Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A is a matrix of order 3 and |A| = 8, then |adj A| = **[1]**

a) 1 b) 2^3

c) 2^6 d) 2

2. A grain wholeseller visits the granary market. While going around to make a good purchase, he takes a handful of rice from random sacks of rice, in order to inspect the quality of farmers produce. The handful rice taken from a sack of rice for quality inspection is a **[1]**

a) Statistic b) Sample

c) Parameter d) Population

3. The present value of a sequence of payment of ₹1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually is **[1]**

a) 2500 b) 15,000

c) 1000 d) 25,000

4. Linear programming of linear functions deals with: **[1]**

a) Minimizing

b) Optimizing

c) Maximizing

d) Normalizing

5. If A $= \begin{bmatrix} -3 & x \\ y & 5 \end{bmatrix}$ and A = A', then **[1]**

a) x = 5, y = -3

b) x = -3, y = 5

c) x = y

d) x = 1, y = 2

6. A dice is thrown twice, the probability of occurring of 5 atleast once is **[1]**

a) $\frac{5}{12}$

b) $\frac{35}{36}$

c) $\frac{11}{36}$

d) $\frac{7}{12}$

7. If **m** is the mean of Poisson distribution, then P(r = 0) is given by: **[1]**

a) e

b) m^{-e}

c) e^{m}

d) e^{-m}

8. Integrating factor of the differential equation $\left(1-y^2\right)\frac{dx}{dy}+xy$ = ay is **[1]**

a) $\frac{1}{\sqrt{1-y^2}}$

b) $\frac{1}{y^2-1}$

c) $\frac{1}{\sqrt{y^2-1}}$

d) $\frac{1}{1-y^2}$

9. If in a 600 m race, A can beat B by 50 m and in a 500 m race, B can beat C by 60 m. Then, in a 400 m race, A will beat C by: **[1]**

a) 70 m

b) $77\frac{1}{2}$ m

c) 77 m

d) 81.33 m

10. The value of $\begin{vmatrix} 2^2 & 2^3 & 2^4 \\ 2^3 & 2^4 & 2^5 \\ 2^4 & 2^5 & 2^6 \end{vmatrix}$ is **[1]**

a) 2^9

b) 2^6

c) 2^{13}

d) 0

11. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol is found to be 26%. The quantity of whisky replaced is **[1]**

a) $\frac{2}{5}$ part

b) $\frac{2}{3}$ part

c) $\frac{3}{5}$ part

d) $\frac{1}{3}$ part

12. If x $\in$ R, |x| $\geq$ -7, then **[1]**

a) x $\in$ [-7, 7]

b) x $\in$ $(-\infty$, -7) $\cup$ [7, ∞)

c) x $\in$ R

d) x $\in$ $(-\infty$, -7) $\cup$ (7, ∞)

13. A tank has a leak that would empty it in 10 hours. A tap is turned on which delivers 4 litre a minute into the tank and now it emptied in 12 hours. The capacity of the tank is **[1]**

a) 1800 litres

b) 648 litres

c) 1440 litres

d) 1200 litres

14. Solution set of inequations $x - 2y \geq 0$, $2x - y \leq -2$, $x \geq 0$, $y \geq 0$ is: **[1]**

a) First quadrant

b) Empty

c) Closed halfplane

d) Infinite

15. Corner points of the feasible region for an LPP are: (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let z = 4x + 6y the objective function. The minimum value of z occurs at **[1]**

a) any point on the line segment joining the points (0, 2) and (3, 0)

b) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

c) (3, 0) only

d) (0, 2) only

16. A specific characteristic of a sample is known as a **[1]**

a) parameter

b) variance

c) statistic

d) population

17. The value of $\int \frac{1}{x+x\log x}dx$ is **[1]**

a) log (1 + log x)

b) x + log x

c) x log (1 + log x)

d) 1 + log x

18. For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are: **[1]**

a) 22, 29, 35, 41

b) 24, 29, 35, 41

c) 24, 28, 35, 41

d) 22, 28, 35, 41

19. **Assertion (A):** If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k such that $A^2 = kA - 2I$, is -1. **[1]**

Reason (R): If A and B are square matrices of same order, then (A + B)(A + B) is equal to $A^2 + AB + BA + B^2$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Let $f(x) = x^4 - 2x^2 + 5$ be defined on [-2, 2]. **[1]**

Assertion (A): The range of f(x) is [2, 13].

Reason (R): The greatest value of f is attained at x = 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Calculate five yearly moving averages of the number of students who have studied in a school given below: **[2]**

Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
No. of students	442	427	467	502	512	515	520	527	515	541

22. A simple interest of ₹ 1000 is paid on a certain sum of money at 10% p.a for 4 years. Find the sum. **[2]**

OR

Jyoti buys a car for which she makes down payment of ₹3,50,000 and the balance is to be paid in 3 years by monthly installments of ₹34,000 each. If the financer charges interest at the rate of 12% per annum and uses flat rate method, find the actual price of the car.

23. By using property of definite integrals, evaluate $\int_0^1 |2x-1| dx$ **[2]**

24. If A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$ **[2]**

OR

Find the values of x, if $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

25. In what ratio must a grocer mix two varieties of tea worth ₹ 60 per kg and ₹ 65 per kg so that by selling the mixture at ₹ 68.20 per kg may gain 10%? **[2]**

Section C

26. Radium decomposes at a rate proportional to the quantity of radium present. It is found that in 25 years, approximately 1.1% of a certain quantity of radium has decomposed. Determine approximately how long it will take for one-half of the original amount of radium to decompose? [Given $\log_e 0.989 = 0.01106$ and $\log_e 2 = 0.6931$] **[3]**

OR

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0.

27. Find the purchase price of a ₹600, 8% bond, dividends payable semi-annually redeemable at par in 5 years, if the yield rate is to be 8% compounded semi-annually. **[3]**

28. A company suffers a loss of ₹1,000 if its product does not sell at all. Marginal revenue and Marginal cost functions for the product are given by MR = 50 - 4x and MC = -10 + x respectively. Determine the total profit function, break-even points and the profit maximization level of output **[3]**

29. The income of a group of 10,000 persons was found to be normally distributed with mean ₹ 750 p.m. and standard deviation ₹ 50. Show that of this group about 95% had income exceeding ₹ 668 and only 5% had income exceeding ₹ 832. What was the lowest income among the richest 100? **[3]**

OR

In a binomial distribution the sum and product of the mean and the variance are $\frac{25}{3}$ and $\frac{50}{3}$ respectively. Find the distribution.

30. Given below are the consumer price index numbers (CPI) of the industrial workers. **[3]**

Year	2014	2015	2016	2017	2018	2019	2020
Index number	145	140	150	190	200	220	230

Find the best fitted trend line by the method of least squares and tabulate the trend values.

31. A random sample of 17 values from a normal population has a mean of 105 cm and the sum of the squares of deviations from this mean is 1225 cm^2. Is the assumption of a mean of 110 cm for the normal population reasonable? Test under 5% and 1% levels of significance. Also, obtain the 95% and 99% confidence limits. (Given t_{16} (0.05) = 2.12 and t_{16} (0.01) = 2.921) **[3]**

Section D

32. A manufacturer has three machines installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas Machine III must operate at least for 5 hours a day. He produces only two items, each requiring the use of three machines. The number of hours required for producing one unit each of the items on the three machines is given in the following table: **[5]**

Item	Number of hours required by the machine

	I	II	III
A	1	2	1
B	2	1	$\frac{5}{4}$

He makes a profit of ₹6.00 on item A and ₹4.00 on item B. Assuming that he can sell all that he produces, how many of each item should he produce so as to maximize his profit? Determine his maximum profit. Formulate this LPP mathematically and then solve it.

OR

A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X, Y and Z), it is necessary to buy two additional products, say, A and B. One unit of product A contains 36 units of X, 3 units of Y, and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs ₹ 20 per unit and product B costs ₹ 40 per unit. Formulate the above as a linear programming problem to minimize the total cost, and solve the problem by using graphical method.

33. Show that the solution set of the following linear in equations is an unbounded set: $x + y \geq 9$, $3x + y \geq 12$, $x \geq 0$, $y \geq 0$ **[5]**

34. Find the probability distribution of the number of green balls drawn when 3 balls are awn, one by one, without replacement from a bag containing 3 green and 5 white balls. **[5]**

OR

A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find each of the following:

i. k
ii. $P(X < 6)$
iii. $P(X \geq 6)$
iv. $P(0 < X < 5)$

35. A loan of ₹ 400000 at the interest rate of 6.75 % p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years. Find **[5]**

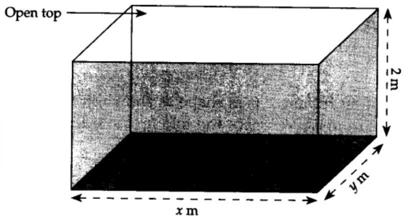
i. the size of each monthly payment.
ii. the principal outstanding at the beginning of 61st month.
iii. the interest paid in 61st payment.
iv. the principal contained in 61st payment.
v. total interest paid.

Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60} = 1.4001$)

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A factory owner wants to construct a tank with rectangular base and rectangular sides, open at the top, so that its depth is 2 m and capacity is 8 m^3. The building of the tank costs ₹280 per square metre for the base and ₹180 per square metre for the sides.



(a) If the length and the breadth of the rectangular base of the tank are x metres and y metres respectively, then find a relation between x and y.

(b) If C (in ₹) is the cost of construction of the tank, then find C as a function of x.

(c) Find the value of x for which the cost of construction of the tank is least.

OR

Find the least cost of construction of the tank.

37. **Read the text carefully and answer the questions:** **[4]**

An equated monthly installment (EMI) is a set monthly payment provided by a borrower to a creditor on a set day, each month. EMIs apply to both interest and principal each month, and the loan is paid off in full over some years.

How is EMI calculated?

There are two ways in which EMI can be calculated. These methods are:

- **The flat rate method:** When the loan amount is progressively being repaid, each interest charge is computed using the original principal amount in the flat rate method.
- **The reducing balance method:** The reducing balance technique, compared to the flat rate method, determines the interest payment according to the outstanding principal.

Example:

A loan of ₹250000 at the interest rate of 6% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years.

(Given $(1.005)^{60} = 1.3489$, $(1.005)^{21} = 1.1104$)

(a) Find the size of each monthly payment.

(b) Find the principal outstanding at beginning of 40th month.

(c) Find interest paid in 40th payment.

OR

Find principal contained in 40th payment.

38. Find the inverse of the matrix : **[4]**

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = 1$.

OR

Using matrix method, solve the following system of equations for x, y and z :

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Solution

Section A

1.

(c) 2^6

Explanation: |A| = d

|adj A| = $|A|^{n-1}$

Here, n = 3, |A| = 8

|adj A| = 8^2

|adj A| = $(2^3)^2 = 2^6$

2.

(b) Sample

Explanation: Sample

3.

(d) 25,000

Explanation: The given annuity is a perpetuity.

present value of perpetuity = $\frac{cash\ flow}{Interest\ rate}$

Here, cash flow = ₹1000

interest rate = $\frac{8/2}{100}$

= $\frac{4}{100}$ = 0.04

So, present value = $\frac{1000}{0.04}$

= ₹25,000

4.

(b) Optimizing

Explanation: Optimizing

5.

(c) x = y

Explanation: $A = \begin{bmatrix} -3 & x \\ y & 5 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -3 & y \\ x & 5 \end{bmatrix}$

$\because$ A = A' $\Rightarrow$ x = y

$\therefore$ Option (x = y) is the correct answer.

6.

(c) $\frac{11}{36}$

Explanation: Here, n = 2, p = $\frac{1}{6}, q = \frac{5}{6}$

P(X $\geq$ 1) = 1 - P(0) = 1 - ${}^2C_0\left(\frac{5}{6}\right)^2 = 1 - \frac{25}{36} = \frac{11}{36}$

7.

(d) e^{-m}

Explanation: Given is the mean of a Passion distribution.

Let's assume that a discrete random Variable.

x follow Poission distribution for over an infinite number of trials and a small finite probability of success then

PMF of this random variable x is $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

Here λ is rate parameter and is equal to the mean

i.e. λ = μ

So posission distribution with its mean 'm'

$P(x) = \frac{m^x e^{-m}}{x!}$

$\therefore P(0)=\frac{m^0e^{-m}}{0!}$

$\Rightarrow$ P(0) = e^{-m}

8. **(a)** $\frac{1}{\sqrt{1-y^2}}$

Explanation: $\frac{dx}{dy}$ + $\frac{y}{1-y^2}$ x = $\frac{ay}{1-y^2}$, which is linear in x.

I.F. = $e^{\int\frac{y}{1-y^2}dy}$ = $e^{-\frac{1}{2}\log\left(1-y^2\right)}$ = $\frac{1}{\sqrt{1-y^2}}$

9.

(d) 81.33 m

Explanation: When A cover 600 m, B cover 550 m

When B cover 500 m, C cover 440 m

When B cover 400 m, C cover = $\frac{440}{500}\times 400$ = 88 $\times$ 4 = 352 m

In a 400 m race,

B beat C by = 400 - 352 = 48 m

When A cover 400 m, B cover = $\frac{550}{600}\times 400=\frac{1100}{3}$ m = 366.67 m

In a 400 m, race A beat B by = 400 - 366.67 = 33.33 m

$\therefore$ In a 400 m race, A beat C by = 48 + 33.33 = 81.33 m

10.

(d) 0

Explanation: Taking 2^2, 2^3 and 2^4 common from R^1, R^2 and R^3 respectively, we get

$2^2\cdot2^3\cdot2^4\begin{vmatrix}1&2&2^2\\1&2&2^2\\1&2&2^2\end{vmatrix}$ (Operate $C_2\rightarrow\frac{1}{2}C_2$)

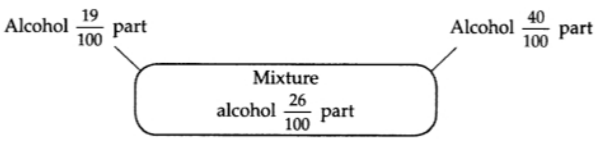
= $2^9\cdot2\begin{vmatrix}1&1&2^2\\1&1&2^2\\1&1&2^2\end{vmatrix}$ = $2^{10}\times 0$ = 0 ($\because$ C_1 and C_2 are same)

$\therefore$ Option (d) is the correct answer.

11.

(b) $\frac{2}{3}$ part

Explanation:



So, ratio $\frac{\frac{40}{100}-\frac{26}{100}}{\frac{26}{100}-\frac{19}{100}}=\frac{14}{7}=\frac{2}{1}$.

$\therefore$ The quantity of whisky replaced by 19% alcohol $=\frac{2}{2+1}$ i.e. $\frac{2}{3}$ part

12.

(c) x $\in$ R

Explanation: x $\in$ R

13.

(c) 1440 litres

Explanation: Let' say the capacity of the cistern is x litres

so it is leaking at $\frac{x}{10}$ litres per hour

Tap fills in 4 litres a min i.e. 60 $\times$ 4 = 240 litres per hour

Now, with tap turned on, the water leakage per hour is ($\frac{x}{10}$ - 240)

It takes, 12 hours to be emptied now, so per hour leakage is $\frac{x}{12}$

so $\frac{x}{10}-240$ = $\frac{x}{12}$

on solving for x,

x = 14400

14.

(b) Empty

Explanation: There will be no common region.

15. **(a)** any point on the line segment joining the points (0, 2) and (3, 0)

Explanation: Here the objective function is given by:

F = 4x + 6y

Corner points	Z = 4x + 6y
(0, 2)	12...(Min.)
(3, 0)	12...(Min.)
(6, 0)	24
(6, 8)	72...(Max.)
(0, 5)	30

Hence, it is clear that the minimum value occurs at any point on the line joining the points (0, 2) and (3, 0)

16.

(c) statistic

Explanation: statistic

17. **(a)** log (1 + log x)

Explanation: $I=\int \frac{1}{x+x \log x} d x$

$I=\int \frac{d x}{x(1+\log x)}$

Put 1 + log x = t

$\Rightarrow \frac{1}{x} d x=d t$

$I=\int \frac{1}{t} d t$

$\Rightarrow$ I = log |t| + C

I = log (1 + log x) + C

18. **(a)** 22, 29, 35, 41

Explanation: 22, 29, 35, 41

19.

(d) A is false but R is true.

Explanation: Assertion: Given, A^2 = kA - 2I

$\Rightarrow$ AA = kA - 2I

$\Rightarrow\left[\begin{array}{ll}3 & -2 \\ 4 & -2\end{array}\right]\left[\begin{array}{ll}3 & -2 \\ 4 & -2\end{array}\right]=k\left[\begin{array}{ll}3 & -2 \\ 4 & -2\end{array}\right]-2\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$

$\Rightarrow\left[\begin{array}{cc}9-8 & -6+4 \\ 12-8 & -8+4\end{array}\right]=\left[\begin{array}{ll}3 k & -2 k \\ 4 k & -2 k\end{array}\right]-\left[\begin{array}{ll}2 & 0 \\ 0 & 2\end{array}\right]$

$\Rightarrow\left[\begin{array}{ll}1 & -2 \\ 4 & -4\end{array}\right]=\left[\begin{array}{cc}3 k-2 & -2 k \\ 4 k & -2 k-2\end{array}\right]$

By definition of equality of matrix, the given matrices are equal and their corresponding elements are equal.

Now, comparing the corresponding elements, we get

3k - 2 = 1 $\Rightarrow$ k = 1

$\Rightarrow$ -2k = -2 = k = 1

$\Rightarrow$ 4k = 4 $\Rightarrow$ k = 1

$\Rightarrow$ -4 = -2A - 2 $\Rightarrow$ k = 1

Hence, k = 1

Reason: We have,

(A + B)(A + B) = A(A + B) + B(A + B)

= A^2 + AB + BA + B^2

20.

(d) A is false but R is true.

Explanation: f(x) = x^4 - $2x^2$ + 5 $\Rightarrow$ f'(x) = $4x^3$ - 4x

$\Rightarrow$ f'(x) = 4x (x - 1) (x + 1)

$\Rightarrow$ f'(x) = 4x (x^2 - 1

$\Rightarrow$ f'(x) = 4x (x -1) (x + 1).

For critical points, f'(x) = 0 $\Rightarrow$ x = 0, -1, 1.

Now, f(-2) = $(-2)^4$ - $2(-2)^2$ + 5 = 16 - 8 + 5 = 13

f(2) = 2^4 - $2(2)^2$ + 5 = 16 - 8 + 5 = 13

f(-1) = $(-1)^4$ - $2(-1)^2$ + 5 = 1 - 2 + 5 = 4

f(0) = 0 - 2 $\times$ 0 + 5 = 5

f(1) = 14 - $2(1)^2$ + 5 = 4

So, the range of f is [4, 13]

$\therefore$ Assertion is false.

Also, f attains it maximum value at x = - 2 and x = 2

$\therefore$ Reason is true.

Section B

21. **Calculation of 5-year moving averages:**

Year	Number of students	5-year moving total	5-year moving average
1993	442	—	—
1994	427	—	—
1995	467	2350	470
1996	502	2423	484.6
1997	512	2516	503.2
1998	515	2576	515.2
1999	520	2589	517.8
2000	527	2618	523.6
2001	515		
2002	541		

($\frac{1}{5}$)

22. Let P be ₹ x, S.I. = ₹ 1000, r = 10% p.a., n = 4 years

$\therefore \frac{x \times 10 \times 4}{100}$ = 1000

$\Rightarrow$ x = 2500

$\therefore$ Sum = ₹ 2500

OR

Let the amount of loan be ₹P

EMI = ₹34000, i = $\frac{12}{12 \times 100}$ = 0.01, n = 3 $\times$12 = 36

EMI = $\frac{\mathrm{P}+\mathrm{P}ni}{n}$

$\Rightarrow$ 34000 = $\frac{\mathrm{P}(1+36 \times 0.01)}{36} = \frac{\mathrm{P} \times 1.36}{36}$

$\Rightarrow$ P = $\frac{34000 \times 36}{1.36}$ $\Rightarrow$ P = 900,000

$\therefore$ Price of the car = down payment + loan

= ₹350000 + ₹900000 = ₹1250000

23. $\int\limits_0^1 |2x-1| dx = \int\limits_0^{\frac{1}{2}} |2x-1| dx + \int\limits_{\frac{1}{2}}^1 |2x-1| dx$

$= \int\limits_0^{\frac{1}{2}} -(2x-1) dx + \int\limits_{\frac{1}{2}}^1 (2x-1) dx$

$= \left[-\left(x^2-x\right)\right]_0^{\frac{1}{2}} + \left[\left(x^2-x\right)\right]_{\frac{1}{2}}^1$

$= -\left(\frac{1}{4}-\frac{1}{2}\right) - 0 + 0 - \left(\frac{1}{4}-\frac{1}{2}\right) = \frac{1}{4} + \left(\frac{1}{4}\right) = \frac{1}{2}$

24. Given, A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$3A^2$ - 2B + I

= $3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$

= $\begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

OR

We have $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow$ (x + 1)(x + 2) - (x - 1)(x - 3) = 4 $\times$ 3 - 1 $\times$ (-1)

$\Rightarrow$ (x^2 + 2x + x + 2) - (x^2 - 3x - x + 3) = 12 + 1

$\Rightarrow$ -2x -1 = 13

$\Rightarrow$ -2x = 14

$\Rightarrow$ x = -7

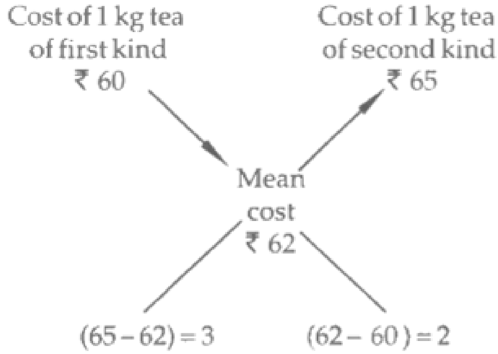
25. We have,

S.P. of mixture = ₹ 68.20 and Gain = 10%

$\therefore$ C.P. = $\frac{SP}{1+\frac{Gain}{100}}$

$\Rightarrow$ C.P. = $\left(\frac{68.20}{1+\frac{10}{100}}\right)$ = $\left(\frac{682}{11}\right)$ = ₹ 62

The allegation grid is as given below:



By using the rule of allegation, we obtain

Tea of first kind: Tea of second kind = 3 : 2

Hence, the grocer must mix in the ratio 3 : 2

Section C

26. Let A be the quantity of radium present at time t and A_0 be the initial quantity of radium.

Therefore, we have,

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt}$ = - 2A

$\frac{dA}{A}$ = - 2dt

$\int \frac{dA}{A} = -\lambda t dt$

log A = $-\lambda t$ + c ...(i)

Now, A = A_0 when t = 0

log A_0 = 0 + c

c = log A_0

Put value of c in equation

log A = $-\lambda$ t + log A_0

$\log\left(\frac{A}{A_0}\right) = -\lambda t$ ...(ii)

Given that, In 25 years, bacteria decomposes 1.1 %, so

A = (100 - 1.1)% = 98.996 %= 0.989 A_0, t = 25

Therefore, (ii) gives,

$\log\left(\frac{0.989A_0}{A_0}\right)$ = - 25λ

log (0.989) = -25λ

$\lambda = -\frac{1}{25}$ log (0.989)

Now, equation (ii) becomes,

$\log\left(\frac{A}{A_0}\right)$ = { $\frac{1}{25}$ log (0.989)} t

Now A = $\frac{1}{2}A_0$

$\log\left(\frac{A}{2A}\right) = \frac{1}{25}$ log (0.989) t

$\frac{-\log 2\times 25}{\log(0.989)}$ = t

$-\frac{0.6931\times 25}{0.01106}$ = t

t = 1567 years

Required time = 1567 years

OR

The given differential equation can be written as

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q, where

P = $\frac{2x}{1+x^2}$ and Q = $\frac{4x^2}{1+x^2}$

$\therefore$ I.F. = $e^{\int Pdx} = e^{\int \frac{2x}{(1+x^2)dx}} = e^{\log(1+x^2)}$ = 1 + x^2

Multiplying both sides of (i) by I.F. = (1 + x^2), we get

(1 + x^2)$\frac{dy}{dx}$ + 2xy = 4x^2

Integrating both sides with respect to x, we get

y(1 + x^2) = $\int$ 4x^2 dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ y (1 + x^2) = $\frac{4x^3}{3}$ + C ...(ii)

It is given that y = 0, when x = 0. Putting x = 0 and y = 0 in (i), we get

0 = 0 + C $\Rightarrow$ C = 0

Substituting C = 0 in (ii), we get y = $\frac{4x^3}{3(1+x^2)}$, which is the required solution.

27. Face value of the bond C = ₹600

Nominal rate of interest i = 8% or 0.08

As dividends are paid semi-annually

Therefore, Rate of interest per period i_d = $\frac{0.08}{2}$ = 0.04

Therefore, periodic dividend payment R = C $\times$ i_d = 600 $\times$ 0.04 = 24

So, semi-annual dividend R is ₹24

Yield rate is 8% = 0.08, compounded semi annually

Therefore i = $\frac{0.08}{2}$ = 0.04

No. of years n = 5

Therefore, no. of dividend periods (n) = 5 $\times$ 2 = 10

Purchase price (V) of the bond is given by

V = $R\left|\frac{1-(1+i)^{-n}}{i}\right| + C(1+i)^{-n}$

$= 24\left|\frac{1-(1+0.04)^{-10}}{0.04}\right| + 600(1+0.04)^{-10}$

$= 24\left|\frac{1-(1.04)^{-10}}{0.04}\right| + 600(1.04)^{-10}$

$= 24\left[\frac{1-0.6755}{0.04}\right| + 600(0.6755)$

= 194.7 + 405.3 = 600

Therefore, purchase price of bond is ₹600.

28. Let P denote the profit function. Then,

$\frac{dP}{dx}$ = MR - MC

$\Rightarrow \frac{dP}{dx}$ = (50 - 4x) - (-10 + x)

$\Rightarrow \frac{dP}{dx}$ = 60 - 5x and $\frac{d^2P}{dx^2}$ = -5

For maximum value of P, we must have

$\frac{dP}{dx} = 0 \Rightarrow$ 60 - 5x = 0 $\Rightarrow$ x = 12

Clearly, $\frac{d^2P}{dx^2}$ = -5 < 0 for all x.

So, profit P is maximum when 12 units are produced. Thus, the profit maximization level of output is 12 units.

Now, $\frac{dP}{dx}$ = 60 - 5x

$\Rightarrow$ P = $\int(60-5x)dx + k$ ... [On intergrating]

$\Rightarrow$ P = 60x - $\frac{5}{2}x^2$ + k ... (i)

where k is the constant of integration

It is given that the company suffers a loss of ₹ 1000, if its product does not sell at all i.e. P = -1000 at x = 0. Substituting these values in (i), we obtain k = -1000.

Putting k = -1000 in (i), we obtain:

P = 60x - $\frac{5}{2}x^2$ + 1000

This is the total profit function. For break-even points

P = 0 $\Rightarrow$ 60x - $\frac{5}{2}x^2$ + 1000 = 0 $\Rightarrow$ $5x^2$ - 120x + 2000 = 0

$\Rightarrow$ x^2 - 24x + 400 = 0

This equation does not give real values of x. So, there is no break-even point.

29. Let X denote the income. Then X is normally distributed with mean μ = ₹ 750 and standard deviation σ = ₹ 50. Let Z be the standard normal variate. Then,

Z = $\frac{X-\mu}{\sigma}$ or, Z = $\frac{X-750}{50}$

When X = 668, we obtain: Z = $\frac{668-750}{50} = -\frac{82}{50}$ = -1.64

Now, P(X > 668)

= P(Z > -1.64)

= P(-1.64 < Z $\leq$ 0) + P(Z $\geq$ 0) = P(0 $\leq$ Z < 1.64) + 0.5 = 0.4495 + 0.5 = 0.9495

Thus, 94.95% persons had income exceeding ₹ 668

When X = 832, we obtain: Z = $\frac{832-750}{50}$ = 1.64

$\therefore$ P(X > 832)

= P(Z > 1.64)

= P(Z $\geq$ 0) - P(0 $\leq$ Z < 1.64) = 0.5 - 0.4495 = 0.0505

Thus, 5.05 % persons had income exceeding ₹ 832

Now, probability of selecting a person out of richest 100 persons = $\frac{100}{10000}$ = 0.01

In order to find the lowest income among the richest 100, we have to find the value k of X such that P (X $\geq$ k) = 0.01

When X = k, we obtain Z = $\frac{k-750}{50}$ = Z_1(Say)

Now, P(X > k) = 0.01

= P(Z $\geq$ Z_1) = 0.01

= 0.5 - P(0 $\leq$ Z $\leq$ Z_1) = 0.01

= P(0 $\leq$ Z $\leq$ Z_1) = 0.49

= Z_1 = 2.33

= $\frac{k-750}{50}$ = 2.33 $\Rightarrow$ k = 750 + 50 $\times$ 2.33 $\Rightarrow$ k = 866.5

Hence, the lowest income among the richest 100 was ₹ 866.50

OR

We have,

Sum of the mean and variance = $\frac{25}{3}$

$\Rightarrow$ np + npq = $\frac{25}{3}$

$\Rightarrow$ np(1 + q) = $\frac{25}{3}$ ...(i)

Product of the mean and variance = $\frac{50}{3}$

$\Rightarrow$ np(npq) = $\frac{50}{3}$ ...(ii)

Dividing eq. (ii) by eq. (i), we have,

$\frac{\mathrm{np(npq)}}{\mathrm{np(1+q)}} = \frac{50}{3} \times \frac{3}{25}$

$\Rightarrow \frac{npq}{1+q} = 2$

$\Rightarrow$ npq = 2(1 + q)

$\Rightarrow$ np(1 - p) = 2(2 - p)

$\Rightarrow np = \frac{2(2-p)}{(1-p)}$

Substituting this value in np + npq = $\frac{25}{3}$, we have,

$\frac{2(2-p)}{(1-p)}(2-p) = \frac{25}{3}$

$\Rightarrow$ 6(4 - 4p + p^2) = 25 - 25p

$\Rightarrow$ $6p^2$ + p - 1 = 0

$\Rightarrow$ (3p - 1) (2p + 1) = 0

$\Rightarrow p = \frac{1}{3}$ or $\frac{-1}{2}$

As p cannot be negative, therefore possible value of p is $\frac{1}{3}$

q = 1 - p $= \frac{2}{3}$

$\Rightarrow$ np + npq = $\frac{25}{3}$

$\Rightarrow n\left(\frac{1}{3}\right)\left(1+\frac{2}{3}\right) = \frac{25}{3}$

$\Rightarrow$ n = 15

$\therefore$ P(X = r) $= {}^{15}C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{15-r}$, r = 0, 1, 2 15

30. Note that the number of years is Odd

$\Rightarrow$ n = odd

Procedure:

i. Take middle year values as As i.e. A = 2017
ii. Find X = x_i - A
iii. Find X^2 and XY

Year	**Index number (Y)**	**X = x_i - A = x_i - 2017**	**X^2**	**XY**	**Trend value Y_t = a + bX**
2014	145	-3	9	-435	182.1 + (-3) $\times$ 16.6 = 132.3
2015	140	-2	4	-280	182.1 + (-2) $\times$ 16.6 = 148.9
2016	150	-1	1	-150	182.1 + (-1) $\times$ 16.6 = 165.5
2017	190	0	0	0	182.1 + (0) $\times$ 16.6 = 182.1
2018	200	1	1	200	182.1 + (1) $\times$ 16.6 = 198.7
2019	220	2	4	440	182.1 + (2) $\times$ 16.6 = 215.3
2020	230	3	9	690	182.1 + (3) $\times$ 16.6 = 231.9
n = 7	$\sum Y$ = 1275	$\sum X$ = 0	$\sum X^2$ = 28	$\sum XY$ = 465	$\sum Y_t$ = 1274.7

$a = \frac{\sum Y}{n} = \frac{1275}{7}$ = 182.14

and $b = \frac{\sum XY}{\sum X^2} = \frac{465}{28}$ = 16.6

Therefore, the required equation of the straight-line trend is given by

y = a + bX

$\Rightarrow$ y = 182.1 + 16.6 X

31. We have,

μ = Population mean = 110, $\bar{X}$ = Sample mean = 105 n = Sample size = 17 and, $\sum_{i=1}^{17}\left(x_i-\bar{X}\right)^2$ = 1225

$\therefore s^2=\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\bar{X}\right)^2$

$\Rightarrow s^2=\frac{1225}{17}$ = 72.0588 ⇒ s = $\sqrt{72.0588}$ = 8.4887

We define, Null Hypothesis H_0: There is no significant difference between the sample mean and population means i.e. assumption that mean of the population is 110 cm is valid.

Alternate hypothesis H_1: Assumption that mean of the population is 110 cm is not valid. Let t be the test statistic given by

t = $\frac{\bar{X}-\mu}{\frac{s}{\sqrt{n-1}}}\Rightarrow$ t = $\frac{105-110}{8.4887}\times\sqrt{17-1}=\frac{-5\times4}{8.4887}$ = -2.3561

⇒ |t| = 2.3561

The sample statistic follows Student's t -distribution with v = (17 - 1) = 16 degrees of freedom.

We shall now compare this calculated value with the tabulated value of t for 16 degrees of freedom at 5% and 1% levels of significance.

At 5% level of significance: It is given that $t_{16}(0.05)$ = 2.12

We find that Calculated |t| = 2.3561 > 2.12 = $t_{16}(0.05)$

i.e. Calculated |t| > Tabulated $t_{16}(0.05)$

So, we reject the null hypothesis at 5% level of significance. Hence, the assumption that the population has a mean of 110 cm is not correct.

The confidence limits at 5% level of significance are

$\bar{X}-\frac{s}{\sqrt{n-1}}$ $t_{16}(0.05)$ and $\bar{X}+\frac{s}{\sqrt{n-1}}$ $t_{16}(0.05)$

or 105 - $\frac{8.4887}{4}\times$ 2.12 and 105 + $\frac{8.4887}{4}\times$ 2.12

or, 105 - 4.499 = 100.501 and 105 + 4.499 = 109.499

The confidence interval is [100.501,109.499]

At 1% level of significance: It is given that $t_{16}(0.01)$ = 2.921

Clearly, calculated |t| < tabulated $t_{16}(0.01)$

So, we accept the null hypothesis at 1% level of significance. Hence, the assumption that the mean of the population is 110 cm is valid.

The confidence limits at 1% level of significance are

$\bar{X}-\frac{s}{\sqrt{n-1}}$ $t_{16}(0.01)$ and $\bar{X}+\frac{s}{\sqrt{n-1}}$ $t_{16}(0.01)$

or, 105 - $\frac{8.4887}{4}\times$ 2.921 and 105 + $\frac{8.4887}{4}\times$ 2.921

or, 105 - 6.199 = 98.801 and 105 + 6.199 = 111.199

The confidence interval at 1% level of significance or at 99% confidence level is [98.801, 111.199]

Section D

32. Let x units of item A and y units of item B be manufactured. Therefore, x, y ≥ 0

As we are given,

Item	Number of hours required by the machine		
	I	II	III
A	1	2	1
B	2	1	$\frac{5}{4}$

Machines I and II are capable of being operated for at most 12 hours whereas Machine III must operate at least for 5 hours a day.

According to the question, the constraints are

x + 2y ≤ 12

2x + y ≤ 12

x + $\frac{5}{4}$y ≥ 5

He makes a profit of ₹6.00 on item A and ₹4.00 on item B. Profit made by him in producing x items of A and y items of B is 6x + 4y

Total profit Z = 6x + 4y which is to be maximized

Thus, the mathematical formulation of the given linear programming problem is

Max Z = 6x + 4y, subject to

x + 2y $\leq$ 12

2x + y $\leq$ 12

x + $\frac{5}{4}$y $\geq$ 5

x, y $\geq$ 0

First, we will convert the inequations into equations as follows:

x+ 2y = 12, 2x + y = 12, x + $\frac{5}{4}$y = 5, x = 0 and y = 0

The region represented by x + 2y $\leq$ 12

The line x + 2y = 12 meets the coordinate axes at A(12, 0) and B(0, 6) respectively. By joining these points, we obtain the line x + y = 12. Clearly (0, 0) satisfies the x + 2y = 12. So, the region which contains the origin represents the solution set of the inequation x + 2y $\leq$ 12

The region represented by 2x + y $\leq$ 12

The line 2x + y = 12 meets the coordinate axes at C(6, 0) and D(0, 12) respectively. By joining these points, we obtain the line 2x + y = 12. Clearly (0, 0) satisfies the 2x + y = 12. So, the region which contains the origin represents the solution set of the inequation 2x + y $\leq$ 12

The region represented by x + $\frac{5}{4}$y $\geq$ 5

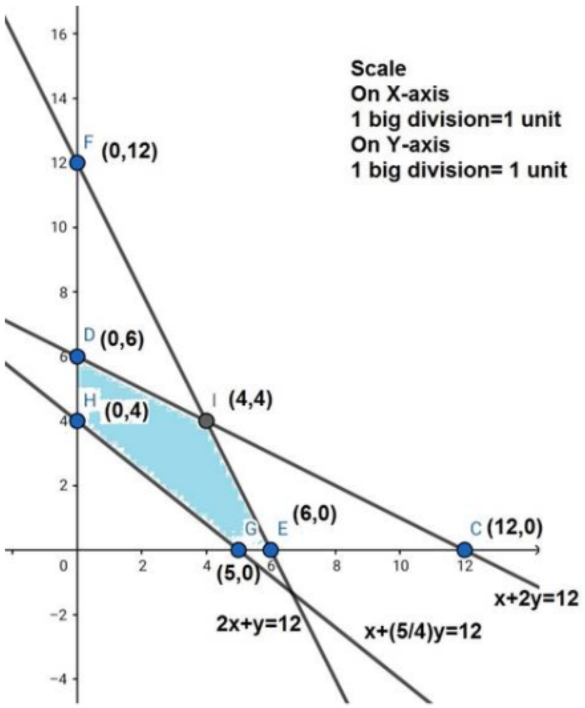
The line x + $\frac{5}{4}$y $\geq$ 5 meets the coordinate axes at E(5, 0) and F(0, 4) respectively. By joining these points, we obtain the line x + $\frac{5}{4}$y = 5. Clearly (0, 0) satisfies the x + $\frac{5}{4}$y $\geq$ 5. So, the region which does not contain the origin represents the solution set of the inequation x + $\frac{5}{4}$y $\geq$ 5

The region represented by x $\geq$ 0, y $\geq$ 0:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0

The feasible region determined by the system of constraints

x + 2y $\leq$ 12, 2x + y $\leq$ 12, x + $\frac{5}{4}$y $\geq$ 5, x, y $\geq$ 0 are as follows:



Thus the maximum profit is of ₹40 obtained when 4 units each of items A and B are manufactured

The corner points are D(0, 6), I(4, 4), C(6, 0), G(5, 0), and H(0, 4). The values of Z at these corner points are as follows:

Corner points	Z = 6x + 4y
D	24
I	40
C	36
G	30
H	16

The maximum value of Z is 40 which is attained at I(4, 4).

OR

The data given in the problem can be summarized in the following tabular form:

Product	Nutrient constituent			Const in ₹
	X	Y	Z	
A	36	3	20	20
B	6	12	10	40
Minimum Required	108	36	100	

Let x units of product A and y units of product B are bought to fulfill the minimum requirement of X, Y and Z and to minimize the cost.

The mathematical formulation of the above problem is as follows:

Minimize Z = 20x + 40y

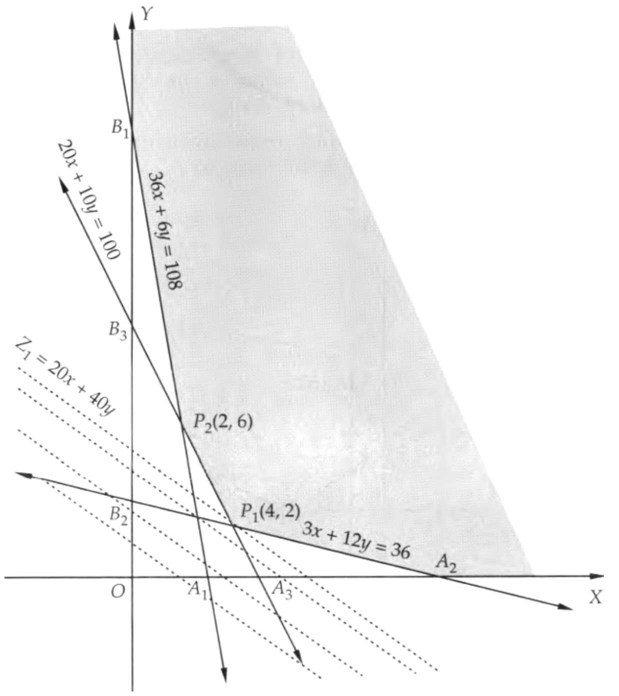
Subject to $36x + 6y \geq 108$

$3x + 12y \geq 36$

$20x + 10y \geq 100$

and, $x, y, z \geq 0$

The set of all feasible solutions of the above LPP is represented by the feasible region shaded darkly in Figure. The coordinates of the corner points of the feasible region are A_2 (12, 0), P_1 (4, 2), P_2 (2, 6) and B_1 (0,18).



Now, we have to find a point or points in the feasible region which give the minimum value of the objective function. For this, let us give some value to Z, say 20, and draw a dotted line 20 = 20x + 40y. Now, draw lines parallel to this line which have at least one point common to the feasible region and locate a line that is nearest to the origin and has at least one point common to the feasible region. Clearly, such a line is Z_1 = 20x + 40y and it has a point P_1(4, 2) common with the feasible region. Thus, Z_1 = 20x + 40y is the minimum value of Z, and the feasible solution which gives this value of Z is the comer P_1(4, 2) of the shaded region.

The values of the variables for the optimal solution are x = 4, y = 2. Substituting these values in Z = 20x + 40y, we get Z = 160 as the optimal value of Z.

Hence, 2 units of product A and 4 units of product B are sufficient to fulfill the minimum requirement at a minimum cost of ₹160

33. First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality. You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y-intercepts always.

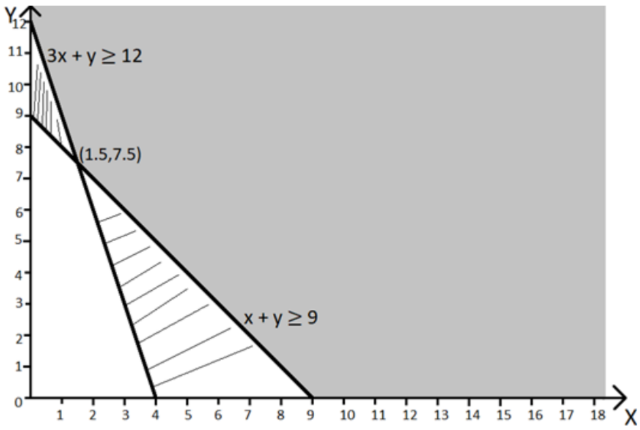
$x + y \geq 9$

x	0	5	9
y	9	4	0

3x + y $\geq$ 12

x	0	2	4
y	12	6	0

x $\geq$ 0, y $\geq$ 0



34. Let X be a random variable denoting the total number of green balls drawn in three draws without replacement. Clearly, there may be all green, 2 green, 1 green or no green at all. Therefore, X can take values 0, 1, 2, and 3. Let G_i denote the event of getting a green ball in i[th] draw.

Now, we have,

P (X = 0) = Probability of getting no green ball in three draws

$\Rightarrow$ P (X = 0) =$P(\overline{G_1} \cap \overline{G_2} \cap \overline{G_3}) = P(\overline{G_1})P(\overline{G_2}/\overline{G_1})P(\overline{G_3}/\overline{G_1} \cap \overline{G_2}) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$

F (X = 1) = Probability of getting one green ball in three draws

$\Rightarrow$ P (X = 1) = $P\left(\left(G_1 \cap \overline{G_2} \cap \overline{G_3}\right) \cup \left(\overline{G_1} \cap G_2 \cap \overline{G_3}\right) \cup \left(\overline{G_1} \cap \overline{G_2} \cap G_3\right)\right)$

$\Rightarrow$ P (X = 1) = $P\left(G_1 \cap \overline{G_2} \cap \overline{G_3}\right) + P\left(\overline{G_1} \cap G_2 \cap \overline{G_3}\right) + P\left(\overline{G_1} \cap \overline{G_2} \cap G_3\right) + P(\overline{G_1})P(\overline{G_2}/\overline{G_1})P\left(G_3/\overline{G_1} \cap \overline{G_2}\right)$

$\Rightarrow$ P (X = 1) = $\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{15}{28}$

P (X = 2) = P$\left(\left(G_1 \cap G_2 \cap \overline{G_3}\right) \cap \left(\overline{G_1} \cap G_2 \cap G_3\right) \cup \left(G_1 \cap \overline{G_2} \cap G_3\right)\right)$

$\Rightarrow$ P (X = 2) = P$(G_1)\, P\,(G_2/G_1)\, P\left(\overline{G_3}/G_1 \cap G_2\right) + P(\overline{G_1})P\left(G_2/\overline{G_1}\right) P\left(G_3/\overline{G_1} \cap G_2\right)$

$+P\,(G_1)\, P\left(\overline{G_2}/G_1\right) P\left(G_3/G_1 \cap \overline{G_2}\right)$

$\Rightarrow$ P (X = 2) = $\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{15}{56}$

and,

P (X = 3) = P$(G_1 \cap G_2 \cap G_3) = P\,(G_1)\, P\left(\frac{G_2}{G_1}\right) P\left(\frac{G_3}{G_1 \cap G_2}\right) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$

Therefore, the probability distribution of the number of green balls is given by

X	0	1	2	3
P(X)	$\frac{5}{28}$	$\frac{15}{28}$	$\frac{15}{56}$	$\frac{1}{56}$

OR

i. We know that the sum of all the probabilities in a probability distribution is always unity. Therefore,we have,

P (X = 0) + P (X = 1) + ...+ P { X = 7) = 1

$\Rightarrow$ 0 + k + 2k + 2k + 3 k + k^2 + $2k^2$ + 7 k^2 + k = 1

$\Rightarrow$$10k^2$ + 9k - 1 = 0

$\Rightarrow$ (10k -1) (k + 1) = 0

$\Rightarrow$10k - 1 = 0

$\Rightarrow k = \frac{1}{10}$

ii. P (X < 6) = P(X = 0) + P(X = 1) + P (X = 2) + P (X = 3) + P (X = 4) + P (X = 5)

$\Rightarrow$P (X < 6) = 0 + k + 2k + 2k + 3k+ k^2

$\Rightarrow$P (X < 6) = k^2 + 8k

$\Rightarrow P(X<6)=\left(\frac{1}{10}\right)^2+\frac{8}{10}$ ...[$\because$ k = $\frac{1}{10}$]

$\Rightarrow P(X<6)=\frac{81}{100}$

iii. P (X $\geq$ 6) = P (X = 6) + P (X = 7)

$\Rightarrow$ P (X $\geq$ 6) = $2k^2+7k^2+k$

$\Rightarrow$ P (X $\geq$ 6) = $9k^2$ + k

$\Rightarrow P(X\geq 6)=\frac{9}{100}+\frac{1}{10}$ ...[$\because$ k = $\frac{1}{10}$]

$\Rightarrow P(X\geq 6)=\frac{19}{100}$

iv. P (0 <X <5) = P (X = 1) + P (X = 2) + P (X = 3) + P (X = 4)

$\Rightarrow$ P (0 < X < 5) = k + 2k + 2k + 3k

$\Rightarrow$ P (0 <X <5) = 8k

$\Rightarrow P(0<X<5)=\frac{8}{10}=\frac{4}{5}$ ...[$\because$ k = $\frac{1}{10}$]

35. i. Given P = ₹ 400000, n = 120, i = $\frac{6.75}{1200}$ = 0.005625

$\therefore$ EMI = $\frac{400000\times 0.005625\times(1.005625)^{120}}{(1.005625)^{120}-1}$

= $\frac{400000\times 0.005625\times 1.9603}{0.9603}$ = ₹4593.

ii. Principal outstanding at the beginning of 61 rnonths

$=\frac{\text{EMI}\left[(1+i)^{n-k+1}-1\right]}{i(1+i)^{n-k+1}}=\frac{4593\left[(1.005625)^{120-61+1}-1\right]}{0.005625(1.005625)^{120-61+1}}$

= $\frac{4593(1.4001-1)}{0.005625\times 1.4001}$ = ₹ 233336.89

iii. Interest paid in 61st payment = $\frac{\text{EMI}\left[(1+i)^{n-k+1}-1\right]}{(1+i)^{n-k+1}}$

$=\frac{4593\times 0.4001}{1.4001}$ = ₹1312.52

iv. Principal paid in 61st payment = EMI - Interest paid in 61st period

= ₹ 4593 - ₹ 1312.52 = ₹ 3280.48

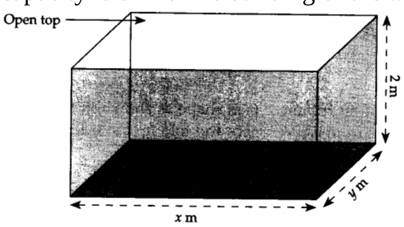
v. Total interest paid = n $\times$ EMI - P

= 120 $\times$ 4593 - 400000 = ₹ 151160.

Section E

36. **Read the text carefully and answer the questions:**

A factory owner wants to construct a tank with rectangular base and rectangular sides, open at the top, so that its depth is 2 m and capacity is 8 m^3. The building of the tank costs ₹280 per square metre for the base and ₹180 per square metre for the sides.



(i) Given volume = 8 m^3 and height = 2 m

So, x $\times$ y $\times$ 2 = 8 $\Rightarrow$ xy = 4

(ii) Area of sides of the tank = 2(x + y) $\times$ h = 4(x + y) m^2

$\therefore$ The cost of construction of the sides of the tank = ₹ 180 $\times$ 4(x + y) = ₹720 (x + y)

The cost of construction of the base of the tank = ₹ x $\times$ y $\times$ 280 = ₹ 4 $\times$ 280 = ₹ 1120 (using part (x $\times$ y $\times$ 2 = 8 $\Rightarrow$ xy = 4))

So, C = ₹ [1120 + 720 (x + y)]

$\Rightarrow$ C = 1120 + 720 $\left(x+\frac{4}{x}\right)$

(iii) $\frac{dC}{dx}$ = = 0 + 720 $\left(1-\frac{4}{x^2}\right)$ and $\frac{d^2C}{dx^2}=\frac{5760}{x^3}$.

Now, $\frac{dC}{dx}$ = 0 $\Rightarrow$ $\left(1-\frac{4}{x^2}\right)$ = 0 $\Rightarrow$ x^2 = 4 $\Rightarrow$ x = 2

$\left[\frac{d^2C}{dx^2}\right]_{x=2}$ = $\frac{5760}{8}$ > 0

$\Rightarrow$ C is minimum when x = 2

OR

The least cost of the tank = ₹ $\left[1120+720\left(2+\frac{4}{2}\right)\right]$

= ₹(1120 + 2880) = ₹ 4000

37. **Read the text carefully and answer the questions:**

An equated monthly installment (EMI) is a set monthly payment provided by a borrower to a creditor on a set day, each month. EMIs apply to both interest and principal each month, and the loan is paid off in full over some years.

How is EMI calculated?

There are two ways in which EMI can be calculated. These methods are:

- **The flat rate method:** When the loan amount is progressively being repaid, each interest charge is computed using the original principal amount in the flat rate method.
- **The reducing balance method:** The reducing balance technique, compared to the flat rate method, determines the interest payment according to the outstanding principal.

Example:

A loan of ₹250000 at the interest rate of 6% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years.

(Given $(1.005)^{60}$ = 1.3489, $(1.005)^{21}$ = 1.1104)

(i) Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 $\times$ 12 = 60

EMI = $\frac{250000\times0.005\times(1.005)^{60}}{(1.005)^{60}-1}$

$=\frac{250000\times0.005\times1.3489}{0.3489}$ = ₹ 4832.69

(ii) Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 $\times$ 12 = 60

Principal outstanding at beginning of 40th month

$=\frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{i(1+i)^{60-40+1}}=\frac{4832.69\times\left[(1.005)^{21}-1\right]}{0.005\times(1.005)^{21}}$

$=\frac{4832.69\times[1.1104-1]}{0.005\times1.1104}$ = $\frac{4832.69\times0.1104}{0.005\times1.1104}$ = ₹ 96096.72

(iii) Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 $\times$ 12 = 60

Interest paid in 40th payment = $\frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{(1+i)^{60-40+1}}$

= $\frac{4832.69\times\left[(1.005)^{21}-1\right]}{(1.005)^{21}}$ = $\frac{4832.69\times0.1104}{1.1104}$ = ₹ 480.48

OR

Given, P = ₹ 250000, i = $\frac{6}{12\times100}$ = 0.005 and n = 5 $\times$ 12 = 60

Principal paid in 40th payment = EMI - Interest paid in 40th payment

= 4832.69 - 480.48 = ₹ 4352.21

38. Here, |A| = -(-4 - 3) - (12 + 1) + 2(9 - 1)

= 7 - 13 + 16 = 10 $\neq$ 0

$$\Rightarrow \text{adj(A)}=\begin{bmatrix}-7&-13&8\\2&-2&2\\3&7&-2\end{bmatrix}^T=\begin{bmatrix}-7&2&3\\-13&-2&7\\8&2&-2\end{bmatrix}$$

Hence $\text{A}^{-1}=\frac{1}{10}\begin{bmatrix}-7&2&3\\-13&-2&7\\8&2&-2\end{bmatrix}$

$$\text{A}A^{-1}=\frac{1}{10}\begin{bmatrix}-1&1&2\\3&-1&1\\-1&3&4\end{bmatrix}\begin{bmatrix}-7&2&3\\-13&-2&7\\8&2&-2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

OR

The matrix equation AX = B is

$$\begin{bmatrix}1&-1&1\\2&1&-3\\1&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\0\\2\end{bmatrix}$$

|A| = 10

$$\text{adj }A=\begin{bmatrix}4&-5&1\\2&0&-2\\2&5&3\end{bmatrix}'=\begin{bmatrix}4&2&2\\-5&0&5\\1&-2&3\end{bmatrix}$$

Here $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Thus, x = 2, y = -1, z = 1