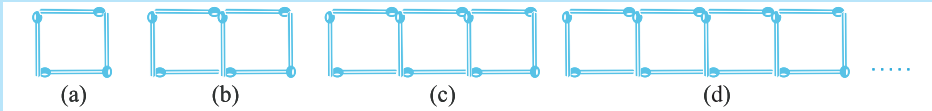
Fig 11.6

</div>

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.

<div align="center">

(a)   (b)   (c)   (d)
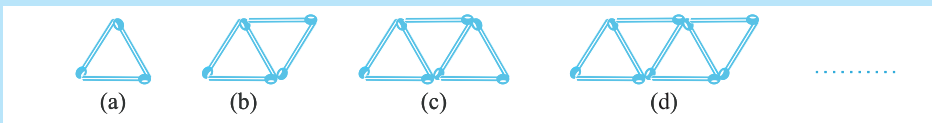
Fig 11.7

</div>

11.6 Use of Variables in Common Rules

Let us now see how certain common rules in mathematics that we have already learnt are expressed using variables.

Rules from geometry

We have already learnt about the perimeter of a square and of a rectangle in the chapter on Mensuration. Here, we go back to them to write them in the form of a rule.

1. Perimeter of a square We know that perimeter of any polygon (a closed figure made up of 3 or more line segments) is the sum of the lengths of its sides. A square has 4 sides and they are equal in length (Fig 11.8). Therefore,

Fig 11.8

The perimeter of a square = Sum of the lengths of the sides of the square = $l \times l \times l \times l = 4l$.
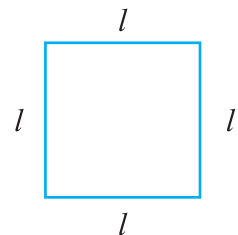
Thus, we get the rule for the perimeter of a square. The use of the variable l allows us to write the general rule in a way that is concise and easy to remember.

We may take the perimeter also to be represented by a variable, say p. Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, $p = 4l$

2. Perimeter of a rectangle We know that a rectangle has four sides. For example, the rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length. Thus, in the rectangle ABCD, let us denote by l, the length of the sides AB or CD and, by b, the length of the sides AD or BC. Therefore,
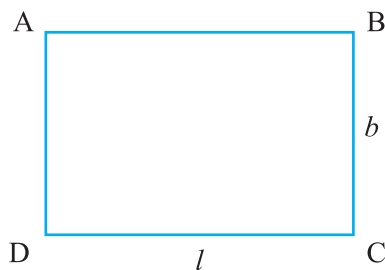
Fig 11.9

Perimeter of a rectangle = length of AB + length of BC

$$+ \text{length of CD} + \text{length of AD}$$
$$= l + b + l + b$$
$$= (l + l) + (b + b)$$
$$= 2l + 2b$$

The rule, therefore, is

The perimeter of a rectangle $= 2l + 2b$

where, l and b are respectively the length and breadth of the rectangle.

Discuss what happens if $l = b$?

If we denote the perimeter of the rectangle by the variable p, the rule for perimeter of a rectangle becomes $p = 2l + 2b$

Note : Here, both l and b are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

In your studies of geometry you will come across several rules and formulas dealing with perimeters and areas of plane figures, and surface areas and volumes of three-dimensional figures. Also, you may obtain formulas for the

sum of internal angles of a polygon, the number of diagonals of a polygon and so on. The concept of variables which you have learnt will prove very useful in writing all such general rules and formulas.

Rules from arithmetic

3. **Commutativity of addition of two numbers:**

   We know that

   $4 + 3 = 7$ and $3 + 4 = 7$

   i.e. $4 + 3 = 3 + 4$

   As we have seen in the chapter on whole numbers, this is true for any two numbers. This property of numbers is known as the **commutativity of addition of numbers**. Commuting means interchanging. Commuting the order of numbers in addition does not change the sum. The use of variables allows us to express the generality of this property in a concise way. Let a and b be two variables which can take any number value.

   Then, $\boldsymbol{a + b = b + a}$

   Once we write the rule this way, all special cases are included in it. If $a = 4$ and $b = 3$, we get $4 + 3 = 3 + 4$. If $a = 37$ and $b = 73$, we get $37 + 73 = 73 + 37$ and so on.

4. **Commutativity of multiplication of two numbers**

   We have seen in the chapter on whole numbers that for multiplication of two numbers, the order of the two numbers being multiplied does not matter. For example,

   $4 \times 3 = 12$, $3 \times 4 = 12$

   Hence, $4 \times 3 = 3 \times 4$

   This property of numbers is known as **commutativity of multiplication of numbers**. Commuting (interchanging) the order of numbers in multiplication does not change the product. Using variables a and b as in the case of addition, we can express the commutativity of multiplication of two numbers as $\boldsymbol{a \times b = b \times a}$

   Note that a and b can take any number value. They are variables. All the special cases like $4 \times 3 = 3 \times 4$ or $37 \times 73 = 73 \times 37$ follow from the general rule.

5. **Distributivity of numbers:**

Suppose we are asked to calculate 7×38. We obviously do not know the table of 38. So, we do the following:

$7 \times 38 = 7 \times (30 + 8)$
$= 7 \times 30 + 7 \times 8$
$= 210 + 56$
$= 266$

Here we assumed $7 \times 38 = 7 \times 30 + 7 \times 8$, i.e. we assumed that multiplication by 7 can be distributed over the addition of 30 and 8. This is always true for any three numbers like 7, 30 and 8. This property is known as distrivutivity of multiplication over addition of numbers. Let us take another example:

We know $9 \times 13 = 117$, $9 \times 11 = 99$ and $9 \times 2 = 18$;

Also $99 + 18 = 117$ and $11 + 2 = 13$.

We may therefore write

$9 \times (11 + 2) \quad = 9 \times 13$
$\qquad\qquad\qquad = 117$
$\qquad\qquad\qquad = 99 + 18$
$\qquad\qquad\qquad = 9 \times 11 + 9 \times 2$
or $9 \times (11 + 2) = 9 \times 11 + 9 \times 2$

Thus, the distributivity property is verified for the numbers 9, 11 and 2. By using variables, we can write this property of numbers also in a general and concise way. Let a, b and c be three variables, each of which can take any number value. Then,

$$a \times (b + c) = a \times b + a \times c$$

Properties of numbers are fascinating. You will learn many of them in your study of numbers this year and in your later study of mathematics. Use of variables allows us to express these properties in a very general and concise way. One more property of numbers is given in question 5 of Exercise 11.2. Try to find more such properties of numbers and learn to express them using variables.

The J & K Board of School Education

EXERCISE 11.2

1. The side of an equilateral triangle is shown by l. Express the perimeter of the equilateral triangle using l.

2. The side of a regular hexagon (Fig 11.10) is denoted by l. Express the perimeter of the hexagon using l. (**Hint :** A regular hexagon has all its six sides equal in length.)
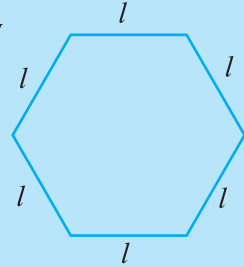
Fig 11.10

3. A cube is a three-dimensional figure as shown in Fig 11.11. It has six faces and all of them are identical squares. The length of an edge of the cube is given by l. Find the formula for the total length of the edges of a cube.
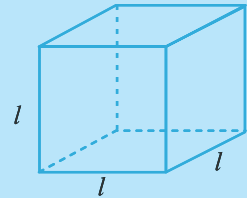
Fig 11.11

4. The diameter of a circle is a line which joins two points on the circle and also passes through the centre of the circle. (In the adjoining figure (Fig 11.12) AB is a diameter of the circle; C is its centre.) Express the diameter of the circle (d) in terms of its radius (r).
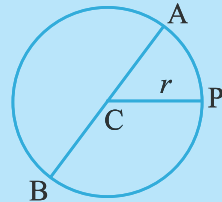
Fig 11.12

5. Consider the sum of three numbers 14, 27 and 13, we may do the sum in two ways:

    (a) We may first add 14 and 27 to get 41 and then add 13 to it to get the total sum 54 or

    (b) We may add 27 and 13 to get 40 and then add it to 14 to get the total sum 54. Thus,

$$(14 + 27) + 13 = 14 + (27 + 13)$$

This can be done for any three numbers. This property is known as the **associativity of addition of numbers**. Express this property which we have already studied in the chapter on Whole Numbers, in a general way, by using variables a, b and c.

11.7 Expressions with Variables

Recall that in arithmetic we have come across expressions like $(2 \times 10) + 3$, $3 \times 100 + (2 \times 10) + 4$ etc. These expressions are formed from numbers like 2, 3, 4, 10, 100 and so on. To form expressions we use all the four number operations of addition, subtraction, multiplication and division. For example, to form $(2 \times 10) + 3$, we have multiplied 2 by 10 and then added 3 to the product. Examples of some of the other arithmetic expressions are :

$$3 + (4 \times 5), \qquad -3 \times 4 + 5,$$
$$8 - 7 \times 2, \qquad 14 - (5 - 2),$$
$$6 \times 2 - 5, \qquad 5 \times 7 - 3 \times 4,$$
$$7 + 8 \times 2 \qquad 5 \times 7 - (3 \times 4 - 7) \text{ etc.}$$

Expressions can be formed from variables too. In fact, we already have seen expressions with variables, for example: $2n$, $5m$, $x + 10$, $x - 3$ etc. These expressions with variables are obtained by operations of addition, subtraction, multiplication and division on variables. For example, the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

We know that variables can take different values; they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication and division can be done on them.

One important point must be noted regarding the expressions containing variables. A number expression like $(4 \times 3) + 5$ can be immediately evaluated as

$$(4 \times 3) + 5 = 12 + 5 = 17$$

But an expression like $(4x + 5)$, which contains the variable x, cannot be evaluated. Only if x is given some value, an expression like $(4x + 5)$ can be evaluated. For example,

when $x = 3$, $4x + 5 = (4 \times 3) + 5 = 17$ as found above.

In the following exercise we shall look at how few simple expressions have been found.

Expression	**How formed?**
(a) $y + 5$	5 added to y
(b) $t - 7$	7 subtracted from t
(c) $10\,a$	a multiplied by 10
(d) $\dfrac{x}{3}$	x divided by 3
(e) $-5\,q$	q multiplied by -5
(f) $3\,x + 2$	first x multiplied by 3, then 2 added to the product
(g) $2\,y - 5$	first y multiplied by 2, then 5 subtracted from the product

Write 10 other such simple expressions and tell how they have been formed.

We should also be able to write an expression through given instruction about how to form it. Look at the following example :

Give expressions for the following :

(a) 12 subtracted from z	$z - 12$
(b) 25 added to r	$r + 25$
(c) p multiplied by 16	$16\,p$
(d) y divided by 8	$\dfrac{y}{8}$
(e) m multiplied by -9	$-9\,m$
(f) y multiplied by 10 and then 7 added to the product	$10\,y + 7$
(g) n multiplied by 2 and l subtracted from the product	$2\,n - l$

Sarita and Ameena decide to play a game of expressions. They take the variable x and the number 3 and see how many expressions they can make. The condition is that they should use not more than one out of the four

number operations and every expression must have x in it. Can you help them?

> Sarita thinks of $(x+3)$.
> Then, Ameena comes up with $(x-3)$.
> Next she suggests $3x$. Sarita then immediately makes $\dfrac{x}{3}$.
> Under the given condition are these the only four expressions that they can get ?

Is $(3x + 5)$ allowed ?
Is $(3x + 3)$ allowed ?

Next they try combinations of y, 3 and 5. The condition is that they should use not more than one operation of addition or subtraction and one operation of multiplication or division. Every expression must have y in it. Check, if their answers are right.

$$y+5, y+3, y-5, y-3,$$
$$3y, \ 5y, \frac{y}{3}, \frac{y}{5}, 3y+5, 3y-5, 5y+3, 5y-3$$

Is $\left(\dfrac{y}{3} + 5\right)$ allowed?
Is $(y + 8)$ allowed?
Is $15y$ allowed?

Can you make some more expressions?

EXERCISE 11.3

1. Make up as many expressions with numbers (no variables) as you can from three numbers 5, 7 and 8. Every number should be used not more than once. Use only addition, subtraction and multiplication.

   (**Hint :** Three possible expressions are $5 + (8 - 7)$, $5 - (8 - 7)$, $(5 \times 8) + 7$; make the other expressions.)

2. Which out of the following are expressions with numbers only?

   (a) $y+3$ (b) $(7 \times 20) - 8z$

   (c) $5(21-7) + 7 \times 2$ (d) 5

   (e) $3x$ (f) $5 - 5n$

   (g) $(7 \times 20) - (5 \times 10) - 45 + p$

3. Identify the operations (addition, subtraction, division, multiplication) in forming the following expressions and tell how the expressions have been formed.

   (a) $z+1, z-1, y+17, y-17$ (b) $17y, \dfrac{y}{17}, 5z$

The J & K Board of School Education

(c) $2y + 17, 2y - 17$ (d) $7m, -7m+3, -7m-3$

4. Give expressions for the following cases.

   (a) 7 added to p (b) 7 subtracted from p

   (c) p multiplied by 7   (d) p divided by 7

   (e) 7 subtracted from $-m$ (f) $-p$ multiplied by 5

   (g) $-p$ divided by 5   (h) p multiplied by -5

5. Give expressions in the following cases.

   (a) 11 added to $2m$ (b) 11 subtracted from $2m$

   (c) 5 times y to which 3 is added   (d) 5 times y from which 3 is subtracted

   (e) y is multiplied by -8

   (f) y is multiplied by -8 and then 5 is added to the result

   (g) y is multiplied by 5 and the result is subtracted from 16

   (h) y is multiplied by -5 and the result is added to 16.

6. (a) Form expressions using t and 4. Use not more than one number operation. Every expression must have t in it.

   (b) Form expressions using y, 2 and 7. Every expression must have y in it. Use only two number operations. These should be different.

11.8 Using Expressions Practically

We have already come across practical situations in which expressions are useful. Let us remember some of them.

	Situation (described in ordinary language)	Variable	Statements using expressions
1.	Sarita has 10 more marbles than Ameena.	Let Ameena have x marbles.	Sarita has $(x + 10)$ marbles.
2.	Balu is 3 years younger than Raju.	Let Raju's age be x years.	Balu's age is $(x - 3)$ years.
3.	Bikash is twice as old as Raju.	Let Raju's age be x years.	Bikash's age is 2x years.
4.	Raju's father's age is 2 years more than 3 times Raju's age.	Let Raju's age be x years.	Raju's father's age is $(3x + 2)$ years.

The J & K Board of School Education

Let us look at some other such situations.

	Situation (described in ordinary language)	Variable	Statements using expressions
5.	How old will Susan be 5 years from now?	Let y be Susan's present age in years.	Five years from now Susan will be (y + 5) years old.
6.	How old was Susan 4 years ago?	Let y be Susan's present age in years.	Four years ago Susan was (y − 4) years old.
7.	Price of wheat per kg is ₹ 5 less than price of rice	Let price of rice per kg be Rs p.	Price of wheat per kg is Rs (p − 5). per kg.
8.	Price of oil per litre is 5 times the price of rice per kg.	Let price of rice per kg be Rs p.	Price of oil per litre is Rs 5p.
9.	The speed of a bus is 10 km/hour more than the speed of a truck going on the same road.	Let the speed of the truck be y km/hour.	The speed of the bus is (y + 10) km/hour.

Try to find more such situations. You will realise that there are many statements in ordinary language, which you will be able to change to statements using expressions with variables. In the next section, we shall see how we use these statements using expressions for our purpose.

EXERCISE 11.4

1. Answer the following:
   (a) Take Sarita's present age to be y years
      (i) What will be her age 5 years from now?
      (ii) What was her age 3 years back?
      (iii) Sarita's grandfather is 6 times her age. What is the age of her grandfather?
      (iv) Grandmother is 2 years younger than grandfather. What is grandmother's age?

    (v) Sarita's father's age is 5 years more than 3 times Sarita's age. What is her father's age?

(b) The length of a rectangular hall is 4 meters less than 3 times the breadth of the hall. What is the length, if the breadth is b meters?

(c) A rectangular box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and the breadth of the box in terms of the height.

(d) Meena, Beena and Leena are climbing the steps to the hill top. Meena is at step s, Beena is 8 steps ahead and Leena 7 steps behind. Where are Beena and Meena? The total number of steps to the hill top is 10 less than 4 times what Meena has reached. Express the total number of steps using s.

(e) A bus travels at v km per hour. It is going from Daspur to Beespur. After the bus has travelled 5 hours, Beespur is still 20 km away. What is the distance from Daspur to Beespur? Express it using v.

2. Change the following statements using expressions into statements in ordinary language.

(For example, Given Salim scores r runs in a cricket match, Nalin scores $(r + 15)$ runs. In ordinary language – Nalin scores 15 runs more than Salim.)

(a) A notebook costs ₹ p. A book costs ₹ $3p$.

(b) Tony puts q marbles on the table. He has $8q$ marbles in his box.

(c) Our class has n students. The school has $20n$ students.

(d) Jaggu is z years old. His uncle is $4z$ years old and his aunt is $(4z - 3)$ years old.

(e) In an arrangement of dots there are r rows. Each row contains 5 dots.

3. (a) Given Munnu's age to be x years, can you guess what $(x - 2)$ may show? (**Hint** : Think of Munnu's younger brother.) Can you guess what $(x + 4)$ may show? What $(3x + 7)$ may show?

(b) Given Sara's age today to be y years. Think of her age in the future or in the past. What will the following expression indicate? $y + 7$, $y - 3$, $y + 4\frac{1}{2}$, $y - 2\frac{1}{2}$.

(c) Given n students in the class like football, what may $2n$ show? What may $\frac{n}{2}$ show? (**Hint** : Think of games other than football).

11.9 What is an Equation?

Let us recall the matchstick pattern of the letter L given in Fig 11.1. For our convenience, we have the Fig 11.1 redrawn here.

(a)   (b)   (c)
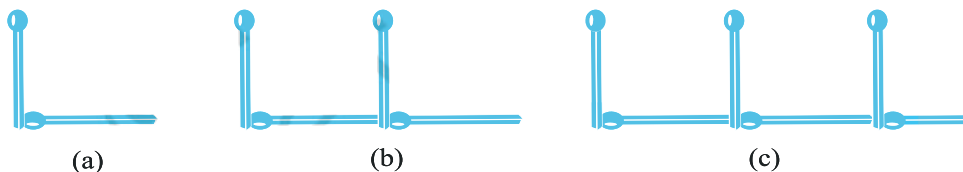
The number of matchsticks required for different number of Ls formed was given in Table 1. We repeat the table here.

Table 1

Number of L's formed	1	2	3	4	5	6	7	8	......	....
Number of matchsticks required	2	4	6	8	10	12	14	16	......	....

We know that the number of matchsticks required is given by the rule, $2n$, if n is taken to be the number of Ls formed.

Appu always thinks differently. He asks, "We know how to find the number of matchsticks required for a given number of Ls. What about the other way round? How does one find the number of Ls formed, given the number of matchsticks"?

We ask ourselves a definite question.

How many Ls are formed if the number of matchsticks given is 10?

This means we have to find the number of Ls (i.e. n), given the number of matchsticks

The J & K Board of School Education

2n = 10   (1)

Here, we have a condition to be satisfied by the variable n. This condition is an example of an equation.

Our question can be answered by looking at Table 1. Look at various values of n. If $n = 1$, the number of matchsticks is 2. Clearly, the condition is not satisfied, because 2 is not 10. We go on checking.

n	$2n$	Condition satisfied? Yes/No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

We find that only if $n = 5$, the condition, i.e. the equation $2n = 10$ is satisfied.

For any value of n other than 5, the equation is not satisfied.

Let us look at another equation.

Balu is 3 years younger than Raju. Taking Raju's age to be x years, Balu's age is $(x - 3)$ years. Suppose, Balu is 11 years old. Then, let us see how our method gives Raju's age.

We have Balu's age, x – 3 = 11   (2)

This is an equation in the variable x. We shall prepare a table of values of $(x - 3)$ for various values of x.

x	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$x-3$	0	1	–	–	–	–	–	–	–	9	10	11	12	13	–	–

Complete the entries which are left blank. From the table, we find that only for $x = 14$, the condition $x - 3 = 11$ is satisfied. For other values, for example for $x = 16$ or for $x = 12$, the condition is not satisfied. Raju's age, therefore, is 14 years.

To summarise, **any equation like the above, is a condition on a variable. It is satisfied only for a definite value of the variable**. For example, the equation $2n = 10$ is satisfied only by the value 5 of the variable n. Similarly, the equation $x - 3 = 11$ is satisfied only by the value 14 of the variable x.

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example : The statement $2n$ is greater than 10, i.e. $2n > 10$, is not an equation. Similarly, the statement $2n$ is smaller than 10 i.e. $2n < 10$, is not an equation. Also, the statements

$(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Now, let us consider $8 - 3 = 5$

There is an equal sign between the LHS and RHS. Neither of the two sides contain a variable. Both contain numbers. We may call this a numerical equation. Usually, the word equation is used only for equations with one or more variables.

Let us do an exercise.

State which of the following are equations with a variable. In the case of equations with a variable, identify the variable.

(a) $x + 20 = 70$ (Yes, x)
(b) $8 \times 3 = 24$ (No, this a numerical equation)
(c) $2p > 30$ (No)
(d) $n - 4 = 100$ (Yes, n)
(e) $20b = 80$ (Yes, b)
(f) $\dfrac{y}{8} < 50$ (No)

Following are some examples of an equation. (The variable in the equation is also identified).

Fill in the blanks as required :

$x + 10 = 30$ (variable x)   (3)
$p - 3 = 7$ (variable p)   (4)

The J & K Board of School Education

$3n = 21$ (variable ____)   (5)
$\frac{t}{5} = 4$ (variable ____)   (6)
$2l + 3 = 7$ (variable ____)   (7)
$2m - 3 = 5$ (variable ____)   (8)

11.10 Solution of an Equation

We saw in the earlier section that the equation

$$2n = 10 \hspace{4cm} (1)$$

was satisfied by $n = 5$. No other value of n satisfies the equation. **The value of the variable in an equation which satisfies the equation is called a solution to the equation**. Thus, $n = 5$ is a solution to the equation $2\,n = 10$.

Note, $n = 6$ is not a solution to the equation $2n = 10$; because for $n = 6$, $2n = 2 \times 6 = 12$ and not 10.

Also, $n = 4$ is not a solution. Tell, why not?

Let us take the equation $x - 3 = 11$ (2)

This equation is satisfied by $x = 14$, because for $x = 14$,

LHS of the equation $= 14 - 3 = 11 = $ RHS

It is not satisfied by $x = 16$, because for $x = 16$,

LHS of the equation $= 16 - 3 = 13$, which is not equal to RHS.

Thus, $x = 14$ is a solution to the equation $x - 3 = 11$ and $x = 16$ is not a solution to the equation. Also, $x = 12$ is not a solution to the equation. Explain, why not?

Now complete the entries in the following table and explain why your answer is Yes/No.

Equation	Value of the variable	Solution (Yes/No)
1. $x + 10 = 30$	$x = 10$	No
2. $x + 10 = 30$	$x = 30$	No
3. $x + 10 = 30$	$x = 20$	Yes
4. $p - 3 = 7$	$p = 5$	No
5. $p - 3 = 7$	$p = 15$	–

6. $p - 3 = 7$	$p = 10$	–
7. $3n = 21$	$n = 9$	–
8. $3n = 21$	$n = 7$	–
9. $\dfrac{t}{5} = 4$	$t = 25$	–
10. $\dfrac{t}{5} = 4$	$t = 20$	–
11. $2l + 3 = 7$	$l = 5$	–
12. $2l + 3 = 7$	$l = 1$	–
13. $2l + 3 = 7$	$l = 2$	–

11.11 Getting a solution to the Equation

In finding the solution to the equation $2n = 10$, we prepared a table for various values of n and from the table, we picked up the value of n which was the solution to the equation (i.e. satisfies the equation). What we used is **a trial and error method**. It is not a **direct** and **practical** way of finding a solution. We shall now look for a direct way of solving an equation, i.e., finding the solution of the equation. We shall learn a more systematic method of solving equations only next year. For the present, we shall learn how to handle simple equations like.

(a) $x + 10 = 30$　　　　　　(b) $x - 3 = 10$

(c) $2n = 10$　　　　　　　　(d) $\dfrac{m}{5} = 4$

Solving $x + 10 = 30$

From earlier classes, we know how to find the number in the box in the statement $\boxed{} + 10 = 30$.

Compare the equation in x　　　　$x + 10 = 30$　　　　(a)

and　　　　　　　　　　　$\boxed{} + 10 = 30$　　　　(b)

If we replace $\boxed{}$ by x in (b), we get the equation. It means that finding the number in the box is the same as finding the value of x which satisfies the equation.

The number in the box is such that 10 added to it gives 30. In other words, it is 10 taken away from 30, i.e., 20. Thus the solution to the equation is $x = 20$.

We may verify the solution:

LHS $= x + 10 = 20 + 10 = 30$ or LHS $=$ RHS

Now, solve this way $\qquad y + 5 = 12$

Compare the equation with $\boxed{} + 5 = 12$

We know $\boxed{} = 12 - 5 = 7$

Therefore, the required solution is $y = \boxed{} = 7$

Solve by the above method $p + 7 = 10$

Solving $x - 3 = 10$

Compare $\qquad x\text{-}3 = 10$

with $\qquad \boxed{} \text{-}3 = 10$

This means that finding the solution to the equation in x is the same as finding the number in the box. Now, the number in the box is given by addition,

$\boxed{} = 10 + 3 = 13$

Therefore, the solution to the equation $x - 3 = 10$ is $x = 13$, which we already know.

We may verify the solution:

LHS $= x - 3 = 13 - 3 = 10 =$ RHS

Solve this way $y - 7 = 5$

Solving $2n = 10$

We note $2n = 2 \times n$

The equation we want to solve is, therefore

$2 \times n = 10$

Compare this with $2 \times \boxed{} = 10$

Solving for n is the same as finding the number in the box. We know that the number in the box can be found by division, $\boxed{} = \dfrac{10}{2} = 5$

The solution to the equation $2n = 10$ is, therefore, $n = 5$, which we know already.

We may verify the solution. LHS $= 2 \times n = 2 \times 5 = 10 =$ RHS

Solving, $\dfrac{m}{5} = 4$

We compare $\dfrac{m}{5} = 4$ with $\dfrac{\boxed{}}{5} = 4$

Solving for *m* is the same as finding the number in the box. We know from earlier classes that this can be done by multiplication,

$$\boxed{} = 5 \times 4 = 20 .$$

Hence the solution to the equation is $m = 20$.

We may verify the solution: LHS $= \dfrac{m}{5} = \dfrac{20}{5} = 4 = $ RHS

11.12 Using an Equation

Appu, Sarita and Ameena are very excited. They tell the class that they have found a method to solve puzzles. They want to explain it to the whole class.

First they ask Sara to hold a number in her mind. Then they ask her to multiply the number by 5 and tell the result. She says 60. Appu immediately says that Sara had 12 in her mind. Sara agrees. The class is surprised.

Ameena explains: .

Sara held some number in mind. It could have been anything. So we took it to be *x*. Now multiplying *x* by 5 gives $5x$. Sara said that she got 60. Thus, we have the condition $5x = 60$

This condition is just a simple equation of the kind we have learnt. We solved this equation by our simple method. We replaced the equation in *x* by

$$5 \times \boxed{} = 60$$

We knew $\boxed{} = \dfrac{60}{5} = 12$

Thus $x = 12$ is the required solution, i.e., it is the number Sara held in her mind.

The whole class clapped. They had learnt how useful equations were. The mathematics teacher complemented Appu, Sarita and Ameena. She added, "Puzzles and problems from everyday life which are much more challenging than that presented by the trio can be solved by using equations. However, for
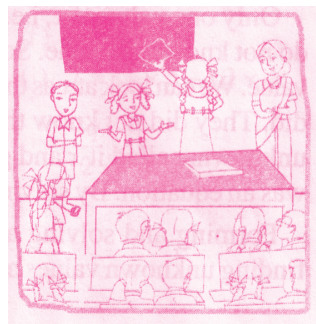
doing that we have to learn a systematic and general method of solving equations. We shall learn such a method next year".

While at the end of the chapter let us closely look at the process of solving an equation. We took an equation to be a condition on the variable in the equation. For example, the equation

$$5x = 60 \text{ is a condition on } x.$$

Only one value of x, i.e. $x = 12$, satisfies the equation. To begin with we do not know this value. Solving the equation means finding this unknown value. We can look at x as this unknown. This is what Appu, Sarita and Ameena did. They did not know the number Sara had in her mind. They took it as unknown, called it x and got the condition that it satisfies. This condition was an equation. By solving the equation, they found the 'unknown'.

Forming and solving an equation, therefore, is a powerful method of finding unknown values and therefore of solving puzzles and problems.

Beginning of Algebra

It is said that algebra as a branch of Mathematics began about 1550 BC, i.e. more than 3500 years ago, when people in Egypt started using symbols to denote unknown numbers.

Around 300 BC, use of letters to denote unknowns and forming expressions from them was quite common in India. Many great Indian mathematicians, **Aryabhatt** (born AD 476), **Brahmagupta** (born AD 598), **Mahavira** (who lived around AD 850) and **Bhaskara II** (born AD 1114) and others, contributed a lot to the study of algebra. They gave names such as *Beeja*, *Varna* etc. to unknowns and used first letters of colour names [e.g., ka from *kala* (black), nee from *neela* (blue)] to denote them.

The Indian name for algebra, *Beejaganit*, dates back to these ancient Indian mathematicians.

The word 'algebra' is derived from the title of the book, **'Aljebar w'al almugabalah'**, written about AD 825 by an Arab mathematician, Mohammed Ibn Al Khowarizmi of Baghdad.

EXERCISE 11.5

1. State which of the following are equations (with a variable). Give reason for your answer. Identify the variable from the equations with a variable.

   (a) $17 = x + 7$ (b) $(t-7) > 5$ (c) $\dfrac{4}{2} = 2$

   (d) $5 \times 4 - 8 = 2x$ (e) $x - 2 = 0$ (f) $2m < 30$

   (g) $2n + 1 = 11$ (h) $7 = (11 \times 2) + p$ (i) $\dfrac{3q}{2} < 5$

   (j) $20 - (10 - 5) = 3 \times 5$ (k) $7 - x = 5$

2. Complete the entries in the third column of the table.

S.No.	Equation	Value of variable Equation satisfied Yes/No
(a)	$10y = 80$	$y = 10$
(b)	$10y = 80$	$y = 8$
(c)	$10y = 80$	$y = 5$
(d)	$4l = 20$	$l = 20$
(e)	$4l = 20$	$l = 80$
(f)	$4l = 20$	$l = 5$
(g)	$b + 5 = 9$	$b = 5$
(h)	$b + 5 = 9$	$b = 9$
(i)	$b + 5 = 9$	$b = 4$
(j)	$h - 8 = 5$	$h = 13$
(k)	$h - 8 = 5$	$h = 8$
(l)	$h - 8 = 5$	$h = 0$
(m)	$p + 3 = 1$	$p = 3$
(n)	$p + 3 = 1$	$p = 1$
(o)	$p + 3 = 1$	$p = 0$
(p)	$p + 3 = 1$	$p = -1$
(q)	$p + 3 = 1$	$p = -2$

3. Pick out the solution from the values given in the bracket next to each equation. Show that the other values do not satisfy the equation.

    (a) $5m = 60$ $(10, 5, 12, 15)$

    (b) $n + 12 = 20$ $(12, 8, 20, 0)$

    (c) $p - 5 = 5$ $(0, 10, 5 - 5)$

    (d) $\dfrac{q}{2} = 7$ $(7, 2, 10, 14)$

    (e) $r - 4 = 0$ $(4, -4, 8, 0)$

    (f) $x + 4 = 2$ $(-2, 0, 2, 4)$

4. (a) Complete the table and by inspection of the table find the solution to the equation $m + 10 = 16$.

m	1	2	3	4	5	6	7	8	9	10	__	__	__
$m + 10$	__	__	__	__	__	__	__	__	__	__	__	__	__

    (b) Complete the table and by inspection of the table, find the solution to the equation $5t = 35$.

t	3	4	5	6	7	8	9	10	11	__	__	__	__
$5t$	__	__	__	__	__	__	__	__	__	__	__	__	__

    (c) Complete the table and find using the table the solution of the equation $z/3 = 4$

x	8	9	10	11	12	13	14	15	16	__	__	__
$\dfrac{z}{3}$	$2\dfrac{2}{3}$	3	$3\dfrac{1}{3}$	__	__	__	__	__	__	__	__	__

    (d) Complete the table and find the solution to the equation $m - 7 = 3$

m	5	6	7	8	9	10	11	12	13	__	__	__
$m - 7$	__	__	__	__	__	__	__	__	__	__	__	__

5. Solve

    (a) $x + 5 = 12$ (b) $y - 2 = 10$

    (c) $7p = 210$ (d) $\dfrac{q}{2} = 5$

    (e) $t + 100 = 125$ (f) $1 - 20 = 30$

(g) $9u = 81$	(h) $\dfrac{k}{8} = 20$
(i) $3y = 33$	(j) $x - 3 = 0$
(k) $\dfrac{k}{8} = 8$	(l) $13y = 65$

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,.... . It is a variable, denoted by some letter like n.

2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.

3. We may use any letter $n, l, m, p, x, y, z,$ etc. to show a variable.

4. A variable allows us to express relations in any practical situation.

5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3, x + 3, 2n, 5m, \dfrac{p}{3}, 2y + 3, 3l - 5,$ etc.

6. Variables allow us to express many common rules in both geometry and arithmetic in a general way. For example, the rule that the sum of two numbers remains the same if the order in which the numbers are taken is reversed can be expressed as $a + b = b + a$. Here, the variables a and b stand for any number, $1, 32, 1000 - 7, -20,$ etc.

7. An equation is a condition on a variable. It is expressed by

saying that an expression with a variable is equal to a fixed number, e.g. $x - 3 = 10$.

8. An equation has two sides, LHS and RHS, between them is the equal (=) sign.

9. The LHS of an equation is equal to its RHS only for a definite value of the variable in the equation. We say that this definite value of the variable satisfies the equation. This value itself is called the solution of the equation.

10. For getting the solution of an equation, one method is the trial and error method. In this method, we give some value to the variable and check whether it satisfies the equation. We go on giving this way different values to the variable, until we find the right value which satisfies the equation.

11. We need a more systematic way of getting a solution of the equation than the trial and error method. In case of very simple equations, the variable can ve replaced by a place holder ⬚ . From earlier classes we know how to get the value of ⬚ . This is the value of the variable which is the solution to the equation.

12. When we are given an equation, its solution, i.e., the value of the variable which satisfies the equation, is unknown to us. Solving the equation means finding the unknown value, we can look at the variable in the equation as an unknown. Starting from the unknown we can see up the equation. Solving the equation is thus a method of finding the unknown. It is therefore a powerful method of solving puzzles and problems.

E3S626