Sample Paper -05
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours | Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. Prove that cos1°cos2°cos3°……cos180° = 0.

2. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?

3. If the lines given by $3x+2ky=2$ and $2x+5y+1=0$ are parallel, then find value of k.

4. Find the mode of the following data:
120, 110, 130, 110, 120, 140, 130, 120, 140, 120

SECTION – B

5. ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.

6. Prove that the polynomial x^2+2x+5 has no zero.

7. $\Delta ABC \sim \Delta DEF$ and their areas be respectively $64cm^2$ *and* $121cm^2$. If $EF = 15.4cm$, find BC.

8. For any positive real number x, prove that there exists an irrational number *y* such that
$0 < y < x$.

9. Find the value of : $3\sin^2 20^o - 2\tan^2 45^o + 3\sin^2 70^o$.

10. If α and β are the zeros of the quadratic polynomial f(x) = x^2 – px + q, then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

SECTION – C

11. If n is an old positive integer, show that (n^2-1) divisible by 8.

12. Find the condition that the zeros of the polynomial $f(x)=x^3-px^2+qx-r$ may be in arithmetic progression.

13. A vertical pole of length 6 cm casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

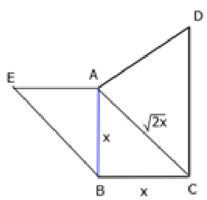
14. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x-5)^\circ$, $\angle B = (y+5)^\circ$, $\angle C = (2y+15)^\circ$ and $\angle D = (4x-7)^\circ$.

15. Find the values of x and y if the total frequency and the median of the following data is 100 and 525, respectively.

Class interval	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	x	12	17	20	y	9	7	4

16. In aΔABC, right angled at B, if AB = 4 and BC = 3, find all the six trigonometric ratios of $\angle$A.

17. ABC is an isosceles triangle right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.

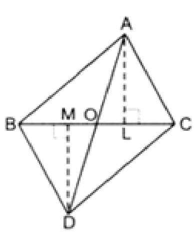


18. I am 3 times as old as my son. 5 years later, I shall be two and a half times as old as my son. How old am I and how old is my son?

19. Prove that $\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan\theta)^2$

20. In the given fig, ΔABC and ΔDBC on the same base BC. If AD intersects BC at O. Prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$$



SECTION – D

21. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

22. Prove that : $\left(\frac{1+\tan^2 A}{1-\tan^2 A}\right)=\left(\frac{1-\tan A}{1+\tan A}\right)^2=\tan^2 A.$

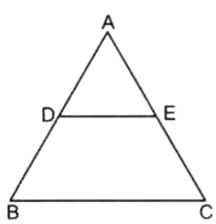
23. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$, prove that $x^2+y^2=1$.

24. Find the mean marks of students from the following cumulative frequency distribution:

Marks	**Number of students**	**Marks**	**Number of students**
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

25. What must be added to $f(x)=4x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is divisible by $g(x)=x^2+2x-3$?

26. Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
Using the above result, do the following:
In given figure, DE||BC and BD = CE. Prove that ΔABC is an isosceles triangle.



27. Draw the graphs of the following equations on the same graph paper.
$2x + y = 2$; $2x + y = 6$
Find the coordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium formed.

28. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

29. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students	Weight (in kg)	Number of students
Less than 38	0	Less than 46	14
Less than 40	3	Less than 48	28
Less than 42	5	Less than 50	32
Less than 44	9	Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

30. Prove $\dfrac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}=\sin^2 A\cos^2 A$

31. Rohan's mother decided to distribute 900 bananas among patients of a hospital on her birthday. If the female patients are twice the male patients and the male patients are thrice the child patients in the hospital, each patient will get only one apple.

(i) Find the number of child patients, male patients and female patients in the hospital.

(ii) Which values are depicted by Rohan's father in the question?

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Time allowed: 3 hours **ANSWERS** Maximum Marks: 90

SECTION – A

1. We have

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots \cos 180^\circ$$

LHS $= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ$ = R.H.S

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots 0 \times \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ = 0$$

2. Product of two numbers = Product of their LCM and HCF

$$\Rightarrow 1800 = 12 \times LCM \Rightarrow LCM = \frac{1800}{12} = 150.$$

3. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \; i.e., k = \frac{15}{4}$$

4. Let us first form the frequency table for the given data as given below:

Value (x_i)	110	120	130	140
Frequency (f_i)	2	4	2	2

We observe that the value 120 has the maximum frequency.
Thus, the mode is 120.

SECTION – B

5. ΔABC is right-angled at C.

$\therefore AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AC^2 + AC^2$ $[\because SC = BC]$

$\Rightarrow AB^2 = 2AC^2$

6. Let f(x) = $x^2 + 2x + 5$

$$= x^2 + 2x + 1 + 4$$

$$= (x+1)^2 + 4$$

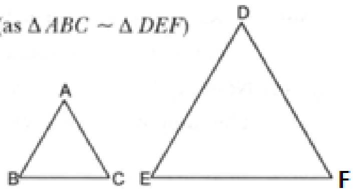
Now, for every real value of x, $(x+1)^2 \geq 0$

$\Rightarrow$ For every real value of x, $(x+1)^2 + 4 \geq 4$

∴ For every real value of x, $f(x) \geq 4$ and hence it has no zero.

7. We have, $\dfrac{area\,of\,\Delta ABC}{area\,of\,\Delta DEF} = \dfrac{BC^2}{EF^2}$

(as $\Delta ABC \sim \Delta DEF$)



$$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{BC}{15.4} = \frac{8}{11}$$

$$\therefore\ BC = \frac{8}{11} \times 15.4 = 11.2cm$$

8. If x is irrational, then $y = \dfrac{x}{2}$ is also an irrational number such that $0 < y < x$.

If x is rational, then $\dfrac{x}{\sqrt{2}}$ is an irrational number such that $\dfrac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

∴ $y = \dfrac{x}{\sqrt{2}}$ is an irrational number such that $0 < y < x$.

9. $3\sin^2 20^o - 2\tan^2 45^o + 3\sin^2 70^o$

$= 3\sin^2 20^o(90^o - 70^o) - 2(1)^2 + 3\sin^2 70^o$ $\quad [\because \tan 45^o = 1]$

$= 3\cos^2 70^o - 2 + 3\sin^2 70^o$ $\quad [\because \sin(90-\theta) = \cos\theta]$

$= 3(\sin^2 70^o + \cos^2 70^o) - 2$

$= 3 \times 1 - 2 = 3 - 2 = 1.$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

10. Since α and β are the zeros of the polynomial $f(x) = x^2 - px + q$,

∴ $\alpha + \beta = -\left(\dfrac{-p}{1}\right) = p$ and $\alpha\beta = \dfrac{q}{1} = q$

Thus, $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{p}{q}$

SECTION – C

11. We know that an old positive integer n is of the form (4q+1) or (4q+3) for some integer q.

Case I When n=(4q+1)

In this case

$n^2 - 1 = (4q+1)^2 - 1 = 16q^2 + 8q = 8q(2q+1)$

Which is clearly divisible by 8.

Case II When n=(4q+3)

In this case, We have

$n^2 - 1 = (4q+3)^2 - 1 = 16q^2 + 14q + 8 = 8(2q^2 + 3q + 1)$

Which is clearly divisible by 8.

12. Let $a - d$, a and $a + d$ be the zeros of the polynomial $f(x)$. Then,

Sum of the zeros = $\frac{Coefficient\ of\ x^2}{Coefficient\ of\ x^3}$

$\Rightarrow \quad (a-d)+a+(a+d) = -\frac{(-p)}{1}$

$\Rightarrow \quad 3a = p \qquad \Rightarrow \qquad a = \frac{p}{3}$

Since a is a zero of the polynomial $f(x)$. Therefore,

$f(a) = 0$

$\Rightarrow \quad a^3 - pa^2 + qa - r = 0$

$\Rightarrow \quad \left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0 \qquad \left[\because a = \frac{p}{3}\right]$

$\Rightarrow \quad p^3 - 3p^3 + 9pq - 27r = 0$

$\Rightarrow \quad 2p^3 - 9pq + 27r = 0$

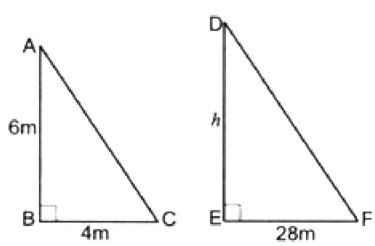
13. Let AB be a vertical pole of length 6 m and BC be its shadow and DE be tower and EF b its shadow. Join AC and DF.

Now, in $\Delta ABC\ and\ \Delta DEF$, we have

$\angle B = \angle C = 90^o$

$\angle C = \angle F$ (Angle of elevation of the sun)

$\therefore \Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)



Thus, $\frac{AB}{DE}=\frac{BC}{EF}$

$\Rightarrow \frac{6}{h}=\frac{4}{28}$ (Let DE=h)

$\Rightarrow \frac{6}{h}=\frac{1}{7} \Rightarrow h=42$

Hence, height of tower, DE = 42 m

14. We know that the sum of the opposite angles of a cyclic quadrilateral is 180°. In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angles.

$\therefore$ $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow$ $2x - 1 + 2y + 15 = 180°$ and $y + 5 + 4x - 7 = 180°$

$\Rightarrow$ $2x + 2y = 166°$ and $4x + y = 182°$

$\Rightarrow$ $x + y = 83°$ ...(i)

And, $4x + y = 182°$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$3x = 99 \Rightarrow x = 33$

Substituting $x = 33$ in equation (i), we get $y = 50$

Hence, $\angle A = (2x-1)° = (2\times33-1)° = 65°$, $\angle B = (y+5)° = (50+5)° = 55°$

$\angle C = (2y+15)° = (2\times50+15)° = 115°$ and $\angle D = (4x-7)° = (4\times33-7)° = 125°$

15.

Calculation of median

Class intervals	**Frequency (*f*)**	**Cumulative frequency (*cf*)**
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$

900-1000	4	$76 + x + y$
		Total = 100

We have, $N = \sum fi = 100$

$\Rightarrow \quad 76 + x + y = 100$

$\Rightarrow \quad x + y = 24$

It is given that the median is 525. Clearly, it lies in the class 500-600.

$\therefore \quad l = 500, h = 100, f = 20, F = 36 + x$ and $N = 100$

Now, median = $l + \dfrac{\frac{N}{2} - F}{f} \times h$

$\Rightarrow \quad 525 = 500 + \dfrac{\frac{100}{2} - (36 + x)}{20} \times 100$

$\Rightarrow \quad 525 - 500 = \dfrac{50 - 36 - x}{20} \times 100$

$\Rightarrow \quad 25 = (14 - x) \times 5$

$\Rightarrow \quad 25 = 70 - 5x$

$\Rightarrow \quad 5x = 45$

$\Rightarrow \quad x = \dfrac{45}{5} \Rightarrow \quad x = 9$

Putting $x = 9$ in $x + y = 24$, we get

$9 + y = 24$

$\Rightarrow \quad y = 24 - 9 = 15$

Thus, $x = 9$ and $y = 15$.

16. We have AB = 4 and BC = 3

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow \quad AC = \sqrt{4^2 + 3^2}$

$\Rightarrow \quad AC = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

Base = AB = 4, Perpendicular = BC = 3 and Hypotenuse = AC = 5

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$, $\tan A = \frac{BC}{AB} = \frac{3}{4}$

$\text{cosec}A = \frac{AC}{BC} = \frac{5}{3}$, $\sec A = \frac{AC}{AB} = \frac{5}{4}$ and $\cot A = \frac{AB}{BC} = \frac{4}{3}$

17. Let AB = BC = x.

It is given that ΔABC is a right-angled at B.

$\therefore \quad AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = x^2 + x^2$

$\Rightarrow \quad AC = \sqrt{2}x$

It is given that

$\Delta ABE \sim \Delta ACD$

$$\Rightarrow \quad \frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{AB^2}{AC^2}$$

$$= \frac{x^2}{\left(\sqrt{2}x\right)^2}$$

$$= \frac{1}{2}$$

18. Suppose my age is x years and my son's age is y years. Then,

$x = 3y$ ...(i)

5 years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$\therefore \quad x + 5 = \frac{5}{2}(y+5)$

$\Rightarrow \quad 2x - 5y - 15 = 0$ ...(ii)

Putting $x = 3y$ in equation (ii), we get

$6y - 5y - 15 = 0 \quad \Rightarrow \quad y = 15$

Putting $y = 15$ in equation (i), we get

$x = 45$

19. L.H.S$= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$ [Rationalising the denominator]

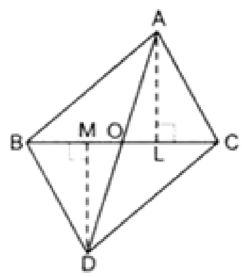
$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta - \tan\theta)^2 = RHS$$

20. Given: $\Delta ABC \; and \; \Delta DBC$ are the on the same base BC and AD intersects BC at O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction**:** Draw AL $\perp$ BC and DM $\perp$ BC



Proof: In $\Delta AOL \; and \; \Delta DMO$, w have

$\angle ALO = \angle DMO = 90^o \; and$

$\angle AOL = \angle DOM$ (Vertically opposite angles)

$\therefore \Delta ALO \sim \Delta DMO$ (By AA-Similarity)

$$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO} \qquad ...(i)$$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AL}{\frac{1}{2} BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO} \text{ (Using (i))}$$

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

SECTION – D

21. Let the numerator and denominator of the fraction be x and y respectively.

Then,

Fraction = $\frac{x}{y}$

It is given that

Denominator = 2(Numerator) + 4

$\Rightarrow \quad y = 2x + 4$

$\Rightarrow \quad 2x - y + 4 = 0$

According to the given condition, we have

$y - 6 = 12(x - 6)$

$\Rightarrow \quad y - 6 = 12x - 72$

$\Rightarrow \quad 12x - y - 66 = 0$

Thus, we have the following system of equations

$2x - y + 4 = 0 \quad$ ...(i)

$12x - y - 66 = 0 \quad$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$10x - 70 = 0$

$\Rightarrow \quad x = 7$

Putting $x = 7$ in equation (i), we get

$14 - y + 4 = 0$

$\Rightarrow \quad y = 18$

Hence, required fraction = $\frac{7}{18}$

22. LH.S = $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

R.H.S. = $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A - 1}\times \tan A\right)^2$$

$(-\tan A)^2 = \tan^2 A$

L.H.S = R.H.S

23. We have,

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow \quad x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\cos\theta$$

$$\Rightarrow \quad x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\cos\theta \qquad [\because x\sin\theta = y\cos\theta]$$

$$\Rightarrow \quad x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta$$

$$\Rightarrow \quad x = \frac{\sin\theta\cos\theta}{\sin\theta} = \cos\theta$$

Now,

$$x\sin\theta = y\cos\theta$$

$$\Rightarrow \quad \cos\theta\sin\theta = y\cos\theta \qquad [\because x = \cos\theta]$$

$$\Rightarrow \quad y = \frac{\cos\theta\sin\theta}{\cos\theta} = \sin\theta$$

$$\therefore \quad x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$$

24. Here, we have the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on.

Thus, we obtain the following frequency distribution:

Marks	Number of students	Marks	Number of students
0-10	3	50-60	15
10-20	5	60-70	12
20-30	7	70-80	6
30-40	10	80-90	2
40-50	12	90-100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.

Computation of mean

Marks (x_i)	Mid-value	Frequency(f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
0-10	5	3	–5	–15
10-20	15	5	–4	–20
20-30	25	7	–3	–21
30-40	35	10	–2	–20
40-50	45	12	–1	–12
50-60	55	15	0	0
60-70	65	12	1	12
70-80	75	6	2	12
80-90	85	2	3	6
90-100	95	8	4	32
Total		$\sum f_i = 80$		$\sum f_i u_i = -26$

We have,

$N = \sum f_i = 80, \sum f_i u_i = -26$, A = 55 and h = 10

$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

$= 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75$ marks

25. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$\Rightarrow \quad f(x) - r(x) = g(x) \times q(x)$

$\Rightarrow \quad f(x) + \{-r(x)\} = g(x) \times q(x)$

Clearly, RHS is divisible by $g(x)$. Therefore, LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$
\begin{array}{r|l}
 & \quad\quad\quad\quad 4x^2-6x+22 \\
x^2+2x-3 & 4x^4+2x^3-2x^2+x-1 \\
 & 4x^4+8x^3-12x^2 \\
 & -\quad -\quad + \\
\hline
 & \quad -6x^3+10x^2+x-1 \\
 & \quad -6x^3-12x^2+18x \\
 & \quad +\quad +\quad - \\
\hline
 & \quad\quad 22x^2-17x-2 \\
 & \quad\quad 22x^2+44x-66 \\
 & \quad\quad -\quad -\quad + \\
\hline
 & \quad\quad\quad -61x+65
\end{array}
$$

$\therefore \quad r(x) = -61x + 65$

Thus, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

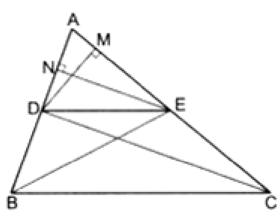
26. Given: A triangle ABC in which a line Parallel to sides BC intersect other two sides AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and then draw DM $\perp$ AC an EN $\perp$ AB.

Proof: Area of $\Delta ADE = \left(\frac{1}{2} base \times height\right)$

So, $ar(\Delta ADE) = \frac{1}{2} AD \times EN$



And $ar(\Delta BDE) = \frac{1}{2} DB \times EN$

Similarly, $ar(\Delta ADE) = \frac{1}{2} AE \times DM$

And $ar(\Delta DEG) = \frac{1}{2} EC \times DM$

Therefore, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ...(i)

And $\dfrac{ar(\Delta ADE)}{ar(\Delta DEC)} = \dfrac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \dfrac{AE}{EC}$ ...(ii)

Now, ΔBDE *and* ΔDEG are on the same base DE and between the same parallel lines BC and DE.

So, $ar(\Delta BDE) = ar(\Delta DEG)$ ...(iii)

Therefore, from (i), (ii) and (iii) we have,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Second Part

As DE||BC

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} + \frac{AE + EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$$

$$\Rightarrow AB = AC \quad (As\, DB = EC)$$

$\therefore$ ΔABC is an isosceles triangle.

27. Graph of the equation $2x + y = 2$:

When $y = 0$, we have $x = 1$

When $x = 0$, we have $y = 2$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 2$.

x	1	0
y	0	2

Graph of the equation $2x + y = 6$:

When $y = 0$, we get $x = 3$

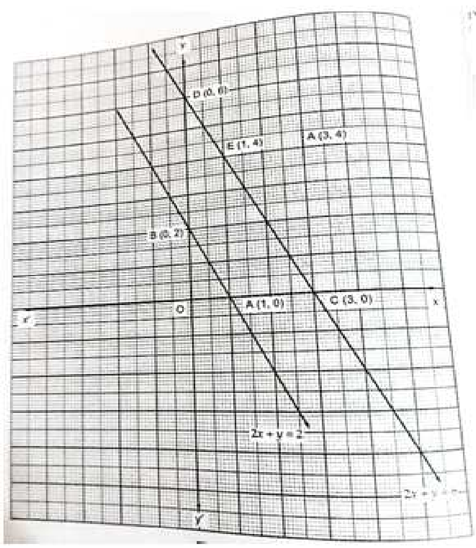
When $x = 0$, we get $y = 6$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$.

x	3	0
y	0	6

Plotting points A(1, 0) and B (0, 2) on the graph paper on a suitable scale and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 2$ as shown in the graph.

Plotting points C(3, 0) and D(0, 6) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 6$ as shown in the graph.



Clearly, lines AB and CD form trapezium ACDB.

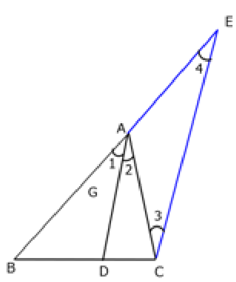
Also, area of trapezium ACDB = Area of ΔOCD – Area of ΔOAB

$$= \frac{1}{2}(OC \times OD) - \frac{1}{2}(OA \times OB)$$

$$= \frac{1}{2}(3 \times 6) - \frac{1}{2}(1 \times 2) = 8 \text{ sq.units}$$

28. Given: AΔABC in which AD is the internal bisector of ∠A and meets BC in D.

To prove: $\frac{BD}{DC} = \frac{AB}{AC}$



Construction: Draw CE || DA to meet BA produced in E.

Proof: Since CE || DA and AC cuts them,

$\therefore \quad \angle 2 = \angle 3$ [Alternate angles] ...(i)

And, $\angle 1 = \angle 4$ [Corresponding angles] ...(ii)

But, $\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle A$]

From (i) and (ii), we get

$\angle 3 = \angle 4$

Thus, in ΔACE, we have

$\angle 3 = \angle 4$

$\Rightarrow \quad AE = AC$ [Sides opposite to equal angles are equal] ...(iii)

Now, in ΔBCE, we have

$DA \parallel CE$

$\Rightarrow \quad \frac{BD}{DC} = \frac{BA}{AE}$ [Using Basic Proportionality Theorem]

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AC}$ [$\because$ BA = AB and AE = AC (From (iii)]

Thus, $\frac{BD}{DC} = \frac{AB}{AC}$

29. To represent the data in the table graphically, we mark the upper limits of the class interval on x-axis and their corresponding cumulative frequency on y-axis choosing a convenient scale.

Now, let us plot the points corresponding to the ordered pair given by (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) on a graph paper and join them by a freehand smooth curve.

Thus, the curve obtained is the less than type ogive.

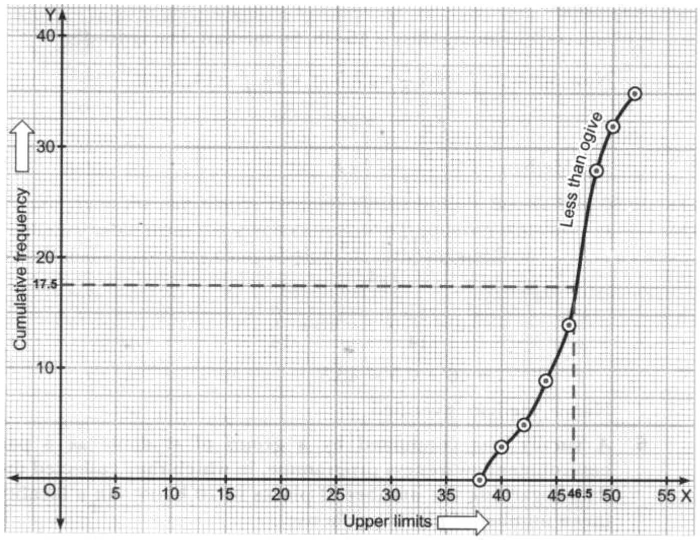


Fig. 6.3

Now, locate $\frac{n}{2} = \frac{35}{2} = 17.5$ on the y-axis,

We draw a line from this point parallel to x-axis cutting the curve at a point from this point, draw a perpendicular line to the x-axis. The point of intersection of this perpendicular with the x-axis gives the median of **the** data. Here it is 46.5

Let us make the following table in order to find median by using formula.

Weight (in kg)	No. of Students frequency (f_i)	Cumulative frequency (cf)
36-38	0	0
38-40	3	6
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Total	$\sum f_i = 35$	

Here, $n = 35, \frac{n}{2} = \frac{35}{2} = 17.5$, cumulative frequency greater than $\frac{n}{2} = 17.5$ is 28 and corresponding class is 46-48. So median class is 46 - 48.

Now, we have $l = 46, \frac{n}{2} = 17.5, cf = 14, f = 14, h = 2$

$$\therefore Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2$$

$$= 46 + \frac{3.5}{14} \times 2 = 46 + \frac{7}{14}$$

$$= 46 + 0.5 = 46.5$$

Hence, median is verified.

30. We have,

LHS $= \dfrac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A}$

$$= \frac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\left(\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}\right)}$$

$$= \frac{\left(1 + \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)(\sin A - \cos A)}{\left(\frac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}\right)}$$

$$= \frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A} \times \frac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A}(\sin A - \cos A)$$

$$= \frac{\sin A \cos A + 1}{\sin^3 A - \cos^3 A} \times (\sin^2 A \cos^2 A)(\sin A - \cos A)$$

$$= \frac{(\sin A \cos A + 1)(\sin^2 A \cos^2 A)(\sin A - \cos A)}{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)} \qquad \left[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\right]$$

$= \dfrac{(\sin A \cos A + 1)\sin^2 A \cos^2 A}{1 + \sin A \cos A}$

$= \sin^2 A \cos^2 A$ = RHS

31. (i) Let the number of child patients in the hospital be x.

Then, the number of male patients = 3x

And, the number of female patients = 2(3x) = 6x

According to the question,

$6x + 3x + x = 900$

$\Rightarrow \quad 10x = 900$

$\Rightarrow \quad x = \dfrac{900}{10} = 90$

Thus, the number of child patients in the hospital is 90.

And, the number of male patients = 3 × 90 = 270

The number of female patients = 2 × 270 = 540

(i) The values depicted by Rohan's father in the question are charity and empathy.