1. Units and Measurements

1. Choose the correct option.

i) $[L^1M^1T^{-2}]$ is the dimensional formula for

(A) Velocity
(B) Acceleration
(C) Force
(D) Work

ii) The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is

(A) 1%
(B) 1/2%
(C) 2%
(D) None of the above.

iii) Light year is a unit of

(A) Time
(B) Mass
(C) Distance
(D) Luminosity

iv) Dimensions of kinetic energy are the same as that of

(A) Force
(B) Acceleration
(C) Work
(D) Pressure

v) Which of the following is not a fundamental unit?

(A) cm
(B) kg
(C) centigrade
(D) volt

2. Answer the following questions.

i) Star A is farther than star B. Which star will have a large parallax angle?

Ans. Star B, since parallax angle is inversely proportional to the distance of the star from the Earth.

ii) What are the dimensions of the quantity
$\boldsymbol{k\sqrt{l/g}}$
***l* being the length and g the acceleration due to gravity?**

Ans. Given, [k] = 1, [l] = [L], [g] = [LT $^{-2}$].
$\therefore \left[k\sqrt{l/g}\right] = [k][l]^{1/2}[g]^{-1/2}$
$= 1[L]^{1/2}[LT^{-2}]^{-1/2} = \left[L^{\frac{3}{2}}T^{1}\right]$
Therefore, the dimensions of
$k\sqrt{l/g}$ are $\frac{3}{2}$
in length and 1 in time.

iii) Define absolute error, mean absolute error, relative error and percentage error.

Ans.

(1) Absolute error: For a set of measurements of the same quantity, the positive difference between each individual value and the most probable value gives the absolute error in that value.

(2) Mean absolute error: For a set of measurements of the same quantity, the arithmetic mean of all the absolute errors is called the mean absolute error in the measurement of that quantity.

(3) Relative error: The ratio of the mean absolute error in the measurement of a physical quantity to its most probable value is called the relative error in the measurement of that quantity.

(4) Mean percentage error: The relative error in a measurement multiplied by 100, gives the mean percentage error in the measurement of a quantity.

iv) Describe what is meant by significant figures and order of magnitude.

Ans. The order of magnitude of a physical quantity is its magnitude expressed to the nearest integral power of ten.
To find the order of magnitude of a physical quantity, its magnitude is expressed as a number that lies between 0.5 and 5 multiplied by an appropriate integral power of 10. The power of 10 along with the unit then gives the order of magnitude of the quantity.

Examples:

(1) One astronomical unit (1 au, the average distance of the Earth from the Sun) =
1.496×10^{11}m
∴ Order of magnitude of 1 au = 10^{11}m.

(2) One light year (1 ly, the distance that light travels in vacuum in 1 year with a speed of 2.997 924 58 x 10^8 m/s)
= 9.46 x 10^{15} m=0.946 x 10^{16} m.
∴ Order of magnitude of 1 ly = 10^{16} m.

(3) Order-of-magnitude sizes of an atomic nucleus and a hydrogen atom are 10^{-14} m and 10^{-10} m, respectively. So that, we can say that a hydrogen atom is four orders of magnitude larger than an atomic nucleus.

v) The measured values of two quantities are a ± Δa and b ± Δb, Δa and Δb being the mean absolute errors. What is the maximum possible error in a ± b?

Ans. Consider two quantities whose measured values are a ± Δa and b ± Δb, where Δa and Δb are their respective mean absolute errors.
(i) If a quantity z is equal to the sum of a and b, and Δz is the maximum absolute error in z, then
z ± Δz = (a ± Δa) + (b ± Δb)
= (a + b) ± (Δa + Δb)
∴Δz = ± Δa ± Δb
The four possible values of Δz are (+ Δa + Δb), (+Δa - Δa), (- Δa + Δb) and (-Δa - Δb).
∴The maximum possible absolute error in z,
Δz = Δa + Δb
(ii) If a quantity z is the difference a - b, and Δz is the maximum absolute error in z, then
z ± Δz = (a ± Δa) - (b ± Δb)
= (a - b) ± Δa ∓ Δb
∴ Δz = ± Δa ∓ Δb
The four possible values of Δz are (+Δa - Δb), (+Δa + Δb), (-Δa -Δb) and (-Δa + Δb).
∴The maximum possible absolute error in z,
Δz = Δa + Δb
Thus, the general rule is: if z = a ± b, the maximum absolute error in z, Δz = Δa + Δb.

vi) Derive the formula for kinetic energy of a particle having mass m and velocity v using dimensional analysis

Ans. The kinetic energy of a body depends upon the mass m and its speed v. To

determine the exact dependence of kinetic energy on m and v, we write
$E = km^x v^y$
where k is dimensionless constant. The values of x and y can be determined using dimensional analysis as follows:
$[E] = [ML^2T^{-2}]$, $[m] = [M]$, $[v] = [LT^{-1}]$
$\therefore [E] = [M^x].[LT^{-1}]^y$
$\therefore [ML^2T^{-2}] = [M^xL^yT^{-y}$
]
Comparing the powers of respective quantities on both the sides,
x = 1 and y = 2
$\therefore E = km\, v^2$
Thus, $E \propto m$ and $E \propto v^2$

Notes: (1) Dimensional analysis can only suggest, and not derive, an expression.

(2) The value of the dimensionless constant k above can only be obtained using the work-energy theorem. It comes out to be
$\frac{1}{2}, so\ that\ E = \frac{1}{2}mv^2.$

3. Solve numarical examples.

i) The masses of two bodies are measured to be 15.7 ± 0.2 kg and 27.3 ± 0.3 kg. What is the total mass of the two and the error in it?

[Ans: 43 kg, ± 0.5 kg]

Solution:

Data: $m_1 = (15.7 \pm 0.2)$ kg, $m_2 = (27.3 \pm 0.3)$ kg
Total mass, m = 15.7kg + 27.3kg = **43.0kg**
Absolute error in m,
$\Delta m = \Delta m_1 + \Delta m_2$ = 0.2kg + 0.3kg = **0.5kg**
ii) The distance travelled by an object in time (100 ± 1) s is (5.2 ± 0.1) m. What is the speed and it's relative error?

[Ans: 0.052 ms^{-1}, ± 0.0292 ms^{-1}]

Solution:

Data: t = (100 ± 1) s, d = (5.2 ± 0.1)m
Average speed, $v \frac{d}{t} = \frac{5.2\ m}{100\ s} = 0.052\ m/s$
Relative error in v, $\frac{\Delta v}{v} = \frac{\Delta d}{d} + \frac{\Delta t}{t}$
$= \frac{0.1\ m}{5.20m} + \frac{1\ s}{100\ s}$

$= 0.01923 + 0.01 = 0.02923$

∴ The absolute error in v,

$\Delta v = (0.02923)\ (0.052\ m/s) = 1.520 \times 10^{-3}\ m/s$

$= 0.001520\ m/s$

= 0.002 m/s, rounded to the precision of v

∴ $v = (0.052 \pm 0.002)\ m/s$

[Note: According to the guidelines of the SI, the final error must be quoted as absolute error.**]**

iii) An electron with charge e enters a uniform. magnetic field
$\vec{B}$ with a velocity
$\vec{V}$ The velocity
is perpendicular to the magnetic field. The force on the charge e is given by
$|\vec{F}| =$ Bev Obtain the dimensions of
$\vec{B}$

[Ans: $[L^0M^1T^{-2}I^{-1}]$]

<u>Solution:</u>

$F = |\vec{F}| = Bev$

$\therefore B = \frac{F}{ev}$ *where* $[F] = [force] = [MLT^{-2}]$,

$[e] = [charge] = [TI]$, $[v] = [speed] = [LT^{-1}]$.

$\therefore [B] = \frac{[F]}{[e][v]} = \frac{[MLT^{-2}]}{[TI][LT^{-1}]}$

$= [MT^{-2}\ I^{-1}]$

iv) A large ball 2 m in radius is made up of a rope of square cross section with edge length 4 mm. Neglecting the air gaps in the ball, what is the total length of the rope to the nearest order of magnitude?

[Ans: $\approx 10^6 m = 10^3 km$]

<u>Solution:</u>

Data: $R = 2\ m,\ d = 4\ mm = 4 \times 10^{-3} m$

Let L be the length of the rope in the ball. The total volume of the rope is roughly the volume V of the ball.

Total volume of the string, V =
(cross-sectional area)(length)

$\therefore V = d^2L = \frac{4}{3}\pi R^3 \approx 4R^3$

$(\because \pi \approx 3)$

$\therefore L = \frac{4R^3}{d^2} = \frac{4(2m)^3}{(4 \times 10^{-3}m)^2} = 2 \times 10^{-6}m$

$\therefore O(L) = 10^6 m = 10^3 km$

v) Nuclear radius R has a dependence on the mass number (A) as R =1.3 × 10^{-15} m) A⅓ For a nucleus of mass number A = 125, obtain the order of magnitude of R expressed in metre.

Solution:

[Note: The formula for nuclear radius is $R \approx R_0 A^{1/3}$, whehe $R_0 \approx 1.3 \times 10^{-15}$ m.]

Data: $A = 125 = 5^3$

$R = (1.3 \times 10^{-15} m)\,(5^3)^{1/3} = (1.3 \times 10^{-15} m)(5)$

$= 6.5 \times 10^{-15}\, m = 0.65 \times 10^{-14}\, m$

$\therefore O(R) = 10^{-14} m$

vi) In a workshop a worker measures the length of a steel plate with a Vernier callipers having a least count 0.01 cm. Four such measurements of the length yielded the following values: 3.11 cm, 3.13 cm, 3.14 cm, 3.14 cm. Find the mean length, the mean absolute error and the percentage error in the measured value of the length.

[Ans: 3.13 cm, 0.01 cm, 0.32%]

Solution:

Data: $n = 4$, $l_1 = 3.11$ cm, $l_2 = 3.13$ cm, $l_3 = 3.14$ cm, $l_4 = 3.14$ cm

(i) Mean (or most probable) length,

$\bar{l} \frac{1}{n}(l1 + l2 + l3 + l4)$

$= \frac{(3.11 + 3.13 + 3.14 + 3.14)cm}{4}$

$= \frac{12.52cm}{4}$

=3.13cm, to the precision of 0.01 cm

(ii)Mean absolute error,

$\Delta l = \frac{1}{n}\sum_i^4 1|l_i - \bar{l}|$

$= \frac{(0.02 + 0.00 + 0.01 + 0.01)\,cm}{4}$

$= \frac{0.04cm}{4} = \mathbf{0.01cm}$

(iii) Percentage error

$\frac{\Delta l}{l} \times 100\%$

$= \frac{0.01cm}{3.13cm} \times 100\%$

$= 0.003195 \times 100\% = 0.3195\%$
$\approx$ **0.32%**

[Notes: (1) Percentage error is expressed in one digit, or if small, in maximum two digits. (2) The answer to percentage error given in the textbook is wrong.**]**

vii) Find the percentage error in kinetic energy of a body having mass 60.0 ± 0.3 g moving with a velocity 25.0 ± 0.1 cm/s.

[Ans: 1.3%]

Solution:

Data: m = (60.0 ± 0.3) g, v = (25.0 ± 0.1) cm/s

Kinetic energy, $K = \frac{1}{2}mv^2$

$= \frac{1}{2}(60.0\ g)(25.0\ cm/s)^2$

$= (3.00 \times 10\ g)[6.25 \times 10^2\ (cm/s)^2]$

$= 1.8750 \times 10^4\ J \simeq 1.88 \times 10^4\ J,$

to 3 significant figures Relative error in K,

$= \frac{\Delta K}{K} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$

$= \frac{0.3g}{60.0g} + 2\left(\frac{0.1\ cm/s}{25.0 cm/s}\right)$

$= 0.005 + 2(0.004) = 0.005 + 0.008$

$= 0.013$

∴ Percentage error in K,

$\frac{\Delta K}{K} \times 100\%$

$= 0.013 \times 100\%$

= 1.3%

viii) In Ohm's experiments, the values of the unknown resistances were found to be 6.12 Ω, 6.09 Ω, 6.22 Ω, 6.15 Ω. Calculate the mean absolute error, relative error and percentage error in these measurements.

[Ans: 0.04 Ω, 0.0065 Ω, 0.65%]

Solution:

Data: $R_1 = 6.12\ \Omega$, $R_2 = 6.09\ \Omega$, $R_3 = 6.22\Omega$, $R_4 = 6.15\ \Omega$

Most probable value of resistance,

$\overline{R} = \frac{R_1 + R_2 + R_3 + R_4}{4}$

$= \frac{(6.12 + 6.09 + 6.22 + 6.15)\Omega}{4}$

$= 6.145\ \Omega \simeq 6.14\ \Omega$

(i) Mean absolute error,

$$\Delta R = \frac{1}{n}\sum_{i=1}^{4}|R_i - \bar{R}|$$

$$= \frac{(0.02\ +\ 0.05\ +\ 0.08\ +\ 0.01)\Omega}{4}$$

$$= \frac{0.16\Omega}{4}$$

= 0.04Ω

(ii) Relative error in R,

$$\frac{\Delta R}{R} = \frac{0.04\Omega}{6.14\Omega}$$

$= 0.006515\ \simeq\ \mathbf{0.0065}$

(iii) Percentage error in R

$$= \frac{\Delta R}{R} \times 100\%$$

$= 0.0065 \times 100\% =$ **0.65%**

ix) An object is falling freely under the gravitational force. Its velocity after travelling a distance h is v. If v depends upon gravitational acceleration g and distance, prove with dimensional analysis that

$\boldsymbol{v = k\sqrt{gh}},$

where k is a constant.

__Solution:__

We assume a power relation where v is the ath power of g and yth power of h.

Then, $v \propto g^x h^y = k g^x h^y$

where k is a dimensionless constant.

$[v] = [LT^{-1}]$, $[g] = [LT^{-2}]$, $[h] = [L]$, $[k] = 1$

$\therefore [v] = 1\,[LT^{-2}]^x[L]^y$

$\therefore [LT^{-1}] = [L^{x+y}T^{-2x}]$

Comparing the powers of the respective quantities on both the sides,

x + y = 1 and -2x= -1

$$\therefore x = \frac{1}{2} and\ y = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore v = kg^{\frac{1}{2}}h^{\frac{1}{2}} = k\sqrt{gh}, as\ required$$

x) $\boldsymbol{v\ =\ at\ + \frac{b}{t+c}v_0}$

is a dimensionally valid equation. Obtain the dimensional formula for a, b and c where v is velocity, t is time and v is initial velocity.

__Solution:__

$[v] = [v_0] = [\text{velocity}] = [LT^{-1}]$, $[t] = [T]$

Given that the equation,

$v = at + \frac{b}{t+c} v^0$,

is dimensionally valid,

$[at] = \left[\frac{b}{t+c}\right] = [v] = [LT^{-1}]$

$\therefore [a] = \frac{[at]}{[t]} = \frac{[LT^{-1}]}{[T]} = [\boldsymbol{LT^{-2}}]$

In an addition, the terms must have the same dimensions.

$\therefore [c] = [t] = \mathbf{[T]}$

$\left[\frac{b}{t+c}\right] = \frac{[b]}{[t+c]} = [LT^{-1}]$

$\therefore [b] = [t + c]\ [LT^{-1}] = [T]\ [LT^{-1}] = \mathbf{[L]}$

xi) The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

<u>Solution:</u>

Data: 1 = 4.234 m, b = 1.005 m,

d = 2.01 cm = 2.01×10^{-2} m

The area of the largest face of the sheet,

A = 1 x b = (4.234 m) (1.005 m) = 4.255 m^2,

correct to 4 significant figures.

The volume of the sheet,

V = lbd = (4.234 m) (1.005 m) (2.01×10^{-2} m)

= $8.553 \times 10^{-2}\ m^2 \simeq \mathbf{8.55 \times 10^{-2}\ m^2}$**,**

rounded to 3 significant figures.

xii) If the length of a cylinder is l = (4.00 ± 0.001) cm, radius r = (0.0250 ± 0.001) cm and mass m = (6.25 ± 0.01) gm. Calculate the percentage error in the determination of density.

[Ans: 8.185%]

<u>Solution:</u>

Data: l = (4.000 ± 0.001) cm,

r = (0.025 ± 0.001) cm,

m = (6.25 +0.01) g

$Density, \rho = \frac{mass}{volume} = \frac{m}{\pi r^2 l}$

$= \frac{6.25\text{g}}{(3.142)\ (2.5 \times 10^{-2}\text{cm})^2\ (4.000\ \text{cm})}$

$= \frac{6.25\ \text{g} \times 10^4 \text{g}}{3.142 (6.25\ \text{cm}^2)(4.000\ \text{cm})}$

$= \frac{2500}{3.124} g/cm^3$

$= 795.7 g/cm^3$

The relative error in ρ,

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta r}{r} + \frac{\Delta l}{l}$$

$$= \frac{0.01g}{6.25g} + 2\left(\frac{0.001\ cm}{0.025\ cm}\right) + \frac{0.001\ cm}{4.000\ cm}$$

$= 0.00016 + 0.08 + 0.0025 = 0.08185$

$\therefore$ *The percentage in* $\rho \frac{\Delta\rho}{\rho} \times 100\%$

$= 0.08185 \times 100\% = 8.185\% \approx$ **8%**

xiii) When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of the Jupiter.

[Ans: 1.428×10^5 km]

<u>Solution:</u>

Data: $D = 824.7 \times 10^6$ km $= 8.247 \times 10^8$ km,

$\alpha = 35.72$as, 1 as $= 4.847 \times 10^{-6}$ rad

$\therefore \alpha = 35.72 \times 4.847 \times 10^{-6}$ rad

Diameter of jupiter,

$d = \alpha D = (35.72 \times 4.847 \times 10^{-6}$ rad)

$(8.247 \times 10^8$ km)

$= 1428 \times 10^2$ km $=$ **1.428×10^5 km**

xiv) If the formula for a physical quantity is

$$\boldsymbol{X = \frac{a^4 b^3}{c^{\frac{1}{3}} d^{\frac{1}{2}}}}$$

and if the percentage error in the measurements of a, b, c and d are 2%, 3%, 3% and 4% respectively. Calculate percentage error in X.

[Ans: 20%]

<u>Solution:</u>

$$X = \frac{a^4 b^3}{c^{\frac{1}{3}} d^{\frac{1}{2}}}$$

$\therefore$ Relative error in X,

$$\frac{\Delta X}{X} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{1}{3}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$

$$\therefore \frac{\Delta X}{X} \times 100\%$$

$$= 4\left(\frac{\Delta a}{a} \times 100\%\right) + 3\left(\frac{\Delta b}{b} \times 100\%\right)$$

$$+ \frac{1}{3}\left(\frac{\Delta c}{c} \times 100\%\right) + \frac{1}{2}\left(\frac{\Delta d}{d} \times 100\%\right)$$

$= 4\,(2\%) + 3\,(3\%) + \frac{1}{3}(3\%)\frac{1}{2}(4\%)$

$= 8\% + 9\% + 1\% + 2\% =$ **20%**

xv) Write down the number of significant figures in the following: 0.003 m^2, 0.1250 gm cm^{-2}, 6.4 x 10^6 m, 1.6 x 10^{-19} C, 9.1 x 10^{-31} kg.

[Ans: 1, 4, 2, 2, 2]

Ans.

Number	Number of significant figures
0.003	1
0.1250	4
6.4×10^6	2
1.6×10^{-19}	2
9.1×10^{-31}	2

xvi) The diameter of a sphere is 2.14 cm. Calculate the volume of the sphere to the correct number of significant figures.

[Ans: 5.13 cm^3]

Solution:

Data: d=2.14 cm
Volume,

$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

$= \frac{1}{6}(3.142)(2.14cm)^3$

$= 5.132$ cm^3

$\simeq$**5.13 cm^3**, correct to 3 significant figures.