JEE MAIN 2024

Sample Paper - 3

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

- All questions are compulsory.
- There are three parts and each part carries 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer. However, there is no negative marking for Section-B (Numerical Questions)

PHYSICS (Section-A)

1. The length, breadth and thickness of a block are given by l = 12 cm, h = 6 cm and t = 2.45 cm. The volume of the block according to the idea of significant figures should be: **[4]**

   a) 1×10^2 cm^3 b) 1.763×10^2 cm^3

   c) 2×10^2 cm^3 d) 3×10^2 cm^3

2. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is: **[4]**

   a) 5.0 m/s b) 4.8 m/s

   c) 5.5 m/s d) 4.0 m/s

3. A body is projected at an angle of 30^o with the horizontal and with a speed of 30 ms^{-1}. What is the angle with the horizontal after 1.5 seconds? (Take g = 10ms^{-2}) **[4]**

   a) 60^o b) 30^o

   c) 90^o d) 0^o

4. A particle is moving in a vertical circle. The tension in the string when passing through two positions at an angle of 30° and 60° from vertical (the lowest position) is T_1 and T_2 respectively, then: **[4]**

   i. $T_1 = T_2$

   ii. $T_1 > T_2$

   iii. $T_1 < T_2$

   iv. tension in the string always remains the same

   a) only ii b) iii and iv

c) i and ii

d) iv and i

5. A particle of mass m is moving with speed 2v and collides with a mass 2m moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be **[4]**

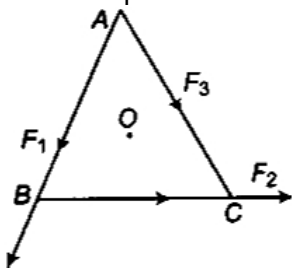
a) $\frac{v}{(2\sqrt{2})}$

b) $\sqrt{2}v$

c) $\frac{v}{\sqrt{2}}$

d) $2\sqrt{2}v$

6. O is the centre of an equilateral triangle ABC. F_1, F_2 and F_3 are three forces acting along the sides AB, BC and AC as shown here. What should be the magnitude of F_3, so that the total torque about 0 is Zero? **[4]**



a) $\frac{(F_1+F_2)}{2}$

b) $2(F_1 + F_2)$

c) $(F_1 + F_2)$

d) $(F_1 - F_2)$

7. Pressure is a scalar quantity because **[4]**

a) it is the ratio of the component of the force normal to the area

b) it depends on the size of the area chosen

c) it is the ratio of force to the area and both force and area are vectors

d) it is the ratio of the magnitude of the force to the area

8. A glass flask of volume one litre at 0^oC is filled, level full of mercury at this temperature. The flask and mercury are now heated to 100^oC. How much mercury will spill out if coefficient of volume expansion of mercury is $1.82 \times 10^{-4}/^oC$ and linear expansion of glass is $0.1 \times 10^{-4}/^oC$ respectively? **[4]**

a) 1.52 cc

b) 15.2 cc

c) 2.12 cc

d) 21.2 cc

9. The volume of an ideal diatomic gas is doubled isothermally. The internal energy: **[4]**

a) increases four times

b) doubles

c) is halved

d) remains unchanged

10. Motion of an oscillating liquid column in a U-tube is: **[4]**

a) periodic but not simple harmonic

b) non-periodic

c) simple harmonic and time period are directly proportional to the density of the liquid

d) simple harmonic and time period are independent of the density of the liquid

11. Given a number of capacitors labelled as 8 pF, 250 V. Find the minimum number of capacitors needed to get an arrangement equivalent to 16 pF, 1000 V. **[4]**

a) 32

b) 16

c) 4

d) 64

12. Two long parallel wires carry currents i_1 and i_2 such that $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 6×10^{-6} T. If the direction of i_2 is reversed, the field becomes 3×10^{-5} T. The ratio of $\frac{i_1}{i_2}$ is : **[4]**

a) 2

b) $\frac{2}{3}$

c) $\frac{3}{2}$

d) $\frac{1}{2}$

13. A bar magnet is 10 cm long and is kept with its north (N)-pole pointing north. A neutral point is formed at a distance of 15 cm from each pole. Given the horizontal component of the earth's field to be 0.4 Gauss, the pole strength of the magnet is: **[4]**

a) 9 A-m

b) 27 A-m

c) 1.35 A-m

d) 6.75 A-m

14. A circular coil of mean radius of 7 cm and having 4000 turns is rotated at the rate of 1800 revolutions per minute in the earth's magnetic field (B = 0.5 gauss), the maximum emf induced in coil will be: **[4]**

a) 1.158 V

b) 0.29 V

c) 5.8 V

d) 0.58 V

15. A transformer has 100 turns in the primary coil and carries 8 A current. If the input power is one kilowatt, the number of turns in the secondary coil to have 500 V output will be: **[4]**

a) 400

b) 300

c) 100

d) 200

16. For plane electromagnetic waves propagating in the z-direction, which one of the following combination gives the correct possible direction for $\vec{E}$ and $\vec{B}$ field respectively? **[4]**

a) $(2\hat{i} + 3\hat{j})$ and $(\hat{i} + 2\hat{j})$

b) $(3\hat{i} + 4\hat{j})$ and $(4\hat{i} - 3\hat{j})$

c) $(\hat{i} + 2\hat{j})$ and $(2\hat{i} - \hat{j})$

d) $(-2\hat{i} - 3\hat{j})$ and $(3\hat{i} - 2\hat{j})$

17. The maximum kinetic energy of the emitted photoelectrons depends upon: **[4]**

a) the voltage applied between the electrodes of the photocell

b) the intensity of incident light

c) the velocity of an incident light photon

d) the frequency of the incident light

18. The ratio of minimum to a maximum wavelength of radiation that an electron in the ground state can cause in a Bohr's hydrogen atom is: **[4]**

a) $\frac{27}{32}$

b) zero

c) $\frac{1}{2}$

d) $\frac{3}{4}$

19. Two nuclei have their mass numbers in the ratio of 1 : 3. The ratio of their nuclear densities would be: **[4]**

a) 3 : 1

b) 1 : 3

c) 1 : 1

d) $(3)^{1/3} : 1$

20. Gallium Arsenide phosphide LED emits light radiation of wavelength about: **[4]**
(Given: E_g of GaAsP LED = 1.9 eV)

a) 3533 $\overset{o}{A}$

b) 4533 $\overset{o}{A}$

c) 5533 $\overset{o}{A}$

d) 6533 $\overset{o}{A}$

PHYSICS (Section-B)

Attempt any 5 questions

21. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is : **[4]**

22. A particle of mass 2g and charge $1\mu C$ is held at a distance of 1m from a fixed charge 1mC. If the particle is released it will be repelled. The speed of particle when it is at a distance of 10 metre from the fixed charge, is(m/sec) **[4]**

23. Two concentric circular coils with radii 1 cm and 1000 cm, and number of turns 10 and 200 respectively are placed coaxially with centers coinciding. The mutual inductance of this arrangement will be _____ $\times 10^{-8}$ H. (Take, π^2 = 10) **[4]**

24. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in kilometer/s _____. **[4]**

25. Two identical piano wires, kept under the same tension T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to **[4]**

occurrence of 6 beats/s when both the wires oscillate together would be _____.

26. A point source of light is placed 4 m below the surface of water of refractive index $\frac{5}{3}$. The minimum diameter of a disc, which should be placed over the source, on the surface of water to cut-off all light coming out of water is _____ m. **[4]**

27. The ratio of magnetic field at the centre of a current carrying coil of radius r to the magnetic field at distance r from the centre of coil on its axis is $\sqrt{x}$: 1. The value of x is _____. **[4]**

28. A string of mass 0.2 kg/m and length L = 0.6 m is fixed at both ends and stretched such that it has a tension of 80 N. The string is vibrating in its third normal mode and has an amplitude of 0.5 cm. Determine the frequency of waves. **[4]**

29. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be _____ nR, where n denotes number of moles of a gas. **[4]**

30. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in the space station is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of a crosssection of the wire = 10^{-2} cm^2) is _____. **[4]**

CHEMISTRY (Section-A)

31. Which one of the following pair of atoms/atom-ion have identical ground state configuration? **[4]**

a) Li^+ and He^+ b) F^+ and Ne

c) Cl^- and Ar d) Na^+ and K^+

32. Which of the following does not represent the correct order of the properties indicated? **[4]**

A. F > Cl > Br > I (EN)

B. $Sc^{3+} > Ti^{2+} > Cr^{1+} >$ Mn (size)

C. O < S > Se > Te (EA)

D. $Fe^{2+} > Co^{2+} > Ni^{2+} > Cu^{2+}$ (unpaired electron)

a) B only b) C only

c) D only d) A only

33. Which of the ions does **not** exist? **[4]**

A. PCl_6^-

B. NH_4^+

C. PBr_6^-

D. I_3^-

a) A b) B

c) D

d) C

34. Temperature and heat are: **[4]**

a) extensive properties

b) extensive and intensive properties respectively

c) intensive properties

d) intensive and extensive properties respectively

35. From separate solutions of four sodium salts NaW, NaX, NaY and NaZ had pH 7.0, 9.0, 10.0 and 11.0 respectively. When each solution was 0.1 M, the strongest acid is: **[4]**

a) HW

b) HZ

c) HX

d) HY

36. Which metal exhibits more than one oxidation states? **[4]**

a) Fe

b) Mg

c) Al

d) Na

37. Due to inert pair effect, _____ decreases from Ga to Tl. **[4]**

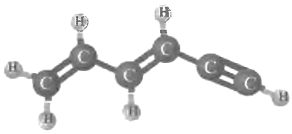
a) density

b) melting point

c) metallic character

d) electronegativity

38. The number of C - C σ bonds and C - C π bonds in the molecule are respectively. **[4]**



a) 5 and 3

b) 6 and 4

c) 5 and 4

d) 6 and 3

39. The intermediate formed when but-1-ene reacts with HBr is _____. **[4]**

a) $\mathrm{CH_3 - CH_2 - CH_2 - \dot{C}H_2}$

b) $\mathrm{CH_3 - CH_2 - CH_2 - \overset{+}{C}H_2}$

c) $\mathrm{CH_3 - CH_2 - \overset{+}{C}H - CH_3}$

d) $\mathrm{CH_3 - CH_2 - \dot{C}H_2 - CH_3}$

40. What is the freezing point of 0.06 molal aqueous solution of a non-volatile and non-electrolyte solute? (K_f = 1.86 K kg mol^{-1}) **[4]**

a) -1.86^oC

b) -0.93^oC

c) 0.112^oC

d) -0.112^oC

41. What is the difference in boiling point of solutions (i) and (ii)? (K_b for water = 0.52 K kg mol^{-1}) **[4]**

i. 40 g of glucose dissolved in 250 g of water?

ii. 25 g of fructose dissolved in 180 g of water?

a) 0.061^oC

b) 0.042^oC

c) 0.088^oC

d) 0.051^oC

42. The rusting of iron-pipe exposed to atmosphere follows the reaction, $2Fe(s) + 3O_2(g) \longrightarrow Fe_2O_3(s)$. This is a: **[4]**

a) may be spontaneous or non-spontaneous depending on conditions

b) spontaneous reaction

c) non-spontaneous reaction

d) neither spontaneous reaction nor non-spontaneous reaction

43. In the following reaction; $xA \rightarrow yB$ **[4]**

$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$

A and B respectively can be

a) N_2O_4 and NO_2

b) C_2H_4 and C_4H_8

c) C_2H_2 and C_6H_6

d) n-butane and iso-butane

44. Which of the following are not postulates of Werner? **[4]**

P. In co-ordination compounds, metals show two types of linkage (valencies) primary and secondary.

Q. The primary valencies are normally ionizable and are satisfied by negative ions.

R. The secondary valencies are non-ionisable. These are satisfied by neutral molecules or negative ions. The secondary valency is equal to the co-ordination number and is fixed for a metal.

S. The ions/groups bound by the primary and secondary linkages to the metal have characteristic spatial arrangements corresponding to different coordination numbers.

T. Octahedral, Tetrahedral and Square planar geometrical shapes are more common in coordination compounds of transition metals.

a) (P), (Q) and (R)

b) (P), (Q), (R) and (S)

c) (S) and (T)

d) (S)

45. Which is the correct order for m.pt.? **[4]**

a) $(CH_3)_3N > NH_3 > PH$

b) $PH_3 > (CH_3)_3N > NH_3$

c) $NH_3 > (CH_3)_3N > PH_3$

d) $NH_3 > PH_3 > (CH_3)_3N$

46. Shape of $Fe(CO)_5$ is: **[4]**

a) square planar

b) trigonal bipyramidal

c) octahedral

d) square pyramidal

47. How many stereoisomers does this molecule have $CH_3CH{=}CHCH_2CHBrCH_3$? **[4]**

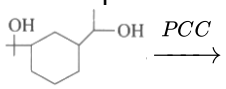
a) 4

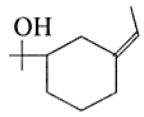
b) 2

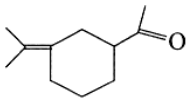
c) 8

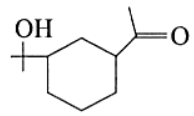
d) 6

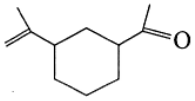
48. What is product of the following reaction? **[4]**

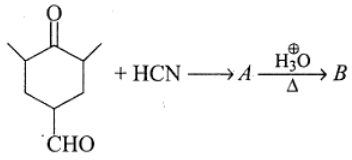


a) 

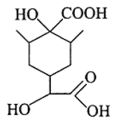
b) 

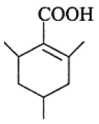
c) 

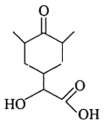
d) 

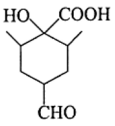
49.  **[4]**

The end product B is:

a) 

b) 

c) 

d) 

50. Which of the following reagents would not be a good choice for reducing an aryl nitro compound to an amine? **[4]**

a) Fe and HCl

b) $LiAlH_4$ in ether

c) Sn and HCl

d) H_2(excess)/Pt

CHEMISTRY (Section-B)

Attempt any 5 questions

51. The orbital angular momentum of an electron in 3s orbital is $\frac{xh}{2\pi}$. The value of x is _____. **[4]**

52. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _____ $\times 10^{-3}$ (Nearest integer) **[4]**
[Given: $pK_b(NH_3)$ = 4.745, log 2 = 0.301, log 3 = 0.477, T = 298K]

53. Maximum number of the nodal plane of π-bond(s) which are in same plane in C_3O_2 (carbon suboxide) is: **[4]**

54. Total number of enthalpy(s) (out of given nine) of M(g) which is/are not associated with conversion of **[4]**
$M^-_{(g)} \longrightarrow M^{4+}_{(g)}$
$I.E._1$, $I.E._2$, $I.E._3$,$I.E._4$, $LE._5$, $I.E._6$, EGE_1, EGE_2, EGE_3
(I. E. = ionization energy, EGE = electron gain enthalpy)

55. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d - d transition. The octahedral splitting energy for the above complex is _____ $\times 10^{-19}$J. (Round off to the nearest integer). h = 6.626×10^{-34} Js; c = 3×10^8 ms^{-1}. **[4]**

56. In the ground state of atomic Fe(Z = 26), the spin-only magnetic moment is _____ $\times 10^{-1}$ BM. (Round off to the nearest integer). **[4]**
[Given: $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

57. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is _____. **[4]**

58. The number of given orbitals which have electron density along the axis is _____. **[4]**
P_x, P_y, P_z, d_{xy}, d_{yz}, d_{xz}, d_{z^2}, $d_{x^2-y^2}$

59. Find number of possible plane(s) in allene (H_2CCCH_2) which contain maximum possible atoms. **[4]**

60. The average S-F bond energy in kJ mol^{-1} of SF_6 is _____. (Rounded off to the nearest integer) **[4]**
[Given: The values of standard enthalpy of formation of SF_6(g), S(g) and F(g) are -1100, 275 and 80 kJ mol^{-1} respectively.]

MATHEMATICS (Section-A)

61. A function f : R $\rightarrow$ R satisfies f(x + y) = f(xy) for all x, y and $f\left(-\frac{1}{2}\right) = -\frac{1}{2}$, then f(2017) + f(2018) equals **[4]**

a) 1 b) -1

c) $-\frac{4035}{2}$ d) 4035

62. If α, β are the roots of the equation $x^2 - \left(5 + 3\sqrt{\log_3 5} - 5\sqrt{\log_5 3}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log 3)^{\frac{2}{3}}} - 1\right) = 0$ **[4]**

Then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$,

a) $3x^2 - 20x + 16 = 0$

b) $3x^2 - 10x - 4 = 0$

c) $3x^2 - 20x - 12 = 0$

d) $3x^2 - 10x + 2 = 0$

63. How many 4-digit even numbers are possible from 1, 2, 3, 4, 5, 6, 7, 8, 9 without repeating a digit? **[4]**

a) 362880

b) 1344

c) 3024

d) 504

64. The sum of the coefficients of all even degree terms is x in the expansion of $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6, (x > 1)$ is equal to **[4]**

a) 32

b) 29

c) 26

d) 24

65. Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is: **[4]**

a) $n^2 + 4n$

b) $3n + 2n^2$

c) $6n^2 - n$

d) $n + 4n^2$

66. Let f: R $\rightarrow$ R be a function such that $|f(x)| \leq x^2$, for all x $\in$ R. Then, at x = 0, f is: **[4]**

a) differentiable but not continuous.

b) continuous as well as differentiable.

c) neither continuous nor differentiable.

d) continuous but not differentiable.

67. A cylindrical container has a capacity of 16π cm^3. The minimum amount of material including the lid, in sq. units, is: **[4]**

a) 96π

b) 48π

c) 16π

d) 24π

68. If the tangent to the curve $y = 1 - x^2$ at $x = \alpha$, where $0 < \alpha < 1$, meets the axes at P and Q. Also α varies, the minimum value of the area of the triangle OPQ is k times the area bounded by the axes and the part of the curve for which 0 < x < 1, then k is equal to : **[4]**

a) $\frac{2}{3}$

b) $\frac{75}{16}$

c) $\frac{2}{\sqrt{3}}$

d) $\frac{25}{18}$

69. If the image of point P(2, 3) in a line L is Q(4, 5), then the image of point R(0, 0) in the same line is: **[4]**

a) (2, 2) b) (7, 7)

c) (4, 5) d) (3, 4)

70. If the area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq units, then c is equal to **[4]**

a) 20 b) -25

c) 25 d) 13

71. Sum of slopes of two normals (other than axis) to the parabola y^2 = 4x which are tangents to x^2 = -12y is: **[4]**

a) 3 b) -3

c) 2 d) $\frac{3}{2}$

72. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$ is: **[4]**

a) 9 b) $\frac{3}{4}$

c) 4 d) $\frac{1}{3}$

73. If 1, 2, 2 and 2, 1, 1 are the direction ratios of two lines and Q is the angle between the lines such that sin 2θ = $\frac{\lambda}{\sqrt{2}}$ for a scalar λ, then λ is: **[4]**

a) $\frac{3}{4}$ b) $\frac{4}{3}$

c) $\frac{4}{\sqrt{3}}$ d) $\frac{2}{3}$

74. Let p, q, r be distinct. If $\begin{vmatrix} p & p^2 & 1+p^3 \\ q & q^2 & 1+q^3 \\ r & r^2 & 1+r^3 \end{vmatrix} = 0$ and the vectors $\overrightarrow{OP}$ = (1, p, p^2), $\overrightarrow{OQ}$ = (1, q, q^2), $\overrightarrow{OR}$ = (1, r, r^2) are non-coplanar, then the value of pqr is **[4]**

a) 1 b) -1

c) 0 d) 2

75. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}$- d each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}$+ d each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals: **[4]**

a) $\frac{2}{3}$ b) $\sqrt{2}$

c) $\frac{\sqrt{5}}{2}$ d) 2

76. Let p = $x_1 x_2 x_3 x_4 x_5$, $x_i \in N$. The probability that p is divisible by 5 or 10 is: **[4]**

a) $\frac{1024}{3125}$ b) $\frac{2101}{3125}$

c) $\frac{16}{25}$ d) $\frac{4}{5}$

77. sin 36° sin 72°sin 108° sin 144° = **[4]**

a) $\frac{3}{4}$ b) $\frac{1}{4}$

c) $\frac{1}{16}$ d) $\frac{5}{16}$

78. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is **[4]**

a) $\frac{13}{8}$ b) $\frac{13}{12}$

c) 2 d) $\frac{13}{6}$

79. Which one of the following is correct? **[4]**

a) A - (B ∩ C) = (A ∩ B') ∩ C) b) A ∪ (B − C) = A ∩ (B ∩ C')

c) A - (B ∪ C) = (A ∩ B') ∩ C') d) A ∩ (B - C) = (A ∩ B) ∩ C)

80. If the system of equations x + y + z = 5, x + 2y + 3z = 9, x + 3y + az = β has infinitely many solutions, then β - α equals **[4]**

a) 8 b) 5

c) 21 d) 18

MATHEMATICS (Section-B)

Attempt any 5 questions

81. The number of distinct real roots of **[4]**
$x^4 - 4x^2 + 12x^2 + x - 1 = 0$ is ....

82. Let k and m be positive real numbers such that the function **[4]**
$f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to _____.

83. The scalar $\vec{A} \cdot [(\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})]$ equals _____. **[4]**

84. Let f(x) be a polynomial of degree 3. If the curve y = f(x) has relative extrema at x = $\frac{\pm 2}{\sqrt{3}}$ and passes through (0, 0) and (1,-2) dividing the circle $x^2 + y^2 = 4$ in two parts, then the area bounded by $x^2 + y^2 = 4$ and $y \geq f(x)$ is $\frac{k\pi}{2}$. Find the value of k. **[4]**

85. Consider a triangle ABC whose vertices are A(0, α, α), B(α, 0, α) and C(α, α, 0), $\alpha > 0$. Let D be a point moving on the line x + z - 3 = 0 = y and G be the centroid of $\triangle$ABC. If the minimum length of GD is $\sqrt{\frac{57}{2}}$, then α is equal to _____. **[4]**

86. Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of 14p is _____. **[4]**

87. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to _____. **[4]**

88. If twice of square of diameter of circumcircle of $\triangle$ABC is equal to sum of square of its sides then find the value of (4 + cos2 A + cos2 B + cos2 C). **[4]**

89. The number of matrices A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d $\in$ {-1, 0, 1, 2, 3,..., 10} such that A = A^{-1}, is _____. **[4]**

90. Let f(x) be a polynomial of degree 3 such that f(k) = $-\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of 52 - 10 f(10) is equal to _____. **[4]**

JEE MAIN 2024

Sample Paper - 3

Solution

PHYSICS (Section-A)

1.

(c) 2×10^2 cm^3

Explanation: Dimensions of the block ,

Length (l) = 12 cm

Breadth (b) = 6 cm

Height (h) = 2.45 cm

volume of the block = lbh

V = $12 \times 6 \times 2.45$

V = 176.4 cm^3

V = $176.4 \times (10^2 \times 10^2)$ cm^3

V = 1.764×10^2 cm^3

V = 2×10^2 cm^3

2.

(d) 4.0 m/s

Explanation: $t_1 = \frac{x/2}{3} = \frac{x}{6}$

$x_1 = 4.5t_2$, $x_2 = 7.5t_2$

Also, the other half distance is :- $x_1 + x_2 = \frac{x}{2} = (4.5 + 7.5)t_2$

That is $t_2 = \frac{x}{24}$

$t = t_1 + 2t_2 = \frac{x}{6} + \frac{2x}{24} = \frac{x}{4}$

$v = \frac{x}{t} = 4\text{m/s}$

3.

(d) 0^o

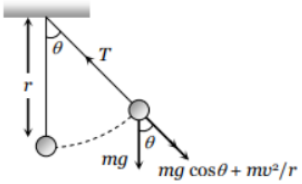
Explanation: The time of flight is given by:

$T = \frac{2u \sin\theta}{g} = \frac{2 \times 30 \times 1}{10 \times 2} = 3\text{sec}$

Thus, after 1.5 sec the body is at the highest point. As the direction of motion is horizontal after 1.5 seconds, the angle with the horizontal is 0^o.

4. **(a)** only ii

Explanation:



T = mg cos $\theta + \frac{mv^2}{r}$

As θ increases T decreases

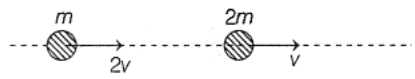
So $T_1 > T_2$

5.

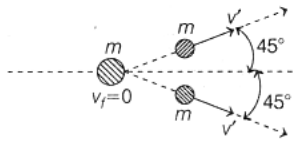
(d) $2\sqrt{2}v$

Explanation: According to the questions,

Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both x and y directions separately.

So,

$(p_x)_{initial} = (P_x)_{final}$

m(2v) + 2m(v) = 0 + mv'cos 45° + mv'cos 45°

$\Rightarrow$ 4mv = $\frac{2m}{\sqrt{2}}v' \Rightarrow$ v' = $2\sqrt{2}v$

So, each particle will move with a speed of $2\sqrt{2}v$

6. **(c)** $(F_1 + F_2)$

   Explanation: It is seen that perpendicular distance of each line of action of force from centre is same as r.

   Now taking momentum about O

   $F_1 \times r + F_2 \times r - F_3 \times r = 0$

   Hence, $F_3 = F_1 + F_2$

7. **(a)** it is the ratio of the component of the force normal to the area

   Explanation: Pressure is a scalar quantity because it is the ratio of the component of the force normal to the area and it is independent of the size of the area chosen.

8. **(b)** 15.2 cc

   Explanation: In case of thermal expansion of liquid, change in volume of liquid relative to container is given by:

   $\Delta V = V(\gamma_L - \gamma_s)\Delta\theta$

   V = 1litre = 1000 cc

   $\gamma s = 3ag = 0.3 \times 10^{-4}/^oC$

   $\therefore \Delta V = 1000(1.82 - 0.3) \times 10^{-4} \times 100 = 15.2$ cc

9. **(d)** remains unchanged

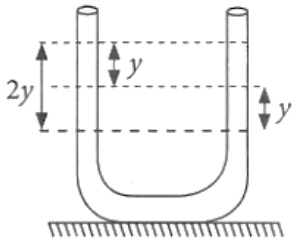
   Explanation: remains unchanged

10. **(d)** simple harmonic and time period are independent of the density of the liquid

    Explanation:

    If the liquid in U-tube is filled to a height h and the cross-section of the tube is uniform and the liquid is incompressible and non-viscous. Initially, the level of liquid in the two limbs will be at the same height equal to h. If the liquid is pressed by y in one limb, it will rise by y along the length of the tube in the other limb, so the restoring force will be developed by hydrostatic pressure difference.

difference.



Restoring force,

F = Weight of liquid column of height 2y

$\Rightarrow$ F = -(A $\times$ 2y $\times$ ρ) $\times$ g

= -2Aρgy

$\Rightarrow$ F $\propto$ - y

Motion is SHM with force constant.

k = 2 Aρg

Time period,

T = $2\pi\sqrt{\frac{m}{k}}$

= $2\pi\sqrt{\frac{A \times 2h \times \rho}{2A\rho g}}$

= $2\pi\sqrt{\frac{h}{g}}$

Which is independent of the density of the liquid.

11. **(a)** 32

Explanation: Let m capacitors are joined in series and n such groups are joined in parallel

So, C = $\frac{8}{m}$ and $C_{equi.}$ = $n \times \frac{8}{m}$ = 16

or, n = 2m

Potential of arrangement, mV = 1000

or, m = $\frac{1000}{250}$ = 4

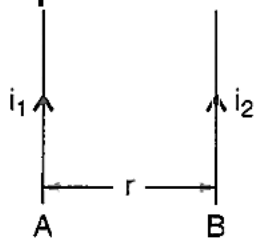
$\therefore$ n = 2 $\times$ 4 = 8

So, total number of capacitors required

= nm = 8 $\times$ 4 = 32

12. **(c)** $\frac{3}{2}$

Explanation:



$B_1 = \frac{\mu_0 i_1}{2\pi r/2} + \frac{\mu_0 i_2}{2\pi r/2} = \frac{\mu_0}{\pi r}(i_2 - i_1) = 6 \times 10^{-6}\text{T}$

When the current is reversed in i_2,

$B_2 = -\frac{\mu_0(i_1+i_2)}{2\pi r/2} = -\frac{\mu_0(i_1+i_2)}{\pi r} = 3 \times 10^{-5}\text{T}$

B_1 = constant (i_2 - i_1) = 6 $\times$ 10^{-6} T

B_2 = constant (i_2 + i_1) = 3 $\times$ 10^{-5} T

$\frac{-(i_1+i_2)}{(i_2-i_1)} = \frac{30}{6} = 5$

$-(i_1 + i_2) = 5i_2 - 5i_1$ or $6i_2 = 4i_1$

$\frac{i_1}{i_2} = \frac{3}{2}$

13.

(c) 1.35 A-m

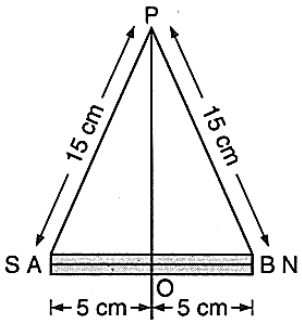
Explanation:

Length of magnet = 10 cm = 10×10^{-2} m

r =15×10^{-2} m

$OP = \sqrt{225 - 25} = \sqrt{200}\text{cm}$

Since, at the neutral point, magnetic field due to the magnet equal to B_H.



$0.4 \times 10^{-4} = 10^{-7} \times \frac{M}{\left(200 \times 10^{-4} + 25 \times 10^{-4}\right)^{3/2}}$

$\frac{0.4 \times 10^{-4}}{10^{-7}} \times \left(225 \times 10^{-4}\right)^{3/2} = M$

$0.4 \times 10^3 \times 10^{-6}(225)^{3/2} = M$

M = 1.35 A-m

14.

(d) 0.58 V

Explanation: We know that,

$e_0 = \omega NBA$

$= (2\pi v)NB(\pi r^2)$

$= 2\pi^2 vNBr^2$

$= 2 \times (3.14)^2 \times \frac{1800}{60} \times 4000 \times 0.5 \times 10^{-4} \times (7 \times 10^{-2})^2$

= 0.58 V

15. **(a)** 400

Explanation: Power = V.I

1000 = 500I

(I) secondary = 2 A

Now, $\frac{I_1}{I_2} = \frac{N_2}{N_1} \Rightarrow \frac{8}{2} = \frac{N_2}{100}$

N_2 = 400

16.

(d) $(-2\hat{i} - 3\hat{j})$ and $(3\hat{i} - 2\hat{j})$

Explanation: As we know, $\vec{E} \cdot \vec{B} = 0 \because [\vec{E} \perp \vec{B}]$ and $\vec{E} \times \vec{B}$ should be along Z diection

As $(-2\hat{i} - 3\hat{j})$ and $(3\hat{i} - 2\hat{j}) = 5\hat{k}$

17.

(d) the frequency of the incident light

Explanation: By Einstein's photoelectric equation, the kinetic energy of photoelectron is given by

$\therefore \frac{1}{2}mv_{max}^2 = h(v - v_0)$

Thus the kinetic energy of photoelectrons depends on the frequency of incident light and is independent light. Hence there is no effect of intensity of light on the kinetic energy of emitted photoelectrons.

18.

(d) $\frac{3}{4}$

Explanation: Energy of radiation that corresponds to the energy difference between two energy levels and is given as:

E = 13.6 $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ eV (numerically)

E is minimum when n_1 = 1 and n_2 = 2

$E_{min.}$ = 13.6 $\left(\frac{1}{1} - \frac{1}{4}\right)$ eV = 13.6 $\times \frac{3}{4}$ eV

E is maximum when n_1 = 1 and $n_2 = \infty$

(The atom is ionized that is known as ionization energy)

$E_{max.}$ = 13.6 (1 - $\frac{1}{\infty}$) = 13.6 eV

$\therefore \frac{E_{min.}}{E_{max}} = \frac{3}{4}$ or $\frac{hc/\lambda_{max}}{hc/\lambda_{min.}} = \frac{3}{4}$

or $\frac{\lambda_{min.}}{\lambda_{max}} = \frac{3}{4}$

19.

(c) 1 : 1

Explanation: $A_1 : A_2$ = 1 : 3

Their radii will be in the ratio $R_0 A_1^{1/3} : R_0 A_2^{1/3} = 1 : 3^{1/3}$

Density = $\frac{A}{\frac{4}{3}\pi R^3}$

$\therefore \quad \rho_{A_1} : \rho_{A_2} = \frac{1}{\frac{4}{3}\pi R_0^3 \cdot 1^3} : \frac{3}{\frac{4}{3}\pi R_0^3 \left(3^{1/3}\right)^3}$

Their nuclear densities will be the same.

20.

(d) 6533 $\overset{o}{A}$

Explanation: As we know that,

$\lambda = \frac{hc}{E_g}$

$\because E_g$ = 1.9 eV

= 1.90 $\times$ 1.6 $\times$ 10^{-19} J

$\therefore \lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.90 \times 1.6 \times 10^{-19}}$

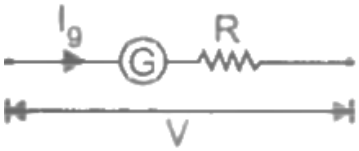
= 6533 $\overset{o}{A}$

PHYSICS (Section-B)

21. 5

Explanation:

Here, Current for full scale deflection, I_g = 0.006 A



Let G be resistance of the galvanometer.

To convert the given galvanometer into a voltmeter of range 0 - 30 V i.e., V = 30 V, a resistance R is connected in series with it such that;

V = I_g (G + R)

30 = 0.006 (G + 4990)

$\frac{30}{0.006}$ = (G + 4990)

$\frac{30\times1000}{6}$ = G + 4990

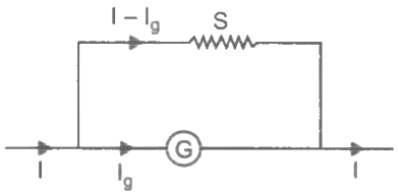
5000 = G + 4990

G = 10Ω

To convert the given galvanometer into ammeter of range of 0 - 1.5 A i.e., I = 1.5 A a resistance of value S is connected in parallel with it such that

$(I - I_g) S = I_g G$

$(1.5 - 0.006) \times \frac{2n}{249} = 0.006 \times 10$



$\frac{2n}{249} = \frac{0.06}{1.494}$

$2n = \frac{0.06\times249}{1.494}$ = 10

or, n = 5

22. 90

Explanation:

Potential at 1 m from the charge,

$V_A = \frac{K\cdot10^{-6}}{1} = K \times 10^{-6}$

Potential at 10 m from the charge,

$V_B = \frac{K\cdot10^{-6}}{10} = K \times 10^{-7}$

Potential diff. = $V_A - V_B = K (10^{-6} - 10^{-7})$

Its velocity at 10 m is v, then

$\frac{1}{2} \times mv^2 = (v_A - v_B) \times q$

$\frac{1}{2} \times 2 \times 10^{-3} \times v^2$

$= K \times 10^{-6} \left(1 - \frac{1}{10}\right) \times 10^{-3}$

$v^2 = \frac{K\times10^{-9}\times9}{10^{-3}\times10}$

$= K \times \frac{9}{10} \times 10^{-6}$

$= 9 \times 10^9 \times \frac{9}{10} \times 10^{-6}$ = 81 × 100

v = 90 m/sec

23. 4.0

Explanation:

For coil (1),

Radius, r_1 = 1 cm, Number of turns, N_1 = 10

For coil (2),

Radius, r_2 = 1000 cm, Number of turns, N_2 = 200

Using $\phi_{1,2}$ = MI

$N_2\vec{B}.N_1\vec{A}_1$ = MI $\left(\because B = \frac{\mu_0 I}{2r_2}\right)$

$\Rightarrow \mathrm{N_1\ N_2} \frac{\mu_0 \mathrm{I}}{2\mathrm{r_2}} \cdot \pi \mathrm{r_1^2} = \mathrm{MI}$

The mutual inductance of this arrangement will be

$\Rightarrow M = \frac{10\times200\times4\pi\times10^{-7}\times\pi\times(0.01)^2}{2\times10}$

$\Rightarrow$ M = 4 $\times$ 10^{-8} H

24. 16

Explanation:

Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$-\frac{GM_Em}{10R}+\frac{1}{2}mV_0^2=-\frac{GM_Em}{R}+\frac{1}{2}mV^2$

[$\because$ Gravitational potential energy = $-\frac{GMm}{r}$]

$\Rightarrow \frac{GM_E}{R}\left(1-\frac{1}{10}\right)+\frac{V_0^2}{2}=\frac{V^2}{2}\Rightarrow V^2=V_0^2+\frac{9}{5}gR$

$\Rightarrow V=\sqrt{V_0^2+\frac{9}{5}gR}\approx$ 16 km/s [$\because$ V_0 = 12 km/s given]

25. 0.02

Explanation:

As $v=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$\therefore \frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}$

$\frac{\Delta T}{T}=2\frac{\Delta v}{v}=2\times\frac{6}{600}=0.02.$

26. 3

Explanation:

$\sin i_c=\frac{1}{\mu}=\frac{3}{5}, \therefore \tan i_c'=\frac{3}{4}$

If x be the diameter of the disc, then

$\tan i_c'=\frac{3}{4}=\frac{x}{4}$ or x = 3 m.

27. 8.0

Explanation:

Magnetic field at centre (B_1) = $\frac{\mu_0 I}{2r}$

Magnetic field on axis (B_2) = $\frac{\mu_0 Ir^2}{2(r^2+x^2)^{3/2}}$

Vlaue of x = r (given)

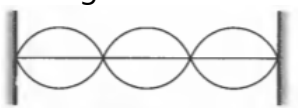
B_2 = $\frac{\mu_0 I}{4\sqrt{2}r}$

$\frac{B_1}{B_2}=\frac{\mu_0 I}{2r}\times\frac{4\sqrt{2}r}{\mu_0 I}=\frac{2\sqrt{2}}{1}=\frac{\sqrt{8}}{1}$ $\because$ x = 8

28. 50

Explanation:

The figure shows the situation corresponding to the question.



L = $\frac{3\lambda}{2}$ = $\frac{3}{2}\times\frac{v}{f}$

or f = $\frac{3v}{2L}$ = $\frac{3}{2\times0.6}\sqrt{\frac{80}{0.2}}$ = 50 Hz

29. 50.0

Explanation:

Given: P = KV^3

$\Rightarrow PV^{-3}$ = Constant K $\because \gamma$ = -3

Work done, $W=\frac{nR(T_1-T_2)}{\gamma-1}=\frac{nR(100-300)}{-3-1}$

or, W = $\frac{200nR}{4}$ = 50 nR

30. 4

Explanation:

Given: Wire length, l = 0.3 m

Mass of the body, m = 10 kg

Breaking stress, σ = 4.8 $\times$ 10^7 Nm^{-2}

Area of cross-section, a = 10^{-2} cm^2

Maximum angular speed ω = ?

T = M$l\omega^2$

$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$

$\frac{ml\omega^2}{A} \leq 48 \times 10^7 \Rightarrow \omega^2 \leq \frac{(48\times10^7)A}{ml}$

$\Rightarrow \omega^2 \leq \frac{(48\times10^7)(10^{-6})}{10\times3} = 16 \Rightarrow \omega_{max}$ = 4 rad/s

CHEMISTRY (Section-A)

31.

(c) Cl^- and Ar

Explanation: Both have $1s^2$, $2s^2$ $2p^6$,$3s^2$$3p^6$ configuration.

32. **(a)** B only

Explanation: Sc^{3+} > Ti^{2+} > Cr^{1+} > Mn (size)

33.

(d) C

Explanation: PBr_6^- don't exist because of steric hindrance.

34.

(d) intensive and extensive properties respectively

Explanation: Temperature is independent of the amount of matter present in the system. Hence, it is an intensive property. Heat depends upon the amount of matter present in the system. Hence, it is an extensive property.

35. **(a)** HW

Explanation: The salt hydrolysis in each case occurs except NaW because its pH = 7. Thus HW is strongest acid.

36. **(a)** Fe

Explanation: +2 and +3

37.

(c) metallic character

Explanation: Due to inert pair effect nuclear charge increases, i.e., the tendency to lose valence electron decreases. Thus, the metallic character decreases.

38.

(c) 5 and 4

Explanation: HC $\equiv$ C - CH = CH - CH = CH_2

Triple bond contributes to two C - C π bonds

Double bond contributes to one C - C π bond

$\therefore$ Number of π bonds = 4

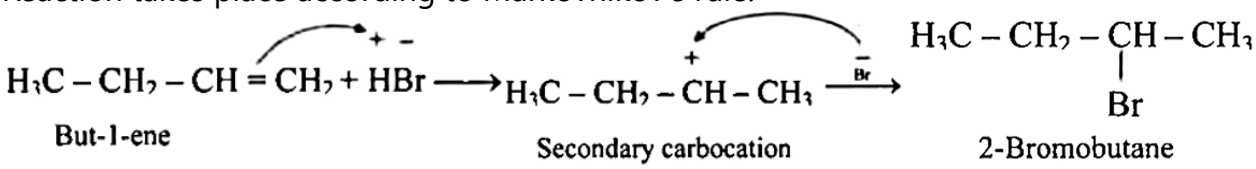
And number of C - C σ bonds = 5

39.

(c) $CH_3 - CH_2 - \overset{+}{C}H - CH_3$

Explanation:

Reaction takes place according to Markovnikov's rule.



2° carbocation is formed as it is more stable than 1° carbocation.

40.

(d) -0.112^{o}C

Explanation: $\Delta T_f = K_f \cdot m$

$= 1.86 \times 0.06 = 0.112$ K or 0.112^{o}C

$\therefore$ Freezing point = 0 - 0.112^{o}C = -0.112^{o}C

41. **(a)** 0.061^{o}C

Explanation: $\Delta T_b = \frac{1000 \times K_B \times W_B}{M_B W_A}$

For glucose solution, $\Delta T_b = \frac{1000 \times 0.52 \times 40}{250 \times 180}$

$= \frac{20800}{45000} = 0.462$ K

For fructose solution, $\Delta T_b = \frac{1000 \times 0.52 \times 25}{180 \times 180}$

$= \frac{13000}{32400} = 0.401$ K

$\therefore$ Boiling point of glucose solution = 100.462^{o}C

$\therefore$ Boiling point of fructose solution = 100.401^{o}C

Difference in boiling point = 0.061^{o}C

42.

(b) spontaneous reaction

Explanation: Rusting takes place with decrease in free energy; E°_{redox} is + ve.

43.

(b) C_2H_4 and C_4H_8

Explanation: In the given reaction; x A $\longrightarrow$ y B

$\log_{10}\left[\frac{-d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right]$ + 0.3010

Value of log2 = 0.3010

Substituting 0.3010 by log2

$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right]$ + log2

Using logarithm rules,

$\left[\frac{-d[A]}{dt}\right] = 2 \times \left[\frac{d[B]}{dt}\right] \Rightarrow -\frac{1}{2}\left[\frac{d[A]}{dt}\right] = \left[\frac{d[B]}{dt}\right]$ ...(i)

Using the rate equation (i) to determine the reaction involved is

2A $\longrightarrow$ B

Option that fits correct in the above reaction is

$2C_2H_4 \longrightarrow C_4H_8$

44.

(d) (S)

Explanation: Spatial arrangements depends only on secondary valencies and not primary valencies.

45.

(c) $NH_3 > (CH_3)_3N > PH_3$

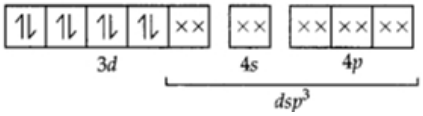
Explanation: NH_3 > $(CH_3)_3N$ > PH_3

46.

(b) trigonal bipyramidal

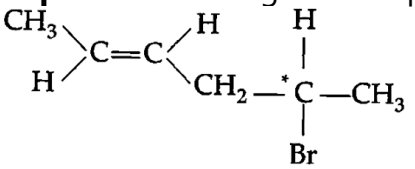
Explanation: In $Fe(CO)_5$, the 'Fe' atom is dsp^3 hybridised, therefore the shape of the molecule is trigonal bipyramidal.

Fe_{26} = [Ar] $3d^6\ 4s^2$



47. **(a)** 4

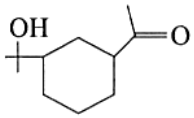
Explanation: The given compound can be written as



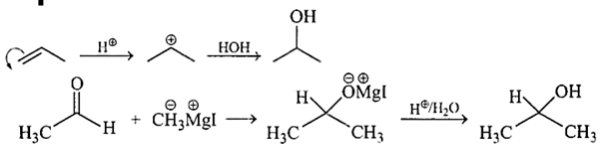
Both geometrical isomerism (cis-trans form) and optical isomerism is possible in the given compound. Number of optical isomer = $2^n = 2^1 = 2$ (where n = no. of asymmetric carbon)
Hence, total number of stereoisomers = 2 + 2 = 4

48.

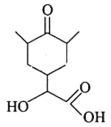
(c)



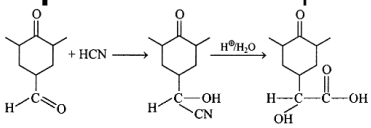
Explanation:



49.

(c) 

Explanation: Nucleophilic addition is faster on aldehyde than ketone.



50.

(b)

$LiAlH_4$ in ether

Explanation:

$LiAlH_4$/ether reduces aryl nitro compounds to azo compounds.

$$\underset{Nitrobenzene}{2C_6H_5NO_2} \xrightarrow{LiAlH_4} \underset{Azobenzene}{C_6H_5N=N-C_6H_5}$$

CHEMISTRY (Section-B)

51. 0

Explanation:

Orbital angular momentum = $\sqrt{1(1+1)}\frac{h}{2\pi}$

Value of 1 for s orbital = 0 $\Rightarrow$ orbital angular momentum = 0.

52. 9.079

Explanation:

0.02 mol Hcl neutralixe 0.02 mol of NH_3.

Rest $\mathbf{n}_{\mathrm{NH_3}}$ = 0.1 - 0.02 = 0.08

$\mathbf{n}_{\mathrm{NH_4Cl}}$ = ${}^{n}\mathrm{NH_4^+}$ = 0.1 + 0.02 = 0.12

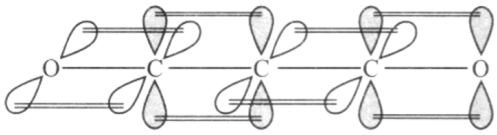
pOH = pK_b + $\log\frac{[\mathrm{NH_4^+}]}{[\mathrm{NH_3}]}$ = 4.745 + $\log\frac{0.12}{0.08}$

= 4.745 + $\log\frac{3}{2}$ = 4.745 + 0.477 - 0.301

pOH = 4.921; pH = 14 - pH = 9.079

53. 2

Explanation:



54. 4

Explanation:

$M^- \rightarrow M$; ΔH = +EGE_1 of M

$M \rightarrow M^+$; ΔH = +$I.E._1$ of M

$M^+ \rightarrow M^{2+}$; ΔH = +$I.E._2$ of M

$M^{2+} \rightarrow M^{3+}$; ΔH = +$I.E._3$ of M

$M^{3+} \rightarrow M^{4+}$; ΔH = +$I.E._4$ of M

55. 4.0

Explanation:

Octahedral splitting energy = $\frac{hc}{\lambda}$

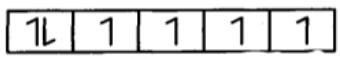
$= \frac{6.626\times10^{-34}\times3\times10^{8}}{498\times10^{-9}}$

= 0.0399 $\times$ 10^{-17}J = 3.99 $\times$ 10^{-19} J $\approx$ 4 $\times$ 10^{-19}.

56. 49.0

Explanation:

Fe (Z = 26) $\Rightarrow$ [Ar] $3d^6$ $4s^2$



Number of unpaired electrons = 4

$\because \mu = \sqrt{n\,(n\,+\,2)}$ BM

$\therefore \mu = \sqrt{4\,(4\,+\,2)} = \sqrt{24}$ BM = 4.89 $\approx$ 49 $\times$ 10^{-1} BM

57. 18

Explanation:

Complete combustion of hydrocarbons can be represented by the following reaction

C_xH_y + (x + $\frac{y}{4}$) O_2 $\longrightarrow$ xCO_2 + $\frac{y}{2}$ H_2O

For propane combustion reaction is

C_3H_8 + (3 + $\frac{8}{4}$) O_2 $\longrightarrow$ 3CO_2 + $\frac{8}{2}$ H_2O

$\therefore$ C_3H_8 + 5O_2 $\longrightarrow$ 3CO_2 + 4H_2O

Similarly, for butane is

$C_4H_{10} + (4 + \frac{10}{4}) O_2 \longrightarrow 4CO_2 + \frac{10}{2} H_2O$

$\therefore C_4H_{10} + \frac{13}{2}, O_2 \longrightarrow 4CO_2 + 5^2H_2O$

$\because$ For 1 mol of C_4H_{10} required $O_2 = \frac{13}{2}$ mol

$\therefore$ For 2 mol of C_4H_{10} required $O_2 = \frac{13}{2} \times 2 = 13$ mol

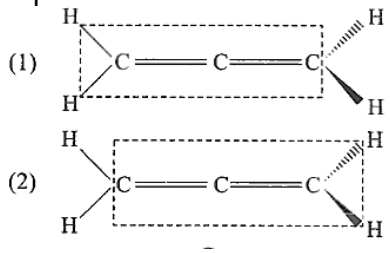
58. 5

Explanation:

P_x, P_y, P_z, d_z^2, and $d_{x^2-y^2}$ are axial orbitals.

59. 2

Explanation:



60. 309

Explanation:

$SF_6(g) \rightarrow S(g) + 6F(g)$

$\Delta H^\circ = \Delta H_f^\circ(S) + 6\Delta H_f^\circ(F) - \Delta H_f^\circ(SF_6)$

$= 275 + 6 \times 80 - (-1100) = 1855$ kJ mol^{-1}

Also, $\Delta H^\circ = 6\Delta H_{S-F}$

$\therefore \Delta H_{S-F} = \frac{1855}{6} = 309.17 = 309$ kJ mol^{-1}.

MATHEMATICS (Section-A)

61.

(b) -1

Explanation: f(x + y) = f(xy) for all x, y

Substituting y = 0, we get f(x) = f(0)

f(0) is a constant $\Rightarrow$ f(x) = constant

$f\left(-\frac{1}{2}\right) = -\frac{1}{2} \Rightarrow f(x) = -\frac{1}{2}$ (as f is a constant function)

$\Rightarrow$ f(2017) + f(2018) $= -\frac{1}{2} - \frac{1}{2} = -1$

62.

(b) $3x^2 - 10x - 4 = 0$

Explanation: Given expression is $x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_3 5}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) =$

0

Take, $3^{\sqrt{\log_3 5}} = 3^{\sqrt{\log_3 5}} \cdot \sqrt{\log_3 5}\sqrt{\log_3 5} = 3^{\sqrt{\log_3 5}} \cdot \sqrt{\log_3 5}$

$= 3(\log_3 5)^{\sqrt{\log_5 3}} = 5^{\sqrt{\log_5 3}}$

$3^{\sqrt[3]{\log_3 5}} = 3^{\log_3 5 \cdot \sqrt[3]{(\log_5 3)^2}} = \left(3^{\log_3 5}\right)^{(\log_5 3)^{\frac{2}{3}}}$

$= 5^{(\log_5 3)^{\frac{2}{3}}}$

After putting the value the equation is $x^2 - 5x - 3 - 0$ and roots are α & β.

Then, $\alpha + \beta = 5$; $\alpha\beta = -3$.

New roots are $\alpha + \frac{1}{\beta}$ & $\beta + \frac{1}{\alpha}$

$\Rightarrow \frac{\alpha\beta+1}{\beta}$ & $\frac{\alpha\beta+1}{\alpha} \Rightarrow \frac{-2}{\beta}$ & $\frac{-2}{\alpha}$

Let $\frac{-2}{\alpha} = p \Rightarrow \alpha = \frac{-2}{p}$

As α^2 - 5α - 3 = 0

$\Rightarrow \left(\frac{-2}{p}\right)^2 - 5\left(\frac{-2}{p}\right)$ - 3 = 0

$\Rightarrow \frac{4}{p^2} + \frac{10}{p}$ - 3 = 0

4 + 10p - 3p^2 = 0 $\Rightarrow$ 3p^2 - 10p - α = 0

Now replace 'p' by 'x' to get required equation

3x^2 - 10x - 4 = 0

63.

(b) 1344

Explanation: For even numbers, the unit's place can be filled with any of 2, 4, 6 or 8

The number of ways to fill a unit's place = 4

There are 8 digits left for ten's place, 7 digits for hundred's place, and 6 digits for thousand's place

$\Rightarrow$ The required number of 4 digit even numbers = $6 \times 7 \times 8 \times 4$

= 1344

64.

(d) 24

Explanation: Given expression is $(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$

$= 2\left[{}^6C_0x^6 + {}^6C_2x^4(\sqrt{x^3 - 1})^2 + {}^6C_4x^2(\sqrt{x^3 - 1})^4 + {}^6C_6(\sqrt{x^3 - 1})^6\right]$

{$\because$ $(a + b)^n + (a - b)^n = 2[{}^nC_0a^n + {}^nC_2a^{n-2}b^2 + {}^nC_4a^{n-4}b^4 + ...]$}

$= 2\left[{}^6C_0x^6 + {}^6C_2x^4\left(x^3 - 1\right) + 6C_4x^2\left(x^3 - 1\right)^2 + {}^6C_6\left(x^3 - 1\right)^3\right]$

The sum of the terms with even power of x

$= 2\left[{}^6C_0x^6 + {}^6C_2\left(-x^4\right) + {}^6C_4x^8 + {}^6C_4x^2 + {}^6C_6\left(-1 - 3x^6\right)\right]$

$= 2\left[{}^6C_0x^6 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 1 - 3x^6\right]$

Now, the required sum of the coefficients of even powers of x in

$(x + \sqrt{x^3 - 1})^6 + (x - \sqrt{x^3 - 1})^6$

$= 2\left[{}^6C_0 - {}^6C_2 + {}^6C_4 + {}^6C_4 - 1 - 3\right]$

= 2[1 - 15 + 15 + 15 - 1 - 3] = 2(15 - 3) = 24

65.

(c) 6n^2 - n

Explanation: Given S_n = 2n + 3n^2

Now, first term = 2 + 3 = 5

second term = 2(2) + 3(4) = 16

third term = 2(3) + 3(9) = 33

66.

(b) continuous as well as differentiable.

Explanation: Let |f(x)| $\leq$ x^2, $\forall x \in R$

Now, at x = 0, | f(0) $\leq$ 0

$\Rightarrow$ f(0) = 0

$\therefore f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h - 0} = \lim_{h \to 0} \frac{f(h)}{h}$ ...(i)

Now, $\left|\frac{f(h)}{h}\right| \leq |h| \left(\because |f(x)| \leq x^2\right)$

$\Rightarrow -|h| \leq \frac{f(h)}{h} \leq |h|$

$\Rightarrow \lim_{h\to 0} \frac{f(h)}{h} \to 0$ ...(ii)

(using sandwich Theorem)

$\therefore$ from (i) and (ii) we get f'(0) = 0,

i.e. - f (x) is differentiable, at x = 0

Since, differentiability $\Rightarrow$ Continuity

$\therefore$ | f(x) | $\leq x^2$, for all x $\in$ R is continuous as well as differentiable at x = 0.

67.

(d) 24π

Explanation:

$V = \pi r^2 h$, $V = 16\pi$

$\Rightarrow r^2 h = 16$ ...(i)

Amount of material (including lid),

$A = 2\pi r (r + h)$

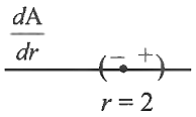
$= 2\pi r^2 + 2\pi r \left(\frac{16}{r^2}\right)$ ...[From (i)]

$= 2\pi \left(r^2 + \frac{16}{r}\right)$

$\Rightarrow \frac{dA}{dr} = 2\pi \left(2r - \frac{16}{r}\right)$

$\Leftrightarrow \frac{d\,A}{dr} = \frac{4\pi}{r^2} (r^3 - 8) = \frac{4\pi}{r^2} (r - 2)(r^2 + 2r + 4)$

$\frac{dA}{dr} = 0 \Leftrightarrow r = 2$ (point of relative minima)



$A_{min} = 2\pi \left(4 + \frac{16}{2}\right) = 24\pi$

68.

(c) $\frac{2}{\sqrt{3}}$

Explanation: $\frac{2}{\sqrt{3}}$

69.

(b) (7, 7)

Explanation: Mid-point of P(2, 3) and Q(4, 5) = (3, 4)

Slope of PQ = 1

Slope of the line L = -1

Mid-point (3, 4) lies on the line L.

Equation of line L,

$y - 4 = -1(x - 3) \Rightarrow x + y - 7 = 0$ ...(i)

Let image of point R(0, 0) be S(x_1, y_1)

Mid-point of RS = $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ lies on the line (i)

$\therefore x_1 + y_1 = 14$ ...(ii)

Slope of RS = $\frac{y_1}{x_1}$

Slope RS $\perp$ line L

$\therefore \frac{y_1}{x_1} \times (-1) = -1 \therefore x_1 = y_1$ ...(iii)

From (ii) and (iii),

$x_1 = y_1 = 7$

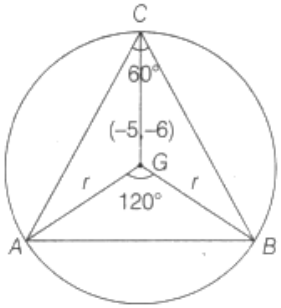
Hence the image of R = (7, 7)

70.

(c) 25

Explanation: Clearly, centre of the circumscribed circle is the centroid (G) of the equilateral triangle ABC.

[∵ in an equilateral triangle, circumcenter and centroid coincide]



Also, we know that

△ AGB ≅ △BGC ≅ △CGA [by SAS congruence rule]

∴ ar(△ABC) = 3 ar(△AGB)

= $3\left(\frac{1}{2}r^2 \sin 120^\circ\right)$ [∵ area of triangle = $\frac{1}{2}$ ab sin (∠C)]

∵ ar(△ABC) = $27\sqrt{3}$ [given]

∴ $\frac{3}{2}r^2\frac{\sqrt{3}}{2} = 27\sqrt{3}$ [sin 120^o = sin (180^o - 60^o) = sin 60^o = $\frac{\sqrt{3}}{2}$]

$\Rightarrow$ r^2 = 4 $\times$ 9

$\Rightarrow$ r = 6

Now, radius of circle,

r = $\sqrt{g^2 + f^2 - c}$

$\Rightarrow$ 6 = $\sqrt{25 + 36 - c}$ [∵ in the given equation of circle 2g = 10 and 2f = 12 $\Rightarrow$ g = 5 and f = 6]

$\Rightarrow$ 36 = 25 + 36 - c

$\Rightarrow$ c = 25

71.

(b) -3

Explanation: -3

72.

(c) 4

Explanation: $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$

$\Rightarrow \left\{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}}\right\}^4 = \left(\frac{d^2y}{dx^2}\right)^{\frac{4}{3}}$

$\Rightarrow \left\{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3\right\}^3 = \left\{\left[\frac{d^2y}{dx^2}\right]^{\frac{4}{3}}\right\}^3$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$

The highes order derivative is $\frac{d^2y}{dx^2}$ with degree 4

$\Rightarrow$ The degree of the differential equation = 4

73.

(b) $\frac{4}{3}$

Explanation: cos θ = $\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

= $\frac{(1)(2)+(2)(1)+(2)(1)}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+1^2+1^2}}$

$\Rightarrow \cos\theta = \frac{2}{\sqrt{6}} \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{4}{6}}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{3}}$

Given that, $\sin 2\theta = \frac{\lambda}{\sqrt{2}}$

$\Leftrightarrow 2\sin\theta\cos\theta = \frac{\lambda}{\sqrt{2}}$

$\Leftrightarrow \frac{4}{\sqrt{18}} = \frac{\lambda}{\sqrt{2}} \Leftrightarrow \lambda = \frac{4}{3}$

74.

(b) -1

Explanation: Given vectors $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}$ are non-coplanar.

$\Leftrightarrow [\overrightarrow{OP}\ \overrightarrow{OQ}\ \overrightarrow{OR}] \neq 0$

$$\Rightarrow \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} \neq 0 \text{ ...(i)}$$

Given , $\begin{vmatrix} p & p^2 & 1+p^3 \\ q & q^2 & 1+q^3 \\ r & r^2 & 1+r^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} p & p^2 & 1 \\ q & q^2 & 1 \\ r & r^2 & 1 \end{vmatrix} + \begin{vmatrix} p & p^2 & p^3 \\ q & q^2 & q^3 \\ r & r^2 & r^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} + pqr \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + pqr) \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix} = 0$$

$\Rightarrow$ (1 + pqr) = 0 ...[From (i)]

$\Rightarrow$ pqr = -1

75.

(b) $\sqrt{2}$

Explanation: Outcomes are $\left(\frac{1}{2}-d\right)$, $\left(\frac{1}{2}-d\right)$, 0..., 10 times, $\frac{1}{2}$, $\frac{1}{2}$, ..., 10 times, $\frac{1}{2}$ + d, $\frac{1}{2}$ + d, 10 times

Mean = $\frac{1}{30}\left(\frac{1}{2}\times 30\right) = \frac{1}{2}$

Variance of the outcomes is,

$\sigma^2 = \frac{1}{30}\Sigma x_i^2 - (\bar{x})^2$

= $\frac{1}{30}\left[\left(\frac{1}{2}-d\right)^2\times 10 + \left(\frac{1}{2}\right)^2\times 10 + \left(\frac{1}{2}+d\right)^2\times 10\right] - \frac{1}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{30}\left[30\times\frac{1}{4} + 20d^2\right] - \frac{1}{4} \Rightarrow \frac{4}{3} = \frac{1}{4} + \frac{2}{3}d^2 - \frac{1}{4}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

76.

(b) $\frac{2101}{3125}$

Explanation: The divisibility of the product of 5 numbers depends upon the last digit of each

number. A digit from 0, 1, 2, ..., 9 can be the last digit of every number.

So, total number of ways of selecting last digit of 5 numbers = 10^5

If the product p is not divisible by 5 and 10, then last digit of each number can be filled in 8 ways.

$\Rightarrow$ Favourable number of ways = 8^5

Probability that product is not divisible by 5 or 10 = $\frac{8^5}{10^5} = \left(\frac{4}{5}\right)^5$

Required probability = $1 - \left(\frac{4}{5}\right)^5 = 1 - \frac{1024}{3125} = \frac{2101}{3125}$

77.

(d) $\frac{5}{16}$

Explanation: sin 36° sin72° sin 108° sin 144°

= sin 36° sin 72° sin(180° - 72°) sin(180° - 36°)

= sin 36° sin 72° sin 72° sin 36° ...[$\because$ sin (180° - θ) = sin θ]

= $\sin^2 36° \sin^2 72°$

= $\left[\frac{\sqrt{10-2\sqrt{5}}}{4}\right]^2 \left[\frac{\sqrt{10+2\sqrt{5}}}{4}\right]^2$

= $\frac{(10-2\sqrt{5})}{16} \cdot \frac{(10+2\sqrt{5})}{4}$

= $\frac{100-4\times5}{16\times16} = \frac{80}{16\times16}$

= $\frac{5}{16}$

78.

(b) $\frac{13}{12}$

Explanation: We know that in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2(e^2 - 1)$, the length of conjugate axis is 2b and distance between the foci is 2ae

According the problem, 2b = 5 and 2ae =13

Now, $b^2 - a^2(e^2 - 1)$

$\Rightarrow \left(\frac{5}{2}\right)^2 = a^2e^2 - a^2$

$\Rightarrow \frac{25}{4} = \frac{(2ae)^2}{4} - a^2$

$\Rightarrow a^2 = \frac{169-25}{4} = \frac{144}{4} = 36$ ($\because$ 2ae=165)

$\Rightarrow$ a = 6

Now, 2ae = 13

$\Rightarrow 2 \times 6 \times e = 13$

$e = \frac{13}{12}$

79.

(c) A - (B $\cup$ C) = (A $\cap$ B') $\cap$ C')

Explanation: A - (B $\cup$ C) = (A $\cap$ B') $\cap$ C')

80. **(a)** 8

Explanation: Since, the system of equations has infinitely many solution, therefore D = D_1 = D_2 = D_3 = 0

Here,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 1(2\alpha - 9) - 1(\alpha - 3) + 1(3 - 2)$$

= α - 5

and $D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 1(2\beta - 27) - 1(\beta - 9) + 5(3 - 2)$

= β - 13

Now, D = 0

$\Rightarrow \quad \alpha - 5 = 0 \Rightarrow \alpha = 5$

and $D_3 = 0 \Rightarrow \beta - 13 = 0$

$\Rightarrow \beta = 13$

$\therefore \quad \beta - \alpha$ = 13 - 5 = 8

MATHEMATICS (Section-B)

81. 2

Explanation:

$f(x) = x^4 - 4x^2 + 12x^2 + x - 1$

$f'(x) = 4x^2 - 12x^2 + 24x + 1$

$f''(x) = 12x^2 - 24x + 24 = 12 (x^2 - 2x + 2)$

$= 12 \{(x - 1)^2 + 1\} > \forall x$

$\Rightarrow$ f'{x) is increasing.

Since, f' (x) is cubic and increasing.

$\Rightarrow$ f' (x) has only one real root and two imaginary roots.

$\therefore$ f(x) cannot have all distinct roots.

$\Rightarrow$ Atmost 2 real roots.

Now, f(- 1) = 15, f(0) = - 1, f(1) = 9

$\therefore$ f(x) must have one root in (- 1, 0) and other in (0, 1).

$\Rightarrow$ 2 real roots

82. 309.0

Explanation:

function is differentiable $\forall$x < 0

$f(x) = 3x^2 + k\sqrt{x+1}$, 0 < x < 1 & $mx^2 + k^2$,x > 1

so $f(1^-) = f(1) = f(1^+)$

$3 + \sqrt{2}k = m + k^2$ ..(i)

& $f'(1^-) = f'(1^+)$

$\Rightarrow 2m = 6 + \frac{k}{2\sqrt{2}}$ ...(ii)

From (i) & (ii) $m = 3 + \frac{k}{4\sqrt{2}}$ ...(iii)

$\Rightarrow k^2 + 3 + \frac{k}{4\sqrt{2}} = 3 + \sqrt{2}k; k = \frac{7}{4\sqrt{2}}, 0$

So, $m = 3 + \frac{7}{32}$[In (iii)]

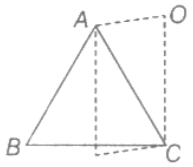
$\Rightarrow m = \frac{103}{32}$ So $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 8 \times \frac{2mx|_{x=8}}{6x + \frac{k}{2\sqrt{x+1}}\Big|_{x=\frac{1}{8}}}$

$= \frac{8 \times 2 \times 8 \times \frac{103}{32}}{\frac{16}{12}} = 309$

83. 0

Explanation:

$\vec{\mathbf{A}} \cdot \{(\vec{\mathbf{B}} + \vec{\mathbf{C}}) \times (\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}})\}$ [$\because$ it is a scalar triple product of three vectors of the form $\vec{\mathbf{A}}, \vec{\mathbf{B}} + \vec{\mathbf{C}}, \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$]



$= \vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{A}} + \vec{\mathbf{B}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}} + \vec{\mathbf{C}} \times \vec{\mathbf{B}} + \vec{\mathbf{C}} \times \vec{\mathbf{C}})$

$= \vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{A}}) + \vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) + \vec{\mathbf{A}} \cdot (\vec{\mathbf{C}} \times \vec{\mathbf{B}})$

$= [\vec{A}\vec{B}\vec{A}] - [\vec{A}\vec{B}\vec{C}] = 0$

84. 4

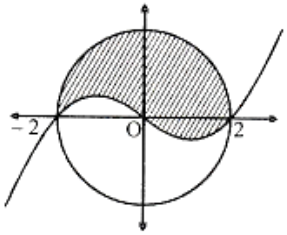
Explanation:

$f'(x) = a\left(x^2 - \frac{4}{3}\right) \Rightarrow f(x) = a\left(\frac{x^3}{3} - \frac{4x}{3}\right) + b$ passes through (0, 0) and (1, -2).

$\therefore b = 0, a = 2$

$f(x) = \frac{2x}{3}(x^2 - 4)$

Required area = $\frac{\pi(2)^2}{2} = 2\pi = 4\frac{\pi}{2} \Rightarrow k = 4$



85. 6.0

Explanation:

Centroid of $\triangle ABC = G\left(\frac{2\alpha}{3}, \frac{2\alpha}{3}, \frac{2\alpha}{3}\right)$

Given equation of line is $\frac{x}{1} = \frac{z-3}{-1} = \frac{y}{0} = \lambda$

$x = \lambda, y = 0, z = -\lambda + 3$

$\therefore D(\lambda, 0, -\lambda + 3)$ be any point on given line

$\therefore GD = \sqrt{\left(\lambda - \frac{2\alpha}{3}\right)^2 + \left(\frac{2\alpha}{3}\right)^2 + \left(-\lambda + 3 - \frac{2\alpha}{3}\right)^2}$

$GD_1 = \left(\lambda - \frac{2\alpha}{3}\right)^2 + \left(\frac{2\alpha}{3}\right)^2 + \left(-\lambda + 3 - \frac{2\alpha}{3}\right)^2$

$\frac{dGD_1}{d\lambda} = 2\left(\lambda - \frac{2\alpha}{3}\right) - 2\left(-\lambda + 3 - \frac{2\alpha}{3}\right)$

$= 4\lambda - 6 = 0 \Rightarrow \lambda = \frac{3}{2}$

$\therefore$ Minimum GD

$= \sqrt{\left(\frac{3}{2} - \frac{2\alpha}{3}\right)^2 + \left(\frac{2\alpha}{3}\right)^2 + \left(-\frac{3}{2} + 3 - \frac{2\alpha}{3}\right)^2}$

$\sqrt{\frac{57}{2}} = \sqrt{\left(\frac{9-4\alpha}{6}\right)^2 + \frac{4\alpha^2}{9} + \left(\frac{9-4\alpha}{6}\right)^2}$

$\Rightarrow \frac{57}{2} = \frac{24\alpha^2 - 72\alpha + 81}{18}$

$\Rightarrow \alpha^2 - 3\alpha\ 18 = 0 \Rightarrow \alpha = -3, 6$

$\therefore \alpha = 6 (\because \alpha > 0).$

86. 8

Explanation:

Prime (2, 3, 5, 7, 11) = {(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6), (6, 5)}

n(odd prime) = 14

$\therefore P(\text{odd prime}) = \frac{14}{36}$

Perfect square = (4, 9) = {(1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3)}

n(perfect square) = 7

$\therefore$ P(perfect square) $= \frac{7}{36}$

and P(odd perfect square) $= \frac{4}{36}$

Required probability

$= \frac{\frac{4}{36}+\frac{14}{36}\times\frac{4}{36}+\left(\frac{14}{36}\right)^2\frac{4}{36}+\cdots}{\frac{7}{36}+\frac{14}{36}\times\frac{7}{36}+\left(\frac{14}{36}\right)^2\frac{7}{36}+\ldots} = P = \frac{4}{7}$

$\therefore$ 14P $= 14 \cdot \frac{4}{7} = 8$

87. 7.0

Explanation:

Let $\sum_{1=1}^{\infty} \frac{a_n}{8^n} = K$

$\because$ $a_{n+2} = 2a_{n+1} + a_n$

Now, divide by 8^n, we get

$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n} \Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{10a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$

$\Rightarrow 64\sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = 16\sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty} \frac{a_n}{8^n}$

$64\left(\mathrm{K} - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(\mathrm{K} - \frac{a_1}{8}\right) + \mathrm{K}$

$\Rightarrow 64\left(\mathrm{K} - \frac{1}{8} - \frac{1}{64}\right) = 16\left(\mathrm{K} - \frac{1}{8}\right) + \mathrm{K}$

64K - 8 - 1 = 16K - 2 + K $\Rightarrow$ 47K = 7

That is, $47\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = 7$

88. 3

Explanation:

3

89. 50.0

Explanation:

Given matrix is A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and A = A^{-1}

Hence, $A^2 = A \cdot A^{-1} = I$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Compare the corresponding elements of the above matrix.

$\therefore$ $a^2 + bc = 1$ ...(i)

ab + bd = 0 ...(ii)

ac + cd = 0 ...(iii)

$bc + d^2 = 1$ ...(iv)

From (i) & (iv),

$a^2 - d^2 = 0 \Rightarrow$ (a + d) = 0 or a - d = 0

Case-I:

a + d = 0 $\Rightarrow$ (a, d) = (-1, 1), (0, 0), (1, -1)

a. (a, d) = (-1, 1) From (i),
1 + bc = 1 $\Rightarrow$ bc = 0
b = 0 c = 12 possibilities
c = 0 b = 12 possibilities
Here, (0, 0) is repeated then total possibilities are 2 $\times$ 12 = 24
Total pairs = 24 - 1 = 23.
b. (a, d) = (1, -1) $\Rightarrow$ be = 0 $\rightarrow$ 23 pairs
c. (a, d) = (0, 0) $\Rightarrow$ bc = 1
$\Rightarrow$ (b, c) = (1, 1) & (-1, -1), 2 pairs

Case-II:

Here, a = d
From (ii) & (iii),
if a $\neq$ 0 then b = c = 0
a^2 = 1
a = ± 1 = d
(a, d) = (1, 1), (-1, -1) $\rightarrow$ 2 pairs
Total number of pairs = 23 + 23 + 2 + 2 = 50 pairs

90. 26.0
Explanation:
Let k f(k) + 2 = λ (k - 2) (k - 3) (k - 4) (k - 5) ...(i)
Put k = 0
we get $\lambda = \frac{1}{60}$
Now, put λ in equation (i)
$\Rightarrow$ kf(k) + 2 = $\frac{1}{60}$(k - 2) (k - 3) (k - 4) (k - 5)
Put k = 10
$\Rightarrow$ 10f(10) + 2 = $\frac{1}{60}$(8)(7)(6)(5) = 28 $\Rightarrow$ 10f(10) = 26
$\Rightarrow$ 52 - 10f(10) = 52 - 26 = 26