Chapter 5 Powers and Exponents

5.1 Ravi asked a question to Mohan; what was the population of India in 2011? He replied; approximately 120 Crores. Ravi again asked; what is the distance between the Sun and the Earth? He immediately replied – Approximately 15 Crore Kilometers. Ravi again asked a question: What is the distance travelled by light covers in a second? He replied – 3 Crore Meters. Ravi once again asked a question: How much is the population of Rajasthan according to 2011 census?

Mohan replied – The population of Rajasthan is around 7 Crores in 2011. Now write them in terms of numbers; then Mohan said, "It is difficult to write in numbers". Can these numbers be read, written and understood easily. We can read and write such big numbers with the help of powers and exponents. In this chapter, we will study the numbers with base as an integer and exponent as whole number.

5.2 Exponent

Let us think of repeated numbers like

$4+4+4+4+4$, $5+5+5+5+5+5$, $7+7+7+7+7+7+7+7$

According to law of multiplication, the sum of repeated numbers can be expressed in the form of products 5×4, 6×5, 8×7.

Can we understand repetition of numbers by multiplication easily? Consider following numbers:

$4 = 2 \times 2$ $\quad$ $8 = 2 \times 2 \times 2$ $\quad$ $16 = 2 \times 2 \times 2 \times 2$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

These could be written as

$2 \times 2 = 2^2$ $\quad$ $2 \times 2 \times 2 = 2^3$ $\quad$ $2 \times 2 \times 2 \times 2 = 2^4$

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Similarly, $100 = 10 \times 10 = 10^2$ $\quad$ $10000 = 10 \times 10 \times 10 \times 10 = 10^4$

$1000 = 10 \times 10 \times 10 = 10^3$ $\quad$ $100000 = 10^5$

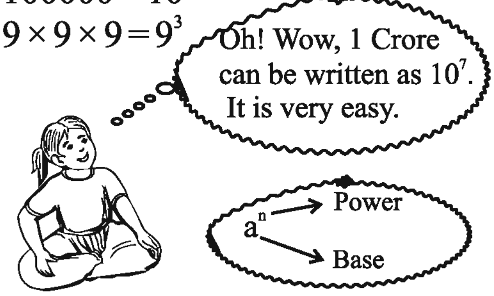
and $9 \times 9 = 9^2$ $\quad$ $9 \times 9 \times 9 = 9^3$

$9 \times 9 \times 9 \times 9 \times 9 \times 9 \ldots$ n factors.

Here, in 2^3 the base is 2 and the power is 3.

In 2^5, the base is 2 and the exponent is 5.

2^5 is read as "2 power 5".



Example 1 Write 64 in terms of exponents.

Solution 64 = 2 x 2 x 2 x 2 x 2 x 2

$64 = 2^6$

Example 2 Which number is greater 3^4 or 4^3 and why?

Solution 3^4 = 3 x 3 x 3 x 3

= 81

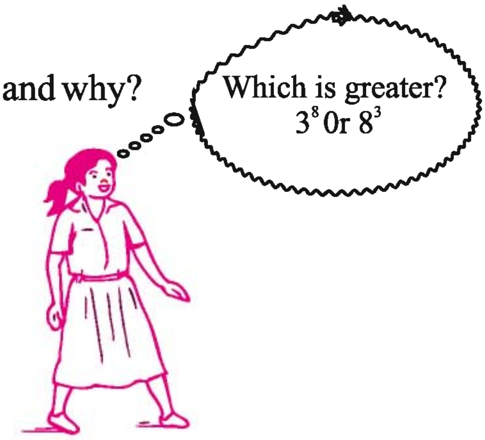
4^3 = 4 x 4 x 4

= 64

you know 81 > 64

$\therefore\ 3^4 > 4^3$

Therefore 3^4 is greater than 4^3



Example 3 Write the following numbers in terms of powers of prime factors:

(i) 36 (ii) 256 (iii) 1000

Solution (i) 36

= 2 x 2 x 3 x 3

$= 2^2 \times 3^2$

2	36
2	18
3	9
3	3
	1

(ii) 256

= 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

$= 2^8$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(iii) 1000

= 2 x 2 x 2 x 5 x 5 x 5

$= 2^3 \times 5^3$

2	1000
2	500
2	250
5	125
5	25
5	5
	1

Example 4 Simplify the following:

Solution

(i) 3×10^3 (ii) $5^2 \times 2^3$

(i) $3 \times 10^3 = 3 \times 10 \times 10 \times 10$

$= 3 \times 1000$

$= 3000$

(ii) $5^2 \times 2^3 = 5 \times 5 \times 2 \times 2 \times 2$

$= 25 \times 8$

$= 200$

Example 5 Find the values of following:

Solution (i) $(-1)^5$ (ii) $(-3)^4$

(i) $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

(ii) $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$

$= 9 \times 9 = 81$

Exercise 5.1

1. Express the following in terms of exponents:

(i) $7 \times 7 \times 7 \times 7 \times 7$ (ii) $3 \times 3 \times 3 \times 7 \times 7$

(iii) $a \times a \times a \times b \times b$ (iv) $5 \times 5 \times t \times t \times t$

2. Express each of the following numbers in the form of exponents:

(i) 32 (ii) 81 (iii) 343 (iv) 125

3. Identify the greater number in the following:

(i) 2^5 or 5^2 (ii) 3^5 or 5^3 (iii) 3^{10} or 10^3 (iv) 7^3 or 3^7

4. Express the following numbers in terms of the powers of prime factors:

(i) 324 (ii) 625 (iii) 1080 (iv) 1800

5. Simplify the following:

(i) 2×3^4 (ii) $7^3 \times 5$ (iii) $5^3 \times 2^2$ (iv) $3^2 \times 10^3$

(v) 0×10^4

6. Find the values of:

(i) $(-1)^3$ (ii) $(-5)^4$ (iii) $(-4)^2 \times (-2)^3$

5.3 Laws of Exponents

Rule 1 Multiplication of exponent numbers with same base.

Example 6 Find the value of $2^3 \times 2^4$.

Solution

$$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^7$$
$$2^3 \times 2^4 = 2^{(3+4)}$$
$$= 2^7$$

Note that here, the base in 2^3 and 2^4 is same and sum of exponents 3 and 4 is 7.

Example 7 Solve $(-5)^2 \times (-5)^3$.

Solution

$$(-5)^2 \times (-5)^3 = [(-5) \times (-5)] \times [(-5) \times (-5) \times (-5)]$$
$$= (-5) \times (-5) \times (-5) \times (-5) \times (-5)$$
$$= (-5)^5$$
$$(-5)^2 \times (-5)^3 = (-5)^{2+3}$$
$$= (-5)^5$$

> In general we can say that if a is a non-zero number, where m and n are positive integers, then $a^m \times a^n = a^{m+n}$

Rule 2: Division of Exponent Numbers with Common Base

Let us divide the numbers with common base and different power.

Example 8 Solve $2^7 \div 2^3$.

Solution

$$2^7 \div 2^3 = \frac{2^7}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2}$$
$$= 2 \times 2 \times 2 \times 2$$
$$= 2^4$$

So, $2^7 \div 2^3 = \frac{2^7}{2^3} = 2^{7-3} = 2^4$

Hence, $2^7 \div 2^3 = 2^4$

Example 9 Find the value of $a^4 \div a^2$.

Solution

$$a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a \times a \times a \times a}{a \times a}$$

So, $a^4 \div a^2 = \frac{a^4}{a^2} = a^{4-2} = a^2$

> If *a* is a non-zero number and *m* and *n* are two positive integers, where *m* >*n*, then
> $a^m \div a^n = a^{m-n}$

Again,

Example 10 Simplify $3^3 \div 3^7$.

Solution $3^3 \div 3^7 = \frac{3^3}{3^7} = \frac{\not{3} \times \not{3} \times \not{3}}{\not{3} \times \not{3} \times \not{3} \times 3 \times 3 \times 3 \times 3}$

$= \frac{1}{3^4}$

$\frac{3^3}{3^7} = \frac{1}{3^{7-3}} = \frac{1}{3^4}$

> If *a* is a non-zero number and *m* and *n* are two positive integers, where *m* <*n*, then
> $a^m \div a^n = \frac{1}{a^{n-m}}$

Zero Exponent:

Look at the following action

$3^2 \div 3^2 = 3^{2-2} = 3^0$

But $3^2 \div 3^2 = \frac{3^2}{3^2} = \frac{3 \times 3}{3 \times 3} = 1$

So, $3^0 = 1$

We have got $3^0 = 1$ above. Similarly, if for any base the exponent is zero, then its value is 1

> If *a* is a non-zero number then $a^0=1$.

Rule 3 Exponent of Exponent Number

Example 11 Find the value of $[(5)^3]^4$..

Solution $[(5)^3]^4 = (5^3) \times (5^3) \times (5^3) \times (5^3)$

$= 5^{3+3+3+3}$

$= 5^{(3\times4)}$

i.e., $[(5)^3]^4 = 5^{3\times4}$

It is concluded from above that

> If *a* is a non-zero number and *m* and *n* are positive integers, then $(a^m)^n = a^{m\times n}$

Rule 4: Multiplication of Numbers with Different Base but Common Exponent

Example 12 Could you simplify $2^4 \times 3^4$?

Solution

$2^4 \times 3^4$

$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= (2 \times 3)^4$

i.e., $2^4 \times 3^4 = (2 \times 3)^4$

It is concluded from the above example that.

> Note: Be aware that
> $a^m + b^m \neq (a+b)^m$
> $a^m - b^m \neq (a-b)^m$
> Example:
> $2^3 + 5^3 \neq (2+5)^3$
> $2^3 - 5^3 \neq (2-5)^3$

> If *a* and *b* are two non-zero number and *m* is a positive integers, then
> $a^m \times b^m = (a \times b)^m$

Rule 5 Division of Numbers with Different Base but Common Exponent

Example 13 Find the value of $8^5 \div 9^5$.

$$8^5 \div 9^5 = \frac{8^5}{9^5} = \frac{8 \times 8 \times 8 \times 8 \times 8}{9 \times 9 \times 9 \times 9 \times 9}$$

$$= \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9}$$

$$= \left(\frac{8}{9}\right)^5$$

i.e., $8^5 \div 9^5 = \frac{8^5}{9^5} = \left(\frac{8}{9}\right)^5$

> If *a* and *b* are two non-zero number and *m* is a positive integer, then
> $$a^m \div b^m = \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Exercise 5.2

1.Solve the following using laws of exponents.

(i)	$3^7 \times 3^8$	(ii)	$(4)^7 \times (4)^2$	(iii)	$a^5 \times a^4$
(iv)	$3^{15} \div 3^9$	(v)	$t^7 \div t^4$	(vi)	$(6^4 \times 6^2) \div 6^5$
(vii)	$(2^6)^3$	(viii)	$(a^5)^4$	(ix)	$5^5 \times 8^5$

(x) $a^3 \times b^3$ (xi) $7^5 \div 6^5$ (xii) $(25^3 \times 25^7) \div 25^{10}$

(xiii) $7^5 \div 7^8$ (xiv) $(9^3)^0$

2 .Simplify the following:

(i) $\{(3^2)^3 \times 3^4\} \div 3^7$ (ii) $16^4 \div 4^2$ (iii) $\frac{5^7}{5^4 \times 5^3}$

(iv) $4^0 \times 5^0 \times 6^0$ (v) $\frac{3^9 \times a^6}{9^2 \times a^3}$ (vi) $(7^3 \times 7)^3$

(vii) $\frac{3^{10}}{3^5 \times 3^7}$ (viii) $\frac{a^9}{a^6} \times a^8$ (ix) $2^0 + 3^0 + 4^0$

3 .Simplify the following:

(i) $\frac{2^3 \times 7^2 \times 13^8}{56 \times 13^7}$ (ii) $\frac{(3^2)^3 \times 5^3}{9^2 \times 25}$ (iii) $\frac{2^5 \times 10^5 \times 5}{5^4 \times 4^3}$

5.4 Denoting Big Numbers in Exponents form

Look at the following:

$$54 = \frac{54 \times 10}{10} = 5.4 \times 10^1$$

$$540 = \frac{540 \times 100}{100} = 5.4 \times 10^2$$

$$5400 = \frac{5400 \times 1000}{1000} = 5.4 \times 10^3$$

$$54000 = \frac{54000 \times 10000}{10000} = 5.4 \times 10^4$$

Here we have expressed 54, 540, 5400, 54000 in standard forms.

The velocity of light is 300,000,000 m/s which can be expressed in the following standard form.

Standard form: 3×10^8 m/s.

When a number is expressed as a product of 1.0 or a decimal number greater than 1.0 and less than 10 and powers of 10, then such form of a number is known as its standard form.

5.5 Expressing Big Numbers in Standard Forms

We know that the big numbers can be expressed conveniently in standard forms by using exponents. Let us express the big numbers in to standard form using exponents.

We write 7465 in standard form.

$7465 = 7.465 \times 1000$

$= 7.465 \times 10^3$.

(Decimal is shifted three places to the left)

Mass of the Earth = 5976, 000, 000, 000, 000, 000, 000, 000 kg
$= 5.976 \times 10^{24}$ kg.

You would be agree with the fact that standard form of a number is easier than a 25 digit number from the point of view of expressing, comparing and understanding.

Example 14 Write the number 150,000,000,000 in to standard form.

Solution $150,000,000,000 = 1.5 \times 10^{11}$
(Decimal is shifted 11 places to the left)

Note: While adding the numbers, numbers should be written in the same powers of 10

Example 15 Write the following numbers in to standard form:

(i) 63000 (ii) 100000 (iii) 425000

Solution

(i) 63000 = 6.3×10000
= 6.3×10^4

(ii) 100000 = 1×100000
= 1×10^5

(iii) 425000 = 4.25×100000
= 4.25×10^5

Example 16 According to census, the population of India in a year was 1,00,84,35,405. Write this in scientific notations.

Solution Population of India = 1,00,84,35,405
= 1.00,84,35,405 × 1,00,00,00,000
= 1.008435405×10^9
= 1.008×10^9 (approx.)

Exercise 5.3

1. Write the following numbers in to standard form:

(i) 50,0000 (ii) 48,30,000
(iii) 3,94,00,00,00,000 (iv) 30000000
(v) 180000

2. Distance of the Sun from the Earth is approximately 15,00,00,000 km. Express this distance in scientific notations.
3. A person gets 3000 Calorie energy from his daily meal. Represent in scientific notations, how much energy he will get in one year.
4. According to an estimation, Indian railways transports approximately 1 Crore 30 Lakhs people from one place to another place daily. How many people travel by train in 30 days? Write your answer in standard form.
5. Write the following in simple form:

   (i) $2.5 \times (10)^{4}$ (ii) $1.75 \times (10)^{6}$

   (iii) $1.21 \times (10)^{-8}$ (iv) $4.50 \times (10)^{-5}$

We Learnt

1. Numbers can be expressed in terms of exponents. Use of exponents makes reading, understanding, comparing and performing operations upon very big and very small numbers easy.
2. Numbers follow certain rules in the exponent form: For non-zero number *a* and *b* and integers *m&n.*

   (i) $a^m \times a^n = a^{m+n}$

   (ii) $a^m \div a^n = a^{m-n}$ if m>n or $a^m \div a^n = \dfrac{1}{a^{n-m}}$ if n > m

   (iii) $(a^m)^n = a^{mn}$

   (iv) $a^m \times b^m = (ab)^m$

   (v) $a^m \div b^m = \left(\frac{a}{b}\right)^m$

   (vi) $a^0 = 1$

3. In order to express a number in scientific notation or standard form, we write the product of a decimal number lying between 1.0 and 10.0 (in which 1.0 is included but 10.0 is not included) and the powers of 10.