Chapter
11
Fluid Mechanics

Fluid is the name given to a substance which begins to flow when external force is applied on it. Liquids and gases are fluids. Fluids do not have their own shape but take the shape of the containing vessel. The branch of physics which deals with the study of fluids at rest is called hydrostatics and the branch which deals with the study of fluids in motion is called hydrodynamics.

Pressure

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it, is called pressure of liquid or hydrostatic pressure.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is $P = F/A$

(1) Units : N/m^2 or Pascal (S.I.) and Dyne/cm (C.G.S.)

(2) Dimension : $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(3) At a point pressure acts in all directions and a definite direction is not associated with it. So pressure is a tensor quantity.

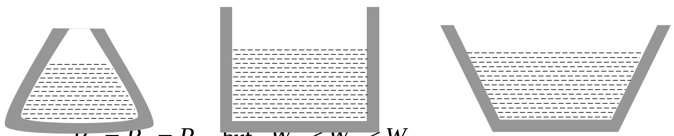
(4) Atmospheric pressure : The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure. Its value on the surface of the earth at sea level is nearly $1.013 \times 10^5 N/m^2$ or Pascal in S.I., other practical units of pressure are atmosphere, bar and torr (*mm* of *Hg*)

$$1atm = 1.01 \times 10^5 Pa = 1.01 \text{bar} = 760 \text{ torr}$$

The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.

(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$

(6) Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.



(A) (B) (C)

$P_A = P_B = P_C$ but $W_A < W_B < W_C$

Fig. 11.2

(7) In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.

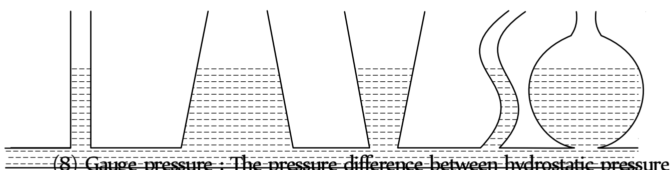


Fig. 11.3

(8) Gauge pressure : The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure.

$$P - P_0 = h\rho g$$

Density

In a fluid, at a point, density ρ is defined as:

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

(1) In case of homogenous isotropic substance, it has no directional properties, so is a scalar.

(2) It has dimensions $[ML^{-3}]$ and S.I. unit *kg/m* while C.G.S. unit *g/cc* with $1g/cc = 10^3 kg/m^3$

(3) Density of substance means the ratio of mass of substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. So for a solid body.

Density of body = Density of substance

While for a hollow body, density of body is lesser than that of substance $[\text{As } V_{\text{body}} > V_{\text{sub.}}]$

(4) When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

(5) Sometimes instead of density we use the term relative density or specific gravity which is defined as :

$$RD = \frac{\text{Density of body}}{\text{Density of water}}$$

(6) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then as

$$m = m_1 + m_2 \text{ and } V = (m_1/\rho_1) + (m_2/\rho_2)$$

$[\text{As } V = m/\rho]$

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\Sigma m_i}{\Sigma(m_i/\rho_i)}$$

If $m_1 = m_2 \quad \rho = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ = Harmonic mean

(7) If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as:

$$m = \rho_1 V_1 + \rho_2 V_2 \text{ and } V = V_1 + V_2 \qquad [\text{As } \rho = m/V]$$

If $V_1 = V_2 = V \qquad \rho = (\rho_1 + \rho_2)/2$ = Arithmetic Mean

(8) With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \qquad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

or $\rho = \dfrac{\rho_0}{(1+\gamma\Delta\theta)} \simeq \rho_0(1-\gamma\Delta\theta)$

(9) With increase in pressure due to decrease in volume, density will increase, *i.e.,*

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \qquad [\text{As } \rho = \frac{m}{V}]$$

But as by definition of bulk-modulus

$$B = -V_0 \frac{\Delta p}{\Delta V} \quad i.e., \; V = V_0\left[1 - \frac{\Delta p}{B}\right]$$

So $\rho = \rho_0\left(1 - \dfrac{\Delta p}{B}\right)^{-1} \simeq \rho_0\left(1 + \dfrac{\Delta p}{B}\right)$

Pascal's Law

It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

or

The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Example : Hydraulic lift, hydraulic press and hydraulic brakes

Working of hydraulic lift : It is used to lift the heavy loads. If a small force *f* is applied on piston of *C* then the pressure exerted on the liquid

$P = f/a \qquad$ [*a* = Area of cross section of the piston in *C*]

This pressure is transmitted equally to piston of cylinder *D*.

Hence the upward force acting on piston of cylinder *D*.

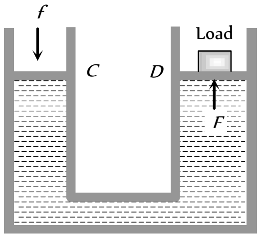


Fig. 11.4

$$F = PA = \frac{f}{a}A = f\left(\frac{A}{a}\right)$$

As $A >> a$, therefore $F >> f$. So heavy load placed on the larger piston is easily lifted upwards by applying a small force.

Archimedes Principle

Accidentally Archimedes discovered that when a body is immersed partly or wholly in a fluid, at rest, it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called Archimedes principle and is a necessary consequence of the laws of fluid statics.

When a body is partly or wholly dipped in a fluid, the fluid exerts force on the body due to hydrostatic pressure. At any small portion of the surface of the body, the force exerted by the fluid is perpendicular to the surface and is equal to the pressure at that point multiplied by the area. The resultant of all these constant forces is called upthrust or buoyancy.

To determine the magnitude and direction of this force consider a body immersed in a fluid of density σ as shown in figure. The forces on the vertical sides of the body will cancel each other. The top surface of the body will experience a downward force.

$$F_1 = AP_1 = A(h_1\sigma g + P_0) \qquad [\text{As } P = h\sigma g + P_0]$$

While the lower face of the body will experience an upward force.

$$F_2 = AP_2 = A(h_2\sigma g + P_0)$$

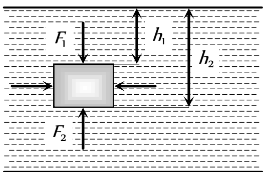


Fig. 11.5

As $h_2 > h_1, F_2$ will be greater than F_1, so the body will experience a net upward force

$$F = F_2 - F_1 = A\sigma g(h_2 - h_1)$$

If *L* is the vertical height of the body $F = A\sigma gL = V\sigma g$

$[\text{As } V = AL = A(h_2 - h_1)]$

i.e., *F* = Weight of fluid displaced by the body.

This force is called upthrust or buoyancy and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy). Though we have derived this result for a body fully submerged in a fluid, it can be shown to hold good for partly submerged bodies or a body in more than one fluid also.

(1) Upthrust is independent of all factors of the body such as its mass, size, density etc. except the volume of the body inside the fluid.

(2) Upthrust depends upon the nature of displaced fluid. This is why upthrust on a fully submerged body is more in sea water than in fresh water because its density is more than fresh water.

(3) Apparent weight of the body of density (ρ) when immersed in a liquid of density (σ).

Apparent weight = Actual weight – Upthrust $= W - F_{up}$

$$= V\rho g - V\sigma g = V(\rho - \sigma)g = V\rho g\left(1 - \frac{\sigma}{\rho}\right)$$

$$\therefore \quad W_{APP} = W\left(1 - \frac{\sigma}{\rho}\right)$$

(4) If a body of volume V is immersed in a liquid of density σ then its weight reduces.

W_1 = Weight of the body in air, W_2 = Weight of the body in water

Then apparent (loss of weight) weight $W_1 - W_2 = V\sigma g$

$$\therefore \quad V = \frac{W_1 - W_2}{\sigma g}$$

(5) Relative density of a body

$$(\text{R.D.}) = \frac{\text{density of body}}{\text{density of water}}$$

$$= \frac{\text{Weight of body}}{\text{Weight of equal volume of water}} = \frac{\text{Weight of body}}{\text{Water thrust}}$$

$$= \frac{\text{Weight of body}}{\text{Loss of weight in water}}$$

$$= \frac{\text{Weight of body in air}}{\text{Weight in air} - \text{weight in water}} = \frac{W_1}{W_1 - W_2}$$

(6) If the loss of weight of a body in water is 'a' while in liquid is 'b'

$$\therefore \quad \frac{\sigma_L}{\sigma_W} = \frac{\text{Upthrust on body in liquid}}{\text{Upthrust on body in water}}$$

$$= \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{a}{b} = \frac{W_{\text{air}} - W_{\text{liquid}}}{W_{\text{air}} - W_{\text{water}}}$$

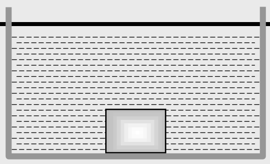
Floatation

(1) **Translatory equilibrium :** When a body of density ρ and volume V is immersed in a liquid of density σ, the forces acting on the body are

Weight of body $W = mg = V\rho g$, acting vertically downwards through centre of gravity of the body.

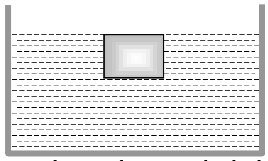
Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

If density of body is greater than that of liquid $\rho > \sigma$



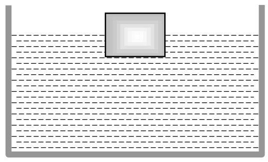
Weight will be more than upthrust so the body will sink

If density of body is equal to that of liquid $\rho = \sigma$



Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium with its top surface in it just at the top of liquid

If density of body is lesser than that of liquid $\rho < \sigma$



Weight will be less than upthrust so the body will, move upwards and in equilibrium will float and partially immersed in the liquid Such that, $W = V_{in}\sigma g \Rightarrow V\rho g = V_{in}\sigma g$

$V\rho = V_{in}\sigma$ Where V_{in} is the volume of body in the liquid

(i) A body will float in liquid only and only if $\rho \le \sigma$

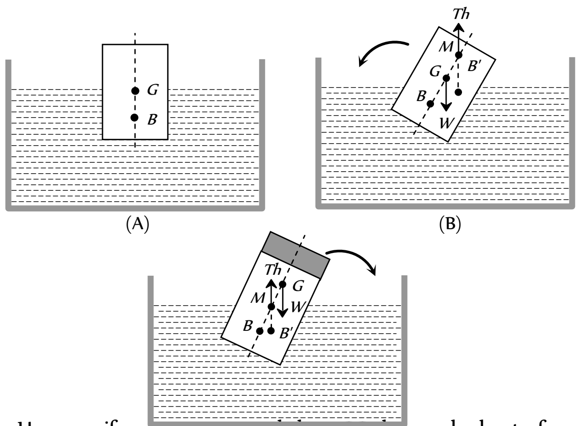
(ii) In case of floating as weight of body = upthrust

So W_{App} = Actual weight – upthrust = 0

(iii) In case of floating $V\rho g = V_{in}\sigma g$

So the equilibrium of floating bodies is unaffected by variations in g though both thrust and weight depend on g.

(2) **Rotatory Equilibrium :** When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta-centre. If the meta-centre M is above the centre of gravity the couple due to forces at G (weight of body W) and at B' (upthrust) tends to bring the body back to its original position. So for rotational equilibrium of floating body the meta-centre must always be higher than the centre of gravity of the body.



(C)

Fig. 11.6

However, if meta-centre goes below CG, the couple due to forces at G and B' tends to topple the floating body.

That is why a wooden log cannot be made to float vertical in water or a boat is likely to capsize if the sitting passengers stand on it. In these situations *CG* becomes higher than *MG* and so the body will topple if slightly tilted.

(3) **Application of floatation**

(i) When a body floats then the weight of body = Upthrust

$$V\rho g = V_{in}\sigma g \;\Rightarrow\; V_{in} = \left(\frac{\rho}{\sigma}\right)V$$

$$\therefore\; V_{\text{out}} = V - V_{in} = \left(1 - \frac{\rho}{\sigma}\right)V$$

i.e., Fraction of volume outside the liquid $f_{\text{out}} = \frac{V_{\text{out}}}{\text{V}} = \left[1 - \frac{\rho}{\sigma}\right]$

(ii) For floatation $V\rho = V_{in}\sigma \;\Rightarrow\; \rho = \frac{V_{in}}{V}\sigma = f_{in}\sigma$

If two different bodies *A* and *B* are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})_A}{(f_{in})_B}$

(iii) If the same body is made to float in different liquids of densities σ_A and σ_B respectively.

$$V\rho = (V_{in})_A\sigma_A = (V_{in})_B\sigma_B \qquad \therefore \qquad \frac{\sigma_A}{\sigma_B} = \frac{(V_{in})_B}{(V_{in})_A}$$

(iv) If a platform of mass *M* and cross-section *A* is floating in a liquid of density σ with its height *h* inside the liquid

$$Mg = hA\sigma g \qquad \text{...(i)}$$

Now if a body of mass *m* is placed on it and the platform sinks by *y* then

$$(M+m)g = (y+h)A\sigma g \qquad \text{...(ii)}$$

Subtracting equation (i) from (ii),

$$mg = A\sigma y g,\ \textit{i.e.,}\ W \propto y \qquad \text{...(iii)}$$

So we can determine the weight of a body by placing it on a floating platform and noting the depression of the platform in the liquid by it.

Streamline, Laminar and Turbulent Flow

(1) **Stream line flow :** Stream line flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceding element passes through that point.

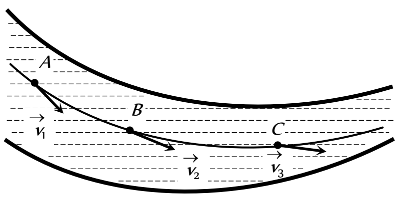


Fig. 11.7

A streamline may be defined as the path, straight or curved, the tangent to which at any point gives the direction of the flow of liquid at that point.

The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

Path *ABC* is streamline as shown in the figure and v_1, v_2 and v_3 are the velocities of the liquid particles at *A*, *B* and *C* point respectively.

(2) **Laminar flow :** If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow.

In this flow, the velocity of liquid flow is always less than the critical velocity of the liquid. The laminar flow is generally used synonymously with streamlined flow.

(3) **Turbulent flow :** When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular. Such a flow is called a turbulent flow.

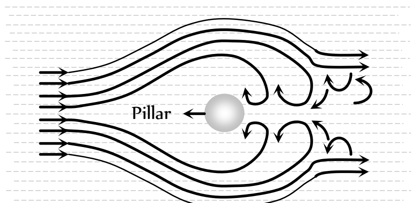


Fig. 11.8

In a turbulent flow, the path and the velocity of the particles of the liquid change continuously and haphazardly with time from point to point. In a turbulent flow, most of the external energy maintaining the flow is spent in producing eddies in the liquid and only a small fraction of energy is available for forward flow. For example, eddies are seen by the sides of the pillars of a river bridge.

Critical Velocity and Reynold's Number

The critical velocity is that velocity of liquid flow upto which its flow is streamlined and above which its flow becomes turbulent.

Reynold's number is a pure number which determines the nature of flow of liquid through a pipe.

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

$$N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}$$

If a liquid of density ρ is flowing through a tube of radius *r* and cross section *A* then mass of liquid flowing through the tube per second $\frac{dm}{dt}$ = volume flowing per second × density = $Av \times \rho$

$\therefore$ Inertial force per unit area = $\frac{dp/dt}{A} = \frac{v(dm/dt)}{A} = \frac{v\,Av\,\rho}{A} = v^2\rho$

Viscous force per unit area $F/A = \frac{\eta v}{r}$

So by the definition of Reynolds number

$$N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{v^2\rho}{\eta v/r} = \frac{v\rho r}{\eta}$$

If the value of Reynold's number

(i) Lies between 0 to 2000, the flow of liquid is streamline or laminar.

(ii) Lies between 2000 to 3000, the flow of liquid is unstable and changing from streamline to turbulent flow.

(iii) Above 3000, the flow of liquid is definitely turbulent.

Equation of Continuity

The equation of continuity is derived from the principle of conservation of mass.

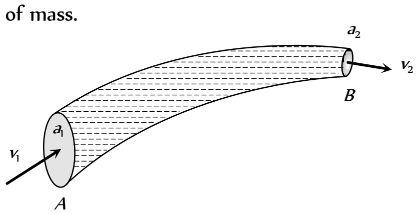


Fig. 11.9

A non-viscous liquid in streamline flow passes through a tube *AB* of varying cross section. Let the cross sectional area of the pipe at points *A* and *B* be a_1 and a_2 respectively. Let the liquid enter with normal velocity v_1 at *A* and leave with velocity v_2 at *B*. Let ρ_1 and ρ_2 be the densities of the liquid at point *A* and *B* respectively.

Mass of the liquid entering per second at *A* = Mass of the liquid leaving per second at *B*

$a_1 v_1 \rho_1 = a_2 v_2 \rho_2$ and $a_1 v_1 = a_2 v_2$

[If the liquid is incompressible $\rho_2 = \rho_1$]

or $av =$ constant or $a \propto \frac{1}{v}$

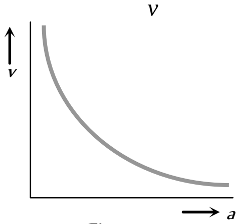


Fig. 11.10

This expression is called the equation of continuity for the steady flow of an incompressible and non-viscous liquid.

(1) The velocity of flow is independent of the liquid (assuming the liquid to be non-viscous)

(2) The velocity of flow will increase if cross-section decreases and vice-versa. That is why :

(a) In hilly region, where the river is narrow and shallow (*i.e.,* small cross-section) the water current will be faster, while in plains where the river is wide and deep (*i.e.*, large cross-section) the current will be slower, and so deep water will appear to be still.

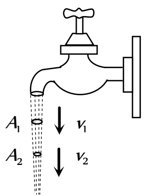


Fig. 11.11

(b) When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (*i.e,* $v_2 > v_1$). So in accordance with continuity equation the cross section of the water stream will decrease (*i.e.,* $A_2 < A_1$), *i.e.,* the falling stream of water becomes narrower.

Energy of a Flowing Fluid

A flowing fluid in motion possesses the following three types of energy

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure without imparting any velocity to it.	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any reference level taken as zero level.	It is the energy possessed by a liquid by virtue of its motion or velocity.
Pressure energy of the liquid *PV*	Potential energy of the liquid *mgh*	Kinetic energy of the liquid $\frac{1}{2}mv^2$
Pressure energy per unit mass of the liquid $\frac{P}{\rho}$	Potential energy per unit mass of the liquid *gh*	Kinetic energy per unit mass of the liquid $\frac{1}{2}v^2$
Pressure energy per unit volume of the liquid *P*	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\frac{1}{2}\rho v^2$

Bernoulli's Theorem

According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.

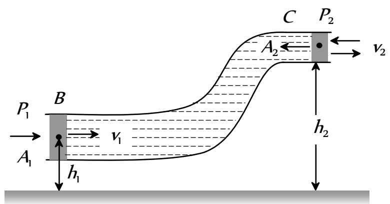


Fig. 11.12

Mathematically for unit volume of liquid flowing through a pipe.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

To prove it, consider a liquid flowing steadily through a tube of non-uniform area of cross-section as shown in fig. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume *V* of incompressible fluid from point B to C through the tube will be

$$W = P_1 V - P_2 V = (P_1 - P_2)V \quad \text{...(i)}$$

This work is used by the fluid in two ways.

(a) In changing the potential energy of mass *m* (in the volume *V*) from mgh_1 to mgh_2

i.e., $\Delta U = mg(h_2 - h_1)$ ...(ii)

(b) In changing the kinetic energy from $\frac{1}{2}mv_1^2$ to $\frac{1}{2}mv_2^2$,

i.e., $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$ ...(iii)

Now as the fluid is non-viscous, by conservation of mechanical energy

$$W = \Delta U + \Delta K$$

i.e., $(P_1 - P_2)V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$

or $P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$ [As $\rho = m/V$]

or $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

or $P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

This equation is the so called Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

(i) Bernoulli's theorem for unit mass of liquid flowing through a pipe can also be written as:

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g}$ = constant

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head. From this equation Bernoulli's theorem can be stated as.

"In stream line flow of an ideal liquid, the sum of pressure head, gravitational head and velocity head of every cross section of the liquid is constant."

Applications of Bernoulli's Theorem

(i) **Attraction between two closely parallel moving boats (or buses)**

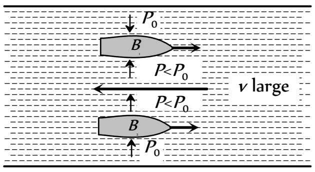


Fig. 11.13

When two boats or buses move side by side in the same direction, the water (or air) in the region between them moves faster than that on the remote sides. Consequently in accordance with *Bernoulli's principle* the pressure between them is reduced and hence due to pressure difference they are pulled towards each other creating the so called attraction.

(ii) **Working of an aeroplane**

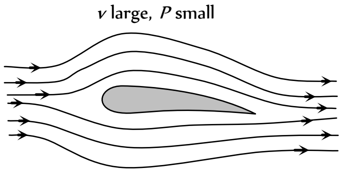


Fig. 11.14

This is also based on Bernoulli's principle. The wings of the aeroplane are of the shape as shown in fig. Due to this specific shape of wings when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' (= pressure difference × area of wing) acts on the plane. If this force becomes greater than the weight of the plane, the plane will rise up.

(iii) **Action of atomiser**

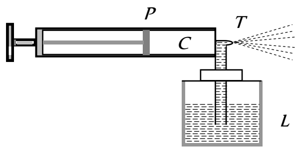


Fig. 11.15

The action of carburetor, paint-gun, scent-spray or insect-sprayer is based on Bernoulli's principle. In all these, by means of motion of a piston P in a cylinder C, high speed air is passed over a tube T dipped in liquid L to be sprayed. High speed air creates low pressure over the tube due to which liquid (paint, scent, insecticide or petrol) rises in it and is then blown off in very small droplets with expelled air.

(iv) **Blowing off roofs by wind storms**

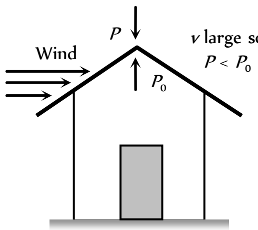


Fig. 11.16

During a tornado or hurricane, when a high speed wind blows over a straw or tin roof, it creates a low pressure (P) in accordance with Bernoulli's principle.

However, the pressure below the roof (*i.e.,* inside the room) is still atmospheric ($= P_0$). So due to this difference of pressure, the roof is lifted up and is then blown off by the wind.

(v) **Magnus effect :** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect and plays as important role in tennis, cricket and soccer, etc. as by applying appropriate spin the moving ball can be made to curve in any desired direction.

If a ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion as shown in fig. then relative to the ball, air will be moving from right to left.

The resultant velocity of air above the ball will be $(v + r\omega)$ while below it $(v - r\omega)$. So in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following the path OA_1 *i.e.,* if a ball is thrown with back-spin, the pitch will curve less sharply prolonging the flight.

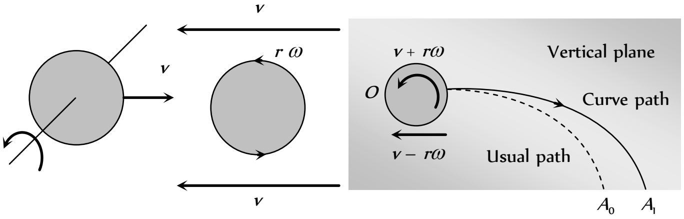


Fig. 11.17

Similarly if the spin is clockwise *i.e.,* the ball is thrown with top-spin, the force due to pressure difference will act in the direction of gravity and so the pitch will curve more sharply shortening the flight.

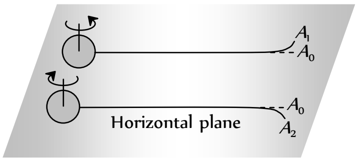


Fig. 11.18

Furthermore, if the ball is spinning about a vertical axis, the curving will be sideways as shown in producing the so called out swing or in swing.

(vi) **Venturimeter :** It is a device based on Bernoulli's theorem used for measuring the rate of flow of liquid through pipes.

It consists of two identical coaxial tubes *A* and *C* connected by a narrow co-axial tube *B*. Two vertical tubes *D* and *E* are mounted on the tubes *A* and *B* to measure the pressure of the flowing liquid.

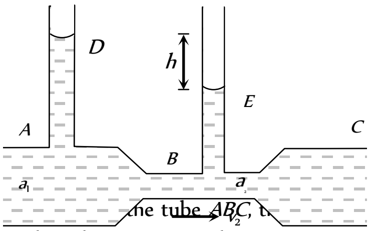


Fig. 11.19

When the ... the tube *ABC*, ... f flow in part *B* will be larger than in the tube *A* or *C*. So the pressure in part *B* will be less than that in tube *A* or *C*. By measuring the pressure difference between *A* and *B*, the rate of flow of the liquid in the tube can be calculated.

Let a_1 and a_2 are area of cross section of tube A and B respectively

v_1, v_2 = Velocity of flow of liquid through A and B respectively

P_1, P_2 = Liquid pressure at A and B respectively

$$\therefore \; P_1 - P_2 = h\rho g \qquad ...(i)$$

[ρ = density of flowing liquid]

From Bernoulli's theorem for horizontal flow of liquid

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad ...(ii)$$

From (i) and (ii) $h\rho g = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho\left[\frac{V^2}{a_2^2} - \frac{V^2}{a_1^2}\right]$

[As $V = a_1 v_1 = a_2 v_2$]

$$\therefore \quad V^2 = \frac{2a_1^2 a_2^2 hg}{a_1^2 - a_2^2} \text{ or } V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$$

Velocity of Efflux

If a liquid is filled in a vessel up to height *H* and a hole is made at a depth *h* below the free surface of the liquid as shown in fig. then taking the level of hole as reference level (*i.e.,* zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get

$$\therefore \qquad (P_0 + h\rho g) + 0 = P_0 + \frac{1}{2}\rho v^2 \text{ or } v = \sqrt{2gh}$$

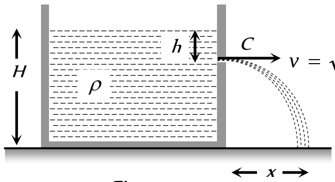


Fig. 11.20

Which is same as the speed that an object would acquire in falling from rest through a distance *h* and is called velocity of efflux or velocity of flow.

This result was first given by Torricelli, so this is known as Torricelli's theorem.

(i) The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

(ii) Greater is the distance of the hole from the free surface of liquid, greater will be the velocity of efflux [*i.e.,* $v \propto \sqrt{h}$]

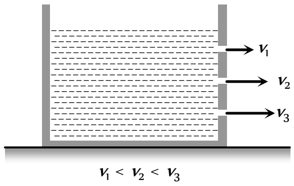


Fig. 11.21

(iii) As the vertical velocity of liquid at the orifice is zero and it is at a height $(H - h)$ from the base, the time taken by the liquid to reach the base-level $t = \sqrt{\frac{2(H-h)}{g}}$

(iv) Now during time *t* liquid is moving horizontally with constant velocity *v*, so it will hit the base level at a horizontal distance *x* (called range) as shown in figure.

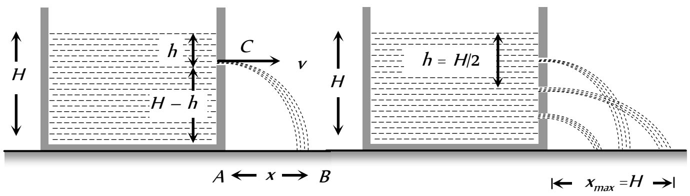


Fig. 11.22

Such that $x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$

For maximum range $\frac{dx}{dh} = 0$

$$\therefore \quad h = \frac{H}{2}$$

i.e., range *x* will be maximum when

$$h = \frac{H}{2}.$$

$\therefore$ Maximum range $x_{max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$

(v)

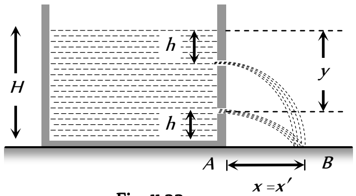


Fig. 11.23

If the level of free surface in a container is at height *H* from the base and there are two holes at depth *h* and *y* below the free surface, then

$x = 2\sqrt{h(H-h)}$ and $x' = 2\sqrt{y(H-y)}$

Now if $x = x'$, *i.e.*, $h(H-h) = y(H-y)$

i.e., $y^2 - Hy + h(H-h) = 0$

or $y = \frac{1}{2}[H \pm (H-2h)]$,

i.e., $y = h$ or $(H-h)$

i.e., the range will be same if the orifice is at a depth h or $(H-h)$ below the free surface. Now as the distance $(H-h)$ from top means $H-(H-h) = h$ from the bottom, so the range is same for liquid coming out of holes at same distance below the top and above the bottom.

(vi)

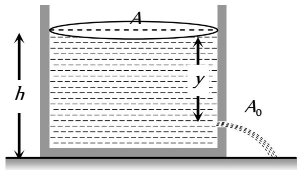


Fig. 11.24

If A_0 is the area of orifice at a depth *y* below the free surface and *A* is that of container, the volume of liquid coming out of the orifice per second will be $(dV/dt) = vA_0 = A_0\sqrt{2gy}$ [As $v = \sqrt{2gy}$]

Due to this, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from *y* to $y - dy$ in time *t* to $t + dt$ then $-dV = A\,dy$

So substituting this value of dV in the above equation

$$-A\frac{dy}{dt} = A_0\sqrt{2gy}$$

i.e., $\int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$

So the time taken for the level to fall from *H to* H'

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H'} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}[\sqrt{H} - \sqrt{H'}]$$

If the hole is at the bottom of the tank, time *t* to make the tank empty :

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$$ [As here $H' = 0$]

Viscosity and Newton's law of Viscous Force.

In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layers in contact, the two layer exert tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called viscous force.

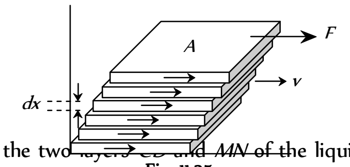


Fig. 11.25

Consider the two layers CD and *MN* of the liquid at distances *x* and *x* + *dx* from the fixed surface *AB*, having the velocities *v* and *v* + *dv* respectively. Then $\frac{dv}{dx}$ denotes the rate of change of velocity with distance and is known as velocity gradient.

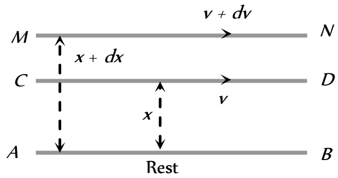


Fig. 11.26

According to Newton's hypothesis, the tangential force *F* acting on a plane parallel layer is proportional to the area of the plane A and the velocity gradient $\frac{dv}{dx}$ in a direction normal to the layer, *i.e.*,

$F \propto A$ and $F \propto \frac{dv}{dx}$

$\therefore F \propto A\frac{dv}{dx}$

or $F = -\eta A\frac{dv}{dx}$

Where η is a constant called the coefficient of viscosity. Negative sign is employed because viscous force acts in a direction opposite to the flow of liquid.

If $A = 1, \frac{dv}{dx} = 1$ then η = *F*.

Hence the coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.

(1) Units : dyne-*s*-*cm* or Poise (C.G.S. system); Newton-*s*-*m* or Poiseuille or decapoise (S.I. system)

1 Poiseuille = 1 decapoise = 10 Poise

(2) Dimension : [*ML T*]

(3) Viscosity of liquid is much greater (about 100 times more) than that of gases i.e. $\eta_L > \eta_G$

Example : Viscosity of water = 0.01 Poise.

While of air = 200 μ Poise

(4) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is practically independent of pressure. The viscosity of water decreases with increase in pressure.

(5) Difference between viscosity and solid friction : Viscosity differs from the solid friction in the respect that the viscous force acting between two layers of the liquid depends upon the area of the layers, the relative velocity of two layers and distance between two layers, but the friction between two solid surfaces is independent of the area of surfaces in contact and the relative velocity between them.

(6) From kinetic theory point of view viscosity represents transport of momentum, while diffusion and conduction represents transport of mass and energy respectively.

(7) The viscosity of thick liquids like honey, glycerin, coaltar *etc.* is more than that of thin liquids like water.

(8) The cause of viscosity in liquids is cohesive forces among molecules where as in gases, it is due to diffusion.

(9) The viscosity of gases increases with increase of temperature, because on increasing temperature the rate of diffusion increases.

(10) The viscosity of liquid decreases with increase of temperature, because the cohesive force between the liquid molecules decreases with increase of temperature

Relation between coefficient of viscosity and temperature; Andrade formula $\eta = \dfrac{A e^{C\rho/T}}{\rho^{-1/3}}$

Where T= Absolute temperature of liquid, ρ = density of liquid, A and C are constants.

Stoke's Law and Terminal Velocity

When a body moves through a fluid, the fluid in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$F = 6\pi\eta r v$

This law is called Stokes law.

If a spherical body of radius r is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity called terminal velocity.

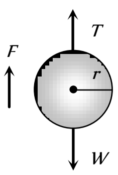


Fig. 11.27

Force on the body

(i) Weight of the body (W) = mg

= (*volume* × *density*) × $g = \frac{4}{3}\pi r^3 \rho g$

(ii) Upward thrust (T) = weight of the fluid displaced

= (*volume* × *density*) of the fluid × $g = \frac{4}{3}\pi r^3 \sigma g$

(iii) Viscous force (F) = $6\pi\eta r v$

When the body attains terminal velocity the net force acting on the body is zero. $\therefore$ $W - T - F = 0$ or $F = W - T$

$$\Rightarrow\ 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$\therefore\ \text{Terminal velocity}\quad v = \frac{2}{9}\frac{r^2(\rho - \sigma) g}{\eta}$$

(i) Terminal velocity depend on the radius of the sphere so if radius is made n - fold, terminal velocity will become n times.

(ii) Greater the density of solid greater will be the terminal velocity

(iii) Greater the density and viscosity of the fluid lesser will be the terminal velocity.

(iv) If $\rho > \sigma$ then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.

(v) If $\rho < \sigma$ then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction. Example : Air bubble in a liquid and clouds in sky.

(vi) Terminal velocity graph :

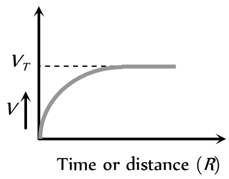


Fig. 11.28

Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l' and radius r, then the volume of liquid coming out of the tube per second is

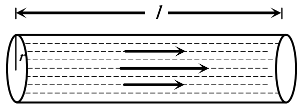


Fig. 11.29

(i) Directly proportional to the pressure difference (P).

(ii) Directly proportional to the fourth power of radius (r) of the capillary tube

(iii) Inversely proportional to the coefficient of viscosity (η) of the liquid.

(iv) Inversely proportional to the length (l) of the capillary tube.

i.e. $V \propto \dfrac{P r^4}{\eta l}$ or $V = \dfrac{K P r^4}{\eta l}$

$$\therefore\ V = \frac{\pi P r^4}{8\eta l}$$

[Where $K = \dfrac{\pi}{8}$ is the constant of proportionality]

This is known as Poiseuille's equation.

This equation also can be written as,

$V = \dfrac{P}{R}$ where $R = \dfrac{8\eta l}{\pi r^4}$

R is called as liquid resistance.

(1) **Series combination of tubes**

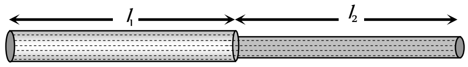


Fig. 11.30

(i) When two tubes of length l_1, l_2 and radii r_1, r_2 are connected in series across a pressure difference P,

Then $P = P_1 + P_2$ ...(i)

Where P_1 and P_2 are the pressure difference across the first and second tube respectively

(ii) The volume of liquid flowing through both the tubes *i.e.* rate of flow of liquid is same.

Therefore $V = V_1 = V_2$

i.e., $V = \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}$ ...(ii)

Substituting the value of P_1 and P_2 from equation (ii) to equation (i) we get

$$P = P_1 + P_2 = V\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]$$

$$\therefore\ V = \frac{P}{\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]} = \frac{P}{R_1 + R_2} = \frac{P}{R_{eff}}$$

Where R_1 and R_2 are the liquid resistance in tubes

(iii) Effective liquid resistance in series combination $R_{eff} = R_1 + R_2$

(2) **Parallel combination of tubes**

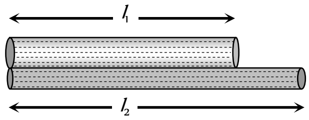


Fig. 11.31

(i) $P = P_1 = P_2$

(ii) $V = V_1 + V_2 = \frac{P\pi r_1^4}{8\eta l_1} + \frac{P\pi r_2^4}{8\eta l_2}$

$$= P\left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right]$$

$$\therefore\ V = P\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{P}{R_{eff}}$$

(iii) Effective liquid resistance in parallel combination

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

Tips & Tricks

✍ When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere.

✍ In a liquid, the pressure is same at the same horizontal level.

✍ Pressure at any point is same in all directions.

✍ The pressure is perpendicular to the surface of the fluid.

✍ The pressure at any point in the liquid depends on depth (h) below the surface, density of liquid and acceleration due to gravity.

It is independent of the shape of the containing vessel, or total mass of the liquid.

✍ Force is a vector quantity but pressure is a tensor quantity

✍ Pressure and density play the same role in case of fluids as force and mass play in case of solids.

✍ Bar and millibar are commonly used units for pressure in meteorology.

✍ Sudden fall in atmospheric pressure predicts possibility of a storm.

✍ Water barometer was constructed in 17th century by Von Guericke and fixed on the outside wall of his house. With the help of this barometer Von Guericke made the first recorded scientific weather forecast. He correctly predicted the severe storm after noting a sudden fall in the height of the water column.

✍ The specific gravity is also known as relative density. Thus, S.G. of a substance $= \frac{\text{density of the substance}}{\text{density of water (at } 4°C)}$

✍ If the specific gravity of the material of a body is x, then its density is

(I) $x\ g\ cm^{-3}$ in C.G.S.

(Ii) $x \times 10^3\ kg\ m^{-3}$ in SI.

✍ The number of moles in a sample of any substance containing N molecules is given by

$$\mu = \frac{N}{N_A}$$

✍ The force between atoms and molecules is electrical in nature. However, it does not obey inverse square law.

✍ If two liquids of masses m_1, m_2 and densities ρ_1, ρ_2 are mixed together, then the density of the mixture is given by

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

✍ If two liquids of same mass but different densities are mixed together, then the density of the mixture is harmonic mean of the densities. That is

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \text{ or } \frac{1}{\rho} = \frac{1}{2}\left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right]$$

✍ It two drops of same volume but different densities are mixed together, then the density of the mixture is arithmetic mean of the densities. That is $\rho = \frac{\rho_1 + \rho_2}{2}$

✍ The density of the liquid changes with pressure as follows : $\rho = \rho_0 \left[1 + \frac{\Delta p}{B}\right]$ where $\Delta p =$ change in pressure and B is the bulk modulus.

✍ The density of a liquid of bulk modules B at depth h is given $\rho_d = \rho_0 \left[1 + \frac{h\rho g}{B}\right]$

Where ρ is the average density of the liquid.

✍ The hydrometer can be used to measure density of the liquid or fluid.

✍ If a vessel contains liquid upto a height H and it has a hole in the side at a height h, then the velocity of efflux is $v = \sqrt{2g(H-h)}$. The time taken by the liquid to reach the ground level is $t = \sqrt{2h/g}$. Horizontal range of the liquid $R = 2[h(H-h)]^{1/2}$. The range is same for the hole at a height h above the bottom or at the depth h below the surface of the liquid.

The range is maximum for $h = H/2$. It is given by :

$$R_{\max} = 2\left[\left(\frac{H}{2}\right)\left(H - \frac{H}{2}\right)\right]^{1/2} = H$$

✍ The cross-section of the water stream from a tap decreases as it goes down in accordance with the equation of continuity.

✍ The upthrust on body immersed in a liquid does not depend on the mass, density or shape of the body. It only depends on the volume of the body.

✍ The weight of the plastic bag full of air is same as that of the empty bag because the upthrust is equal to the weight of the air enclosed.

✍ Upthrust depends on the density of the fluid, not the density of the body.

✍ If two bodies have equal upthrust in a liquid, both have the same volume.

✍ When air blows over a roof, the force on the roof is upwards.

✍ If one floats one's back on the surface of water, the apparent weight is zero.

✍ If a body just floats in liquid (density of the body is equal to the density of liquid) then the body sinks if it is pushed downwards.

✍ The line joining the centre of gravity and centre of buoyancy is called central line.

✍ The point where the vertical line through centre of huoyancy intersects the central line is called metacentre.

✍ The floating body is in stable equilibrium where the metacentre is above the centre of gravity. (Centre of gravity is below the centre of buoyancy)

✍ The floating body is in unstable equilibrium when the metancetre lies below the centre of gravity. (Centre of gravity is above the centre of buoyancy).

✍ The floating body is in the neutral equilibrium when centre of gravity coincides with the metacentre. (Centre of gravity coincides with the centre of buoyancy).

✍ The wooden rod cannot float vertically in a pond of water because centre of gravity lies above the metacentre.

✍ Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

✍ The faster the air, the lower the pressure.

✍ Wings of an aeroplane are shaped to make air travel further and faster over their top surfaces.

✍ The lift force on a wing or aerofoil is proportional to the square of the speed of flow.

✍ Viscous force between the layers of a liquid is analogous to friction between two solid surfaces.

✍ With increase in temperature, the coefficient of viscosity of liquids decreases but that of gases increases. The reason is that as temperature rises, the atoms of the liquid become more mobile, whereas in case of a gas, the collision frequency of atoms increases as their motion becomes more random.

✍ We cannot sip a drink with a straw on the moon because there is no atmosphere on the moon.