CHAPTER 2

Linear Equations in One Variable

Learning Objectives

In this chapter you will learn:

- *To solve the different types of linear equations.*
- *To use the linear equations in daily life.*
- *To tackle the practical life situations using variables.*

2.1 Introduction :-

In class VII, we have already discussed the concept of an equation in one variable and its solution.
e.g. $4x = 12$, $3y = 15$, $2y + 1 = 9$ etc.

In this section, we shall discuss this concept in detail.

2.2 Equation in one Variable:-

The equation in which algebraic expression have one variable is called equation in one variable. e.g. $2y + 5 = 9$, $3z^2 - 1 = 7$, $4x^2 + 5x = 8$ etc.

In above examples, all the equations have one variable.

But $2x + 3y = 7$, $2a + b + c = 7$, $4abc = 5$ etc. are not equations in one variable as

- 1st equation has two variables i.e. x and y
- 2nd equation has three variables i.e. a, b and c.
- 3rd equation also has three variables i.e. a, b and c.

2.2.1 Linear Equation in one variable:-

The equation in one variable having degree (highest power of variable) 1 is called linear equation in one variable e.g.(i) $2x + 3 = 12$ is linear equation in one variable (x).

(ii) $5y - 3 = 8$ is linear equation in one variable (y).

(iii) $2x + 3 = 5y$ is not a linear equation in one variable. Here, we have two variables (x and y)

(iv) $3z^2 - 1 = 7$ is not a linear equation in one variable because its degree is 2.

2.2.2 Solution of Linear Equation in one variable:-

In equation, Equal (=) sign divides the equation in two parts i.e. Left part is called Left Hand Side (LHS) and Right part is called Right Hand Side (RHS) i.e. value of left part = value of right part. This is true for particular values of the variable and these particular values are called the solutions of the equation. Linear equation in one variable has one solution. That means only one value of the variable satisfies the equation.

e.g: $4x = 8$

$LHS = 4x$

(i) If $x = 1$

$4x = 4 \times 1 = 4$

$RHS = 8$

$LHS \neq RHS$

For $x = 1$,

Value of left part $\neq$ value of right part

$\therefore$ $x = 1$ is not a solution of the given equation.

(ii) If $x = 2$ then LHS $= 4x = 4(2) = 8$

RHS $= 8$

LHS $=$ RHS

Here , value of left part = value of right part

$\therefore$ $x = 2$ is a solution of the given equation.

2.3 Solving equations having variables on both sides

In last section, we have discussed the equation having variable on one side and In this section, we shall discuss the equations having variables on both sides.

First, we adjust variable on one side and constants on other side then we shall solve as discussed in previous sections.

Example 2.1 : Solve:-

(i) $4x - 3 = 3x + 2$ **(ii)** $3x + 4 = x - 6$

(iii) $4x - 5 = 7x + 8$

Sol. (i) We have $4x - 3 = 3x + 2$

$4x = 3x + 2 + 3$ (transposing -3 to RHS)

$4x = 3x + 5$

transposing $3x$ to LHS we get

$4x - 3x = 5$

$\Rightarrow \quad x = 5$

which is the required answer.

Or We can solve this equation by transposing variable and constant together.

we have $4x - 3 = 3x + 2$

Transposing -3 to RHS and $3x$ to LHS, we get

$4x - 3x = +2 + 3$

$\Rightarrow \quad x = 5$

which is the required solution.

(ii) we have $3x + 4 = x - 6$

Transposing $+4$ to RHS and x to LHS, we get

$3x - x = -6 - 4$

$\Rightarrow \quad 2x = -10 \quad \Rightarrow \quad x = \dfrac{-10}{2} = -5$

which is the required solution.

(iii) we have, $4x - 5 = 7x + 8$

Transposing -5 to RHS and $7x$ to LHS, we get

$4x - 7x = +8 + 5$

$-3x = 13$

$x = \dfrac{13}{-3} \Rightarrow x = \dfrac{-13}{3}$

which is the required solution.

Example 2.2 : Solve the following equations and verify the answer:-

(i) $8x + 4 = 3(x - 1) + 7$ **(ii)** $2x - 5 = 14 - (x - 2)$

(iii) $3(\ell - 3) = 5(2\ell + 1)$

Sol. (i) We have, $8x + 4 = 3(x-1)+7$

$\Rightarrow 8x + 4 = 3x - 3 + 7$

$\Rightarrow 8x + 4 = 3x + 4$

Transposing +4 to RHS and 3x to LHS, we get

$8x - 3x = +4 - 4$

$\Rightarrow \quad 5x = 0$

Transposing 5 to RHS, we get

$$x = \frac{0}{5} = 0$$

Verification

put $x = 0$

in the given equation, we get

L.H.S.	R.H.S.
$8(0) + 4$	$3(0 - 1) + 7$
$= 0 + 4$	$= -3 + 7$
$= 4$	$= 4$

L.H.S. = R.H.S.

Both sides are equal. Hence, the solution is verified.

(ii) We have, $2x - 5 = 14 - (x - 2)$

$\Rightarrow 2x - 5 = 14 - x + 2$

$\Rightarrow 2x - 5 = 16 - x$

Transposing -5 to RHS and $-x$ to LHS, we get.

$\Rightarrow 2x + x = 16 + 5$

$\Rightarrow 3x = 21$

$$\Rightarrow x = \frac{21}{3} = 7$$

Verification

Put $x = 7$ in the given equation, we get

L.H.S.	R.H.S.
$2(7) - 5$	$14 - (7 - 2)$
$= 14 - 5$	$= 14 - 5$
$= 9$	$= 9$

L.H.S. = R.H.S.

Hence the solution is verified.

(iii) We have $3(\ell - 3) = 5(2\ell + 1)$

$3\ell - 9 = 10\ell + 5$

Transpose -9 to RHS and 10ℓ to LHS, we get

$3\ell - 10\ell = 5 + 9$

$-7\ell = 14$

Transpose -7 to RHS, we get

$$\ell = \frac{14}{-7} = -2$$

Verification

Put $\ell = -2$ in the given equation,

LHS	RHS
$3(-2 - 3)$	$5[2(-2) + 1]$
$= 3(-5)$	$= 5(-4 + 1)$
$= -15$	$= 5(-3)$
	$= -15$

LHS = RHS

Hence, the solution is verified

Exercise 2.1

Solve the following equations and verify the result :-

(1) $2x - 3 = x + 2$ **(2)** $5x - 6 = 2x + 9$ **(3)** $5a - 3 = 3a - 5$

(4) $5x + 9 = 5 + 3x$ **(5)** $4y + 3 = 6 + 2y$ **(6)** $3x - 1 = 15 - x$

(7) $4x + 3 = 2(x - 1) + 5$ **(8)** $3\ell - 5 = 4(\ell + 2) - 6$ **(9)** $6x = 5(x + 10) - 2$

2.4 Some Practical Applications

Example 2.3 : A positive number is 5 times another number. If 21 is added to both the numbers then larger number becomes twice the shorter number. What are the numbers?

Sol. Suppose the shorter number be x

As given : Larger number = 5 × (shorter number) = 5x

According to another condition

If 21 is added to both numbers then numbers are (x + 21) and (5x + 21)

Now, larger number = 2 × shorter number

$\Rightarrow 5x + 21 = 2(x + 21)$

$\Rightarrow 5x + 21 = 2x + 42 \Rightarrow 5x - 2x = 42 - 21$

$\Rightarrow 3x = 21$

$$\Rightarrow x = \frac{21}{3} = 7$$

So, numbers are x and 5x i.e. 7 and 5 × 7 = 35

Example 2.4 : A number consists of two digits and sum of digits is 8. If 18 is added to the number, its digits are reversed. Find the number.

Sol. We are given a two-digit number which consists ones place digit and tens place digit.

Given condition:- Sum of digits = 8

i.e. sum of ones digit & tens digit = 8

Let ones digit be x

then tens digit be 8 – x

∴ Two digit number = 10 × (tens digit) + (ones digit)

$= 10(8 - x) + x = 80 - 10x + x$

$= 80 - 9x$

Now, number obtained by reversing the digits

= 10 × (ones digit) + (tens digit)

$= 10 \times x + (8 - x) = 9x + 8$

2nd condition :- 18 added to original number = number obtained by reversing the digits

i.e. 18 + (Two-Digit number) = number obtained by reversing digits

$\Rightarrow 18 + (80 - 9x) = 9x + 8$

$\Rightarrow 98 - 9x = 9x + 8$

$\Rightarrow 98 - 8 = 9x + 9x \Rightarrow 90 = 18x$

$$\Rightarrow x = \frac{90}{18} = 5$$

∴ Two-digit number = 80 – 9x = 80 – 9 (5)

= 80 – 45 = 35

Example 2.5 : Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

Sol. Given Condition:-

Shobo's mother's present age = 6 × Shobo's present age

Suppose shobo's present age be x years

Shobo's mother's present age be 6x years

Now,

Shobo's age five years from now = $\frac{1}{3}$ of Mother's present age

$\Rightarrow x + 5 = \frac{1}{3} \times 6x \Rightarrow x + 5 = 2x$

$\Rightarrow 5 = 2x - x = x \quad \Rightarrow \quad x = 5$

Hence, Shobo's present age = 5 years

and Shobo's mother's present age = 6 × 5 = 30 years

Exercise 2.2

1. A number is such that it is as much greater than 84 as it is less than 108. Find it.
2. Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.
3. Find a number such that when 5 is subtracted from 5 times the number, the result is 4 more than twice the number.
4. The digits of a two digit number are differ by 3. If the digits are inter changed and the resulting number is added to the original number, we get 143. Find the original number.
5. Sum of digits of a two digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. Find the two-digit number.
6. Preet is 6 years older than Abdul. Six years ago, Preet's age was four times Abdul's age. Find their present ages.
7. After 12 years, I Shall be 3 times as old as was 4 years ago. Find my present age.
8. Jiya is twice as old as Kavya. If six years is subtracted from Kavya's age and four years added to Jiya's age, then Jiya will be four times Kavya's age. Find their present ages.

2.5 Reducing Equation to simplest form :

Example 2.6 : Solve the following equations:-

(i) $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ (ii) $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + 2$

(iii) $\frac{3x}{4} - \frac{3}{2} - \frac{-2}{3} - \frac{5x}{6}$

Sol. (i) We have, $\frac{2x}{3}+1=\frac{7x}{15}+3$

Multiply both sides of the equation by LCM of denominators (3 & 15) i.e. 15, we get.

$$\left(\frac{2x}{3}+1\right)\times 15=\left(\frac{7x}{15}+3\right)\times 15$$

$$\Rightarrow \frac{2x}{3}\times 15+1\times 15=\frac{7x}{15}\times 15+3\times 15$$

$\Rightarrow 10x + 15 = 7x + 45$

Transposing + 15 to RHS and 7x to LHS, we get

$\Rightarrow 10x - 7x = + 45 - 15$

$\Rightarrow 3x = 30$

$$\Rightarrow x=\frac{30}{3}=10$$

(ii) We have $\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+2$

Multiply both sides of the equation by LCM of denominators (2, 5, 3) i.e. 30, we get

$$\left(\frac{x}{2}-\frac{1}{5}\right)\times 30=\left(\frac{x}{3}+2\right)\times 30$$

$$\Rightarrow \frac{x}{2}\times 30-\frac{1}{5}\times 30=\frac{x}{3}\times 30+2\times 30$$

$\Rightarrow 15x - 6 = 10x + 60$

Transposing –6 to RHS and 10x to LHS, we get

$15x - 10x = 60 + 6$

$\Rightarrow \quad 5x = 66$

$$\Rightarrow \quad x=\frac{66}{5}$$

which is the required solution.

(iii) We have $\frac{3x}{4}-\frac{3}{2}=\frac{-2}{3}-\frac{5x}{6}$

Multiply both sides of the equation by LCM of denominators (4, 2, 3, 6) i.e. 12, we get

$$\left(\frac{3x}{4}-\frac{3}{2}\right)\times 12=\left(\frac{-2}{3}-\frac{5x}{6}\right)\times 12$$

$$\Rightarrow \frac{3x}{4}\times 12-\frac{3}{2}\times 12=\frac{-2}{3}\times 12-\frac{5x}{6}\times 12$$

$\Rightarrow 9x - 18 = -8 - 10x$

Transposing –18 to RHS and–10x to LHS, we get

$9x + 10x = -8 + 18$

$\Rightarrow 19x = 10 \Rightarrow x = \frac{10}{19}$

which is the required solution.

Example 2.7 : Solve the following equations:-

(i) $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$ (ii) $\frac{5x}{3} - \frac{x-1}{4} = \frac{x-3}{5}$

(iii) $\left(\frac{3a-2}{4}\right) - \frac{2a+3}{3} = \frac{2}{3} - a$

Sol. (i) We have $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$

Multiply both sides of the equation by LCM of denominators (2, 3) i.e. 6, we get

$\left(\frac{6x+1}{2} + 1\right) \times 6 = \left(\frac{7x-3}{3}\right) \times 6$

$\Rightarrow \left[\frac{6x+1}{2}\right] \times 6 + 1 \times 6 = \left[\frac{7x-3}{3}\right] \times 6$

$\Rightarrow 3(6x + 1) + 6 = 2\ (7\ x - 3)$

$\Rightarrow 18\ x + 3 + 6 = 14x - 6$

$\Rightarrow 18x + 9 = 14x - 6$

$\Rightarrow 18x - 14x = -6 - 9$

$\Rightarrow 4x = -15$

$\Rightarrow x = \frac{-15}{4}$

(ii) We have, $\frac{5x}{3} - \frac{x-1}{4} = \frac{x-3}{5}$

Multiply both sides of the equation by the LCM of denominators (3, 4, 5) i.e. 60, we get

$\left(\frac{5x}{3} - \frac{x-1}{4}\right) \times 60 = \left(\frac{x-3}{5}\right) \times 60$

$\Rightarrow \frac{5x}{3} \times 60 - \left[\frac{x-1}{4}\right] \times 60 = \left[\frac{x-3}{5}\right] \times 60$

$\Rightarrow 100x - 15\ (x - 1) = 12\ (x - 3)$

$\Rightarrow 100x - 15x + 15 = 12x - 36$

$\Rightarrow 85x + 15 = 12x - 36$

$\Rightarrow 85x - 12x = -36 - 15$

$\Rightarrow 73x = -51$

$$\Rightarrow x = \frac{-51}{73}$$

(iii) We have, $\frac{3a-2}{4} - \frac{2a+3}{3} = \frac{2}{3} - a$

Multiply both sides of the equation by LCM of denominators (4, 3, 3) i.e. 12, we get

$$\left[\frac{3a-2}{4} - \frac{(2a+3)}{3}\right] \times 12 = \left(\frac{2}{3} - a\right) \times 12$$

$$\Rightarrow \left(\frac{3a-2}{4}\right) \times 12 - \left[\frac{2a+3}{3}\right] \times 12 = \frac{2}{3} \times 12 - a \times 12$$

$$\Rightarrow 3(3a-2) - 4(2a+3) = 8 - 12a$$

$$\Rightarrow 9a - 6 - 8a - 12 = 8 - 12a$$

$$\Rightarrow a - 18 = 8 - 12a$$

$$\Rightarrow a + 12a = 8 + 18 \Rightarrow 13a = 26$$

$$\Rightarrow a = \frac{26}{13} = 2$$

Example 2.8 : Solve the equation $15(y-4) - 2(y-9) + 5(y+6) = 0$

Sol: $15(y-4) - 2(y-9) + 5(y+6) = 0$

$15y - 60 - 2y + 18 + 5y + 30 = 0$

$15y - 2y + 5y - 60 + 18 + 30 = 0$

$18y - 12 = 0$

$18y = 12$

$$y = \frac{12}{18}$$

$$y = \frac{2}{3}$$

Exercise 2.3

Solve the following Equations :

(1) $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$

(2) $\frac{l}{2} - \frac{1}{5} = \frac{l}{3} + \frac{1}{4}$

(3) $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

(4) $5x - 2(2x-7) = 2(3x-1) + \frac{7}{2}$

(5) $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

(6) $\frac{3a}{4} - \frac{a-1}{2} = \frac{a-2}{3}$

(7) $4(x+2) - 5 = 2(x-1) + 7$

(8) $7(2a-3) = 4 - 3(1-a)$

(9) $3(5x-7)-2(9x-11)=4(8x-13)-17$

(10) $15(a-4)-2(a-9)+5(a+6)=0$

Example 2.23 : The Present ages of Hari and Harry are in the ratio 5:7. Four years from

Learning Outcomes

After completion of the chapter, students are now able to:

- *Solve different types of linear equations.*
- *Use linear equations in daily life.*
- *Tackle the pratical life situations using variables*

Answers

Exercise 2.1

1. $x=5$ 2. $x=5$ 3. $a=-1$ 4. $x=-2$ 5. $y=\frac{3}{2}$
6. $x=4$ 7. $x=0$ 8. $\ell=-7$ 9. $x=48$

Exercise 2.2

1. 96 2. 14, 20 3. 3 4. 85 or 58 5. 36
6. Preet 's Present Age = 14 Years
   Abdul's Present Age = 8 years
7. 12 Years
8. Kavya's Present Age = 14 Years
   Jiya's Present Age = 28 Years

Exercise 2.3

1. $n=36$ 2. $\ell=-\frac{27}{10}$ 3. $m=-\frac{7}{5}$ 4. $x=\frac{5}{2}$ 5. $x=-1$
6. $a=14$ 7. $x=1$ 8. $a=2$ 9. $x=2$ 10. $a=\frac{2}{3}$