CUET (UG)

Mathematics Sample Paper - 09

Solved

Time Allowed: 50 minutes **Maximum Marks: 200**

General Instructions:

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

Section A

1. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$, $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then A - B is equal to **[5]**

a) 2I
b) I
c) O
d) $\frac{1}{2}I$

2. $A = \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}$ then **[5]**

a) $O(A^3) = 9 \times 9$
b) $O(A^2) = 4 \times 4$
c) $O(A^2) = 2 \times 2$
d) $O(A^3) = 2 \times 2$

3. If A and B are square matrices of the same order, then $(A + B)^2 = A^2 + 2AB + B^2$ implies **[5]**

a) none of these
b) AB = BA
c) AB + BA = O
d) AB = O

4. The function f(x) = cos x -2 λx is monotonic decreasing when **[5]**

a) $\lambda > 2$
b) $\lambda < 1/2$
c) $\lambda > 1/2$
d) $\lambda < 2$

5. The function/(x) = 3x + cos 3x is **[5]**

a) Strictly increasing on R
b) Strictly decreasing on R
c) Increasing on R
d) Decreasing on R

6. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when **[5]**

a) x > 2
b) 1 < x < 2
c) x < 2
d) x > 3

7. $\int_{-8}^{8} (\sin^{93}x + x^{295})\, dx$ is equal to **[5]**

a) $2(8^{295} + 1)$
b) 1
c) $2 + 8^{295}$
d) 0

8. $\int \sec^4 x \tan x dx = ?$ **[5]**

a) None of these
b) $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

c) $\frac{1}{2}\sec^2 x + \frac{1}{4}\sec^4 x + C$

d) $\frac{1}{2}\sec x + \log|\sec x + \tan x| + C$

9. Integration of $\dfrac{1}{1+\left(\log_e x\right)^2}$ with respect to $\log_e x$ is **[5]**

a) $\dfrac{\tan^{-1} x}{x} + C$

b) $\dfrac{\tan^{-1}\left(\log_e x\right)}{x} + c$

c) none of these

d) $\tan^{-1}\left(\log_e x\right) + C$

10. The area bounded by the curve $y^2 = 8x$ and $x^2 = 8y$ is **[5]**

a) $\frac{3}{16}$ sq. units

b) $\frac{3}{14}$ sq. units

c) $\frac{64}{3}$ sq. units

d) $\frac{14}{3}$ sq. units

11. For the differential equation $\left(\dfrac{dy}{dx}\right)^2 - x\left(\dfrac{dy}{dx}\right) + y = 0$, which one of the following is not its solution? **[5]**

a) $4y = x^2$

b) $y = -x - 1$

c) $y = x - 1$

d) $y = x$

12. The differential equation satisfied by $ax^2 + by^2 = 1$ is **[5]**

a) $xyy_2 + xy_1^2 - yy_1 = 0$

b) None of these

c) $xyy_2 + y_1^2 + yy_1 = 0$

d) $xyy_2 - xy_1^2 + yy_1 = 0$

13. The maximum value of Z = 4x + 3y subject to constraint $x + y \leq 10$, $xy \geq 0$ is **[5]**

a) 40

b) 36

c) 20

d) 10

14. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that all the five cards are spades? **[5]**

a) $\frac{5}{1024}$

b) $\frac{3}{1024}$

c) $\frac{7}{1024}$

d) $\frac{1}{1024}$

15. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is: **[5]**

a) 0

b) $\frac{1}{36}$

c) $\frac{1}{3}$

d) $\frac{1}{12}$

Section B

Attempt any 25 questions

16. If $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$ then (f o f)(x) =? **[5]**

a) x

b) 2x - 3

c) 2x + 3

d) $\frac{4x-6}{3x+4}$

17. The value of $\sec^{-1}(\sec \frac{4\pi}{3})$ is **[5]**

a) $\frac{4\pi}{3}$

b) $\frac{-\pi}{3}$

c) $\frac{\pi}{3}$

d) $\frac{2\pi}{3}$

18. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is **[5]**

a) a symmetric matrix

b) a unit matrix

c) a diagonal matrix

d) a skew- symmetric matrix

19. If ω is a complex cube root of unity then the value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$. **[5]**

a) 2

b) 0

c) 4

d) -3

20. A(adj A) is equal to **[5]**

a) None of these

b) I

c) |A|I

d) O

21. For what value of λ the following system of equations does not have a solution x + y + z = 6, 4x + λy - λz = 0, 3x + 2y – 4z = - 5 ? **[5]**

a) 1

b) -3

c) 0

d) 3

22. If $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ then $\frac{dy}{dx} = ?$ **[5]**

a) None of these

b) $\frac{-2}{\left(1+x^2\right)}$

c) $\frac{2}{\left(1+x^2\right)}$

d) $\frac{2x}{\left(1+x^2\right)}$

23. For a real number x, let [x] denote the greatest integer less than or equal to x and f (x) = $\frac{\tan\left(\pi[x-\pi]\right)}{1+[x]^2}$,then **[5]**

a) f '(x) exists for all x but f ''(x) does not exist

b) f '(x) exists for all x

c) continuous for some x

d) continuous at all x but f '(x) does not exist

24. If $y = \sqrt{\frac{1+x}{1-x}}$ then $\frac{dy}{dx} = ?$ **[5]**

a) $\frac{2}{(1-x)^2}$

b) $\frac{x}{(1-x)^{\frac{3}{2}}}$

c) None of these

d) $\frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$

25. If x = a sec θ, y = b tan θ then $\frac{dy}{dx}$ = ? **[5]**

a) $\frac{b}{a}\sec\theta$

b) None of these

c) $\frac{b}{a}cosec\theta$

d) $\frac{b}{a}\cot\theta$

26. The value of k for which f(x) $= \begin{cases} \frac{3x + 4\tan x}{2}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous at x = 0, is **[5]**

a) 3

b) 7

c) None of these

d) 4

27. Tangents to the curve y= x^3 + 3x at x = -1 and x = 1 are **[5]**

a) intersecting at right angles

b) intersecting at an angle of 45^o.

c) intersecting obliquely but not at an angle of 45^o

d) parallel

28. The curve $y = x^{1/5}$ has at (0, 0) **[5]**

a) a vertical tangent

b) oblique tangent

c) a horizontal tangent

d) no tangent

29. The equation of the tangent to the curve $y^2 = 4ax$ at the point $(at^2, 2at)$ is **[5]**

a) $ty = x + at^2$

b) none of these

c) $tx + y = a t^3$

d) $ty = x - at^2$

30. The point on the curve $y^2 = 4x$ which is nearest to the point (2,1) is **[5]**

a) $(1, 2\sqrt{2})$

b) (-2, 1)

c) (1, -2)

d) (1, 2)

31. $\int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1 + x^2}dx = ?$ **[5]**

a) $\frac{19}{6}$

b) $\frac{19}{3}$

c) $\frac{9}{4}$

d) $\frac{38}{3}$

32. $\int \frac{1}{x(\log x)} dx = ?$ **[5]**

a) $(\log x)^2 + c$

b) $\frac{-2}{x^2} + C$

c) log |log x| + C

d) log |x|+C

33. $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx = ?$ **[5]**

a) None of these

b) $\log|\sin x + \sqrt{\sin^2 x - 2\sin x - 3}| + C$

c) $\log|(\sin x - 1) - \sqrt{\sin^2 x - 2\sin x - 3}| + C$

d) $\log|(\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3}| + C$

34. $\int \frac{\cos x}{\left(1 + \sin^2 x\right)} dx = ?$ **[5]**

a) $-\tan^{-1}(\cos x) + C$

b) $\tan^{-1}(\cos x) + C$

c) $-\tan^{-1}(\sin x) + C$

d) $\tan^{-1}(\sin x) + C$

35. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to **[5]**

a) $\pi^2 ab$

b) πab

c) πab^2

d) $\pi a^2 b$

36. Find a particular solution of $\frac{dy}{dx} = y\tan x$; y =1 when x = 0 **[5]**

a) y = tan x

b) y = sec x

c) y = sin x

d) y = cos x

37. Integrating factor of the differential equation cos x $\frac{dy}{dx}$ + y sin x = 1 is **[5]**

a) tan x

b) sin x

c) sec x

d) cos x

38. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, respectively, are **[5]**

a) 2 and 4

b) 2 and 2

c) 2 and 3

d) 3 and 3

39. ABCD is a parallelogram with AC and BD as diagonals. Then, $\overrightarrow{AC} - \overrightarrow{BD} =$ **[5]**

a) $3\overrightarrow{AB}$

b) $\overrightarrow{AB}$

c) $4\overrightarrow{AB}$

d) $2\overrightarrow{AB}$

40. If $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$, then $\vec{a} \times \vec{b}$ is **[5]**

a) $10\hat{i} - 3\hat{j} + 11\hat{k}$ b) $10\hat{i} - 2\hat{j} - 10\hat{k}$

c) $10\hat{i} + 3\hat{j} + 11\hat{k}$ d) $10\hat{i} + 2\hat{j} + 11\hat{k}$

41. If the vectors $\alpha\hat{i} + \alpha\hat{j} + \gamma\hat{k}$, $\hat{i} + \hat{k}$ and $\gamma\hat{i} + \gamma\hat{j} + \beta\hat{k}$ lie on a plane, where α, β and γ are distinct non-negative numbers, then γ is **[5]**

a) arithmetic mean of α and β b) harmonic mean of α and β

c) mean of α and β d) geometric mean of α and β

42. The vector with initial point P (2, -3, 5) and terminal point Q(3, -4, 7) is **[5]**

a) $-\hat{i} + \hat{j} - 2\hat{k}$ b) $\hat{i}-\hat{j}+2\hat{k}$

c) $5\hat{i}- 7\hat{j}+12\hat{k}$ d) none of these

43. The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is defined as **[5]**

a) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ b) $\vec{a}.\vec{b} = 2|\vec{a}||\vec{b}|\cos\theta$

c) $\vec{a}.\vec{b} = 2|\vec{a}||\vec{b}|\sin\theta$ d) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\sin\theta$

44. Equation of a plane that cuts the coordinates axes at (a, 0, 0), (0, b, 0) and (0, 0, c) is **[5]**

a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{2c} = 1$ b) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

c) $\frac{x}{2a} + \frac{y}{b} + \frac{z}{c} = 1$

d) $\frac{x}{a} + \frac{y}{2b} + \frac{z}{c} = 1$

45. The foot of the perpendicular from the point A(7, 14, 5) to the plane 2x + 4y - z = 2 is **[5]**

a) (5, -3, -4)

b) (3, -3, 5)

c) (3, 1, 8)

d) (1, 2, 8)

46. A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are **[5]**

a) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$

b) $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$

c) $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-4}{6}$

d) none of these

47. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, the probability of getting two heads is **[5]**

a) None of these

b) $\frac{15}{2^8}$

c) $\frac{15}{2^{13}}$

d) $\frac{2}{15}$

48. One hundred identical coins, each with probability p of showing heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, the value of p is **[5]**

a) $\frac{49}{101}$

b) $\frac{51}{101}$

c) None of these

d) $\frac{1}{2}$

49. If E and F are independent, then _____ **[5]**

a) P (E ∩ F) = P (E) P (E|F)

b) P (E ∩ F) = P (E) P (F)

c) P (E ∩ F) = P (E) P (F|E)

d) P (E ∩ F) =P (E ∪ F)

50. If A and B are events such that P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6, then P(A/B) = ? **[5]**

a) 0.3

b) 0.5

c) 0.2

d) 0.4

Solutions

Section A

1.

(d) $\frac{1}{2}I$

Explanation: In the given question, B = $\begin{bmatrix} -\frac{1}{\pi}\cos^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & -\frac{1}{\pi}\tan^{-1}\pi x \end{bmatrix}$

and A = $\begin{bmatrix} \frac{1}{\pi}\sin^{-1}x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\cot^{-1}\pi x \end{bmatrix}$

$\therefore$ A - B = $\begin{bmatrix} \frac{1}{\pi}\left(\sin^{-1}x\pi + \cos^{-1}x\pi\right) & 0 \\ 0 & \frac{1}{\pi}\left(\cot^{-1}\pi x + \tan^{-1}\pi x\right) \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{\pi}\cdot\frac{\pi}{2} & 0 \\ 0 & \frac{1}{\pi}\cdot\frac{\pi}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}I$

2.

(d) $O(A^3) = 2 \times 2$

Explanation: $O(A) = 2 \times 2$

$A^2 = \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 12 \\ -4 & 1 \end{vmatrix}$

$\therefore$ $O(A^3) = 2 \times 2$

$A^3 = A \times A^2 = \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 12 \\ -4 & 1 \end{vmatrix} = \begin{vmatrix} -10 & 27 \\ -9 & -10 \end{vmatrix}$

$O(A^3) = 2 \times 2$

3.

(b) AB = BA

Explanation: If A and B are square matrices of same order, then, product of the matrices is not commutative.Therefore, the given result is true only when AB = BA.

4.

(c) $\lambda > 1/2$

Explanation: $\lambda > 1/2$

5.

(c) Increasing on R

Explanation: Given,f'(x) = 3x + cos 3x

f'(x) = 3 - 3sin 3x

f'(x) = 3(t - sin 3x)

Sin 3x varies from [-1, 1]

When sin 3x is 1 f'(x) = 0 and sin 3x is -1 f'(x) = 6

As the function is increasing in 0 to 6.

∴ The function is increasing of R.

6.

(b) 1 < x < 2

Explanation: 1 < x < 2

7.

(d) 0

Explanation: Given function is an odd function. Whenever f(x) is an odd function

$\int_{-a}^{a} f(x)dx = 0$

8.

(b) $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

$\Rightarrow \int \sec^2 x \sec^2 x \tan x dx = \int (1+\tan^2 x) \sec^2 x\, x dx$

$= \int \sec^2 x \tan x dx + \int \tan^3 x \sec^2 x dx$

Put tan x = t

$\Rightarrow \sec^2 x dx = dt$

$= \int t^1 dt + \int t^3 dt$

$$\frac{t^2}{2} + \frac{t^4}{4} + c\frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$$

9.

(d) $\tan^{-1}\left(\log_e x\right) + C$

Explanation: $\int \frac{1}{1+\left(\log_e x\right)^2} d\left(\log_e x\right)$

Put $\log_e x = t$

$$\int \frac{dt}{1+t^2} = \tan^{-1} t + c = \tan^{-1}\left(\log_e x\right) + c$$

10.

(c) $\frac{64}{3}$ sq. units

Explanation: The area bounded by the parabolic curve $y^2 = 8x$ and $x^2 = 8y$

$\Rightarrow$ $y^2 = 8x$ and $x^4 = 64y^2$

$\Rightarrow 8x = \frac{x^4}{64}$

$\Rightarrow$ x = 0 or x = 8 is given by

$$A = \int_0^8 \left(\sqrt{8x} - \frac{x^2}{8}\right) dx$$

$$A = 2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)^8 - \frac{1}{8}\left(\frac{x^3}{3}\right)^8$$

$$A = \frac{4\sqrt{2}}{3} \times 8\sqrt{8} - \frac{1}{24} \times 8^3$$

$$A = \frac{64}{3} \text{ sq units.}$$

Which is the required solution.

11.

(d) y = x

Explanation: The given differential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0 \text{ ...(i)}$$

$$y = x \Rightarrow \frac{dy}{dx} = -1$$

From Eq. (i), $(1)^2 + x(1) + x = 1 \neq 0$

So, y = x is not a solution of Eq. (i).

12. **(a)** $xyy_2 + xy_1^2 - yy_1 = 0$

Explanation: We have ,

$$ax^2 + by^2 = 1$$

$$\Rightarrow 2ax + 2by\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-ax}{by}$$

Consider,

$$ax + by\frac{dy}{dx} = 0$$

$$a + by\frac{d^2y}{dx^2} + b\left(\frac{dy}{dx}\right)^2 = 0$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{a}{b}$$

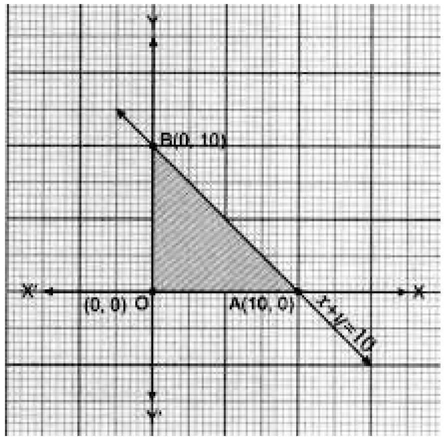
$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

$$\Rightarrow w_2 + \left(y_1\right)^2 = \frac{y}{x}y_1$$

$$\Rightarrow xyy_2 + x\left(y_1\right)^2 = yy_1$$

$$\Rightarrow xyy_2 + x\left(y_1\right)^2 - yy_1 = 0$$

13. **(a)** 40

Explanation: 

Feasible region is shaded region shown in figure with corner points 0(0, 0), A(10, 0), B(0, 10), Z(0, 0) = 0, Z (10, 0) = 40 $\longrightarrow$ maximum Z (0, 10) = 30

14.

(d) $\frac{1}{1024}$

Explanation: Here , probability of getting a spade from a deck of 52 cards = $\frac{13}{52} = \frac{1}{4}$. p = $\frac{1}{4}$, q = $\frac{3}{4}$. let , x is the number of spades, then x has the binomial distribution with n = 5 , p = $\frac{1}{4}$, q = $\frac{3}{4}$.

P(all 5 cards are spades)= P (x = 5) = $^5C_5\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$.

15.

(b) $\frac{1}{36}$

Explanation: Clearly, n(s) = 36. Favourable cases are {2, 2} Therefore required probability = $\frac{1}{36}$

Section B

16. **(a)** x

Explanation: x

17.

(d) $\frac{2\pi}{3}$

Explanation: $\sec^{-1}\left(\sec\frac{4\pi}{3}\right) = \sec^{-1}\left(\sec\left(\pi + \frac{\pi}{3}\right)\right)$

$= \sec^{-1}(-\sec\frac{\pi}{3}) = \sec^{-1}(-2) = \pi - \sec^{-1} 2$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

18. **(a)** a symmetric matrix

Explanation: Symmetric matrix. Since, A' = A, therefore, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

19.

(c) 4

Explanation: $1 + \omega + \omega^2 = 0 \Rightarrow (1 + \omega) = -\omega^2$. Put$(1 + \omega) = -\omega^2$ and expand.

20.

(c)

|A|I

Explanation: Since, we know that

$A^{-1} = \frac{adjA}{|A|}$

pre multiply by A,

$AA^{-1} = \frac{AadjA}{|A|}$

$I = \frac{AadjA}{|A|} \Rightarrow AadjA = |A|I$ (since $AA^{-1}=I$)

21.

(d) 3

Explanation: The given system of equations does not have solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4+\lambda & 2\lambda & -\lambda \\ 7 & 6 & 4 \end{vmatrix} = 0 \Rightarrow (24 + 6\lambda - 14\lambda) = 0 \Rightarrow \lambda = 3$$

22.

(b) $\frac{-2}{\left(1+x^2\right)}$

Explanation: Given that $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y = \frac{x^2-1}{x^2+1}$ or $\sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1$$

$$= \frac{4x^2}{\left(x^2-1\right)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

Using $-\tan x = \tan(-x)$, we obtain

$= -2\theta$

$= -2\tan^{-1} x$

Differentiating with respect to x, we obtain

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

23.

(b) f '(x) exists for all x

Explanation: Since $[x-\pi]$ is an integer for all $x \in R$ & $\tan n\pi = 0\ \forall\ n \in I$. Therefore, f(x)= 0 for all x in R. So, f(x) is a constant and hence derivatives of f(x) of all order exist.

24.

(d) $\frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$

Explanation: Given that $y = \sqrt{\frac{1+x}{1-x}}$

Let $x = -\cos\theta \Rightarrow \theta = \cos^{-1}(-x)$

Using $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$ and $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$, we obtain

$$y = \sqrt{\frac{2\sin^2 \frac{\theta}{2}}{2\cos^2 \frac{\theta}{2}}} = \tan\left(\frac{\theta}{2}\right)$$

Differentiating with respect to x, we obtain

$$\frac{dy}{dx} = \sec^2\left(\frac{\theta}{2}\right) \times \frac{1}{2}\frac{d\theta}{dx} - (1)$$

Since $x = -\cos\theta \Rightarrow 2\cos^2\frac{\theta}{2} = 1+\cos\theta = 1 - x$ or $\sec^2\left(\frac{\theta}{2}\right) = \frac{2}{1-x} - (2)$

Also, since $\theta = \cos^{-1}(-x)$, therefore $\frac{d\theta}{dx} = \frac{1}{\sqrt{1-x^2}} - (3)e$

Substituting (ii) and (iii) in (i), we obtain

$$\frac{dy}{dx} = \frac{2}{1-x} \times \frac{1}{2} \times \frac{1}{\sqrt{1-x^2}} = \frac{1}{(1-x)\sqrt{1-x^2}} = \frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

25.

(c) $\frac{b}{a} cosec\theta$

Explanation: x = a secθ ,we get

$$\therefore \frac{dx}{d\theta} = a\sec\theta \cdot \tan\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{a\sec\,\theta \cdot \tan\,\theta}$$

y = b tan θ ,we get

$$\therefore \frac{dy}{d\theta} = b \cdot \sec^2\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = b \cdot \sec^2\theta \times \frac{1}{a\sec\,\theta \cdot \tan\,\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec\,\theta}{a\tan\,\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b \cdot \frac{1}{\cos\theta}}{a \cdot \frac{\sin\theta}{\cos\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b}{a} cosec\theta$$

26.

(b) 7

Explanation: $\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{3x+4\tan x}{x}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{3x}{x} + \frac{4\tan x}{x}$$

$$\Rightarrow f(x) = 3 + 4 \lim_{x \to 0} \frac{\tan x}{x}$$

$\Rightarrow$ f(x) = 3 + 4

$\therefore$ k = 7

27.

(d) parallel

Explanation: Given y =x^3 + 3x

$$\Rightarrow \frac{dy}{dx} = 3x^2 + 3$$

Slope of tangent at x = 1 = 6 and

Slope of tangent at x = -1 = 6

Hence, the two tangents are parallel.

28. **(a)** a vertical tangent

Explanation: $y = x^{\frac{1}{5}}$

$$\frac{dy}{dx} = \frac{1}{5} x^{\frac{-4}{5}}$$

when x= 0, Slope of the tangent $\frac{dy}{dx} = \infty$

Which means the tangent is parallel to Y - axis implies the tangent is vertical.

29. **(a)** ty = x +at^2

Explanation: $y^2 = 4ax$

$\Rightarrow 2y\frac{dy}{dx} = 4a$

$\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

$\Rightarrow \frac{dy}{dx}$ at $(at^2, 2at)$ is $\frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow$ Slope of tangent $= m = \frac{1}{t}$

Hence, equation of tangent is $y - y_1 = m\left(x - x_1\right)$

$\Rightarrow y - 2at = \frac{1}{t}(x - at^2)$

$\Rightarrow yt - 2at^2 = x - at^2$

$\Rightarrow yt = x + at^2$

30.

(d) (1, 2)

Explanation: $y^2 = 4x \Rightarrow x = \frac{y^2}{4}$

$\Rightarrow d = \sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow d^2 = (x - 2)^2 + (y - 1)^2$

$\Rightarrow d^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$

Let $u = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$

$\Rightarrow \frac{du}{dy} = 2\left(\frac{y^2}{4} - 2\right)\frac{y}{2} + 2(y - 1)$

To find minima

$\frac{du}{dy} = 0$

$2\left(\frac{y^2}{4} - 2\right)\frac{y}{2} + 2(y - 1) = 0$

$\Rightarrow$ y = 2 $\Rightarrow$ x = 1 $\left(x = \frac{y^2}{4}\right)$

$$\frac{d^2u}{dy^2} = \frac{3y^2}{4}$$

$$\Rightarrow \left(\frac{d^2u}{dy^2}\right)_{(1,2)} = 3 > 0$$

Hence, nearest point is (1, 2).

31.

(b) $\frac{19}{3}$

Explanation: $y = \int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1 + x^2}dx$

Let, $x^2 = t$

Differentiating both sides with respect to t

$$2x\frac{dx}{dt} = 1$$

$$\Rightarrow xdx = \frac{1}{2}dt$$

At $x = \sqrt{3}$, t = 3

At $x = \sqrt{8}$, t = 8

$$y = \frac{1}{2}\int_3^8 \sqrt{1 + t}dt$$

$$= \frac{1}{2}\left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right)_3^8$$

$$= \frac{1}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$

$$= \frac{1}{3}(27 - 8)$$

$$= \frac{19}{3}$$

32.

(c) log |log x| + C

Explanation: Given integral is $\int \frac{1}{x(\log x)}$

Let, logx = z

$\Rightarrow \frac{dx}{x} = dz$

So,$\int \frac{1}{x(\log x)} dx$

$= \int \frac{1}{z} dz$

=log z+c

=log (log x)+c

where c is the integrating constant.

33.

(d) $\log|(\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3}| + C$

Explanation: The given integral is $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$

Putting sin x = t and cos x dx = dt, we get

$$l = \int \frac{dt}{\sqrt{t^2 - 2t - 3}} = \int \frac{dt}{\sqrt{\left(t^2 - 2t + 1\right) - 4}} = \int \frac{dt}{\sqrt{(t-1)^2 - 2^2}}$$

$$= \log|(t - 1) + \sqrt{(t - 1)^2 - 2^2}| + C = \log|(t - 1) + \sqrt{t^2 - 2t - 3}| + C$$

$$= \log|(\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3}| + C$$

34.

(d) $\tan^{-1}(\sin x) + C$

Explanation: $\int \frac{\cos x}{(\sin x)^2 + 1^2} dx$

sin x = t

cos x dt = dt

$= \int \frac{dt}{t^2 + 1^2}$

We know, $\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$= \tan^{-1} \frac{t}{1} + c$

put t = sin x

= $\tan^{-1}$ (sin x) + c

35.

(b) πab

Explanation: Area of standard ellipse is given by :πab.

36.

(b) y = sec x

Explanation: $\frac{dy}{y} = tanxdx$

$\int \frac{dy}{y} = \int tanxdx$

$log|y| = log|secx| + logc$

$log|y| = log|csecx|$

$y = csecx$

here y=1 and x=0 gives $1 = csec0$

hence c = 1

$\therefore y = secx$

37.

(c) sec x

Explanation: We have, cos x $\frac{dy}{dx}$ + y sin x = 1

$\Rightarrow \frac{dy}{dx}$ + y tan x = sec x

This is a linear differential equation.

On comparing it with $\frac{dy}{dx}$ + Py = Q, we get

P = tan x and Q = sec x

I.F = $e^{\int Pdx} = e^{\int tanxdx} = e^{-\log\cos x} = e^{log\,(cosx)^{-1}} = (cosx)^{-1} = \frac{1}{cosx}$ = sec x

38. **(a)** 2 and 4

Explanation: We have

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} = -x^{1/5}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$$

$\Rightarrow \frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$

$\therefore$ Order = 2, Degree = 4

39.

(d) $2\overrightarrow{AB}$

Explanation: Given: ABCD, a parallelogram with diagonals AC and BD.

Then,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$\Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

$\therefore \overrightarrow{AC} - \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} - (\overrightarrow{AD} - \overrightarrow{AB}) = \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AD} + \overrightarrow{AB} = 2\overrightarrow{AB}$ [$\because \overrightarrow{AD} = \overrightarrow{BC}$]

40.

(c) $10\hat{i} + 3\hat{j} + 11\hat{k}$

Explanation: Given $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$\Longrightarrow \hat{i}(6+4) - \hat{j}(-4+1) + \hat{k}(8+3)$

$= 10\hat{i} + 3\hat{j} + 11\hat{k}$

41.

(d) geometric mean of α and β

Explanation: Since, the vectors are coplanar.

$$\therefore \begin{vmatrix} \alpha & \alpha & \gamma \\ 1 & 0 & 1 \\ \gamma & \gamma & \beta \end{vmatrix} = 0$$

$\Rightarrow \alpha(0 - \gamma) - \alpha(\beta - \gamma) + \gamma(\gamma) = 0$

$\Rightarrow \gamma^2 = \alpha\beta \Rightarrow \gamma = \sqrt{\alpha\beta}$

Hence, γ is GM of α and β.

42.

(b) $\hat{i} - \hat{j} + 2\hat{k}$

Explanation: To find the vector we need to find the $\overrightarrow{PQ}$

$=3\hat{i} - 4\hat{j} + 7\hat{k} - (2\hat{i} + 3\hat{j} - 5\hat{k})$.

Hence, the vector formed by above points is with the following (1,-1,2).

43. **(a)** $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Explanation: The scalar product of two nonzero vectors $\vec{a}$ and $\vec{b}$ is defined as:

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

44.

(b) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Explanation: Equation of a plane that cuts the coordinates axes at (a, 0, 0), (0, b, 0) and (0, 0, c) is called the equation of plane in intercept form having intercepts a , b , and c on coordinate axis i.e. at x- axis , y – axis and z – axis respectively is given by :

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

45.

(d) (1, 2, 8)

Explanation: Let N be the foot of the perpendicular drawn from the point A(7, 14, 5) and perpendicular to the plane 2x + 4y - z = 2.

Then, the equation of the line PN is $\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda(\text{ say })$

Let the coordinates of N be $N(2\lambda + 7, 4\lambda + 14, -\lambda + 5)$

Since N lies on the plane 2x+ 4y - z = 2, so

$2(2\lambda + 7) + 4(4\lambda + 14) - (-\lambda + 5) = 2$

$\Rightarrow 21\lambda = -63 \Rightarrow \lambda = -3$

$\therefore$ required foot of the perpendicular is

N (-6 + 7, - 12 + 14, 3 + 5),

i.e.,N (1, 2, 8)

46.

(d) none of these

Explanation: To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

Parallel vector $= ((2-1)\hat{\imath} + (-1-2)\hat{\jmath} + (4+2)\hat{\mathrm{k}})$

$= \hat{\imath} - 3\hat{\jmath} + 6\hat{\mathrm{k}}$

Or $= -(\hat{\imath} - 3\hat{\jmath} + 6\hat{\mathrm{k}})$

Fixed point is $2\hat{i} - \hat{j} + 4\hat{k}$

Equation of line :-

$\frac{x-2}{1} = \frac{y-(-1)}{-3} = \frac{z-4}{6}$

$$\frac{x-2}{1}=\frac{y+1}{-3}=\frac{z-4}{6}$$

Or

$$\frac{x-2}{-1}=\frac{y+1}{3}=\frac{z-4}{-6}$$

47.

(c) $\frac{15}{2^{13}}$

Explanation: Let X be the number of heads.

$p=\frac{1}{2}\Rightarrow q=\frac{1}{2}$ .... (i)

P(X = 7) = P(X = 9)

$^{n}C_{7}p^{7}q^{n-7}={}^{n}C_{9}^{9}{}_{g}q^{n-9}$

$$\frac{\frac{n!}{7!(n-7)!}}{\frac{n!}{9!(n-9)!}}=q^{-2}p^{2}$$

$$\frac{9!(n-9)!}{7!(n-7)!}=\frac{p^{2}}{q^{2}}$$

$$\frac{9\times8\times7|(n-9)!}{7!(n-7)(n-8)(n-9)!}=1 \text{ .... [} \because \text{ from (i)]}$$

9 × 8 = (n - 7)(n - 8)

Comparing both sides,

n - 7 = 9 ⇒ n = 16

$\Rightarrow P(X=2)=16C_{2}\times0.5^{2}\times0.5^{14}$

$\Rightarrow P(X=2)=\frac{15}{2^{13}}$

48.

(b) $\frac{51}{101}$

Explanation: Let X denote the number of coins showing head.

Therefore, X follows a binomial distribution with p and n as parameters.

Given that P(X = 50) = P(X = 51)

$\Rightarrow {}^{100}C_{50}p^{50}q^{50}={}^{100}C_{51}p^{51}q^{49}$

on simplifying we get,

$$\frac{51}{50}=\frac{p}{q}$$

$\Rightarrow \frac{51}{50} = \frac{p}{1-p}$ (Since p + q = 1)

$\Rightarrow p = \frac{51}{101}$

49.

(b) P (E ∩ F) = P (E) P (F)

Explanation: we know that $P(E/F) = \frac{P(E \cap F)}{P(F)}$

If E and F are independent, then P(E/F) = P(E) and P(F/E) = P(F)

P (E ∩ F) = P (E) P (F).

50. **(a)** 0.3

Explanation: P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6

$P(B/A) = \frac{P(A \cap B)}{P(A)} = 0.6$

$P(A \cap B) = 0.24$

$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.3$