Relations and Functions

Relation : If A and B are two non-empty sets, then any subset R of $A \times B$ is called Relation from set A to B. i.e. $R : A \to B \Leftrightarrow R \subseteq A \times B$

If $(x, y) \in R$ then we write xRy (read as x is R related to y) and
If $(x, y) \notin R$ then we write $x \not R y$ (read as x is not R related to y)

Domain and Range of a Relation : If R is any relation from set A to set B then,

- Domain of R is the set of all first coordinates of elements of R and is denoted by Dom(R).
- Range of R is the set of all second coordinates of R and it is denoted by Range (R)

A relation R on set A means, the relation from A to A i.e., $R \subseteq A \times A$

Empty Relation : A Relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e. $R = \phi \subset A \times A$

Universal Relation : A Relation R in a set A is called universal relation each of A is related to every element of A, i.e. $R = A \times A$

Identity Relation : $R = \{(x, y) : x \in A,\ y \in A,\ x = y\}$ OR $R = \{(x, x) ; x \in A\}$

A Relation R in a set A is called -

Reflexive Relation : If $(a, a) \in A$, for every $a \in A$

Symmetric Relation : If $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$

Transitive Relation : If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$

Equivalence Relation : If R is reflexive, symmetric and transitive

Antisymmetric Relation : A relation R in a set A is antisymmetric.
if $(a, b) \in R, (b, a) \in R \Rightarrow a = b \ \forall\ a, b \in R$ or aRb and $bRa \Rightarrow a = b,\ \forall\ a, b \in R$.

Inverse Relation : If A and B are two non-empty sets and R be a relation from A to B, such that $R = \{(a, b) : a \in A, b \in B\}$, then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b\ a) : (a, b) \in R\}$

Equivalence class : Let R be an equivalence relation on a non-empty set A. For all $a \in A$, the equivalence class of 'a' is defined as the set of all such elements of A which are related to 'a' under R. It is denoted by $[a]$.
i.e. $[a]$ = equivalence class of 'a' $= \{x \in A : (x, a) \in R\}$

Function : Let X and Y be two non-empty sets. Then a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y, is called a function from X to Y and written as $f : X \to Y$ where, $f(x)$ is called image of x and x is called the pre-image of $f(x)$ and set Y is called the co-domain of f and $f(X) = \{f(x) : x \in X\}$ is called the range of f.

One-One or Injective Function : A function $f : X \to Y$ is defined to be one-one if the images of distinct element of X under f are distinct;
i.e $x_1, x_2 \in X : f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ Otherwise f is called many-one.

Onto OR Surjective : A function $f: X \to Y$ is said to be onto if every element of Y is the image of some element of X under f ; i.e. for every $y \in Y$, there exists an element x in X such that $f(x) = y$

One - One and onto OR Bijective : A function $f: X \to Y$ is said to be one - one and onto, if f is both one - one and onto.

Note : $f: X \to Y$ is onto if and only if Range of $f = Y$

Composition of function : Let $f: A \to B$ and $g: B \to C$ be two function then the composition of f and g denoted by gof and defined as the function $gof: A \to C$

$$\boxed{gof = g[f(x)], \ \forall \ x \in A}$$

Invertible function : A function $f: X \to Y$ is defined to be invertible, if there exists a function $g: Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.

Binary operation : A binary operation $*$ on a set A is a function $*: A \times A \to A$. we denote $*(a,b)$ by $a * b$

- A binary operation $*$ on a set A is called commutative, if $a * b = b * a$, for every $a, b \in A$.
- A binary operation $*: A \times A \to A$ is said to be associative if $(a * b) * c = a * (b * c)$, $\forall \ a, b, c \in A$
- A binary operation $*: A \times A \to A$, an element $e \in A$, if it exists, is called identity for the operation $*$, if $a * e = e * a$, $\forall \ a \in A$

  identity element

- A binary operation $*: A \times A \to A$ with the identity element e in A, an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a * b = e = b * a$ and b is called the inverse of a and is denoted by a^{-1}.

No. of function : Let $f: A \to B$ be any mapping and $|A| = n$ and $|B| = m$ where,
$|A|$ represent no. of elements in Set A
$|B|$ represent no. of elements in Set B

Then ; Total no. of function from A to B $= m^n$

- **Case (i)** If $n = m$; then
  Total no. of mapping $= n^n$
  Total no. of one - one mapping $= n!$
  Total no. of onto mapping $= n!$

- **Case (ii)** If $n < m$; then
  Total no. of mapping $= m^n$
  Total no. of one - one mapping $= {}^mC_n \, n!$
  Total no. of onto mapping $= 0$

- **Case (iii)** If $n > m$; then
  Total no. of mapping $= m^n$
  Total no. of one - one mapping $= 0$
  Total no. of onto mapping $= \sum_{r=0}^{m-1} (-1)^r \, {}^mC_r \, (m-r)^n$