EXERCISE 21 (Pg.No.: 982)

Find the general solution for each of the following differential equation

1. $\frac{dy}{dx}+\frac{1}{x}\cdot y=x^2$

Sol. Given differential equation is $\frac{dy}{dx}+\frac{1}{x}\cdot y=x^2$

This is of the form $\frac{dy}{dx}+py=Q$

Where $P=\frac{1}{x}$ and $Q=x^2$

This the given differential equation is a linear differential equation

Now $I.F=e^{\int P dx}$

$=e^{\int \frac{1}{x} dx}=e^{\log x}=x$

Three fore the solution is given by $y(I.F)=\int (I.F)Q\,dx+C$

$\Rightarrow y\cdot x=\int x\cdot x^2 dx+C$

$\Rightarrow xy=\frac{x^4}{4}+C \Rightarrow y=\frac{x^3}{4}+\frac{C}{x}$

This is the required solution of given differential equation

2. $x\frac{dy}{dx}+2y=x^2$

Sol. Given differential equation is $x\cdot\frac{dy}{dx}+2y=x^2$

$\Rightarrow \frac{dy}{dx}+\frac{2}{x}\cdot y=x$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=\frac{2}{x}$ and $Q=x$

This the given differential equation is linear differential equation

Now $I.F=e^{\int P dx}$

$\Rightarrow I.F=e^{\int \frac{2}{x} dx}$

$\Rightarrow I.F=e^{2\log x} \Rightarrow I.F=x^2$

Therefore the solution is given by $y\cdot(I.F)=\int (I.F)x\,dx+C$

$\Rightarrow y\cdot x^2=\int x^2\cdot x\,dx+C$

$$\Rightarrow x^2 y = \frac{x}{4} + C \Rightarrow y = \frac{x^2}{4} + \frac{C}{x^2}$$

This is the required solution of given differential equation

3. $2x\frac{dy}{dx} + y = 6x^3$

Sol. Given differential equation is $2x\frac{dy}{dx} + y = 6x^3$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2x} \cdot y = 3x^2$$

This is of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{1}{2x}$ and $Q = 3x^2$

Thus the differential equation is linear

Now $I.F = e^{\int P\,dx}$

$$= e^{\int \frac{dx}{2x}} = e^{\frac{1}{2}\log x} = x^{1/2}$$

There fore the solution is given by

$$y(I.F) = \int (I.F) Q\,dx + C$$

$$\Rightarrow y \cdot x^{1/2} = \int x^{1/2} \cdot 3x^2 dx + C$$

$$\Rightarrow y \cdot x^{1/2} = 3\int x^{5/2} dx + C \Rightarrow y \cdot x^{1/2} = 3 \cdot \frac{x}{7/2} + C$$

$$\Rightarrow y \cdot x^{1/2} = \frac{6}{7} x^{7/2} + C \Rightarrow y = \frac{6}{7} \cdot x^3 + \frac{C}{\sqrt{x}}$$

This is the required solution of given differential equation

4. $x\frac{dy}{dx} + y = 3x^2 - 2, x > 0$

Sol. Given differential equation is $x \cdot \frac{dy}{dx} + y = 3x^2 - 2$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x} \cdot y = 3x - \frac{2}{x}$$

This is of the form $\frac{dy}{dx} + Py = Q,$ where $P = \frac{1}{x}$ and $Q = 3x - \frac{2}{x}$

This the differential equation is linear

Now $I.F = e^{\int P dx}$

$$\Rightarrow I.F = e^{\int \frac{1}{x} dx} \Rightarrow I.F = e^{\log x} \Rightarrow I.F = x$$

There fore the solution is given by $y.(I.F) = f(I.F) dx + C$

$$\Rightarrow y \cdot x = \int x\left(3x - \frac{2}{x}\right) dx + C$$

$$\Rightarrow x \cdot y = \int (3x^2 - 2) dx + C$$

$$\Rightarrow x \cdot y = x^3 - 2x + C \Rightarrow y = x^2 - 2 + \frac{C}{x}$$

This is the required solution of given differential equation

5. $x\frac{dy}{dx}-y=2x^3$

Sol. Given differential equation is $x\cdot\frac{dy}{dx}-y=2x^3$

$\Rightarrow \frac{dy}{dx}-\frac{1}{x}y=2x^2$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=-\frac{1}{x}$ and $Q=2x^2$

Now, $I.F=e^{\int Pdx}$

$\Rightarrow I.F=e^{\int\left(-\frac{1}{x}\right)dx}$

$\Rightarrow I.F=e^{-\log x} \Rightarrow I.F=\frac{1}{x}$

Therefore the solution is $y(I-F)=\int(I-F)Qdx+C$

$\Rightarrow y.\frac{1}{x}=\int\frac{1}{x}\left(2x^2\right)dx+C$

$\Rightarrow \frac{y}{x}=\int 2x\,dx+C \Rightarrow \frac{y}{x}=x^2+C \Rightarrow y=x^3+Cx$

This is the required solution of given differential equation

6. $x\frac{dy}{dx}-y=x+1$

Sol. The given equation may be written as $\frac{dy}{dx}-\frac{1}{x}y=\frac{x+1}{x}$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=\frac{-1}{x}$, $Q=\frac{x+1}{x}$

I.F. $=e^{\int P\,dx}=e^{-\int\frac{1}{x}dx}=e^{-\log|x|}=\frac{1}{x}$

So, the required solution is given by, $y\times \text{I.F}=\int Q\times \text{I.F}\,dx$

$\Rightarrow y.e^{2x}=\int x.e^{4x}.e^{2x}dx \Rightarrow y.e^{2x}=\int x.e^{6x}dx \Rightarrow y.e^{2x}=x\int e^{6x}dx-\int\left[\frac{d(x)}{dx}\int e^{5x}dx\right]dx$

$\Rightarrow y.e^{2x}=x.\frac{e^{6x}}{6}-\int\frac{e^{6x}}{6}dx \Rightarrow ye^{2x}=\frac{x}{6}e^{6x}-\frac{e^{6x}}{36}+c \quad \therefore y=\frac{x\,e^{4x}}{6}-\frac{e^{4x}}{36}+c\,e^{-2x}$

7. $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{\left(1+x^2\right)}$

Sol. Given differential equation is $\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$

$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{1}{\left(1+x^2\right)^2}$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=\frac{2x}{1+x^2}$ and $Q=\frac{1}{\left(1+x^2\right)^2}$

This the given differential equation is linear

Now. $I.F=e^{\int P dx} \Rightarrow I.F=e^{\int \frac{2x\,dx}{1+x^2}}$

$=e^{\log\left(1+x^2\right)}=1+x^2$

Therefore the solution is given by $y\cdot(I.F)=\int(I.F)Q+C$

$\Rightarrow y\left(1+x^2\right)=\int\left(1+x^2\right)\frac{1}{\left(1+x^2\right)^2}dx+C$

$\Rightarrow y\left(1+x^2\right)=\int\frac{dx}{1+x^2}+C$

$\Rightarrow y\cdot\left(1+x^2\right)=\tan^{-1}x+C$

this is the required solution of given differential equation

8. $\left(1-x^2\right)\frac{dy}{dx}+xy=x\sqrt{1-x^2}$

Sol. Given differential equation is $\left(1-x^2\right)\cdot\frac{dy}{dx}+xy=x\sqrt{1-x^2}$

$\Rightarrow \frac{dy}{dx}+\frac{x}{1-x^2}\cdot y=\frac{x}{\sqrt{1-x^2}}$

This is of the form $\frac{dy}{dx}+Py=Q$. where

$P=\frac{x}{1-x^2}$ and $Q=\frac{x}{\sqrt{1-x^2}}$

This the given differential equation is linear

Now, $I.F=e^{\int P dx}$

$\Rightarrow I.F=e^{\int\frac{x}{1-x^2}dx}$

$\Rightarrow I.F=e^{\frac{1}{2}\int\frac{2x}{1-x^2}dx} \Rightarrow I.F=e^{-\frac{1}{2}\log\left(1-x^2\right)}$

$\Rightarrow I.F=\left(1-x^2\right)^{-\frac{1}{2}} \Rightarrow I.F=\frac{1}{\sqrt{1-x^2}}$

Therefore the solution is given by

$(I.F)\cdot y=\int(I.F)Q+C$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}\cdot y=\int\frac{1}{\sqrt{1-x^2}}\cdot\frac{x}{\sqrt{1-x^2}}dx+C$

$\Rightarrow \frac{y}{\sqrt{1-x^2}}=\int\frac{x}{1-x^2}dx+C \Rightarrow \frac{y}{\sqrt{1-x^2}}=-\frac{1}{2}\int\frac{-2x}{1-x^2}dx+C$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{1}{2}\log\left(1-x^2\right)+C \Rightarrow y = -\frac{1}{2}\sqrt{1-x^2}\cdot\log\left(1-x^2\right)+C$$

This is the required solution of given differential equation

9. $\left(1-x^2\right)\frac{dy}{dx}+xy = ax$

Sol. Given differential equation is $\left(1-x^2\right)\frac{dy}{dx}+xy = ax$

$$\Rightarrow \frac{dy}{dx}+\frac{x}{1-x^2}\cdot y = \frac{ax}{1-x^2}$$

This is of the form $\frac{dy}{dx}+py = Q$

Where $P = \frac{x}{1-x^2}$ and $Q = \frac{ax}{1-x^2}$

This the given differential equation is linear

Now, $I.F = e^{\int P\,dx}$

$$= e^{\int \frac{x\,dx}{1-x^2}}$$

$$= e^{-\frac{1}{2}\int \frac{-2x}{1-x^2}dx} = e^{-\frac{1}{2}\log\left(1-x^2\right)} = \left(1-x^2\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-x^2}}$$

Solution is

$$(I.F)\cdot y = \int (I.F)Q\,dx + C$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}\cdot y = \int \frac{1}{\sqrt{1-x^2}}\cdot\frac{ax}{1-x^2}dx + C$$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{\left(1-x^2\right)^{3/2}}dx + C$$

$$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{a}{2}\int \frac{-2x}{\left(1-x^2\right)^{3/2}}dx + C \quad \text{..... (i)}$$

Now, $I = \int \frac{-2x}{\left(1-x^2\right)^{3/2}}$

Let $t = 1-x^2$

$\Rightarrow dt = -2x\,dx$

$$\therefore I = \int \frac{dt}{t^{3/2}} \Rightarrow I = \int t^{-3/2}dt \Rightarrow I = \int t^{-3/2}dt$$

$$\Rightarrow I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} \Rightarrow I = -2\cdot\frac{1}{\sqrt{t}} = \frac{-2}{\sqrt{1-x^2}}$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = -\frac{2}{x}\times\frac{-2}{\sqrt{1-x^2}}+C \Rightarrow y = a + C\sqrt{1-x^2}$$

This is the required solution of given differential equation

10. $\left(x^2+1\right)\frac{dy}{dx}-2xy = \left(x^2+1\right)\left(x^2+2\right)$

Sol. Given differential equation is $\left(x^2+1\right)\frac{dy}{dx}-2xy=\left(x^2+1\right)\left(x^2+2\right)$

$\Rightarrow \frac{dy}{dx}+\left(\frac{-2x}{x^2+1}\right)y=x^2+2$

This is of the form $\frac{dy}{dx}+Py=Q,$ where

$P=\frac{-2x}{x^2+1}$ and $Q=x^2+2$

Thus the given differential equation is linear differential equation

Now $I.F=e^{\int P\,dx}$

$=e^{\int \frac{-2x}{x^2+1}dx}$

$=e^{-\log\left(x^2+1\right)}=\left(x^2+1\right)^{-1}=\frac{1}{x^2+1}$

Therefore the solution is given by

$(I.F)\cdot y=\int (I.F)Q+C$

$\Rightarrow \frac{1}{x^2+1}\cdot y=\int \frac{1}{x^2+1}\left(x^2+2\right)dx+C$

$\Rightarrow \frac{y}{x^2+1}=\int \frac{x^2+2}{x^2+1}dx+C$

$\Rightarrow \frac{y}{x^2+1}=\int \frac{x^2+1+1}{x^2+1}dx+C$

$\Rightarrow \frac{y}{x^2+1}=\int \left\{1+\frac{1}{x^2+1}\right\}dx+C$

$\Rightarrow \frac{y}{x^2+1}=x+\tan^{-1}x+C$

$\Rightarrow y=\left(x^2+1\right)\left(x+\tan^{-1}x+C\right)$

This is the required solution of given differential equation

11. $\frac{dy}{dx}+2y=6x^x$

Sol. Given differential equation is $\frac{dy}{dx}+2y=6e^x$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=2$ and $Q=6\cdot e^x$

Thus the given differential equation is linear differential equation

Now $I.F=e^{\int P dx}$

$\Rightarrow I.F=e^{\int 2dx} \Rightarrow I.F=e^{2x}$

Solution is $y\cdot(I.F)=\int (I.F)Q\,dx+C$

$\Rightarrow y\cdot e^{2x}=\int e^{2x}\cdot 6\cdot e^x dx+C \Rightarrow y\cdot e^{2x}=6\int e^{3x}dx+C$

$\Rightarrow y\cdot e^{2x}=\frac{6}{3}\cdot e^{3x}+C \;\Rightarrow y=2\cdot e^{x}+C\cdot e^{-2x}$

This is the required solution of given differential equation

12. $\frac{dy}{dx}+3y=e^{-2x}$

Sol. Given differential equitation is $\frac{dy}{dx}+3y=e^{-2x}$

This is of the form $\frac{dy}{dx}+Py=Q,$ where $P=3$ and $Q=e^{-2x}$

Now $I.F=e^{\int 3dx}=e^{3x}$

Therefore the solution is $y\cdot(I.F)=\int(I.F)Q\,dx+C$

$\Rightarrow y\cdot e^{3x}=\int e^{3x}\cdot e^{-2x}dx+C \;\Rightarrow y\cdot e^{3x}=\int e^{x}dx+C$

$\Rightarrow y\cdot e^{3x}=e^{x}+C \;\Rightarrow y=e^{-2x}+C\cdot e^{-3x}$

This is the required solution of given differential equation

13. $\frac{dy}{dx}+8y=5e^{-3x}$

Sol. Given differential equation is $\frac{dy}{dx}+8y=5\cdot e^{-3x}$,

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=8$ and $Q=e^{-3x}$

Thus the given differential equation is linear

Now. $I.F=e^{\int 8dx}=e^{8x}$

Therefore the solution is $(I.F)\cdot y=\int(I.F)Q\;dx+C$

$\Rightarrow e^{8x}\cdot y=\int e^{8x}\cdot e^{-3x}dx+C \;\Rightarrow e^{8x}\cdot y=\int e^{5x}dx+C$

$\Rightarrow e^{8x}\cdot y=\frac{1}{5}\cdot e^{5x}+C \;\Rightarrow y=\frac{e^{-3x}}{5}+C\cdot e^{-8x}$

This is the required solution of given differential equation

14. $x\frac{dy}{dx}-y=(x-1)e^{x}, x>0$

Sol. Given differential equation is $x\cdot\frac{dy}{dx}-y=(x-1)k.e^{x}$

$\Rightarrow \frac{dy}{dx}+\frac{-1}{x}\cdot y=\frac{x-1}{x}e^{x}$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=-\frac{1}{x}$ and $Q=\frac{x-1}{x}$

Thus the given differential equation is linear

Now $I.F=e^{\int P\,dx}$

$$\Rightarrow I.F = e^{\int\left(-\frac{1}{x}\right)dx}$$

$$\Rightarrow I.F = e^{-\log(x)} = e^{\log\left(\frac{1}{x}\right)}$$

$$\Rightarrow I.F = \frac{1}{x}$$

Solution is

$$(I.F)\cdot y = \int (I.F)\cdot Q\,dx + C$$

$$\Rightarrow \frac{1}{x}\cdot y = \int \frac{1}{x}\cdot\frac{x-1}{x}\cdot e^x dx + C$$

$$\Rightarrow \frac{y}{x} = \int e^x\left\{\frac{1}{x} - \frac{1}{x^2}\right\}dx + C$$

$$\Rightarrow \frac{y}{x} = e^x\cdot\frac{1}{x} + C \;\Rightarrow y = e^x + Cx$$

This is the required solution of given differential equation

15. $\frac{dy}{dx} - y\tan x = e^x \sec x$

Sol. Given differential equation is

$$\frac{dy}{dx} - y\tan x = e^x \sec x$$

This is of the form $\frac{dy}{dx} + Py = Q$ where

$P = -\tan x$ and $Q = e^x \sec x$

Now, $I.F = e^{\int P\,dx}$

$= e^{\int(-\tan x)dx}$

$= e^{\log(\cos x)} = \cos x$

Solution is $y\cdot(I.F) = \int (I.F)Q\,dx + C$

$$\Rightarrow y\cdot\cos x = \int \cos x\cdot e^x\cdot \sec x dx + C$$

$$\Rightarrow y\cdot\cos x = \int e^x\,dx + C \qquad \Rightarrow y\cdot\cos x = e^x + C$$

This is the req solution of given differential equation

16. $(x\log x)\frac{dy}{dx} + y = 2\log x$

Sol. Given differential equation is $(x\log x)\frac{dy}{dx} + y = 2\log x$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$$

This is of the form $\frac{dy}{dx} + py = Q$

Where $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$

Thus the given differential equation is linear differential equation

Now, $I.F = e^{\int P\,dx} = e^{\int \frac{dx}{x\log x}} = e^{\log(\log x)} = \log x$

Now solution is $y\cdot(I.F) = \int (I.F)Q\,dx + C$

$\Rightarrow y\cdot(\log x) = \int \log x.\frac{2}{x}dx + C \Rightarrow y(\log x) = (\log x)^2 + C$

This is the required solution of given differential equation

17. $x\dfrac{dy}{dx} + y = x\log x$

Sol. Given differential equation is $x\dfrac{dy}{dx} + y = x\log x$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{1}{x}y = \log x$

This is of the form $\dfrac{dy}{dx} + Py = Q$

Where $P = \dfrac{1}{x}$ and $q = \log x$

Thus the given differential equation is linear

Now, $I.F = e^{\int P\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

The solution is $y\cdot(I.f) = \int (I.f)Q\,dx + C$

$\Rightarrow y\cdot(I.F) = \int (I.F)Q\,dx + C$

$\Rightarrow \quad xy = \log x\int x\,dx - \int\left\{\dfrac{d}{dx}(\log x)\,dx\right\}dx + C$

$\Rightarrow xy = \dfrac{x^2}{2}\log x - \int \dfrac{1}{x}\cdot\dfrac{x^2}{2}dx + C$

$\Rightarrow xy = \dfrac{x^2}{2}\cdot\log x - \dfrac{x^2}{4} + C \Rightarrow 4xy = 2x^2\log x - x^2 + C$

This is the required solution of given differential equation

18. $x\dfrac{dy}{dx} + 2y = x^2\log x$

Sol. Given differential equation is $x\cdot\dfrac{dy}{dx} + 2y = x^2\log x$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{2}{x}\cdot y = x\log x$

This is of the form $\dfrac{dy}{dx} + Py = Q.$

Where $P = \dfrac{2}{x}$ and $Q = x\cdot\log x$

Thus the differential equation is linear

$I.F = e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$

There fore the solution is given by $y\cdot(I.F) = \int (I.F)Q\,dx + C$

$\Rightarrow y(x^2) = \int x^2\cdot x\log x\,dx + C \Rightarrow y(x^2) = \int \log x\cdot x^3 dx + C$

$$\Rightarrow x^2y = \log x\cdot\int x^3dx - \int\left\{\frac{d}{dx}\log x\int x^3dx\right\}dx + C$$

$$\Rightarrow x^2y = \frac{x^4}{4}\log x - \int\frac{1}{x}\cdot\frac{x^4}{4}dx + C$$

$$\Rightarrow x^2y = \frac{1}{4}x^4\log x - \int\frac{x^3}{4}dx + C \Rightarrow x^2y = \frac{1}{4}x^4\log x - \frac{x^4}{16} + C$$

$$\Rightarrow x^2y = \frac{x^4}{16}(4\log x - 1) + C \Rightarrow y = \frac{x^2}{16}(4\log x - 1) + \frac{e}{x^2}$$

This is the required solution of given differential equation

19. $(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$

Sol. Given differential equation is $(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{1+x}y = e^{3x}(1+x)$$

This differential equation is of the form $\frac{dy}{dx} + Py = Q$

Where $P = -\frac{1}{1+x}$ and $Q = e^{3x}(1+x)$

Now $I.F = e^{\int Pdx} = e^{\int\frac{-dx}{1+x}} = e^{-\log(1+x)} = \frac{1}{1+x}$

Solution is $(I.F)\cdot y = \int(I.F)Q\,dx + C$

$$\Rightarrow \frac{1}{1+x}\cdot y = \int\frac{1}{1+x}\,e^{3x}(1+x)dx + C$$

$$\Rightarrow \frac{1}{1+x}\cdot y = \int e^{3x}dx + C \Rightarrow \frac{y}{1+x} = \frac{1}{3}\cdot e^{3x} + C$$

$$\Rightarrow y = \frac{1}{3}\cdot e^{3x}(1+x) + C(1+x)$$

This is the required solution of given differential equation

20. $\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(x^2+1)^2} = 0$

Sol. Given differential equation is $\frac{dy}{dx} + \frac{4x}{x^2+1}\cdot y + \frac{1}{(x^2+1)^2} = 0$

$$\Rightarrow \frac{dy}{dx} + \frac{4x}{x^2+1}\cdot y = \frac{-1}{(x^2+1)^2}$$

This is of the form $\frac{dy}{dx} + Py = Q$,

Where $P = \frac{4x}{x^2+1}$ and $Q = \frac{-1}{(x^2+1)^2}$

Thus the given differential equation is linear

Now, $I.F = e^{\int \frac{4x\,dx}{x^2+1}} = e^{2\log(x^2+1)} = (x^2+1)^2$

Therefore the solution is given by $(I.F).y = \int (I.F)Q\; dx + C$

$$\Rightarrow (x^2+1)^2 \cdot y = \int (x^2+1)^2 \cdot \frac{-1}{(x^2+1)^2} dx + C$$

$$\Rightarrow (x^2+1)^2 y = -\int dx + C \;\Rightarrow (x^2+1)^2 \cdot y = -x + C$$

$$\Rightarrow y = \frac{-x}{(x^2+1)^2} + \frac{C}{(x^2+1)^2}$$

This is the required solution of given differential equation

21. $\left(y+3x^2\right)\dfrac{dx}{dy} = x$

Sol. Given differential equation is $\left(y+3x^2\right)\dfrac{dx}{dy} = x$

$$\Rightarrow \frac{dy}{dx} = \frac{y+3x^2}{x} \;\Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 3x$$

This is of the form $\dfrac{dy}{dx} + Py = Q$

Where $P = -\dfrac{1}{x}$ and $Q = 3x$

Thus the given differential equation is linear differential equation

Now $I.F = e^{\int P dx} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \dfrac{1}{x}$

Now the solution is

$$y \cdot (I.F) = \int (I.F) \cdot Q\, dx + C$$

$$\Rightarrow y \cdot \frac{1}{x} = \int \frac{1}{x} 3x\, dx + C$$

$$\Rightarrow \frac{y}{x} = 3x + C \;\Rightarrow y = 3x^2 + Cx$$

This is the required solution of given differential equation

22. $xdy - \left(y+2x^2\right)dx = 0$

Sol. Given differential equation is $x\,dy - \left(y+2x^2\right)dx = 0$

$$\Rightarrow xdy = \left(y+2x^2\right)dx \;\Rightarrow \frac{dy}{dx} = \frac{y+2x^2}{x} \;\Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x$$

This is of the form $\dfrac{dy}{dx} + Py = Q$, where $P = -\dfrac{1}{x}$ and $Q = 2x$

Thus the given differential equation is linear differential equation

Now $I.F = e^{\int \frac{-1}{x} dx} = e^{-\log x} = \dfrac{1}{x}$

Now solution is $y \cdot (I.F) = \int (I.F)Q\; dx + C$

$$\Rightarrow y\cdot\frac{1}{x}=\int\frac{1}{x}\cdot 2x\,dx+C$$

$$\Rightarrow \frac{y}{x}=2x+C \;\Rightarrow y=2x^2+Cx$$

This is the required solution of given differential equation

23. $xdy+\left(y-x^3\right)dx=0$

Sol. $x\,dy+\left(y-x^3\right)dx=0$

$$\Rightarrow x\,dy=\left(x^3-y\right)dx$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^3-y}{x} \;\Rightarrow \frac{dy}{dx}+\frac{1}{x}\cdot y=x^2$$

This is of the form $\frac{dy}{dx}+Py=Q,$

Where $P=\frac{1}{x}$ and $Q=x^2$

Thus the given differential equation is linear differential equation

Now, $I.F=e^{\int Pdx}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

Now, Solution is $y\cdot(I.F)=\int(I.F)Q\;dx+C$

$$\Rightarrow y\cdot x=\int x\cdot x^2dx+C$$

$$\Rightarrow xy=\int x^3dx+C \;\Rightarrow xy=\frac{x^3}{4}+C$$

$$\Rightarrow y=\frac{x^3}{4}+Cx$$

This is the required solution of given differential equation

24. $xdv-\left(y+2x^2\right)dx=0$

Sol. Given differential equation is $\frac{dy}{dx}+2y=\sin x$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=2$ and $Q=\sin x$

$I.F=e^{\int Pdx}=e^{\int 2dx}=e^{2x}$

Solution is $(I.F)y=\int(I.F)Q\;dx+C$

$$\Rightarrow e^{2x}\cdot y=\int e^{2x}\cdot\sin dx+C$$

$$\Rightarrow e^{2x}\cdot y=I_1+C \quad \text{(let)} \qquad \text{..... (i)}$$

Now, $I_1=\int e^{2x}\cdot\sin x\;dx$

$$\Rightarrow I_1=e^{2x}\int\sin x\,dx-\int\left\{\frac{d}{dx}\left(e^{2x}\right)\int\sin x\,dx\right\}dx$$

$$\Rightarrow I_1=e^{2x}\int\sin x\,dx+2\int e^{2x}\cos x\,dx$$

$$\Rightarrow I_1 = -e^{2x}\cdot\cos x + 2\left[e^{2x}\int\cos x\,dx - \int\left\{\frac{d}{dx}e^{2x}\int\cos x\,dx\right\}dx\right]$$

$$\Rightarrow I_1 = -e^{2x}\cos x + 2\left[e^{2x}\sin x - 2\int e^{2x}\cdot\sin x\,dx\right]$$

$$\Rightarrow I_1 = -e^{2x}\cdot\cos x + 2\cdot e^{2x}\sin x - 4I_1 \Rightarrow 5I_1 = 2\cdot e^{2x}\cdot\sin x - e^{2x}\cdot\cos x$$

$$\Rightarrow I_1 = \frac{1}{5}\cdot e^{2x}\{2\sin x - \cos x\} \qquad \text{.... (ii)}$$

From (i) and (ii) we have

$$e^{2x}\cdot y = \frac{1}{5}\cdot e^{2x}\{2\sin x - \cos x\} + C$$

$$\Rightarrow y = \frac{1}{5}\{2\sin x - \cos x\} + C\cdot e^{-2x}$$

This is the required solution of given differential equation

25. $\dfrac{dy}{dx} + y = \cos x - \sin x$

Sol. This is of the form $\dfrac{dy}{dx} + Py = Q$, where $P = 1$, $Q = \cos x - \sin x$

I.F $= e^{\int P\,dx} = e^{\int 1\,dx} = e^x$

So, the required solution is given by, $y\times \text{I.F} = \int Q\times(\text{I.F})\,dx$

$$\Rightarrow y.e^x = \int(\cos x - \sin x)\,e^x\,dx \quad \Rightarrow y.e^x = e^x\cos x + c$$

$$\Rightarrow y = \cos x + c\,e^{-x} \quad \therefore \int\left(f(x) + f'(x)\right)e^x dx = f(x)e^x + c$$

26. $\sec x\dfrac{dy}{dx} - y = \sin x$

Sol. Given differential equation is $\sec x\cdot\dfrac{dy}{dx} - y = \sin x$

$$\Rightarrow \frac{dy}{dx} - (\cos x)y = \sin x\cdot\cos x$$

This is of the form $\dfrac{dy}{dx} + Py = Q$, where $P = -\cos x$ and $Q = \sin x\cdot\cos x$

This the given differential equation is linear

$I.F = e^{\int P\,dx} = e^{\int(-\cos x)dx} = e^{-\sin x}$

Solution is $(I.F)\cdot y = \int(I.F)Q\,dx + C$

$$\Rightarrow e^{-\sin x}y = \int e^{-\sin x}\cdot\sin x\cdot\cos x\,dx + C$$

$$\Rightarrow e^{-\sin x}\cdot y = I_1 + C \quad \{\text{Let}\} \text{ .... (i)}$$

Now, $I_1 = \int e^{-\sin x}\cdot\sin x\cos x\,dx$

Let $-\sin x = z$

$\Rightarrow -\cos x = dz$

$\therefore I_1 = \int e^z z\,dz$

$$\Rightarrow I_1 = \left[z\int e^z dz - \int\left\{\frac{dz}{dz}\int e^z dz\right\}dz\right]$$

$\Rightarrow I_1 = z \cdot e^z - e^z \quad \Rightarrow I_1 = \sin x \cdot e^{-\sin x} - e^{-\sin x}$

$\Rightarrow I_1 = e^{-\sin x}(\sin x - 1)$ ..... (ii)

From (i) and (ii) we have $e^{-\sin x} \cdot y = e^{-\sin x} \cdot (\sin x - 1) + C$

$\Rightarrow y = (\sin x - 1) + C \cdot e^{\sin x}$

This is the required solution of given differential equation

27. $(1+x^2)dy + 2xydx = \cot x$

Sol. Given differential equation is $(1+x^2)dy + 2xy\,dx = \cot x\,dx$

$\Rightarrow (1+x^2)\frac{dy}{dx} + 2xy = \cot x \Rightarrow \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{2x}{1+x^2}$ and $Q = \frac{\cot x}{1+x^2}$

This the given differential equation is linear

$I.F = e^{\int P\,dx}$

$\Rightarrow I.F = e^{\int \frac{2x\,dx}{1+x^2}} \Rightarrow I.F = e^{\log(1+x^2)} \Rightarrow I.F = (1+x^2)$

Now solution is $(I.F)y = \int (I.F)Q\,dx + C$

$\Rightarrow (1+x^2) \cdot y = \int (1+x^2)\frac{\cot x}{1+x^2}dx + C$

$\Rightarrow (1+x^2)y = \int \cot x \cdot dx + C \Rightarrow (1+x^2) \cdot y = \log|\sin x| + C$

This is the required solution of given differential equation

28. $(\sin x)\frac{dy}{dx} + (\cos x)y = \cos x \sin^2 x$

Sol. Given differential equation is $(\sin x)\frac{dy}{dx} + (\cos x)y = \cos x \cdot \sin^2 x$

$\Rightarrow \frac{dy}{dx} + \cot x \cdot y = \cos x \cdot \sin x$

This is of the form $\frac{dy}{dx} + Py = Q$,

Where $P = \cot x$ and $Q = \cos x \cdot \sin x$

This the given differential equation is linear

Now, $I.F = e^{\int P dx} = e^{\int \cot x dx} = e^{\log(\sin x)} = \sin x$

Therefore the required solution $y \cdot (I.F) = \int (I.F)Q\,dx + C$

$\Rightarrow y \cdot (\sin x) = \int \sin x \cdot \cos x \cdot \sin x\,dx + C$

$\Rightarrow y \cdot (\sin x) = \int (\sin^2 x) \cdot \cos x\,dx + C \Rightarrow y(\sin x) = \frac{\sin^3 x}{3} + C,$

This is the required solution of given differential equation

29. $\frac{dy}{dx} + 2y\cot x = 3x^2 \operatorname{cosec}^2 x$

Sol. Given differential equation is $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2 x$

This is of the form $\frac{dy}{dx}+Py=Q$ where $P=2\cot x$ and $Q=3x^2\cdot\operatorname{cosec}^2 x$

This the given differential equation linear

Now, $I.F=e^{\int P\,dx}$

$\Rightarrow I.F=e^{\int 2\cot x\,dx}$

$\Rightarrow I.F=e^{2\log(\sin x)}$ $\Rightarrow I.F=\sin^2 x$

Now the solution is $y\cdot(I.F)=\int(I.F)Q\;dx+C$

$\Rightarrow y(\sin^2 x)=\int\sin^2 x\cdot 3x^2\cdot\operatorname{cosec}^2 x dx+C$

$\Rightarrow y(\sin^2 x)=\int 3x^2 dx+C$ $\Rightarrow y(\sin^2 x)=x^3+C$

This is the required solution of given differential equation

30. $x\frac{dy}{dx}-y=2x^2\sec x$

Sol. The given equation may be written as $\frac{dy}{dx}-\frac{1}{x}y=2x\sec x$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$, $Q=2x\sec x$

$\text{I.F}=e^{\int P\,dx}=e^{-\int\frac{1}{x}dx}=e^{-\log|x|}=\frac{1}{x}$

So, the required solution is given by, $y\times\text{I.F}=\int Q\times(\text{I.F})dx$

$\Rightarrow y.\frac{1}{x}=\int 2x\sec x.\frac{1}{x}\,dx$ $\Rightarrow \frac{y}{x}=2\int\sec x\,dx$

$\Rightarrow \frac{y}{x}=2\log|\sec x+\tan x|+c$ $\therefore\ y=2x\log|\sec x+\tan x|+cx$

31. $\frac{dy}{dx}=y\tan x-2\sin x$

Sol. This is of the form $\frac{dy}{dx}+Py=Q$ where $P=-\tan x,\ Q=-2\sin x$

$\text{I.F}=e^{-\int\tan x\,dx}=e^{\log|\cos x|}=\cos x$

So, the required solution is given by, $y\times\text{I.F}=\int Q\times(\text{I.F})\,dx$ $\Rightarrow y\cos x=\int -2\sin x\cos x\,dx$

$\Rightarrow y\cos x=-\int\sin 2x\,dx$ $\Rightarrow y\cos x=\frac{\cos 2x}{2}+c$ $\therefore\ 2y\cos x=\cos 2x+c$

32. $\frac{dy}{dx}+y\cot x=\sin 2x$

Sol. This is of the form $\frac{dy}{dx}+Py=Q$, where $P=\cot x,\ Q=\sin 2x$

$\text{I.F}=e^{\int\cot x\,dx}=e^{\log|\sin x|}=\sin x$

So, the required solution is given by, $y\times\sin x=\int\sin 2x.\sin x\,dx$

$$\Rightarrow y \sin x = 2\int \sin x \cos x \sin x \, dx \quad \Rightarrow y \sin x = 2\int \sin^2 x \cos x \, dx$$

Put $\sin x = t \Rightarrow \cos x = \frac{dt}{dx} \Rightarrow \cos x\, dx = dt \Rightarrow y \sin x = 2\int t^2 \, dt$

$$\Rightarrow y\sin x = \frac{2t^3}{3} + c \quad \therefore\ y\sin x = \frac{2}{3}\sin^3(x) + c$$

33. $\frac{dy}{dx} + 2y\tan x = \sin x$

Sol. Given differential equation is $\frac{dy}{dx} + 2y \cdot \tan x = \sin x$

This is of the form $\frac{dy}{dx} + 2y\tan x = \sin x$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = 2\tan x$ and $Q = \sin x$

Thus the given differential equation is linear

Now $I.F = e^{\int P dx} = e^{\int 2\tan x dx} = e^{2\log(\sec x)} = \sec^2 x$

Now the solution is $(I.f) \cdot y = \int (I.f) Q \, dx + C$

$$\Rightarrow \sec^2 x \cdot y = \int \sec^2 x \cdot \sin x \, dx + C \Rightarrow \sec^2 xy = \int \sec x \tan x dx + C$$

$$\Rightarrow \sec^2 x \cdot y = \sec x + C \Rightarrow y = \cos x + C\cos^2 x$$

This is the required solution of given differential equation

34. $\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$

Sol. Given differential equation is $\frac{dy}{dx} + y\cot x - x^2 \cot x + 2x$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \cot x$ and $Q = x^2 \cot x + 2x$

Thus the given differential equation is linear

Now, $I.F = e^{\int P dx} \Rightarrow I.F = e^{\int \cot x dx} \Rightarrow I.F = e^{\log(\sin x)} \Rightarrow I.F = \sin x$

Therefore the solution is $(I.F)y = \int (I.F) Q \, dx + C$

$$\Rightarrow (\sin x)y = \int \sin x\left(x^2 \cot x + 2x\right)dx + C$$

$$\Rightarrow \sin x \cdot y = \int \left(x^2 \cos x + 2x \cdot \sin x\right)dx + C$$

$$\Rightarrow \sin x \cdot y = \int x^2 \cos x \, dx + \int 2x \cdot \sin x dx + C$$

$$\Rightarrow \sin x \cdot y = x^2 \int \cos x \, dx - \int \left\{\frac{d}{dx}(x^2) \int \cos x \, dx\right\} dx + \int 2x \sin x \, dx + C$$

$$\Rightarrow \sin x \cdot y = x^2 \sin x - \int 2x \sin x \, dx + \int 2x \cdot \sin x \, dx + C$$

$$\Rightarrow \sin x \cdot y = x^2 \sin x + C \Rightarrow y = x^2 + C\,\mathrm{cosec}\, x$$

This is the required solution of given differential equation

Find a particular solution satisfying the given condition for each of the following differential equation

35. $x\frac{dy}{dx}+y=x^3$, given that $y=1$ when $x=2$

Sol. Given differential equation is $x\cdot\frac{dy}{dx}+y=x^3$

$\Rightarrow \frac{dy}{dx}+\frac{1}{x}\cdot y=x^2$

This is of the form $\frac{dy}{dx}+Py=Q$

Where $P=\frac{1}{x}$ and $Q=x^2$

Now, $I.F=e^{\int Pdx}=e^{\int \frac{1}{x}dx}=e^{\log x}=x$

Now, solution is $(I.F)\cdot y=\int (I.F)Q\,dx+C$

$\Rightarrow x\cdot y=\int x\cdot x^2dx+C$

$\Rightarrow xy=\frac{x}{4}+C$ .... (i)

Putting $x=2$ and $y=1$ we have $2=\frac{16}{4}+C$

$xy=\frac{x^4}{4}-2$

$\Rightarrow y=\frac{x^3}{4}-\frac{2}{x}$

This is the required solution of given differential equation

36. $\frac{dy}{dx}+y\cot x=4x\operatorname{cosec} x$, given that $y=0$ when $x=\frac{\pi}{2}$

Sol. Given differential equation is $\frac{dy}{dx}+y\cdot\cot x=4x\cdot\operatorname{cosec} x$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=\cot x$ and $Q=4x\operatorname{cosec} x$

Thus the given differential equation is linear

$I.F=e^{\int Pdx}=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$

Solution is given by $y\cdot(I.f)Q\,dx+C$

$\Rightarrow y\cdot\sin x=\int \sin x\cdot 4x\cdot\operatorname{cosec} x dx+C$

$\Rightarrow y\cdot\sin x=4\int x\,dx+C \Rightarrow y\sin x=4\cdot\frac{x^2}{2}+C$

$\Rightarrow y\sin x=2x^2+C$ ..... (i)

Putting $x=\frac{\pi}{2}$ and $y=0$ we have $0\cdot\sin\frac{\pi}{2}=2\cdot\frac{\pi^2}{4}+C$

$\Rightarrow 0=\frac{\pi^2}{2}+C \Rightarrow C=-\frac{\pi^2}{2}$

Putting $C=-\frac{\pi^2}{2}$ in (i) we have $y\cdot\sin x=2x^2-\frac{\pi^2}{2}$

This is the required solution of given differential equation

37. $\frac{dy}{dx}+2xy=x,$ given that $y=1$ when $x=0$

Sol. Given differential equation is $\frac{dy}{dx}+2xy=x$

Here $P=2x$ and $Q=x,$ thus the given differential equation is linear $\therefore I.F=e^{\int Pdx}$

$I.F=e^{\int 2x\,dx}=e^{x^2}$

Therefore the solution is $y\cdot(I.F)=\int(I.F)Q\,dx+C$

$\Rightarrow y\cdot e^{x^2}=\int e^{x^2}\cdot x\,dx+C$

$\Rightarrow y\cdot e^{x^2}=I_1+C$ (Let) ..... (i)

Now $I_1=\int e^{x^2}\cdot x\,dx+C$

Let $x^2=z$

$\Rightarrow 2x=dz \quad \Rightarrow x\cdot dz=\frac{1}{2}dz$

$\therefore I_1=\frac{1}{2}\int e^z dz+C$

$\Rightarrow I_1=\frac{1}{2}\cdot e^z+C$

$\Rightarrow I_1=\frac{1}{2}\cdot e^{x^2}+C$ ..... (ii)

From (i) and (ii) we have $y\cdot e^{x^2}=\frac{1}{2}\cdot e^{x^2}+C$ ..... (iii)

Putting $x=0$ and $y=1$ we have $1=\frac{1}{2}+C$

$\Rightarrow C=\frac{1}{2}$

From (iii) we have $y\cdot e^{x^2}=\frac{1}{2}e^{x^2}+\frac{1}{2}\Rightarrow 2y=1+e^{-x^2}$

This is the required solution of given differential equation

38. $\frac{dy}{dx}+2y=e^{-2x}\sin x,$ given that $y=0$ when $x=0$

Sol. Given differential equation is $\frac{dy}{dx}+2y=e^{-2x}\cdot\sin x$

This is of the form $\frac{dy}{dx}+Py=Q,$

Where $P=2$ and $Q=e^{-2x}\cdot\sin x$

Thus the given differential equation is linear

Now $I.F=e^{\int 2dx}=e^{2x}$

Solution is $(I.F)\cdot y=\int (I.F)Q\,dx+C$

$\Rightarrow e^{2x}\cdot y=\int e^{2x}\cdot e^{-2x}\cdot \sin x\,dx+C$

$\Rightarrow e^{2x}.y=\int \sin x\,dx+C$

$\Rightarrow e^{2x}.y=-\cos x+C \quad \ldots \text{(i)}$

Putting $x=0$ and $y=0$ in (i) we have $0=-\cos 0+C$

$\Rightarrow 0=-+C \Rightarrow C=1$

Putting $C=1$ in (i) we have $e^{2x}\cdot y=-\cos x+1$

$\Rightarrow y=e^{-2x}(1-\cos x)$

This is the required solution of given differential equation

39. $(1+x^2)\dfrac{dy}{dx}+2xy=4x^2$, given that $y=0$ when $x=0$

Sol. Given differential equation is $(1+x^2)\dfrac{dy}{dx}+2xy=4x^2$

$\Rightarrow \dfrac{dy}{dx}+\dfrac{2x}{1+x^2}\cdot y=\dfrac{4x^2}{1+x^2}$

This is of the form $\dfrac{dy}{dx}+Py=Q$, where $P=\dfrac{2x}{1+x^2}$ and $Q=\dfrac{4x^2}{1+x^2}$

Now, $I.F=e^{\int P dx}=e^{\int \frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

Solution is $y\cdot(I.F)=\int (I.F)Q\,dx+C$

$\Rightarrow y\cdot(1+x^2)=\int (1+x^2)\cdot\dfrac{4x^2}{1+x^2}dx+C$

$\Rightarrow y\cdot(1+x^2)=\dfrac{4x^3}{3}+C \quad \ldots\ldots \text{(i)}$

Putting $x=0$ and $y=0$, in (i) we have $C=0$

Putting $C=0$ in we have $y(1+x^2)=\dfrac{4x^3}{3}$

$\Rightarrow y=\dfrac{4x^3}{3(1+x^2)}$

This is the required solution of given differential equation

40. $x\dfrac{dy}{dx}-y=\log x$, given that $y=0$ when $x=1$

Sol. Given differential equation is $x\cdot\dfrac{dy}{dx}-y=\log x$

$\Rightarrow \dfrac{dy}{dx}+\left(-\dfrac{1}{x}\right)y=\dfrac{1}{x}\log x$

This is of the form $\dfrac{dy}{dx}+Py=Q$

Where $P=-\dfrac{1}{x}$ and $Q=\dfrac{1}{x}\cdot\log x$

This the given differential equation is linear

Here $I.F = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \frac{1}{x}$

Therefore the solution is given by $y \cdot (I.F) = \int (I.F) Q \, dx + C$

$\Rightarrow y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot \frac{1}{x} \cdot \log x \, dx + C \Rightarrow \frac{y}{x} = \int \frac{1}{x^2} \cdot \log x \, dx + C$

$\Rightarrow \frac{y}{x} = \log x \int \frac{1}{x^2} dx - \int \left\{ \frac{d}{dx}(\log x) \int \frac{1}{x^2} dx \right\} dx + C$

$\Rightarrow \frac{y}{x} = -\frac{1}{x} \cdot \log x + \int \frac{1}{x} \cdot \frac{1}{x} dx + C$

$\Rightarrow \frac{y}{x} = -\frac{1}{x} \cdot \log x - \frac{1}{x} + C$ ..... (i)

Putting $x = 1$ and $y = 0$

$0 = -\log 1 - 1 + C$

$C = 1$

Putting $C = 1$ in equation (i) we have $\frac{y}{x} = -\frac{1}{x} \cdot \log x - \frac{1}{x} + 1$

$\Rightarrow y = x - 1 - \log x$

This is the required solution of given differential equation

41. $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, given that $y = 1$ when $x = 0$

Sol. Given differential equation is $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$

This is of the form $\frac{dy}{dx} + Py = Q$

Where $P = \tan x$ and $Q = 2x + x^2 \tan x$

Thus the given differential equation is linear

Now, $I.F = e^{\int P dx} = e^{\int \tan x \, dx} = e^{\log(\sec x)} = \sec x$

Now, the solution is $y \cdot (I.F) = \int (I.F) Q \, dx + C$

$\Rightarrow y \cdot \sec x = \int \sec x \cdot \left(2x + x^2 \tan x\right) dx + C$

$\Rightarrow y \cdot \sec x = \int \left(2x \cdot \sec x + x^2 \sec x \cdot \tan x\right) dx + C$

$\Rightarrow y \cdot \sec x = \sec x \int 2x \, dx - \int \left\{ \frac{d}{dx}(\sec x) \int 2x \, dx \right\} dx + \int x^2 \sec x \cdot \tan x \, dx + C$

$\Rightarrow y \cdot \sec x = x^2 \sec x - \int x^2 \cdot \sec x \cdot \tan x \, dx + \int x^2 \cdot \sec x \cdot \tan x \, dx + C$

$\Rightarrow y \cdot \sec x = x^2 \cdot \sec x + C$ ..... (i)

Putting $x = 0$ and $y = 1$ in (i) we have $1 = C$

Putting $C = 1$ in equation (i) we have $y \sec x = x^2 \cdot \sec x + 1$

$\Rightarrow y = x^2 + \cos x$

This is the required solution of given differential equation

42. A curve passes through the origin and the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point. Find the equation of the curve

Sol. Slope of tangent to a curve at a point (x, y) is

$$m = \frac{dy}{dx}$$

According to question $\frac{dy}{dx} = x + y$

$$\Rightarrow \frac{dy}{dx} - y = x$$

This is of the form $\frac{dy}{dx} + Py = Q,$

Where $P = -1$ and $Q = x$

$$\therefore I.F = e^{\int(-1)dx} = e^{-x}$$

Now, solution is $y \cdot (I.F) = \int (I.F) \cdot Q \, dx + C$

$$\Rightarrow y \cdot e^{-x} = \int e^{-x} \cdot x \, dx + C$$

$$\Rightarrow y \cdot e^{-x} = x\int e^{-x}dx - \int\left\{\frac{dx}{dx}\int e^{-x}dx\right\}dx + C$$

$$\Rightarrow y \cdot e^{-x} = -x \cdot e^{-x} + \int e^{-x}dx + C$$

$$\Rightarrow y \cdot e^{-x} = -x \cdot e^{-x} - e^{-x} + C \quad \text{.... (i)}$$

Since the curve passes through origin

$$\therefore 0 = 0 - e^0 + C$$

$$\Rightarrow C = 1$$

Putting $C = 1$ in (i) we have $y \cdot e^{-x} = -x \cdot e^{-x} - e^{-x} + 1$

$$\Rightarrow y = -x - 1 + e^{-x} \Rightarrow x + y + 1 = e^{-x}$$

This is the required equation of curve

43. A curve passes through the point $(0, 2)$ and the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. Find the equation of the curve

Sol. Slope of tangent at a point (x, y) is given by

$$m = \frac{dy}{dx}$$

According to question $x + y - \frac{dy}{dx} = 5$

$$\Rightarrow \frac{dy}{dx} - y = x - 5$$

This if of the form $\frac{dy}{dx} + Py = Q,$ where $P = -1$ and $q = x - 5$

Now, $I.F = e^{-\int dx} = e^{-x}$

Solution is $y \cdot (I.F) = \int (I.F) Q \, dx + C$

$$\Rightarrow y\cdot e^{-x}=\int e^{-x}\cdot(x-5)dx+C$$

$$\Rightarrow y\cdot e^{-x}=\int x\cdot e^{-x}dx-5\int e^{-x}dx+C$$

$$\Rightarrow y\cdot e^{-x}=x\int e^{-x}dx-\int\left\{\frac{dx}{dx}\int e^{-x}dx\right\}dx+5\cdot e^{-x}+C$$

$$\Rightarrow y\cdot e^{-x}=-x\cdot e^{-x}+\int e^{-x}dx+5\cdot e^{-x}+C$$

$$\Rightarrow y\cdot e^{-x}=-x\cdot e^{-x}-e^{-x}+5\cdot e^{-x}+C$$

$$\Rightarrow y=-x-1+5+C\cdot e^{x}\ \Rightarrow y+x-4=C\cdot e^{x}$$

Since the curve passes through $(0,2)$ we have $2+0-4=C\cdot e^{0}\ \Rightarrow C=-2$

Hence the equation of curve is $x+y=4-2\cdot e^{x}$

This is the required solution of given differential equation

Find the general solution for each of the following differential equations

44. $ydx-\left(x+2y^{2}\right)dy=0$

Sol. The given equation may be written as $\frac{dx}{dy}=\frac{x+y^{2}}{y}\quad\Rightarrow\ \frac{dx}{dx}-\frac{1}{y}x=y$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{y}$, $Q=y$

$\text{I.F}=e^{\int P\,dy}=e^{-\int\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

So, the required solution is given by, $x\times\text{I.F}=\int Q.(\text{I.F})\,dy$

$$\Rightarrow x\times\frac{1}{y}=\int y.\frac{1}{y}\,dy\quad\Rightarrow\frac{x}{y}=y+c\quad\therefore\ x=y^{2}+c\,y$$

45. $ydx+\left(x-y^{2}\right)dy=0$

Sol. The given equation may be written as $\frac{dx}{dy}=\frac{y^{2}-x}{y}\quad\Rightarrow\ \frac{dx}{dy}+\frac{1}{y}x=y$

This is of the form $\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{y}$, $Q=y$

$\text{I.F}=e^{\int P\,dy}=e^{\int\frac{1}{y}dx}=e^{\log y}=y$

So the required solution is given as, $x\times\text{I.F}=\int Q.\,(\text{I.F})\,dy$

$$\Rightarrow x\,y=\int y.y\,dy\quad\Rightarrow x\,y=\frac{y^{3}}{3}+c\quad\therefore\ 3xy=y^{3}+c$$

46. $\left(x-y^{3}\right)\frac{dy}{dx}+y=0$

Sol. The given equation may be written as $\frac{dx}{dy}=\frac{y^{3}-x}{y}\quad\Rightarrow\ \frac{dx}{dy}+\frac{1}{y}x=y^{2}$

This is of the form $\frac{dx}{dy}+Py=Q$, where $P=\frac{1}{y}$, $Q=y^{2}$

$$\text{I.F} = e^{\int P\,dy} = e^{\int \frac{1}{y}dy} = e^{\log(y)} = y$$

So the required solution is given by, $x \times \text{I.F} = \int Q.\,(\text{I.F})\,dy$

$$\Rightarrow x\,y = \int y^2.\,y\,dy \quad \Rightarrow xy = \frac{y^4}{4} + c \quad \therefore\ 4\,xy = y^4 + c$$

47. $\left(x+3y^2\right)\dfrac{dy}{dx} = y, (y>0)$

Sol. Given differential equation is $\left(x+3y^2\right)\dfrac{dy}{dx} = y$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x+3y^2} \ \Rightarrow \frac{dx}{dy} = \frac{x+3y^2}{y}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \ \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$$

This is of the form $\dfrac{dx}{dy} + Py = Q$

Where $P = -\dfrac{1}{y}$ and $Q = 3y$

Now, I.F $= e^{\int P\,dy} = e^{-\int \frac{1}{y}dy} = e^{-\log y} = \dfrac{1}{y}$

Therefore, the solution is $(I.F)\cdot x = \int (I.F)\times Q\,dy + C$

$$\Rightarrow \frac{1}{y}\cdot x = \int \frac{1}{y}\times 3y\,dy + C \ \Rightarrow \frac{x}{y} = \int 3dy + C$$

$$\Rightarrow \frac{x}{y} = 3y + C \ \Rightarrow x = 3y^2 + Cy$$

This is the required solution of given differential equation

48. $(x+y)\dfrac{dy}{dx} = 1$

Sol. Given differential equation $(x+1)\dfrac{dy}{dx} = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x+y} \quad \Rightarrow \frac{dx}{dy} = x + y \ \Rightarrow \frac{dx}{dy} - x = y$$

This is of the form $\dfrac{dy}{dx} + Px = Q$

Where $P = -1$ and $Q = y$

Now $I.F = e^{\int P\,dy} = e^{-\int dy} = e^{-y}$

Solution is $(I.F)x = \int (I.F)Q\,dy + c$

$$\Rightarrow e^{-y}\cdot x = \int e^{-y}\cdot y\cdot dy + C$$

$$x\cdot e^{-y} = y\int e^{-y}dy - \int\left\{\frac{dy}{dy}\int e^{-y}dy\right\}dy + C$$

$$\Rightarrow x\cdot e^{-y} = -y\cdot e^{-y} + \int e^{-y}dy + C$$

$\Rightarrow x\cdot e^{-y}=-y\cdot e^{-y}-e^{-y}+C$

$\Rightarrow x=-y-1+C\cdot e^{-y} \quad \Rightarrow x+y+1=C\cdot e^{y}$

This is the required solution of given differential equation

49. $(x+y+1)\dfrac{dy}{dx}=1$

Sol. The given equation may be written as $\dfrac{dx}{dy}=\dfrac{x+y+1}{1} \quad \Rightarrow \dfrac{dx}{dy}-x=(1+y)$

This is of the form $\dfrac{dx}{dy}+Px=Q$, where $P=-1, Q=1+y$

I.F $=e^{-\int dy}=e^{-y}$

So the required solution is given by, $x\times \text{I.F}=\int Q\times(\text{I.F})\,dy \quad \Rightarrow x\,e^{-y}=\int e^{-y}(y+1)\,dy$

$\Rightarrow x\,e^{-y}=\int e^{-y}dy+\int y\,e^{-y}\,dt \quad \Rightarrow x\,e^{-y}=-e^{-y}+y\int e^{-y}\,dy-\int\left[\dfrac{dy}{dx}\int e^{-y}\,dy\right]dy$

$\Rightarrow x\,e^{-y}=-e^{-y}-y\,e^{-y}+\int 1.e^{-y}\,dy \quad \Rightarrow x\,e^{-y}=-e^{-y}-y\,e^{-y}-e^{-y}+c$

$\Rightarrow y\,e^{-y}=-2\,e^{-y}-y\,e^{-y}+c \quad \therefore\ x=c\,e^{y}-y-2$

50. Solve $(x+1)\dfrac{dy}{dx}=2e^{-y}-1$, given that $x=0$ when $y=0$

Sol. Given differential equation is $(x+1)\cdot\dfrac{dy}{dx}=2\cdot e^{-y}-1$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{2\cdot e^{-y}-1}{x+1} \Rightarrow \dfrac{dx}{dy}=\dfrac{x+1}{2\cdot e^{-y}-1}$

$\Rightarrow \dfrac{dx}{dy}=\dfrac{1}{2\cdot e^{-y}-1}x+\dfrac{1}{2e^{-y}-1}$

$\Rightarrow \dfrac{dx}{dy}=\dfrac{e^{y}}{2-e^{y}}x+\dfrac{e^{y}}{2-e^{y}} \Rightarrow \dfrac{dx}{dy}-\dfrac{e^{y}}{2-e^{y}}x=\dfrac{e^{y}}{2-e^{y}}$

Now $I.F=e^{\int\frac{-e^{y}}{2-e^{y}}dy}=e^{\log(2-e^{y})}=2-e^{y}$

Solution is $x\cdot(2-e^{y})=\int(2-e^{y})\cdot\dfrac{e^{y}}{2-e^{y}}dy+C$

$x\cdot(2-e^{y})=e^{y}+C$

Now, $y=0$ and $x=0 \Rightarrow C=1$

$\therefore x(2-e^{y})=e^{y}-1$

$\Rightarrow 2x-x\cdot e^{y}=e^{y}-1 \Rightarrow 2x+1=e^{y}(x+1)$

$\Rightarrow e^{y}=\dfrac{2x+1}{x+1} \Rightarrow y=\log\left(\dfrac{2x+1}{x+1}\right)$

This is the required solution of given differential equation

51. Solve $\left(1+y^2\right)dx+\left(x-e^{-\tan^{-1}y}\right)dy=0$ given that when $y=0$, then $x=0$

Sol. Given differential equation is $\left(1+y^2\right)dy+\left(x-e^{-\tan^{-1}y}\right)dy=0$

$$\Rightarrow \left(1+y^2\right)\cdot\frac{dx}{dy}+x-e^{-\tan^{-1}y}=0$$

$$\Rightarrow \left(1+y^2\right)\cdot\frac{dx}{dy}+x=e^{-\tan^{-1}y}$$

$$\Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{e^{-\tan^{-1}y}}{1+y^2}$$

This is of the form $\frac{dx}{dy}+Px=Q$,

Where $P=\frac{1}{1+y^2}$ and $Q=\frac{e^{-\tan^{-1}y}}{1+y^2}$

Now, $I.F=e^{\int P\,dy}=e^{\int\frac{dy}{1+y^2}}=e^{\tan^{-1}y}$

Solution is $y\cdot(I.F)=\int(I.F)Q\cdot dy+C$

$$\Rightarrow y\cdot e^{\tan^{-1}y}=\int e^{\tan^{-1}y}\cdot\frac{e^{-\tan^{-1}y}}{1+y^2}dy+C$$

$$\Rightarrow y\cdot e^{\tan^{-1}y}=\int\frac{dy}{1+y^2}+C$$

$$\Rightarrow y\cdot e^{\tan^{-1}y}=\tan^{-1}y+C \quad \ldots\ldots \text{(i)}$$

Putting $x=0$ and $y=0$ in equation (i) we have

$$0\times e^{\tan^{-1}0}=\tan^{-1}0+C$$

$$\Rightarrow C=0$$

Hence, the required solution is $y\cdot e^{\tan^{-1}y}=\tan^{-1}y$