Chapter 2

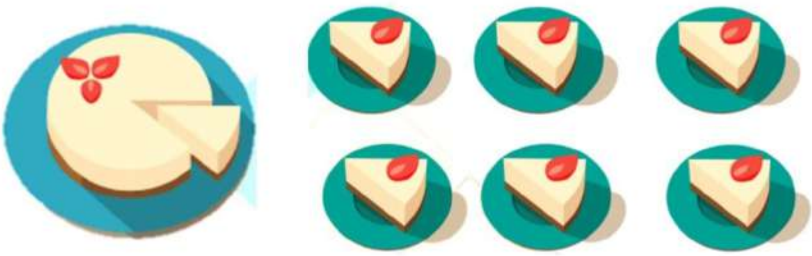
Fractions and Decimals

Introduction to Fractions

Ethan is celebrating his birthday at home. His mother has baked a cake for his birthday. When his friends came home, he cuts the cake. Now, his mother wants to distribute the cake equally among all his friends.

There are 6 people (including Ethan's mother) at the party.

So, his mother cuts the cake into 6 equal parts.



Can you tell what fraction of the cake does Ethan gets?

Total number of slices of cake = 6

Ethan got $\frac{1}{6}$ (one-sixth) part of the cake. So, Ethan ate one part out of **six parts of the cake.**

Now, Ethan and his friends had learnt about fractions at school.

So, one of Ethan's friends while eating the cake, cuts his slice of cake into two equal pieces and asked Ethan what fraction of the whole slice was that piece?

So Ethan said that each equal piece is one – half $\frac{1}{2}$ of one whole slice and the two pieces together will be $\frac{2}{2}$ or 1 whole slice.

We can say that a fraction is a number representing part of the whole.
The whole may be a single object or a group of objects.

A fraction is a number of the form $\frac{p}{q'}$ such that $q \neq 0$

In the fraction $\frac{p}{q'}$

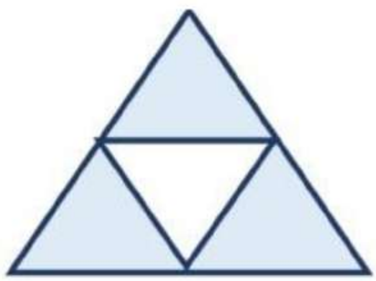
- p and q are whole numbers.

- p is called the numerator and q is called the denominator.

Example: Write the fraction representing the shaded portion.

a) The given figure is divided into 4 equal parts.
Number of shaded parts = 3
Total number of equal parts = 4

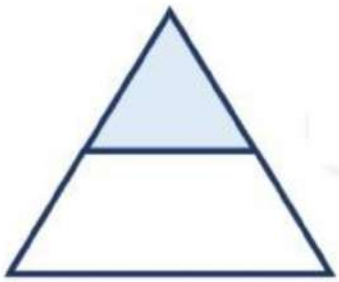
Fraction representing the shaded portion = $\frac{3}{4}$



b) The given figure is divided into 4 equal parts.
Number of shaded parts = 1
Total number of equal parts = 4

Fraction representing the shaded portion = $\frac{1}{4}$

c) The given figure is not divided into equal parts. For making fractions the figure should be divided into equal parts.



Types of Fraction

Proper Fraction

- The numerator of a fraction is smaller than the denominator
- $\frac{1}{9}, \frac{6}{11}, \frac{10}{30}$

Improper Fraction

- The numerator of a fraction is greater than the denominator.
- $\frac{11}{7}, \frac{34}{27}, \frac{101}{100}$

Mixed Fraction

- Fractions having whole numbers along with proper fraction
- $5\frac{6}{7}, 3\frac{4}{11}, 8\frac{2}{13}$

Like Fraction

- Fractions with same denominators are called like fractions
- $\frac{5}{7}, \frac{23}{7}, \frac{44}{7}$

Unlike Fraction

- Fractions with different denominators are called unlike fractions
- $\frac{3}{7}, \frac{16}{15}, \frac{100}{9}$

Addition and Subtraction of Fractions

1: Addition and Subtraction of two like fractions

When we add or subtract like fractions, we add or subtract their numerators and the denominator remains the same.

1) $\frac{5}{9} + \frac{2}{9}$

The two fractions are like fractions, so we add their numerators and keep the denominator the same.

$\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9}$

2) $\frac{14}{25}$ - $\frac{9}{25}$

Here, the given fractions are like fractions. So, we subtract their numerators and keep the denominator the same.

$\frac{11}{25}$ - $\frac{11}{25} = \frac{14-11}{25} = \frac{3}{25}$

2) Addition and Subtraction of two unlike fractions

When we add or subtract unlike fractions, we follow the following steps:

1. **Find the LCM of their denominators.**
2. **Convert the given fractions into like fractions.**
3. **Then add or subtract the like fractions.**

1) $\frac{5}{8} + \frac{11}{24}$

2	8, 24
2	4, 12
2	2, 6
3	1, 3
	1, 1

The given fractions are unlike fractions, so we first find LCM of their denominators.

LCM of 8 and 24 = 2 × 2 × 2 × 3 = 24

Now, we convert the fractions into like fractions.

(changing the denominator of fractions to 24)

$\frac{5x3}{8x3} = \frac{15}{24}$ and $\frac{11}{24}$

$\frac{15}{24} + \frac{11}{24} = \frac{15+11}{24} = \frac{26}{24}$

2) $\frac{11}{15} - \frac{5}{27}$

3	15, 27
3	5, 9
3	5, 3
5	5, 1
	1, 1

As the given fractions are unlike fractions, we find the LCM of their denominator.

LCM of 15 and 27 = 3 × 3 × 3 × 5 = 135

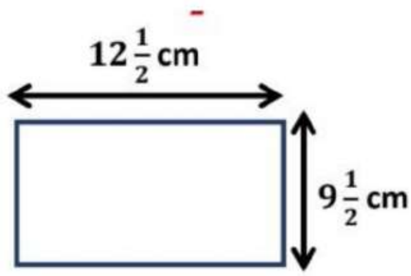
Next, we convert the fractions into like fractions

(fractions with the same denominator)

$\frac{11x9}{15x9} = \frac{99}{135}$ and $\frac{11x5}{27x5} = \frac{55}{135}$

$$\frac{99}{135} - \frac{55}{135} = \frac{99-55}{135} = \frac{44}{135}$$

Example: A rectangular sheet of paper is $12\frac{1}{2}$ cm long and $9\frac{1}{2}$ cm wide. Find its perimeter.



Length of the rectangular sheet = $12\frac{1}{2}$ cm

$$12\frac{1}{2} = \frac{12x2+1}{2} = \frac{24+1}{2} = \frac{25}{2}$$

Breadth of the rectangular sheet = $9\frac{1}{2}$ cm

$$9\frac{1}{2} = \frac{9x2+1}{2} = \frac{18+1}{2} = \frac{19}{2}$$

$$a\frac{b}{c} = \frac{a \times c + b}{c}$$

Perimeter of a rectangle = $2(l + b)$

Perimeter is the distance around a closed figure.

Perimeter of rectangular sheet of paper

$$= 2\left(\frac{25}{2} + \frac{19}{2}\right) = 2\left(\frac{25+19}{2}\right) = 2\left(\frac{44}{2}\right) = 44 \text{ cm}$$

Example: Michael finished coloring a picture in $\frac{7}{12}$ hour. Vaibhav finished colouring the same picture in $\frac{3}{4}$ hour. Who worked longer? By what fraction was it longer?

Time taken by Michael to colour the picture = $\frac{7}{12}$ hour

Time taken by Vaibhav to colour the same picture = $\frac{3}{4}$ hour

The two fractions are unlike, so we first convert them to like fractions (fractions having same denominator).

$$\frac{7}{12}, \frac{3}{4}$$

LCM of 12 and 4 = 2 × 2 × 3 =

2	12, 4
2	6, 2
3	3, 1
	1, 1

12

$\frac{7}{12}$ and $\frac{3x3}{4x3} = \frac{9}{12}$

On comparing the two fractions we get, $\frac{9}{12} > \frac{7}{12}$

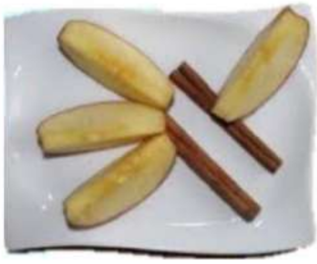
Therefore, Vaibhav worked longer by

$\frac{9}{12} - \frac{7}{12} = \frac{(9-7)}{12} = \frac{(2)}{12} = \frac{(1)}{6}$ hour

Multiplication of Fractions

1) Multiplication of a fraction by a whole number

Suppose there is an apple, you cut it into four equal parts.



Each part represents one – fourth of an apple.

So, $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1}{4} = 1$ (whole apple)

Or $4 \times \frac{1}{4} = 1$

$$w \times \frac{a}{b} = \frac{w \times a}{b}$$

Therefore, we can say that multiplication is repeated addition.

When we multiply a fraction by a whole number, we multiply the numerator of the fraction with the whole number keeping the denominator the same.

i) $3 \times \frac{2}{7}$

Here, we are multiplying a whole number by a proper fraction. So, we multiply the numerator of the fraction with the whole number and keep the denominator the same.

$$3 \times \frac{2}{7} = \frac{(3x2)}{7} = \frac{6}{7}$$

Let us see one example where we are multiplying a whole number by an improper fraction.

ii) $4 \times \frac{7}{5}$

Here, the whole number is being multiplied by an improper fraction (numerator is greater than denominator). Again we multiply the numerator of the fraction with the whole number and keep the denominator the same.

$$4 \times \frac{7}{5} = \frac{(4x7)}{5} = \frac{28}{5}$$

As the product is an improper fraction, we express it as a mixed fraction.

$$\frac{28}{5} = \frac{5x5 + 3}{5} = \frac{25}{5} + \frac{3}{5} = 5 + \frac{3}{5} = 5\,\frac{3}{5}$$

Example: Multiply and reduce to lowest form:

i) $8 \times \frac{3}{5}$ ii) $\frac{2}{9} \times 5$ iii) $11 \times \frac{7}{4}$ iv) $14 \times \frac{1}{3}$

i) $8 \times \frac{3}{5}$

Here, we are multiplying a whole number by a proper fraction (numerator is smaller than denominator). So, we multiply the numerator of the fraction with the whole number and keep the denominator the same.

$$8 \times \frac{3}{5} = \frac{(8x3)}{5} = \frac{24}{5}$$

$$\frac{24}{5} = \frac{5x4 + 4}{5} = \frac{20}{5} + \frac{4}{5} = 4 + \frac{4}{5} = 4\,\frac{4}{5}$$

ii) $\frac{2}{9} \times 5$

A proper fraction is being multiplied by a whole number. So we multiply the numerator of the fraction with the whole number and keep the denominator the same.

$$\frac{2}{9} \times 5 = \frac{(2x5)}{9} = \frac{10}{9}$$

As the product is an improper fraction (numerator is greater than denominator), we express it as a mixed fraction.

$$\frac{10}{9} = \frac{9x1+1}{9} = \frac{9}{9} + \frac{1}{9} = 1 + \frac{1}{9} = 1\frac{1}{9}$$

$$\frac{2}{9} \times 5 = 1\frac{1}{9}$$

iii) $11 \times \frac{7}{4}$

We are multiplying a whole number by an improper fraction. So, we multiply the numerator of the fraction by the whole number and the denominator is kept the same.

$$11 \times \frac{7}{4} = \frac{(11x7)}{4} = \frac{77}{4}$$

Now, the product is an improper fraction, so we express it as a mixed fraction.

$$\frac{77}{4} = \frac{4x19+1}{4} = \frac{4x19}{4} + \frac{1}{4} = 19 + \frac{1}{4} = 19\frac{1}{4}$$

$$11 \times \frac{7}{4} = 19\frac{1}{4}$$

iv) $14 \times \frac{1}{3}$

Here, we are multiplying a whole number by an improper fraction. So, we multiply the numerator of the fraction by the whole number and the denominator is kept the same.

$$14 \times \frac{1}{3} = \frac{(14x1)}{3} = \frac{14}{3}$$

The product is an improper fraction, so we express it as a mixed fraction.

$$\frac{14}{3} = \frac{3x4+2}{3} = \frac{3x4}{3} + \frac{2}{3} = 4 + \frac{2}{3} = 4\,\frac{2}{3}$$

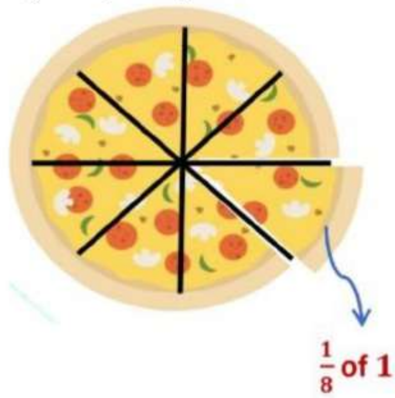
$$14 \times \frac{1}{3} = 4\,\frac{2}{3}$$

Fraction as an operator 'of'

A pizza is divided into 8 equal slices.

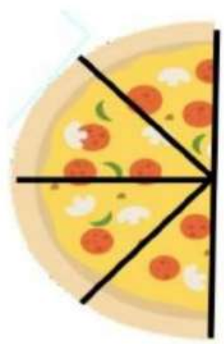
Each slice represents $\frac{1}{8}$ th of pizza.

On combining four slices of pizza, we get



$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

So, we say that $\frac{1}{8}$ of $4 = \frac{1}{2}$



We say that 'of' represents multiplication.

$\frac{1}{2}$ of 10 is $\frac{1}{2}$ x 10

We know that when we multiply a whole number by a fraction, we multiply the numerator of the fraction by the whole number and the denominator is kept the same

$\frac{1}{2} \times 10 = \frac{(1x10)}{2} = 5$

Example: Find

i) $\frac{2}{3}$ of 27 ii) $\frac{4}{5}$ of 35 iii) 3 $\frac{5}{6}$ of 5

i) $\frac{2}{3}$ of 27

$\frac{2}{3}$ of 27 is $\frac{2}{3}$ x 27

We multiply the numerator of the fraction by 27 and keep the denominator the same.

$\frac{2}{3} \times 27 = \frac{(2x27)}{3} = \frac{54}{3} = 18$

ii) $\frac{4}{5}$ of 35

$\frac{4}{5}$ of 35 is $\frac{4}{5}$ x 35

We multiply the numerator of the fraction by 35 and keep the denominator the same.

$\frac{4}{5} \times 35 = \frac{(4x35)}{5} = \frac{140}{5} = 28$

iii) $3\frac{5}{6}$ of 5

$3\frac{5}{6}$ of 5 is $3\frac{5}{6} \times 5$

$$3\frac{5}{6} = \frac{3 \times 6 + 5}{6} = \frac{18 + 5}{6} = \frac{23}{6}$$

$\frac{23}{6} \times 5$

We again multiply the numerator of the fraction by 23 and keep the denominator the same.

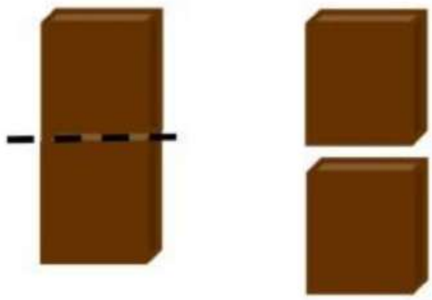
$$\frac{23}{6} \times 5 = \frac{(23 \times 5)}{6} = \frac{115}{6}$$

The product is an improper fraction, so we express it as a mixed fraction.

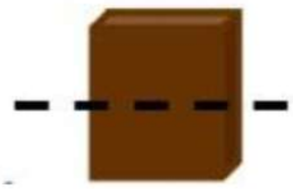
$$\frac{115}{6} = \frac{6 \times 19 + 1}{6} = \frac{6 \times 19}{6} + \frac{1}{6} = 19 + \frac{1}{6} = 19\frac{1}{6}$$

Multiplication of a Fraction by Another Fraction

John has a bar of chocolate. He divided the chocolate bar into two equal parts and gave one part to his brother, Jason.



This part of chocolate represents $\frac{1}{2}$ of whole or $\frac{1}{2}$ of 1.



Now, Jason again divided his share of chocolate into two equal parts, then each of the two parts represents $\frac{1}{2}$ of $\frac{1}{2}$.

$\frac{1}{2}$ **of** $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

When we multiply a fraction by a fraction, we multiply their numerators and denominators.

Product of two fractions = $\frac{\textit{Product of Numerators}}{\textit{Product of Denominators}}$

$$\frac{1}{5} \text{ x } \frac{1}{7}$$

Here, we are multiplying two fractions, so we multiply their numerators and denominators.

$\frac{1}{5}$ **x** $\frac{1}{7} = \frac{(1x1)}{(5x7)} = \frac{1}{35}$

Value of the Products

Consider the two proper fractions, $\frac{1}{5}$ and $\frac{2}{7}$

Product of $\frac{1}{5}$ and $\frac{2}{7}$ = $\frac{1}{5}$ x $\frac{2}{7}$

We multiply the numerators and denominators of the two fractions.

$$\frac{1}{5} \times \frac{2}{7} = \frac{(1x2)}{(5x7)} = \frac{2}{35}$$

Now, we compare the two fractions, $\frac{1}{5}$ and $\frac{2}{7}$ with their product, $\frac{2}{35}$Converting the two fractions to like fractions we get,

$\frac{1x7}{5x7}$and $\frac{2x5}{7x5}$

$=\frac{7}{35}$ and $\frac{10}{35}$

$\frac{10}{35} > \frac{7}{35} > \frac{2}{35}$

We see that the value of the product of two proper fractions is smaller than each of the two fractions.

Now, again consider two improper fractions, $\frac{3}{2}$ and $\frac{8}{7}$

Product of ,$\frac{3}{2}$and $\frac{8}{7}$= ,$\frac{3}{2}$ x $\frac{8}{7}$

We multiply the numerators and denominators of the two fractions.

$$\frac{3}{2} \times \frac{8}{7} = \frac{(3x8)}{(7x2)} = \frac{24}{14}$$

Now, comparing the two fractions, $\frac{3}{2}$and $\frac{8}{7}$ with their product $\frac{24}{14}$

$\frac{3x7}{2x7}$ and, $\frac{8x2}{7x2}$ (Converting the two fractions to like fractions)

$= \frac{21}{14}$ and $\frac{16}{14}$

$$\frac{24}{14} > \frac{21}{14} > \frac{21}{16}$$

We see that the value of the product of two improper fractions is more than each of the two fractions.

Example: Multiply and reduce to lowest form, tell whether the fraction obtained is proper or improper and if the fraction obtained is improper then convert it into a mixed fraction.

i) $\frac{2}{7} \times \frac{7}{9}$ ii) $\frac{9}{5} \times \frac{3}{5}$ iii) $\frac{2}{3} \times 2\frac{2}{3}$ iv) $\frac{9}{2} \times \frac{7}{4}$

i) $\frac{2}{7} \times \frac{7}{9}$

As we are multiplying two fractions, we multiply their numerators and denominators.

$$\frac{2}{7} \times \frac{7}{9} = \frac{(2 \times 7)}{(7 \times 9)} = \frac{14}{63}$$

$\frac{14}{63}$ is a proper fraction because the numerator is smaller than the denominator.

ii) $\frac{9}{5} \times \frac{3}{5}$

We multiply the numerators and denominators of the two fractions.

$$\frac{9}{5} \times \frac{3}{5} = \frac{(3 \times 9)}{(5 \times 5)} = \frac{27}{25}$$

$\frac{27}{25}$ is an improper fraction because the numerator is greater than the denominator and so we convert it into a mixed fraction.

$$\frac{27}{25} = \frac{25x1+2}{25} = \frac{25x1}{25} = \frac{2}{25} = 1\ \frac{2}{25} = 1\ \frac{2}{25}$$

iii) $\frac{2}{3}$ x 2 $\frac{2}{3}$

We first change the mixed fraction to an improper fraction.

$$2\ \frac{2}{3} = \frac{3x2+2}{3} = \frac{6+2}{3} = \frac{8}{3}$$

$\frac{2}{3}$ x $\frac{8}{3}$ = $\frac{(2x8)}{(3x3)}$ (multiplying the numerators and denominators of the two fractions)

$$= \frac{16}{9}$$

As $\frac{16}{9}$ is an improper fraction (Numerator > Denominator), we convert it into a mixed fraction.

$$\frac{16}{9} = \frac{9x1+7}{9} = \frac{9x1}{9} + \frac{7}{9} = 1 + \frac{7}{9} = 1\ \frac{7}{9}$$

iv) $\frac{9}{2}$ x $\frac{7}{4}$

$\frac{9}{2}$ x $\frac{7}{4}$ = $\frac{(9x7)}{(2x4)}$ (multiplying the numerators and denominators of the two fractions)

$= \frac{63}{8}$ ' (Improper Fraction (Numerator > Denominator)

$$\frac{63}{8} = \frac{8x7+7}{8} = \frac{8x7}{8} + \frac{7}{8} = 7 + \frac{7}{8} = 7\ \frac{7}{8}$$

Example: Saahat reads $\frac{1}{3}$ part of a book in 1 hour. How many parts of the book will he read in 2 $\frac{1}{4}$ hours?

Part of the book read by Saahat in 1 hour = $\frac{1}{3}$

Part of the book read by Saahat in 2 $\frac{1}{4}$ hours = $\frac{1}{3}$ x 2 $\frac{1}{4}$

We first change the mixed fraction to an improper fraction.

$2\frac{1}{4} = \frac{2x4+1}{4} = \frac{8+1}{4} = \frac{9}{4}$

$\frac{1}{3} \text{ x } \frac{9}{4} = \frac{(1x9)}{(3x4)}$ (multiplying the numerators and denominators of the two fractions)

$= \frac{3}{4}$

Therefore, Saahat read $\frac{3}{4}$ part of the book in 2 $\frac{1}{4}$ hours.

Division of Whole Numbers by a Fraction

Ethan's mother brings a jar full of lemonade and pours $\frac{1}{3}$ liters into each glass.

Can you tell how many glasses of lemonade she will get if the capacity of the jar of lemonade is 2 liters?

To find the number of glasses we divide 2 liters (capacity of the jar) by $\frac{1}{3}$ (quantity of lemonade in each glass).

$2 \div \frac{1}{3}$ = Number of glasses obtained when 2 liters is divided into equal parts

$2 \div \frac{1}{3} = 2 \text{ x } \frac{3}{1} = \frac{2x3}{1} = \frac{6}{1} = 6$

Here 3 is the reciprocal of $\frac{1}{3}$.

The non-zero numbers whose product with each other is 1, are called reciprocals of each other.

$$a \times \frac{1}{a} = 1$$

So, reciprocal of $\frac{7}{9}$ is $\frac{9}{7}$, since $\frac{7}{9} \times \frac{9}{7} = 1$

1) When we divide a whole number by any fraction, we multiply that whole number by the reciprocal of that fraction.

$7 \div \frac{2}{5}$

Reciprocal

of $\frac{2}{5}$ is $\frac{5}{2}$.

We see that the reciprocal of proper fraction $(\frac{2}{5})$ is an improper fraction $(\frac{5}{2})$

Now, we multiply 7 by the reciprocal of $\frac{2}{5}$.

$7 \div \frac{2}{5} = 7 \times \frac{5}{2}$

$7 \times \frac{5}{2} = \frac{7x5}{2}$ (Multiplying numerator of the fraction by the whole number)

$= \frac{35}{2}$

$7 \div \frac{2}{5} = \frac{35}{2}$

2) While dividing a whole number by a mixed fraction, first convert the mixed fraction into an improper fraction and then solve it.

$4 \div 2\frac{2}{5}$

We first convert the mixed fraction into an improper fraction.

$2\frac{2}{5} = \frac{2x5 + 2}{5} = \frac{10 + 2}{5} = \frac{12}{5}$

$4 \div 2\frac{2}{5}$

We see that the reciprocal of improper fraction ($\frac{12}{5}$) is aproper fraction ($\frac{5}{12}$)

Reciprocal of $\frac{12}{5}$ is $\frac{5}{12}$

Now, we multiply 4 by the reciprocal of $\frac{12}{5}$.

$4 \times \frac{5}{12} = \frac{4 \times 5}{12}$

(Multiplying numerator of the fraction by the whole number)

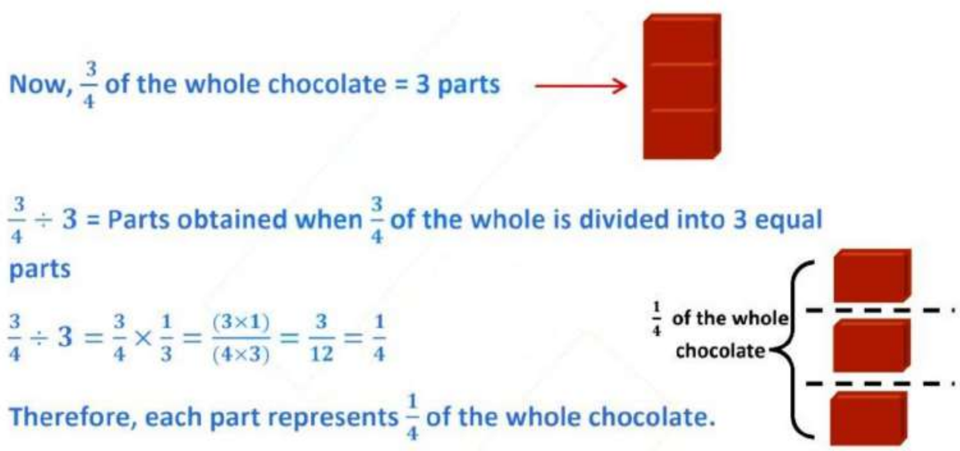
$= \frac{20}{12}$

$4 \div \frac{12}{5} = \frac{20}{12}$

Division of Fraction by a Whole Number

Suppose you have a bar of chocolate.
Now, you have to divide three – fourth of the chocolate into three equal parts.
How will you do it?

Let’s do it step by step.



When we divide a fraction by a whole number, we multiply the fraction by the reciprocal of the whole number.

$\frac{a}{b} \div w = \frac{a}{b} x \frac{1}{w} = \frac{(ax1)}{(bxw)} = \frac{a}{bxw}$

$\frac{6}{13} \div 7$

We will multiply the fraction by the reciprocal of the whole number. Reciprocal of 7 = $\frac{1}{7}$

$= \frac{6}{13} x \frac{1}{7} = \frac{6}{91}$

Example: Find,

i) $\frac{4}{9} \div 5$ ii) $\frac{5}{7} \div 6$ iii) $3\frac{1}{2} \div 4$ iv) $4\frac{3}{7} \div 7$

i) $\frac{4}{9} \div 5$

Reciprocal of 5 = $\frac{1}{5}$

Next, we multiply the fraction with reciprocal of 5.

$\frac{4}{9} x \frac{1}{5} = \frac{(4x1)}{9x5)}$ (Multiplying the numerators and denominators of fractions)

$=\frac{4}{45}$

ii) $\frac{5}{7} \div 6$

Reciprocal of 6 = $\frac{1}{6}$

$\frac{5}{7} \div 6 = \frac{5}{7} x \frac{1}{6}$ **(multiplying the fraction with reciprocal of 5)**

$\frac{5}{7}$ x $\frac{1}{6}$ = $\frac{(5x1)}{(7x6)}$ (Multiplying the numerators and denominators of fractions)

$= \frac{5}{12}$

$\frac{5}{7} \div 6 = \frac{5}{12}$

iii) $3\frac{1}{2} \div 4$

$3\frac{1}{2}$
$= \frac{2x3+1}{2} = \frac{6+1}{2} = \frac{7}{2}$

While dividing mixed fractions by whole numbers, we first convert the mixed fractions into improper fractions.

$\frac{7}{2} \div 4$

Reciprocal of 4 = $\frac{1}{4}$

$\frac{7}{2} \div 4 = \frac{7}{2}$ x $\frac{1}{4}$ (multiplying the fraction with reciprocal of 4)

$\frac{7}{2}$ x $\frac{1}{4}$ = $\frac{(7x1)}{(2x4)}$ (multiplying the numerators and denominators of fractions)

$= \frac{7}{8}$

$\frac{7}{2} \div 4 = \frac{7}{8}$

iv) $4\frac{3}{7} \div 7$

$4\frac{3}{7} = \frac{4x7+3}{7} = \frac{28+3}{7} = \frac{31}{7}$ (converting mixed fraction into improper fraction)

$\frac{31}{7} \div 7$

Reciprocal of $7 = \frac{1}{7}$

$\frac{31}{7} \div 7 = \frac{31}{7} \times \frac{1}{7}$ (multiplying the fraction with reciprocal of 7)

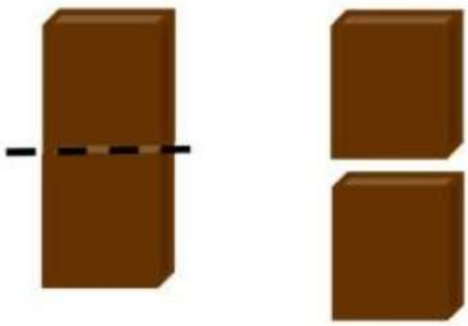
$\frac{31}{7} \times \frac{1}{7} = \frac{(31x1)}{(7x7)}$ (multiplying the numerators and denominators of fractions)

$= \frac{31}{49}$

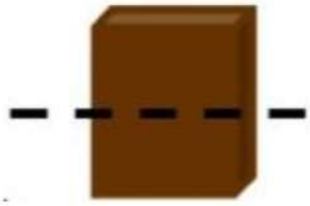
$\frac{31}{7} \div 7 = \frac{31}{49}$

<u>Division of a Fraction by Another Fraction</u>

John has a bar of chocolate. He divided the chocolate bar into two equal parts and gave one part to his brother, Jason.



This part of chocolate represents $\frac{1}{2}$ of whole or 1 divided by 2.



Now, Jason again divided his share of chocolate into two equal parts, then each of the two parts represents $\frac{1}{2}$ divided by 2 .

$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

When we divide a fraction by a fraction, we multiply the numerator by denominator and denominator by numerator.

Division of two fractions

$= \frac{x}{y} \div \frac{a}{b} = \frac{x \times b}{y \times a}$

$$\frac{x}{y} \div \frac{a}{b} = \frac{x \times b}{y \times a}$$

For Example, $\frac{1}{5} \div \frac{1}{7}$

Here, we are multiplying two fractions, so we multiply their numerators and denominators.

$\frac{1}{5} \div \frac{1}{7} = \frac{1x7}{5x1} = \frac{7}{5}$

Example: Solve the following

i) $\frac{2}{7} \div \frac{9}{7}$ ii) $\frac{9}{5} \div \frac{3}{5}$ iii) $\frac{2}{3} \div 2\frac{2}{3}$ iv) $\frac{9}{2} \div \frac{7}{4}$

i) It is given that $\frac{2}{7} \div \frac{9}{7}$

Now

$\frac{2}{7} \div \frac{9}{7} = \frac{2}{7} \text{ x } \frac{7}{9} = \frac{2}{9}$

ii) It is given that $\frac{9}{5} \div \frac{3}{5}$

Now

$\frac{9}{5} \div \frac{3}{5} = \frac{9}{5} \text{ x } \frac{5}{3} = 3$

iii) It is given that $\frac{2}{3} \div 2\frac{2}{3}$

$2\frac{2}{3} = \frac{8}{3}$

Now

$\frac{2}{3} \div 2\frac{2}{3} = \frac{2}{3} \div \frac{8}{3} = \frac{2}{3} \text{ x } \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$

iv) It is given that $\frac{9}{2} \div \frac{7}{4}$

Now

$\frac{9}{2} \div \frac{7}{4} = \frac{9}{2} \text{ x } \frac{4}{7} = \frac{18}{7}$

Example: Sushant reads $\frac{1}{3}$ part of a book in 2 hours. How many parts of the book will he read in 1 hour?

Part of the book read by Sushant in 2 hours = $\frac{1}{3}$

Part of the book read by Sushant in 1 hour = $\frac{1}{3} \div 2$

$$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Therefore, Sushant read $\frac{1}{6}$ part of the book in 1 hour.

Introduction to Decimals

The numbers expressed in decimal forms are called decimals.

Decimals have a decimal part and a whole number part. The point is used to separate these parts.

The number on the left side of decimal is the whole number part and the number formed by the digits at the right side of the decimal is called decimal

part. **1.5 litres of milk**

35.5 inches

53.4

53 → Whole number

4 → Decimal Part

Place Values of Decimals

Let's revise the place value chart of decimal numbers

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Example: Arrange the given decimal numbers in the place value chart and also write their expanded form

i) 21.6

ii) 305.64

iii) 3.289

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	2	1	6	0	0
3	0	5	6	4	0
		3	2	8	9

$$21.6 = 2 \times 10 + 1 \times 1 + 6 \times \frac{1}{10}$$

$$305.64 = 3 \times 100 + 0 \times 10 + 5 \times 1 + 6 \times \frac{1}{10} + 4 \times \frac{1}{100}$$

$$3.289 = 3 \times 1 + 2 \times \frac{1}{10} + 8 \times \frac{1}{100} + 9 \times \frac{1}{1000}$$

Comparing Decimals

Consider the decimals, 28.43 and 28.67.

If we have to compare the given decimals, we follow the following steps.

1) We first compare the whole-number part (starting from the leftmost digit)

In the given decimals, 28.43 and 28.67 we see that the digits, 2 and 8 to the left of the decimal point are the same in both the decimals.

2) If the whole number parts are equal, then we compare the digits on the right of the decimal point starting from the tenths place.

Digits at tenths place of the decimals, 28.43 and 28.67 are 4 and 6 respectively.

Now, $6 > 4$

Therefore $28.67 > 28.43$

Example: Which is greater?

i) 0.5 or 0.05 ii) 1.37 or 1.49 iii) 0.8 or 0.88

i) 0.5 or 0.05

We compare the whole number parts (digit to the left of the decimal point) of the decimals, 0.5 and 0.05. Clearly, it is the same in both the decimals.

Next, we compare the digits at the tenths place of 0.5 and 0.05.

Now, $5 > 0$.

Therefore, $0.5 > 0.05$.

ii) 1.47 or 1.49

Comparing the whole number parts of the decimals, 1.47 and 1.49 (digit to the left of the decimal point) we see that it is the same in both the numbers.

Next, we compare the tenths digits of the decimals, 1.47 and 1.49. Clearly, the tenths digit is also the same in both the numbers.

We compare the hundredths digit of the decimals, 1.47 and 1.49.

Now, $9 > 7$. So, $1.49 > 1.47$

iii) 0.8 or 0.88

As the number of digits to the right of the decimals is not the same, so we add a 0 to the right of 0.8.

Therefore, the two decimals are 0.80 and 0.88.

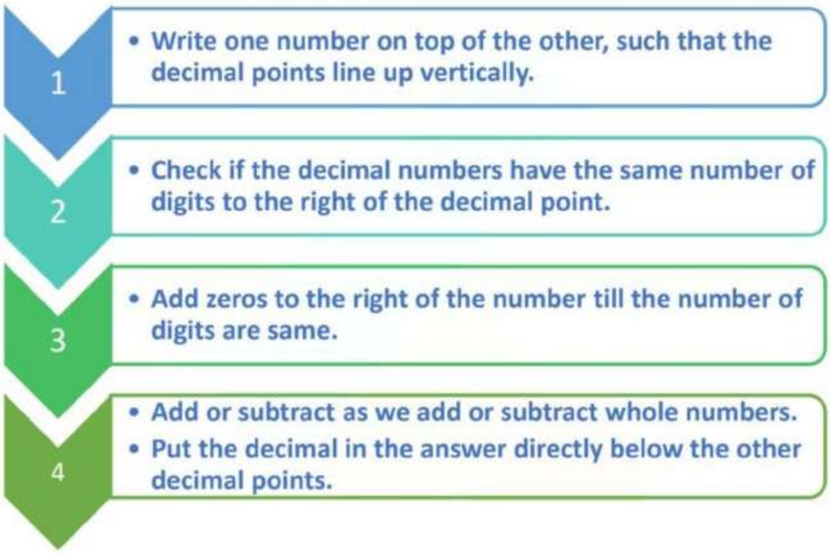
We see that the whole number parts of the decimals, 0.80 and 0.88 are the same.

On comparing the tenths digits of the decimals, 0.80 and 0.88 we see that they are also the same.

Next, we compare the hundredths digit of the decimals, 0.80 and 0.88.

Now, $8 > 0$. So, $0.88 > 0.80$

Addition and Subtraction of Decimals



Add: $0.19 + 2.3$
Decimal numbers, 0.19 and 2.3 have two digits and one digit respectively to the right of the decimal point. So, we add a zero to the right of 2.3.

```
  0.19
+ 2.30
------
  2.49
```

9 + 0 = 9

1 + 3 = 4

0 + 2 = 2

Subtract: 39.87 – 21.98

Decimals numbers 39.87 and 21.98 have the same number of zeros after the decimal point.

```
   8 171
   39.87
 - 21.98
 -------
   17.89
```

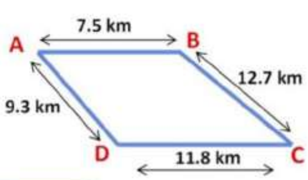
Digit at the hundredths place of the top number is smaller than that of the bottom number.
Digit at tenths place of the top number is also smaller than that of the bottom number (8<9).
So, we borrow from ones digit in order to do the subtraction.
17 – 8 = 9
17 – 9 = 8
8 – 1 = 7
3 – 2 = 1

Example: Dinesh went from place A to place B and from there to place C. A is 7.5 km from B and B is 12.7 km from C. Ayub went from place A to place D and from there to place C. D is 9.3 km from A and C is 11.8 km from D. Who travelled more and by how much?

Distance travelled by Dinesh

= Distance from A to B + Distance from B to C

=7.5 km + 12.7 km




```
   1
    7.5
  1
 + 12.7
 ------
   20.2
```

5 + 7 = 12

1 + 7 + 2 = 10

1 + 1 = 2

3 + 8 = 11

1 + 9 + 1 = 11

1 + 1 = 2

Distance travelled by Dinesh = 20.2 km

Distance travelled by Ayub

=Distance from A to D + Distance from D to C

= 9.3 km + 11.8 km

$$\begin{array}{r} \overset{1}{}9.3 \\ +\ \overset{1}{1}1.8 \\ \hline 21.1 \end{array}$$

Distance travelled by Ayub = 21.1 km

We see that the distance travelled by Ayub is more than the distance travelled by Dinesh.

Difference = 21.1 km – 20.2 km

$$\begin{array}{r} \overset{0}{2}\overset{1}{\not{1}}.1 \\ 20.2 \\ \hline 00.9 \end{array}$$

11- 2 = 9

0 – 0 = 0

2 – 2 = 0

So, Ayub travelled 0.9 km more than Dinesh.

Multiplication of Decimal Numbers

We will now learn the multiplication of two decimal numbers.

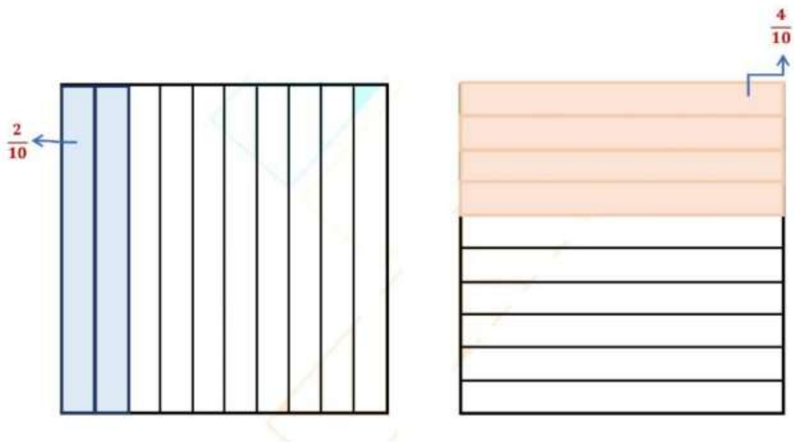
Consider two decimal numbers, 0.2 and 0.4.

Let us now find 0.2 × 0.4,

1) Take a square and divide it into 10 equal parts.

2) 0.2 or $\frac{2}{10}$ represents 2 parts out of 10 equal parts

3) Similarly, 0.4 or $\frac{4}{10}$ represents 4 parts out of 10 equal parts.



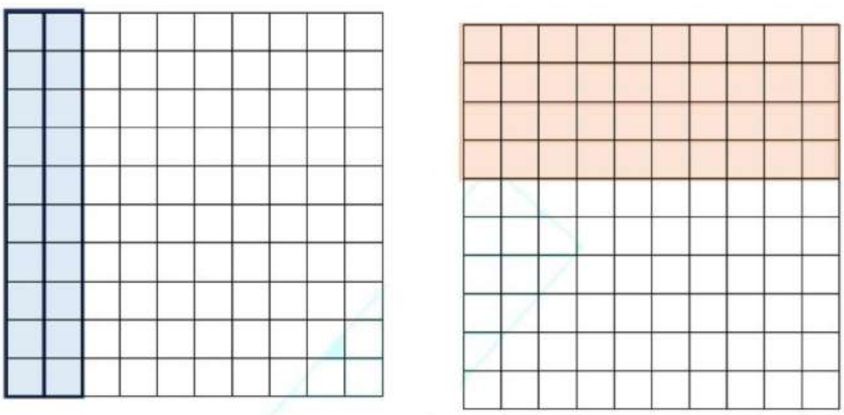
4) If we divide each small rectangle into 10 equal parts, we get 100 small squares.

We know,

0.2 is same as 0.20

2 tenths = 2 hundredths (20 small squares out of hundred)

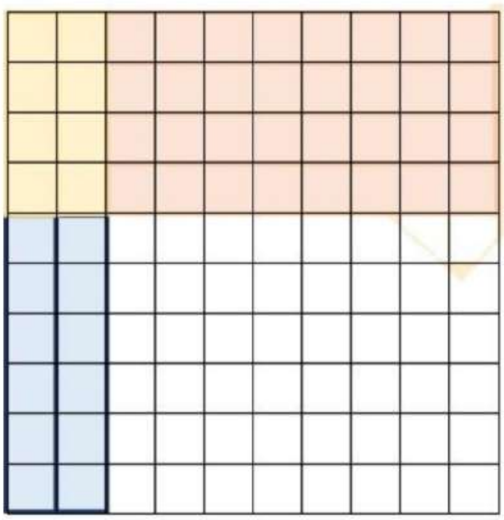
Similarly, 4 tenths = 4 hundredths (40 small squares out of a hundred)



If we overlap the two grids, we see that 8 small squares out of 100 are common to both. ($\frac{8}{100}$ or 0.08)

The yellow region represents 0.2×0.4

Thus, $0.2 \times 0.4 = 0.08$



Multiplication of Decimal Numbers

1) Multiply the given decimal numbers without a decimal point just like whole numbers.

2)Put the decimal point in the product by counting as many places from right to left as the sum of the decimal places of the decimals being multiplied.

Find 2.7×1.3

First, multiply the given decimals as whole numbers.

On multiplying 27 and 13 we get,

$$\begin{array}{r} 27 \\ \times\ 13 \\ \hline 351 \end{array}$$

We see that in 2.7 and 1.3, there is 1 digit to the right of the decimal point.

Now, $1 + 1 = 2$. So, we count 2 digits from the rightmost digit (i.e., 1) in 351 and move towards left and put the decimal point there.

$$\begin{array}{r} 2.7 \\ \times\ 1.3 \\ \hline 3.51 \end{array}$$

Example: Find 10.05×1.05

10.05×1.05

We first multiply the given decimals as whole numbers.

Number of decimal places in $10.05 = 2$
Number of decimal places in $1.05 = 2$

$$\begin{array}{r} 1005 \\ \times\ 105 \\ \hline 5025 \\ 00000 \\ 100500 \\ \hline 105525 \end{array}$$

Now, $2 + 2 = 4$
So, we count 4 digits from the rightmost digit in 105525 and put the decimal point there.

$$\begin{array}{r} 10.05 \\ \times\ 1.05 \\ \hline 10.5525 \end{array}$$

Multiplication of Decimal Numbers by 10, 100, 1000

When a decimal number is multiplied by 10, 100 or 1000, the digits in the product are the same as in the decimal number but the decimal point in the product is shifted to the right by as many places as there are zeros over one.

i) $1.3 \times 10 = 1.30 = 13.0$

On multiplying a decimal by 10, we shift the decimal point to the right by one place.

ii) $3.62 \times 100 = 3.6200 = 362$

On multiplying a decimal by 100, we shift the decimal point to the right by two places.

iii) $56.3 \times 1000 = 56.3000 = 56300$

On multiplying a decimal by 1000, we shift the decimal point to the right by three places.

Example: Find the product:

i) 36.75×10 ii) 166.07×10 iii) 3.62×100 iv) 154.1×100

v) 0.03×1000

i) $36.75 \times 10 = 367.5$

When we multiply a decimal number by 10, we shift the decimal point to the right by 1 place

ii) $166.07 \times 10 = 1660.7$ (Shifting decimal point to the right by 1 place)

iii) $3.62 \times 100 = 362.0$
On multiplying a decimal number by 100, we shift the decimal point to the right by 2 places.

iv) $154.1 \times 100 = 15410.0$(Shifting decimal point to the right by 2 places)

v) $0.03 \times 1000 = 030.0 = 30$

When we multiply a decimal number by 1000, we shift the decimal point to the right by 3 places.

Division of Decimal Numbers by 10, 100, 1000

While dividing a number by 10, 100 or 1000, the digits of the number and the quotient are the same but the decimal point in the quotient shifts to the left by as many places as there are zeros over one.

i) 4.8 ÷ 10 = 04.8 = 0.48

On dividing a decimal by 10,we shift the decimal point to the left by one place.

ii) 2.7÷ 100 = 002.7 = 0.027

On dividing a decimal by 100,we shift the decimal point to the left by two places.

iii) 128.0÷1000 = 0128.0 = 0.128

On dividing a decimal by 1000,we shift the decimal point to the left by three places.

Example: Find:

i) 33.2 ÷ 10 ii) 2.8 ÷ 100 iii) 127.9 ÷ 1000 iv) 0.6 ÷ 1000

i) 33.2 ÷ 10 = 3.32 (Shifting decimal point to the left by 1 place)

ii) 2.8 ÷ 100 = 0.028 (Shifting decimal point to the left by 2 places)

iii) 127.9 ÷ 1000 = 0.1279(Shifting decimal point to the left by 3 places)

iv) 0.6 ÷ 1000 = 0.0006 (Shifting decimal point to the left by 3 places)

Division of a Decimal Number by Another Whole Number

1) Divide the decimal number, treating it as a whole number by the given whole number.

2) Put the decimal point at the same number of decimal places as in the given decimal.

Divide: $65.4 \div 6$

Dividing the decimal number as the whole number by the given whole number we get,

$654 \div 6 = 109$

In the decimal number 65.4, the number of decimal places is 1. So, we put the decimal point at the same place.

$65.4 \div 6 = 10.9$

Example: Find

i) $651.2 \div 4$ ii) $14.49 \div 7$

i) $651.2 \div 4$

We divide the decimal number as the whole number by the given whole number,

$6512 \div 4 = 1628$

We put the decimal point at the same decimal place as in 651.2.

$651.2 \div 4 = 162.8$

ii) $14.49 \div 7$

We divide the decimal number as the whole number by the given whole number,

$1449 \div 7 = 207$

Putting the decimal point at the same decimal place as in 14.49 we get,

$14.49 \div 7 = 2.07$

Division of a Decimal Number by Another Decimal Number

1) We multiply the dividend and divisor by 10, 100 or 1000 etc. to convert the divisor into a whole number.

2) Now, divide the new dividend by the whole number.

Divide: 3.25÷ 0.5

As the number of decimal places in the divisor, 0.5 is 1 we multiply the numerator and denominator by 10.

$$3.25 \div 0.5 = \frac{3.25}{0.5} = \frac{3.25x10}{0.5x10} = \frac{32.5}{5} = 6.5$$

Example:

i) 0.5÷ 0.25 ii) 37.8 ÷ 1.4

i) 0.5 ÷ 0.25

The number of decimal places in the divisor, 0.25 is 2 so we multiply the numerator and denominator by 100.

$$0.5 \div 0.25 = \frac{0.5}{0.25} = \frac{0.5x100}{0.25x100} = \frac{50}{25} = 2$$

ii) 37.8 ÷ 1.4

Here, the number of decimal places in the divisor, 1.4 is 1 so we multiply the numerator and denominator by 10.

$$37.8 \div 1.4 = \frac{37.8}{1.4} = \frac{37.8x10}{1.4x10} = \frac{3780}{14} = 27$$