Chapter 23. Trigonometrical Ratios of Standard Angles [Including Evaluation of an Expression Involving Trigonometric Ratios]

Exercise 23(A)

Solution 1:

(i) $\sin 30^\circ \cos 30^\circ = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

(ii) $\tan 30^0 \tan 60^0 = \frac{1}{\sqrt{3}}\left(\sqrt{3}\right) = 1$

(iii) $\cos^2 60^0 + \sin^2 30^0 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(iv) $\text{cosec}^2 60^\circ - \tan^2 30^\circ = \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3} - \frac{1}{3} = 1$

(v) $\sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 = \frac{1}{4} + \frac{3}{4} + 1 = 2$

(vi)

$$\cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 1^2$$
$$= \frac{1}{4} + \frac{4}{3} + 1$$
$$= \frac{3+16+12}{12}$$
$$= \frac{31}{12}$$
$$= 2\frac{7}{12}$$

Solution 2:

(i) $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + 1^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3 = \frac{13}{3} = 4\frac{1}{3}$

(ii) $\frac{\tan 45^0}{\text{cos ec}30^0} + \frac{\sec 60^0}{\cot 45^0} - \frac{5 \sin 90^0}{2 \cos 0^0} = \frac{1}{2} + \frac{2}{1} - \frac{5}{2} = \frac{1+4-5}{2} = 0$

(iii) $3 \sin^2 30^0 + 2 \tan^2 60^0 - 5 \cos^2 45^0$

$$= 3\left(\frac{1}{2}\right)^2 + 2\left(\sqrt{3}\right)^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3}{4} + 6 - \frac{5}{2} = \frac{3+24-10}{4} = 4\frac{1}{4}$$

Solution 3:

(i) LHS=sin 60° cos 30° + cos 60°. sin 30°

$= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + \frac{1}{2}\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1 = \text{RHS}$

(ii) LHS=cos 30°. cos 60° - sin 30°. sin 60°

$= \frac{\sqrt{3}}{2}\frac{1}{2} - \frac{1}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$ =RHS

(iii) LHS= cosec^2 45° - cot^2 45°

$= \left(\sqrt{2}\right)^2 - 1^2 = 2 - 1 = 1$ =RHS

(iv) LHS= cos^2 30° - sin^2 30°

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \cos 60^0$ =RHS

(v) LHS= $\left(\frac{\tan 60^o + 1}{\tan 60^o - 1}\right)^2$

$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 = \frac{4+2\sqrt{3}}{4-2\sqrt{3}} = \frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{1+\cos 30^0}{1-\cos 30^0}$ =RHS

(vi) LHS= $3\ \text{cosec}^2 60^o - 2\cot^2 30^o + \sec^2 45^o$

$= 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 = 4 - 6 + 2 = 0$ =RHS

Solution 4:

(i)

$RHS =$

$$\frac{2\tan 30°}{1+\tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}$$

$LHS = \sin(2 \times 30°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\therefore LHS = RHS$

(ii)

RHS,

$$\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2}$$

LHS,

$$\cos(2\times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

LHS = *RHS*

(iii)

RHS,

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

LHS,

$$\tan(2\times 30^\circ) = \tan 60^\circ = \sqrt{3}$$

LHS = *RHS*

Solution 5:

Given that AB = BC = x

$$\therefore AC = \sqrt{AB^2+BC^2} = \sqrt{x^2+x^2} = x\sqrt{2}$$

(i) $\sin 45^\circ = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

(ii) $\cos 45^\circ = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$

(iii) $\tan 45^\circ = \frac{AB}{BC} = \frac{x}{x} = 1$

Solution 6:

(*i*) $LHS = \sin 60° = \frac{\sqrt{3}}{2}$

$RHS = 2\sin 60° \cos 60° = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$LHS = RHS$

(*ii*) $LHS = 4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 + (1)^4\right]$

$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] = \frac{4 \times 2}{16} + 3 \times \frac{1}{2} = 2$

$RHS = 2$

$LHS = RHS$

Solution 7:

(i)

The angle, x is acute and hence we have, 0<x<90 degrees

We know that

$$\cos^2 x + \sin^2 x = 1$$

$$\Rightarrow 2\sin^2 x = 1 \quad [\text{since } \cos x = \sin x]$$

$$\Rightarrow \sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = 45°$$

(ii)

$$\sec A = \operatorname{cosec} A$$

$$\cos A = \sin A$$

$$\cos^2 A = \sin^2 A$$

$$\cos^2 A = 1 - \cos^2 A$$

$$2\cos^2 A = 1$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$A = 45^0$$

(iii)

$$\tan\theta = \cot\theta$$

$$\tan\theta = \frac{1}{\tan\theta}$$

$$\tan^2\theta = 1$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45^0$$

$$\theta = 45^0$$

(iv)

$\sin x = \cos y = \sin(90° - y)$

If x and y are acute angles,

$x = 90° - y$

$\Rightarrow x + y = 90°$

Hence x and y are complementary angles

Solution 8:

(i)

$$\sin x = \cos y = \sin\left(\frac{\pi}{2} - y\right)$$

if x and y are acute angles,

$$x = \frac{\pi}{2} - y$$

$$x + y = \frac{\pi}{2}$$

$\therefore$ x + y = 45° is false.

(ii)

$$\sec\theta \cdot \cot\theta = \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

Secθ. Cot θ = cosec θ is true

(iii)

$$\sin^2\theta + \cos^2\theta = \sin^2\theta + 1 - \sin^2\theta = 1$$

Solution 9:

(i) For acute angles, remember what sine means: opposite over hypotenuse. If we increase the angle, then the opposite side gets larger. That means "opposite/hypotenuse" gets larger or increases.

(ii) For acute angles, remember what cosine means: base over hypotenuse. If we increase the angle, then the hypotenuse side gets larger. That means "base/hypotenuse" gets smaller or decreases.

(iii) For acute angles, remember what tangent means: opposite over base. If we decrease the angle, then the opposite side gets smaller. That means "opposite /base" gets decreases.

Solution 10:

(i) $\sin 60^0 = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.87$

(ii) $\frac{2}{\tan 30^0} = \frac{2}{\frac{1}{\sqrt{3}}} = 2\sqrt{3} = 2 \times 1.732 = 3.46$

Solution 11:

(i) Given that A= 15^0

$$\frac{\cos 3A - 2\cos 4A}{\sin 3A + 2\sin 4A} = \frac{\cos\left(3\times 15^0\right) - 2\cos\left(4\times 15^0\right)}{\sin\left(3\times 15^0\right) + 2\sin\left(4\times 15^0\right)}$$

$$= \frac{\cos 45^0 - 2\cos 60^0}{\sin 45^0 + 2\sin 60^0}$$

$$= \frac{\frac{1}{\sqrt{2}} - 2\left(\frac{1}{2}\right)}{\frac{1}{\sqrt{2}} + 2\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}} + \sqrt{3}}$$

$$= \frac{1-\sqrt{2}}{1+\sqrt{6}}$$

$$= \frac{1}{5}\left(\sqrt{6} - 1 - 2\sqrt{3} + \sqrt{2}\right)$$

(ii) Given that B= 20^0

$$\frac{3\sin 3B + 2\cos\left(2B + 5^0\right)}{2\cos 3B - \sin\left(2B - 10^0\right)} = \frac{3\sin 3\times 20^0 + 2\cos\left(2\times 20^0 + 5^0\right)}{2\cos 3\times 20^0 - \sin\left(2\times 20^0 - 10^0\right)}$$

$$= \frac{3\sin 60^0 + 2\cos 45^0}{2\cos 60^0 - \sin 30^0}$$

$$= \frac{3\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{\sqrt{2}}\right)}{2\left(\frac{1}{2}\right) - \frac{1}{2}}$$

$$= \frac{\frac{3\sqrt{3}}{2} + \sqrt{2}}{2}$$

$$= 3\sqrt{3} + 2\sqrt{2}$$

Exercise 23(B)

Solution 1:

Given A = 60^0 and B = 30^0

(i)

$$\begin{aligned} LHS &= \sin(A+B) \\ &= \sin(60^0+30^0) \\ &= \sin 90^0 \\ &= 1 \\ RHS &= \sin A \cos B + \cos A \sin B \\ &= \sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0 \\ &= \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{1}{2} \\ &= \frac{3}{4}+\frac{1}{4} \\ &= 1 \\ LHS &= RHS \end{aligned}$$

(ii)

$$\begin{aligned} LHS &= \cos(A+B) \\ &= \cos(60^0+30^0) \\ &= \cos 90^0 \\ &= 0 \\ RHS &= \cos A \cos B - \sin A \sin B \\ &= \cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0 \\ &= \frac{1}{2}\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\frac{1}{2} \\ &= \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} \\ &= 0 \\ LHS &= RHS \end{aligned}$$

(iii)

$$\begin{aligned} LHS &= \cos(A-B) \\ &= \cos(60^0-30^0) \end{aligned}$$

$$= \cos 30^0$$

$$= \frac{\sqrt{3}}{2}$$

$$RHS = \cos A \cos B + \sin A \sin B$$

$$= \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$$

$$= \frac{1}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$LHS = RHS$$

(iv)

$$\text{LHS} = \tan(A - B)$$

$$= \tan\left(60^0 - 30^0\right)$$

$$= \tan 30^0$$

$$= \frac{1}{\sqrt{3}}$$

$$RHS = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$$

$$= \frac{\tan 60^0 - \tan 30^0}{1 + \tan 60^0.\tan 30^0}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{2}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$LHS = RHS$$

Solution 2:

Given A= 30^0

(i)

$$\sin 2A = \sin 2\left(30^0\right) = \sin 60^0 = \frac{\sqrt{3}}{2}$$

$$2\sin A \cos A = 2\sin 30^0 \cos 30^0$$
$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

$$\frac{2\tan A}{1+\tan^2 A} = \frac{2\tan 30^0}{1+\tan^2 30^0}$$
$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$
$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$
$$= \frac{\sqrt{3}}{2}$$

$$\therefore \sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$$

(ii)

$$\cos 2A = \cos 2\left(30^0\right) = \cos 60^0 = \frac{1}{2}$$

$$\cos^2 A - \sin^2 A = \cos^2 30^0 - \sin^2 30^0$$
$$= \frac{3}{4} - \frac{1}{4}$$
$$= \frac{1}{2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \tan^2 30^0}{1 + \tan^2 30^0}$$
$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

$$\therefore \cos 2A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

(iii)

$$2\cos^2 A - 1 = 2\cos^2 30^0 - 1$$
$$= 2\left(\frac{3}{4}\right) - 1$$
$$= \frac{3}{2} - 1$$
$$= \frac{1}{2}$$

$$1 - 2\sin^2 A = 1 - 2\sin^2 30^0$$
$$= 1 - 2\left(\frac{1}{4}\right)$$
$$= \frac{1}{2}$$

$$1 - 2\sin^2 A = 1 - 2\sin^2 30^0$$
$$= 1 - 2\left(\frac{1}{4}\right)$$
$$= \frac{1}{2}$$

$\therefore 2\cos^2 A - 1 = 1 - 2\sin^2 A$

(iv)

$$\sin 3A = \sin 3\left(30^0\right)$$
$$= \sin 90^0$$
$$= 1$$

$$3\sin A - 4\sin^3 A = 3\sin 30^0 - 4\sin^3 30^0$$
$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$
$$= \frac{3}{2} - \frac{1}{2}$$
$$= 1$$

$\therefore \sin 3A = 3\sin A - 4\sin^3 A$

Solution 3:

Given that $A = B = 45^0$

(i)

$$LHS = \sin(A - B)$$
$$= \sin(45^0 - 45^0)$$
$$= \sin 0^0$$
$$= 0$$
$$RHS = \sin A \cos B - \cos A \sin B$$
$$= \sin 45^0 \cos 45^0 - \cos 45^0 \sin 45^0$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= 0$$
$$LHS = RHS$$

(ii)

$$LHS = \cos(A + B)$$
$$= \cos(45^0 + 45^0)$$
$$= \cos 90^0$$
$$= 0$$
$$RHS = \cos A \cos B - \sin A \sin B$$
$$= \cos 45^0 \cos 45^0 - \sin 45^0 \sin 45^0$$
$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= 0$$
$$LHS = RHS$$

Solution 4:

Given that A = 30°

(i)

$$\begin{aligned} LHS &= \sin 3A \\ &= \sin 3(30^0) \\ &= \sin 90^0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} RHS &= 4 \sin A \sin (60° - A) \sin (60° + A) \\ &= 4 \sin 30^0 \sin (60° - 30^0) \sin (60° + 30^0) \\ &= 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1) \\ &= 1 \end{aligned}$$

$$LHS = RHS$$

(ii)

$$\begin{aligned} LHS &= (\sin A - \cos A)^2 \\ &= (\sin 30^0 - \cos 30^0)^2 \\ &= \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} + \frac{3}{4} - \frac{\sqrt{3}}{2} \\ &= 1 - \frac{\sqrt{3}}{2} \\ &= \frac{2 - \sqrt{3}}{2} \end{aligned}$$

$$RHS = 1 - \sin 2A$$
$$= 1 - \sin 2(30^0)$$
$$= 1 - \sin 60^0$$
$$= 1 - \frac{\sqrt{3}}{2}$$
$$= \frac{2-\sqrt{3}}{2}$$
$$LHS = RHS$$

(iii)

$$LHS = \cos 2A$$
$$= \cos 2(30^0)$$
$$= \cos 60^0$$
$$= \frac{1}{2}$$
$$RHS = \cos^4 A - \sin^4 A$$
$$= \cos^4 30^0 - \sin^4 30^0$$
$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$$
$$= \frac{9}{16} - \frac{1}{16}$$
$$= \frac{1}{2}$$
$$LHS = RHS$$

(iv)

$$LHS = \frac{1-\cos 2A}{\sin 2A}$$
$$= \frac{1-\cos 2(30^0)}{\sin 2(30^0)}$$

$$= \frac{1 - \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}}$$

$$RHS = \tan A$$

$$= \tan 30^0$$

$$= \frac{1}{\sqrt{3}}$$

$$LHS = RHS$$

(v)

$$LHS = \frac{1 + \sin 2A + \cos 2A}{\sin A + \cos A}$$

$$= \frac{1 + \sin 2(30^0) + \cos 2(30^0)}{\sin 30^0 + \cos 30^0}$$

$$= \frac{1 + \frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}}$$

$$= \frac{3 + \sqrt{3}}{\sqrt{3} + 1}\left(\frac{\sqrt{3} - 1}{\sqrt{3} - 1}\right)$$

$$= \frac{3\sqrt{3} - 3 + 3 - \sqrt{3}}{2}$$

$$= \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

$$RHS = 2 \cos A$$

$$= 2 \cos (30^0)$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}$$

(vi)

$$LHS = 4\cos A \cos\left(60^\circ - A\right).\cos\left(60^\circ + A\right)$$
$$= 4\cos 30^0 \cos\left(60^\circ - 30^0\right).\cos\left(60^\circ + 30^0\right)$$
$$= 4\cos 30^0 \cos 30^0 \cos 90^0$$
$$= 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)(0)$$
$$= 0$$
$$RHS = \cos 3A$$
$$= \cos 3\left(30^0\right)$$
$$= \cos 90^0$$
$$= 0$$
$$LHS = RHS$$

(vii)

$$LHS = \frac{\cos^3 A - \cos 3A}{\cos A} + \frac{\sin^3 A + \sin 3A}{\sin A}$$
$$= \frac{\cos^3 30^0 - \cos 3\left(30^0\right)}{\cos 30^0} + \frac{\sin^3 30^0 + \sin 3\left(30^0\right)}{\sin 30^0}$$
$$= \frac{\left(\frac{\sqrt{3}}{2}\right)^3 - 0}{\frac{\sqrt{3}}{2}} + \frac{\left(\frac{1}{2}\right)^3 + 1}{\frac{1}{2}}$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{\frac{9}{8}}{\frac{1}{2}}$$
$$= \frac{3}{4} + \frac{9}{4}$$
$$= \frac{12}{4}$$
$$= 3$$
$$= RHS$$

Exercise 23(C)

Solution 1:

(i)

$2 \sin A = 1$

$\sin A = \frac{1}{2}$

$\sin A = \sin 30^0$

$A = 30^0$

(ii)

$2 \cos 2A = 1$

$\cos 2A = \frac{1}{2}$

$\cos 2A = \cos 60^0$

$2A = 60^0$

$A = 30^0$

(iii)

$\sin 3A = \frac{\sqrt{3}}{2}$

$\sin 3A = \sin 60^0$

$3A = 60^0$

$A = 20^0$

(iv)

$\sec 2A = 2$

$\sec 2A = \sec 60^0$

$2A = 60^0$

$A = 30^0$

(V)

$\sqrt{3} \tan A = 1$

$\tan A = \frac{1}{\sqrt{3}}$

$\tan A = \tan 30^0$

$A = 30^0$

(vi)

$$\tan 3A = 1$$
$$\tan 3A = \tan 45^0$$
$$3A = 45^0$$
$$A = 15^0$$

(vii)

$$2\sin 3A = 1$$
$$\sin 3A = \frac{1}{2}$$
$$\sin 3A = \sin 30^0$$
$$3A = 30^0$$
$$A = 10^0$$

(viii)

$$\sqrt{3}\cot 2A = 1$$
$$\cot 2A = \frac{1}{\sqrt{3}}$$
$$\cot 2A = \cot 60^0$$
$$2A = 60^0$$
$$A = 30^0$$

Solution 2:

(i)

$$(\sin A - 1)(2\cos A - 1) = 0$$

$$(\sin A - 1) = 0 \quad and \quad 2\cos A - 1 = 0$$

$$\sin A = 1 \quad and \quad \cos A = \frac{1}{2}$$

$$\sin A = \sin 90^0 \quad and \quad \cos A = \cos 60^0$$

$$A = 90^0 \quad and \quad A = 60^0$$

(ii)

$$(\tan A - 1)(\text{cosec } 3A - 1) = 0$$

$$\tan A - 1 = 0 \quad and \quad \text{cosec } 3A - 1 = 0$$

$$\tan A = 1 \quad and \quad \text{cosec} 3A = 1$$

$$\tan A = \tan 45^0 \quad and \quad \text{cosec} 3A = \text{cosec} 90^0$$

$$A = 45^0 \quad \quad A = 30^0$$

(iii)

$$(\sec 2A - 1)(\text{cosec } 3A - 1) = 0$$

$$\sec 2A - 1 = 0 \quad and \quad \text{cosec } 3A - 1 = 0$$

$$\sec 2A = 1 \quad and \quad \text{cosec} 3A = 1$$

$$\sec 2A = \sec 0^0 \quad and \quad \text{cosec} 3A = \text{cosec} 90^0$$

$$A = 0^0 \quad \quad A = 30^0$$

(iv)

$$\cos 3A.\ (2\sin 2A - 1) = 0$$

$$\cos 3A = 0 \quad and \quad 2\sin 2A - 1 = 0$$

$$\cos 3A = \cos 90^0 \quad and \quad \sin 2A = \frac{1}{2}$$

$$3A = 90^0 \quad and \quad \sin 2A = \sin 30^0$$

$$A = 30^0 \quad \quad 2A = 30^0 \Rightarrow A = 15^0$$

(v)

$$(\text{cosec } 2A - 2)(\cot 3A - 1) = 0$$

$$\text{cosec } 2A - 2 = 0 \quad and \quad \cot 3A - 1 = 0$$

$$\text{cosec } 2A = 2 \quad and \quad \cot 3A = 1$$

$$\text{cosec } 2A = \text{cosec} 30^0 \quad and \quad \cot 3A = \cot 45^0$$

$$2A = 30^0 \quad and \quad 3A = 45^0$$

$$A = 15^0 \quad and \quad A = 15^0$$

Solution 3:

(i)

$$2\sin x^\circ - 1 = 0$$
$$\sin x^\circ = \frac{1}{2}$$

(ii)

$$\sin x^\circ = \frac{1}{2}$$
$$\sin x^\circ = \sin 30^0$$
$$x^0 = 30^0$$

(iii)

$$\cos x^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$
$$\tan x^\circ = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Solution 4:

(i)

$$4\cos^2 x^\circ - 1 = 0$$
$$4\cos^2 x^\circ = 1$$
$$\cos^2 x^\circ = \left(\frac{1}{2}\right)^2$$
$$\cos x^0 = \frac{1}{2}$$
$$\cos x^0 = \cos 60^0$$
$$x^0 = 60^0$$

(ii)

$$\sin^2 x^\circ + \cos^2 x^\circ = \sin^2 60^\circ + \cos^2 60^\circ$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= 1$$

(iii)

$$\frac{1}{\cos^2 x^0} - \tan^2 x^0 = \frac{1}{\cos^2 60^0} - \tan^2 60^0$$
$$= \frac{1}{\left(\frac{1}{2}\right)^2} - \left(\sqrt{3}\right)^2$$
$$= 4 - 3$$
$$= 1$$

Solution 5:

$$4\sin^2\theta - 1 = 0$$
$$\sin^2\theta = \frac{1}{4}$$
$$\sin\theta = \frac{1}{2}$$
$$\sin\theta = \sin 30^0$$
$$\theta = 30^0$$

$$\cos^2\theta + \tan^2\theta = \cos^2 30^0 + \tan^2 30^0$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= \frac{3}{4} + \frac{1}{3}$$
$$= \frac{9+4}{12}$$
$$= \frac{13}{12}$$

Solution 6:

$$\sin 3A = 1$$
$$\sin 3A = \sin 90^0$$
$$3A = 90^0$$
$$A = 30^0$$

(i)

$$\sin A = \sin 30^0$$
$$\sin A = \frac{1}{2}$$

(ii)

$$\cos 2A = \cos 2\left(30^0\right)$$
$$= \cos 60^0$$
$$= \frac{1}{2}$$

(iii)

$$\tan^2 A - \frac{1}{\cos^2 A} = \tan^2 30^0 - \frac{1}{\cos^2 30^0}$$
$$= \left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$$
$$= \frac{1}{3} - \frac{4}{3}$$
$$= \frac{-3}{3}$$
$$= -1$$

Solution 7:

(i)

$$2\cos 2A = \sqrt{3}$$
$$\cos 2A = \frac{\sqrt{3}}{2}$$
$$\cos 2A = \cos 30^0$$
$$2A = 30^0$$
$$A = 15^0$$

(ii)

$$\sin 3A = \sin 3\left(15^0\right)$$
$$= \sin 45^0$$
$$= \frac{1}{\sqrt{2}}$$

(iii)

$$\sin^2\left(75^\circ - A\right) + \cos^2\left(45^\circ + A\right) = \sin^2\left(75^\circ - 15^0\right) + \cos^2\left(45^\circ + 15^0\right)$$
$$= \sin^2 60^0 + \cos^2 60^0$$
$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= 1$$

Solution 8:

(i)

Given that $x = 30^0$

$$\sin x + \cos y = 1$$
$$\sin 30^0 + \cos y = 1$$
$$\cos y = 1 - \sin 30^0$$
$$\cos y = 1 - \frac{1}{2}$$
$$\cos y = \frac{1}{2}$$
$$\cos y = \cos 60^0$$
$$y = 60^0$$

(ii)

Given that $B = 90^0$

$$3 \tan A - 5 \cos B = \sqrt{3}$$
$$3 \tan A - 5 \cos 90^0 = \sqrt{3}$$
$$3 \tan A - 0 = \sqrt{3}$$
$$\tan A = \frac{\sqrt{3}}{3}$$
$$\tan A = \frac{1}{\sqrt{3}}$$
$$\tan A = \tan 30^0$$
$$A = 30^0$$

Solution 9:

(i)

$$\cos x^\circ = \frac{10}{20}$$

$$\cos x^\circ = \frac{1}{2}$$

(ii)

$$\cos x^\circ = \frac{1}{2}$$

$$\cos x^\circ = \cos 60^0$$

$$x^\circ = 60^0$$

(iii)

$$\frac{1}{\tan^2 x^0} - \frac{1}{\sin^2 x^0} = \frac{1}{\tan^2 60^0} - \frac{1}{\sin^2 60^0}$$

$$= \frac{1}{\left(\sqrt{3}\right)^2} - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{3} - \frac{4}{3}$$

$$= -1$$

(iv)

$$\tan x^\circ = \tan 60^\circ$$

$$= \sqrt{3}$$

We know that $\tan x^\circ = \frac{AB}{BC}$

$$\Rightarrow \tan x^\circ = \frac{y}{10}$$

$$\Rightarrow y = 10 \tan x^\circ$$

$$\Rightarrow y = 10 \tan 60^\circ$$

$$\Rightarrow y = 10\sqrt{3}$$

Solution 10:

(i)

$$\tan\theta^0 = \frac{5}{5} = 1$$

(ii)

$$\tan\theta^0 = 1$$
$$\tan\theta^0 = \tan 45^0$$
$$\theta^0 = 45^0$$

(iii)

$$\sin^2\theta^\circ - \cos^2\theta^0 = \sin^2 45^\circ - \cos^2 45^0$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= 0$$

(iv)

$$\sin\theta^0 = \frac{5}{x}$$
$$\sin 45^0 = \frac{5}{x}$$
$$x = \frac{5}{\sin 45^0}$$
$$x = \frac{5}{\frac{1}{\sqrt{2}}}$$
$$x = 5\sqrt{2}$$

Solution 11:

(i)

$$2\sin A\cos A - \cos A - 2\sin A + 1 = 0$$
$$2\sin A\cos A - \cos A = 2\sin A - 1$$
$$(2\sin A - 1)\cos A - (2\sin A - 1) = 0$$
$$(2\sin A - 1) = 0 \quad and \quad \cos A = 1$$
$$\sin A = \frac{1}{2} \quad and \quad \cos A = \cos 0^0$$
$$A = 30^0 \quad and \quad A = 0^0$$

(ii)

$$\tan A - 2\cos A\tan A + 2\cos A - 1 = 0$$
$$\tan A - 2\cos A\tan A = 1 - 2\cos A$$
$$\tan A(1 - 2\cos A) - (1 - 2\cos A) = 0$$
$$(1 - 2\cos A)(\tan A - 1) = 0$$
$$1 - 2\cos A = 0 \quad and \quad \tan A - 1 = 0$$
$$\cos A = \frac{1}{2} \quad and \quad \tan A = 1$$
$$A = 60^0 \quad and \quad A = 45^0$$

(iii)

$$2\cos^2 A - 3\cos A + 1 = 0$$
$$2\cos^2 A - \cos A - 2\cos A + 1 = 0$$
$$\cos A(2\cos A - 1) - (2\cos A - 1) = 0$$
$$(2\cos A - 1)(\cos A - 1) = 0$$
$$2\cos A - 1 = 0 \quad and \quad \cos A - 1 = 0$$
$$\cos A = \frac{1}{2} \quad and \quad \cos A = 1$$
$$A = 60^0 \quad and \quad A = 0^0$$

(iv)

$$2\tan 3A\cos 3A - \tan 3A + 1 = 2\cos 3A$$
$$2\tan 3A\cos 3A - \tan 3A = 2\cos 3A - 1$$
$$\tan 3A(2\cos 3A - 1) = 2\cos 3A - 1$$
$$(2\cos 3A - 1)(\tan 3A - 1) = 0$$
$$2\cos 3A - 1 = 0 \quad and \quad \tan 3A - 1 = 0$$
$$\cos 3A = \frac{1}{2} \quad and \quad \tan 3A = 1$$
$$3A = 60^0 \quad and \quad 3A = 45^0$$
$$A = 20^0 \quad and \quad A = 15^0$$

Solution 12:

(i)

$$2\cos 3x - 1 = 0$$
$$\cos 3x = \frac{1}{2}$$
$$3x = 60^0$$
$$x = 20^0$$

(ii)

$$\cos\frac{x}{3} - 1 = 0$$
$$\cos\frac{x}{3} = 1$$
$$\frac{x}{3} = 0^0$$
$$x = 0^0$$

(iii)

$$\sin\left(x + 10^\circ\right) = \frac{1}{2}$$
$$\sin\left(x + 10^\circ\right) = \sin 30^0$$
$$x + 10^\circ = 30^0$$
$$x = 20^0$$

(iv)

$$\cos\left(2x - 30^\circ\right) = 0$$
$$\cos\left(2x - 30^\circ\right) = \cos 90^0$$
$$2x - 30^\circ = 90^0$$
$$2x = 120^0$$
$$x = 60^0$$

(v)

$$2\cos\left(3x - 15^\circ\right) = 1$$

$$\cos\left(3x - 15^\circ\right) = \frac{1}{2}$$

$$\cos\left(3x - 15^\circ\right) = \cos 60^0$$

$$3x - 15^\circ = 60^0$$

$$3x = 75^0$$

$$x = 25^0$$

(vi)

$$\tan^2\left(x - 5^\circ\right) = 3$$

$$\tan\left(x - 5^\circ\right) = \sqrt{3}$$

$$\tan\left(x - 5^\circ\right) = \tan 60^0$$

$$x - 5^0 = 60^0$$

$$x = 65^0$$

(vii)

$$3\tan^2\left(2x - 20^\circ\right) = 1$$

$$\tan\left(2x - 20^\circ\right) = \frac{1}{\sqrt{3}}$$

$$\tan\left(2x - 20^\circ\right) = \tan 30^0$$

$$2x - 20^\circ = 30^0$$

$$2x = 50^0$$

$$x = 25^0$$

(viii)

$$\cos\left(\frac{x}{2}+10^0\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{x}{2}+10^0\right)=\cos 30^0$$

$$\frac{x}{2}+10^0=30^0$$

$$x=40^0$$

(ix)

$$\sin^2 x+\sin^2 30^o=1$$

$$\sin^2 x=1-\sin^2 30^o$$

$$\sin^2 x=1-\frac{1}{4}$$

$$\sin^2 x=\frac{3}{4}$$

$$\sin x=\frac{\sqrt{3}}{2}$$

$$x=60^0$$

(x)

$$\cos^2 30^o+\cos^2 x=1$$

$$\cos^2 x=1-\cos^2 30^o$$

$$\cos^2 x=1-\frac{3}{4}$$

$$\cos x=\frac{1}{2}$$

$$x=60^0$$

(xi)

$$\cos^2 30^\circ + \sin^2 2x = 1$$
$$\sin^2 2x = 1 - \cos^2 30^\circ$$
$$\sin^2 2x = 1 - \frac{3}{4}$$
$$\sin 2x = \frac{1}{2}$$
$$2x = 30^0$$
$$x = 15^0$$

(xii)

$$\sin^2 60^\circ + \cos^2\left(3x - 9^\circ\right) = 1$$
$$\cos^2\left(3x - 9^\circ\right) = 1 - \sin^2 60^\circ$$
$$\cos^2\left(3x - 9^\circ\right) = 1 - \frac{3}{4}$$
$$\cos^2\left(3x - 9^\circ\right) = \frac{1}{4}$$
$$\cos\left(3x - 9^\circ\right) = \frac{1}{2}$$
$$3x - 9^\circ = 60^0$$
$$3x = 69^0$$
$$x = 23^0$$

Solution 13:

(i)

$$4\cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30^0$$

(ii)

$$\cos^2 x + \cot^2 x = \cos^2 30^0 + \cot^2 30^0$$

$$= \frac{3}{4} + 3$$

$$= \frac{15}{4}$$

$$= 3\frac{3}{4}$$

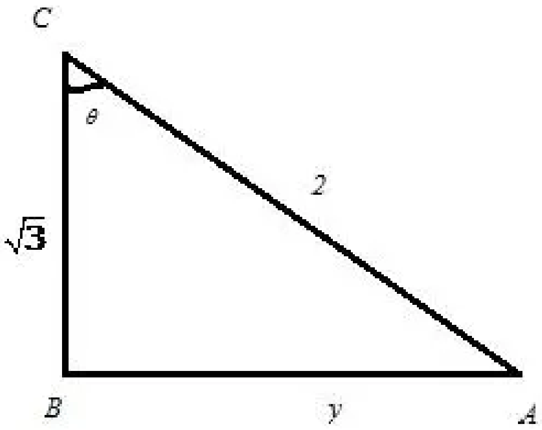
(iii)

$$\cos 3x = \cos 3\left(30^0\right) = \cos 90^0 = 0$$

(iv)

$$\sin 2x = \sin 2\left(30^0\right) = \sin 60^0 = \frac{\sqrt{3}}{2}$$

Solution 14:



(i)

From ΔABC

$$\sin x^{\circ} = \frac{\sqrt{3}}{2}$$

(ii)

$$\sin x^{\circ} = \frac{\sqrt{3}}{2}$$

$$\sin x^{0} = \sin 60^{0}$$

$$x^{0} = 60^{0}$$

(iii)

$$\tan x^{\circ} = \tan 60^{\circ}$$

$$= \sqrt{3}$$

(iv)

$$\cos x^{\circ} = \frac{y}{2}$$

$$\cos 60^{0} = \frac{y}{2}$$

$$\frac{1}{2} = \frac{y}{2}$$

$$y = 1$$

Solution 15:

$$2\cos(A + B) = 1$$

$$\cos(A + B) = \frac{1}{2}$$

$$\cos(A + B) = \cos 60^0$$

$$A + B = 60^0 \quad \ldots\ldots\ldots(1)$$

$$2\sin(A - B) = 1$$

$$\sin(A - B) = \frac{1}{2}$$

$$A - B = 30^0 \quad \ldots\ldots\ldots(2)$$

Adding (1) and (2)

$$A + B + A - B = 60^0 + 30^0$$

$$2A = 90^0$$

$$A = 45^0$$

$$A + B = 60^0$$

$$B = 60^0 - A$$

$$B = 60^0 - 45^0$$

$$B = 15^0$$