ICSE 2025 EXAMINATION

Sample Question Paper - 5

Mathematics

Time Allowed: 2 hours and 30 minutes **Maximum Marks: 80**

General Instructions:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes.
- This time is to be spent reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- **Attempt all questions from Section A and any four questions from Section B**.
- All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answers.
- Omission of essential work will result in a loss of marks.
- The intended marks for questions or parts of questions are given in brackets [ ]
- Mathematical tables are provided.

Section A

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) A retailer purchases a fan for ₹1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, the cost of the fan to the consumer inclusive of tax is: **[1]**

a) ₹1848 b) ₹1830

c) ₹1650 d) ₹1800

(b) A trader bought a number of articles for ₹1200. Ten were damaged and he sold each of the rest at ₹2 more than what he paid for it, thus cleaning a profit of ₹60 on the whole transaction. If x denotes the number of articles he bought, then the value of x is **[1]**

a) 60 b) 80

c) 110 d) 100

(c) When $x^3 - 3x^2 + 5x - 7$ is divided by x - 2, then the remainder is **[1]**

a) 0 b) -1

c) 2 d) 1

(d) If α and β are the roots of the equation $x^2 + x - 6 = 0$ such that $\beta > \alpha$, then the product of the matrices $\begin{bmatrix} 0 & \alpha \\ \alpha & \beta \end{bmatrix}$ and $\begin{bmatrix} \beta + 1 & 0 \\ -\beta & \alpha \end{bmatrix}$ is **[1]**

a) b)

$\begin{bmatrix} -5 & 4 \\ -9 & -2 \end{bmatrix}$ $\begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 4 \\ 9 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 6 & 13 \\ 9 & 6 \end{bmatrix}$

(e) An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is **[1]**

a) 3 b) 6

c) 5 d) 2

(f) If the image of the point P under the reflection in the X-axis is (-3, 2), then the coordinates of the point P are **[1]**

a) (-3, -2) b) (3, 2)

c) (-3, 0) d) (3, -2)

(g) O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q, then OP is equal to **[1]**

a) OP = $\frac{1}{3}$OQ b) OP = OQ

c) OQ = 2OP d) OP = 2OQ

(h) A sphere of radius a units is immersed completely in water contained in a right circular cone of semi-vertical angle 30° and water is drained off from the cone till its surface touches the sphere. Then, the volume of water remaining in the cone will be **[1]**

a) $\frac{5}{3}\pi a^2$ b) $\frac{5\pi}{3}a^3$

c) $\frac{\pi a^3}{3}$ d) $5\pi a^3$

(i) The maximum value of 23 - |2x + 3| is **[1]**

a) 23 b) 20

c) 26 d) 17

(j) The probability that a non leap year selected at random will have 53 Sundays is: **[1]**

a) 4/7 b) 2/7

c) 1/7 d) 3/7

(k) If both A + B and AB are defined, then which one of the following is true? **[1]**

a) A and B are square matrices of same order b) A and B are square matrices of different order

c) A and B are rectangular matrices of same order d) A and B are rectangular matrices of different order

(l) Join two points P(2, 2) and 0(4, 2) in the point P and rotate the line PQ in anti-clockwise direction at an angle of 270°. Then, the new coordinates of point Q and the area formed by this figure will be **[1]**

a) (2, 4); 2.94 sq units b) (4, 2); 4.92 sq units

c) (2, 4); 9.42 sq units d) (4, 2); 9.42 sq units

(m) The coordinates of the vertices of ΔABC are respectively (–4, –2), (6, 2) and (4, 6). The centroid G of ΔABC is: **[1]**

a) (2, 3) b) (0, –1)

c) (2, 2) d) (3, 3)

(n) If x < y < 2x, then the median and mean of x, y and 2x are 27 and 33, respectively. The mean of x and y is **[1]**

a) 25.2 b) 25.5

c) 25 d) 25.1

(o) **Assertion (A):** Three consecutive terms 2k + 1, 3k + 3 and 5k - 1 form an AP than k is equal to 6. **[1]**
Reason (R): In an AP a, a + d, a + 2d, ...the sum of n terms of the AP be $S_n = \frac{n}{2}(2a + (n-1)d)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

2. **Question 2** **[12]**

(a) Sanya has a Recurring Deposit Account in a bank of ₹2000 per month at the rate of 10% per annum. If she gets ₹83100 at the time of maturity, then find the total time for which the account was held. **[4]**

(b) Find the fourth proportional of the following. **[4]**

i. $3a^2b^2$, a^3, b^3

ii. $a^2 - 5a + 6$, $a^2 + a - 6$, $a^2 - 9$

(c) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, then prove that $q(p^2 - 1) = 2p$. **[4]**

3. **Question 3** **[13]**

(a) A solid cylinder of radius 7 cm and height 14 cm is melted and recast into solid spheres each of radius 3.5 cm. Find the number of spheres formed. **[4]**

(b) Three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2) find: **[4]**

i. the coordinates of the fourth vertex D.

ii. length of diagonal BD.

iii. equation of side AB of the parallelogram ABCD.

(c) Use graph paper to answer this question. **[5]**

i. Plot the points A(4, 6) and B(1, 2).

ii. A' is the image of A, when reflected in X-axis

iii. B' is the image of B, when B is reflected in X = 4.

iv. Give the geometrical name for the figure ABA'B'.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) Mr. Verma goes to a shop and buy a Jacket having cost ₹ 1180 (list price). The rate of GST 18%. He tells the shopkeeper to reduce the price such an extent that he has to pay ₹ 1180 inclusive of GST. Find the reduction needed in the price of the jacket. **[3]**

(b) Solve the following quadratic equation and give the answer correct to two significant figures. **[3]**

$4x^2 - 7x + 2 = 0$

(c) In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below: **[4]**

Marks	1	2	3	4	5	6	7	8	9	10
Number of students	1	2	3	3	6	10	5	4	3	3

Calculate the following for the given distribution:

i. Median

ii. Mode

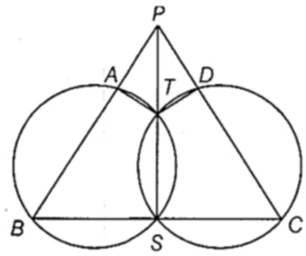
5. **Question 5** **[10]**

(a) If $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$, then find the following. **[3]**

i. 3A

ii. (-2)A

(b) In the given figure, two circles intersect at S and T. STP, BSC and BAP are straight lines. Prove that PATD is a cyclic quadrilateral. **[3]**



(c) Show that (x - 5) is a factor of $2x^2 - 9x - 5$. Hence, factorise $2x^2 - 9x - 5$. **[4]**

6. **Question 6** **[10]**

(a) The mid-point of the line joining (3a, 4) and (- 2, 2b) is (2, 2a + 2). Find the values of a and b. **[3]**

(b) Prove the following identities. **[3]**

i. (sec A - sin A) (cosec A + cos A) = $\sin^2$ A tan A + cot A

ii. (1 + cot A + tan A) (sin A - cos A) = $\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$

(c) Find the sum of all two-digit odd positive numbers. **[4]**

7. **Question 7** **[10]**

(a) A trader buys x articles for a total cost of ₹ 600. **[5]**

i. Write down the cost of one article in terms of x. If the cost per article were ₹ 5 more, the number of articles that can be bought for ₹ 600, would be four less.

ii. Write down the equation in x for the above situation and solve it to find x.

(b) The following distribution represents the height of 160 students of a school. **[5]**

Height (in cm)	**Number of Students**
140 - 145	12
145 - 150	20
150 - 155	30
155 - 160	38
160 - 165	24

165 - 170	16
170 - 175	12
175 - 180	8

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:

i. the median height
ii. the inter quartile range
iii. the number of students, whose height is above 172 cm.

8. **Question 8** **[10]**

(a) A child has a die whose six faces shows the letters as given below **[3]**

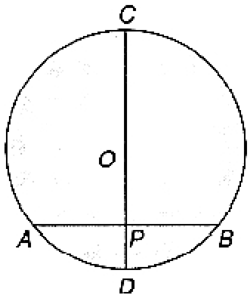
A B C D E F

The die is thrown once. What is the probability of getting

i. A
ii. D

(b) The total surface area of a sphere is 616 cm^2. Find the radius and volume of the sphere. [Take $\pi = \frac{22}{7}$] **[3]**

(c) In the figure given below, CD is the diameter of the circle which meets the chord AB at P such that AP = BP = 12 cm. If DP = 8 cm, find the radius of the circle. **[4]**



9. **Question 9** **[10]**

(a) An integer is such that one-third of the next integer is atleast 2 more than one-fourth of the previous integer. Find the smallest value of the integer. **[3]**

(b) The mean of the following data is 14. Find the value of k. **[3]**

x	5	10	15	20	25
f	7	k	8	4	5

(c) In $\triangle$ABC, D and E are points on the sides AB and AC respectively, such that DE || BC. If AD = 4x - 3, AE = 8x - 7, BD = 3x - 1 and CE = 5x - 3, then find the value of x. **[4]**

10. **Question 10** **[10]**

(a) If x : y :: y : z, prove that x : z :: $x^2 : y^2$. **[3]**

(b) Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and $\angle ABC = 120^o$. **[3]**

a. Construct circle circumscribing the triangle ABC.
b. Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C.

(c) The shadow of a vertical tower on a level ground increases by 10 m, when the altitude of the sun changes from 45^o to 30^o. Find the height of the tower correct to two decimal places. **[4]**

Solution

Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(a)** ₹1848

Explanation: {

Here, selling price of fan = ₹1650

GST on fan = 12% of ₹ 1650

= 1650 $\times \frac{12}{100}$

= 198

Thus, cost of a fan to the consumer inclusive of tax

= ₹(1650 + 198) = ₹1848

(ii) **(d)** 100

Explanation: {

As the CP of x articles is ₹1200

$\therefore$ CP of one article = ₹$\frac{1200}{x}$

As the selling price of each article is ₹2 more than its CP.

$\therefore$ SP of each article = ₹$\left(\frac{1200}{x}+2\right)$

Since 10 articles were damaged, therefore

Number of articles left for selling = x - 10

$\therefore$ SP of all articles (worth selling) = ₹(x - 10) $\left(\frac{1200}{x}+2\right)$

As the trader earns a net profit of ₹60.

$\therefore$ (x - 10) $\left(\frac{1200}{x}+2\right)$ = 1200 + 60

$\Rightarrow$ (x - 10) $\left(\frac{1200+2x}{x}\right)$ = 1260

$\Rightarrow$ (x - 10)(1200 + 2x) = 1260x

$\Rightarrow$ 1200x + 2x^2 - 12000 - 20x - 1260x = 0

$\Rightarrow$ 2x^2 - 80x - 12000 = 0

$\Rightarrow$ x^2 - 40x - 6000 = 0 [dividing both sides by 2]

$\Rightarrow$ x^2 - 100x + 60x - 6000 = 0 [splitting the middle term]

$\Rightarrow$ x (x - 100) + 60 (x - 100) = 0

$\Rightarrow$ (x - 100) (x + 60) = 0

$\Rightarrow$ x = 100 or x = - 60

$\therefore$ x = 100 [$\because$ number of articles cannot be negative]

(iii) **(b)** -1

Explanation: {

f(x) = x^3 - 3x^2 + 5x - 7

g(x) = x - 2, if x - 2 = 0, then x = 2

Remainder will be

$\therefore$ f(2) = $(2)^3$ - 3$(2)^2$ + 5 $\times$ 2 - 7

= 8 - 12 + 10 - 7 = 18 - 19 = -1

$\therefore$ Remainder = -1

(iv) **(b)** $\begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

Explanation: {

Given quadratic equation is x^2 + x - 6 = 0

$\Rightarrow$ x^2 + 3x - 2x - 6 = 0 [by splitting the middle term]

$\Rightarrow$ x(x + 3) - 2(x + 3) = 0

$\Rightarrow$ (x + 3) (x - 2) = 0

$\Rightarrow$ x = -3, 2

Also, given $\beta > \alpha$

$\therefore$ We take β = 2 and α = -3

Now, $\begin{bmatrix} 0 & \alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} \beta+1 & 0 \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} 0-\alpha\beta & 0+\alpha^2 \\ \alpha\beta+\alpha-\beta^2 & 0+\alpha\beta \end{bmatrix}$

$= \begin{bmatrix} -(-3)(2) & (-3)^2 \\ (-3)(2)-3-(2)^2 & (-3)(2) \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 \\ -6-3-4 & -6 \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 \\ -13 & -6 \end{bmatrix}$

(v) **(d)** 2

Explanation: {

Given, S_{11} = 33

$\Rightarrow \frac{11}{2}$ (2a + 10d) = 33 [$\because S_n = \frac{n}{2}$ [2a + (n - 1)d]

$\Rightarrow$ a + 5d = 3

i.e. $a_6 = 3 \Rightarrow a_4 = 2$ [$\because$ alternate terms are integers and the given sum is possible]

(vi) **(a)** (-3, -2)

Explanation: {

Clearly, point P will be the reflection of (-3, 2) in the X-axis. Thus, (-3, -2) is the required point.

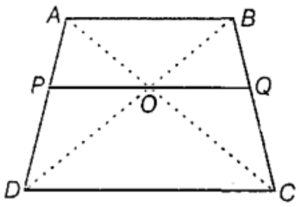
(vii) **(b)** OP = OQ

Explanation: {

Given ABCD is a trapezium. Diagonals AC and BD intersect at O.

$\therefore$ PQ || AB || DC

To prove PO = QO



Proof In $\triangle$ABD and $\triangle$POD,

PO || AB [$\because$ PQ || AB]

$\angle$ADB = $\angle$PDO [common angle]

$\angle$ABD = $\angle$POD [corresponding angles]

$\therefore \triangle ABD \sim \triangle POD$ [by AA similarity criterion]

Then, $\frac{OP}{AB} = \frac{PD}{AD}$ ...(i)

In $\triangle$ABC and $\triangle$OQC, OQ || AB [$\because$ PQ || AB]

$\angle$ACB = $\angle$OCQ [common angle]

and $\angle$BAC = $\angle$QOC [corresponding angles]

$\therefore \triangle ABC \sim \triangle OQC$ [by AA similarity criterion]

Then, $\frac{OQ}{AB} = \frac{QC}{BC}$ ...(ii)

Now, in $\triangle$ADC, OP || DC

$\therefore \frac{AP}{PD} = \frac{OA}{OC}$ ...(iii) [by basic proportionality theorem]

In $\triangle$ABC, OQ || AB

$\therefore \frac{BQ}{QC} = \frac{OA}{OC}$ ...(iv) [by basic proportionality theorem]

From Eqs. (iii) and (iv),

$\frac{AP}{PD} = \frac{BQ}{QC}$

On adding 1 to both sides, we get

$\frac{AP}{PD} + 1 = \frac{BQ}{QC} + 1$

$\Rightarrow \frac{AP+PD}{PD} = \frac{BQ+QC}{QC}$

$\Rightarrow \frac{AD}{PD} = \frac{BC}{QC}$

$\Rightarrow \frac{PD}{AD} = \frac{QC}{BC}$ [reciprocal the terms]

$\Rightarrow \frac{OP}{AB} = \frac{OQ}{AB}$ [from Eqs. (i) and (ii)]

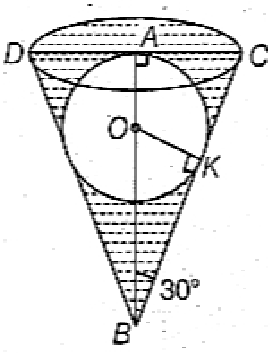
$\Rightarrow$ OP = OQ

(viii) **(b)** $\frac{5\pi}{3}a^3$

Explanation: {

Let radius of sphere be a, i.e. OK = OA = a.

Then, the centre O of a sphere will be centroid of the $\triangle$BCD



$\therefore$ OA = $\frac{1}{3}$AB $\Rightarrow$ AB = 3(OA)

In right angled $\triangle$OKB,

sin 30^o = $\frac{OK}{OB}$ = $\frac{a}{OB}$

$\Rightarrow \frac{1}{2} = \frac{a}{OB}$

$\Rightarrow$ OB = 2a

Now, AB = OA + OB = a + 2a = 3a

Now, in right angled $\triangle$BAC,

$\frac{AC}{AB}$ = tan 30^o $\Rightarrow \frac{AC}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ AC = $\frac{AB}{\sqrt{3}} = \frac{3a}{\sqrt{3}}$

$\therefore$ AC = $\sqrt{3a}$ units

Now, volume of a cone BCD = $\frac{1}{3}\pi$ (AC)$^2 \times$ AB

= $\frac{1}{3}\pi(a\sqrt{3})^2 \times 3a = 3\pi a^3$

$\therefore$ Volume of water remaining in the cone = Volume of the cone BCD - Volume of a sphere

= $3\pi a^3 - \frac{4}{3}\pi a^3 = \frac{5\pi}{3}a^3$ cu units

(ix) **(a)** 23

Explanation: {

Since, |x| is always non-negative.

$\therefore$ a - |x| $\leq$ a

So, the maximum value of 23 - |2x + 3| is 23.

(x) **(c)** 1/7

Explanation: {

Non-leap year contains 365 days = 364 days + 1 day weeks + 1 day = 52 weeks + 1 remaining day = 52 Sundays + 1 remaining day

We will have 53 Sundays if 1 remaining day is a Sunday.

Possible outcomes = {(Monday), (Tuesday), (Wednesday), (Thursday), (Friday), (Saturday), (Sunday)}

Number of Total outcomes = 7

Number of possible outcomes = 1

Required Probability= 1/7

(xi) **(a)** A and B are square matrices of same order

Explanation: {

Since, A + B is defined, therefore both A and B are of the same type.

Suppose that both A and B are of order m $\times$ n.

Also, AB is defined.

Thus, the number of columns in the pre-factor A must be equal to the number of rows in the post-factor B, i.e. n = m

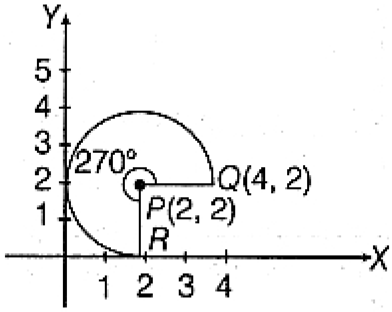
Hence, both A and B are of order n $\times$ n, i.e. A and B are square matrices of the same type.

(xii) **(d)** (4, 2); 9.42 sq units

Explanation: {

When we rotate the line PQ in anti-clockwise direction at an angle of 270°, then the new coordinates of point Q will be at R, which touches the X-axis at (2, 0).

Hence, the coordinates of R are (2, 0).



Now, PQ = $\sqrt{(4-2)^2+(2-2)^2}$ = $\sqrt{2^2+0}$ = 2 units [$\because$ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

Area of the figure = $\frac{3}{4}\pi r^2$ = $\frac{3}{4}$ $\times$ 3.14 $\times$ 4 = 9.42 sq units

(xiii) **(c)** (2, 2)

Explanation: {

Centroid : x= $(x_1+x_2+x_3)/3$

y= $(y_1+y_2+y_3)/3$

(xiv) **(b)** 25.5

Explanation: {

As x, y and 2x are in ascending order, therefore median is y.

$\therefore$ y = 27

Also, mean = $\frac{x+y+2x}{3}$ = 33

$\Rightarrow$ $\frac{3x+27}{3}$ = 33 $\Rightarrow$ x = 24

$\therefore$ Mean of x and y = $\frac{x+y}{2} = \frac{24+27}{2} = \frac{51}{2}$ = 25.5

(xv) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: {

For 2k + 1, 3k + 3 and 5k - 1 to form an AP

(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)

k + 2 = 2k - 4

2 + 4 = 2k - k = k

k = 6

So, both assertion and reason are correct but reason does not explain assertion.

2. Question 2

(i) p = ₹ 2000 per month

r = 10%

m.v. = ₹83,100

n = ?

m.v = $pn + \frac{prn(n+1)}{2400}$

$\Rightarrow$ 83,100 = 2000 $n + \frac{2000\times10\times n(n+1)}{2400}$

$\Rightarrow$ 83,100 = $2000n + \frac{25n(n+1)}{3}$

$\Rightarrow$ 83,100 = $2000n + \frac{25n^2+25n}{3}$

$\Rightarrow$ 83,100 $\times$ 3 = 6000n + $25n^2$ + 25n

$\Rightarrow$ $25n^2 + 6025n - 249300 = 0$

$\Rightarrow$ $25(n^2 + 241n - 9972) = 0$

$\Rightarrow$ $n^2 + 241n - 9972 = 0$

$\Rightarrow$ n = $\frac{-241 \pm \sqrt{(241)^2 - 4(1)(-9972)}}{2 \times 1}$

$n = \frac{-241 \pm \sqrt{97969}}{2}$

$n = \frac{-241 \pm 313}{2}$

n = $\frac{-241 + 313}{2}$

n = 36 months

n = 3 years

Neglecting -ve value. As no. of months can't be -ve.

(ii) i. $3a^2b^2, a^3, b^3$

Let the fourth proportional to $3a^2, a^3, b^3$ be x.

$\Rightarrow$ $3a^2b^2 : a^3 :: b^3 : x$

$\Rightarrow$ $3a^2b^2 \times x = a^3 \times b^3$

$\Rightarrow$ x = $\frac{a^3 \times b^3}{3a^2b^2}$

$\Rightarrow$ x = $\frac{1}{3}$ab

ii. $a^2 - 5a + 6, a^2 + a - 6, a^2 - 9$

Let the fourth proportional to $a^2 - 5a + 6$, $a^2 + a - 6$ and $a^2 - 9$ be x.

$\Rightarrow$ $a^2 - 5a + 6 : a^2 + a - 6 :: a^2 - 9 : x$

$\Rightarrow$ $(a^2 - 5a + 6)\, x = (a^2 + a - 6)(a^2 - 9)$

$\Rightarrow x = \frac{(a^2+a-6)(a^2-9)}{(a^2-5a+6)}$

$x = \frac{(a^2+3a-2a-6)(a^2-3^2)}{a^2-3a-2a+6}$

$x = \frac{[a(a+3)-2(a+3)](a+3)(a-3)}{a(a-3)-2(a-3)}$

$= \frac{(a+3)(a-2)(a+3)(a-3)}{(a-3)(a-2)}$ = $(a + 3)^2$

x = $(a + 3)^2$

(iii) Given, sin θ + cos θ = p

sec θ + cosec θ = q

then prove that $q(p^2 - 1) = 2p$

sec θ + cosec θ = q

$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q$

$\Rightarrow$ $\frac{\sin\theta+\cos\theta}{\sin\theta\cdot\cos\theta} = q$ [given, sin θ + cos θ = p]

$\Rightarrow$ $\frac{p}{\sin\theta\cdot\cos\theta} = q$

LHS.

$q(p^2 - 1)$

$\Rightarrow$ $\frac{p}{\sin\theta\cdot\cos\theta}\{(\sin\theta + \cos\theta)^2 - 1\}$

$\Rightarrow$ $\frac{p}{\sin\theta\cdot\cos\theta}(\sin^2\theta \cos^2\theta + 2 \sin\theta\cdot\cos\theta - 1)$

$\Rightarrow$ $\frac{P}{\sin\theta-\cos\theta}(1 + 2 \sin\theta\cdot\cos\theta - 1)$

$\Rightarrow$ $\frac{p}{\sin\theta-\cos\theta}(2\sin\theta \cdot \cos\theta)$

$\Rightarrow$ 2p

Here, LHS = RHS

$q(p^2 - 1) = 2p$ Proved

3. Question 3

(i) Given, radius of cylinder (r) = 7 cm

height (h) = 14 cm

radius of one sphere (R) = 3.5 cm

Let the number of spheres formed be n.

$\therefore$ Volume of n spheres formed = Volume of cylinder.

$n \times \frac{4}{3}\pi R^3 = \pi r^2 h$

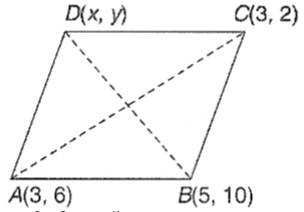
$n \times \frac{4}{3} \times \pi \times (3.5)^3 = \pi \times 7 \times 7 \times 14$

n = $\frac{7\times7\times14\times3}{4\times3.5\times3.5\times3.5}$ = 12

Hence, number of spheres formed = 12

(ii) i. Let the fourth vertex be D(x, y), then

Mid point of AC = Mid point of BD



$\therefore \frac{3+3}{2} = \frac{5+x}{2}$ (for x)

(as mid points are equal)

and $\frac{6+2}{2} = \frac{10+y}{2}$ (for y)

$\therefore$ x = 1 and y = -2

$\therefore$ D(1, -2).

ii. Length of BD = $\left|\sqrt{(10+2)^2+(5-1)^2}\right|$

[By using distance formula, $\left|\sqrt{(x_2-x_1)^2+y_2-y_1)^2}\right|$]

= $\left|\sqrt{12^2+4^2}\right| = |\sqrt{160}|$

= $4\sqrt{10}$ units

iii. Slope of AB = $\frac{10-6}{5-3} = \frac{4}{2}$ = 2

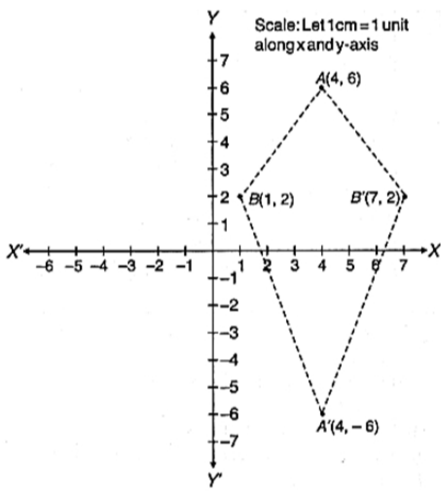
$\therefore$ Eqn. of AB : y - 6 = 2(x - 3)

[By using equation of a line, y - y_1 - m(x - x_1)]

y - 6 = 2x - 6

or 2x - y = 0

(iii)

i. 

ii. The image of A on reflection in X-axis is A'(4, - 6)

iii. The image of B on reflection in the line AA' is B' (7, 2)

iv. ABA'B' is kite.

Section B

4. Question 4

(i) Let the reduced price of jacket be ₹x.

Then, amount of GST on ₹x = 18% of x = $\frac{18}{100} \times$ x

$\therefore$ Mr. Verma pays the amount for jacket,

= ₹x + ₹ $\frac{18}{100}$x = ₹(1 + $\frac{18}{100}$)x

$= ₹ \left(1+\frac{9}{50}\right)x = ₹ \left(\frac{59}{50}\right)x$

According to the given condition,

$\frac{59}{50}x$ = 1180

$\Rightarrow$ x = $\frac{1180 \times 50}{59}$ = ₹1000

$\therefore$ Reduced price of the jacket = ₹1000

Thus, the reduction needed in the price of jacket

= ₹(1180 - 1000) = ₹180

(ii) $4x^2 - 7x + 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

a = 4, b = -7, c = 2

D = b^2 - 4ac

= $(-7)^2 - 4 \times 4 \times 2$

= 49 - 32

= 17

Roots are

x = $\frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow$ x = $\frac{7 \pm \sqrt{17}}{2 \times 4}$

$\Rightarrow$ x = $\frac{7 \pm \sqrt{17}}{8} = \frac{7 \pm 4 \cdot 123}{8}$

$\Rightarrow$ Either x = $\frac{7 + 4 \cdot 123}{8}$ or $\frac{7 - 4 \cdot 123}{8}$

$\Rightarrow$ x = $\frac{11 \cdot 123}{8}$ = 1.39 = 1.4

or, x = $\frac{2 \cdot 877}{8}$ = 0.359 = 0.36

$\Rightarrow$ x = 1.4

or, x = 0.36

(iii)

Marks (x)	**No. of students (f)**	**cf**
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40
	Σf = 40	

i. Median = 6

ii. Mode = 6

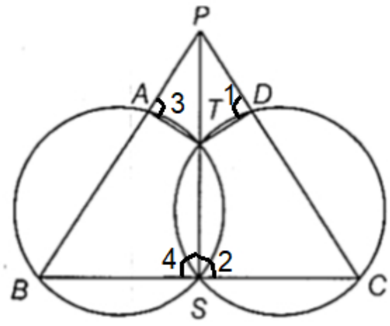
5. Question 5

(i) Given, $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

i. 3 A = $3 \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 \times 3 & -1 \times 3 \\ 0 \times 3 & 3 \times 3 \end{bmatrix}$ = $\begin{bmatrix} 6 & -3 \\ 0 & 9 \end{bmatrix}$

ii. (-2) A = $(-2) \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 \times 2 & -2 \times (-1) \\ -2 \times 0 & -2 \times 3 \end{bmatrix}$ = $\begin{bmatrix} -4 & 2 \\ 0 & -6 \end{bmatrix}$

(ii)



Let $\angle 1 = \angle PDT$

$\angle 2 = \angle TSC$

$\angle 3 = \angle PAT$

$\angle 4 = \angle TSB$

Here, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ {$\because$ exterior angle of cyclic quadrilateral is equal to the interior opposite angle}

$\angle 2 + \angle 4 = 180^o$ {$\because$ BSC is a straight line}

$\therefore \angle 1 + \angle 3 = 180^o$

Hence PATD is a cyclic quadrilateral.

{$\because \angle 1 + \angle 3 = 180^o$}

Hence proved.

(iii) Let $p(x) = 2x^2 - 9x - 5$ and $g(x) = x - 5$

Now, $g(x) = 0$

$x - 5 = 0, x = 5$

By factor theorem, g(x) will be factor of p(x) if p(5) = 0

And, $p(5) = 2 \times (5)^2 - 9 \times 5 - 5$

$= 50 - 45 - 5 = 50 - 50 = 0$

p(5) = 0, So g(x) is a factor of p(x)

Now, $2x^2 - 9x - 5$

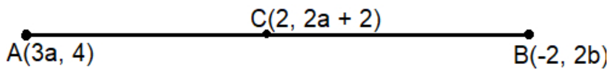
$2x^2 - 10x + x - 5$ [$\because (-5) \times 2x^2 = -10x^2$ and $-10x \times x = 10x^2$]

$2x(x - 5) + 1(x - 5)$

$(2x + 1)(x - 5)$

6. Question 6

(i) Let the coordinate of A and B are (3a, 4) and (-2, 2b) respectively.



C divides, AB in the ratio 1 : 1

$2 = \frac{3a-2}{2}$

$\Rightarrow 3a - 2 = 4$

$\therefore a = 2$

Again, $2a + 2 = \frac{4+2b}{2}$

$\Rightarrow 4 + 2 = \frac{4+2b}{2}$

$\Rightarrow 4 + 2b = 12$

$\therefore b = 4$

Hence the value of a and b are 2 and 4.

(ii) i. LHS = (sec A - sin A) (cosec A + cos A)

$= \left(\frac{1}{\cos A} - \sin A\right)\left(\frac{1}{\sin A} + \cos A\right)$

$= \left(\frac{1-\sin A \cos A}{\cos A}\right)\left(\frac{1+\sin A \cos A}{\sin A}\right)$

$= \frac{(1-\sin^2 A \cos^2 A)}{\sin A \cos A}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$= \frac{\sin^2 A+\cos^2 A-\sin^2 A \cos^2 A}{\sin A \cos A}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \frac{\sin^2 A-\sin^2 A \cos^2 A+\cos^2 A}{\sin A \cos A}$

$= \frac{\sin^2 A(1-\cos^2 A)+\cos^2 A}{\sin A \cos A}$

$= \frac{\sin^2 A \sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{\sin^4 A + \cos^2 A}{\sin A \cos A}$

$= \frac{\sin^4 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A} = \frac{\sin^3 A}{\cos A} + \frac{\cos A}{\sin A}$

$= \sin^2 A \cdot \frac{\sin A}{\cos A}$ + cot A

= $\sin^2$ A · tan A + cot A = RHS

Hence proved.

ii. LHS = (1 + cot A + tan A) (sin A - cos A)

$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)$ (sin A - cos A)

$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A}\right)$ (sin A - cos A)

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin A \cos A}$

$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$ [∵ (a - b)(a^2 + b^2 + ab) = a^3 - b^3] ...(i)

and RHS = $\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}}$

$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} = \frac{\sin^3 A - \cos^3 A}{\cos A \sin A}$ ...(ii)

From Eqs. (i) and (ii),

LHS = RHS

Hence proved.

(iii) 11 + 13 + 15 + ... + 99 = ?

a = 11, d = 2, a_n = 99

a_n = a + (n - 1)d

99 = 11 + (n - 1)(2)

88 = (n - 1)2

(n - 1) = 44

n = 44 + 1

n = 45

$S_n = \frac{n}{2}$[a + l]

$S_{45} = \frac{45}{2}$[11 + 99]

$= \frac{45}{2} \times 110$

= 2475

7. Question 7

(i) Number of articles = x

The total cost of article = ₹600

$\frac{600}{x-4} - \frac{600}{x} = 5$

$600\left[\frac{1}{x-4} - \frac{1}{x}\right] = 5$

$\frac{x-x+4}{x(x-4)} = \frac{5}{600}$

$\frac{4}{x(x-4)} = \frac{1}{120}$

480 = x^2 - 4x

x^2 - 4x - 480 = 0

x^2 - 24x + 20x - 480 = 0

x(x - 24) + 20(x - 24) = 0

(x - 24)(x + 20) = 0

x - 24 = 0; x + 20 = 0

x = 24, x = -20

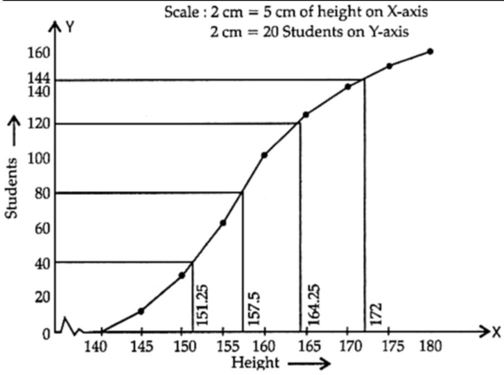
Since the number of article cannot be negative.

So x = 24

(ii) The cumulative frequency table for the given continuous distribution is given below:

Height	Number of Students	Cumulative frequency (c.f.)
140 - 145	12	12

145 - 150	20	32
150 - 155	30	62
155 - 160	38	100
160 - 165	24	124
165 - 170	16	140
170 - 175	12	152
175 - 180	8	160
	N = 160	



i. Median = $\frac{N}{2}$ th term = $\left(\frac{160}{2}\right)^{th}$ term = 80^{th} term

   Median = 157.5 cm

ii. Lower quartile, Q_1 = $\frac{N}{4}$ th term = $\frac{160}{4}$ th term

   = 40th term = 151.25

   Upper quartile, Q_3 = $\left(\frac{3N}{4}\right)^{th}$ term = $\frac{3\times160}{4}$

   = 120^{th} term = 164.25

   Inter quartile range = Q_3 - Q_1

   = 164.25 - 151.25

   = 13

iii. The number of students, whose height is above 172 cm

   = 160 - 144

   = 16.

8. Question 8

(i) n(s) = 6' = 6

$\because$ We know that:

When 'n' dice are throw, total no. of sample space is equals to 6^n

i.e n(s) = 6^n

But here only one dice is thrown, $\therefore$ n = 1

So n(s) = 6' = 6

The only differs is instead of no. letters are written here.

Let E_1 be the event of getting 'A' and E_2 be the event of getting D

$P(E_1) = \frac{n(E_1)}{n(s)} = \frac{1}{6}$

$P(E_1) = \frac{n(E_2)}{n(s)} = \frac{1}{6}$

(ii) Surface area of sphere = 616 cm^2

r = ?, V = ?

$\because$ Surface area of sphere = $4\pi r^2$

616 = $4 \times \frac{22}{7} \times r^2$

$r^2 = \frac{616\times7}{88}$

$r^2 = 49$

r = 7 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

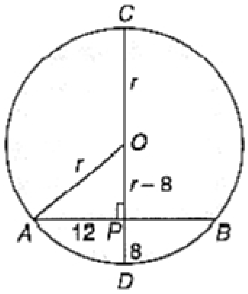
$= \frac{4}{3} \times \frac{22}{7} \times 7^3$

= 205.33 cm^3

Radius of sphere = 7 cm and volume of sphere = 205.33 cm

(iii) Let the radius of circle be r cm.

$\therefore$ OP = r - 8



If a diameter bisects a chord, then it must be perpendicular to the chord.

$\therefore \angle APO = 90°$

In $\triangle APO$ $r^2 = (12)^2 + (r - 8)^2$

(Using Pythagoras theorem)

$\Rightarrow r^2 = 144 + r^2 - 16r + 64$

$\Rightarrow$ 16r = 208

$\Rightarrow r = \frac{208}{16}$

= 13 cm

9. Question 9

(i) Let the integer be x, then one-third of the next integer is $\frac{x+1}{3}$ and one-fourth of the previous integer is $\frac{x-1}{4}$.

According to the question,

$\frac{x+1}{3} \geq \frac{x-1}{4} + 2 \Rightarrow \frac{12(x+1)}{3} \geq \frac{12(x-1)}{4} + 2 \times 12$ [multiplying both sides by 12]

$\Rightarrow 4(x + 1) \geq 3(x - 1) + 24$

$\Rightarrow 4x + 4 \geq 3x - 3 + 24$

$\Rightarrow 4x + 4 - (3x + 4) \geq 3x + 21 - (3x + 4)$ [subtracting (3x + 4) from both sides]

$\Rightarrow 4x + 4 - 3x - 4 \geq 3x + 21 - 3x - 4$

$\Rightarrow x \geq 17$

Hence, the smallest value of x is 17.

(ii) Table for the given data is

x_i	f_i	$f_i x_i$
5	7	35
10	k	10k
15	8	120
20	4	80
25	5	125
Total	$\sum f_i = k + 24$	$\sum f_i x_i = 10k + 360$

Here, $\sum f_i = k + 24$ and $\sum f_i x_i = 10k + 360$

Given, mean = 14

$\therefore \frac{\Sigma f_i x_i}{\Sigma f_i} = 14 \Rightarrow \frac{10k+360}{k+24} = 14$

$\Rightarrow 10k + 360 = 14(k + 24)$

$\Rightarrow 10k + 360 = 14k + 336$

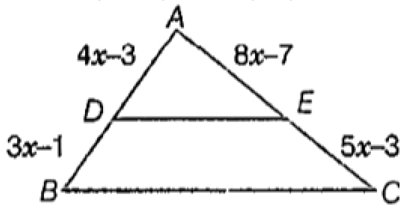
$\Rightarrow$ 14k - 10k = 360 - 336

$\Rightarrow$ 4k = 24 $\Rightarrow$ k = $\frac{24}{4}$ = 6

(iii) Given, in $\triangle$ABC, DE || BC

By Thales theorem, we get

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$ [$\because$ AD = 4x - 3, DB = 3x - 1, AE = 8x - 7, EC = 5x - 3]



$\Rightarrow$ (4x - 3) (5x - 3) = (8x - 7) (3x - 1)

$\Rightarrow$ $20x^2$ - 12x + 9 - 15x = $24x^2$ - 21x - 8x + 7

$\Rightarrow$ $4x^2$ - 2x - 2 = 0

$\Rightarrow$ $2x^2$ - x - 1 = 0 [dividing both sides by 2]

$\Rightarrow$ $2x^2$ - 2x + x - 1 = 0 [by splitting the middle term]

$\Rightarrow$ 2x(x - 1) + 1(x - 1) = 0 $\Rightarrow$ (2x + 1) (x - 1) = 0

$\therefore$ x = $-\frac{1}{2}$ or x = 1

If x = $-\frac{1}{2}$, then AD = 4 $\times$ $-\frac{1}{2}$ - 3 = -5 < 0 [not possible]

Hence, x = 1 is the required value.

10. Question 10

(i) Let $\frac{x}{y} = \frac{y}{2}$ = k

x = yk and y = kz

L.H.S.

$\frac{x}{2} = \frac{yk}{(\frac{y}{k})}$

$\frac{x}{2}$ = k^2

R.H.S.

$\frac{x^2}{y^2} = \frac{(yk)^2}{(kz)^2}$

$= \frac{y^2k^2}{k^2z^2}$

$= \frac{\left(k^2z^2\right)k^2}{k^2z^2}$

= k^2 : LHS

$\because \frac{x}{z} = \frac{x^2}{y^2}$ = k^2

$\therefore$ x : z : : x^2 : y^2

Proved

(ii) a. **Steps of construction:**

i. Draw a line segment BC = 6 cm.

ii. Construct $\angle$CBX = 120^o.

iii. Cuts BX at 5.5 cm.

iv. Join A to C.

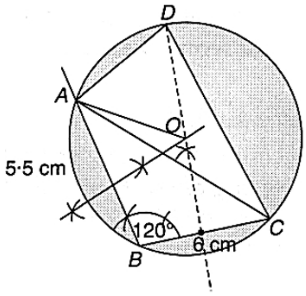
v. Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO.

vi. Taking O as centre, and OA as radius draw a circle, passing through A, B, and C.

b. i. Extend the right bisector of BC, which intersect the circle at D.

ii. Join A to D and C to D.

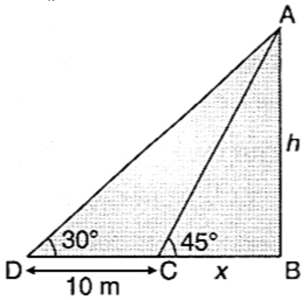
iii. ABCD is required cyclic quadrilateral.



(iii)Let the height of the tower is h m.

In $\triangle$ABC, tan 45^o = $\frac{AB}{BC}$

1 = $\frac{h}{x}$



x = h ...(i)

In $\triangle$ADB, tan 30^o = $\frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{10+x}$

10 + x = $\sqrt{3}h$

10 + h = $\sqrt{3}h$ [from (i)]

$\sqrt{3}h$ - h = 10

h($\sqrt{3}$ - 1) = 10

h = $\frac{10}{\sqrt{3}-1}$

$= \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{10\times(\sqrt{3}+1)}{2}$

= 5 $\times$ ($\sqrt{3}$ + 1)

= 5(1.732 + 1)

= 5 $\times$ 2.732

= 13.66 m