CHAPTER 10

Limit State of Serviceability-Deflection and Cracking

10.1 Introduction

As discussed in the previous chapters, there are two limit states in the limit state method of design viz. the *limit state of collapse* and the *limit state of serviceability.* After designing a structure for the limit state of collapse (flexure, shear, torsion etc.), it needs to be checked for deflections, vibrations, cracking etc. i.e. collectively known as *limit state of serviceability.* These *limit states of serviceability* play a crucial role in the design of structures and they ensure that the structure performs its intended function very well with safety with respect to limit state of collapse.

Deflection and cracking are two important limit states of serviceability. For a check on deflection, we need to consider the both short term and long term deflection. We will cover the factors which are primarily responsible to influence these limit states of serviceability. As we can see during normal day to day experiences, normally a structure gets fail in serviceability and NOT in collapse.

10.2 The Limit States of Serviceability-Deflection and Cracking

As per limit state design philosophy, there are two types of limit state that are to be considered in design viz. ultimate limit states and serviceability limit states. The former i.e. the ultimate limit state deals with safety in terms of strength, overturning, sliding, buckling, fatigue fracture, etc., and the latter i.e. serviceability limit state deals with serviceability in terms of deflection, cracking, vibration, etc. The major consideration in structural design by the LSM is to ensure both safety and serviceability, so that the structure performs its intended function satisfactorily.

Use of limit state method of design with higher grades of concrete and steel has lead to smaller sections, which are otherwise large in the traditional working stress method of design. Thus those structures must be checked for deflection and cracking. Cl. 42 and 43 of IS 456: 2000 stipulates not to check every normal structure for deflection and crack widths provided codal provisions for limiting *l/d* ratios and maximum permissible reinforcement spacing recommendations are followed. However many a times (like where the dead and live loads are very heavy), it is specifically required to calculate the crack widths and deflections in the structure.

Remember: Structures often fail in terms of serviceability and rarely fail in terms of safety.

Cl. 35.1.1 of IS 456: 2000 states that the designer should consider all the relevant limit states to ensure a suitable degree of safety and serviceability. **Cl. 35.3.1 of IS 456: 2000** specifies limit state of serviceability comprising **deflection** and **Cl. 35.3.2 of IS 456: 2000** specifies limit state of serviceability comprising **cracking**. **Cl. 8 of IS 456: 2000** refers to **durability** consideration of concrete. Stability of the structure against **overturning** and **sliding (as per Cl. 20 of IS 456: 2000)** and **fire resistance** (as per Cl. 21 of IS 456: 2000) are other indispensable aspects to be taken into account while designing reinforced concrete structures.

Use of limit state design with improved grades of concrete and steel have resulted in concrete sections that are thinner in size and higher stress levels at service loads. Consequently, the structures undergo larger deflections, crack widths, vibrations etc. Thus the requirement of serviceability limit state check has attained a greater importance now.

10.3 Limit State of Serviceability: Deflection

The limits on deflection of flexural members are prescribed in order to have:

1. Aesthetic and psychological comfort to occupants.
2. A limit on crack width in the structures.
3. Proper functioning of attached and unattached floor and other structural elements.
4. No ponding in roof slabs.

Short Term Deflection: It refers to immediate deflection after casting of concrete and application of partial or full service loads. The following factors affect the short term deflection of structures:

1. Live loads magnitude and its distribution on the structure.
2. Span and type of end supports.
3. Cross sectional areas of the elements of the structure.
4. Amount of reinforcement and stress developed in the reinforcement.
5. Characteristic strengths of concrete and steel used in the structure.
6. Amount and extent of cracks developed.

Long Term Deflection: It occurs over a long period of time mainly due to creep and shrinkage. **Long term deflection is almost 2-3 times the short term deflection.** The following factors affect the long term deflection of concrete:

1. Humidity and temperature range during the time of curing.
2. Age of concrete at the time of loading.
3. Type and the size of aggregates, water-cement ratio, member sizes, amount of compressive steel which ultimately affects the creep and shrinkage of concrete.

10.3.1 Control of Deflection

Cl. 23.2 of IS 456: 2000 specifies limiting deflection under two groups as given below:

1. The maximum final deflection should not normally exceed **span/250** due to all loads including the long term effects of creep, shrinkage and temperature and measured from as cast level of the support of floors, roofs and other horizontal elements of the structure.
2. The maximum deflection should not normally exceed the lesser of **span/350 or 20 mm** including the effects of temperature, creep and shrinkage occurring after the erections of partitions and application of finishes.

Both of the above requirements must be satisfied by a structure.

10.3.2 Selection of Preliminary Dimensions of the Structural Elements

The above two requirements for deflection control are checked after designing the structure. But the structural design has to be revised if it fails to satisfy either condition (1) or (2) or both as stated in **section 10.3.1**. In order to avoid all this repetitive type of work, **IS 456: 2000** recommends to assume some initial dimensions of the structural elements which will generally satisfy the deflection limits. **Cl. 23.2.1 of IS 456: 2000** specifies different ratios of effective span to effective depth and **Cl. 23.3 of IS 456: 2000** limits slenderness of simply supported, continuous and cantilever beams.

For deflection requirement : Basic values of effective span to effective depth ratios for span upto 10 m and different support conditions are specified in **IS 456: 2000,** which is given as below:

Table 10.1: *Limiting span to depth ratios*

S. No.	Item	Cantilever	Simply Supported	Continuous
1.	Basic values of effective span to effective depth ratios for spans up to 10 m	7	20	26
2.	Modification factor for span > 10 m	Not applicable as deflection calculations are to be done	Multiply basic values of row 1 by (10/span in m)	
3.	Modification factor depending on area and stress in steel	Multiply values of Row 1 or Row 2 (as the case may be) with modification factors given in Figure 4 of IS 456: 2000		
4.	Modification factor depending on area of compression steel	Further multiply earlier respective value with modification factors given in Figure 5 of IS 456: 2000		
5.	Modification factors for flanged beams	(i) Modify values of Row 1 or Row 2 (as the case may be) as per values given in Fig. 6 of IS 456: 2000 (ii) Further modify as per Row 3 and/or Row 4 where reinforcement percentage to be used on area of section equal to $b_r d$ where b_r is the width of rib.		

For lateral stability : Cl. 23.3 of IS 456: 2000 specifies the following requirements

1. For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the minimum of **60*b* or 250*b*²/*d***, where b is the width of compression face of the beam midway between the lateral restraints and d is the effective depth of the beam.
2. For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the minimum of **25*b*** or **100*b*²/*d***.

All limits prescribed in **IS 456: 2000 (Cl. 23.2)** with reference to deflections are based on service loads and thus partial factor of safety to characteristic loads should be unity (one). **Cl. 36.4.2.2 of IS 456: 2000** states that the modulus of elasticity and other relevant parameters to be considered in deflection calculations should be based on *characteristic strengths of steel and concrete.*

Do You Know?

Why Precise Prediction of Deflection is Difficult?

Precise prediction of deflections in reinforced concrete members is difficult because:

1. There are uncertainties in predicting the flexural rigidity (*EI*) of the member which depends on degree of tensile cracking, amount of flexural reinforcement, modulus of elasticity of concrete, modulus of rupture of concrete.

2. There are other environmental and time dependent effects which influence shrinkage and creep.
3. Concrete members shows inelastic behavior also.
4. There are large variations in observed deflections even under laboratory conditions.

Due to the above said reasons, approximations and simplifications are necessary for calculation of deflections. The deflection calculation is considered in two parts viz. immediate or short term deflection and long term deflection. Calculation of short term deflection involves the consideration of full dead and live loads. However for calculating the long term deflection due to creep and shrinkage, the permanent load i.e. dead load and that part of live load which is sustained to the structure are considered. Even after making suitable approximations and simplifications, the deflection calculations are quite rigorous and complicated.

10.3.3 Calculation of Short-term Deflection

Short term deflection under service loads are calculated assuming linear elastic behavior of concrete and steel. Usual expressions for maximum deflections in beams subjected to different types of loading are covered in the subject of *Structural Analysis*. In the deflection calculations, an important parameter is the flexural rigidity (EI) where 'I' is the second moment of area of the member section and 'E' is the short term modulus of elasticity of concrete which is expressed as:

$$E = 5000\sqrt{f_{ck}} \qquad \text{(As per IS 456: 2000)}$$

$$E = 5700\sqrt{f_{ck}} \qquad \text{(As per IS 456: 1978)}$$

The second moment of area of the section (I) depends on amount of reinforcement and amount of flexural cracking which in turn depends on the applied bending moment and the modulus of rupture of concrete.

Cl. C-2 of Annexure C of IS 456: 2000 specifies the steps of calculating the short term deflections. The said expression is based on earlier version of the British Code. The code recommends the usual methods for elastic deflections using short term modulus of elasticity of concrete (E_c) and effective moment of inertia (I_{eff}) which is given as:

$$I_{eff} = \frac{I_{cr}}{1.2 - \frac{M_{cr}}{M}\left(\frac{z}{d}\right)\left(1-\frac{x}{d}\right)\left(\frac{b_w}{b}\right)}$$

But $\qquad I_{cr} \le I_{eff} \le I_{gr}$

where,
I_{cr} = Moment of inertia of cracked section
M_{cr} = Cracking moment ($=f_{cr}I_{gr}/y_t$) where f_{cr} is the modulus of rupture of concrete
I_{gr} = Moment of inertia of gross section about the centroidal axis ignoring the reinforcement
y_t = Distance of extreme fibre in tension from the centroidal axis of gross section ignoring the reinforcement
M = Maximum moment under service loads
z = Lever arm
x = Depth of neutral axis
d = Effective depth
b_w = Width of web
b = Width of compression face

For continuous beams, the values of I_{gr}, I_r and M_r has to be modified by the following equation:

$$X_e = k_1\left[\frac{X_1+X_2}{2}\right]+(1-k_1)X_0$$

where,
X_e = Modified value of X, $\qquad X_1, X_2$ = Values of X at supports
X_o = Value of X at mid span, $\qquad k_1$ = Coefficient given in **Table 25 of IS 456: 2000**
X = Values of I_{gr}, I_r or M_r as desired

Table 10.2 : *Values of Coefficient k_1*

k_2	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_1	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Here k_2 is given by:

$$k_2 = \frac{M_1+M_2}{M_{F1}+M_{F2}}$$

where, M_{F1}, M_{F2} = Fixed end moments
M_1, M_2 = Support moments

Additional Short-term Deflection due to Live Loads alone

As described at the start of this chapter that the deflection calculations require calculation of deflections after the structure has been constructed (including the long term effects of creep and shrinkage). This needs the calculation of deflections due to live loads alone.

Now because of the fact that with the variation in flexural rigidity (EI) with applied moment, the load deflection behavior of reinforced concrete member is non-linear. This implies that principle of superposition cannot be applied in deflection calculations.

Thus deflection due to live loads alone is computed from the difference of deflection due to both dead and live loads and the deflection due to dead loads alone.

10.3.4 Long-term Deflections

The deflection of reinforced concrete members increase with time because of the following reasons:

1. Variation in temperature which results in differential shrinkage in concrete members.
2. Creep due to sustained loading.

The combined long term deflection due to creep, shrinkage and temperature may be about 2-3 times the short term deflection. However, the excessive short term deflections can be nullified by providing a camber (just like in slab casting), but this cannot be applied for long term deflections in the structure.

NOTE

If observed deflections in a structure just after its completion are within the prescribed limits, it cannot be said with 100% surety that the same structure will meet the deflection requirements in long run also. In fact, long term deflections gradually build up over a period of time to such an extent that the total deflection exceeds the prescribed deflection limits.

Other factors which increase the long term deflection are:

1. Formation of new cracks.
2. Widening/enlarging of existing cracks.
3. Repeated load cycles acting on the structure.

10.3.5 Calculation of Deflection Due to Shrinkage

As usually observed, in unrestrained concrete members, drying shrinkage of concrete shortens the member but the embedded reinforcing steel does not shorten to same extent. This introduces compressive stress in reinforcing steel and tensile stress in the surrounding concrete.

When reinforcement is placed symmetrically in a concrete section, except in indeterminate structures, no curvature results due to shrinkage. But unsymmetrically placed reinforcement in concrete section (which is usually the case) results differential strains across the section.

Cl. C-3 of Annexure-C of IS 456: 2000 describes the method of calculating the deflection due to shrinkage (α_{cs}) as:

$$\alpha_{cs} = k_3 \Psi_{cs} l^2$$

where, k_3 = Constant which depends on support conditions
= 0.5 for cantilevers
= 0.125 for simply supported members
= 0.086 for members continuous at one end
= 0.063 for fully continuous members

Ψ_{cs} = shrinkage curvature = $k_4(\varepsilon_{cs}/D)$ where ε_{cs} is the ultimate shrinkage strain of concrete

$$k_4 = 0.72\left(\frac{P_t - P_c}{\sqrt{P_t}}\right) \le 1.0 \quad \text{for } 0.25 \le (P_t - P_c) < 1.0$$

$$= 0.65\left(\frac{P_t - P_c}{\sqrt{P_t}}\right) \le 1.0 \quad \text{for } (P_t - P_c) \ge 1.0$$

where, $P_t = 100\dfrac{A_{st}}{bd}$ and $p_c = 100\dfrac{A_{sc}}{bd}$

D is the total depth of section and l is the span length

10.3.6 Calculation of Deflection Due to Creep

It is a usual fact that under sustained loading, the compressive strain in concrete keeps increasing in a non-linear fashion. This phenomenon is referred as *creep*. The creep coefficient is defined as:

$$C_t = \frac{\text{Creep strain } (\epsilon_{cp})}{\text{Initial elastic strain } (\alpha_i)}$$

This creep coefficient provides a method to calculate creep in concrete.

The maximum value of creep coefficient (C_t) is called as the **ultimate creep coefficient (denoted as θ in IS 456: 2000).** This ultimate creep coefficient (θ) is used for assessing the maximum deflection in a concrete member due to creep.

Cl. 6.2.5.1 of IS 456: 2000 states that in the absence of any data on factors influencing creep, the value of ultimate creep coefficient (θ) can be taken as 2.2, 1.6 and 1.1 respectively for loading ages of 7 days, 28 days and 1 year (or 365 days).

The distribution of creep strain across the depth of concrete section is non uniform and it in fact varies linearly similar to the flexural strain produced by applied bending moment.

NOTE Creep strain is primarily related to increase in compressive strain of concrete but there is a marginal increase in tensile strain in steel. Due to creep in concrete, there is small increase in the depth of neutral axis thereby reducing the lever arm. In order to maintain the static equilibrium with the applied loading (or applied moment), there is a little increase in the steel stress which in turn increases the strain in steel.

Cl. C-4 of Annexure C of IS 456: 2000 describes the following method of calculating the deflection due to creep. The creep deflection due to permanent loads is given by:

$$\alpha_{cc(\text{perm})} = \alpha_{i,cc(\text{perm})} - \alpha_{i(\text{perm})}$$

where, $\alpha_{icc(\text{perm})}$ = Initial deflection plus creep deflections due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity E_{ce}, which is given by:

$$E_{ce} = \frac{E_c}{(1+\theta)}, \text{ where } \theta \text{ is the creep coefficient}$$

$\alpha_{i(\text{perm})}$ = Short term deflection due to permanent load using E_c

10.3.7 Deflection Due to the Effect of Temperature

From the *Structural Analysis*, it is found that variation in temperature causes stresses in indeterminate structures. The calculation of bending moments due to variation in temperatures can be done by any of the principles described in *Structural Analysis*. This requires a suitable value of coefficient of thermal expansion. In addition to overall stresses in a structure, local stresses in members may also get induced due to unsymmetrical reinforcement. The calculation of such deflections is identical to calculations done for assessing the deflection due to differential shrinkage.

In common design practice, deflections due to temperature is usually NOT computed because such deflections are not significant and are also reversible in nature owing to increase or decrease in temperature. But tensile stresses and thus resulting tensile cracks induced by the restraints against temperature changes can be quite significant which require special attention by proper detailing of reinforcement.

10.3.8 Check on Total Deflection

Once we have calculated the deflections due to various components viz. short term deflection and long term deflection, the overall total deflection need to be checked with the limit prescribed in **Cl. 23.2 of IS 456: 2000.**

$$\Delta_{DL+LL} + \Delta_{long\text{-}term} \le \frac{l}{250}$$

$$\Delta_{LL} + \Delta_{long\text{-}term} \le \frac{l}{350} \text{ or } 20 \text{ mm} \qquad \text{(whichever is less)}$$

where, $\Delta_{long\text{-}term} = \Delta_{shrinkage} + \Delta_{creep} + \Delta_{temperature}$

and l = Effective span of the member

If these limits exceed, then the member needs to be redesigned by increasing the depth (and thereby stiffness) of the member. If there are limitations on the depth of the member to be provided, then grade of concrete has to be improved thereby increasing the modulus of elasticity of concrete. Provision of compression reinforcement also reduces the deflection in members.

10.4 Limit State of Serviceability: Cracking

Cracking in concrete occurs whenever the tensile stress of concrete exceeds the permissible limit. The tensile strength of concrete is usually very low and thus it is generally not possible to have almost zero cracks in concrete. But proper design of concrete member can control the cracks in terms of crack widths and spacing of cracks.

Disadvantages of Cracking in Concrete

The occurrence of cracks in concrete brings with it the following drawbacks:

1. Cracks in concrete spoil aesthetic of structure.
2. Cracks reduce the durability of concrete.
3. Development of cracks in concrete leads to corrosion of reinforcement.
4. In liquid retaining structures, appearance of cracks can in fact adversely affect the functional requirement of the structure at all.

Thus it is always tried to limit the crack width in concrete structures.

Cracking in concrete occurs due to the following reasons:

1. Flexural tensile stresses due to the applied loadings.
2. Shear and torsion induces diagonal tension in concrete members.
3. Restraints against volume changes due to variation in temperatures.
4. Very high compressive stresses induce lateral tensile strains owing to Poisson's effect.
5. Direct tensile stress under the applied loads (like hoop tension in circular tanks).

As stated above, cracking in concrete occurs in tension resulting in splitting of concrete at the surface which penetrates inwards. The consequent appearance of cracks in concrete, their spacing and widths depend on magnitude of tensile stresses, reinforcement detailing, grade and properties of concrete and the thickness/depth of the concrete section/member. It is always desirable to have closely spaced cracks with smaller crack widths rather than widely spaced cracks with large crack widths. The later type of cracks are associated with relatively low percentages of steel, widely spaced reinforcing bars of high strength steel (Fe 415 and Fe 500).

Factors Affecting Crack Widths

The following factors affect the wodth of cracks in reinforced concrete members:

1. Tensile stresses in the reinforcing bars.
2. Bond strength and tensile strength of concrete.
3. Depth of the concrete member and the location of neutral axis.
4. Diameter and spacing of reinforcing bars.
5. Concrete cover.

10.4.1 Limits on Crack Widths

Cl. 35.3.2 of IS 456: 2000 limits the width of cracks to a maximum limit of 0.3 mm for mild exposure conditions. This limit is based purely on aesthetic considerations. Fortunately, this limit on crack width is also found to be adequate for durability purpose also.

A lower limit of crack width of 0.2 mm is prescribed for structures exposed to continuous moisture conditions or which are in direct contact with soil or the ground water ('moderate' exposure condition). This limit on crack width is further reduced to 0.1 mm for severe, very severe and extreme exposure conditions.

Earlier versions of IS 456: 2000 didn't provide any procedure to calculate the crack width. But however IS 456: 2000 (Annexure F) provides a procedure to calculate crack widths. Cl. 43.1 of IS 456: 2000 states that explicit calculations for crack widths are required where reinforcement spacing exceeds the prescribed limits irrespective of the value of cover to reinforcement provided.

The MAJOR DRAWBACK in IS 456: 2000 is that the earlier limit (IS 456: 1978) on clear cover of 75 mm has been eliminated. Increased cover to reinforcement implies increased cracks in concrete especially in flexural concrete members.

'Annexure F' of IS 456: 2000 describes the expression for calculating the crack widths as,

$$w_{cr} = \frac{3a_{cr}\,\varepsilon_m}{1+2(a_{cr}-C_{\min})/(D-x)}$$

where,
D = Overall depth of the member
$C_{\min}$ = Minimum cover to the main longitudinal bar
x = Depth of neutral axis
a_{cr} = Distance from the point considered to the surface of the nearest longitudinal bar
ε_m = Average strain at the level where cracking is being considered (steel level)

10.5 Other Limit States of Serviceability

Some other limit states of serviceability are:

1. **Tension Stiffening Effect:** The increase in the stiffness of concrete over and above the cracked section stiffness due to the ability of concrete between the cracks to take tension is called as **tension stiffening effect.**
2. **Creep in steel:** Steel itself is NOT at all subjected to creep but due to creep in concrete, there is a slight increase in the actual depth of neutral axis thereby reducing the lever arm of the member section. In order to maintain the static equilibrium with the applied moments, there is slight increase in the steel stress and consequently the steel strain.
3. **Cracking in concrete:** Practically it is very difficult to avoid cracks in concrete and thus it is always beneficial to have a large number of well distributed hair line cracks in concrete rather than a few wide cracks. **This is the main objective of Limit State of Serviceability of Cracking for design of a member by Limit State Method.**

Example 10.1 There is a cantilever beam of span 6000 mm of cross-section 200 × 500 mm. Check the beam for deflection and lateral stability.

Solution:

Check for deflection

For cantilever, minimum effective depth required

$$= \frac{\text{span}}{7} = \frac{6000}{7} = 857 \text{ mm}$$

But effective depth available (d) = 450 mm (Assuming 50 mm effective cover)

$\therefore$ Beam fails in deflection

Check for lateral stability

For lateral stability, span $\not>$ $\left\{\begin{array}{l}(1)\ 25b = 25 \times 200 \text{ mm} = 5000 \text{ mm} \\ (2)\ \dfrac{100b^2}{d} = \dfrac{100 \times 200^2}{450} = 8888.9 \text{ mm}\end{array}\right\}$ whichever is small

$= 5000$ mm

But, actual span $= 6000 \text{ mm} > 5000 \text{ mm}$

∴ Beam fails in lateral stability

∴ The above beam fails in both deflection and lateral stability.

Objective Brain Teasers

Q.1 In case of rectangular RC beams, shrinkage deflection can be reduced by

(a) providing less compression steel than tension steel

(b) providing more compression steel than tension steel

(c) providing equal tension and compression steel

(d) tension steel greater than compression steel by $33\frac{1}{3}\%$

Q.2 In composite construction, the effect of creep and shrinkage

(a) can be ignored at the ultimate limit state since large inelastic strains are developed

(b) can be ignored at limit state of serviceability

(c) can be totally eliminated by removing the props after 28 days

(d) insufficient data

Q.3 Unequal top and bottom reinforcement in a beam section leads to

(a) large deflection (b) shrinkage deflection

(c) creep deflection (d) long term deflection

Answers

1. (c) 2. (a) 3. (b)

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