Sample Question Paper - 8
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

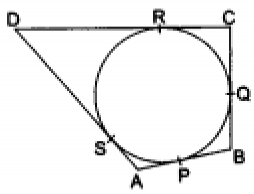
1. Find the sum of the APs: –37, –33, –29, ..., to 12 terms. **[2]**

OR

Verify a, 2a + 1, 3a + 2, 4a + 3,...forms an A.P, and then write its next three terms.

2. Find the roots of quadratic equation by the factorisation method: $2x^2 + \frac{5}{3}x - 2 = 0$ **[2]**

3. A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that AB + CD = AD + BC. **[2]**



4. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice. **[2]**

Aisha, a juise seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3cm. The height of both the glasses was 10cm.

First type: A Glass with hemispherical raised bottom.

Second type: A glass with conical raised bottom of height 1.5cm.

Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

5. Following table shows the weight of 12 students: **[2]**

Weight (in kgs):	67	70	72	73	75
Number of students:	4	3	2	2	1

Find the mean weight of the students.

6. Find the value of k for which the given quadratic equation has real and distinct roots : $kx^2 + 2x + 1 = 0$ **[2]**

OR

Solve: $\frac{14}{x+3} - 1 = \frac{5}{x+1}, x \neq$ -3, -1

Section B

7. The percentage of marks obtained by 100 students in an examination are given below: **[3]**

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	28	23	18	8	3

Determine the median percentage of marks.

8. Draw a circle of radius 2.5 cm and take a point P outside it, Without using the centre of the circle, draw two tangents to the circle from the point P. **[3]**

9. Find the mode of the following distribution: **[3]**

Class Interval	**Frequency**
0 - 10	5
10 - 20	8
20 - 30	7
30 - 40	12
40 - 50	28
50 - 60	20
60 - 70	10
70 - 80	10

10. A moving boat observed from the top of a 150 m high cliff, moving away from the cliff. The angle of depression of the boat changes from 60^o to 45^o in 2 minutes. Find the speed of the boat. **[3]**

OR

The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of

$1500\sqrt{3}$m, find the speed of the plane in km/hr.

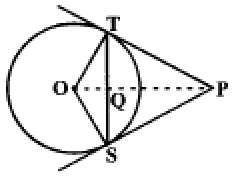
Section C

11. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area in hectare will it irrigate in 30 minutes if 8 cm of standing water is needed? **[4]**

12. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ. **[4]**

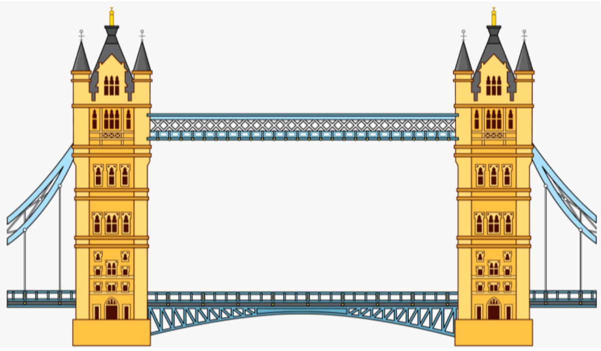
OR

In the adjoining figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that $\angle OTS = \angle OST = 30^\circ$



13. Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping. **[4]**

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^0 and 30^0 respectively.



i. Neeta used some applications of trigronomatry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

ii. Also find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

14. The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. **[4]**

i. How much distance did she cover in completing this job and returning to collect her books?
ii. What is the maximum distance she travelled carrying a flag?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. Here, a = -37

   d = -33 - (-37) = -33 + 37 = 4

   n = 12

   We know that

   $S_n = \frac{n}{2}[2a + (n-1)d]$

   $\Rightarrow S_{12} = \frac{12}{2}[2(-37) + (12-1)4]$

   $\Rightarrow S_{12} = 6[-74 + 44]$

   $\Rightarrow S_{12} = 6[-30]$

   $\Rightarrow S_{12} = -180$

   So, the sum of the first 12 terms of the given AP is -180.

   OR

   Here a_1 = a

   a_2 = 2a + 1

   a_3 = 3a + 2

   a_4 = 4a + 3

   a_2 - a_1 = a + 1

   a_3 - a_2 = a + 1

   a_4 - a_3 = a + 1

   As difference of successive terms are equal therefore it is an A.P with common difference $\sqrt{3}$

   Next three term will be:

   4a + 3 + a + 1, 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)

   5a + 4, 6a + 5, 7a + 6

2. Given, $2x^2 + \frac{5}{3}x - 2 = 0$

   $\Rightarrow$ $6x^2$ + 5x - 6 = 0

   By splitting the middle term, we have

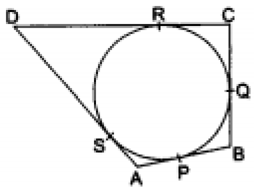
   $6x^2$ + 9x - 4x - 6 = 0

   $\Rightarrow$ 3x (2x + 3) - 2 (2x + 3) = 0

   $\Rightarrow$ (2x + 3) (3x - 2) = 0

   $\therefore$ 2x + 3 = 0 or 3x - 2 = 0

   $\therefore \text{x} = -\frac{3}{2}$ or $\text{x} = \frac{2}{3}$

3. 

   We know that the lengths of tangents drawn from an exterior point to a circle are equal.

   AP = AS, ... (i) [tangents from A]

   BP = BQ, ... (ii) [tangents from B]

   CR = CQ, ... (iii) [tangents from C]

   DR = DS. ... (iv) [tangents from D]

   AB + CD = (AP + BP) + (CR + DR)

   = (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

   = (AS + DS) + (BQ + CQ)

   = AD + BC.

   Hence, AB + CD = AD + BC.

4. Capacity of first glass $= \pi r^2 H - \frac{2}{3}\pi r^3$
$= \pi \times 9(10 - 2) = 72\pi cm^3$
Capacity of second glass $= \pi r^2 H - \frac{1}{3}\pi r^2 h$
$= \pi \times 3 \times 3(10 - 0.5) = 85.5\pi cm^3$
$\therefore$ Suresh got more quantity of juice.
5. **Calculation of Arithmetic Mean**

Weight (in kgs) x_i	**Frequency** f_i	$f_i x_i$
67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
	$N = \sum f_i = 12$	$\sum f_i x_i = 843$

$\therefore \quad \text{Mean} = \overline{X} = \frac{\sum f_i x_i}{N} = \frac{843}{12} = 70.25\text{kg}$
6. We have, $kx^2 + 2x + 1 = 0$
here, a = k, b=2, c=1
$\therefore D = b^2 - 4ac = (2)^2 - 4(k)(1) = 4 - 4k$
The given equation will have real and distinct roots,
$D > 0$
$\Rightarrow 4 - 4k > 0 \Rightarrow k < 1$

OR

We have,
$\frac{14}{x+3} - \frac{5}{x+1} = 1$
Taking LCM
$\Rightarrow \frac{14(x+1) - 5(x+3)}{(x+3)(x+1)} = 1$
$\Rightarrow \frac{(9x-1)}{x^2+4x+3} = 1$
$\Rightarrow x^2 + 4x + 3 = 9x - 1$
$\Rightarrow x^2 - 5x + 4 = 0$
Factorise the equation,
$\Rightarrow x^2 - 4x - x + 4 = 0$
$\Rightarrow x(x - 4) - 1(x - 4) = 0$
$\Rightarrow (x - 4)(x - 1) = 0$
$\Rightarrow x - 4 = 0$ or $x - 1 = 0$
$\Rightarrow x = 4$ or $x = 1$.

Section B

7.

Marks (Class)	Number of Students (Frequency)	Cumulative frequency
30-35	14	14
35-40	16	30
40-45	18	48
45-50	23	71 (Median class)
50-55	18	89
55-60	8	97

60-65	3	100

Here, N = 100

Therefore, $\frac{N}{2}$ = 50, This observation lies in the class 45-50.

l (the lower limit of the median class) = 45

cf (the cumulative frequency of the class preceding the median class) = 48

f (the frequency of the median class) = 23

h (the class size) = 5

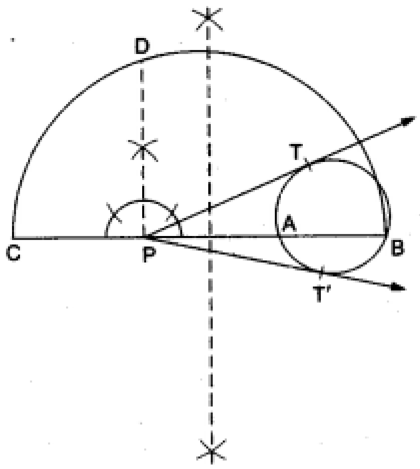
Median $= l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) h$

$= 45 + \left(\frac{50-48}{23}\right) \times 5$

$= 45 + \frac{10}{23} = 45.4$

So, the median percentage of marks is 45.4.

8. **STEPS OF CONSTRUCTION**
   1. Draw a circle of radius 2.5 cm and take a point p outside it.
   2. Through P draw a secant PAB to intersect the circle at A and B.



   3. Produce AP to a point C such that PA = PC
   4. Draw a semicircle with CB as diameter.
   5. Draw PD $\perp$ CB, intersecting the semicircle at D.
   6. With P as centre and PD as radius, draw arcs to intersect the circle at T and T'.
   7. Join PT and PT'.

   Then, PT and PT' are the required tangents.
9. Modal Class 40- 50,

   Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

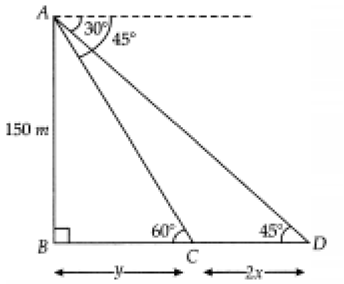
   Mode = $40 + \left(\frac{28-12}{2\times28-12-20}\right) \times 10$

   = 40 + $\left(\frac{16}{56-32}\right) \times 10$

   = 40 + $\left(\frac{16}{24}\right) \times 10$

   = 40 + $\frac{20}{3}$

   $= 46.666 = 46.67$

10. 

Let the speed of the boat be x m/min.

∴Distance covered in 2 minutes = 2x

∴ CD = 2x

Let BC = y

In △ABD,

$\frac{AB}{BC} = \tan 60^\circ$

$\Rightarrow \quad \frac{150}{y} = \sqrt{3}$

$\Rightarrow \quad y = \frac{150}{\sqrt{3}}$

$\Rightarrow \quad y = 50\sqrt{3}$.......(i)

In △ABD,

$\frac{AB}{BD} = \tan 45^\circ$

$\Rightarrow \quad \frac{150}{y+2x} = 1$

$\Rightarrow \quad y + 2x = 150$........(ii)

Substituting the value of y from (i) in (ii) we get

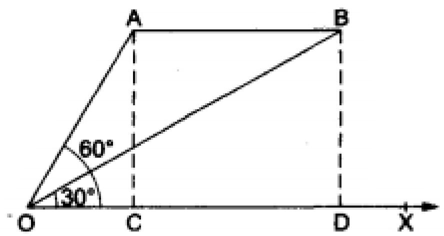
$50\sqrt{3} + 2x = 150$

$x = 75 - 25\sqrt{3} = 25(3 - \sqrt{3})$ m/sec

$= \frac{25(3-\sqrt{3})\times 60}{1000} = \frac{3}{2} \times (3 - \sqrt{3})$ km/min

OR

Let A and B be the two positions of the aeroplane.



Let $AC \perp OX$ and $BD \perp OX$. Then,

$\angle COA = 60^\circ, \angle DOB = 30^\circ$

and AC = BD = $1500\sqrt{3}$m.

From right ΔOCA, we have

$\frac{OC}{AC} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{OC}{1500\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow OC = 1500\text{m}$

From right ΔODB,we have

$\frac{OD}{BD} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{OD}{1500\sqrt{3}\text{m}} = \sqrt{3}$

$\Rightarrow OD = (1500 \times 3)\text{m} = 4500\text{m}.$

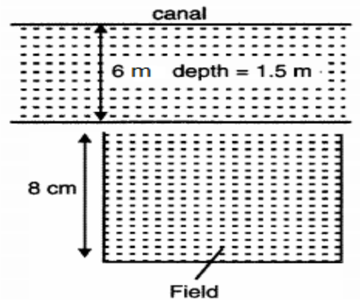
$\therefore CD = (OD - OC) = (4500 - 1500)m = 3000m.$

Thus, the aeroplane covers 300m in 15 seconds.

∴ speed of the aeroplance $= \left(\frac{3000}{15} \times \frac{60\times 60}{1000}\right)\text{km/hr}$

= 720 km /hr.

Section C

11. 

Water flows in 1 hr = 10 km

Water flows in $\frac{1}{2} hr = \frac{10}{2}$

= 5km

= 5000 m

Now volume of water flows in $\frac{1}{2} hr$

= lbh

$= 5000 \times 6 \times 1.5 \mathrm{m}^3$

$= 45000 \mathrm{m}^3$

From the question,we are given that,

Volume of water in $\frac{1}{2} hr$

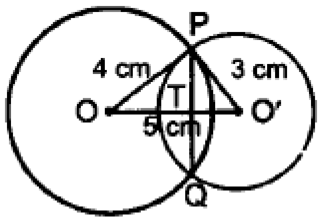
= area of irrigated field $\times \frac{8}{100} \mathrm{m}$

$\Rightarrow 45000 = \text{Area} \times \frac{8}{100}$

$\therefore$ Area $= \frac{45000 \times 100}{8} = 562500 \mathrm{m}^2$

= 56.25 $hectare$

12. Given, OP is tangent of the circle having center O'



So, $\angle$OPO' = 90^{o}

In right angled $\triangle$OPO'

OP = 4 cm

O'P = 3 cm

By pythagoras theorem, we get

$\mathrm{OO'}^2 = \mathrm{OP}^2 + \mathrm{O'P}^2$

$= 4^2 + 3^2$

= 16 + 9 = 25

$OO' = 5cm.$

Let O'T = x , then OT = 5 - x

In right angled $\triangle$PTO

By pythagoras theorem, we get

$OP^2 = OT^2 + PT^2$

$\Rightarrow PT^2 = OP^2 - OT^2$

$PT^2 = 4^2 - (5-x)^2$...(i)

In right angled $\triangle$PTO'

By pythagoras theorem, we get

$O'P^2 = O'T^2 + PT^2$

$\Rightarrow PT^2 = O'P^2 - O'T^2$

$PT^2 = 3^2 - x^2$...(ii)

From (i) and (ii) , we get

$3^2 - x^2 = 4^2 - (5-x)^2$

$9 - x^2 = 16 - 25 - x^2 + 10x$

18 = 10x

⇒ x = $\frac{18}{10}$ = 1.8

Substitute x in (ii), we get

$PT^2 = 3^2 - 1.8^2 = 9 - 3.24 = 5.76$

$PT = \sqrt{5.76} = 2.4$

⇒ PQ = 2 PT

= 2 $\times$ 2.4

∴ PQ= 4.8cm

OR

In the given figure,

OP = 2r (given)

and $\angle OTP$ = 90° (Radius is perpendicular to the tangent at the point of contact)

Now In $\triangle OTP$

sin $\angle OPT$ = $\frac{OT}{OP} = \frac{R}{2R} = \frac{1}{2}$

⇒ $\angle OPT$ = 30°

Therefore $\angle TOP$ = 60°

In $\triangle OTS$

OT = OS (both are radii of same circle)

Therefore $\triangle OTS$ is an isosceles triangle.

$\angle OTS = \angle OST$ (angles oppposite to equal sides of an isosceles triangle are equal)

In $\triangle OTQ$ and $\triangle OSQ$

OS = OT (radii of same circle)

OQ = OQ (common)

$\angle OTQ$ = $\angle OSQ$ (angles opposite to equal sides of an isosceles triangle are equal)

Therefore $\triangle OTQ$ ≅ $\triangle OSQ$

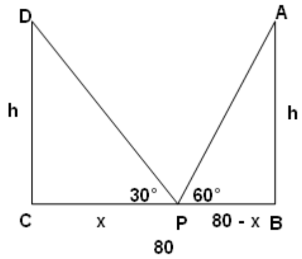
$\angle TOQ$ = $\angle SOQ$ ((by C.A.C.T)

$\angle TOS$ = 120°

⇒ $\angle OTS$ + $\angle OST$ = 180° - 120 =° 60°

∴ $\angle OTS$ = $\angle OST$ = 60°/ 2 =30°

Hence proved.

13. 

Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x) . Also, ∠APB = 60^0 and ∠DPC =30^0

In right angled triangle DCP,

tan 30^0 = $\frac{CD}{CP}$

⇒ $\frac{h}{x} = \frac{1}{\sqrt{3}}$

⇒ $h = \frac{x}{\sqrt{3}}$ ....(1)

In right angled triangle ABP,

Tan 60^0 = AB/AP $\frac{AB}{AP}$

⇒ $\frac{h}{80-x} = \sqrt{3}$

⇒ $h = \sqrt{3}(80 - x)$

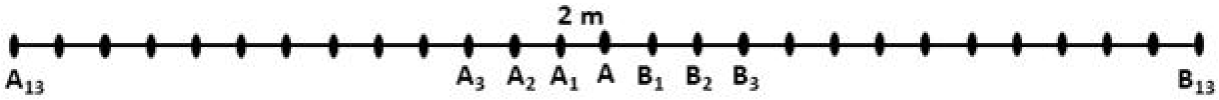
⇒ $\frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

⇒ x = 3(80 – x)

⇒ x = 240 – 3x

⇒ x + 3x = 240

⇒ 4x = 240
⇒ x = 60
Height of the tower, h = x/√3 = 60/√3 = 20√3.
Thus, the position of the point P is 60 m from C and the height of each tower is 20√3 m.

14.


Let A be the position of the middle - most flag.
Now, there are 13 flags(A_1, A_2..........A_{12}) to the left of A and 13 flags (B_1, B_2, B_3.............B_{13}) to the right of A.
Distance covered in fixing flag to A_1 = 2 + 2 = 4 m
Distance covered in fixing flag to A_2 = 4 + 4 = 8 m
Distance covered in fixing flag to A_3 = 6 + 6 = 12 m
......
Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m
This forms an A.P. with,
First term, a = 4
Common difference, d = 4
and n= 13
∴ Distance covered in fixing 13 flags to the left of A = S_{13}
$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$
$= \frac{13}{2} \times [8 + 48]$
$= \frac{13}{2} \times 56$
= 364
Similarly, distance covered in fixing 13 flags to the right of A = 364
Total distance covered by Ruchi in completing the task
= 364 + 364 = 728 m
Maximum distance travelled by Ruchi in carrying a flag
= Distance from A_{13} to A or B_{13} to A = 26 m