Playing with Numbers





Exercise 3.1

Page No. 50

1.	Write all the fa	ctors of the following n	umbers			
	(a) 24	(b) 15	(c) 21	(d) 27		
	(e) 12	(f) 20	(g) 18	(h) 23		
	(i) 36					
Sol.	(a) We have, 24					
	$24 = 1 \times 24; 24 =$	$2 \times 12; 24 = 3 \times 8; = 4 \times 6$				
	∴ Factors of 24	are 1,2,3, 4, 6, 8, 12 and	24.			
	(b) We have, 15					
	15=1×15; 15=3	× 5				
	∴ Factors of 15	are 1, 3, 5 and 15.				
	(c) We have, 21					
	21=1×21; 21=3	× 7				
	∴ Factors of 21	are 1, 3, 7 and 21.				
	(d) We have, 27					
	27=1×27:27=3	× 9				
	∴ Factors of 27	are 1, 3, 9 and 27.				
	(e) We have, 12					
	12=1×12; 12=2	×6; 12=3×4				
	∴ Factors of 12	are 1, 2, 3, 4, 6 and 12.				
	(f) We have, 20					
	20= 1×20; 20=2	2×10; 20=4×5				
	∴ Factors of 20	are 1, 2, 4, 5, 10 and 20).			
	(g) We have <i>,</i> 18					
	18 = 1 × 18; 18 =	= 2× 9; 18 = 3×6				
	∴ Factors of 18	are 1, 2, 3, 6, 9 and 18.				
	(h) We have <i>,</i> 23					
	23=1×23					
	∴ Factors of 23	are 1 and 23.				
	(i) We have <i>,</i> 36					
	$36 = 1 \times 36$; $36 = 2 \times 18$; $36 = 3 \times 12$; $36 = 4 \times 9$; $36 = 6 \times 6$					
	∴ Factors of 36	are 1, 2, 3, 4, 6, 9, 12, 1	8 and 36.			

2. Write first five multiples of (b) 8

(a) 5

(c) 9

Sol. (a) Multiples of 5, 5×1= 5; 5×2=10; 5×3= 15; 5×4=20; 5×5=25 \therefore First five multiples of 5 are 5, 10, 15, 20 and 25. (b) Multiples of 8,

 $8 \times 1 = 8$; $8 \times 2 = 16$; $8 \times 3 = 24$; $8 \times 4 = 32$; $8 \times 5 = 40$

 \therefore First five multiples of 8 are 8, 16, 24, 32 and 40.

(c) Multiples of 9, 9 \times 1 = 9; 9 \times 2 = 18; 9 \times 3 =27; 9 \times 4 = 36; 9 x 5 = 45

 \therefore First five multiples of 9 are, 9, 18, 27, 36 and 45.

Note We can also write the Table' of required number to get the multiples of that number.

Match the items in column 1 with the items in column 2.

Column I	Column II	
(i) 35	(a) Multiple of 8	
(ii) 15	(b) Multiple of 7	
(iii) 16	(c) Multiple of 70	
(iv) 20	(d) Factor of 30	
(v) 25	(e) Factor of 50	
	(f) Factor of 20	

Sol. (i) Factors of 35 are 5 and 7 and we know that, a number is a multiple of each of its factor. Hence, 35 is a multiple of 7.

(ii) We know that, a factor of a number is an exact divisor of that number.

Here, 30 is divided by 15. So, 15 is a factor of 30.

(iii) Factors of 16 are 2 and 8 and we know that, a number is a multiple of each of its factor. Hence, 16 is a multiple of 8.

(iv) We know that, every number is a factor of itself. So, 20 is a factor of itself i.e. 20.

(v) We know that, a factor of a number is an exact divisor of that number. Here, 50 is divided by 25. So, 25 is a factor of 50.

Now, matching of these items is as follows:

(i) \rightarrow (b)	(ii) \rightarrow (d)	(iii) \rightarrow (a)	$(iv) \rightarrow (f)$
$(v) \rightarrow (e)$			

Page No. 51

3.

Find all the multiples of 9 up to 100. Sol. Multiples of 9 up to 100 are as follows:

501.	Multiples of 9 up to 100 are as follows:				
	9x1=9;	9x4=36;	9x7= 63;	9x10=90	
	9x2=18;	9x5=45;	9x8=72;	9x11=99	
	9x3=27;	9x6=54;	9x9=81		
	Hence, all the	multiples of 9 up	to 100 are 9, 18	, 27, 36, 45, 54, 63, 72,81, 90 and 99.	

Exercise 3.2

Page No. 53

1. What is the sum of any two

- (a) odd numbers?
- (b) even numbers?

TIPS

We know that a number is called an even number, if it is completely divided by 2 (i.e. remainder = 0). Otherwise, it is an odd number.

- Sol. (a) Let the two odd numbers be 5 and 7. Sum =5+7=12 (even number) Taking one more example, Let the two odd numbers be 3 and 7. Sum = 3 + 7=10 (even number) Hence, we can say, sum of any two odd numbers is an evennumber. (b) Let the two even numbers be 4 and 8. Sum =4+8=12 (even number) Taking one more example, Let the two even numbers be 6 and 20. Sum = 6 + 20 = 26(even number) Hence, we can say, sum of any two even numbers is an evennumber.
- 2. State whether the following statements are true or false.
 - (a) The sum of three odd numbers is even.
 - (b) The sum of two odd numbers and one even number is even.
 - (c) The product of three odd numbers is odd.
 - (d) If an even number is divided by 2, the quotient is alwaysodd.
 - (e) All prime numbers are odd.
 - (f) Prime numbers do not have any factor.
 - (g) Sum of two prime numbers is always even.
 - (h) 2 is the only even prime number.
 - (i) All even numbers are composite numbers.
 - (j) The product of two even numbers is always even.
 - (a) False, because the sum of three odd numbers is always odd.

e.g. 3+5+7=15 and9+11+13=33 (odd)

(b) True, because the sum of two odd numbers and one ever numberis always an even number.

3 + 7 + 6 = 16

e.g. $\downarrow \downarrow \downarrow \downarrow$ (even)

Odd Odd Even

(c) True, because product of three odd numbers is always odd.

e.g. 3 x 5 x 7 = 105 (odd)

(d) False, because if an even number is divided by 2, the quotient isalways an even number.

e.g.
$$\frac{24}{2} = 12$$
 (even)

(e) False, because 2 is only even prime number. So, all primenumbers are not odd.

(f) False, because prime numbers have two factors, which are 1 and number itself.

e.g. 5 is a prime number and has two factors 1 and 5.

(g) False, because sum of two prime numbers is either odd or even.

e.g. 2+3=5 (odd) and 3 + 7 = 10 (even)

(h) True,

Sol.

(i) False, because all even numbers are not composite numbers.

e.g. 2 has the factors 1 and 2 only, so it is a prime number but nota composite number.

(j) True, the product of two even numbers is always even.

e.g. 2x4=8 (even)

3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100.

 Sol.
 All prime numbers upto 100 are 2, 3, 5, 7, 11, 13, 17, 19,23, 29, 31,37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97. Out of theseprime numbers, a pair of prime numbers having same digits are

 (i) 13, 31
 (ii) 17, 71
 (iii) 37, 73
 (iv) 79, 97

 Hence, there are 4 pairs of such types.
 (iv) 79, 97
 (iv) 79, 97

4. Write down separately the prime and compositenumbers less than 20.

Sol. Prime numbers are those numbers whose only factors are 1 and thenumber itself.
 So, prime numbers less than 20 are 2, 3, 5, 7,11,13,17 and 19.
 Composite numbers are those numbers which have more than twofactors.
 So, composite numbers less than 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16and 18.

5. What is the greatest prime number between 1 and 10?

Sol. Prime numbers between 1 and 10 are 2, 3, 5 and 7. Therefore, thegreatest prime number between 1 and 10 is 7.

6.	Express the following as the sum of two odd primes.						
	(a) 44	(b) 36	(c) 24	(d) 18			
Sol.	We know that	at, every prime number ex	cept 2 are odd numbers				
	We have, 44						
	44=13+31 or	44=3+41					
	(b) We have,	36					
	36=5+31 or 3	36=13+23					
	(c) We have,	24					
	24 =5+19 or	24=11+13					
	(d) We have,	18					
	18=7+11 or 1	18=5+13					
7.	Give three p	airs of prime numbers wh	ose difference is 2.				
	[Remark Two	o prime numbers whose o	difference is 2 arecalled	twin primes].			
Sol.	Three pairs o	of prime numbers whose d	lifference is 2 are as follo	ows:			
	(i) 5 <i>,</i> 7	i.e. 7 – 5=2					
	(ii) 11 <i>,</i> 13	i.e. 13 – 11=2					
	(;;;;) 17 10	i.e. 19 – 17=2					

8. Which of the following numbers are prime?
(a) 23 (b) 51 (c) 37 (d) 26

TIPS

To check these given numbers, firstly we divide each numberindividually by the prime numbers, less than each.

If any number is not completely divided by prime numbers, it would bea prime number.

Sol. (a) We find that 23 is not exactly divisible; by any of the primenumbers 2, 3, 5, 7, 11, 17 and 19. So, it is a prime number.

(b) We find that 51 is divisible by 3. So, it is not a prime number.

(c) We find that 37 is not exactly divisible by any of the primenumbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31. So, it is a primenumber.

(d) We find that 26 is exactly divisible by 2 and 13. So, it is not aprime number.

- 9. Write seven consecutive composite numbers less than100 so that there is no prime number between them.
- Sol. Seven composite numbers of such type are as follows: 90,91,92, 93, 94, 95 and 96

Page No. 54

10.	Express each of	the following	numbers as the	sum ofthree o	dd primes.	
	(a) 21	(b) 31		(c) 53	(d) 61	
Sol.	(a) We have, 21	⇒21=3+5+13	3			
	where 3, 5 and 1					
	(b) We have, 31	•				
	where 3, 5 and 2					
	(c) We have, 53	•	= 13 + 17 + 23			
	where 13, 17 an					
	(d) We have, 61					
	where 7, 13 and					
	,	•				
11.	Write five pairs	of prime num	bers less than 2	0, whosesum is	s divisible by 5. (Hint 3 + 7 = 10)	
Sol.	Prime numbers	less than 20 a	re 2, 3, 5, 7, 11,	13, 17 and 19.		
	Here, 2+3=5 (d	divisible by 5);	2+13=15 (divi	sible by 5)		
	3+7=10 (divisible by 5); 3 + 17 = 20 (divisible by 5)					
	7+13= 20 (divisible by 5)					
	Hence, five pairs	of prime nun	nbers whose sur	n is divisible by	5 are	
	(i) 2 <i>,</i> 3	(ii) 2 <i>,</i> 13	(iii) 3 <i>,</i> 7	(iv) 3, 17		
	(v) 7, 13					
12.	Fill in the blanks					
	(a) A number w					
	(b) A number which has more than two factors is called a					
	(c) 1 is neither nor					
	(d) The smallest prime number is (e) The smallest composite number is					
				•		
	(f) The smallest					
Sol.	(a) A number wh					
	(b) A number wh			s is called a com	posite number.	
	(c) 1 is neither p		•			
	(d) The smallest	•				
	(e) The smallest	•				
	(f) The smallest	even number	is 2.			

Exercise 3.3

Page No. 57

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by9; by 10; by 11 (say, yes or no)?

Number		Divisible by							
	2	3	4	5	6	8	9	10	11
(i) 128	Yes	No	Yes	No	No	Yes	No	No	No
(ii) 990									
(iii) 1586									
(iv) 275									
(v) 6686									
(vi) 639210									
(vii) 429714									
(viii) 2856									
(ix) 3060									
(x) 406839									

- **Sol.** We know that, a number is divisible by
 - 2, if it has digits 0, 2, 4, 6 or 8 in ones place.
 - 3, if the sum of the digits is a multiple of 3 or it is, divisible by 3.
 - 4, if last two digits of the number is completely divisible by 4.
 - 5, if a number has 0 or 5 in its ones place.
 - 6, if it is divisible by 2 and 3 both.
 - 8, if last three digits of the number is completely divisible by 8.
 - 9, if the sum of the digits of the number is divisible by 9.
 - 10, if a number has 0 in its ones place.

11, if the difference of sum of the digits of even place and, sum of the digits at odd place is either 0 or multiple of II.

Now, complete table is shown as below

Number		Divisible by								
		2	3	4	5	6	8	9	10	11
(i)	128	Yes	No	Yes	No	No	Yes	No	No	No
(ii)	990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
(iii)	1586	Yes	No							
(iv)	275	No	No	No	Yes	No	No	No	No	Yes
(v)	6686	Yes	No							
(vi)	639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
(vii)	429714	Yes	Yes	No	No	Yes	No	Yes	No	No
(viii)	2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
(ix)	3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
(x)	406839	No	Yes	No						

2. Using divisibility tests, determine which of the following numbers are divisible by 4 and 8? (a) 572 (b) 726352 (c) 5500 (d) 6000 (g) 21084 (e) 12159 (f) 14560 (h) 31795072 (i) 1700 (j) 2150 We know that, a number is divisible by 4, if the number formed by last two digits i.e. tens and ones place Sol. digits are divisible by 4. Number is divisible by 8, if the number formed by last three digits i.e. its hundreds, tens and ones place digits are divisible by 8. (a) We have, 572 (i) Divisibility by 4 Number formed by last two digits = 72 On dividing 72 by 4, we get Remainder =0 \therefore 72 is divisible by 4, so 572 is also divisible by 4. 4)72(18 4 32 32 × (ii) Divisibility by 8 Number formed by last three digits On dividing572 by 8, we get Remainder $\neq 0$ \therefore 572 is not divisible by 8. 8)572(71 56 12 8 $\overline{4}$ (b) We have, 726352 (i) Divisibility by 4 Number formed by last two digits = 52 On dividing 52 by 4, we get Remainder = 0 :: 52 is divisible by 4, so 726352 is also divisible by 4. 4)52(13 4 12 12 4 (ii) Divisibility by 8 Number formed by last three digits = 352 On dividing 352 by 8, we get Remainder =0

:352 is divisible by 8 \therefore 726352 is also divisible by 8. 8)352(44 32 32 32 × (c) We have, 5500 (i) Divisibility by 4 Number formed by last two digits = 00 which is divisible by4. :. 5500 is divisible by 4 (ii) Divisibility by 8 Number formed by last three digits = 500 On dividing 500 by 8, we get Remainder $\neq 0$:: 500 is not divisible by 8 \therefore 5500 is not divisible by 8. 8)500(62 48 20 16 4 (d) We have, 6000 (i) Divisibility by 4 Number formed by last two digits = 00 which is divisible by 4. \therefore 6000 is divisible by 4. (ii) Divisibility by 8 Number formed by last three digits = 000 which is divisible by 8. \therefore 6000 is divisible by 8. (e) We have, 12159 (i) Divisibility by 4 Number formed by last two digits = 59 On dividing 59 by 4, we get Remainder $\neq 0$ \therefore 59 is not divisible by 4, so 12159 is not divisible by 4. 4)59(14 4 19 16 3 (ii) Divisibility by 8 Number formed by last three digits = 159 On dividing 159 by 8, we get

Remainder $\neq 0$ \therefore 159 is not divisible by 8, \therefore 12159 is not divisible by 8. 8)159(19 8 79 72 7 (f) We have, 14560 (i) Divisibility by 4 Number formed by last two digits = 60 On dividing 60 by 4, we get Remainder = 0 \therefore 60 is divisible by 4, so 14560 is also divisible by 4. 4)60(15 4 20 20 × (ii) Divisibility by 8 Number formed by last three digits = 560 On dividing 560 by 8, we get Remainder =0 \therefore 560 is divisible by 8. \therefore 14560 is also divisible by 8. 8)560(70 56 0 0 × (g) We have, 21084 (i) Divisibility by 4 Number formed by last two digits = 84 On dividing 84 by 4, we get Remainder = 0 \therefore 84 is divisible by 4, so 21084 is divisible by 4. 4)84(21 8 4 4 × (ii) Divisibility by 8 Number formed by last three digits = 084 = 84 On dividing 084 by 8, we get

Remainder $\neq 0$:: 084 is not divisible by 8. \therefore 21084 also not divisible by 8. (h) We have, 31795072 (i) Divisibility by 4 Number formed by last two digits = 72 On dividing 72 by 4, we get Remainder =0 :: 72 is divisible by 4. \therefore 31795072 is also divisible by 4. 4)72(18)4 32 32 \times (ii) Divisibility by 8 Number formed by last three digits = 072 = 72On dividing 072 by 8, we get Remainder = 0 \therefore 072 is divisible by 8, so 31795072 is also divisible by 8. 8)72(9 72 × (i) We have, 1700 (i) Divisibility by 4 Number formed by last two digits = 00, which is divisible by 4. \therefore 1700 is also divisible by 4. (ii) Divisibility by 8 Number formed by last three digits = 700 On dividing 700 by 8, we get Remainder $\neq 0$ \therefore 700 is not divisible by 8. \therefore 1700 is also not divisible by 8. 8)700(87 64 60 56 4 (j) We have, 2150 (i) Divisibility by 4 Number formed by last two digits =50 On dividing 50 by 4, we get Remainder $\neq 0$ \therefore 50 is not divisible by 4, so 2150 is also not divisible by 4.

4)50(12 4 $\overline{10}$ 8 2 (ii) Divisibility by 8 Number formed by last three digits =150 On dividing 150 by 8, we get Remainder $\neq 0$ \therefore 150 is not divisible by 8 \therefore 2150 is also divisible by 8. 8)150(18 8 70 64 6 3. Using divisibility tests, determine which of the following numbers are divisible by 6? (a) 297144 (b) 1258 (c) 4335 (d) 61233 (e) 901352 (f) 438750 (g) 1790184 (h) 12583 (i) 639210 (j) 17852 Sol. We know that, a number is divisible by 6, if it is divisible by 2 and 3both. Also, a number is divisible by 2 if it has any of the digits 0, 2, 4, 6or 8 in its ones place and number is divisible by 3, if the sum of the digits is a multiple of 3. (a) We have, 297144 (i) Divisibility by 2 \therefore Units digit of number = 4, so 297144 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number = 2 + 9 + 7 + 1 + 4 + 4 = 27 \therefore 27 is divisible by 3, so 297144 is also divisible by 3. Now, we see that 297144 is divisible by 2 and 3 both. Hence, it is divisible by 6. (b) We have, 1258 (i) Divisibility by 2 \therefore Units digit of number = 8, so 1258 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =1+2+5+8=16 16 is not divisible by 3, so 1258 is not divisible by 3. Now, we see that 1258 is divisible by 2 but not divisible by 3. Hence, it is not divisible by 6. (c) We have, 4335 (i) Divisibility by 2 :: Units digit of number = 5 which is not any of the digits 0, 2, 4, 6 or 8. \therefore 4335 is not divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =4+3+3+5=15 \therefore 15 is divisible by 3, so 4335 is divisible by 3. Now, we see that 4335 is divisible by 3 but not divisible by 2.

Hence, it is not divisible by 6. (d) We have, 61233 (i) Divisibility by 2 \therefore Units digit of number = 3which is not any of the digits 0, 2, 4, 6 or 8. \therefore 61233 is not divisible by 2. Now, we have no need to check the given number is divisible by 3 or not because it is not divisible by one of the factors of 6. Hence, the given number 61233 is not divisible by 6. (e) We have, 901352 (i) Divisibility by 2 \therefore Units digit of number = 2, so 901352 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =9+0+1+3+5+2=20 \therefore 20 is not divisible by 3, so 901352 is not divisible by 3. Now, we see that 901352 is divisible by 2 but not divisible by 3. Hence, it is not divisible by 6. (f) We have, 438750 (i) Divisibility by 2 \therefore Units digit of number = 0, so 438750 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =4+3+8+7+5+0= 27 \therefore 27 is divisible by 3, so 438750 is divisible by 3. Now, we see that 438750 is divisible by 2 and 3 both. Hence, it is divisible by 6. (g) We have, 1790184 (i) Divisibility by 2 \therefore Units digit of number = 4, so 1790184 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =1+7+9+0+1+8+4= 30 \therefore 30 is divisible by 3, so 1790184 is divisible by 3. Now, we see that 1790184 is divisible by 2 and 3 both. Hence, it is divisible by 6. (h) We have, 12583 (i) Divisibility by 2 \therefore Units digit of number = 3which is not any of the digits 0, 2, 4, 6 or 8. \therefore 12583 is not divisible by 2. Now, we have no need to check the given number is divisible by 3 or not because it is not divisible by one of the factors of 6. Hence, the given number 12583 it not divisible by 6. (i) We have, 639210 (i) Divisibility by 2 \therefore Units digit number = 0, so 639210 is divisible by 2. (ii) Divisibility by 3 Sum of digits of given number =6+3+9+2+1+0=21 \therefore 21 is divisible by 3, so 639210 is divisible by 3. Now, we see that 639210 is divisible by 2 and 3 both. Hence, it is divisible by 6. (j) We have, 17852

(i) Divisibility by 2
Units digit of number = 2, so 17852 is divisible by 2.
(ii) Divisibility by 3
Sum of digits of given number =1+7+8+5+2=23
∴ 23 is not divisible by 3, so 17852 is not divisible by 3.
Now, we see that 17852 is divisible by 2 but not divisible by 3.
Hence, it is not divisible by 6.

4. Using divisibility tests, determine which of the following numbers are divisible by II? (a) 5445 (b) 10824 (c) 7138965 (d) 70169308 (e) 10000001 (f) 901153

TIPS

Firstly, find the difference between the sum of the digits at odd places '(from the right) and the sum of the digits at even places (from the right) of the number. If the difference is either 0 or divisible by 11, then thenumber is divisible by 11, otherwise not.

Sol. (a) We have, 5445

5 4 4 5

 $\downarrow \hspace{0.1cm} \downarrow \hspace{0.1cm} \downarrow \hspace{0.1cm} \downarrow \hspace{0.1cm} \downarrow \hspace{0.1cm} \downarrow$

 $E \quad O \quad E \quad O$

where, O = Odd and E = EvenSum of digits at odd places from right =5+4=9 Sum of digits at even places from right =4+5=9 Now, difference = 9 – 9= 0, so 5445 is divisible by 11. (b) We have, 10824

1 0 8 2 4 | | | | |

 $O \quad E \quad O \quad E \quad O$

Sum of digits at odd places from right =4+8+1=13Sum of digits at even places from right =2+0=2Now, difference =13-2=11, so 10824 is divisible by 11. (c) We have, 7138965

7	1	3	8	9	6	5	

```
O \quad E \quad O \quad E \quad O \quad E \quad O
```

Sum of digits at odd places from right =5+9+3+7=24Sum of digits at even places from right =6+8+1=15Now, difference = 24 - 15 = 9 $\therefore 9$ is not a multiple of 11, so 7138965 is not divisible by 11. (d) We have, 70169308

Sum of digits at odd places from right =8+3+6+0=17 Sum of digits at even places from right =0+9+1+7=17 Now, difference = 17 - 17 = 0 $\therefore 70169308$ is divisible by 11. (e) We have, 10000001 $1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1$ $| \quad | \quad | \quad | \quad | \quad | \quad | \quad |$ $E \quad O \quad E \quad O \quad E \quad O \quad E \quad O$

Sum of digits at odd places from right =1+0+0+0=1 Sum of digits at even places from right =0+0+0+ 1=1 Now, difference = 1 - 1 = 0, so 10000001 is divisible by 11. (f) We have, 901153 9 0 1 1 5 3

| | | | | | E O E O E O

Sum of digits at odd places from right =3+1+0=4Sum of digits at even places from right =5+1+9=15Now, difference = 15 - 4 = 11, so 901153 is divisible by 11.

5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers, so that the number formed is divisible by 3:

(a) <u>6724</u> (b) 4765 <u>2</u>

TIPS

Firstly, find the sum of digits of given number, then subtract it from those multiples of 3, which are greater than this sum to get smallest and greatest digit.

Sol. (a) We have, _____6274 Sum of the given digits =6+7+2+4=19 ∴ Multiples of 3 greater than 19 are 21, 24, 27, 30,... ∴ 21-19=2; 24 - 19=5; 27 - 19=8; 30-19= 11 But 11 is not a single digit. ∴ Smallest digit = 2 and greatest digit = 8 (b) We have, 4765 _____2 Sum of the given digits =4+7+6+5+2=24 ∴ 24 is a multiple of 3, so smallest digit = 0 Now, multiples of 3 greater than 24 are 27, 30, 33, 36, ... ∴ 27 - 24=3; 30 - 24=6; 33 - 24=9; 36 - 24=12 But 12 is not a single digit. ∴ Smallest digit = 0 and greatest digit = 9

Page No. 58

6. Write a digit in the blank space of each of the following numbers, so that the number formed is divisible by 11.
(a) 92 ____ 389 (b) 8 ____ 9484

TIPS

Firstly, assume the blank space digit as x, then find the sum of odd places and even places digits (from the right) separately. Now, take difference of sum of odd places digits and sum of even places digits equal to 0 or 11 and simplify to get the value of x.

Sol. (a) Let the required unknown digit be xThen, number be 2 x 3 8 9 9 \downarrow $\downarrow \downarrow$ $\downarrow \downarrow \downarrow \downarrow$ E O E O E Owhere, O = odd and E = evenSum of digits at odd places from right = 9 + 3 + 2 = 14Sum of digits at even places from right = 8 + x + 9 = 17 + x:: Number is divisible by 11. ∴ Difference of digits will be 0 or 11. (17 + x) - 14 = 0 or 11 \Rightarrow 17 + x - 14 = 0 or $11 \implies x + 3 = 0$ or 11 \Rightarrow Taking difference 0, 0, x + 3 = 0x = 0 - 3 = -3 (not possible) \Rightarrow Taking difference 11, x = 3 = 11So, required digit to write in the blank space is (b) Let the required unknown digit be x. Then, number be 8 x 9 4 8 4 \downarrow $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$ EOEOEO Sum of digits at odd places from right =4+4+x=8+xSum of digits at even places from right =8+9+8=25 :: Number is divisible by 11. ∴ Difference of digits will be 0 or 11. $\Rightarrow 25 - (8 + x) = 0 \text{ or } 11$ 25-8-x=0 or $11 \Longrightarrow 17-x=0$ or 11 \Rightarrow Taking difference 0, $17 - x = 0 \Longrightarrow x = 17 + 0 \Longrightarrow x = 17$ [but 17 is not a single digit number, so it is not possible] Taking difference 11, 17 - x = 11x = 17 - 11 = 6 \Rightarrow So, required digit to write in the blank space is

Exercise 3.4

(d) 56 and 120

Page No. 59

1.	Find the common factors of					
	(a) 20 and 28 (b) 15 a	and 25 (c) 35 and 5	50			
Sol.	(a) Factors of 20 and 28 are as f	ollows:				
	$20 = 1 \times 20; \ 20 = 2 \times 10;$	$20 = 4 \times 5$				

and $28 = 1 \times 28$; $28 = 2 \times 14$; $28 = 4 \times 7$ Now, all factors of 20 = 1, 2, 4, 5, 10, 20and all factors of 28 = 1, 2, 4, 7, 14, 28 \therefore Common factors =1, 2, 4 Hence, common factors of 20 and 28 are 1, 2 and 4. (b) Factors of 15 and 25 are as follows: $15 = 1 \times 15; 15 = 3 \times 5$ and $25 = 1 \times 25; 25 = 5 \times 5$ Now, all factors of 15 = 1, 3, 5, 15and all factors of 25 = 1, 5, 25∴ Common factors =1,5 Hence, common factors of 15 and 25 are 1 and 5. (c) Factors of 35 and 50 are as follows: $35 = 1 \times 35; 35 = 5 \times 7$ $50 = 1 \times 50; 50 = 2 \times 25; 50 = 5 \times 10$ and Now, all factors of 35 = 1, 5, 7and all factors of 50 = 1, 2, 5, 10, 25, 50∴ Common factors =1, 5 Hence, common factors of 35 and 50 are 1 and 5. (d) Factors of 56 and 120 are as follows: $56 = 1 \times 56$; $56 = 2 \times 28$; $56 = 4 \times 14$; $56 = 7 \times 8$ $20 = 1 \times 1$ 20; $120 = 2 \times 60$; $120 = 3 \times 40$; $120 = 4 \times 30$ and $120=5 \times 24$; $120=6\times 20$; $120=8\times 15$; $120=10\times 12$ Now, all factors of 56 = 1, 5, 4, 7, 8, 14, 28, 56and all factors of 120 = 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120 \therefore Common factors = 1,2,4,8 Hence, common factors of 56 and 120 are 1, 2, 4 and 8.

2. Find the common factors of (a) 4, 8 and 12

(b) 5, 15 and 25

```
Sol. (a) We have 4, 8 and 12

4=1\times4; 4=2\times2

\therefore Factors of 4 are 1, 2 and 4.

Now, 8=1\times8; 8=2\times4

\therefore Factors of 8 are 1, 2, 4 and 8.

And 12=1\times12; 12=2\times6; 12=3\times4

\therefore Factors of 12 are 1, 2, 3, 4, 6 and 12.

Now, all factors of 4=1, 2, 4, 4

all factors of 8=1, 2, 4, 4

all factors of 8=1, 2, 4, 4

\therefore Factors of 12 are 1, 2, 3, 4, 6 and 12.

Now, all factors of 4=1, 2, 4, 4

\Rightarrow and all factors of 4=1, 2, 4, 4

\Rightarrow and 12=1\times12; 12=2\times6; 12=3\times4

\therefore Factors of 12 are 1, 2, 3, 4, 6 and 12.

Now, all factors of 4=1, 2, 4, 4

\Rightarrow and 12=1\times12; 12=2\times6; 12=3\times4

\Rightarrow \therefore Factors of 12 are 1, 2, 3, 4, 6 and 12.

Now, all factors of 4=1, 2, 3, 4, 6, 12

Hence, common factors of 4, 8 and 12 are 1, 2 and 4.

(b) We have 5, 15 and 25

5=1\times5
```

∴ Factors of 5 are 1 and 5. Now, $15=1\times15; 15=3\times5$ ∴ Factors of 15 are 1, 3, 5 and 15. and $25=1\times25; 25=5\times5$ ∴ Factors of 25 are 1, 5 and 25. Now, all factors of 5=1, 5all factors of 15=1, 3, 5, 15and all factors of 25=1, 5, 25Hence, common factors of 5, 15 and 25 are 1 and 5.

3. Find first three common multiples of (a) 6 and 8 (b) 12 and 18

Sol. (a) Some multiples of 6 = 6,12,18, 24, 30,36,42, 48,54,60,66, 72, ...Some multiples of 8 = 8,16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, ...Hence, first three common multiples are 24, 48 and 72. (b) Some multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, ...Some multiples of 18 = 18, 36, 54, 72, 90, 108, 126, 144, ...Hence, first three common multiples are 36, 72 and 108.

4. Write all the numbers less than 100 which are commonmultiples of 3 and 4.

Sol. Multiples of 3 which are less than 100 are as follows: 3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,66,69,72, 75,78,81,84,87,90,93,96,99
Multiples of 4 which are less than 100 are as follows: 4,8,12,16,20,24,28,32,36,40,44,48,52,56,60,64,68,72,76,80,84,88,92,96
Hence, common multiples of 3 and 4 are 12, 24, 36, 48, 60, 72, 84and 96.

5.	Which of the following numbers are co-prime?						
	(a) 18 and 35	(b) 15 and 37	(c) 30 and 415	(d) 17 and 68			
	(e) 216 and 215	(f) 81 and 16					

TIPS

Firstly, write the factors of given numbers. If two numbers have only 1 ascommon factor, then they are called co-prime numbers otherwise not.

Sol. (a) We have, 18 and 35 $18=1\times18; 18=2\times9; 18=3\times6$ Factors of 18 are 1, 2, 3, 6, 9 and 18. and $35=1\times35; 35=5\times7$ Factors of 35 are 1, 5, 7 and 35. \therefore Common factor of 18 and 35 is only 1. So, 18 and 35 are co-prime numbers. (b) We have, 15 and 37 $15=1\times15; 15=3\times5$

```
Factors of 15 are 1, 3, 5 and 15.
and
         37 = 1 \times 37
Factors of 37 are I and 37.
: Common factor of 15 and 37 is only 1.
So, 15 and 37 are co-prime numbers.
(c) We have, 30 and 415
30 = 1 \times 30; 30 = 2 \times 15; 30 = 3 \times 10; 30 = 5 \times 6
Factors of 30 are I, 2, 3, 5, 6, 10, 15 and 30.
         415 = 1 \times 415; 415 = 5 \times 83
and
Factors of 415 are 1, 5, 83 and 415.
: Common factors of 30 and 415 are 1 and 5.
So, 30 and 415 are not co-prime numbers.
(d) We have, 17 and 68
         17 = 1 \times 17
Factors of 17 are 1 and 17.
         68 = 1 \times 68; 68 = 2 \times 34; 68 = 4 \times 17
and
∴ Factors of 68 are 1, 2, 4, 17, 34 and 68.
: Common factors of 17 and 68 are 1 and 17.
So, 17 and 68 are not co-prime numbers.
(e) We have, 216 and 215
216 = 1 \times 216; 216 = 2 \times 108; 216 = 3 \times 72; 216 = 4 \times 54
216 = 6 \times 36; 216 = 8 \times 27; 216 = 9 \times 24; 216 = 12 \times 18
Factors of 216 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72, 108 and 216.
         215 = 1 \times 215; 215 = 5 \times 43
and
Factors of 215 are 1, 5, 43 and 215
:: Common factor of 216 and 215 is only 1.
So, 216 and 215 are co-prime numbers.
(f) We have, 81 and 16
         81 = 1 \times 81; 81 = 3 \times 27; 81 = 9 \times 9
Factors of 81 are 1, 3, 9, 27 and 81.
and
         16 = 1 \times 16; 16 = 2 \times 8; 16 = 4 \times 4
Factors of 16 are 1, 2, 4, 8 and 16.
: Common factor of 81 and 16 is only 1.
So, 81 and 16 are co-prime numbers.
```

6.	A number is divisible by both 5 and 12. By which othernumber will that number be always divisible?
Sol.	The given number will be divisible by the product of 5 and 12.
	i.e., it is always divisible by $5 \times 12 = 60$.
	Note If a number is divisible by two co-prime numbers, then it is divisible bytheir product also.

7. A number is divisible by 12. By what other numbers willthat number be divisible?

Sol. If any number is divisible by 12, then this number will also bedivisible by the factors of 12. i.e. $12=1\times12; 12=2\times6; 12=3\times4$ Factors of 12 are 1, 2, 3, 4, 6 and 12. \therefore Number will be divisible by 1, 2, 3, 4, 6 and 12.

Exercise 3.5

Page No. 61

- 1. Which of the following statements are true?
 - (a) If a number is divisible by 3, it must be divisible by 9.
 - (b) If a number is divisible by 9, it must be divisible by 3.
 - (c) A number is divisible by 18, if it is divisible by both 3and 6.
 - (d) If a number is divisible by 9 and 10 both, then it must bedivisible by 90.
 - (e) If two numbers are co-prime, atleast one of them mustbe prime.
 - (f) All numbers which are divisible by 4 must also bedivisible by 8.
 - (g) All numbers which are divisible by 8 must also bedivisible by 4.
 - (h) If a number exactly divides two numbers separately, itmust exactly divide their sum.

(i) If a number exactly divides the sum of two numbers, itmust exactly divide the two numbers separately.

Sol. (a) False, because there are plenty of numbers, which are divisible by 3 but not divisible by 9. e.g. 30 is divisible by 3, but not divisible by 9.

(b) True, because if a number is divisible by any number, then it is divisible by each factor of that number. Here, 3 is a factor of 9.

e.g.
$$\frac{27}{9} = 3$$
 and $\frac{27}{3} = 9$

(c) False, e.g. Number 30 is divisible by 3 and 6 both but not divisible by 18.

(d) True, because if a number is divisible by two co-prime numbers, then it is divisible by their product also.

(e) False, we know that, two numbers having only 1 as a commonfactor are called co-prime numbers. So, it is not necessary thatone of them must be prime.

e.g. numbers 8 and 15 are co-prime numbers, since both haveonly 1 as a common factor, but no one is a prime number.

(f) False, e.g. number 36 is divisible by 4 but not divisible by 8.

(g) True because if a number is divisible by any number, then it isdivisible by each factor of that number. Here, 4 is a factor of 8.

So, all numbers divisible by 8 must also be divisible by 4.

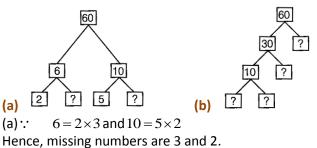
e.g. Number 56 is divisible by 8 as well as divisible by 4.

(h) True, if two given numbers are divisible by a number, then theirsum is also dividible by that number, e.g. number 13 is exactly divides number 52 and 65 also divide their sum 117.

False, e.g. Number 5 is exactly divides the sum of number 2 and 3 but not exactly divides these two numbers.

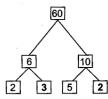
Page No. 62

2. Here are two different factor trees for 60. Write themissing numbers.



Sol.

(b) $\therefore 60 = 30 \times 2$ $30 = 10 \times 3$ and $10 = 5 \times 2$ \therefore Hence, missing numbers are 5, 2, 3 and 2.



3. Which factors are not included in the prime factorization of a composite number?

Sol. Factor 1 and that number itself are not included in the primefactorisation of a composite number.

4. Write the greatest 4-digit number and express it interms of its prime factors.

Sol. The greatest 4-digit number = 9999

3	9999	
3	3333	
11	1111	
101	101	
	1	

 \therefore Prime factors of $= 3 \times 3 \times 11 \times 101$

5. Write the smallest 5-digit number and express it in theform of its prime factors.

Sol. The smallest 5-digit number = 10000

Now,

2	10000
2	5000
2	5000
2	1250
5	625
5	125
5	25
5	5
	1

 $\therefore \textbf{Prime factors } of 10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

6. Find all the prime factors of 1729 and arrange them inascending order. Now, state the relation, if any, between twoconsecutive prime factors.

Sol. Prime factors of $1729 = 7 \times 13 \times 19$ Ascending order of prune factors of 1729 are 7,13 and 19. Here, 19 - 13 = 6 and 13 - 7 = 6So, it is clear that difference of two consecutive prime factors is 6.

7	1729
13	247
19	19
	1

- 7. The product of three consecutive numbers is, alwaysdivisible by 6. Verify this statement with the help of some examples.
- Sol. We know that, a number is divisible by 6, if the number is divisible by 2 and 3 both. **Example 1** Let the three consecutive numbers be 7, 8 and 9. Product of numbers $=7 \times 8 \times 9 = 504$ Units digit of number = 4, so it is divisible by 2. Now, sum of the digit =5+0+4=9, which is a multiple of 3. So, 504 is divisible by 3. :: 504 is divisible by both 2 and 3, so 504 is divisible by 6. Example 2 Let the three consecutive numbers be 11, 12 and 13. Product of numbers $=11 \times 12 \times 13 = 1716$ Units digit of number = 6, so it is divisible by 2. Now, sum of the digit =1+7+1+6=15, which is a multiple of 3. So, 1716 is divisible by 3. \therefore 1716 is divisible by both 2 and 3. \therefore 1716 is divisible by 6. Hence, verified.
- 8. The sum of two consecutive odd numbers is divisible by4. Verify this statements with the help of some examples.
- Sol. Let us consider the following examples

Consecutive odd numbers	tive odd numbers 9 and 11	
Sum	9 + 11 = 20	103 + 105 = 208
Number formed from the	20, which is	08, which is divisible
last two digits of the sum	divisible by 4	by 4

We observed that, the numbers formed from the last two digits isdivisible by 4. Hence, the sum of two consecutive odd numbers are also divisible by 4. **Hence, verified.**

9. In which of the following expressions, primefactorisation has been done?

(a) $24 = 2 \times 3 \times 4$ (b) $56 = 7 \times 2 \times 2 \times 2$ (c) $70 = 2 \times 5 \times 7$ (d) $54 = 2 \times 3 \times 9$

Sol. (a) We have, $24 = 2 \times 3 \times 4$

Here, 4 is not a prime number.

So, number 24 does not have prime factorisation.

(b) We have, $56 = 7 \times 2 \times 2 \times 2$

Here, 2 and 7 are prime numbers.

- So, number 56 have prime factorisation.
- (c) We have, $70 = 2 \times 5 \times 7$, Here, 2, 5 and 7 all are prime numbers.
- So, number 70 have prime factorisation.
- (d) We have, $54 = 2 \times 3 \times 9$, Here, 9 is not a prime number.
- So, number 54 does not have prime factorisation.

10. Determine, if 25110 is divisible by 45.

[Hint 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].

TIPS

If a number is divisible by two co-prime numbers, then it is divisible by their product also.

- Sol. We have, 45 = 5×9, also 5 and 9 are co-prime. Now, 25110 is divisible by 5 because its units digit is 0. Again, sum of the digits =2+5+1+1+0=9 ∴ Sum of the digits is divisible by 9, so 25110 is also divisible by 9. Since, 25110 is divisible by both 5 and 9, where 5 and 9 are co-primes. ∴ 25110 is divisible by 5×9 i.e. by 45.
- 11. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6.Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer.
- Sol. We know that, if a number is divisible by two co-prime numbers, then it is divisible by their product also. Here, 2 and 3 are co-prime numbers. So, 18 is divisible by their product i.e. 6 but 4 and 6 are not co-prime numbers, so the number divisible by 4 and 6 will not be divisible by $4 \times 6 = 24$. e.g. Take the number 60.

It is divisible by both 4 and 6, but it is not divisible by its product. i.e. $6 \times 4 = 24$

12. I am the smallest number, having four different prime factors. Can you find me?

Sol. Since the number is smallest, so different four smallest prime factors are 2, 3, 5 and 7 \therefore Smallest number having four different prime factors = $2 \times 3 \times 5 \times 7 = 210$

Exercise 3.6

Page No. 63

1.	Find the HCF of the fo	llowing numbers.		
	(a) 18, 48	(b) 30, 42	(c) 18, 60	(d) 27, 63
	(e) 36, 84	(i) 34, 102	(g) 70, 105,175	(h) 91, 112, 49
	(i) 18, 54, 81	(j) 12, 45, 75		
Sol.	(a) Prime factorisation	of $18 = 2 \times 3 \times 3$		
	Prime factorisation of	$48 = 2 \times 2 \times 3 \times 2 \times 2$		
	Common factors of 18	and 48 are 2 and 3.		
	Thus, HCF of 18 and 43	$8 = 2 \times 3 = 6$		
	(b) Prime factorisation	of $30 = 2 \times 3 \times 5$		
	Prime factorisation of	$42 = 2 \times 3 \times 7$		
	Common factors of 30	and 42 are 2 and 3.		
	Thus, HCF of 30 and 42	$2 = 2 \times 3 = 6$		
	(c) Prime factorisation	of $18 = 2 \times 3 \times 3$		
	Prime factorisation of	$60 = 2 \times 3 \times 3 \times 5$		
	Common factors of 18	and 60 are 2 and 3.		
	Thus, HCF of 18 and 60	$0 = 2 \times 3 = 6$		
	(d) Prime factorisation	of $27 = 3 \times 3 \times 3$		

Prime factorisation of $63 = 3 \times 3 \times 7$ Common factor of 27 and 63 is 3 (occurring twice) Thus, HCF of 27 and $63 = 3 \times 3 = 9$ (e) Prime factorisation of $36 = 2 \times 2 \times 2 \times 3$ Prime factorisation of $84 = 2 \times 2 \times 3 \times 7$ Common factors of 36 and 84 are 2 (occurring twice) and 3. Thus, HCF of 36 and $84 = 2 \times 2 \times 3 = 12$ (f) Prime factorisation of $34 = 2 \times 17$ Prime factorisation of $102 = 2 \times 17 \times 3$ Common factors of 34 and 102 are 2 and 17. Thus, HCF of 34 and $102 = 2 \times 17 = 34$ (g) Prime factorisation of $70 = 2 \times 5 \times 7$ Prime factorisation of $105 = 3 \times 5 \times 7$ Prime factorisation of $175 = 3 \times |5| \times |7|$ Common factors of 70, 105 and 175 are 5 and 7. Thus, HCF of 70, 105 and $175 = 5 \times 7 = 35$ (h) Prime factorisation of $91 = 7 \times 13$ Prime factorisation of $112 = 7 \times 2 \times 2 \times 2 \times 2$ Prime factorisation of $49 = 7 \times 7$ Common factor of 91, 112 and 49 is 7 Thus, HCF of 91, 112 and 49 = 7. (i) Prime factorisation of $18 = 2 \times 3 \times 3$ Prime factorisation of $54 = 2 \times 3 \times 3 \times 3$ Prime factorisation of $81 = 3 \times 3 \times 3 \times 3 \times 3$ Common factor of 18, 54 and 81 is 3 (occurring twice). Thus, HCF of 18, 54 and $81 = 3 \times 3 = 9$ (j) Prime factorisation of $12 = 2 \times 2 \times |3|$ Prime factorisation of $45 = 5 \times 3 \times 3$ Prime factorisation of $75 = 5 \times 5 \times 3$ Common factor of 12, 45 and 75 is 3. Thus, HCF of 12, 45 and 75 = 3 What is the HCF of two consecutive (a) numbers? (b) even numbers?

(c) odd numbers?

2.

Sol. (a) Let the two consecutive numbers be 8 and 9. Now, prime factorisation of $8 = 2 \times 2 \times 2$ Prime factorisation of $9 = 3 \times 3$ Common factor of 8 and 9 = 1Thus, HCF of 8 and 9 is 1. Note The HCF of two consecutive numbers is 1. (b) Let the two consecutive even numbers be 10 and 12. Now, prime factorisation of $10 = 2 \times 5$ Prime factorisation of $12 = 2 \times 2 \times 3$ Common factor of 10 and 12=2 Thus, HCF of 10 and 12 is 2. **Note** The HCF of two consecutive even numbers is 2. (c) Let two consecutive odd numbers be 3 and 5. Prime factorisation of $3 = 1 \times 3$ Prime factorisation of $5 = 1 \times 5$ Common factor of 3 and 5 = 1Thus, HCF of 3 and 5 is 1. **Note** The HCF of two consecutive odd numbers is 1.

Page No. 64

- 3. HCF of co-prime numbers 4 and 15 was found as follows by factorisation. $4=2\times 2$ and $15=3\times 5$, since there is no common prime factor, so HCF of 4 and 15 is 0. Is the answer correct? If not, what is the correct HCF?
- Sol. No, the answer is not correct. Because 0 (zero) cannot be factor of any number. If there is no common factor means 1 is the common factor.

∴ HCF of 4 and 15=1

Exercise 3.7

Page No. 67

- 1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight, which can measure the weight of the fertiliser exact number of times.
- **Sol.** For getting the maximum value of weight, we have to find out the HCF of 75 kg and 69 kg.

i.e.

3	75	3	69
5	25	23	23
5	5		1
	1		

 \therefore Prime factorisation of $75 = 3 \times 5 \times 5$

Prime factorisation of $69 = 3 \times 23$

Common factor of 75 and 69 is 3.

Thus, HCF of 75 kg and 69 kg = 3 kg

Hence, required maximum value of weight is 3 kg.

- 2. Three boys step off together from the same spot. Theirsteps measure 63 cm, 70 cm and 77 cm, respectively. Whatis the minimum distance each should cover, so that all cancover the distance in complete steps?
- **Sol.** Here, the required minimum distance will be equal to LCM of measures of their steps, because, the minimum distance each boyshould walk must be the least common multiple of the measure of their steps.

So, we have to find out the LCM of 63, 70 and 77 cm. i.e.

2	63,70,77
3	63,35,77
3	21,35,77
5	7,35,77
7	7,7,77
11	1,1,11
	1,1,1

: LCM of 63, 70 and $77 = 2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930$ Hence, required minimum distance = 6930 cm

- 3. The length, breadth and height of a room are 825 cm,675 cm and 450 cm, respectively. Find the longest tape, which can measure the three dimensions of the roomexactly.

Sol.	For the longest tape,	which can	measure	dimensions	of room	exactly,we	have to	find out	the HCF	of
	dimensions, i.e.									

3	825		3	675		2	450
	275	•	2	225	•	2	225
5	275		5	225		5	225
5	55		3	75		3	75
11	11		5	25		5	25
	1		5	5		5	5
				1			1

 \therefore Prime factorisation of $825 = 3 \times 5 \times 5 \times 11$

Prime factorisation of $675 = 3 \times 5 \times 5 \times 3 \times 3$

Prime factorisation of $450 = 3 \times 5 \times 5 \times 3 \times 2$

.:. Common factors of 825, 675 and 450 are 3 and 5 (occurringtwice).

Thus, HCF of 825, 675 and $450 = 3 \times 5 \times 5 = 75$

Hence, required measure of longest tape = 75 cm

- Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12. 4.
- Sol. Firstly, we have to find out the LCM of 6, 8 and 12.

2	6,8,12
2	3,4,6
2	3,2,3
3	3,1,3
	1,1,1

 $\therefore LCM = 2 \times 2 \times 2 \times 3 = 24$

We know that, smallest three digit number is 24)100(4

96

4

On dividing 100 by 24,

We find that, when 100 is divided by 24, we get remainder 4. Now, required three digit number which is exactly divisible by 24 = (Smallest 3-digit numbers + Divisor – Remainder)

=100+24 - 4=120

5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

Sol. Firstly, we have to find out the LCM of 8, 10 and 12.

	2	8,10,12	
	2	4,5,6	_
	2	2,5,3	_
	3	1,5,3	_
	5	1,5,1	-
		1,1,1	-
00(8			
<u>_</u>			

120)900(8

960

39

 $\therefore LCM = 2 \times 2 \times 2 \times 3 \times 5 = 120$

We know that, largest 3-digit number = 999

On dividing 999 by 120.

We find that, when 999 is divided by 120, we get remainder 39.

Now, required three digit number which is exactly divisible by120 = (Greatest 3-digit number - Remainder) = 999 - 39 = 960

- 6. The traffic lights at three different road crossings change after every 48 seconds 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?
- **Sol.** The time period after which the traffic lights at three different road crossing changes simultaneously again will be the LCM of 48, 72 and 108.

R	48,36,54
2	24, 36, 54
2	12,18,27
2	6,9,27
3	3,9,27
3	1,3,9
3	1,1,3
	1,1,1

 $\therefore LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

 $=\frac{432}{60}$ min = 7 minute 12 second

 $\begin{bmatrix} \because 1\min = 60\\ and 1s = \frac{1}{60}\min \end{bmatrix}$

So, time when they will change again 07:00:00

$$= \frac{+ 07:12}{07:07:12}$$

i.e. 7 min 12 s past 7 a.m.

- 7. Three tankers contain 403 Litres 434 Litres and 465Litres of diesel, respectively. Find the maximum capacity of a container, that can measure the diesel of the three containers exact number of times.
- Sol. Here, maximum quantity of container will be equal to the HCF of 403, 434 and 465.

13	403	2	434	3	465
31	31	7	217	5	155
	1	31	31	31	31
	1		1		1

Prime factorisation of $403 = 13 \times 31$

Prime factorisation of $434 = 2 \times 7 \times 31$

Prime factorisation of $465 = 3 \times 5 \times 31$

Common factor of 403, 434 and 465 is 31.

... Maximum capacity of container = HCF of 403, 434, and 465 =31 Litres

8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

Sol. First, we have to find out the LCM of 6, 15,18.

:. LCM = $2 \times 3 \times 3 \times 5 = 90$ Now, required number = LCM + Remainder = 90 + 5 = 95Hence, the required number is 95.

9. Find the smallest 4-digit number which is divisible ay18, 24 and 32.

First, we have to find out the LCM of 18, 24, and 32. \therefore LCM = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$ We know that, smallest 4-digit number is On dividing 1000 by 288

288)1000(3									
	864								
	136								
2	18,24,32								
2	9,12,16								
2	9,6,8								
2	9,3,4								
2	9,3,2								
3	9,3,1								
3	3,1,1								
	· · · · ·								

Sol.

1,1,1

Sol.

When we divide 1000 by 288, we get 136 as remainder. ∴ Required smallest 4-digit number which is exactly divisible by 18, 24 and 32 = (Smallest 4-digit number + Divisor - Remainder) =1000+288 - 136=1152

10. Find the LCM of the following numbers

(a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4 Observe a common property in the obtained LCM. Is LCM the product of two numbers in each case?

(a) LCM of 9 and 4 2 9,4 2 9,2 2 9,1 3 3,1 1,1 $\therefore LCM = 2 \times 2 \times 3 \times 3 = 36$ (b) LCM of 12 and 5 2 12,5 2 6,5 2 3,5 5 1,5 1,1 \therefore LCM = 2×2×3×5 = 60 (c) LCM of 6 and 5 2 6,5 3 3,5 5 1,5 1,1 \therefore LCM = 2×3×5 = 30 (d) LCM of 15 and 4 2 15,4 2 15,2 3 15,1 5 5,1 1,1 $\therefore LCM = 2 \times 2 \times 3 \times 5 = 60$

Here, we see that in each case LCM of given numbers is amultiple of 3 and 2. Yes, in each case LCM = Product of two given numbers.

11.Find the LCM of the following numbers in which one number is the factor of the other.(a) 5, 20(b) 6, 18(c) 12,48(d) 9,45

What do you observe in the results obtained?

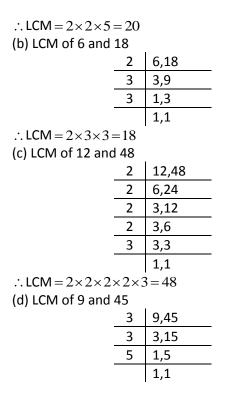
Sol. (a) LCM of 5 and 20

 2
 5,20

 2
 5,10

 5
 5,5

 1,1



 \therefore LCM = 3×3×5 = 45

Here, we observe that in all parts, LCM of the given numbers is the larger of two numbers because one number is the factor of the other number.



Directions In questions 1 to 9, out of the four options, only one is correct. Write the correct answer.

1.	Number of even n (a) 10	numbers between 58 and 80 (b) 11	is (c) 12	(d) 13
Sol.	We know that, all So, even numbers	the multiples of 2 are called between 58 and 80 are 60, 9 n numbers between 58 and 8	even numbers. 62, 64, 66, 68, 70, 72, 74,	
2.		er of primes between 16 to		(1) 46
Sol.		(b) 18 ne numbers other than 1 wh	(c) 17 nose any factors are 1 a	(d) 16 nd the number itself are called prime
	∴ Number of prim And prime numbe	ne numbers between16 to 80 ers between 90 to 100 is 1. i.e of the number of primes betw	0 = 16 e. 97	47, 53, 59, 61, 67, 71, 73, 79. 00 = 16 +1 = 17.
3.		owing statement is not true?		
		o distinct prime numbers is	1.	
		o co-prime numbers is 1.		
		o consecutive even numbers even and an odd number is		
Sol.		en and an odd number is an c		
		umber be 2 and an odd num		
	-	d 3=1 (which is odd). Hence,		
4	The number of di-	ationst maining forstown of the la		
4.	(a) 2	stinct prime factors of the la (b)3	(c) 5	(d) 11
Sol.	• •	gest 4-digit number = 9999	(0) 5	(0) 11
	Prime factors of 99			
	3	1		
	3	3333		
	11	1 1111		
	103			
		1		
		$3 \times 3 \times 11 \times 101$	langest 4 digit succession	2 Honor ontion (b) is some st
	The number of (distinct prime factors of the	largest 4-digit numberis	3. Hence, option (b) is correct.

5.The number of distinct prime factors of the smallest 5-digit number is
(a) 2(b) 4(c) 6(d) 8

Sol. We know that, smallest 5-digit number = 10000Prime factors of 10000

			_
	2	10000	
	2	5000	-
	2	2500	
	2	1250	
	5	625	
	5	125	
	5	25	-
	5	5	-
		1	-
•		2.2.5	_

i.e. $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$ \therefore The number of distinct prime factors of the smallest 5-digitnumber is 2. Hence, option (a) is correct.

6. If the number 7254*98 is divisible by 22, the digit at * is (a) 1 (b) 2 (c) 6

Sol. We know that, if a number is divisible by two co-prime numbers, then it is divisible by their product also. Here, the number 7254 *98 is divisible by 22. It means the givennumber is divisible by 2 and 11 also. The units place of given number is 8. Hence, it is divisible by 2.

(d) 0

Now, we check this number for 11.

 $O \quad E \quad O \quad E \quad O \quad E \quad O$

Sum of the digits at odd places from right =8+ *+5+7=20+* Sum of the digits at even places from right =9+4+2=15 According to the test of divisibility by 11, $20+*-15=11 \Rightarrow 5+*=11$ *=11-5 \Rightarrow *=6 Hence, the value of * = 6 Hence, option (c) is correct.

Note A number is divisible by 11, if the difference of the sum of its digits inodd places (from the right) and the sum of its digits in even places (from the right) is either 0 or a multiple of 11.

7.The largest number which always divides the sum of any pair of consecutive odd numbers is
(a) 2(b) 4(c) 6(d) 8

Sol. The largest number which always divides the sum of any pair of onsecutive odd numbers is 4.
e.g. Let any pair of consecutive odd number is 3 and 5.
Sum of 3 and 5 = 3 + 5 = 8 which is divisible by 4.
So, 4 is the largest number, which always divides the sum of any pair of consecutive odd number. Hence, option (b) is correct.

8. A number is divisible by 5 and 6. It may not be divisible by (a) 10 (b) 15 (c) 30 (d) 60 Sol. The given number will be divisible by the product of 5 and 6. i.e. it is always divisible by $5 \times 6 = 30$

But $30 \div 60$ (not possible). Hence, option (d) is correct.

9. The sum of the prime factors of 1729 is (a) 13 (b) 19

(c) 32

(d) 39

Hence, option (d) is correct.

Directions In questions 10 to 21, state whether the given statements are true (T) or false (F).

10. Sum of two consecutive odd numbers is always divisible by 4.

Sol. True, e.g. Let two consecutive odd numbers be 3 and 5.

1

Sum = 3 + 5 = 8; 8 - 4 = 2

11. If a number is divisible both by 2 and 3, then it is divisible by 12.

- Sol. False, e.g. Let a number be 48. Here, units digit = 8, which is divisible by 2. So, 48 is divisible by 2. Now, sum of the digits of 48 = 4 + 8 = 12 which is divisible by 3. Now again, check it for 12. Here, tens digit = 4 and units digit = 8. So, 48 is divisible by 12. Now, take another example. Let a number be 210. Here, units digit = 0. So, 210 is divisible by 2. Now, sum of the digit of 210 = 2 + 1 + 0 = 3 which is divisible by 3 Here, tens digit = 1 and units digit = 0. So, 10 is not divisible by 4. Thus, 210 is not divisible by 12. Hence, we can say that it is an exceptional case.
- 12. A number with three or more digits is divisible by 6, if the number formed by its last two digits (i.e. ones and tens) is divisible by 6.
- Sol. False, e.g. Let two numbers be 1284 and 1384. Its last two digits(ones and tens digit) are divisible by 6. But 1284 is divisible by 6 and 1384 is not divisible by 6.

13. All numbers which are divisible by 4 may not be divisible by 8.

Sol. True, e.g. 16, 20 and 24 are divisible by 4 but 20 is not divisible by 8.

14. The highest common factor of two or more numbers is greater than their lowest common multiple.

Sol. False, the highest common factor of two or more numbers is notgreater than their lowest common multiple. e.g. Let two numbers be 24 and 30.

2 24 2 30

i.e. HCF is not greater than LCM.

15. LCM of two numbers is 28 and their HCF is 8.

Sol. False, as we know that, LCM of two or more numbers is alwaysdivisible by the HCF of those number. Here, LCM of two numbers (28) is not divisible by HCF (8).

16. LCM of two or more numbers may be one of the numbers.

- Sol. True, LCM of two or more numbers may be one of the numbers. e.g. Let two numbers be 24 and 120. LCM of 24 and 120
 - 224,120212,6026,3033,1551,51,1

 \therefore LCM of 24 and $120 = 2 \times 2 \times 2 \times 3 \times 5 = 120$

17. HCF of two or more numbers may be one of the numbers.

Sol. True, HCF of two or more numbers may be one of the numbers.

e.g. Let two numbers be 12 and 24. Prime factorisation of 12 and 24

2	12	2	24			
2	6	2	12			
3	3	2	6			
	1	3	3			
12 =						
24 =	$=2\times2\times2\times3$					
Common factors = 2, 2						
and	$3 \therefore HCF = 2 \times 2 \times 3 = 12$					

18. If the HCF of two numbers is one of the numbers, then their LCM is the other number.

Sol. True, let two numbers be 18 and 36.

2	18	2	2	36
3	9	2	2	18
3	3	3	3	9
	1	3	3	3
				1

 $18 = 2 \times 3 \times 3$ $36 = 2 \times 3 \times 3 \times 2$ Common factors are 2, 3 and 3. $\therefore HCF = 2 \times 3 \times 3 = 18 \text{ and}$ $LCM = 2 \times 3 \times 3 \times 2 = 36$ It is clear that, if the HCF of two numbers is one of the numbers, then their LCM is the other number.

19. The HCF of two numbers is smaller than the smaller of the numbers.

Sol. True, the HCF of two numbers is smaller than the smaller of the numbers, e.g. Let two numbers be 14 and 30.

Prime factorisation of 14 and 30

2	14	2	30
7	7	3	15
7	7	5	5
	1		1

 $14 = \boxed{2} \times 7$ $30 = \boxed{2} \times 3 \times 5$ Here, common factor is 2. $\therefore \text{HCF}=2$

It is clear that, the HCF of two numbers is smaller than the smaller of the numbers.

20. The LCM of two numbers is greater than the larger of the numbers.

Sol. False, the LCM of two numbers is not always greater than the larger of the numbers, e.g. Let two numbers be 14 and 28.

LCM of 14 and 28

$$\begin{array}{r}
 2 \quad 14,28 \\
 2 \quad 7,14 \\
 \overline{} \quad 7,7 \\
 1,1 \\
 7 \quad -28 \\$$

 $\mathsf{LCM} = 2 \times 2 \times 7 = 28$

21. The LCM of two co-prime numbers is equal to the product of the numbers.

Sol. True, e.g. Let two co-prime numbers be 2 and 3.

 \therefore Their product =2x3=6 and LCM of 2 and 3=6 It is clear that, the LCM of two co-prime numbers is equal to the product of the numbers.

Directions In questions 22 to 24, fill in the blanks to make the statements true.

- 22. Two numbers having only 1 as a common factor are called..... number.
- Sol. co-prime

23. The LCM of two or more given numbers is the lowest of their common

Sol. multiple

24. The HCF of two or more given numbers is the highest of their common

Sol. factors

25. Determine the least number which when divided any 3, 4 and 5 leaves remainder 2 in each case.

Sol. We have,

:. LCM of 3, 4 and $5 = 2 \times 2 \times 3 \times 5 = 60$ Since, 60 is the least number exactly divisible by 3, 4 and 5. To get the remainder 2, The least number =60+2= 62

2	3,4,5
2	3,2,5
3	3,1,5
5	1,1,5
	1,1,1

- 26. A merchant has 120 L of oil of one kind, 180 L of another kind and 240 L of a third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?
- Sol. The greatest capacity of the required measure will be equal to the HCF of 120, 180 and 240 L. Prime factorisation of 120, 180 and 240.

2	120	2	180		2	240
2	60	2	90		2	120
2	30	3	45		2	60
3	15	3	15		2	30
5	5	5	5	_	З	15
	1		1	_	5	5
				-		1

 $120 = 2 \times 2 \times 3 \times 5$

 $180 = 2 \times 2 \times 3 \times 3 \times 5$

 $240 = 2 \times 2 \times 2 \times 3 \times 5$

Common factors of 120, 180 and $240 = 2 \times 2 \times 3 \times 5 = 60$ Hence, greatest capacity of tin = 60 L

27. A box contains 5 strips having 12 capsules of 500 mg medicine in each capsule. Find the total weight in grams of medicine in 32 such boxes.

- Sol. Given, in each strip their are 12 capsule Also, given weight of one capsule is 500 mg Weight of 12 capsule $=12 \times 500 = 6000 mg$
 - \therefore weight of 1 strip = weight of 12 capsule= 6000 mg
 - : weight of 5 strips = $5 \times$ weight of 1 strip = $5 \times 6000 = 30000 mg$
 - \therefore weight of 1 box = weight of 5 strips = 30000 mg
 - : weight of 32 box = $32 \times$ weight of 1 box = $32 \times 30000 mg$
 - = 960000 mg

 $=\frac{960000}{1000} grams \left[\therefore 1 gram = 1000 mg \\ \therefore 1 mg = \frac{1}{1000} gram \right]$

= 960 grams Hence, the total weight of 32 medicine box is 960 grams.

28. Find a 4-digit odd number using each of digits 1, 2, 4 and 5 only once such that when the first and last digits are interchanged, it is divisible by 4.

Sol. We know that 4-dight number is said to be an add number, if the place digit is an odd number (i.e. 1 or 5) Total such add numbers are, 4125,4215,1245,1425,2145,2415,4251,4521,5241,5421,2451,2541
Also we know that any four digits number be divisible by 4, if the last two digits number is divisible by 4. Consider a number 4521, if we interchange the first and the last digit, the new number will be 1524. Here we see that the last two dights (i.e.24) which is divisibley 4. Hence the number 1524 is divisible by 4. Hence, the required four digits number is 4521.

- 29. The floor of a room is 8m 96cm long and 6m 72cm broad. Find the minimum number of square tiles of the same size needed to cover the entire floor.
- Sol. Given, length of the floor = 8 m 96 cm
 =8x100 cm+96 cm [∵1 m = 100 cm]
 = (800 + 96) cm = 896 cm and breadth of the floor = 6 m 72 cm
 = 6 x 100 cm + 72 cm [∵1 m = 100 cm]
 = (600 + 72) cm = 672 cm
 Now, size of the square tile = HCF of 896 and 672
 Prime factorisation of 896 and 672

$$\frac{2}{2} \frac{896}{2448} \qquad \frac{2}{2} \frac{672}{336} \\ \frac{2}{2} \frac{224}{224} \qquad \frac{2}{2} \frac{336}{2} \\ \frac{2}{2} \frac{112}{2} \qquad \frac{2}{2} \frac{84}{2} \\ \frac{2}{2} \frac{56}{2} \frac{2}{42} \\ \frac{2}{2} \frac{28}{2} \\ \frac{2}{2} \frac{28}{2} \\ \frac{2}{2} \frac{14}{777} \\ \frac{7}{77} \\ \frac{1}{1} \\ \frac{1}{1} \\ 896 = [2] \times [2] \times [2] \times [2] \times [2] \times 2 \times 27] \\ 672 = [2] \times [2] \times [2] \times [2] \times [2] \times 2 \times 27] \\ 672 = [2] \times [2] \times [2] \times [2] \times [2] \times [2] \times 3 \\ \times [7] \\ Common factors of 896 and $672 = 2 \times 2 \times 2 \times 2 \times 2 \times 7 = 224 \\ \therefore \text{ Minimum number of square tiles } = \frac{\text{Area of floor}}{\text{Area of square tile}} \\ = \frac{896 \times 672}{224 \times 224} \quad [\because \text{ area of square = (side)}^2] \\ = \frac{896 \times 3}{224} = 4 \times 3 = 12$$$

- **30.** In a school library, there are 780 books of English and 364 books of Science. Ms Yakang, the librarian of the school wants to store these books in shelves such that each shelf should have the same number of books of each subject. What should be the minimum number of books in each shelf?
- **Sol.** Given, number of English books = 780 Number of Science books = 364

For getting the minimum number of books in each shelf, we have to find the HCF of 780 and 364. \Rightarrow Prime factorisation of 780 and 364

	2	780			2	364	
	2	390		-	2	182	
	3	195			7	91	
	5	65			13	13	
	13	13		_		1	
		1					
780 = 2	$\times 2$						
364 = 2	$\times 2$	$\times 7 \times 13$	3				
_	~						

Common factors of 780 and $364 = 2 \times 2 \times 13 = 52$

- 31. In a colony of 100 blocks of flats numbering 1 to 100, a school van stops at every sixth block while a school bus stops at every tenth block. On which stops will both of them stop if they start from the entrance of the colony?
- **Sol.** Given, in a colony block numbering 1 to 100. Here school van and bus stop at the same stopage means it is the LCM of both stopage

 \therefore LCM of 6 and $10 = 2 \times 3 \times 5 = 30$

It shows that first time they both meets at 30 the stopage and the next time they again meet at a multiples of 30.

Hence, the both of them meet the stopage on 30, 60 and 90.