Inverse: ** (Inverse, adj, Determinent apply only on square matrices)

Q)

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}, \text{ find } A^{-1} = ?$$

** $\Rightarrow$ $$\boxed{A^{-1} = \frac{1}{|A|} \text{adj } A}$$

$$|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -1(1-9) + 2(1-6)$$
$$= 8 - 10$$
$$= -2$$

$\text{adj}(A) \rightarrow A^T \rightarrow [\ ]$

$$A^T = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & +1 & -1 \\ +8 & -6 & +1 \\ -5 & +3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 1 \\ -5 & 3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 1 \\ -5 & 3 & -1 \end{bmatrix}$$ Ans

** $\boxed{AA^{-1} = I}$

$\rightarrow$ every mtx not necessary to have inverse.

ex) find, x, y, z using inverse. $\boxed{|A| \neq 0}$

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$A^{-1} \cdot B = ?$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A$$

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix}$$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$= (1+1) - 2(-1+3)$$
$$= 2 - 4 = -2$$

$-1+3$

$-1-3$

Adj A

$$A^T = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$-2+1$

$2+1$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

$$A^{-1} \cdot B = \begin{bmatrix} -9+6+4 \\ 3-1 \\ 9-3-3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1, y = 2, z = 3$

Properties –

(I) $\text{adj}A^T = (\text{adj}A)^T$

(II) A is symmetric then Adj A is also symmetric.

(III) ~~non~~ if non-singular mtx 'A' is symmetric, then, A^{-1} is also symmetric.

non-singular means, determinant of 'A' is not 'zero'. $(\Delta \neq 0)$

(IV) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ $(\Delta \neq 0)$

(V) A and B two square mtx.
if $AB = I$ then $BA = I$.

(VI) $A^2 = A$ and $A \neq I$ then $|A| = 0$
idempotent

(VII) $AB = 0$ and $|B| \neq 0$, prove $A = 0$

$$AB = 0$$
$$\Rightarrow ABB^{-1} = 0 \cdot B^{-1}$$
$$\Rightarrow AI = 0$$
$$= A = 0$$

(VIII) $\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$.

(IX) $(AB)^{-1} = B^{-1}A^{-1}$ $(|A| \neq 0 \ \&\& \ |B| \neq 0)$
no-singular

(X) $(A^T)^{-1} = (A^{-1})^T$
$(|A| \neq 0)$