Probability

- **Probability:**
  - The empirical (or experimental) probability of an event A is given by

$$P(A) = \frac{\text{Number of trials in which event A has occured}}{\text{Total number of trials}}$$

Example:
When a coin is tossed 500 times and on the upper face of the coin tail comes up 280 times. What is the probability of getting head on the upper face of the coin?
Solution:
Let A be the event of getting head on the upper face of the coin.
Total number of trials = 500
Number of trials in which tail comes up = 280
Number of trials in which head comes up = 500 – 280 = 220

$$\therefore P(A) = \frac{220}{500} = \frac{11}{25}$$

  - The probability of an event always lies between 0 and 1 (0 and 1 inclusive).

- **Experimental Probability:** The probability obtained from the result of an experiment when we actually perform the experiment is called experimental (or empirical) probability.
- **Theoretical Probability:** The probability we find through the theoretical approach without actually performing the experiment is called theoretical probability.
- The theoretical probability (or classical probability) of an event E, is denoted by P(E) and is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- Experimental probability may or may not be equal to the theoretical probability.

- **Complementary events**

For an event E such that $0 \le P(E) \le 1$ of an experiment, the event $\bar{E}$ represents 'not E' , which is called the complement of the event E. We say, E and $\bar{E}$ are **complementary** events.

$P(E) + P(\bar{E}) = 1$

$\Rightarrow P(\bar{E}) = 1 - P(E)$

Example:
A pair of dice is thrown once. Find the probability of getting a different number on each die.
Solution:
When a pair of dice is thrown, the possible outcomes of the experiment can be listed as:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The number of all possible outcomes = 6 × 6 = 36
Let E be the event of getting the same number on each die.
Then, $\bar{E}$ is the event of getting different numbers on each die.

Now, the number of outcomes favourable to E is 6.

$\therefore P(\bar{E}) = 1 - P(E) = 1 - \frac{6}{36} = \frac{5}{6}$

Thus, the required probability is $\frac{5}{6}$.