CBSE Class 10 Mathematics Standard

Sample Paper - 05 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. Has the rational number $\frac{441}{2^2 \times 5 \times 7^2}$ a terminating or a non-terminating decimal representation?

OR

Show that 12^n cannot end with digit 0 or 5 for any natural number n.

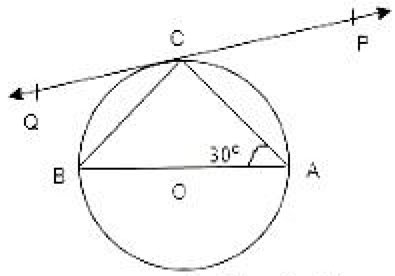
2. Determine the nature of the roots of the quadratic equation:

$4x^2 - 4x + 1 = 0$.

3. If x = a, y = b is the solution of the pair of equations x – y = 2 and x + y = 4, find the values

of a and b.

4. In the following figure, PQ is a tangent at a point C to circle with centre O. If AB is a diameter and $\angle CAB = 30^o$, then find $\angle PCA$.



5. For the A.P $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots$ write the first term and the common difference.

OR

Find the next term in AP: 3, 1, -1, -3.

6. Write the next two terms of the AP: 1, -1, -3, -5,...
7. State whether $\sqrt{3}x^2 - 2x + \frac{1}{2} = 0$ is a quadratic equation or not?

OR

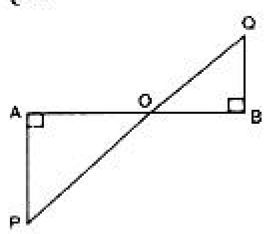
Find the discriminant of the Quadratic Equation:
$3x^2 + 2x - 1 = 0$

8. Distance between two parallel lines is 14 cm. Find the radius of the circle which will touch both the lines.
9. From an external point P, tangents PA = PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

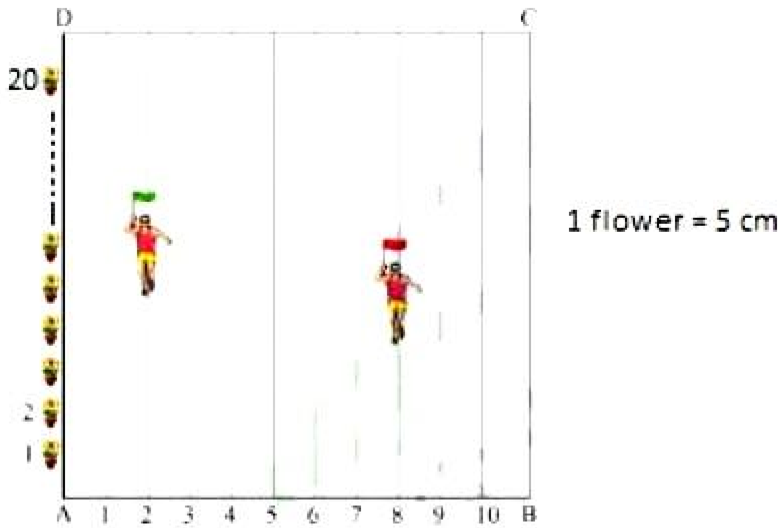
OR

A quadrilateral ABCD is drawn to circumscribe a circle. If AB = 12 cm, BC = 15 cm and CD = 14 cm, find AD.

10. In the given figure, if ∠A = 90°, ∠B = 90°, OB = 4.5 cm, OA = 6 cm and AP = 4 cm, then find QB.



11. What is the common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$?
12. Evaluate $\frac{\tan 45^\circ}{cosec 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$.
13. Write the value of $4\tan^2\theta - \frac{4}{\cos^2\theta}$.
14. A metallic solid cone is converted into a solid cylinder of equal radius. If the height of the cylinder is 5 cm, then find the height of the cone.
15. Write the value of $a_{30} - a_{10}$ for the A.P. 4, 9,14,19,.....
16. A bag contains 4 red, 5 black and 6 white balls. A ball is drawn at random. Find the probability that the ball drawn is red or white.
17. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs the distance AD on the 2nd line and posts a green flag. Preet runs the distance AD on the eighth line and posts a red flag. (take the position of feet for calculation)



i. In the distance, Niharika posted the green flag:
   a. 5
   b. 15
   c. 25
   d. 20

ii. The coordinates of the green flag are:
   a. (2, 15)
   b. (25, 2)
   c. (2, 5)

d. (2, 25)

iii. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

a. 20.5 m on the 5th line

b. 22.5 m on the 5th line

c. 25.5 m on the 5th line

d. 24.5 m on the 5th line

iv. What is the distance between both the flags?

a. $\sqrt{61}m$

b. $\sqrt{63}m$

c. $\sqrt{60}m$

d. $\sqrt{62}m$

v. The coordinates of the Red flag are:

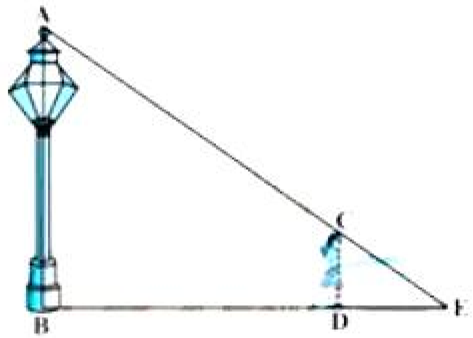
a. (8, 4)

b. (4, 8)

c. (8, 20)

d. (2, 25)

18. Some kindergarten students were playing near a lamp-post. They were so excited to see their shadows and trying to show that their shadow is the longest. The lamp was 3.6 m above the ground. One of the girl of height 90 cm was walking away from the base of a lamp-post at a speed of 1.2 m/s.



i. Which of the following line segment shows the length of the shadow?

a. CE

b. BE

c. DE

d. CD

ii. What would be the length of her shadow after 4 seconds?

a. 1.2 m

b. 1.6 m

c. 2.3 m

d. 1.4 m

iii. How far is the girl from the lamp-post?

a. 4.8 m

b. 1.2 m

c. 12 m

d. 6.4 m

iv. Triangle ABE and CDE are similar because:

a. All sides are equal

b. The shadow of a girl is equal to the height of the lamp-post

c. Angle B and Angle E are common

d. Both are related to the same length of the shadow

v. AB denotes the _______ and CD _______ after walking for 4 seconds away from the lamp-post.

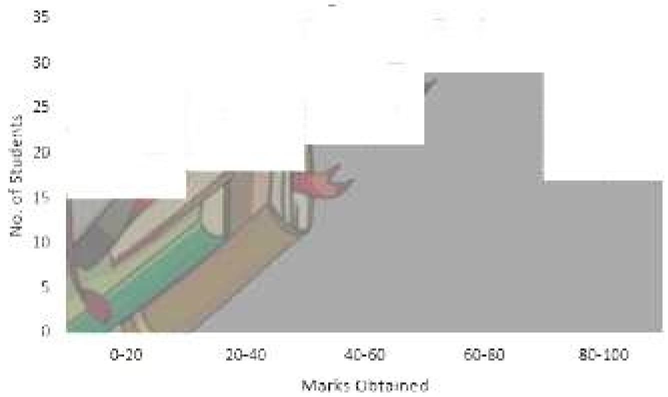
a. the girl, lamp-post

b. the shadow, the girl

c. the girl, the shadow

d. lamp-post, the girl

19. Recently the half-yearly examination was conducted in DAV public school. The mathematics teacher maintains a record of the marks of 100 students. On the basis of the recorded data of the marks obtained in Mathematics, the histogram is given below:



On the basis of the above histogram, answer the following questions:

i. Identify the modal class from the given graph.

a. 80 - 100

b. 20 - 40
c. 60 - 80
d. 40 - 60

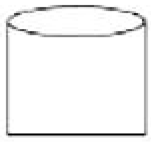
ii. Find the mode of the distribution of marks obtained by the students in an examination.
a. 78
b. 68
c. 48
d. 58

iii. Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.
a. 78
b. 68
c. 48
d. 58

iv. The construction of the cumulative frequency table is useful in determining the
a. Median
b. Mean
c. Mode
d. All of the above

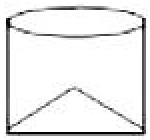
v. What will be the upper limit of the modal class?
a. 100
b. 80
c. 40
d. 60

20. Ganesh a juice seller has his juice shop near Kutub Minar in Delhi. He has three types of glasses, Type A - A glass with a plane bottom, Type B - A glass with a hemispherical raised bottom, and Type C - A glass with the conical raised bottom of height 1.5 cm. The inner diameter of all types of glass is the same as 5cm to serve the customer. The height of the glasses is 10cm (use pi = 3.14)



Type A

Type B



Type C

i. The volume of the glass of type A:

a. 196.25 cm^3

b. 169.52 cm^3

c. 187.25 cm^3

d. 172.55 cm^3

ii. The volume of the hemisphere in the glass of type B:

a. 37.71cm^3

b. 32.71 cm^3

c. 33.71 cm^3

d. 43.34 cm^3

iii. The volume of a glass of type B:

a. 136.54 cm^3

b. 166.45 cm^3

c. 163.54 cm^3

d. 176.54 cm^3

iv. The volume of the cone in the glass of type C:

a. 8.33 cm^3

b. 9.81 cm^3

c. 10.81 cm^3

d. 11.88 cm^3

v. The volume of a glass of type C:

a. 188.88 cm^3

b. 189.99 cm^3

c. 196.89 cm^3

d. 186.44 cm^3

Part-B

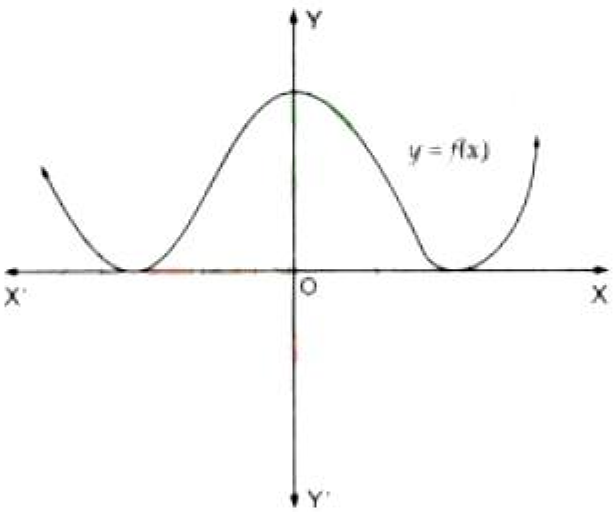
21. Show that $5-2\sqrt{3}$ is an irrational number.

22. Find the coordinates of the point of trisection of the line segment joining the points A(2, – 2) and B(– 7, 4).

OR

If A (-2,-1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram, find the values of a and b.

23. Write the number of real zeros of $f(x)$ where graph of a polynomial $f(x)$ is shown in Fig.



24. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

25. If 3 tan θ =4, find the value of $\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$

OR

Prove that: $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{cosecA-1}{cosecA+1}$

26. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
27. Prove that 2 + 5$\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
28. Solve the quadratic equations by factorization: $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}, x\neq-2, \frac{3}{2}$

OR

Find the value of k for which the roots are real and equal of equation:

$(3k + 1)x^2 + 2(k + 1)x + k = 0$

29. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.
30. The diagonal BD of a parallelogram ABCD intersects the line-segment AE at the point F, where E is any point on the side BC. Prove that DF × EF = FB × FA

OR

A ladder is placed in such a way that its foot is at a distance of 15 m from a wall and its top reaches a window 20 m above the ground. Find the length of the ladder.

31. In a bag there are 44 identical cards with figure of circle or square on them. There are 24 circles, of which 9 are blue and rest are green and 20 squares of which 11 are blue and rest are green. One card is drawn from the bag at random. Find the probability that it has the figure of
    i. square
    ii. green colour,
    iii. blue circle and
    iv. green square.
32. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

33. The daily income of a sample of 50 employees are tabulated as follows:

Income (in Rs.)	1-200	201-400	401-600	601-800
No. of employees	14	15	14	7

Find the mean daily income of employees.

34. A chord of a circle subtends an angle of θ at the centre of the circle. The area of the minor segment cut off by the chord is one eighth of the area of the circle. Prove that $8\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$.

35. Vijay had some bananas, and he divided them into two lots A and B. He sold first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 per five bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

36. A man in a boat moving away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3}$ = 1.732]

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Sample Paper - 05 (2020-21)

Solution

Part-A

1. Clearly denominator = $2^3 \times 7^2 \times 5$

 and numerator 441 = 7 × 7 × 3 × 3

 Number = $\frac{441}{2^2 \times 5 \times 7^2} = \frac{7 \times 7 \times 3 \times 3}{2^2 \times 5 \times 7^2} = \frac{3 \times 3}{2^2 \times 5}$

 When we simplify it we get $\frac{3^2}{2^2 \times 5}$

 So, it is in the form of $2^n \times 5^m$.

 If $\frac{p}{q}$ is a rational number such that the prime factorization of q is of the form $2^n \times 5^m$, where n, m are non-negative integers. Then, given rational number has a decimal expansion which terminates.

 Hence it is a terminating decimal.

OR

 $12 = 2^2 \times 3$

 $\therefore 12^n = (2^2 \times 3)^n = (2^2)^n \times 3^n$

 So, only primes in the factorisation of 12^n are 2 and 3 and, not 5.

 Hence, 12^n cannot end with digit 0 or 5.

2. The given equation is $4x^2 - 4x + 1 = 0$. Here, a = 4, b = -4 and, c = 1

 $\therefore D = b^2 - 4ac = (4)^2 - 4 \times 4 \times 1 = 0$

 We find that D = 0. Therefore, the roots of the given equation are real and equal.

3. Here we have

 x – y = 2 .... (i)

 x + y = 4 .... (ii)

 Add (i) and (ii)

 2x = 6

 x = 3

Put x = 3 in (ii), we get

3 + y = 4

y = 1

So, x = 3, y = 1

∴ a = 3, b = 1

4. Given,PQ is a tangent at a point C to circle with centre O and $\angle CAB = 30^o$.

   Join OC.

   $\therefore OA = OC$ [∵ radii of a circle]

   $\Rightarrow \angle OCA = \angle OAC$ [∵ angles opposite to equal sides of a triangle are equal]

   $\Rightarrow \angle OCA = 30^o$ $[\angle OAC = 30^o]$...(i)

   A tangent is perpendicular to the radius at the point of contact.

   $\therefore OC \perp PQ$

   $\Rightarrow \angle OCP = 90^o$

   $\Rightarrow \angle OCA + \angle PCA = \angle OCP$

   $\Rightarrow \angle OCA + \angle PCA = 90^o$

   $\Rightarrow 30^o + \angle PCA = 90^o$ [∵Using Eq(i)]

   $\Rightarrow \angle PCA = 90^o - 30^o$

   $\Rightarrow \angle PCA = 60^o$

5. Let a be the 1st term of the given A.P and d be a common difference then we have

   $a = \frac{1}{5}$,

   $d = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

OR

   first term, a=3 and common difference,

   d = 1 - 3 = - 2

   Clearly, next term of given AP

   $a_5 = a + 4d = 3 + 4(-2) = 3 - 8 = -5$

   Aliter:

   $a_5 = a_4 + d = -3 + (-2) = -3 - 2 = -5$

6. $a_1 = 1$

   $d = a_2 - a_1 = -1 - 1 = -2$

   $a_5 = a_1 + 4d$

$= 1 + (4)(-2) = 1 - 8 = -7$

$a_6 = a_5 + d = -7 - 2 = -9$

Next two terms are -7 and -9.

7. A polynomial equation is a quadratic equation if it is of the form $ax^2 + bx + c = 0$ such that $a \neq 0$

$\therefore \sqrt{3}x^2 - 2x + \frac{1}{2} = 0$ is a quadratic equation.

OR

The given equation is $3x^2 + 2x - 1 = 0$

Here, $a = 3, b = 2\ and c = -1$

$\therefore D = b^2 - 4ac = 2^2 - 4 \times 3 \times -1 = 4 + 12 = 16$

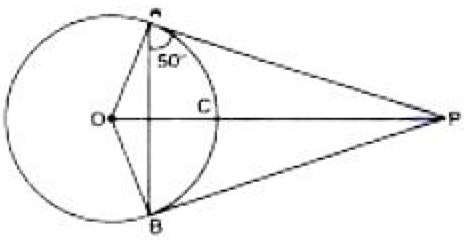
8. Circle touches both the parallel lines

Given, Distance between the parallel lines = 14 cm

We know that, Diameter of circle = Distance between the parallel lines

$\therefore$Radius = $\frac{14}{2}$ = 7 cm

9.



Since AO is the radius and AP is the tangent from an external point P

Therefore, $\angle OAP = 90^0$ (Theorem: Tangents and radius are always perpendicular to each other at the point of contact)

$\Rightarrow \angle OAB = 90^\circ - 50^\circ$

$\Rightarrow \angle OAB = 40^\circ$

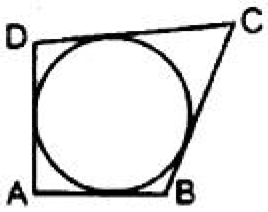
$\angle OAB = \angle OBA = 40^\circ$ (OA and OB are radii)

$\therefore \angle AOB + 40^\circ + 40^\circ = 180^\circ$

$\angle AOB = 180^\circ - 80^\circ = 100^\circ$

Hence $\angle AOB = 100^\circ$

OR



Now,

$AB + CD = BC + AD$

$\Rightarrow\ 12 + 14 = 15 + \text{AD}$

$\Rightarrow AD = 11cm$

10. In $\triangle$PAO and $\triangle$QBO

$\angle$A = $\angle$B = 90°

$\angle$POA = $\angle$QOB (Vertically Opposite Angle)

$\triangle$PAO ~ $\triangle$QBO,(by AA criteria)

$\frac{OA}{OB} = \frac{PA}{QB}$

or, $\frac{6}{4.5} = \frac{4}{QB}$

or, $QB = \frac{4\times4.5}{6}$

Therefore, QB = 3 cm

11. Common difference(d)= a_2 - a_1 = $\frac{(1-p-1)}{p} = \frac{-p}{p} = -1$

therefore, d = -1

12. We know that, tan45° = 1 = Cot45° = Sin90° = Cos0°, Cosec30° = 2 = Sec 60°, putting these values in the given expression, we get:-

$\frac{\tan 45^{\circ}}{cosec 30^{\circ}} + \frac{\sec 60^{\circ}}{\cot 45^{\circ}} - \frac{5\sin 90^{\circ}}{2\cos 0^{\circ}}$

$= \frac{1}{2} + \frac{2}{1} - \frac{5\times1}{2\times1}$

$= \frac{1}{2} + \frac{2}{1} - \frac{5}{2}$

$= \frac{1+4-5}{2}$

$= \frac{0}{2} = 0$

13. $4\tan^2\theta$ - $\frac{4}{\cos^2\theta}$

$= 4\frac{\sin^2\theta}{\cos^2\theta} - \frac{4}{\cos^2\theta}$

$= \frac{4\sin^2\theta-4}{\cos^2\theta}$

$= \frac{4(\sin^2\theta-1)}{\cos^2\theta} = \frac{-4(1-\sin^2\theta)}{\cos^2\theta}$

$= \frac{-4\times(\cos^2\theta)}{\cos^2\theta}$

= -4

14. Let height of cone = h_1 cm.

Given, height of cylinder (h) = 5 cm.

Hence, the volume of a cylinder = $\pi r^2 h$ ......(i)

And , the volume of cone = $\frac{1}{3}\pi r^2 h$ ......(ii)

According to the question ,

$\frac{1}{3}\pi r^2 h_1 = \pi r^2 h$ (given radius of both are same)

$\Rightarrow h_1 = 3h$

$\Rightarrow h_1 = 3 \times 5 = 15$ cm. Ans.

15. Given A.P. 4, 9,14,19, ...

Here a = 4 , d = 9 - 4 = 5

In general n^{th} term of A.P. is $a_n = a + (n - 1)d$

Now $a_{30} - a_{10}$

$= a + 29d - a - 9d = 20d = 20 \times 5 = 100$

16. Total number of balls = 4 + 5 + 6 = 15

Number of all possible outcomes =15

Probability of the event = $\frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Number of favorable outcomes for red or white ball = 4 + 6 = 10

Probability for getting red or white ball = $\frac{10}{15} = \frac{2}{3}$

17. i. (c) It can be observed that Niharika posted the green flag at 5th position flower of the distance AD i.e., 5×5 = 25m from the starting point of 2nd line.

ii. (d) The coordinates of the Green flag are (2, 25).

iii. (b) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A(x, y)

Now by midpoint formula,

$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$x = \frac{2+8}{2} = 5$

$y = \frac{25+20}{2} = 22.5$

Hence, A(x, y) = (5, 22.5)

Therefore, Rashmi should post her blue flag at 22.5 m on the 5th line.

iv. (a) According to the distance formula, Distance between these flags by using the distance formula, D

= $[(8 - 2)^2 + (25 - 20)^2]^{1/2} = (36 + 25)^{1/2} = \sqrt{61}m$

v. Preet posted a red flag at the distance of 4th flower position of AD i.e., 4×5 = 20m from the starting point of the 8th line. Therefore, the coordinates of this point R are (8, 20).

18. i. (c) DE

ii. (b) 1.6 m

iii. (a) 4.8 m

iv. (c) Angle B and Angle E are common

v. (d) lamp-post, the girl

19. First, we will convert the graph into the tabular form as shown below:

Marks obtained	0 - 20	20 - 40	40 - 60	60 - 80	80 -100
Number of students	15	18	21	29	17

i. (c) Modal class is the class having a maximum number of frequency.

Here, the maximum frequency is 29 and it belongs to class 60-80, so Modal class = 60-80

ii. (b) Mode = $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Here, l = 60, f_1 = 29, f_0 = 21, f_2 = 17 and h = 20

Mode = $60 + \frac{29-21}{2\times29-21-17} \times 20$

$= 60 + \frac{8}{58-38} \times 20$ = 68

iii. (d) Mode = 3 median - 2 mean

Mode = 68 and mean = 53 (given)

$\therefore$ 3 median = mode + 2 mean

3 median = 68 + 2 $\times$ 53

Median = $\frac{174}{3}$ = 58

Hence, Median = 58x

iv. (a) Median

v. (b) 80

20. i. (a) 196.25 cm^3

ii. (b) 32.71 cm^3

iii. (c) 163.54 cm^3

iv. (b) 9.81 cm^3

v. (d) 186.44 cm^3

Part-B

21. Let us assume that $5-2\sqrt{3}$ is a rational number.

Then, there must exist positive co primes a and b such that

$\Rightarrow 5-2\sqrt{3}=\frac{a}{b}$

$\Rightarrow -2\sqrt{3}=\frac{a}{b}-5$

$\Rightarrow 2\sqrt{3}=5-\frac{a}{b}$

$\Rightarrow 2\sqrt{3}=\frac{5b-a}{a}$

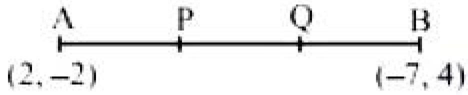
$\Rightarrow \sqrt{3}=\frac{5b-a}{2a}$

A right side $\frac{5b-a}{2a}$ is a rational number so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number.

Hence, our assumption is incorrect and $5-2\sqrt{3}$ is an irrational number.

22. Let P and Q be the points of trisection of AB i.e., AP = PQ = QB



Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula, are

$\left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$, i.e., $(-1,0)$

Now, Q also divides AB internally in the ratio 2 : 1. So, the coordinates of Q are

$\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$, i.e., $(-4,2)$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are (–1, 0) and (– 4, 2).

OR

Given, A (-2,-1), B (a, 0), C (4, b) and D (1, 2)

We know that the diagonals of a parallelogram bisect each other.

Therefore, the coordinates of the mid-point of AC are the same as the coordinates of the mid-point of BD i.e.

$\left(\frac{-2+4}{2}, \frac{-1+b}{2}\right)=\left(\frac{a+1}{2}, \frac{0+2}{2}\right)$

$\Rightarrow \quad \left(1, \frac{b-1}{2}\right) = \left(\frac{a+1}{2}, 1\right)$

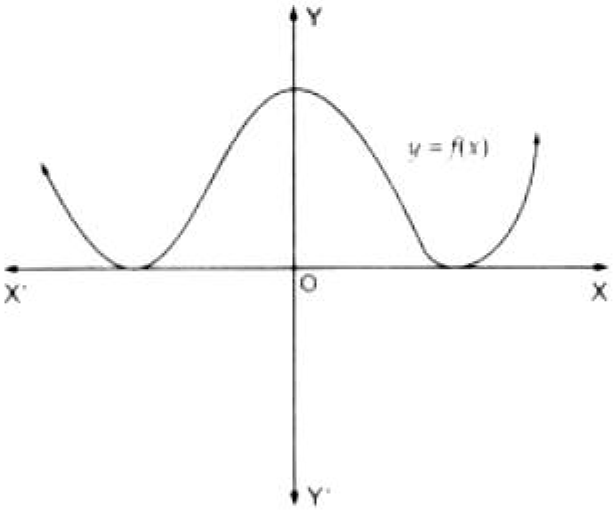
$\Rightarrow \frac{a+1}{2} = 1$ and $\frac{b-1}{2} = 1$

$\Rightarrow a + 1 = 2 \; and \; b - 1 = 2$

$\Rightarrow$ a = 1 and b = 3

Hence, a = 1 and b = 3

23. The graph of a polynomial f(x) touches the x−axis at two points.



We know that if a curve touches the x-axis at two points then it has two distinct roots each repeated two times.

Therefore, the number of zeros of this polynomial is 4

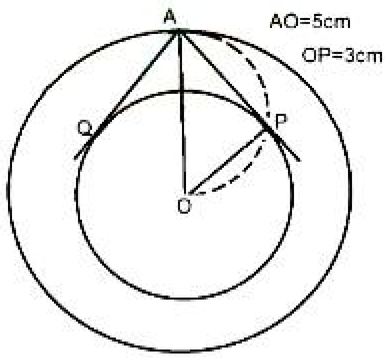
24. A tangent to a circle is perpendicular to the radius at the point of tangency.

Using this property and Pythagoras Theorem,

$AO^2 = AP^2 + OP^2$

$5^2 = AP^2 + 4^2$

Thus, AP = 4



25. $3 \tan \theta = 4$

$\tan \theta = \frac{4}{3}$

Now given expression is $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$

Dividing numerator and denominator by $\cos \theta$, we get

$$\frac{\frac{5 \sin \theta}{\cos \theta} - \frac{3 \cos \theta}{\cos \theta}}{\frac{5 \sin \theta}{\cos \theta} + \frac{2 \cos \theta}{\cos \theta}} = \frac{5 \tan \theta - 3}{5 \tan \theta + 2}$$

Putting $\tan \theta = \frac{4}{3}$, we get, $\frac{5 \times \frac{4}{3} - 3}{5 \times \frac{4}{3} + 2} = \frac{11}{26}$

OR

LHS = $\frac{\cot A - \cos A}{\cot A + \cos A}$

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\frac{\cos A - \sin A \cos A}{\sin A}}{\frac{\cos A + \sin A \cos A}{\sin A}}$$

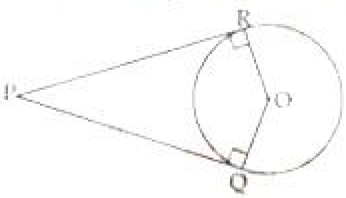
$$= \frac{\cos A(1 - \sin A)}{\cos A(1 + \sin A)}$$

$$= \frac{1 - \sin A}{1 + \sin A}$$

$$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$$

$$= \frac{cosec A - 1}{cosec A + 1}$$ = RHS

26. Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radii and tangent respectively at contact point R.

Therefore, $\angle PRO = 90°$

Similarly $\angle PQO = 90°$

In quadrilateral QOPR, we have

$\angle P + \angle R + \angle O + \angle Q = 360°$

$\Rightarrow \angle P + \angle 90° + \angle O + \angle 90° = 360°$

$\Rightarrow \angle P + \angle O = 360° - 180° = 180°$

These are opposite angles of quadrilateral QORP and are supplementary.

Therefore, Quadrilateral QORP is cyclic. hence, proved.

27. If possible, let us suppose that 2 + 5$\sqrt{3}$ is a rational number

Then, we can write

$2+5\sqrt{3}=\frac{p}{q}$ (Where p and q are co-prime)

$\implies 5\sqrt{3}=\frac{p}{q}-2$

$\implies 5\sqrt{3}=\frac{p-2q}{q}$

$\implies \sqrt{3}=\frac{p-2q}{5q}$

$\implies \sqrt{3}=\frac{integer}{integer}$ (Since p and q are integers)

$\implies \sqrt{3}$ is rational number

which is a contradiction to the given fact that $\sqrt{3}$ is irrational.

$\therefore$ 2 + 5$\sqrt{3}$ cannot be rational

Hence, 2 + 5$\sqrt{3}$ is irrational.

28. We have,

$\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

(x + 3)(2x - 3) = (3x - 7)(x + 2)

$\Rightarrow x(2x-3)+3(2x-3)=3x(x+2)-7(x+2)$

$\Rightarrow 2x^2-3x+6x-9=3x^2+6x-7x-14$

$\Rightarrow 2x^2+3x-9=3x^2-x-14$

$\Rightarrow 3x^2-2x^2-x-3x-14+9=0$

$\Rightarrow x^2-4x-5=0$

In order to factorize x^2-4x-5, we have to find two numbers 'a' and 'b' such that.

a + b = -4 and ab = -5

Clearly, -5 + 1 = -4 and (-5)(1) = -5

$\therefore$ a = -5 and b = 1

Now,

x^2 - 4x - 5 = 0

$\Rightarrow$ x^2 - 5x + 1x - 5 = 0

$\Rightarrow$ x(x - 5) + 1(x - 5) = 0

$\Rightarrow$ (x - 5)(x + 1) = 0

$\Rightarrow$ x - 5 = 0 or x + 1 = 0

$\Rightarrow$ x = 5 or x = -1

OR

According to the question,

$(3k+1)x^2+2(k+1)x+k=0$

Here, a = 3k + 1, b = 2(k + 1) and c = k

$\therefore D=b^2-4ac$

= $[2(k+1)]^2 - 4 \times (3k+1) \times (k)$

= $4(k+1)^2 - 4 \times (3k^2+k)$

= $4[k^2+1+2k]-12k^2-4k$

= $4k^2+4+8k-12k^2-4k$

= $-8k^2+4k+4$

$\Rightarrow D=-8k^2+4k+4$

The given equation will have real and equal roots, if

D = 0

$\Rightarrow -8k^2 + 4k + 4 = 0$

$\Rightarrow 8k^2 - 4k - 4 = 0$

$\Rightarrow 4[2k^2 - k - 1] = 0$

$\Rightarrow 2k^2 - k - 1 = 0$

$\Rightarrow 2k^2 - 2k + 1k - 1 = 0$

$\Rightarrow$ 2k(k - 1) + 1(k - 1) = 0

$\Rightarrow$ (k - 1)(2k + 1) = 0

$\Rightarrow$ k - 1 = 0 or 2k + 1 = 0

$\Rightarrow$ k = 1 or $k=-\frac{1}{2}$

29. i. $p(x) = 2x^3 - 11x^2 + 17x - 6$

    $p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$

    = 16 - 44 + 34 - 6

    = 50 - 50

$= 0$

ii. Again $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$= 54 - 99 + 51 - 6$

$= 105 - 105 = 0$

iii. Again $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$

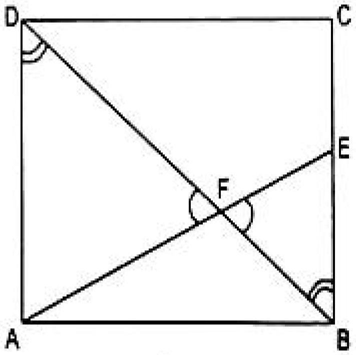
$= \frac{1-11+34-6\times4}{4}$

$= \frac{35-35}{4} = \frac{0}{4} = 0$

Hence, 2,3, $\frac{1}{2}$ are the zeroes of p(x).

30. Given: The diagonal BD of a parallelogram ABCD intersects the line segment AE at the point F, where E is any point on the side BC

To prove: DF $\times$ EF = FB $\times$ FA



Proof: In $\triangle$ FBE and $\triangle$ FDA,

$\angle$ FBE = $\angle$ FDA ....(1) ....[Alt.Int. $\angle$ s]

$\angle$ BFE = $\angle$ AFD (2) ....[Vert. opp. $\angle$ s]

In view of (1) and (2),

$\triangle$FBE ~ $\triangle$FDA ....AA similarity criterion

$\therefore \frac{EF}{AF} = \frac{FB}{FD}$ .... $\because$ Corresponding sides of two similar

$\Rightarrow \frac{EF}{FA} = \frac{FB}{DF}$

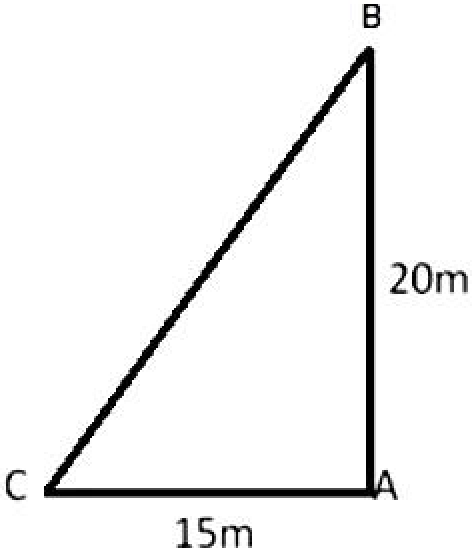
$\Rightarrow$ DF $\times$ EF = FB $\times$ FA

OR

Let AB be the wall where window is at B, CB be the ladder and AC be the distance between the foot of the ladder and wall.

Then,

AB = 20m, AC = 15m, and $\angle CAB = 90^{o}$



By pythagoras theorem, we have

$CB^2 = AB^2 + AC^2$

$= [(20)^2 + (15)^2]m^2$

$= (400 + 225)m^2$

$= 625m^2$

$CB = \sqrt{625}\ m = 25m$

Hence, the length of ladder is 25m.

31. Number of identical cards = 44

Out of 44 cards, one card can be drawn in 44 ways.

∴ Total number of elementary events = 44

Number of circles = 24

Number of blue circles = 9

∴ Number of green circles = 24 - 9 = 15

Number of squares = 20

Number of blue squares = 11

∴ Number of green squares = 20 - 11 = 9

i. Number of square = 20

   ∴ Favourable number of elementary events = 20

   Hence, required probability = $\frac{20}{44} = \frac{5}{11}$

ii. Number of green figures = Number of green circles + Number of green square

= 15 + 9 = 24

∴ Favourable number of elementary events = 24

Hence, required probability = $\frac{24}{44}$ = $\frac{6}{11}$

iii. Number of blue circles = 9

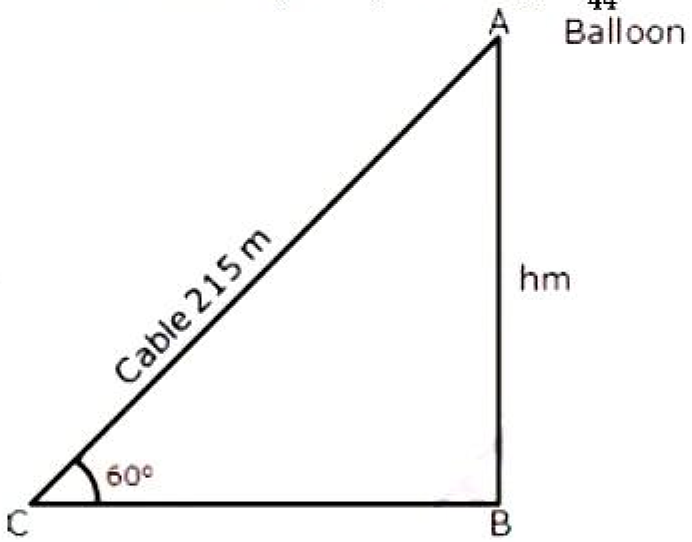
∴ Favourable number of elementary events = 9

Hence, required probability = $\frac{9}{44}$

iv. Number of green squares = 9

∴ Favourable number of elementary events = 9

Hence, the required probability = $\frac{9}{44}$.

32. 

Let the height of balloon from ground = h m

Length of cable $= 215m$

In ΔABC

$\sin 60^o = \frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{215}$

$\Rightarrow h = \frac{215\sqrt{3}}{2}$

$\Rightarrow h = \frac{215\times1.73}{2} = 185.9$

33. The given series is an inclusive series.

Making it an exclusive series, we get

Class interval	**Frequency fi**	**Mid-value X_i**	$u_i = \frac{x_i - A}{h} = \frac{x_i - 500.5}{200}$	$f_i \times u_i$
0.5-200.5	14	100.5	-2	-28
200.5-400.5	15	300.5	-1	-15

400.5-600.5	14	500.5 =A	0	0
600.5-800.5	7	700.5	1	7
	$\sum f_i = 50$			$\sum f_i u_i = -36$

Thus, A=500.5, h =200, ΣF_i =50 and $\Sigma F_i u_i$=-36

$Mean = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$

$= 500.5 + \left\{200 \times \frac{-36}{50}\right\}$

= 500.5 - 144

= 356.5

Thus, the mean daily income of employees is Rs.356.50.

34. Given

Area of minor segment cut off by $AB = \frac{1}{8} \times$ Area of circle

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta = \frac{1}{8} \times \pi r^2$

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi r^2 = \frac{1}{8} \pi r^2 + \frac{1}{2} r^2 \sin\theta$

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi = \frac{\pi}{8} + \frac{1}{2} \sin\theta \quad \left[\text{Divide by } r^2\right]$

$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 4\sin\theta \quad$ [Multiply by 8]

$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 4 \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \quad [\sin 2\theta = 2\sin\theta\cos\theta]$

$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 8\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Hence Proved.

35. Let the number of bananas in lot A is x and in lot B be y.

Condition I: In lot A, cost of 3 bananas = Rs. 2

∴ In lot A, the cost of x bananas = ₹ $x \times \frac{2}{3}$ = ₹ $\frac{2x}{3}$

In lot B, cost of 1 banana = ₹ 1

∴ In lot B, cost of y bananas = ₹ y

The total cost of lot A and lot B = ₹ 400

$\therefore \frac{2x}{3} + y = 400$

∴ 2x + 3y = 1200 ....(1)

Condition II: In lot A, cost of 1 banana = ₹ 1

∴ In lot A, cost of x bananas = ₹ x

In lot B, cost of 5 bananas = ₹ 4

∴ In lot B, cost of y bananas = ₹ $y \times \frac{4}{5}$ = ₹ $\frac{4y}{5}$

Total cost of lot A and lot B = ₹ 460

$\therefore x+\frac{4}{5}y=460$

$\therefore 5x+4y=2300$ ...(2)

Multiplying equation (1) by 5,

$\Rightarrow 10x+15y=6000$ ....(3)

Multiplying equation (2) by 2,

$\Rightarrow 10x+8y=4600$ ...(4)

Subtracting eq. (3) from eq. (4),

$\Rightarrow -7y=-1400$

$\therefore y=\frac{1400}{7}=200$

Substituting the value of y in (1), $\Rightarrow 2x+3\times 200=1200$

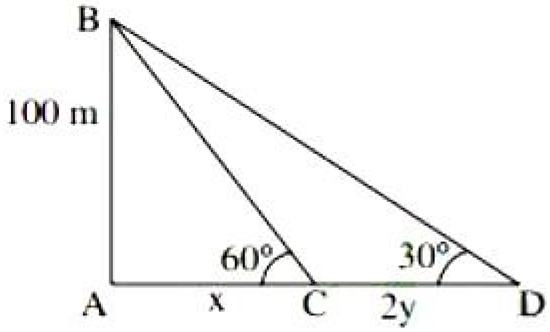
$\therefore 2x=1200-600$

$x=\frac{600}{2}=300$

A number of bananas in lot A = 300.

Number of bananas in lot B = 200

Therefore, the total number of bananas = $300+200=500$

36. 

Let the speed of the boat be y m/min

$\therefore$ CD = 2y

$\tan 60^\circ=\sqrt{3}=\frac{100}{x}\Rightarrow x=\frac{100}{\sqrt{3}}$

$\tan 30^\circ=\frac{1}{\sqrt{3}}=\frac{100}{x+2y}\Rightarrow x+2y=100\sqrt{3}$

$\therefore \text{y}=\frac{100\sqrt{3}}{3}=57.73$

or speed of boat = 57.73 m/min