# **JEE Advanced 2024**

# Sample Paper - 4

Time Allowed: 3 hours Maximum Marks: 180

### **General Instructions:**

This question paper has THREE main sections and four sub-sections as below.

#### **MRQ**

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

#### **MCQ**

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

#### NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

#### **MATCH**

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

## **Mathematics (MRQ)**

1. Tangents are drawn to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$ , parallel to the straight line 2x - y = 1. **[4]** The points of contact of the tangents on the hyperbola are

a) 
$$(3\sqrt{3}, -2\sqrt{2})$$

b) 
$$\left(-\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

c) 
$$(-3\sqrt{3},2\sqrt{2})$$

d) 
$$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

- 2. Let a,  $b \in R$  and  $f: R \to R$  be defined by  $f(x) = a \cos(|x^3 x|) + b |x| \sin(|x^3 + x|)$ . Then f is
  - a) NOT differentiable at x = 0 if a = 1b = 0
- b) NOT differentiable at x = 1 if a = 1 and b = 1
- c) differentiable at x = 0 if a = 0 and b = 1
- d) differentiable at x = 1 if a = 1 and b = 0

[4]

3. Let  $f: R \Rightarrow (0, 1)$  be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

a) 
$$x^9-f(x)$$
 b)  $e^x-\int\limits_0^x f(t)\sin tdt$ 

c) 
$$x-\int\limits_{0}^{rac{\pi}{2}-x}f(t)\cos tdt$$

d) 
$$f(x)+\int\limits_0^{rac{\pi}{2}}f(t)\sin tdt$$

### **Mathematics (MCQ)**

4. If 
$$n - 1C_r = (k^2 - 3) {}^{n}C_{r+1}$$
, then k belongs to

[3]

a) 
$$[2,\infty)$$

b) 
$$(-\infty, -2]$$

c) 
$$(\sqrt{3}, 2]$$

d) 
$$[-\sqrt{3}, \sqrt{3}]$$

5. The total number of local maxima and local minima of the function f(x) =

[3]

$$\left\{egin{array}{ll} (2+x)^3, & -3 < x \leq -1 \ x^{2/3}, & -1 < x < 2 \end{array}
ight.$$
 is

a) 1

b) 3

c) 2

d) 0

6. The points  $\left(0, \frac{8}{3}\right)$ ,  $\left(1, 3\right)$  and (82, 30) are vertices of

[3]

- a) A right angled triangle
- b) An obtuse angled triangle

c) None of these

d) An acute angled triangle

7. Let  $f'(x) = \frac{x}{(1+x^n)^{1/n}}$  for  $n \ge 2$  and  $g(x) = \underbrace{(\text{ fofo...of})}_{f \text{ occurs } n \text{ times}} (x)$ . Then  $\int x^{n-2} g(x) dx$  equals. [3]

a) 
$$rac{1}{n(n+1)}(1+nx^n)^{1+rac{1}{n}}+K$$

b) 
$$rac{1}{n(n-1)}(1+nx^n)^{1-rac{1}{n}}+K$$

c) 
$$\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$$

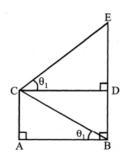
d) 
$$\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$$

### **Mathematics (NUM)**

8. Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP (1; 3) AP (2; 5) AP (3; 7) = AP (a; d) then a + d equals \_\_\_\_\_.

9. Let the mirror image of a circle  $c_1: x^2 + y^2 - 2x - 6y + \alpha = 0$  in line y = x + 1 be  $c_2: 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$ . If r is the radius of circle  $c_2$ , then  $\alpha + 6r^2$  is equal to \_\_\_\_\_.

10. In the figure,  $\theta_1 + \theta_2 = \frac{\pi}{2}$  and  $\sqrt{3}(BE) = 4(AB)$ . If the area of  $\triangle CAB$  is  $2\sqrt{3} - 3$  unit<sup>2</sup>, when  $\frac{\theta_2}{\theta_1}$  is the largest, then the perimeter (in unit) of  $\triangle CED$  is equal to \_\_\_\_\_.



- 11. Consider the set of eight vectors  $V = \{a\hat{i} + \hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$ . Three non-coplanar vectors can be chosen from V in  $2^p$  ways. Then p is:
- 12. Let  $f_1:(0,\infty)\to\mathbb{R}$  and  $f_2:(0,\infty)\to\mathbb{R}$  be defined by  $f_1(x)=\int\limits_0^x\prod_{j=1}^{21}(t-j)^jdt, x>0$  and  $f_2(x)=98$   $(x-1)^{50}$   $600(x-1)^{49}$  + 2450, x>0, where for any positive integer n and real numbers  $a_1$ ,  $a_2$ ,...  $a_n$ ,  $\prod_{i=1}^n a_i$  denotes the product of  $a_1$ ,  $a_2$ ,...  $a_n$ , Let  $m_i$  and  $n_i$ , respectively, denote the number of points of local minima and the number of points of local maxima of function  $f_i$ , i=1,2, in the interval  $(0,\infty)$ . The value of  $6m_2+4n_2+8m_2n_2$  is \_\_\_\_\_.
- 13. The number of all possible values of  $\theta$  where  $0 < \theta < \pi$ , for which the system of equations [4]  $(y + z) \cos 3\theta = (xyz) \sin 3\theta$   $x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$  (xyz)  $\sin 3\theta = (y + 2z) \cos 3\theta$  +y  $\sin 3\theta$  have a solution (x<sub>0</sub>, y<sub>0</sub>, z<sub>0</sub>) with y<sub>0</sub>z<sub>0</sub>  $\neq$  0, is

## **Mathematics (MATCH)**

14. Let 
$$z_k = \cos(\frac{2k\pi}{10}) + i\sin(\frac{2k\pi}{10})$$
; k = 1, 2, ..., 9.

List-I	List-II
(P) For each $z_k$ there exists as $z_j$ such that $z_k.z_j = 1$	(1) True
(Q) There exists a $k \in \{1, 2,, 9\}$ such that $z_1.z = z_k$ has no solution z in the set of complex numbers	(2) False
(R) $\frac{ 1-z_1  1-z_2   1-z_0 }{10}$ equal	(3) 1
$\left(S) \ 1 - \sum\limits_{k=1}^{9} \cos\!\left(rac{2k\pi}{10} ight) \ equal  ight.$	(4) 2

[3]

[3]

15. Let H:  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , here a > b > 0, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of  $60^{\rm O}$  at one of its vertices N. Let the area of the triangle LMN be  $4\sqrt{3}$ .

List I	List II
P. The length of the conjugate axis of H is	1. 8

List I	List II
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	4. 4

a) P 
$$ightarrow$$
 4; Q  $ightarrow$  3; R  $ightarrow$  1; S  $ightarrow$  2

b) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3

c) P 
$$\rightarrow$$
 4; Q  $\rightarrow$  1; R  $\rightarrow$  3; S  $\rightarrow$  2

d) P 
$$\rightarrow$$
 3; Q  $\rightarrow$  4; R  $\rightarrow$  2; S  $\rightarrow$  1

16. Let p, q, r be nonzero real numbers that are, respectively, the  $10^{th}$ ,  $100^{th}$  and  $1000^{th}$  terms of a harmonic progression. Consider the system of linear equations x + y + z = 1

x + y + z = 110x + 100y + 1000z = 0

qr x + pr y + pq z = 0.

List-I	List-II
(I) If $\frac{q}{r} = 10$ , then the system of linear equations has	(P) $x = 0$ , $y = \frac{10}{9}$ , $z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq$ 100, then the system of linear equations has	(Q) $x = \frac{10}{9}$ , $y = -\frac{1}{9}$ , $z = 0$ solution
(III) If $\frac{p}{q} \neq$ 10, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q}$ = 10, then the system of linear equations has	(S) no solution
	(T) at least one solution

a) (I) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (R)

b) (I) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (R)

c) (I) 
$$\rightarrow$$
 (T); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (T)

d) (I) 
$$\rightarrow$$
 (T); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (T)

17. Let  $\ell_1$  and  $\ell_2$  be the lines  $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$  and  $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{k})$ , respectively. Let X **[3]** be the set of all the planes H that contain the line  $\ell_1$ . For a plane H, let d(H) denote the smallest possible distance between the points of  $\ell_2$  and H. Let H<sub>0</sub> be a plane in X for which d(H<sub>0</sub>) is the maximum value of d(H) as H varies over all planes in X. Match each entry in List-I to the correct entries in List-II.

(1) $\sqrt{3}$
(2) $\frac{1}{\sqrt{3}}$
(3) 0

(S) The distance of origin from the point of intersection of planes $y = z$ , $x = H_0$ is	1 and (4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

- a) (P)  $\rightarrow$  (5), (Q)  $\rightarrow$  (1), (R)  $\rightarrow$  (4), (S)  $\rightarrow$  b) (P)  $\rightarrow$  (5), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  (1)
- c) (P)  $\rightarrow$  (2), (Q)  $\rightarrow$  (1), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  d) (P)  $\rightarrow$  (2), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (5), (S)  $\rightarrow$  (1)

## Physics (MRQ)

18. The moment of inertia of a thin square plate ABCD, Fig., of uniform thickness about an axis **[4]** passing through the centre O and perpendicular to the plane of the plate is



where  $I_1$ ,  $I_2$ ,  $I_3$  and  $I_4$  are respectively the moments of intertial about axis 1, 2, 3 and 4 which are in the plane of the plate.

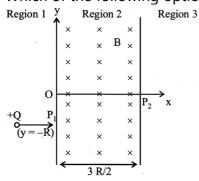
a) 
$$I_1 + I_3$$

b) 
$$1_3 + 1_4$$

c) 
$$11 + 12$$

d) 
$$l_1 + l_2 + l_3 + l_4$$

- 19. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation,  $y(x, t) = (0.01 \text{ m}) \sin [(62.8 \text{ m}^{-1})x] \cos [(628 \text{ s}^{-1})t]$ . Assuming n = 3.14, the correct statement(s) is (are)
  - a) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
- b) The fundamental frequency is 100 Hz
- c) The length of the string is  $0.25\ m$
- d) The number of nodes is 5
- 20. A uniform magnetic field B exists in the region between x = 0 and  $x = \frac{3R}{2}$  (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P<sub>1</sub> (y = -R). Which of the following option(s) is/are correct?



- a) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P<sub>1</sub> and the point of re -entry into region 1 is inversely proportional to the mass of the particle
- 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P<sub>1</sub> and the farthest point from y-axis is  $\frac{p}{\sqrt{2}}$

b) When the particle re-enters region

- c) For B =  $\frac{8}{13} \frac{p}{QR}$ , the particle will enter region 3 through the point P<sub>2</sub> on x axis.
- d) For B >  $\frac{2}{3} \frac{p}{QR}$ , the particle will reenter region 1

[3]

[4]

## Physics (MCQ)

21. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density:

a) 
$$[FL^{-5}T^2]$$

b) 
$$[FL^{-3}T^{3}]$$

c) 
$$[FL^{-4}T^2]$$

d) 
$$[FL^{-3}T^{2}]$$

- 22. A block of mass 0.1 kg is held against a wall applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is
  - a) 0.98 N

b) 4.9 N

c) 0.49 N

- d) 2.5 N
- 23. The escape velocity of a body on the surface of the earth is 11.2 km/sec. If the earth's mass increases to twice its present value and radius of the earth become half, the escape velocity becomes:
  - a) 11.2 km/s

b) 5.6 km/s

c) 44.8 km/s

- d) 22.4 km/s
- 24. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength  $\frac{81\pi}{7} \times 10^5 \mathrm{Vm}^{-1}$ . When the field is switched off, the drop is observed to fall with terminal velocity  $2 \times 10^{-3} \mathrm{\ ms}^{-1}$ . Given  $g = 9.8 \mathrm{\ ms}^{-1}$ , viscosity of the air =  $1.8 \times 10^{-5} \mathrm{\ Ns\ m}^{-2}$  and the density of oil =  $900 \mathrm{\ kg\ m}^{-3}$ , the magnitude of q is
  - a)  $1.6 \times 10^{-19}$  C

b)  $4.8 \times 10^{-19}$  C

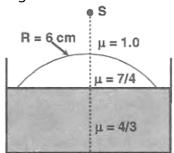
c)  $8.0 \times 10^{-19}$  C

d)  $3.2 \times 10^{-19}$  C

## Physics (NUM)

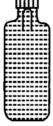
25. Water (with refractive index =  $\frac{4}{3}$ ) in a tank is 18 cm deep. Oil of refractive index  $\frac{7}{4}$  lies on water making a convex surface of radius of curvature **R** = **6** cm as shown. Consider oil to act as a thin lens. An object **S** is placed 24 cm above water surface. The location of its

image is at  $\mathbf{x}$  cm above the bottom of the tank. Then  $\mathbf{x}$  is:



- 26. 300 grams of water at 25 °C is added to 100 grams of ice at 0 °C. The final temperature of the mixture is \_\_\_\_\_ °C.
- 27. Four solid spheres each of diameter  $\sqrt{5}$  cm and mass 0.5 kg are placed with their centres at the comers of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N  $\times$  10<sup>-4</sup> kg-m<sup>2</sup>, then N is
- 28. A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure  $p_0 = 10^5$  Pa so that the volume of the trapped air is  $v_0 = 3.3$  cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure  $P_0 + \Delta p$  without changing its orientation. At this pressure, the volume of the trapped air is  $v_0 \Delta v$ .

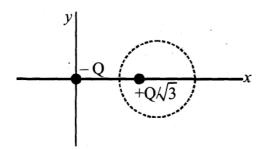
Let  $\Delta v = X$  cc and  $\Delta p = Y \times 10^3$  Pa.



The value of X is \_\_\_\_\_.

- 29. A Hydrogen-like atom has atomic number Z. Photons emitted in the electronic transitions from level n = 4 to level n = 3 in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1.95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of Z is \_\_\_\_\_.

  [Given: hc = 1240 eV-nm and Rhc = 13.6 eV, where R is the Rydberg constant, h is the Planck's constant and c is the speed of light in vacuum]
- 30. Two point charges -Q and  $+\frac{Q}{\sqrt{3}}$  are placed in the xy-plane at the origin (0, 0) and a point [4] (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.



The value of b is \_\_\_\_\_ meter.

## **Physics (MATCH)**

List I describes thermodynamic processes in four different systems. List II gives the 31. magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process.

List-I	List-
(I) $10^{-3}$ kg of water at $100^{\circ}$ C is converted to steam at the same temperature, at a pressure of $10^{5}$ Pa. The volume of the system changes from $10^{-6}$ m <sup>3</sup> to $10^{-3}$ m <sup>3</sup> in the process. Latent heat of water = $2250$ kJ/kg.	(P) 2 kJ
(II) 0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume $R = 8.0 \text{ Jmol}^{-1} \text{ K}^{-1}$ .	(Q) 7 kJ
(III) One mole of a monatomic ideal gas is compressed adiabatically from volume $V=rac{1}{3}m^3$ and pressure 2 kPa to volume $rac{V}{8}$ .	(R) 4 kJ
(IV) Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion.	(S) 5 kJ
	(T) 3 kJ

Which one of the following options is correct?

a) (I) 
$$\rightarrow$$
 (P); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (T); (IV)  $\rightarrow$  (Q)

b) (I) 
$$\rightarrow$$
 (S); (II)  $\rightarrow$  (P); (III)  $\rightarrow$  (T); (IV)  $\rightarrow$  (P)

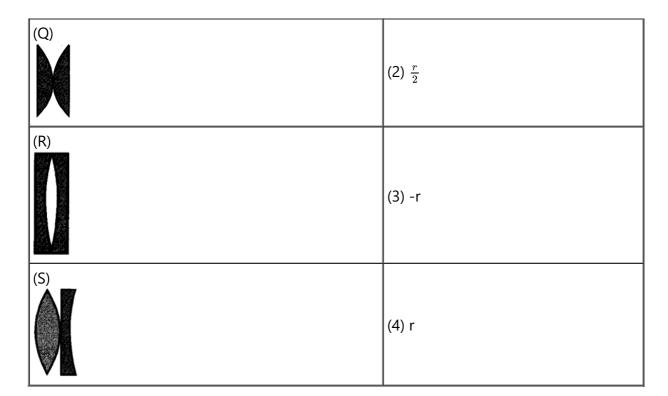
c) (I) 
$$\rightarrow$$
 (T); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (Q)

d) (I) 
$$\rightarrow$$
 (Q); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (T)

32. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists.

(P)	(1) 2r

[3]



- a) P 4, Q 1, R 2, S 3
- b) P 2, Q 4, R 3, S 1

- c) P 2, Q 1, R 3, S 4
- d) P 1, Q 2, R 3, S 4
- 33. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

List I	List II
P. Alpha decay	$1{8}^{15}O \rightarrow_{7}^{15}O + \dots$
Q. β+ decay	$ ext{2.} \ ^{138}_{92} ext{U}  ightarrow ^{234}_{90} ext{Th} + \dots$
R. Fission	$3{83}^{185} { m Bi}  ightarrow _{82}^{184} { m Pb} + \dots$
S. Proton emission	$4.~^{239}_{94}{ m Pu} ightarrow ^{140}_{57}{ m La}+\ldots$

- a) (P) (2); (Q) (1); (R) (4); (S) (3)
- b) (P) (4); (Q) (3); (R) (2); (S) (1)
- c) (P) (4); (Q) (2); (R) (1); (S) (3)
- d) (P) (1); (Q) (3); (R) (2); (S) (4)
- 34. A musical instrument is made using four different metal strings 1,2,3 and 4 with mass per unit length  $\mu$ ,  $2\mu$ ,  $3\mu$  and  $4\mu$  respectively. The instrument is played by vibrating the strings by varying the free length in between the range L<sub>0</sub> and 2L<sub>0</sub>. It is found that in string-1 ( $\mu$ ) at free length L<sub>0</sub> and tension T<sub>0</sub> the fundamental mode frequency is f<sub>0</sub>. List I gives the above four strings while list II lists the magnitude of some quantity.

List-I	List-II
(I) String - 1 (μ)	(P) 1
(II) String - 2 (2μ)	(Q) $\frac{1}{2}$
(III) String - 3 (3 $\mu$ )	(R) $\frac{1}{\sqrt{2}}$

List-I	List-II
(IV) String - 4 (4μ)	$(S)\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) 1/16

If the tension in each string is  $T_0$ , the correct match for the highest fundamental frequency in  $f_0$  units will be,

- a) (I)  $\rightarrow$  (P), (II) $\rightarrow$  (Q), (III) $\rightarrow$  (T), (IV) $\rightarrow$  (S)
- b) (I)  $\rightarrow$  (Q), (II) $\rightarrow$  (P), (III) $\rightarrow$  (R), (IV) $\rightarrow$  (T)
- c) (I)  $\rightarrow$  (Q), (II) $\rightarrow$  (S), (III) $\rightarrow$  (R), (IV) $\rightarrow$  (P)
- d) (I)  $\rightarrow$  (P), (II) $\rightarrow$  (R), (III) $\rightarrow$  (S), (IV) $\rightarrow$  (Q)

## **Chemistry (MRQ)**

- 35. Which of the following compounds will give a yellow precipitate with iodine and alkali? [4]
  - a) 2-Hydroxypropane

b) acetophenone

c) methyl acetate

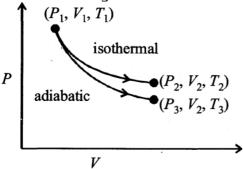
d) acetamide

36. A catalyst:

[4]

[4]

- a) decreases the activation energy
- b) alters the reaction mechanism
- c) increases the frequency of collisions of reacting species
- d) increases the average kinetic energy ofreacting molecules
- 37. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct?



a)  $W_{isothermal} > W_{adiabatic}$ 

b)  $T_3 > T_1$ 

c)  $T_1 = T_2$ 

d)  $\Delta U_{isothermal} > \Delta U_{adiabatic}$ 

## Chemistry (MCQ)

38. The isoelectronic set of ions is

- a)  $F^{-}$ , Li<sup>+</sup>, Na<sup>+</sup> and Mg<sup>2+</sup>
- b)  $N^{3-}$ ,  $O^{2-}$ ,  $F^{-}$  and  $Na^{+}$

	<sup>c)</sup> Li <sup>+</sup> , Na <sup>+</sup> , O <sup>2-</sup> and F <sup>-</sup>	d) $N^{3}$ -, Li <sup>+</sup> , $Mg^{2}$ + and $O^{2}$ -	
39.	39. Which one is more acidic in aqueous solution?		[3]
	a) FeCl <sub>3</sub>	b) BeCl <sub>2</sub>	
	c) AICI <sub>3</sub>	d) NiCl <sub>2</sub>	
40.	A solution when diluted with H <sub>2</sub> O and boiled, it gives a white precipitate. On the addition of excess NH <sub>4</sub> Cl/NH <sub>4</sub> OH, the volume of precipitate decreases leaving behind a white gelatinous precipitate. Identify the precipitate which dissolves in NH <sub>4</sub> OH/NH <sub>4</sub> Cl.		
	a) Al(OH) <sub>3</sub>	b) Mg(OH) <sub>2</sub>	
	c) Ca(OH) <sub>2</sub>	d) Zn(OH) <sub>2</sub>	
41.	Which of the following acids has the smalle	est dissociation constant?	[3]
	a) CH <sub>3</sub> CHFCOOH	b) FCH <sub>2</sub> CH <sub>2</sub> COOH	
	c) BrCH <sub>2</sub> CH <sub>2</sub> COOH	d) CH <sub>3</sub> CHBrCOOH	
42.	For the following reaction scheme, percentage $Mg_2C_3 \xrightarrow{H_2O} P \xrightarrow{NaNH_2} Q \xrightarrow{\text{red hot iron tube } 873 \text{ K}} R$ $(4.0 \text{ g}) \xrightarrow{75\%} Q \xrightarrow{\text{tube } 873 \text{ K}} R$ $(4.0 \text{ g}) \xrightarrow{75\%} Q \xrightarrow{\text{tube } 873 \text{ K}} R$ $(4.0 \text{ g}) \xrightarrow{75\%} Q \xrightarrow{\text{tube } 873 \text{ K}} R$ $(4.0 \text{ g}) \xrightarrow{\text{tube } 873 \text{ K}} R$ $(4$	ly.	[4]
43.	The vapour pressure of pure benzene at a c	tertain temperature is 640 mm of Hg. A non- 5 g is added to 39.0 g of benzene. The vapour molar mass of solid substance is g	[4]
44.			[4]
45.		ed below. The number of metals that will show nm wavelength falls on the metal is	[4]

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
$\Phi(eV)$	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

46. Consider the following reversible reaction,

 $A(g) + B(g) \leftrightharpoons AB(g)$ 

The activation energy of the backward reaction exceeds that of the forward reaction by 2RT (in J mol<sup>-1</sup>). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of  $\Delta G^{\circ}$  (in J mol<sup>-1</sup>) for the reaction at 300 K is

(Given;  $ln(2) = 0.7 RT = 2500 J mol^{-1}$  at 300 K and G is the Gibbs energy)

47. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula [4]  $C_4H_6$  is

### **Chemistry (MATCH)**

48. Match items of column I and II

Column I (Mixture of compounds)	Column II (Separation Technique)	
(A) $\frac{\mathrm{H_2O}}{\mathrm{CH_2Cl_2}}$	(i) Crystallization	
$(B) \bigcirc \bigcirc$	(ii) Differential solvent extraction	
(C) Kerosene/Naphthalene	(iii) Column chromatography	
(D) $\frac{\mathrm{C_6H_{12}O_6}}{\mathrm{NaCl}}$	(iv) Fractional Distillation	

49. Match each set of hybrid orbitals from LIST-I with complex(es) given in LIST-II

List-I	List-II	
(A) dsp <sup>2</sup>	(p) $[\mathrm{FeF}_6]^4$	
(B) sp <sup>3</sup>	(q) $[\mathrm{Ti}(\mathrm{H_2O})_3\mathrm{Cl}_3]$	
(C) $sp^3 d^2$	(r) $\left[\mathrm{Cr}(\mathrm{NH_3})_6 ight]^{3+}$	
(D) $d^2sp^3$	(s) ${ m [FeCl_4]}^{2-}$	
	(t) [Ni(CO) <sub>4</sub> ]	
	$\left[  ext{(w)} \left[  ext{Ni}( ext{CN})_4  ight]^{2-}$	

[3]

[4]

50. The standard reduction potential data at 25°C is given below:

$$E^{O}(Fe^{3+}, Fe^{2+}) = +0.77 \text{ V}; E^{O}(Fe^{2+}, Fe) = -0.44 \text{ V}; E^{O}(Cu^{2+}, Cu) = +0.34 \text{ V}; E^{O}(Cu^{+}, Cu) = +0.52 \text{ V}$$

$$E^{O}[O_{2}(g) + 4H^{+} + 4e^{-} \rightarrow 2H_{2}O] = +1.23 \text{ V}; E^{O}[O_{2}(g) + 2H_{2}O + 4e^{-} \rightarrow 4OH^{-}] = + 0.40 \text{ V}$$
  $E^{O}(Cr^{3+}, Cr) = -0.74 \text{ V}; E^{O}(Cr^{2+}, Cr) = -0.91 \text{ V}$ 

Match E<sup>O</sup> of the redox pair in List I with the values given in List II and select the correct answer using the code given below the lists:

List I	List II
(P) E <sup>O</sup> (Fe <sup>3+</sup> , Fe)	(1) - 0.18 V
(Q) $E^{O}(4H_{2}O \rightleftharpoons 4H^{+} + 4OH^{-})$	(2) -0.8 V
(R) $E^{O}(Cu^{2+} + Cu \rightarrow 2Cu^{+})$	(3) -0.04 V
(S) $E^{O}(Cr^{3+}, Cr^{2+})$	(4) -0.83 V

51. Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option.

List- I	List- II
$(P) P2O3 + 3H2O \rightarrow$	(1) P(O)(OCH <sub>3</sub> )Cl <sub>2</sub>
(Q) $P_4$ + 3NaOH + 3H <sub>2</sub> O $\rightarrow$	(2) H <sub>3</sub> PO <sub>3</sub>
(R) PCI <sub>5</sub> + CH <sub>3</sub> COOH $\rightarrow$	(3) PH <sub>3</sub>
(S) $H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$	(4) POCI <sub>3</sub>
	(5) H <sub>3</sub> PO <sub>4</sub>

a) P 
$$ightarrow$$
 2; Q  $ightarrow$  3; R  $ightarrow$  1; S  $ightarrow$  5

b) P 
$$\rightarrow$$
 2; Q  $\rightarrow$  3; R  $\rightarrow$  4; S  $\rightarrow$  5

c) P 
$$\rightarrow$$
 3; Q  $\rightarrow$  5; R  $\rightarrow$  4; S  $\rightarrow$  2

d) P 
$$\rightarrow$$
 5; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3

# **JEE Advanced 2024**

# Sample Paper - 4

### **Solution**

## **Mathematics (MRQ)**

1. **(b)** 
$$\left(-\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

(d) 
$$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

**Explanation:** If slope of tangents to hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is m, then equations of tangent to

the hyperbola is  $y = mx \pm \sqrt{a^2m^2 - b^2}$  with the points of contact  $\left(\frac{\pm a^2m}{\sqrt{a^2m^2 - b^2}}, \frac{\pm b^2}{\sqrt{a}\frac{m^2 - b^2}}\right)$ 

- $\therefore$  Tangent to hyperbola  $\frac{x^2}{9} \frac{y^2}{4} = 1$  is parallel to 2x y = 1,
- ∴ Slope of tangent = 2
- $\therefore$  Points of contact are  $\left(\frac{\pm 9 \times 2}{\sqrt{9 \times 4 4}}, \frac{\pm 4}{\sqrt{9 \times 4 4}}\right)$

i.e. 
$$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$
 and  $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ 

- 2. (c) differentiable at x = 0 if a = 0 and b = 1
  - (d) differentiable at x = 1 if a = 1 and b = 0

**Explanation:**  $f(x) = a \cos(|x^3 - x|) + b |x| \sin(|x^3 + x|)$ 

a. If 
$$a = 0$$
,  $b = 1$ 

$$\Rightarrow f(x) = |x| \sin |x^3 + x|$$

$$= x \sin(x^3 + x)$$

Which is differentiable every where.

- b. (c) If a = 1,  $b = 0 \Rightarrow f(x) = \cos(|x^3 x|) = \cos(x^3 x)$ Which is differentiable every where.
- c. When a = 1, b = 1,  $f(x) = \cos(x^3 x) + x \sin(x^3 + x)$ Which is differentiable at x = 1

Hence only options (differentiable at x = 0 if a = 0 and b = 1) and (differentiable at x = 1 if a = 1 and b = 0) are the correct options.

3. **(a)** 
$$x^9 - f(x)$$

(c) 
$$x - \int\limits_0^{\frac{\pi}{2}-x} f(t) \cos t dt$$

**Explanation:** Let us check the given options one by one.

i. Let 
$$g(x) = x^9$$
-  $f(x)$ 

$$\Rightarrow$$
 g(0) = -f(0) < 0 [:: f(x) \in (0, 1)]

And 
$$g(1) = 1 - f(1) > 0$$

$$\therefore x^9$$
 - f(x) = 0 for some x  $\in$  (0, 1)

ii. Let 
$$\mathrm{h}(\mathrm{x}) = \mathrm{x} - \int\limits_0^{\frac{\pi}{2} - \mathrm{x}} \mathrm{f}(\mathrm{t}) \cos \mathrm{t} \ d\mathrm{t}$$

$$h(0) = -\int_{0}^{\frac{\pi}{2}} f(t) \cos t dt < 0$$

and h(1)= 
$$1-\int\limits_0^{\frac{\pi}{2}-1}f(t)\cos tdt>0$$

$$\therefore$$
 h(0) < 0 and h(1) > 0  $\Rightarrow$  h(x) = 0 at some x  $\in$  (0, 1)

$$\therefore \mathrm{h}(\mathrm{x}) = \mathrm{x} - \int\limits_{0}^{rac{\pi}{2} - \mathrm{x}} \mathrm{f}(\mathrm{t}) \cos \mathrm{t} d\mathrm{t} = 0$$

at some  $x \in (0, 1)$ 

iii. 
$$e^{\mathrm{x}}-\int\limits_{0}^{\mathrm{x}}\mathrm{f}(\mathrm{t})\sin\mathrm{t}d\mathrm{t}$$

$$\therefore x \in (0,1) \Rightarrow e^x \in (1,e)$$

and 0 < f(t) < 1 and  $0 < \sin t < 1$ ,  $\forall x \in (0, 1)$ 

$$\therefore 0 < \int\limits_0^{\mathrm{x}} \mathrm{f}(\mathrm{t}) \sin \mathrm{t} d\mathrm{t} < 1$$

$$\therefore e^x - \int_0^x f(t) \sin t dt \neq 0$$
 for any  $x \in (0, 1)$ 

$$\therefore e^x - \int_0^x f(t) \sin t dt \neq 0 \;\; ext{for any } x \in (0, 1)$$
iv.  $f(x) + \int_0^{rac{\pi}{2}} f(t) \sin t dt \; ext{is always positive} \; orall x \in (0, 1)$ 

### **Mathematics (MCQ)**

4.

(c) 
$$(\sqrt{3}, 2]$$

**Explanation:** Given, 
$$^{n-1}C_r = (k^2 - 3) ^n C_{r+1}$$

$$\Rightarrow$$
  $^{n-1}C_r=\left(k^2-3
ight)rac{n}{r+1}^{n-1}C_r$ 

$$\Rightarrow \quad k^2-3=rac{r+1}{n}$$
 [since,  $n\geq r\Rightarrowrac{r+1}{n}\leq 1$  and n, r > 0]

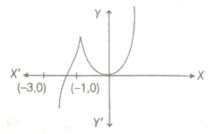
$$\Rightarrow \quad 0 < k^2 - 3 \leq 1 \Rightarrow 3 < k^2 \leq 4$$

$$\Rightarrow \quad k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$$

5.

**Explanation:** Given, 
$$f(x) = \left\{ \begin{array}{ll} (2+x)^3, & \text{if } -3 < x \leq -1 \\ x^{2/3}, & \text{if } -1 < x < 2 \end{array} \right.$$
  $\Rightarrow \quad f'(x) = \left\{ \begin{array}{ll} 3(x+2)^2, & \text{if } -3 < x \leq -1 \\ \frac{2}{3}x^{\frac{1}{3}} & , & \text{if } -1 < x < 2 \end{array} \right.$ 

$$\Rightarrow \quad f'(x) = \left\{ egin{array}{ll} 3(x+2)^2, & ext{if } -3 < x \leq -1 \ rac{2}{3}x^{rac{1}{3}} & , ext{ if } -1 < x < 2 \end{array} 
ight.$$



Clearly, f'(x) changes its sign at x = -1 from +ve to -ve and so f(x) has local maxima at x = -1.

Also, f'(0) does not exist but  $f'(0^-) < 0$  and  $f'(0^+) < 0$ .

It can only be inferred that f(x) has a possibility of a minima at x = 0. Hence, the given function has one local maxima at x = -1 and one local minima at x = 0.

6.

(c) None of these

**Explanation:** Since, vertices of a triangle are  $(0, \frac{8}{3})$ , (1, 3) and (82, 30)

Now, 
$$\frac{1}{2} \begin{vmatrix} 0 & \frac{8}{3} & 1 \\ 1 & 3 & 1 \\ 82 & 30 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \left[ -\frac{8}{3} (1 - 82) + 1(30 - 246) \right]$$
$$= \frac{1}{2} [216 - 216] = 0$$

... Points are collinear.

7

**(b)** 
$$\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$$

**Explanation:** Given:  $f(x) = \frac{x}{(1+x^n)^{1/n}}$  for  $n \geq 2$ 

$$\Rightarrow fof(x) = f[f(x)] = f\left[rac{x}{(1+x^n)^{1/n}}
ight]$$

$$=rac{rac{x}{(1+x^n)^{1/n}}}{\left\lceil 1+rac{x^n}{1+x^n}
ight
ceil^{1/n}}=rac{x}{(1+2x^n)^{1/n}}$$

Similarly, f o f o f(x) 
$$= \frac{x}{(1+3x^n)^{1/n}}$$

Proceeding in the same way, we get

g(x) = f o f o f .... o f(x) = 
$$\frac{x}{(1+nx^n)^{1/n}}$$
 (f occurs n times)

Now, 
$$\mathrm{I}=\int x^{n-2}g(x)dx=\intrac{x^{n-1}}{(1+nx^n)^{1/n}}dx$$

Let 
$$1 + nx^n = t \Rightarrow n^2x^{n-1}dx = dt$$

$$\therefore I = rac{1}{n^2} \int t^{-1/n} dt = rac{1}{n^2} \cdot rac{t^{-rac{1}{n}+1}}{-rac{1}{n}+1} + K$$

$$=rac{1}{n}\cdotrac{t^{1-1/n}}{n-1}+K=rac{(1+nx^n)^{1-1/n}}{n(n-1)}+K$$

## **Mathematics (NUM)**

8. 157

**Explanation:** 

AP (1, 3); 1, 4, 7, 10, 13 ...

AP (2, 5): 2, 7, 12, 17, 22 ...

AP (3, 7): 3, 10, 17, 24, 31 ...

For AP  $(1, 3) \cap AP (2, 5) \cap AP (3, 7)$ 

first term will be the minimum common value of a term

... we need to find that minimum number which when divided by 7 leaves remainder 3 (7m + 3)

and when divided by 5 leaves remainder 2 (5p + 2)

and when divided by 3 leaves remainder 1 (3q +1)

By hit and trial 52 is such number  $(7 \times 7 + 3)$ 

:. first term 'a' of intersection AP = 52

Also common difference 'd' of intersection AP

$$= LCM (7, 5, 3) = 105$$

9. 12.0

**Explanation:** 

Image of centre  $c_1 \equiv (1, 3)$  in x - y + 1 = 0 is given by

$$\frac{x_1 - 1}{1} = \frac{y_1 - 3}{-1} = \frac{-2(1 - 3 + 1)}{1^2 + 1^2}$$

$$\Rightarrow$$
 x<sub>1</sub> = 2, y<sub>1</sub> = 2

 $\therefore$  Centre of circle  $c_2 \equiv (2, 2)$ 

∴ Equation of c<sub>2</sub> be 
$$x^2 + y^2 - 4x - 4y + \frac{38}{5} = 0$$

Now radius of c<sub>2</sub> is  $\sqrt{4+4-\frac{38}{5}}$  r<sub>2</sub> =  $\sqrt{\frac{2}{5}}$ 

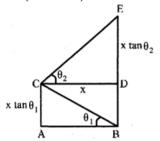
 $(radius of C_1)^2 = (radius of c_2)^2$ 

$$\Rightarrow$$
 10 -  $\alpha = \frac{2}{5}$   $\Rightarrow$   $\alpha = \frac{48}{5}$   $\therefore \alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$ 

10.6.0

**Explanation:** 

$$\sqrt{3}BE = 4AB \ {
m Ar}(\triangle CAB) = 2\sqrt{3} - 3$$



$$\frac{1}{2}x^2 \cdot \tan heta_1 = 2\sqrt{3} - 3$$

$$BE = BD + DE$$

$$=x\left( an heta_{1}+ an heta_{2}
ight)$$

$$\mathrm{BE} = \mathrm{AB} \left( \tan heta_1 + \cot heta_1 
ight) \left[ \because \sqrt{3} \, \mathsf{BE} = \mathsf{4} \, \mathsf{AB} \right]$$

$$rac{4}{\sqrt{3}}=( an heta_1+\cot heta_1)\Rightarrow an heta_1=\sqrt{3},rac{1}{\sqrt{3}}$$

$$heta_1=rac{\pi}{6},\quad heta_2=rac{\pi}{3} ext{ or } heta_1=rac{\pi}{3},\quad heta_2=rac{\pi}{6},$$

As 
$$\frac{\theta_2}{\theta_1}$$
 is largest  $\theta_1 = \frac{\pi}{6}$ ;  $\theta_2 = \frac{\pi}{3}$ 

$$\therefore x^2=rac{(2\sqrt{3}-3) imes 2}{ an heta_1}=rac{\sqrt{3}(2-\sqrt{3}) imes 2}{ anrac{\pi}{a}}$$
 [In (i)]

$$\Rightarrow x^2 = 12 - 6\sqrt{3} = (3 - \sqrt{3})^2 \Rightarrow x = 3 - \sqrt{3}$$

Now perimeter of  $\triangle CED = CD + DE + CE$ 

$$=3\sqrt{3}+(3-\sqrt{3})\sqrt{3}+(3-\sqrt{3})\times 2=6$$

11.5

**Explanation:** 

Given 8 vectors are

$$(1, 1, 1), (-1, -1, -1); (-1, 1, 1), (1, -1, -1); (1, -1, 1), (-1, 1, -1); (1, 1, -1), (-1, -1, 1)$$

These are the 4 diagonals of a cube and their opposites.

For 3 non-coplanar vectors first, we select 3 groups of diagonals and their opposite in  ${}^4C_3$  ways. Then one vector from each group can be selected in 2  $\times$  2  $\times$  2 ways.

$$\therefore$$
 Total ways =  ${}^4C_3 \times 2 \times 2 \times 2 = 32 = 2^5$ 

12.6.0

**Explanation:** 

$$f_2(x) = 98 (x - 1)^{50} - 600(x - 1)^{49} + 2450$$

$$\Rightarrow f_2(x) = 2 \times 49 \times 50(x - 1)^{49} - 50 \times 12 \times 49(x - 1)^{48}$$

$$= 50 \times 49 \times 2(x - 1)^{48} (x - 1 - 6)$$

$$= 50 \times 49 \times 2 (x - 1)^{48} (x - 7)$$

 $f_2$  (x) has local minimum at x = 7 and no local maxima

$$\Rightarrow$$
 m<sub>2</sub> = 1, n<sub>2</sub> = 0

$$= 6 m_2 + 4n_2 + 8 m_2n_2$$
  
=  $6 \times 1 + 4 \times 0 + 8 \times 1 = 6$ 

13.3

**Explanation:** 

Given equations are

$$xyzsin 3\theta = (y + z)cos3\theta ...(i)$$

$$xyzsin3\theta = 2zcos3\theta + 2y sin 3\theta$$
 ...(ii)

$$xyz \sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$$
 ...(iii)

On subtracting eq. (ii) from (i), we get

$$(\cos 3\theta - 2 \sin \theta)y - (\cos 3\theta)z = 0 ...(iv)$$

On subtracting eq. (i) from (iii), we get

$$\sin 3\theta y + (\cos 3\theta)z = 0 ...(v)$$

Eq. (iv) and (v) from the homogeneous system of linear equation.

But 
$$y \neq 0$$
,  $z \neq 0$ 

$$egin{array}{l} rac{\cos 3 heta - 2\sin 3 heta}{\sin 3 heta} = -rac{\cos 3 heta}{\cos 3 heta} \Rightarrow \cos 3 heta = \sin 3 heta \ \Rightarrow an 3 heta = 1 \Rightarrow 3 heta = n\pi + rac{\pi}{4} \Rightarrow heta = (4n+1)rac{\pi}{12}, n \in Z \end{array}$$

For 
$$heta \in (0,\pi) \Rightarrow heta = rac{\pi}{12}, rac{5\pi}{12}, rac{3\pi}{4}$$

... Three such solutions are possible.

## **Mathematics (MATCH)**

**Explanation:** (P) 
$$\to$$
 (1) :  $z_k = \cos \frac{2k\pi}{10} + i \sin \frac{2k\pi}{10}$ , k = 1 to 9

$$\therefore z_k = e^{irac{2k\pi}{10}}$$

Now 
$$z_k z_j = 1 \Rightarrow z_j = \frac{1}{z_k} = e^{-i \frac{2k\pi}{10}} = \overline{z_k}$$

We know if  $z_k$  is  $10^{th}$  root of unity so will be  $\bar{z}_k$ .

$$\therefore$$
 For every  $z_k$ , there exist  $z_i = \bar{z}_k$ 

Such that 
$$z_k \cdot z_j = z_k \cdot \bar{z}_k = 1$$

Hence the statement is true.

(Q) 
$$ightarrow$$
 (2) z<sub>1</sub>= z<sub>k</sub>  $\Rightarrow$   $z=rac{z_k}{z_1}$  for  $z_1
eq 0$ 

$$\therefore$$
 We can always find a solution of  $z_1.z = z_k$ 

Hence the statement is false.

(R) 
$$\rightarrow$$
 (3): We know  $z^{10}$  -1 = (z - 1)(z - z<sub>1</sub>)...(z - z<sub>9</sub>)

$$\Rightarrow$$
 (z - z<sub>1</sub>) (z - z<sub>2</sub>)...(z - z<sub>9</sub>) =  $\frac{z^{10}-1}{z-1}$ 

$$= 1 + z + z^2 + ...z^9$$

For 
$$z = 1$$
, we get  $(1 - z_1)(1 - z_2)...(1 - z_9) = 10$ 

$$\therefore \frac{|1-z_1||1-z_2|\dots|1-z_9|}{10} = 1$$

(S) 
$$\rightarrow$$
 (4): 1, Z<sub>1</sub>, Z<sub>2</sub>, ..., Z<sub>9</sub> are 10th roots of unity.

$$\therefore Z^{10} - 1 = 0$$

From equation 
$$1 + Z_1 + Z_2 + ... + Z_9 = 0$$
,

$$Re(1) + Re(Z_1) + Re(Z_2) + ... + Re(Z_9) = 0$$

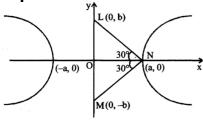
$$\Rightarrow \text{Re}(Z_1) + \text{Re}(Z_2) + ... \text{Re}(Z_9) = -1$$

$$\Rightarrow \sum_{K=1}^{9} \cos \frac{2k\pi}{10} = -1 \Rightarrow 1 - \sum_{K=1}^{9} \cos \frac{2k\pi}{10} = 2$$

Hence ((P) - (1), (Q) - (2), (R) - (3), (S) - (4)) is the correct option.

15. **(a)**  $P \rightarrow 4$ ;  $Q \rightarrow 3$ ;  $R \rightarrow 1$ ;  $S \rightarrow 2$ 

**Explanation:** Area of  $\triangle$ LMN =  $4\sqrt{3}$  (given)



$$\Rightarrow \frac{1}{2} \times \mathsf{LM} \times \mathsf{ON} = 4\sqrt{3} \Rightarrow \frac{1}{2}(\mathsf{2b})(\sqrt{3}b) = 4\sqrt{3}$$

$$\therefore b^2 = 4 \Rightarrow b = 2$$

So, length of the conjugate axis of hyperbola = 2b = 4

Now tan 30° = 
$$\frac{OL}{ON} = \frac{a}{b} \Rightarrow a = \sqrt{3}b \Rightarrow a = 2\sqrt{3}$$

$$\therefore b^2 = a^2 (e^2 - 1) \Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

$$\therefore$$
 The eccentricity of hyperbola = e =  $\frac{2}{\sqrt{3}}$  and

The distance between the foci of hyperbola = 2ae

$$=2\times2\sqrt{3}\times=\tfrac{2}{\sqrt{3}}=8$$

And length of latus ractum of hyperbola

$$=\frac{2b^2}{a}=\frac{2\times 4}{2\sqrt{3}}=\frac{4}{\sqrt{3}}$$

16. (a) (l) 
$$\rightarrow$$
 (Q); (ll)  $\rightarrow$  (S); (lll)  $\rightarrow$  (S); (lV)  $\rightarrow$  (R)

**Explanation:** We have system of linear equations

$$x + y + z = 1 ...(i)$$

$$10x + 100y + 1000z = 0$$

$$x + 10y + 100z = 0 ...(ii)$$

$$qrx + pry + pqz = 0 ...(iii)$$

$$\Rightarrow \frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$$
 (: p, q, r  $\neq 0$ )

Let 
$$p = \frac{1}{a+9d}$$
,  $q = \frac{1}{a+99d}$ ,  $r = \frac{1}{a+999d}$ 

Now, equation (iii) is

$$(a + 9d)x + (a + 99d)y + (a + 999d)z = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 10 & 100 \\ a + 9d & a + 99d & a + 999d \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 10 & 100 \\ 0 & a + 99d & a + 999d \end{vmatrix} = 900(d - a)$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 100 \\ a + 9d & 0 & a + 999d \end{vmatrix} = 990(a - d)$$

$$\Delta_z = egin{array}{cccc} 1 & 1 & 1 & 1 \ 1 & 10 & 0 \ a + 9d & a + 99d & 0 \ \end{bmatrix} = 90 (\mathsf{d} - \mathsf{a})$$

Let option I: If  $\frac{q}{r} = 10 \Rightarrow a = d$ 

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

Since eq. (i) and eq. (ii) represents non-parallel planes and eq. (ii) and eq. (iii) represents same plane

⇒ Infinitely many solutions

So, option I  $\rightarrow$  P, Q, R, T

Option II:  $\frac{p}{r} \neq 100 \Rightarrow a \neq d$ 

$$\Delta$$
 = 0,  $\Delta_x$ ,  $\Delta_y$ ,  $\Delta_z \neq$  0

No solution

So, option II  $\rightarrow$  S

Option III:  $\frac{p}{q} \neq 10 \Rightarrow a \neq d$ 

No solution

So, option III  $\rightarrow$  S

Option IV: If  $\frac{p}{a} = 10 \Rightarrow a = d$ 

Infinitely many solution

Hence, IV  $\rightarrow$  P, Q , R , T

17.

**(b)** (P) 
$$\rightarrow$$
 (5), (Q)  $\rightarrow$  (4), (R)  $\rightarrow$  (3), (S)  $\rightarrow$  (1)

**Explanation:** For largest possible distance between plane  $H_0$  and  $I_2$ , the line  $I_2$  must be parallel to plane  $H_0$ .

 $\therefore$  H<sub>0</sub> will be the plane containing the line I<sub>1</sub> and parallel to I<sub>2</sub>

Normal vector 
$$\overrightarrow{\mathbf{n}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{k}$$

∴ H<sub>0</sub> : x - z = 
$$\frac{c}{(0,0,0)}$$
 ⇒ c = 0

∴ 
$$H_0$$
:  $x - z = 0$ 

(P) Distance of point (0,1, -1) from H<sub>0</sub>.

d (H<sub>0</sub>) = 
$$\left| \frac{0 - (1)}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$$

- (Q) The distance of the point (0, 1, 2) from  $H_0 = \left| \frac{0-2}{\sqrt{2}} \right| = \sqrt{2}$
- (R) The distance of origin from  $H_0 = \left| \frac{0}{\sqrt{2}} \right| = 0$
- (S) Point of intersection of planes y = z, x = 1 and  $H_0$  is (1, 1, 1).

Distance 
$$=\sqrt{1+1+1}=\sqrt{3}$$
.

# Physics (MRQ)

**Explanation:** Since, ABCD is a square lamina hence by symmetry  $I_1 = I_2$  and  $I_3 = I_4$  From perpendicular axes theorem,

Moment of inertia about an axis perpendicular to square plate and passing from centre, O

$$I_0 = I_1 + I_2 = I_3 + I_4$$

or 
$$I_0 = 2I_2 = 2I_3 : I_2 = I_3$$

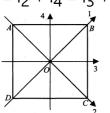
$$I_1 = I_2 = I_3 = I_4$$

Therefore, I<sub>O</sub> can be obtained by adding any two

i.e., 
$$I_0 = I_1 + I_2 = I_1 + I_3$$

$$= I_1 + I_4 = I_2 + I_3$$

$$= 1_2 + 1_4 = 1_3 + 1_4$$



- 19. **(a)** The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
  - (c) The length of the string is 0.25 m

**Explanation:**  $y = [0.01 \sin (62.8x)] \cos (6281)$ . [Given]

From the given equation, k  $= rac{2\pi}{\lambda} = 62.8$   $\therefore \lambda = rac{2\pi}{62.8} = 0.1 ext{ m}$ 

Length of string, l  $= 5 imes rac{\lambda}{2} = 5 imes rac{1}{20} = 0.25 \ \mathrm{m}$ 

The midpoint M is an antinode and has the maximum displacement = 0.01 m

The fundamental frequency, v =  $\frac{\rm v}{2l} = \frac{\frac{\omega}{k}}{2l} = \frac{628}{2\times0.25\times62.8} = 20~{\rm Hz}$ 

- 20. **(c)** For B =  $\frac{8}{13} \frac{p}{QR}$ , the particle will enter region 3 through the point P<sub>2</sub> on x axis.
  - **(d)** For B >  $\frac{2}{3} \frac{p}{QR}$ , the particle will re-enter region 1

## **Explanation:**

a. For the charge +Q to return region 1.

$$rac{mv^2}{\left(rac{3R}{2}
ight)}$$
 = QvB  $\Rightarrow rac{2p}{3R}$  = QB [Here, radius  $r=rac{3}{2}R$ ]

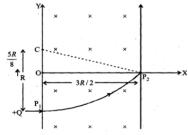
$$\therefore \mathsf{B} = \frac{2p}{3QR}$$

Therefore for B  $\geq \frac{2p}{2QR}$ , the particle will re-enter region 1.

b. When B =  $\frac{8p}{13QR}$ 

$$\frac{mv^2}{r} = \operatorname{Qv}\left(\frac{8p}{13QR}\right) : r = \frac{13R}{8}$$

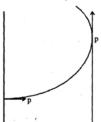
Thus 'C' is the of the centre of circular path of radius  $\frac{13R}{8}$ 



Also CP<sub>2</sub> = 
$$\sqrt{CO^2 + OP_2^2} \sqrt{\left(\frac{5R}{8}\right)^2 + \left(\frac{3R}{2}\right)^2}$$

$$\therefore CP_2 = \frac{13R}{8}$$

Thus the particle will enter region 3 through the point P<sub>1</sub> on X - axis



c. Change in momentum =  $\sqrt{2}$  p

d. Further  $\frac{mv^2}{r} = qvB : r \propto m$ 

i.e., Distance is directly proportional to mass.

## Physics (MCQ)

21.

(c) 
$$[FL^{-4} T^2]$$

**Explanation:** As, density =

$$[F]^a[L^b][T^c]$$

$$[ML^{-3}] = [MLT^{-2}]^a [L]^b [T]^c$$

$$[ML^{-3}] = [M^aL^aT^{-2a}L^bT^c]$$

$$[M^{1}L^{-3}] = [M^{a}L^{a+b}T^{-2a+c}]$$

On comparing

$$a = 1$$
,  $a + b = -3$ ,  $1 + b = -3$ ,  $b = -4$ 

$$-2a + c = 0 \Rightarrow c = 2a$$

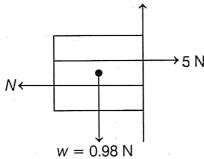
$$c = 2$$
: Density =  $[F^1L^{-4}T^2]$ 

22. (a) 0.98 N

## **Explanation:**

$$N = 5N$$

$$(f)_{max} = \mu N = (0.5)(5) = 2.5 N$$



For vertical equilibrium of the block,

$$f = mg = 0.98 N < (f)_{max}$$

23.

**Explanation:** Escape velocity, 
$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \mathrm{km/s}$$

For 
$$R' = \frac{R_e}{2}M' = 2M_e$$

$$v_e' = \sqrt{rac{2G(2M_e)}{rac{R_e}{2}}}$$
 = 2 $v_e$  = 22.4km/s

24.

(c) 
$$8.0 \times 10^{-19}$$
 C

**Explanation:** 
$$qE = mg ...(i)$$

$$6\pi\eta \ rv = mg$$

$$rac{4}{2}\pi r^3
ho g=mg$$
 ...(ii)

$$\therefore$$
  $r=\left(rac{3mg}{4\pi
ho g}
ight)^{1/3}$ 

Substituting the value of r in Eq. (ii) we get,

$$6\pi\eta v \Bigl(rac{3mg}{4\pi
ho g}\Bigr)^{1/3} = mg$$

or 
$$(6\pi\eta v)^3\left(rac{3mg}{4\pi
ho g}
ight)=(mg)^3$$

Again substituting mg = qE we get,

$$(qE)^2=\left(rac{3}{4\pi
ho g}
ight)(6\pi rv)^3$$

or 
$$qE=\left(rac{3}{4\pi
ho g}
ight)^{1/2}(6\pi r/v)^{3/2}$$

$$\therefore \quad q = \frac{1}{E} \left( \frac{3}{4\pi\rho q} \right)^{1/2} (6\pi r v)^{3/2}$$

Substituting the values we get,

$$q = \frac{7}{81\pi \times 10^5} \sqrt{\frac{3}{4\pi \times 900 \times 9.8} \times 216\pi^3} \times \sqrt{\left(18 \times 10^{-5} \times 2 \times 10^{-3}\right)^3}$$

$$= 8.0 \times 10^{-19} \text{ C}$$

## **Physics (NUM)**

### 25.2

**Explanation:** 

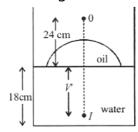
For the convex spherical refracting surface i.e., air-oil interface

u = -24 cm, v = ?, u<sub>1</sub> = 1, 
$$\mu_2 = \frac{7}{4}$$
 and R = 6 cm

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore \frac{-1}{(-24)} + \frac{\frac{7}{4}}{v} = \frac{\frac{7}{4} - 1}{6}$$

This image will not as object for the water-oil interface



u = 21 cm, v = 
$$v'$$
,  $\mu_1=rac{7}{4}$ ,  $\mu_2=rac{4}{3}$  anf R =  $\infty$ 

$$\frac{\frac{-7}{4}}{+21} + \frac{\frac{4}{3}}{V'} = 0$$

$$\therefore v' = 16 \text{ cm}$$

Therefore the distance of the image from the bottom of the tank = 18 - 16 = 2 cm

#### 26.0

**Explanation:** 

The heat required for 100 g of ice at  $0^{\circ}$  C to change into water at  $0^{\circ}$  C = mL =  $100 \times 80 \times 4.2$  = 33,600 J

The heat released by 300 g of water at 25° C to change its temperature to 0° C = mc $\Delta$ T = 300  $\times$  4.2  $\times$  25 = 31,500 J

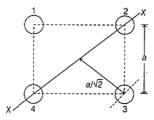
Hence complete ice will not melt, so the final temperature of the mixture will be 0°C.

#### 27.9

**Explanation:** 

$$r = \frac{d}{2} = \frac{\sqrt{5}}{2} \text{ cm} = \frac{\sqrt{5}}{2} \times 10^{-2} \text{ m} \Rightarrow \text{m} = 0.5 \text{ kg}$$

$$a = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$$



$$I_{XX} = I_1 + I_2 + I_3 + I_4$$

$$=\left[rac{2}{5}mr^2+m\Big(rac{a}{\sqrt{2}}\Big)^2
ight]+rac{2}{5}mr^2+\left[rac{2}{5}mr^2+m\Big(rac{a}{\sqrt{2}}\Big)^2
ight]+rac{2}{5}mr^2$$

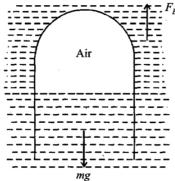
Substituting the values, we get

$$I_{XX} = 9 \times 10^{-4} \text{ kg m}^2$$

### 28. 0.3

**Explanation:** 

When tube + air system starts sinking



$$F_B = mg$$

$$\Rightarrow 
ho_0(V_{
m glass}\,+V_{
m gas}\,)$$
 = m

$$1(2 + V_{gas}) = 5$$

$$\Rightarrow$$
 V<sub>gas</sub> = 3cc

Hence 
$$\Delta V = V_0 - V_{gas}$$

$$= 3.3 \text{ cc} - 3\text{cc} = 0.3 \text{ cc}.$$

$$\therefore x = \Delta V = 0.3$$

### 29. 3.0

**Explanation:** 

$$K_{max}$$
 = E - W  $\Rightarrow$  E<sub>4</sub>  $\rightarrow$  3 =  $K_{max}$  + W = 1.95 +  $\frac{hc}{\lambda}$  = 1.95 +  $\frac{1240}{310}$  = 5.95 eV

$$13.6 \ \mathsf{Z}^2 \left( \frac{1}{3^2} - \frac{1}{4^2} \right) = 5.95$$

$$13.6 \ Z^2 \left(\frac{7}{9 \times 16}\right) = 5.95 \Rightarrow Z^2 = \frac{5.95 \times 9 \times 16}{13.6 \times 7} = 9$$

$$\therefore Z = 3$$

30. 3.0

**Explanation:** 

let us consider a point P on the circle

$$\begin{array}{c|c}
 & & & & P(x,y) \\
\hline
 & & & & & \\
\hline
 & & & & \\
\hline
 & & & & \\
 & & & & \\
\hline
 & & & \\
\hline
 & & & & \\
\hline
 &$$

$$\begin{aligned} & \mathsf{Vp} = 0 = \frac{k(-Q)}{r_1} + \frac{\frac{kQ}{\sqrt{3}}}{r_2} \Rightarrow \frac{kQ}{r_1} = \frac{\frac{kQ}{\sqrt{3}}}{r_2} \\ & \Rightarrow \frac{1}{\sqrt{x^2 + y^2}} - \frac{1}{\sqrt{3}\sqrt{(x - 2)^2 + y^2}} \\ & \Rightarrow 3(x - 2)^2 + 3y^2 = x^2 + y^2 \\ & \Rightarrow 3(x^2 + 4 - 4x) - x^2 + 2y^2 = 0 \Rightarrow 2x^2 + 12 - 12x + 2y^2 = 0 \\ & \Rightarrow x^2 + 6 - 6x + y^2 = 0 \Rightarrow (x - 3)^2 + y^2 = (\sqrt{3})^2 \\ & \mathsf{or} \ (x - b)^2 + y^2 = (\sqrt{3})^2 = R^2 \\ & \therefore \ \mathsf{R} = \sqrt{3} = 1.73 \ \mathsf{and} \ \mathsf{b} = 3 \end{aligned}$$

### **Physics (MATCH)**

31. (a) (l) 
$$\rightarrow$$
 (P); (II)  $\rightarrow$  (R); (III)  $\rightarrow$  (T); (IV)  $\rightarrow$  (Q)

### **Explanation:**

I. By first law of thermodynamics.

$$\begin{split} \Delta U &= \Delta Q - \Delta W \\ &= ML_V - P\Delta V \\ &= 10^{-3} \times 2250 \times 10^3 - 10^5 \times \left(10^{-3} - 10^{-6}\right) \\ &= 2250 \text{ - } 100 = 2150 \text{ J} \\ &= 2.15 \text{ kJ. So, (I)} \rightarrow \text{(P)} \\ \text{II. } P &= \frac{nRT}{V} = \frac{0.2 \times 8 \times 500}{V} = \frac{800}{V} \text{ Pa} \end{split}$$

. 
$$P = \frac{nRT}{V} = \frac{0.2 \times 8 \times 500}{V} = \frac{800}{V} \text{ Pa}$$

$$\Delta U = \frac{f}{2} P \Delta V = \frac{5}{2} \times \frac{800}{V} \times 2 \text{ V} = 4000 \text{ J} = 4 \text{ kJ}$$
So (II)  $\rightarrow$  (R)

III. 
$$ext{PV}^{\gamma} = ext{const} \ \Rightarrow ext{P}_1 \ ext{V}_1^{\gamma} = ext{P}_2 \ ext{V}_2^{\gamma} \ \Rightarrow 2 \ ext{V}^{\gamma} = ext{P}_2 \left( \frac{ ext{V}}{8} \right)^{\gamma} \Rightarrow ext{P}_2 = 2 \times 8^{\gamma} = 2 \times 8^{5/3} = 64 \ ext{kPa} \ ext{So, } \Delta U = rac{f}{2} (P_2 V_2 - P_1 V_1) \ = rac{3}{2} \left( 64 imes rac{1}{24} - 2 imes rac{1}{3} \right) imes 10^3 = 3 \ ext{kJ} \ ext{So, (III)} \ o (T)$$

IV. Here f = 7

So, 
$$\Delta U=nC_V\Delta T=\frac{t}{2}nR\Delta T=\frac{7}{2}nR\Delta T$$
 and,  $\Delta Q=nC_V\Delta T=\left(\frac{f}{2}+1\right)nR\Delta T=\frac{9}{2}nR\Delta T=\frac{9}{2}\times\frac{2}{7}\;\Delta U=\frac{9}{7}\Delta U$  So,  $\Delta U=\frac{7}{9}\Delta Q=\frac{7}{9}\times 9$  = 7 kJ. So (IV)  $\rightarrow$  (Q)

32.

**Explanation:** For double convex lens, (P)  $\frac{1}{f} = (\mu - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$   $\Rightarrow (1.5 - 1) \left( \frac{1}{r} - \frac{1}{r} \right) = (1.5 - 1) \left[ \frac{2}{r} \right] = \frac{1}{r} \Rightarrow f = r$   $\frac{1}{F_{\text{eq.}}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}$   $\therefore$   $F_{\text{eq}} = \frac{r}{2}$ 

For (Q) plano-convex lens 
$$\frac{1}{f}$$
 =  $(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ 

= 
$$(1.5 - 1) \left[ \frac{1}{\infty} - \frac{1}{-r} \right] = \frac{0.5}{r} = \frac{1}{2r} : f = 2r$$
  
 $\frac{1}{F_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{2r} + \frac{1}{2r} = \frac{2}{2r} = \frac{1}{r} : F_{eq} = r$ 

For (R) plano-concave lens

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-r} - \frac{1}{\infty}\right) \Rightarrow f = -2r$$

$$\frac{1}{F_{\mathrm{eq.}}}$$
 =  $\frac{1}{f}$  +  $\frac{1}{f}$  =  $\frac{1}{-2r}$  +  $\frac{1}{-2r}$   $\Rightarrow$  Feq. = -r

For (S) combination of one double convex and one planoconcave lens

$$\frac{1}{F_{\text{eq.}}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow F_{\text{eq.}} = 2r$$

**Explanation:** In  $B^+$  - decay mass number (Z) decreases by 1 and mass number (A) remains unchanged.

$$^{15}_{8}O \longrightarrow ^{15}_{7}N + ^{0}_{1}\beta$$

In  $\alpha$ -decay mass number (A) decreases by 4 unit and atomic number (Z) by 2 unit.

$$^{238}_{92}\mathrm{U} \longrightarrow ^{234}_{90}\mathrm{Th} + ^{4}_{2-\,\mathrm{particle}}\mathrm{He}$$

In proton  $\binom{1}{1}H$  emission both (A) and (Z) decreases by 1.

$$^{185}_{83}\mathrm{Bi}\longrightarrow ^{184}_{82}\mathrm{Pb}+^{1}_{1}\mathrm{H}$$

In fission process heavier nucleus breaks into two fragments.

$$^{239}_{94}\mathrm{Pu} \longrightarrow ^{140}_{57}\mathrm{La} + ^{99}_{37}\mathrm{X}$$

34.

(d) (l) 
$$\rightarrow$$
 (P), (II) $\rightarrow$  (R), (III) $\rightarrow$  (S), (IV) $\rightarrow$  (Q)

**Explanation:** Frequency,  $v = \frac{1}{2\ell} \sqrt{\frac{T}{m}}$  for first mode of vibration

For 'v' to be maximum, 'l' should be minimum.

String - 1 
$$f_0=rac{1}{2\,L_0}\sqrt{rac{T_0}{\mu}}$$

String - 2 
$$f_2=\frac{1}{2\,L_0}\sqrt{\frac{T_0}{2\mu}}=\frac{f_0}{\sqrt{2}}$$

String - 3 
$$f_3=rac{1}{2L_0}\sqrt{rac{T_0}{4\mu}}=rac{f_0}{\sqrt{3}}$$

String - 4 
$$f_4=\frac{1}{2\,L_0}\sqrt{\frac{T_0}{4\mu}}=\frac{f_0}{2}$$

## Chemistry (MRQ)

- 35. (a) 2-Hydroxypropane
  - (b) acetophenone

Explanation: lodoform reaction is given by the compounds containing - COCH3, -

CH(OH)CH<sub>3</sub> group and also CH<sub>3</sub>CH<sub>2</sub>OH and CH<sub>3</sub>CHO.

2-Hydroxypropane ( $CH_3CHOHCH_3$ ) contains the grouping  $CH_3CHOH$  - and acetophenone ( $C_6H_5COCH_3$ ) contains the grouping  $CH_3CO$  -.

- 36. (a) decreases the activation energy
  - (b) alters the reaction mechanism

**Explanation:** A catalyst provides a new path of lower activation energy. The catalyst reacts with the reactants to form an activated complex of low activation energy. The activated complex then decomposes to form the products along with regeneration of catalyst. Thus, the reaction mechanism changes completely.

- 37. (a)  $W_{isothermal} > W_{adiabatic}$ 
  - (c)  $T_1 = T_2$
  - (d)  $\Delta U_{isothermal} > \Delta U_{adiabatic}$

**Explanation:**  $T_1 = T_2$  because process is isothermal.

Work done in adiabatic process is less than in isothermal process because area covered by isothermal curve is more than the area covered by the adiabatic curve.

In adiabatic process expansion occurs by using internal energy, hence, it decreases while in isothermal process temperature remains constant, that's why no change in internal energy.

## **Chemistry (MCQ)**

38.

**(b)** 
$$N^{3-}$$
,  $O^{2-}$ ,  $F^{-}$  and  $Na^{+}$ 

**Explanation:** The species with its atomic number and number of electrons are as follows:

Species (ions)	At. no. (Z)	No. of electrons
N <sup>3</sup> -	7	7 + 3 = 10
O <sup>2-</sup>	8	8 + 2 = 10
F <sup>-</sup>	9	9 + 1 = 10
Na <sup>+</sup>	11	11 - 1 = 10
Li <sup>+</sup>	3	3 - 1 = 2
Mg <sup>2+</sup>	12	12 - 2 = 10

Thus, option ( $N^{3-}$ ,  $O^{2-}$ ,  $F^{-}$  and  $Na^{+}$ ) contains isoelectronic set of ions.

39.

## **(c)** AICI<sub>3</sub>

**Explanation:** AlCl<sub>3</sub> is more acidic in aqueous solution as on hydrolysis, it gives weak base and strong acid.

$$AICI_3 + 3H_2O \rightarrow AI(OH)_3 + 3HCI$$

40.

**Explanation:** 
$$\operatorname{Zn}^{2+} + 2\operatorname{H}_2\operatorname{O} \longrightarrow \operatorname{Zn}(\operatorname{OH})_2 \downarrow + 2\operatorname{H}^+$$

41.

# (c) BrCH2CH2COOH

### **Explanation:**

- i. The acidity increases with the increase in electronegativity of the halogen present.
- ii. The inductive effect decreases with increase in distance of halogen atom from the carboxylic group and hence, the strength of acid proportionally decreases.

Smallest dissociation constant means weakest acid, which is BrCH<sub>2</sub> CH<sub>2</sub>COOH because here Br (less electronegative than F) is two carbon atoms away from - COOH.

## **Chemistry (NUM)**

42. 3.2

**Explanation:** 

$$CH_{3}C - C \equiv CH \xrightarrow{333K, 100\%}$$

$$(0.1) \text{ mole} \qquad Kucherov \text{ reaction}$$

$$CH_{3} - C - CH_{3}$$

$$(S) \qquad Ba(OH)_{2}/\Delta$$

$$H_{3}C \qquad NaOC1 \qquad NaOC1 \qquad 80\%$$

$$H_{3}C \qquad C = CH - CO - OH + CHC1_{3}$$

$$H_{3}C \qquad (U)$$

From 0.1 mol of P, 0.032 mol of U is produced.

So, the value of  $y = 0.032 \times 100 = 3.2 g$ 

### 43.65.25

**Explanation:** 

Given, 
$$P^0 = 640$$
 mm,  $P_S = 600$ mm.

w = 2.175g, W = 390g, 
$$M_{C_{\theta}H_{6}}=78$$

$$\therefore \frac{(P^0 - P_S)}{P_S} = \frac{(w \times M)}{(m \times W)}$$

$$\therefore \frac{(640-600)}{600} = \frac{(2.175\times78)}{(m\times39)}$$

$$\therefore$$
 m = 65.25g mol<sup>-1</sup>

#### 44.557

**Explanation:** 

$$\Delta H = \Delta U + \Delta (PV) = \Delta U + V\Delta P \ (\because \Delta V = 0)$$
  
or  $\Delta U = \Delta H - V\Delta P = -560 - [1(40 - 70) \times 0.1]$   
= -560 + 3 = -557 kJ mol<sup>-1</sup>

So, the magnitude is 557 kJ mol<sup>-1</sup>

### 45. 4.14

**Explanation:** 

**Energy of Photon** 

$$=\frac{hc}{\lambda}\mathbf{J}=\frac{hc}{e\lambda}$$
eV  $=\frac{6.625\times10^{-34}\times3\times10^{8}}{300\times10^{-9}\times1.602\times10^{-19}}$  = 4.14 eV

For the photoelectric effect to occur, the energy of incident photons must be greater than the work function of the metal. Hence, only Li, Na, K, and Mg have work functions less than 4.14 V.

#### 46. -8500

**Explanation:** 

For the reaction,

$$A(g) + B(g) \hookrightarrow AB(g)$$

Given 
$$E_{ab} = E_{af} + 2RT$$
 or  $E_{ab} - E_{af} = 2RT$ 

Further

$$A_f = 4A_b \text{ or } \frac{A_f}{A_b} = 4$$

Now, the rate constant for forward reaction,

$$k_f = A_f e^{-E_{af}/RT}$$

Likewise, rate constant for backward reaction,

$$k_b = A_b e^{-E_{ab}/RT}$$

At equilibrium, Rate of forward reaction = Rate of backward reaction

i.e, 
$$\mathsf{k_f} = \mathsf{k_b} \ \mathsf{or} \ rac{k_f}{k_h} = k_{eq}$$

so k
$$_{eq}$$
 =  $rac{A_f e^{-E_{a_f}/RT}}{A_b e^{-E_{a_b}/RT}}$  =  $rac{A_f}{A_b} e^{-\left(E_{a_f}-E_{a_b}
ight)/RT}$ 

After putting the given values

$$k_{eq}=4e^2$$
 (as  $\mathsf{E}_{\mathsf{ab}}$  -  $\mathsf{E}_{\mathsf{af}}$  = 2RT and  $rac{A_f}{A_b}$  = 4)

Now, 
$$\Delta G^{\circ} = -RT \ln K_{\mathrm{eq}}$$
 = -2500 In (4e<sup>2</sup>)

$$= -25000 (ln 4 + ln e^{2}) = -2500 (1.4 + 2)$$

$$= -2500 \times 3.4 = -8500 \text{ J/mol}$$

Absolute value = -8500 J/mol

47.5

**Explanation:** 



### **Chemistry (MATCH)**

48. (a) A - (ii), B - (iii), C - (iv), D - (i)

**Explanation:** Density of CH<sub>2</sub>Cl<sub>2</sub> is greater than H<sub>2</sub>O. Therefore they can be separated by differential solvent extraction. Due to H-bonding in p-nitrophenol it can be separated from other component by column chromatography.

Due to different boiling point of kerosene and Naphthalene, it can be separated by fractional distillation. NaCl (ionic compound) and  $C_6H_{12}O_3$  can be separated by crystallisation.

49.

**Explanation:** (p)  $[FeF_6]^{4-}$ ,  $Fe^{2+} = 3d^6$  and  $F^-$  is weak field ligand

- $\therefore$  Hybridization is  $sp^3d^2$  (high spin complex)
- (q)  $[Ti(H_2O)_3Cl_3]$ ,  $Ti^{3+}=3d^1$  (No effect of ligand field strength)
- $\therefore$  Hybridization is  $d^2sp^3$
- (r)  $\left[\mathrm{Cr}(\mathrm{NH_3})_6\right]^{3+},\mathrm{Cr}^{3+}=3\text{d}^3$  (No effect of ligand field strength)
- $\therefore$  Hybridization is  $d^2sp^3$
- (s)  $[{
  m FeCl_4}]^{2-}, 3d^6$  and  ${
  m Cl}^-$  is weak field ligand
- $\therefore$  Hybridization is  $sp^3$
- (t)  $[Ni(CO)_4]$ ,  $Ni = 3d^{10}$  and CO is strong field ligand
- $\therefore$  Hybridization is  $sp^3$
- (w)  $[Ni(CN)_4]^{2-}$ ,  $Ni^{2+}=3d^8$  and CN is strong field ligand
- $\therefore$  Hybridization is  $dsp^2$
- 50. **(a)** (P) (3), (Q) (4), (R) (1), (S) (2)

**Explanation:** 

Fe<sup>3+</sup> 
$$\xrightarrow{44.77}$$
 Fe<sup>2+</sup>  $\xrightarrow{-4.44}$  Fe

p.  $XV$ 
 $n = 3$ 

$$\Delta G_{Fe^{3+}/Fe}^{o} = \Delta G_{Fe^{3+}/Fe^{2+}}^{o} + \Delta G_{Fe^{2+}/Fe}^{o}$$

$$\Rightarrow -3 \times FE_{(Fe^{+3}/Fe)}^{o} = -1 \times FE_{(Fe^{+3}/Fe^{+2})}^{o} + \left(-2 \times FE_{Fe^{+2}/Fe}^{o}\right)$$

$$\Rightarrow 3 \times x = 1 \times 0.77 + 2 \times (-0.44)$$

$$\Rightarrow x = -\frac{0.11}{3}V \simeq -0.04 V.$$

$$2H_2O \longrightarrow O_2 + 4H^+ + 4e^- \quad E^\circ = -1.23V$$
q.  $\frac{4e + O_2 + 2H_2O \longrightarrow 4OH^-}{4H_2O \longrightarrow 4H^+ + 4OH^-} \quad E^\circ = -0.83V$ 

$$Cu^{2+} + 2e \longrightarrow Cu \qquad E^\circ = +0.34V$$
r.  $\frac{2Cu \longrightarrow 2Cu^+ + 2e}{Cu^{2+} + Cu} \xrightarrow{-0.91V} Cr$ 
s.  $\frac{-0.74V}{n=1} \times Cr^{2+} \xrightarrow{-0.91V} Cr$ 
s.  $\frac{-0.74V}{n=2} \times Cr^{2+} \xrightarrow{-0.91V} Cr$ 

$$x - 1.82 = -2.22 \Rightarrow x = -0.4 V$$

51.

**(b)** P 
$$\to$$
 2; Q  $\to$  3; R  $\to$  4; S  $\to$  5

**Explanation:** (P)  $P_2O_3 + 3H_2O \rightarrow 2H_3PO_3$ 

(Q) 
$$P_4$$
 +  $3NaOH$  +  $3H_2O$   $\rightarrow$   $3NaH_2PO_2$  +  $PH_3$ 

(R) 
$$PCI_5 + CH_3COOH \rightarrow CH_3COCI + POCI_3 + HCI$$

(S) 
$$\text{H}_3\text{PO}_2$$
 +  $2\text{H}_2\text{O}$  +  $4\text{AgNO}_3 \rightarrow 4\text{Ag}$  +  $4\text{HNO}_3$  +  $\text{H}_3\text{PO}_4$