

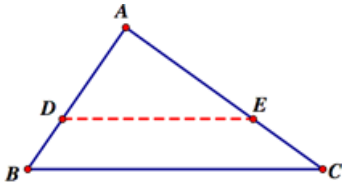
6. Geometry

Exercise 6.1

1 A. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, then find AC.



Answer

Given $DE \parallel BC$ and $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm

Required: $AC = ?$

Here, $DE \parallel BC$, $\therefore$ By Thales theorem $\frac{AD}{DB} = \frac{AE}{EC}$

$$\therefore \frac{6}{9} = \frac{8}{EC}$$

$$\Rightarrow EC = \frac{8 \times 9}{6}$$

$$\Rightarrow EC = \frac{8 \times 3}{2}$$

$$\Rightarrow EC = 4 \times 3 = 12$$

$$\therefore EC = 12 \text{ cm}$$

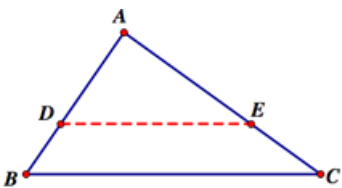
$$\therefore AC = AE + EC = 8 + 12 = 20 \text{ cm}$$

$$\therefore \text{Length of } AC = 20 \text{ cm}$$

1 B. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

If $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm, then find CE.



Answer

Given $DE \parallel BC$ and $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm

Required: $CE = ?$

Here, $DB = AB - AD = 12 - 8 = 4$ cm

Also, $DE \parallel BC$, $\therefore$ By Thales theorem $\frac{AD}{DB} = \frac{AE}{CE}$

$$\therefore \frac{8}{4} = \frac{12}{CE}$$

$$\Rightarrow CE = \frac{12 \times 4}{8}$$

$$\Rightarrow CE = \frac{12}{2}$$

$$\Rightarrow CE = 6$$

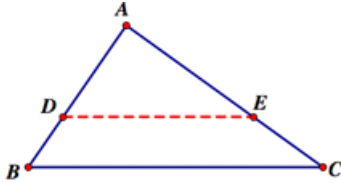
$$\therefore CE = 6\text{cm}$$

$$\therefore \text{Length of CE} = 6\text{cm}$$

1 C. Question

In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

If $AD = 4x-3$, $BD = 3x-1$, $AE = 8x-7$ and $EC = 5x-3$, then find the value of x.



Answer

Given $DE \parallel BC$ and $AD = 4x-3$, $BD = 3x-1$, $AE = 8x-7$ and $EC = 5x-3$

Required: $x = ?$

Here, $DE \parallel BC$, $\therefore$ By Thales theorem $\frac{AD}{BD} = \frac{AE}{EC}$

$$\therefore \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$$

$$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$\Rightarrow 24x^2 - 20x^2 + 12x + 15x - 8x - 21x - 9 + 7 = 0$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$

$$\Rightarrow 2(2x^2 - x - 1) = 0$$

$$\Rightarrow 2x^2 - x - 1 = 0$$

$$\Rightarrow 2x^2 - 2x + x - 1 = 0$$

$$\Rightarrow 2x(x-1) + 1(x-1) = 0$$

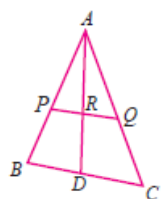
$$\Rightarrow (2x+1)(x-1) = 0$$

$$\Rightarrow x = 1 \text{ or } \frac{-1}{2}$$

$\therefore$ Values x can have are $x = 1$ and $\frac{-1}{2}$

2. Question

In the figure, $AP = 3\text{ cm}$, $AR = 4.5\text{ cm}$, $AQ = 6\text{ cm}$, $AB = 5\text{ cm}$, and $AC = 10\text{ cm}$. Find the length of AD.



Answer

Given: $AP = 3\text{ cm}$, $AR = 4.5\text{ cm}$, $AQ = 6\text{ cm}$, $AB = 5\text{ cm}$, and $AC = 10\text{ cm}$

Required: Find the length of AD

Consider the $\triangle ABC$

Here, $PB = AB - AP = 5 - 3 = 2\text{cm}$ and $QC = AC - AQ = 10 - 6 = 4\text{cm}$

Now, $\frac{AP}{PB} = \frac{3}{2}$ and $\frac{AQ}{QC} = \frac{6}{4} = \frac{3}{2}$

Here, we can clearly see that $\frac{AP}{PB} = \frac{AQ}{QC}$

$\therefore$ By Converse Thales theorem $PQ \parallel BC$ ($\because \frac{AP}{PB} = \frac{AQ}{QC}$)

Now Consider $\triangle ABD$

Here, $PR \parallel BD$

$\therefore$ BY Thales theorem $\frac{AP}{PB} = \frac{AR}{RD}$

$$\Rightarrow \frac{3}{2} = \frac{4.5}{RD}$$

$$\Rightarrow RD = \frac{4.5 \times 2}{3}$$

$$\Rightarrow RD = 1.5 \times 2$$

$$\Rightarrow RD = 3\text{cm}$$

$$\therefore AD = AR + RD = 4.5 + 3 = 7.5\text{ cm}$$

$$\therefore \text{Length of AD} = 7.5\text{cm}$$

3 A. Question

E and F are points on the sides PQ and PR respectively, of a $\triangle PQR$. For each of the following cases, verify $EF \parallel QR$.

PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm.

Answer

Given: E and F are points on the sides PQ and PR respectively, of a $\triangle PQR$. PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

Required: To verify if $EF \parallel QR$

Consider the $\triangle PQR$

Here, $\frac{PE}{EQ} = \frac{3.9}{3} = \frac{1.3}{1}$ and $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2}$

We can clearly see that $\frac{PE}{EQ} \neq \frac{PF}{FR}$

$\therefore$ Converse Thales theorem is also not satisfied.

$\therefore$ No, EF is not parallel to QR

3 B. Question

E and F are points on the sides PQ and PR respectively, of a $\triangle PQR$. For each of the following cases, verify $EF \parallel QR$.

PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9cm.

Answer

Given: E and F are points on the sides PQ and PR respectively, of a $\triangle PQR$. PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9cm

Required: To verify if $EF \parallel QR$

Consider the ΔPQR

$$\text{Here, } \frac{PE}{EQ} = \frac{4}{4.5} \text{ and } \frac{PF}{FR} = \frac{8}{9} = \frac{4}{4.5}$$

$$\text{We can clearly see that } \frac{PE}{EQ} = \frac{PF}{FR}$$

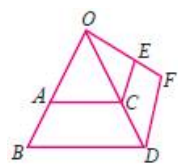
$\therefore$ Converse Thales theorem is satisfied and $EF \parallel QR$.

$\therefore$ Yes, EF is parallel to QR ($EF \parallel QR$)

4. Question

In the figure,

$AC \parallel BD$ and $CE \parallel DF$. If $OA = 12\text{cm}$, $AB = 9\text{ cm}$, $OC = 8\text{ cm}$ and $EF = 4.5\text{ cm}$, then find FO.



Answer

Given: $AC \parallel BD$ and $CE \parallel DF$. $OA = 12\text{cm}$, $AB = 9\text{ cm}$, $OC = 8\text{ cm}$ and $EF = 4.5\text{ cm}$

Required: Length of FO

Consider ΔBOD

Here, $AC \parallel BD$

$$\therefore \text{ By Thales theorem } \frac{OA}{AB} = \frac{OC}{CD}$$

$$\Rightarrow \frac{12}{9} = \frac{8}{CD}$$

$$\Rightarrow CD = \frac{8 \times 9}{12} = 6$$

$$\therefore CD = 6\text{cm}$$

Now, Consider ΔDOF

Here, $CE \parallel DF$

$$\therefore \text{ By Thales theorem } \frac{OC}{CD} = \frac{OE}{EF}$$

$$\Rightarrow \frac{8}{6} = \frac{OE}{4.5}$$

$$\Rightarrow OE = \frac{8 \times 4.5}{6} = 6$$

$$\therefore OE = 6\text{cm}$$

$$\therefore FO = OE + EF = 6 + 4.5 = 10.5\text{cm}$$

$$\therefore \text{ Length of FO} = 10.5\text{ cm.}$$

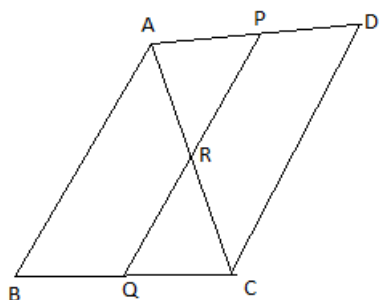
5. Question

ABCD is a quadrilateral with AB parallel to CD. A line drawn parallel to AB meets AD at P and BC at Q. Prove

$$\text{that } \frac{AP}{PD} = \frac{BQ}{QC}.$$

Answer

Given: ABCD is a quadrilateral with AB parallel to CD and line drawn parallel to AB meets AD at P and BC at Q



Required: To prove $\frac{AP}{PD} = \frac{BQ}{QC}$

Construction: draw a diagonal AC, which intersects PQ at R

Now, in $\triangle ABC$

Here, $QR \parallel AB$

$\therefore$ By Thales theorem $\frac{QC}{BQ} = \frac{CR}{AR}$

$$\Rightarrow \frac{BQ}{QC} = \frac{AR}{RC} \text{ --eq(1)}$$

Now, consider $\triangle ACD$

Here, $RP \parallel DC$

$\therefore$ By Thales theorem $\frac{AR}{RC} = \frac{AP}{PD} \text{ --eq(2)}$

From --eq(1) and --eq(2)

We have,

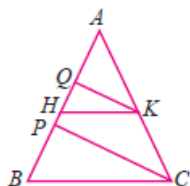
$$\frac{AP}{PD} = \frac{BQ}{QC}$$

$$\therefore \frac{AP}{PD} = \frac{BQ}{QC}$$

Hence Proved

6. Question

In the figure, $PC \parallel QK$ and $BC \parallel HK$. If $AQ = 6$ cm, $QH = 4$ cm, $HP = 5$ cm, $KC = 18$ cm, then find AK and PB.



Answer

Given: $PC \parallel QK$ and $BC \parallel HK$, $AQ = 6$ cm, $QH = 4$ cm, $HP = 5$ cm, $KC = 18$ cm

Required: Length of AK and PB

Consider the $\triangle APC$

Here, $PC \parallel QK$

$\therefore$ By Thales theorem $\frac{AQ}{QP} = \frac{AK}{KC}$

Here, $QP = QH + HP = 4 + 5 = 9$ cm

$$\Rightarrow \frac{6}{9} = \frac{AK}{18}$$

$$\Rightarrow AK = \frac{6 \times 18}{9} = 12$$

$$\therefore AK = 12\text{cm}$$

Now, Consider $\triangle ABC$

Here, $BC \parallel HK$

$$\therefore \text{By Thales theorem } \frac{AH}{HB} = \frac{AK}{KC}$$

$$\text{Here, } AH = AQ + QH = 6 + 4 = 10\text{cm}$$

$$\Rightarrow \frac{10}{HB} = \frac{12}{18}$$

$$\Rightarrow HB = \frac{10 \times 18}{12} = 15$$

$$\Rightarrow HB = 15\text{cm}$$

$$\Rightarrow PB = HB - HP = 15 - 5 = 10\text{cm}$$

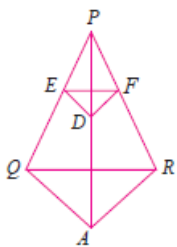
$$\therefore PB = 10\text{cm}$$

$\therefore$ Length of AK and PB are 12 cm and 10 cm respectively.

7. Question

In the figure, $DE \parallel AQ$ and $DF \parallel AR$

Prove that $EF \parallel QR$.



Answer

Given: $DE \parallel AQ$ and $DF \parallel AR$

Required: To prove $EF \parallel QR$.

Consider $\triangle APQ$

Here, $ED \parallel QA$

$$\therefore \text{By Thales theorem } \frac{PE}{EQ} = \frac{PD}{DA} \text{ --eq(1)}$$

Now, Consider $\triangle PAR$

Here, $DF \parallel AR$

$$\therefore \text{By Thales theorem } \frac{PD}{DA} = \frac{PF}{FR} \text{ --eq(2)}$$

From -eq(1) and -eq(2), we can see that

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

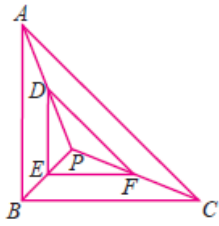
$\therefore$ By Converse Thales theorem we can say that $EF \parallel QR$ in $\triangle PQR$ ($\because \frac{PE}{EQ} = \frac{PF}{FR}$)

Hence Proved

8. Question

In the figure $DE \parallel AB$ and $DF \parallel AC$.

Prove that $EF \parallel BC$.



Answer

Given: $DE \parallel AC$ and $DF \parallel AB$

Required: To prove $EF \parallel BC$.

Consider $\triangle APB$

Here, $DE \parallel AB$

$\therefore$ By Thales theorem $\frac{PE}{EB} = \frac{PD}{DA}$ --eq(1)

Now, Consider $\triangle PAC$

Here, $DF \parallel AC$

$\therefore$ By Thales theorem $\frac{PD}{DA} = \frac{PF}{FC}$ --eq(2)

From -eq(1) and -eq(2), we can see that

$$\frac{PE}{EB} = \frac{PF}{FC}$$

$\therefore$ By Converse Thales theorem we can say that $EF \parallel BC$ in $\triangle PBC$ ($\because \frac{PE}{EB} = \frac{PF}{FC}$)

Hence Proved

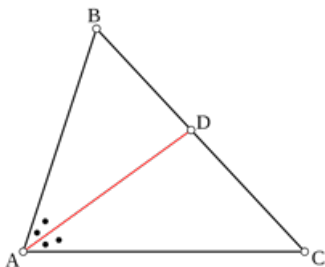
9 A. Question

In a $\triangle ABC$, AD is the internal bisector of $\angle A$, meeting BC at D.

If $BD = 2$ cm, $AB = 5$ cm, $DC = 3$ cm find AC.

Answer

Given: A $\triangle ABC$ with AD as internal bisector of $\angle A$, meeting BC at D. and $BD = 2$ cm, $AB = 5$ cm, $DC = 3$ cm



Required: The length of AC

Here, In $\triangle ABC$ AD is the internal bisector of $\angle A$

$\therefore$ By angle bisector theorem $\frac{BD}{DC} = \frac{AB}{AC}$

$$\Rightarrow \frac{2}{3} = \frac{5}{AC}$$

$$\Rightarrow AC = \frac{5 \times 3}{2} = 7.5$$

$$\therefore AC = 7.5 \text{ cm}$$

$$\therefore \text{Length of AC} = 7.5 \text{ cm}$$

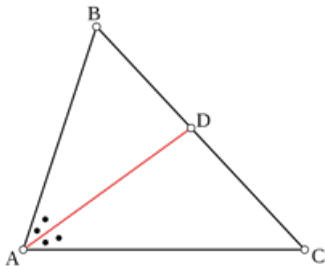
9 B. Question

In a $\triangle ABC$, AD is the internal bisector of $\angle A$, meeting BC at D.

If $AB = 5.6 \text{ cm}$, $AC = 6 \text{ cm}$ and $DC = 3 \text{ cm}$ find BC.

Answer

Given: A $\triangle ABC$ with AD as internal bisector of $\angle A$, meeting BC at D. and $AB = 5.6 \text{ cm}$, $AC = 6 \text{ cm}$, $DC = 3 \text{ cm}$



Required: The length of BC

Here, In $\triangle ABC$ AD is the internal bisector of $\angle A$

$$\therefore \text{By angle bisector theorem } \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{BD}{3} = \frac{5.6}{6}$$

$$\Rightarrow BD = \frac{5.6 \times 3}{6} = 2.8$$

$$\therefore BD = 2.8 \text{ cm}$$

$$\therefore \text{Length of BD} = 2.8 \text{ cm}$$

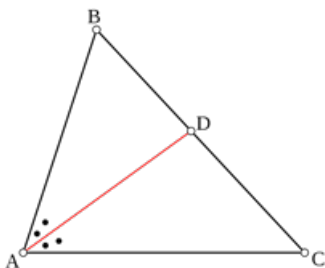
9 C. Question

In a $\triangle ABC$, AD is the internal bisector of $\angle A$, meeting BC at D.

If $AB = x$, $AC = x-2$, $BD = x+2$ and $DC = x-1$ find the value of x .

Answer

Given: A $\triangle ABC$ with AD as internal bisector of $\angle A$, meeting BC at D. and $AB = x$, $AC = x-2$, $BD = x+2$, $DC = x-1$



Required: The length of BC

Here, In $\triangle ABC$ AD is the internal bisector of $\angle A$

$$\therefore \text{By angle bisector theorem } \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \frac{x+2}{x-1} = \frac{x}{x-2}$$

$$\Rightarrow (x + 2)(x - 2) = x(x - 1)$$

$$\Rightarrow x^2 - 4 = x^2 - x \quad (\because (a + b)(a - b) = a^2 - b^2)$$

$$\Rightarrow x = 4$$

$\therefore$ The value of $x = 4$ cm

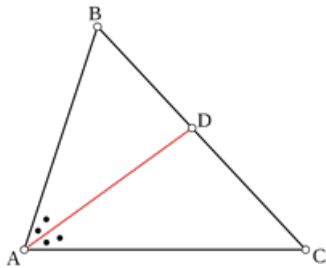
10 A. Question

Check whether AD is the bisector of $\angle A$ of $\triangle ABC$ in each of the following.

AB = 4 cm, AC = 6 cm, BD = 1.6 cm, and CD = 2.4 cm.

Answer

Given: AB = 4 cm, AC = 6 cm, BD = 1.6 cm, and CD = 2.4 cm



Required: To verify if AD is the bisector of $\angle A$ of $\triangle ABC$

Consider the $\triangle ABC$,

$$\text{Here, } \frac{BD}{DC} = \frac{1.6}{2.4} = \frac{2}{3} \text{ and } \frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Here, we can clearly see that } \frac{BD}{DC} = \frac{AB}{AC}$$

$\therefore$ We can say that AD is the angle bisector of $\angle A$ using Converse angle bisector theorem.

$\therefore$ Yes, AD is the angle bisector of $\angle A$

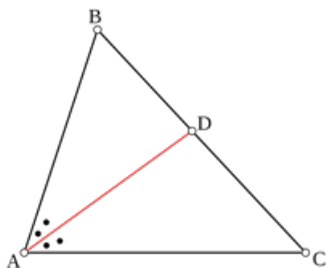
10 B. Question

Check whether AD is the bisector of $\angle A$ of $\triangle ABC$ in each of the following.

AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 3 cm.

Answer

Given: AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 3 cm



Required: To verify if AD is the bisector of $\angle A$ of $\triangle ABC$

Consider the $\triangle ABC$,

$$\text{Here, } \frac{BD}{DC} = \frac{1.5}{3} = \frac{1}{2} \text{ and } \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$$

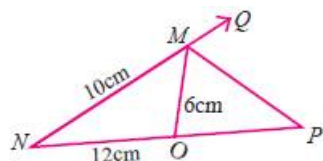
$$\text{Here, we can clearly see that } \frac{BD}{DC} \neq \frac{AB}{AC}$$

$\therefore$ We can say that AD is not the angle bisector of $\angle A$ using Converse angle bisector theorem.

∴ No, AD is not the angle bisector of $\angle A$

11. Question

In a $\triangle MNO$, MP is the external bisector of $\angle M$ meeting NO produced at P. If MN = 10 cm, MO = 6 cm, NO = 12 cm, then find OP.



Answer

Given: A $\triangle MNO$ with MP as external bisector of $\angle M$ meeting NO produced at P, and MN = 10 cm, MO = 6 cm, NO = 12 cm

Required: Length of OP

In $\triangle MNP$, MP is the external bisector of $\angle M$ meeting NO and produced at P.

Let OP = x cm, Now by angle bisector theorem, we have $\frac{NP}{OP} = \frac{MN}{MO}$

$$\Rightarrow \frac{x+12}{x} = \frac{10}{6}$$

$$\Rightarrow (x+12) \times 6 = 10x$$

$$\Rightarrow 6x + 72 = 10x$$

$$\Rightarrow 10x - 6x = 72$$

$$\Rightarrow 4x = 72$$

$$\Rightarrow x = \frac{72}{4} = 18$$

∴ The value of x = 18 cm.

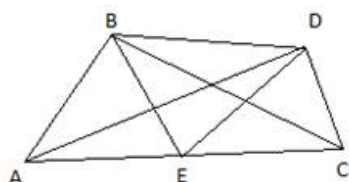
12. Question

In a quadrilateral ABCD, the bisectors of $\angle B$ and $\angle D$ intersect on AC at E.

Prove that $\frac{AB}{BC} = \frac{AD}{DC}$.

Answer

Given: quadrilateral ABCD with bisectors at B and D intersecting at e on AC



Required: To Prove $\frac{AB}{BC} = \frac{AD}{DC}$

Consider $\triangle ABC$, here BE is the angle bisector of $\angle B$

∴ By Angle bisector theorem we have, $\frac{AE}{EC} = \frac{AB}{BC}$ --eq(1)

Now, Consider $\triangle ADC$, here DE is the angle bisector of $\angle D$

∴ By Angle bisector theorem we have, $\frac{AE}{EC} = \frac{AD}{DC}$ --eq(2)

From -eq(1) and -eq(2)

We have,

$$\frac{AB}{AC} = \frac{AD}{DC}$$

Hence proved

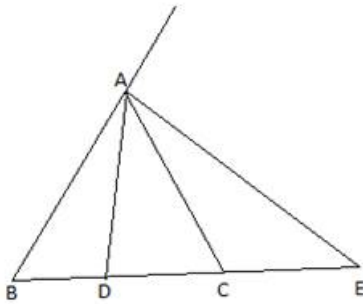
13. Question

The internal bisector of $\angle A$ of $\triangle ABC$ meets BC at D and the external bisector of $\angle A$ meets BC produced at E .

Prove that $\frac{BD}{BE} = \frac{CD}{CE}$.

Answer

Given: The internal bisector of $\angle A$ of $\triangle ABC$ meets BC at D and the external bisector of $\angle A$ meets BC produced at E



Required: To prove $\frac{BD}{BE} = \frac{CD}{CE}$

Consider $\triangle ABC$, here AD is the internal angle bisector of $\angle A$

$\therefore$ By Angle bisector theorem we have, $\frac{BD}{CD} = \frac{AB}{AC}$ --eq(1)

Again Consider $\triangle ABC$, Now AE is the External angle bisector of $\angle A$

$\therefore$ By Angle bisector theorem we have, $\frac{BE}{CE} = \frac{AB}{AC}$ --eq(2)

From -eq(1) and -eq(2)

We have,

$$\frac{BD}{CD} = \frac{BE}{CE}$$

$$\Rightarrow \frac{BD}{BE} = \frac{CD}{CE}$$

$$\therefore \frac{BD}{BE} = \frac{CD}{CE}$$

Hence Proved

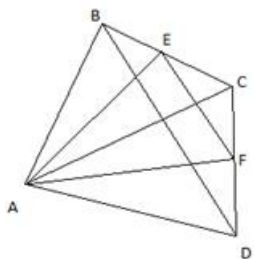
14. Question

$ABCD$ is a quadrilateral with $AB = AD$. If AE and AF are internal bisectors of $\angle BAC$ and $\angle DAC$ respectively, then prove that $EF \parallel BD$

Answer

Given: $ABCD$ is a quadrilateral with $AB = AD$. If AE and AF are internal bisectors of $\angle BAC$ and $\angle DAC$ respectively

Required: To prove $EF \parallel BD$



Consider the $\triangle ABC$,

Here, AE is the angle bisector of $\angle A$

$\therefore$ By angle bisector theorem $\frac{CE}{EB} = \frac{AC}{AB}$ --eq(1)

Now, In $\triangle ACD$,

Here, AF is the angle bisector of $\angle A$

$\therefore$ By angle bisector theorem $\frac{CF}{FD} = \frac{AC}{AD}$

$\Rightarrow \frac{CF}{FD} = \frac{AC}{AB}$ ($\because AD = AB$) --eq(2)

From --eq(1) and --eq(2)

We have,

$$\frac{CE}{EB} = \frac{CF}{FD}$$

Now, Consider $\triangle BCD$

Here, $\frac{CE}{EB} = \frac{CF}{FD}$

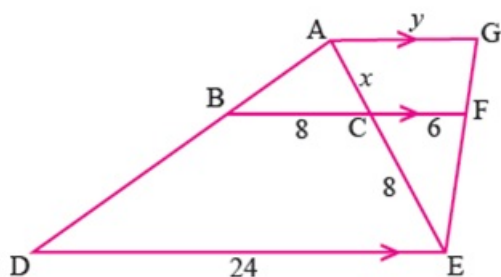
$\therefore$ We can say that $EF \parallel BD$ by Converse Thales theorem.

Hence Proved

Exercise 6.2

1 A. Question

Find the unknown values in each of the following figures. All lengths are given in centimeters. (All measures are not in scale)



Answer

In this question we must find the value of x and y, i.e. AC and AG respectively.

In such questions where we are given with some sides or angles of one or more triangles and we need find the other remaining sides and angles of triangles. The concept generally used is of congruency and similarity.

In this question we will be dealing only with sides.

Here,

In $\triangle ADE$ and $\triangle ABC$

$\angle A$ is common

$\angle ACB = \angle AED$ by correspondence ($BC \parallel DE$),

$\angle ABC = \angle ADE$ by correspondence ($FC \parallel DA$)

$\therefore \triangle ADE \sim \triangle ABC$

Similarly, $\triangle EGA \sim \triangle EFC$

$$\Rightarrow \frac{AC}{AE} = \frac{BC}{DE} (\because \triangle ADE \sim \triangle ABC)$$

$$\Rightarrow \frac{x}{x+8} = \frac{8}{24}$$

$$\Rightarrow 24x = 8x + 64$$

$$\Rightarrow 16x = 64$$

$$\Rightarrow x = 4 \text{ cm}$$

Also,

$$\frac{EC}{EA} = \frac{FC}{GA} (\because \triangle EGA \sim \triangle EFC)$$

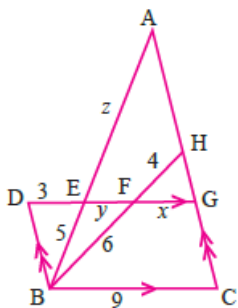
$$\Rightarrow \frac{8}{12} = \frac{6}{y} (\because EA = 8 + x, GA = y)$$

$$\Rightarrow y = 9 \text{ cm}$$

$$\Rightarrow y = 4 \text{ cm}$$

1 B. Question

Find the unknown values in each of the following figures. All lengths are given in centimeters. (All measures are not in scale)



Answer

We need to find x , y and z . i.e. FG and EF respectively.

Now,

In $\triangle HFG$ and $\triangle HBC$

$\angle HFG = \angle HBC$ by corresponding angles ($DG \parallel BC$)

$\angle HGF = \angle HCB$ by corresponding angles ($FG \parallel BC$)

$\therefore \triangle HFG \sim \triangle HBC$

$$\Rightarrow \frac{FG}{BC} = \frac{HF}{HB} (\because \triangle HFG \sim \triangle HBC)$$

$$\Rightarrow \frac{x}{9} = \frac{4}{4+6}$$

$$\Rightarrow x = 3.6 \text{ cm}$$

Now,

In $\triangle FHG$ and $\triangle FBD$

$\angle DFB = \angle HFG$ by vertically opposite angles

$\angle FDB = \angle FGH$ by alternate angles ($DB \parallel AC$)

$\therefore \triangle FHG \sim \triangle FBD$

$$\Rightarrow \frac{x}{3+y} = \frac{4}{6}$$

$$\Rightarrow 6 \times 3.6 = 12 + 4y \quad (\because x=3.6)$$

$$\Rightarrow y = 2.4 \text{ cm}$$

Now,

In $\triangle AEG$ and $\triangle ABC$

$\angle AEG = \angle ABC$ by corresponding angles ($EG \parallel BC$)

$\angle AGE = \angle ACB$ by corresponding angles ($EG \parallel BC$)

$\therefore \triangle AEG \sim \triangle ABC$

$$\Rightarrow \frac{AE}{AB} = \frac{EG}{BC} \quad (\because \triangle AEG \sim \triangle ABC)$$

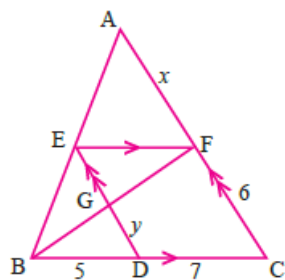
$$\Rightarrow \frac{z}{z+5} = \frac{x+y}{9}$$

$$\Rightarrow \frac{z}{z+5} = \frac{6}{9} \quad (\because x=3.6 \text{ and } y=2.4)$$

$$\Rightarrow z = 10 \text{ cm}$$

1 C. Question

Find the unknown values in each of the following figures. All lengths are given in centimeters. (All measures are not in scale)



Answer

To find x and y i.e. AF and GD respectively.

Now,

In $\triangle FBC$ and $\triangle GBD$

$\angle FBC = \angle GBD$ (common)

$\angle BFC = \angle BGD$ by corresponding angles ($DE \parallel CF$)

$\therefore \triangle FBC \sim \triangle GBD$

$$\Rightarrow \frac{GD}{FC} = \frac{BD}{DC} \quad (\because \triangle FBC \sim \triangle GBD)$$

$$\Rightarrow \frac{y}{6} = \frac{5}{7+5}$$

$$\Rightarrow y = 2.5 \text{ cm}$$

$EF = DC$ ($\because$ EFCD is a parallelogram)

$$\Rightarrow EF = 7 \text{ cm}$$

Now,

In $\triangle AEF$ and $\triangle ABC$

$\angle AEF = \angle ABC$ by corresponding angles ($EF \parallel BC$)

$\angle AFE = \angle ACB$ by corresponding angles ($EF \parallel BC$)

$\therefore \triangle AEF \sim \triangle ABC$

$$\Rightarrow \frac{AF}{AC} = \frac{EF}{BC} \quad (\because \triangle AEF \sim \triangle ABC)$$

$$\Rightarrow \frac{x}{x+6} = \frac{7}{12}$$

$$\Rightarrow x = 8.4 \text{ cm}$$

2. Question

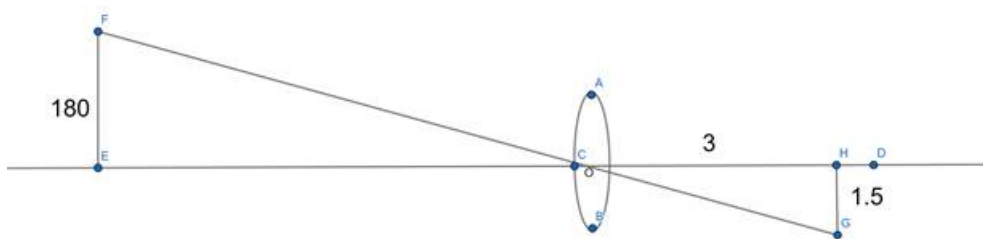
The image of a man of height 1.8 m, is of length 1.5 cm on the film of a camera. If the film is 3 cm from the lens of the camera, how far is the man from the camera?

Answer

The figure is given below. Here, FE is the man whose height is 180 cm.

HG is the image of the man, height of image = 1.5 cm

AB is the camera lens.



We need to find the length of EO.

In $\triangle FEO$ and $\triangle HGO$

$\angle GOH = \angle FOE$ (Vertically Opposite)

$\angle GHO = \angle FEO$ by alternate angles ($FE \parallel GH$)

$\therefore \triangle FEO \sim \triangle HGO$

$$\Rightarrow \frac{HG}{EF} = \frac{HO}{EO} \quad (\because \triangle FEO \sim \triangle HGO)$$

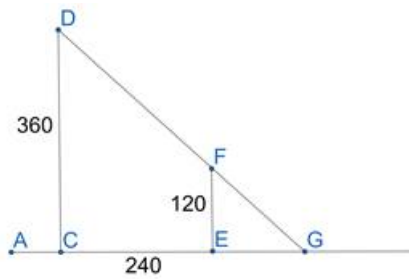
$$\Rightarrow \frac{1.5}{180} = \frac{3}{EO}$$

$$\Rightarrow EO = 3.6 \text{ m}$$

3. Question

A girl of height 120 cm is walking away from the base of a lamp-post at a speed of 0.6 m/sec. If the lamp is 3.6 m above the ground level, then find the length of her shadow after 4 seconds.

Answer



Here,

In the figure above,

FE is the height of the girl.

DC is the height of the lamp.

CE is the distance traveled by the girl in 4 seconds.

GE is the shadow of the girl after 4 seconds.

We need to find GE.

$$CE = 4 \times 0.6 \text{ m}$$

$$CE = 240 \text{ cm}$$

In ΔGFE and ΔGDC

$\angle EFG = \angle CDG$ by corresponding angles ($DC \parallel FE$)

$\angle GFE = \angle GDC$ by corresponding angles ($DC \parallel FE$)

$\therefore \Delta GFE \sim \Delta GDC$

$$\Rightarrow \frac{GE}{GC} = \frac{FE}{DC} (\because \Delta GFE \sim \Delta GDC)$$

$$\Rightarrow \frac{GE}{240 + GE} = \frac{120}{360}$$

$$\Rightarrow \frac{GE}{240 + GE} = \frac{1}{3}$$

$$\Rightarrow 3 \times GE = 240 + GE$$

$$\Rightarrow 2 \times GE = 240$$

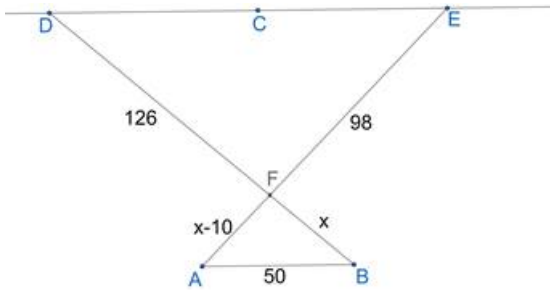
$$\Rightarrow GE = 120 \text{ cm}$$

$$\Rightarrow GE = 1.2 \text{ m}$$

4. Question

A girl is in the beach with her father. She spots a swimmer drowning. She shouts to her father who is 50 m due west of her. Her father is 10 m nearer to a boat than the girl. If her father uses the boat to reach the swimmer, he has to travel a distance 126 m from that boat. At the same time, the girl spots a man riding a water craft who is 98 m away from the boat. The man on the water craft is due east of the swimmer. How far must the man travel to rescue the swimmer? (Hint: see figure).

Answer



Here,

In the above figure,

B is the girl.

A is her father.

F is the boat.

D is the Drowning Man.

E is the man driving water craft.

We need to find the distance ED.

Now,

In ΔFAB and ΔFED

$\angle AFB = \angle EFD$ (vertically opposite angle)

$\angle FAB = \angle FED$ by alternate angles ($AB \parallel DE$)

$\therefore \Delta FAB \sim \Delta FED$

$$\Rightarrow \frac{BF}{DF} = \frac{FA}{FE} (\because \Delta FAB \sim \Delta FED)$$

$$\Rightarrow \frac{x}{126} = \frac{x-10}{98}$$

$$\Rightarrow x = 45 \text{ m}$$

$$\frac{FB}{FE} = \frac{AB}{ED} (\because \Delta FAB \sim \Delta FED)$$

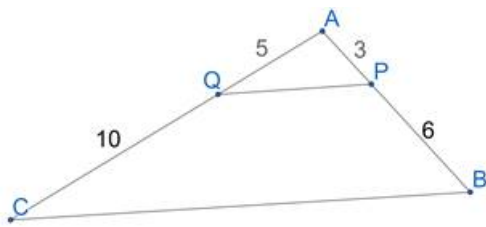
$$\Rightarrow \frac{x}{98} = \frac{50}{ED}$$

$$\Rightarrow ED = 140 \text{ m} (\because x = 45 \text{ m})$$

5. Question

P and Q are points on sides AB and AC respectively, of ΔABC . If $AP = 3 \text{ cm}$, $PB = 6 \text{ cm}$, $AQ = 5 \text{ cm}$ and $QC = 10 \text{ cm}$, show that $BC = 3 PQ$.

Answer



Now,

In $\triangle APQ$ and $\triangle ABC$,

$\angle QAP = \angle CAB$ (common)

$$\frac{AP}{AB} = \frac{AQ}{AC} \text{ (given)}$$

$\therefore \triangle ACB \sim \triangle AQP$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} \text{ } (\because \triangle ACB \sim \triangle AQP)$$

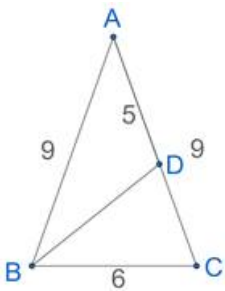
$$\Rightarrow \frac{3}{3+6} = \frac{PQ}{BC}$$

$$\Rightarrow BC = 3PQ$$

6. Question

In $\triangle ABC$, $AB = AC$ and $BC = 6$ cm. D is a point on the side AC such that $AD = 5$ cm and $CD = 4$ cm. Show that $\triangle BCD \sim \triangle ACB$ and hence find BD .

Answer



Here,

$$DC = (9 - 5) \text{ cm}$$

$$DC = 4 \text{ cm}$$

Now,

In $\triangle BCD$ and $\triangle ACB$

$\angle DCB = \angle ACB$ (common)

$$\frac{DC}{BC} = \frac{BC}{AC} = \frac{2}{3}$$

$\therefore \triangle BCD \sim \triangle ACB$

$$\Rightarrow \frac{BD}{AB} = \frac{DC}{BC} \text{ } (\because \triangle BCD \sim \triangle ACB)$$

$$\Rightarrow \frac{BD}{9} = \frac{4}{6}$$

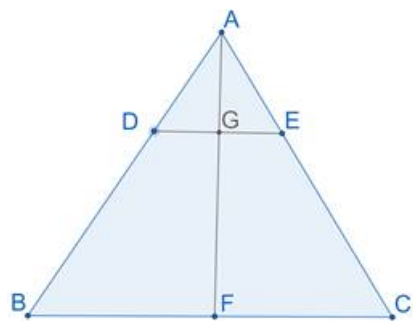
$$\Rightarrow BD = 6 \text{ cm}$$

7. Question

The points D and E are on the sides AB and AC of $\triangle ABC$ respectively, such that $DE \parallel BC$. If $AB = 3 AD$ and the

area of ΔABC is 72 cm^2 , then find the area of the quadrilateral DBCE.

Answer



Here,

In ΔADE and ΔABC

$\angle ADE = \angle ABC$ by corresponding angles ($DE \parallel BC$)

$\angle DEA = \angle BCA$ by corresponding angles ($DE \parallel BC$)

$\therefore \Delta AED \sim \Delta ACB$

Similarly,

$\Delta AGD \sim \Delta AFB$ (where $AF \perp BC$)

$\Rightarrow AF = 3AG$ ($\because AB = 3AD$ which is given) ----(1)

Similarly,

$\Rightarrow BC = 3 \times DE$ ----(2)

$\Rightarrow \frac{1}{2} \times BC \times AF = \frac{1}{2} \times (3 \times DE) \times (3 \times AG)$ {by (1) and (2)}

$\Rightarrow \frac{1}{2} BC \times AF = 9 \times \frac{1}{2} DE \times AG$

$\Rightarrow \text{Area of } \Delta ABC = 9 \times \text{Area of } \Delta ADE$

$\Rightarrow \text{Area of } \Delta ADE = \frac{72}{9} \text{ cm}^2$ ($\because \text{Area of } \Delta ABC = 72 \text{ cm}^2$)

$\Rightarrow \text{Area of } \Delta ADE = 8 \text{ cm}^2$

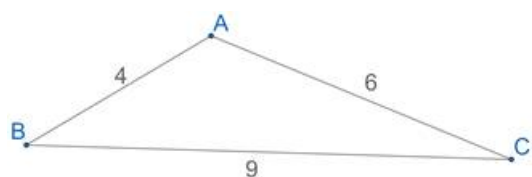
$\Rightarrow \text{Area of DCEB} = \text{Area of } \Delta ABC - \text{Area of } \Delta ADE$

$\Rightarrow \text{Area of DCEB} = 72 - 8 = 64 \text{ cm}^2$

8. Question

The lengths of three sides of a triangle ABC are 6 cm, 4 cm and 9 cm. $\Delta PQR \sim \Delta ABC$. One of the lengths of sides of ΔPQR is 35cm. What is the greatest perimeter possible for ΔPQR ?

Answer



One of the length should be 35cm.

So, for perimeter to be greatest the smallest length of PQR should be 35 cm so that the other 2 sides could be greater than 35 cm hence greatest Perimeter achieved in that case.

$\Delta PQR \sim \Delta ABC$ (given)

$\Rightarrow PQ = 35 \text{ cm}$ ($\because$ It corresponds AB which is the smaller side of $\triangle ABC$)

$$\Rightarrow \frac{PQ}{PR} = \frac{AB}{AC}$$

$$\Rightarrow \frac{35}{PR} = \frac{4}{6}$$

$$\Rightarrow PR = 52.5 \text{ cm}$$

$$\Rightarrow \frac{PQ}{QR} = \frac{AB}{BC}$$

$$\Rightarrow \frac{35}{QR} = \frac{4}{9}$$

$$\Rightarrow QR = 78.75 \text{ cm}$$

$$\Rightarrow \text{Perimeter} = PQ + PR + QR$$

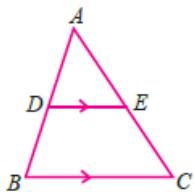
$$\Rightarrow \text{Perimeter} = 35 + 52.5 + 78.75$$

$$\Rightarrow \text{Perimeter} = 166.25 \text{ cm}$$

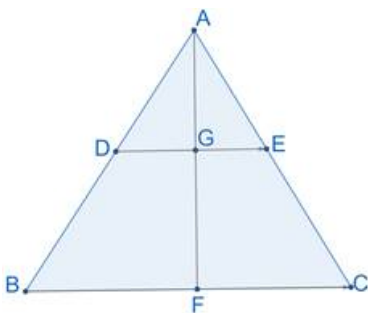
9. Question

In the figure, $DE \parallel BC$ and $\frac{AD}{BD} = \frac{3}{5}$, calculate the value of

(i) $\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC}$ (ii) $\frac{\text{area of trapezium BCED}}{\text{area of } \triangle ABC}$



Answer



i) In $\triangle ADG$ and $\triangle ABF$

$\angle ADE = \angle ABC$ by corresponding angles ($DG \parallel BF$)

$\angle AED = \angle ACB$ by corresponding angles ($DG \parallel BF$)

$\therefore \triangle ADE \sim \triangle ABC$

Let $AD = 3x$, so $BD = 5x$.

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{3}{8} = \frac{DE}{BC} \text{----(1)}$$

Similarly, $\triangle ADG \sim \triangle ABF$

$$\frac{AD}{AB} = \frac{AG}{AF}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{AG}{AF}$$

$$\Rightarrow \frac{3}{8} = \frac{AG}{AF} \text{----(2)}$$

$$\text{Now, } \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \frac{\frac{1}{2} DE \times AG}{\frac{1}{2} AC \times AF}$$

$$\Rightarrow \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \frac{9}{64} \text{ (by (1) and (2)) ----(3)}$$

ii) Area of BCED = Area of $\triangle ABC$ - Area of $\triangle ADE$

$$\text{Area of BCED} = \text{Area of } \triangle ABC - \frac{9}{64} \times \text{Area of } \triangle ABC \text{ (by (3))}$$

$$\text{Area of BCED} = \frac{55}{64} \times \text{Area of } \triangle ABC$$

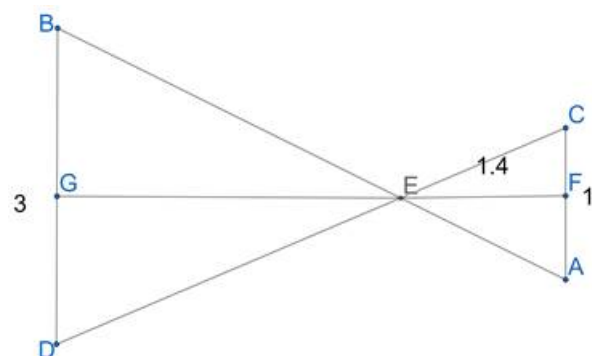
$$\frac{\text{Area of BCED}}{\text{Area of } \triangle ABC} = \frac{55}{64}$$

10. Question

The government plans to develop a new industrial zone in an unused portion of land in a city. The shaded portion of the map shown on the right, indicates the area of the new industrial zone. Find the area of the new industrial zone.



Answer



To find the Area of $\triangle EBD$.

$$EF = 1.4 \text{ Km}$$

In $\triangle EBD$ and $\triangle EAC$

$$\angle EBD = \angle EAC \text{ by alternate angles (BD} \parallel \text{CA)}$$

$$\angle EDB = \angle ECA \text{ by alternate angles (BD} \parallel \text{CA)}$$

$$\therefore \triangle EBD \sim \triangle EAC$$

$$\Rightarrow \frac{BE}{AE} = \frac{BD}{AC}$$

$$\Rightarrow \frac{BE}{AE} = 3 (\because BD = 3, AC = 1)$$

Similarly,

$$\triangle EBG \sim \triangle EAF$$

$$\Rightarrow \frac{EG}{EF} = \frac{3}{1} \text{ ---(1)}$$

$$\Rightarrow \frac{\text{Area of } \triangle EBD}{\text{Area of } \triangle EAC} = \frac{\frac{1}{2} \times EG \times BD}{\frac{1}{2} \times EF \times AC}$$

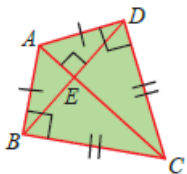
$$\Rightarrow \frac{\text{Area of } \triangle EBD}{\text{Area of } \triangle EAC} = 9 \text{ (by (1))}$$

$$\Rightarrow \text{Area of } \triangle EBD = 9 \times \frac{1}{2} \times 1.4 \times 1 \text{ Km}^2$$

$$\Rightarrow \text{Area of } \triangle EBD = 6.3$$

11. Question

A boy is designing a diamond shaped kite, as shown in the figure where $AE = 16$ cm, $EC = 81$ cm. He wants to use a straight cross bar BD . How long should it be?



Answer

To find DB

$$\text{Let } \angle EAD = x \text{ ---(1)}$$

$$\Rightarrow \angle ADE = 90 - x (\because \angle AED = 90 \text{ and sum of angles of triangle is } 180)$$

$$\Rightarrow \angle EDC = x (\because \angle ADC = 90) \text{ ---(2)}$$

$$\Rightarrow \angle EAD = \angle EDC \text{ (by (1) and (2)) ---(3)}$$

Now,

In $\triangle AED$ and $\triangle DEC$

$$\angle AED = \angle DEC \text{ (both perpendicular to AC)}$$

$$\angle EAD = \angle EDC \text{ (by (3))}$$

$$\therefore \triangle AED \sim \triangle DEC$$

$$\Rightarrow \frac{EA}{ED} = \frac{ED}{EC} (\because \triangle AED \sim \triangle DEC)$$

$$\Rightarrow 16 \times 81 = ED^2$$

$$\Rightarrow ED = 36 \text{ cm}$$

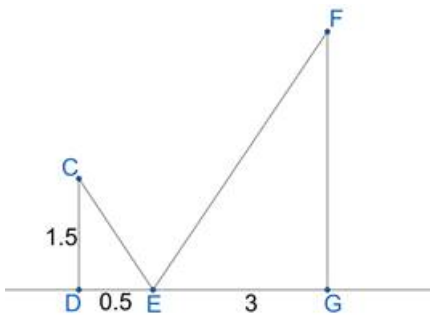
$$\Rightarrow 2 \times ED = DB = 72 \text{ cm}$$

12. Question

A student wants to determine the height of a flagpole. He placed a small mirror on the ground so that he can

see the reflection of the top of the flagpole. The distance of the mirror from him is 0.5 m and the distance of the flagpole from the mirror is 3 m. If his eyes are 1.5 m above the ground level, then find the height of the flagpole. (The foot of student, mirror and the foot of flagpole lie along a straight line).

Answer



Here,

In the above Figure,

DE is the distance of student and mirror.

EG is the distance of mirror and flagpole.

CD is the height of student till its eyes.

FG is the height of flagpole which we need to find.

Now,

In $\triangle CDE$ and $\triangle FGE$

$\angle FEG = \angle CED$ (by mirror property)

$\angle CDE = \angle FGE$ (both perpendicular given)

$\therefore \triangle CDE \sim \triangle FGE$

$$\Rightarrow \frac{DE}{GE} = \frac{CD}{FG} (\because \triangle CDE \sim \triangle FGE)$$

$$\Rightarrow \frac{0.5}{3} = \frac{1.5}{FG}$$

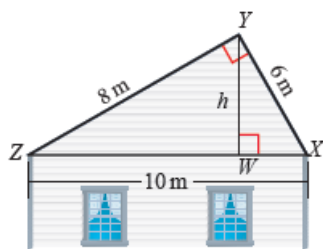
$$\Rightarrow FG = 9 \text{ m}$$

13. Question

A roof has a cross section as shown in the diagram,

(i) Identify the similar triangles

(ii) Find the height h of the roof.



Answer

In $\triangle YZW$

Let $\angle YZW = x$

$$\Rightarrow \angle ZYW = 90 - x (\because \text{It is a right-angled triangle})$$

$$\Rightarrow \angle XYW = x \quad (\because \angle ZYX \text{ is } 90^\circ)$$

$$\Rightarrow \angle YZW = \angle XYW \text{ ----(1)}$$

Similarly,

$$\angle XYW = \angle XZY \text{ ----(2)}$$

Now,

In $\triangle YWZ$ and $\triangle XYZ$

$$\angle WXY = \angle ZXY \text{ (common)}$$

$$\angle YWZ = \angle XYZ \text{ (by (1))}$$

$$\therefore \triangle YWZ \sim \triangle XYZ \text{ ----(3)}$$

Now,

In $\triangle XWY$ and $\triangle XYZ$

$$\angle YXW = \angle ZXY \text{ (common)}$$

$$\angle XYW = \angle XZY \text{ (by (2))}$$

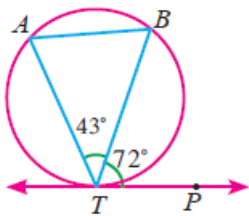
$$\therefore \triangle XWY \sim \triangle XYZ \text{ ----(4)}$$

$$\Rightarrow \triangle XWY \sim \triangle XYZ \sim \triangle YWZ \text{ (by (3) and (4))}$$

Exercise 6.3

1. Question

In the figure TP is a tangent to a circle. A and B are two points on the circle. If $\angle BTP = 72^\circ$ and $\angle ATB = 43^\circ$ find $\angle ABT$.



Answer

The alternate segment theorem (also known as the tangent-chord theorem) states that in any circle, the angle between a chord and a tangent through one of the end points of the chord is equal to the angle in the alternate segment.

Therefore by above theorem,

$$\angle BAT = \angle BTP = 72^\circ$$

$$\text{Sum of all angles of a triangle} = 180^\circ$$

In $\triangle ABT$,

$$\angle ABT + \angle BTA + \angle TAB = 180^\circ$$

$$\Rightarrow \angle ABT + 43^\circ + 72^\circ = 180^\circ$$

$$\Rightarrow \angle ABT + 115^\circ = 180^\circ$$

$$\Rightarrow \angle ABT = 180^\circ - 115^\circ$$

$$\Rightarrow \angle ABT = 65^\circ$$

Hence, $\angle ABT = 65^\circ$

2. Question

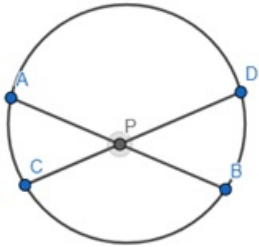
AB and CD are two chords of a circle which intersect each other internally at P.

(i) If $CP = 4$ cm, $AP = 8$ cm, $PB = 2$ cm, then find PD.

(ii) If $AP = 12$ cm, $AB = 15$ cm, $CP = PD$, then find CD

Answer

If two chords of a circle intersect each other internally or externally, then area of rectangle contained by the segment of one chord is equal to the area of rectangle contained by the segment of other chord.



Using above theorem,

$$PA \times PB = PC \times PD \dots(1)$$

(i) Given: $CP = 4$ cm

$$AP = 8 \text{ cm}$$

$$PB = 2 \text{ cm}$$

Putting the values in (1),

$$8 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm} \times PD$$

$$\Rightarrow PD = \frac{8 \text{ cm} \times 2 \text{ cm}}{4 \text{ cm}}$$

$$\Rightarrow PD = \frac{16}{4} \text{ cm}$$

$$\Rightarrow PD = 4 \text{ cm}$$

Hence, $PD = 4$ cm

(ii) Given: $AP = 12$ cm

$$AB = 15 \text{ cm}$$

$$CP = PD$$

$$\therefore AB = AP + PB$$

$$\Rightarrow PB = AB - AP$$

$$\Rightarrow PB = 15 \text{ cm} - 12 \text{ cm}$$

$$\Rightarrow PB = 3 \text{ cm}$$

Putting the values in (1),

$$12 \text{ cm} \times 3 \text{ cm} = PD \times PD$$

$$\Rightarrow PD^2 = 36 \text{ cm}^2 \Rightarrow PD = 6 \text{ cm}$$

Hence, $PD = 6$ cm

3. Question

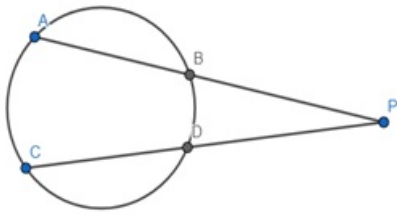
AB and CD are two chords of a circle which intersect each other externally at P

(i) If $AB = 4$ cm $BP = 5$ cm and $PD = 3$ cm, then find CD.

(ii) If $BP = 3$ cm, $CP = 6$ cm and $CD = 2$ cm, then find AB.

Answer

If two chords of a circle intersect each other internally or externally, then area of rectangle contained by the segment of one chord is equal to the area of rectangle contained by the segment of other chord.



Using above theorem,

$$PA \times PB = PC \times PD \dots(1)$$

(i) Given: $AB = 4 \text{ cm}$

$$BP = 5 \text{ cm}$$

$$PD = 3 \text{ cm}$$

$$\therefore AP = AB + BP$$

$$\Rightarrow AP = 4 \text{ cm} + 5 \text{ cm}$$

$$\Rightarrow AP = 9 \text{ cm}$$

Putting the values in (1),

$$9 \text{ cm} \times 5 \text{ cm} = PC \times 3 \text{ cm}$$

$$\Rightarrow PC = \frac{9 \text{ cm} \times 5 \text{ cm}}{3 \text{ cm}}$$

$$\Rightarrow PC = \frac{45}{3} \text{ cm}$$

$$\Rightarrow PC = 15 \text{ cm}$$

$$\therefore PC = CD + DP$$

$$\Rightarrow 15 \text{ cm} = CD + 3 \text{ cm}$$

$$\Rightarrow CD = 15 \text{ cm} - 3 \text{ cm}$$

$$\Rightarrow CD = 12 \text{ cm}$$

Hence, $CD = 12 \text{ cm}$

(ii) Given: $BP = 3 \text{ cm}$

$$CP = 6 \text{ cm}$$

$$CD = 2 \text{ cm}$$

$$\therefore CP = CD + DP$$

$$\Rightarrow DP = CP - CD$$

$$\Rightarrow DP = 6 \text{ cm} - 2 \text{ cm}$$

$$\Rightarrow DP = 4 \text{ cm}$$

Putting the values in (1),

$$PA \times 3 \text{ cm} = PC \times PD$$

$$\Rightarrow PA \times 3 \text{ cm} = 6 \text{ cm} \times 4 \text{ cm} \Rightarrow PA = \frac{6 \text{ cm} \times 4 \text{ cm}}{3 \text{ cm}}$$

$$\Rightarrow PA = \frac{24}{3} \text{ cm}$$

$$\Rightarrow PA = 8 \text{ cm}$$

$$\because AP = AB + BP$$

$$\Rightarrow 8 \text{ cm} = AB + 3 \text{ cm}$$

$$\Rightarrow AB = 8 \text{ cm} - 3 \text{ cm}$$

$$\Rightarrow AB = 5 \text{ cm}$$

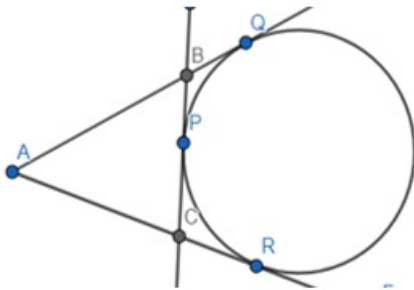
Hence, $AB = 5 \text{ cm}$

4. Question

A circle touches the side BC of $\triangle ABC$ at P, AB and AC produced at Q and R respectively, prove that $AQ = AR$

$$= \frac{1}{2} (\text{perimeter of } \triangle ABC)$$

Answer



Lengths of tangents drawn from an exterior point to the circle are equal.

From the above theorem,

$$AR = AQ \dots(1)$$

$$BP = BQ \dots(2)$$

$$CP = CR \dots(3)$$

Now,

$$\text{Perimeter of } \triangle ABC = AB + BC + CA$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = AB + BP + PC + CA$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = (AB + BQ) + (CR + CA) \text{ [Using (2) and (3)]}$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = AQ + AR$$

$$\Rightarrow \text{Perimeter of } \triangle ABC = AQ + AQ$$

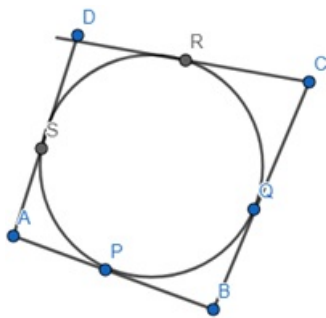
$$\Rightarrow \text{Perimeter of } \triangle ABC = 2AQ \text{ [Using (1)]}$$

$$\Rightarrow \frac{\text{Perimeter of } \triangle ABC}{2} = AQ \because AQ = AR = \frac{1}{2} \text{Perimeter of } \triangle ABC$$

Hence, Proved.

5. Question

If all sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.



Answer

Lengths of tangents drawn from an exterior point to the circle are equal.

From the above theorem,

$$AP = AS$$

$$DS = DR$$

$$CR = CQ$$

$$BQ = BP$$

Adding above equations,

$$AP + DR + CR + BP = AS + DS + CQ + BQ$$

$$\Rightarrow (AP + BP) + (DR + CR) = (AS + DS) + (CQ + BQ)$$

$$\Rightarrow AB + CD = AD + BC$$

Also,

$$AB = CD$$

$$AD = BC$$

[$\because$ Parallel sides of a parallelogram are equal]

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow AB + AB = BC + BC$$

$$\Rightarrow 2AB = 2BC$$

$$\Rightarrow AB = BC$$

$$\Rightarrow AB = BC = CD = AD$$

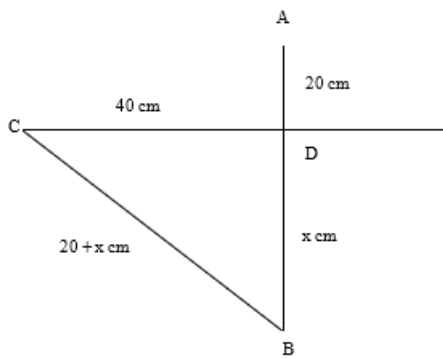
$$\Rightarrow ABCD \text{ is a rhombus}$$

Hence, Proved

6. Question

A lotus is 20 cm above the water surface in a pond and its stem is partly below the water surface. As the wind blew, the stem is pushed aside so that the lotus touched the water 40 cm away from the original position of the stem. How much of the stem was below the water surface originally?

Answer



Let AB be the length of stem

D is the point of lotus that touch the ground

x cm be the length of stem that lies below the ground

$$AD = 20 \text{ cm}$$

Clearly,

$$AB = AD + DB$$

$$\Rightarrow AB = 20 \text{ cm} + x \text{ cm}$$

$$\Rightarrow AB = (20 + x) \text{ cm}$$

Also,

$$BC = AB [\because \text{length of stem}]$$

$$\Rightarrow BC = (20 + x) \text{ cm}$$

Now by Pythagoras theorem,

$$BC^2 = BD^2 + DC^2$$

$$\Rightarrow (20 + x)^2 = x^2 + 40^2$$

$$\Rightarrow 20^2 + x^2 + 2 \times 20 \times x = x^2 + 40^2$$

$$\Rightarrow 400 + x^2 + 40x = x^2 + 1600$$

$$\Rightarrow x^2 + 1600 - x^2 - 400 - 40x = 0$$

$$\Rightarrow x^2 + 1600 - x^2 - 400 - 40x = 0$$

$$\Rightarrow 1200 - 40x = 0$$

$$\Rightarrow 40x = 1200$$

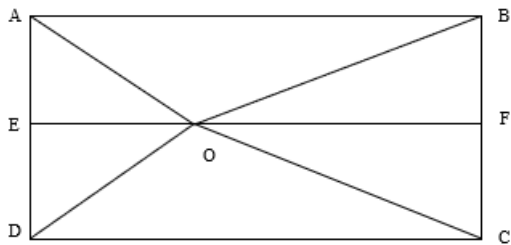
$$\Rightarrow x = 30$$

Hence, length of stem below water = 30 cm

7. Question

A point O in the interior of a rectangle ABCD is joined to each of the vertices A, B, C and D. Prove that $OA^2 + OC^2 = OB^2 + OD^2$

Answer



Draw $EF \parallel AB \parallel DC$ passing through O.

Also $AB = EF = DC$

$\therefore$ ABCD and CDEF are rectangles

Now by Pythagoras theorem,

$$BO^2 = BF^2 + OF^2 \dots(1)$$

And,

$$OD^2 = OE^2 + ED^2 \dots(2)$$

Adding (1) and (2),

$$BO^2 + OD^2 = BF^2 + OF^2 + OE^2 + ED^2$$

$$\Rightarrow BO^2 + OD^2 = AE^2 + OF^2 + OE^2 + CF^2$$

$$\Rightarrow BO^2 + OD^2 = AE^2 + OE^2 + OF^2 + CF^2$$

$$\Rightarrow BO^2 + OD^2 = AO^2 + OC^2$$

Hence, Proved

Exercise 6.4

1. Question

If a straight line intersects the sides AB and AC of a $\triangle ABC$ at D and E respectively and is parallel to BC, then

$$\frac{AE}{AC} =$$

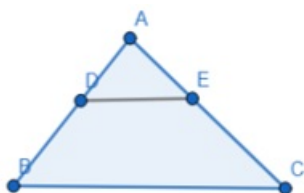
A. $\frac{AD}{DB}$

B. $\frac{AD}{AB}$

C. $\frac{DE}{BC}$

D. $\frac{AD}{EC}$

Answer



Given: DE intersect AB and AC and $DE \parallel BC$

Using Basic Proportionality theorem which states that if a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the ratio

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$$

$$\Rightarrow \frac{EC}{AE} = \frac{DB}{AD} \text{ {Reciprocal of above}}$$

$$\Rightarrow \frac{EC}{AE} + 1 = \frac{DB}{AD} + 1 \text{ {Adding 1 on both sides}}$$

$$\Rightarrow \frac{EC + AE}{AE} = \frac{DB + AD}{AD}$$

$$\Rightarrow \frac{AC}{AE} = \frac{AB}{AD}$$

$$\Rightarrow \frac{AE}{AC} = \frac{AD}{AB} \text{ {Reciprocal of above}}$$

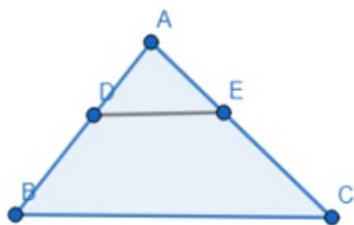
2. Question

In $\triangle ABC$, DE is $\parallel$ to BC , meeting AB and AC at D and E .

If $AD = 3$ cm, $DB = 2$ cm and $AE = 2.7$ cm, then AC is equal to

- A. 6.5 cm
- B. 4.5 cm
- C. 3.5 cm
- D. 5.5 cm

Answer



Given: DE intersect AB and AC and $DE \parallel BC$

$AD = 3$ cm, $DB = 2$ cm and $AE = 2.7$ cm

Using Basic Proportionality theorem which states that if a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the ratio

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$$

Putting the values,

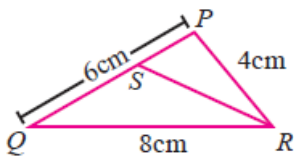
$$\Rightarrow \frac{2.7}{EC} = \frac{3}{2}$$

$$\Rightarrow EC = \frac{2.7 \times 2}{3} = 1.8 \text{ cm}$$

Now, $AC = AE + EC = 2.7 + 1.8 = 4.5$ cm

3. Question

In $\triangle PQR$, RS is the bisector of $\angle R$. If $PQ = 6$ cm, $QR = 8$ cm, $RP = 4$ cm then PS is equal to



- A. 2 cm
- B. 4 cm
- C. 3 cm
- D. 6 cm

Answer

Given: $\angle PRS = \angle SRQ$ and $PQ = 6$ cm, $QR = 8$ cm, $RP = 4$ cm

Using Angle Bisector Theorem which states that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

Here, RS is the bisector of $\angle A$ so

$$\frac{PS}{SQ} = \frac{PR}{RQ}$$

Let PS be x then SQ will be $6 - x$.

$$\Rightarrow \frac{x}{6-x} = \frac{4}{8}$$

$$\Rightarrow 8x = 24 - 4x$$

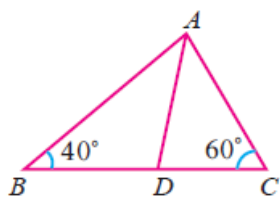
$$\Rightarrow 12x = 24$$

$$\Rightarrow x = 2$$

So, PS = 2 cm

4. Question

In figure, if $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 40^\circ$, and $\angle C = 60^\circ$, then $\angle BAD =$



- A. 30°
- B. 50°
- C. 80°
- D. 40°

Answer

Given: $\frac{AB}{AC} = \frac{BD}{DC}$ and $\angle B = 40^\circ$, $\angle C = 60^\circ$

We know that Converse of Angle Bisector Theorem states that if a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line

bisects the angle internally at the vertex.

Here, $\frac{AB}{AC} = \frac{BD}{DC}$ {Given}

So, AD is the bisector of $\angle A$

$\therefore \angle A + \angle B + \angle C = 180^\circ$ {Angle sum property}

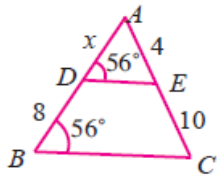
$$\Rightarrow \angle A = 180^\circ - 40^\circ - 60^\circ$$

$$\Rightarrow \angle A = 80^\circ$$

$$\text{Also, } \angle BAD = \angle DAC = \frac{\angle A}{2} = 40^\circ$$

5. Question

In the figure, the value x is equal to



A. $4 \cdot 2$

B. $3 \cdot 2$

C. $0 \cdot 8$

D. $0 \cdot 4$

Answer

Here, DE intersect AB and AC and $DE \parallel BC$

$AD = x$, $DB = 8$, $EC = 10$ and $AE = 4$

Using Basic Proportionality theorem which states that if a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the ratio

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB}$$

Putting the values,

$$\Rightarrow \frac{4}{10} = \frac{x}{8}$$

$$\Rightarrow x = \frac{4 \times 8}{10} = 3.2$$

6. Question

In triangles ABC and DEF, $\angle B = \angle E$, $\angle C = \angle F$, then

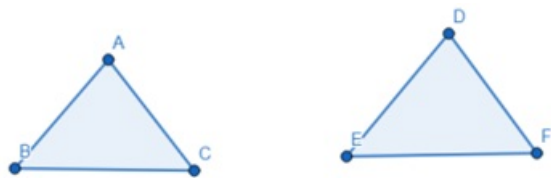
A. $\frac{AB}{DE} = \frac{CA}{EF}$

B. $\frac{BC}{EF} = \frac{AB}{FD}$

C. $\frac{AB}{DE} = \frac{BC}{EF}$

D. $\frac{CA}{FD} = \frac{AB}{EF}$

Answer



Given: $\angle B = \angle E$ and $\angle C = \angle F$

By the criterion of similar triangle AA which says that

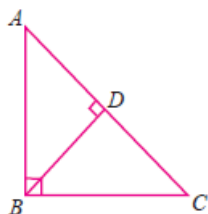
if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

Hence, $ABC \sim DEF$ (AA similarity criterion)

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \text{ \{Corresponding sides of the similar triangles are in same ratio\}}$$

7. Question

From the given figure, identify the wrong statement.



A. $\triangle ADB \sim \triangle ABC$

B. $\triangle ABD \sim \triangle ABC$

C. $\triangle BDC \sim \triangle ABC$

D. $\triangle ADB \sim \triangle BDC$

Answer

In $\triangle ABC$ and $\triangle ADB$,

$$\angle A = \angle A \text{ \{Common\}}$$

$$\angle B = \angle D \text{ \{90}^\circ \text{ each\}}$$

By the criterion of similar triangle AA which says that

if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

$$\triangle ABC \sim \triangle ADB \text{ ...(1)}$$

In $\triangle ABC$ and $\triangle BDC$,

$$\angle C = \angle C \text{ \{Common\}}$$

$$\angle B = \angle D \text{ \{90}^\circ \text{ each\}}$$

By the criterion of similar triangle AA which says that

if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

$$\triangle ABC \sim \triangle BDC \text{(2)}$$

From (1) and (2),

$$\triangle BDC \sim \triangle ADB$$

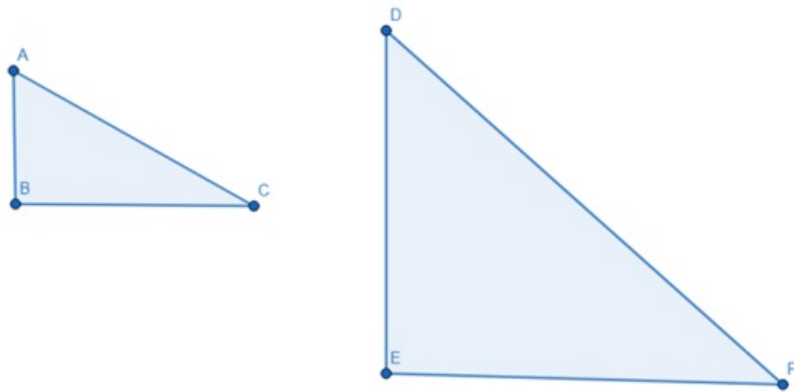
Hence, option (B) is wrong.

8. Question

If a vertical stick 12 m long casts a shadow 8 m long on the ground and at the same time a tower casts a shadow 40 m long on the ground, then the height of the tower is

- A. 40 m
- B. 50 m
- C. 75 m
- D. 60 m

Answer



Let AB be the stick of length 12 m and BC be the shadow 8 m long. Similarly DE be the tower with shadow 40 m long.

In $\triangle ABC$ and $\triangle DEF$,

$$\angle ABC = \angle DEF \{90^\circ \text{ each}\}$$

$$\angle BCA = \angle EFD \{\text{angular elevation is same at the same instant}\}$$

By the criterion of similar triangle AA which says that

if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

$$\triangle ABC \sim \triangle DEF$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} \{\text{Corresponding sides of the similar triangles are in same ratio}\}$$

$$\Rightarrow \frac{12}{8} = \frac{DE}{40}$$

$$\Rightarrow DE = 60 \text{ m}$$

9. Question

The sides of two similar triangles are in the ratio 2:3, then their areas are in the ratio

- A. 9:4
- B. 4:9
- C. 2:3
- D. 3:2

Answer

Given: Sides of two similar triangles are in the ratio 2:3

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of

their corresponding sides

$$\text{Ratio of their area} = \frac{2^2}{3^2} = \frac{4}{9} = 4:9$$

10. Question

Triangles ABC and DEF are similar. If their areas are 100 cm^2 and 49 cm^2 respectively and BC is 8.2 cm then EF =

- A. 5.47 cm
- B. 5.74 cm
- C. 6.47 cm
- D. 6.74 cm

Answer

Given: $\text{ar}(\text{ABC}) = 100 \text{ cm}^2$, $\text{ar}(\text{DEF}) = 49 \text{ cm}^2$ and $\text{BC} = 8.2 \text{ cm}$

Areas of two similar triangles are in the ratio 100:49

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides

$$\text{Ratio of their corresponding sides } \frac{\text{BC}}{\text{EF}} = \frac{\sqrt{100}}{\sqrt{49}} = \frac{10}{7}$$

Putting the values

$$\Rightarrow \frac{8.2}{\text{EF}} = \frac{10}{7}$$

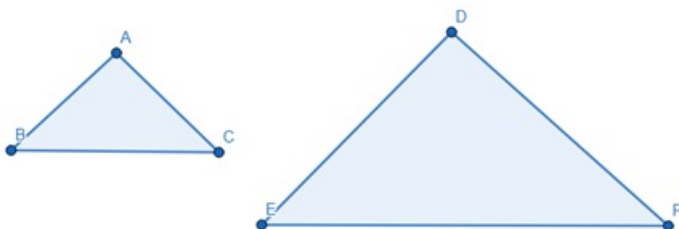
$$\Rightarrow \text{EF} = \frac{57.4}{10} = 5.74 \text{ cm}$$

11. Question

The perimeters of two similar triangles are 24 cm and 18 cm respectively. If one side of the first triangle is 8 cm, then the corresponding side of the other triangle is

- A. 4 cm
- B. 3 cm
- C. 9 cm
- D. 6 cm

Answer



Given: $\text{Perimeter}(\text{ABC}) = 24 \text{ cm}$, $\text{Perimeter}(\text{DEF}) = 18 \text{ cm}$ and $\text{BC} = 8 \text{ cm}$

Perimeters of two similar triangles are in the ratio $24:18 = 4:3$

We know that if two triangles are similar, then the ratio of the corresponding sides is equal to the ratio of the corresponding perimeters.

Ratio of their corresponding sides $\frac{BC}{EF} = \frac{4}{3}$

Putting the values

$$\Rightarrow \frac{8}{EF} = \frac{4}{3}$$

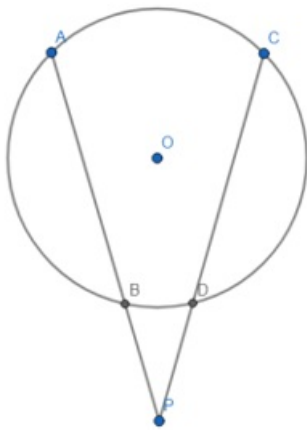
$$\Rightarrow EF = 2 \times 3 = 6 \text{ cm}$$

12. Question

AB and CD are two chords of a circle which when produced to meet at a point P such that AB = 5 cm, AP = 8 cm, and CD = 2 cm then PD =

- A. 12 cm
- B. 5 cm
- C. 6 cm
- D. 4 cm

Answer



Given: AB = 5 cm, AP = 8 cm, and CD = 2 cm

We know that two chords AB and CD intersect at P inside the circle with centre at O.

Then $PA \times PB = PC \times PD$.

$$PB = AP - AB = 8 - 5 = 3 \text{ cm}$$

Let PD = x then PC = PD + CD = 2 + x

$$\Rightarrow 8 \times 3 = x(2+x)$$

$$\Rightarrow 2x + x^2 = 24$$

$$\Rightarrow x^2 + (6-4)x - 24 = 0$$

$$\Rightarrow (x+6)(x-4) = 0$$

$$\Rightarrow x = 4 \text{ and } -6$$

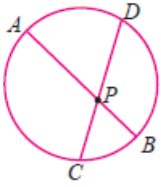
$x = -6$ is not possible because length is always positive

$$\Rightarrow PD = 4 \text{ cm}$$

13. Question

In the adjoining figure, chords AB and CD intersect at P.

If AB = 16 cm, PD = 8 cm, PC = 6 and AP > PB, then AP =



- A. 8 cm
- B. 4 cm
- C. 12 cm
- D. 6 cm

Answer

Given: $AB = 16$ cm, $PD = 8$ cm, and $PC = 6$ cm

We know that two chords AB and CD intersect at P inside the circle with centre at O.

Then $PA \times PB = PC \times PD$.

Let $PA = x$ then $PB = AB - AP = 16 - x$

$$\Rightarrow x \times (16 - x) = 6 \times 8$$

$$\Rightarrow 16x - x^2 = 48$$

$$\Rightarrow x^2 - (16 + 4)x + 48 = 0$$

$$\Rightarrow (x - 12)(x - 4) = 0$$

$$\Rightarrow x = 4 \text{ and } 12$$

$x = 4$ is not possible because $AP > PB$

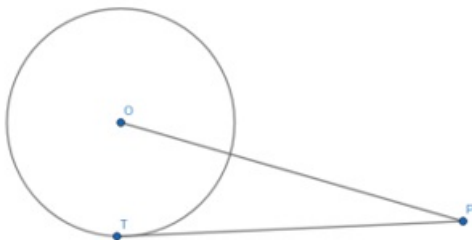
$$\Rightarrow AP = 12 \text{ cm}$$

14. Question

A point P is 26 cm away from the centre O of a circle and PT is the tangent drawn from P to the circle is 10 cm, then OT is equal to

- A. 36 cm
- B. 20 cm
- C. 18 cm
- D. 24 cm

Answer



Given: $OP = 26$ cm, $PT = 10$ cm

We know that a tangent at any point on a circle is perpendicular to the radius through the point of contact

$$\Rightarrow \angle OTP = 90^\circ$$

Using Pythagoras theorem,

$$OP^2 = PT^2 + OT^2$$

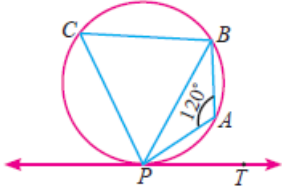
$$\Rightarrow OT^2 = OP^2 - PT^2$$

$$\Rightarrow OT^2 = 26^2 - 10^2$$

$$\Rightarrow OT = 24 \text{ cm}$$

15. Question

In the figure, if $\angle PAB = 120^\circ$ then $\angle BPT =$



A. 120°

B. 30°

C. 50°

D. 60°

Answer

From the figure, we find that ABCP is a cyclic quadrilateral

$\therefore$ Opposite angles are supplementary

$$\therefore \angle BAP + \angle PCB = 180^\circ$$

$$\Rightarrow \angle PCB = 180^\circ - \angle BAP$$

$$\Rightarrow \angle PCB = 180^\circ - 120^\circ$$

$$\Rightarrow \angle PCB = 60^\circ$$

Now, using Tangent-Chord theorem

$$\angle PCB = \angle BPT$$

$$\Rightarrow \angle BPT = 60^\circ$$

16. Question

If the tangents PA and PB from an external point P to circle with centre O are inclined to each other at an angle of 40° then $\angle POA =$

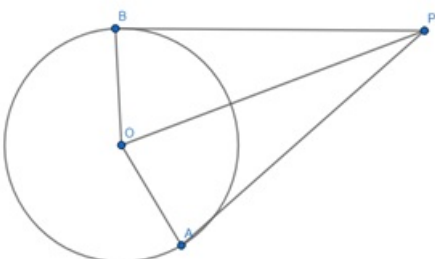
A. 70°

B. 80°

C. 50°

D. 60°

Answer



In $\triangle AOP$ and $\triangle BOP$,

$$OP = OP \text{ \{Common\}}$$

$$PB = PA \text{ \{Tangents from the same external point are equal\}}$$

$$OB = OA \text{ \{Radii\}}$$

$$\Rightarrow \triangle AOP \cong \triangle BOP$$

$$\Rightarrow \angle BPO = \angle APO \text{ \{Corresponding parts of congruent triangles\}}$$

$$\text{Given: } \angle BPA = 40^\circ$$

$$\Rightarrow \angle APO = 20^\circ$$

Also, $\angle OAP = 90^\circ$ {a tangent at any point on a circle is perpendicular to the radius through the point of contact}

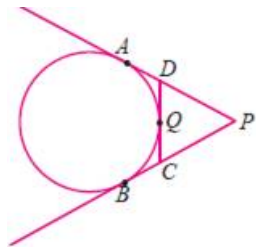
So, By angle sum property

$$\angle POA = 180^\circ - 90^\circ - 20^\circ$$

$$\Rightarrow \angle POA = 70^\circ$$

17. Question

In the figure, PA and PB are tangents to the circle drawn from an external point P. Also CD is a tangent to the circle at Q. If PA = 8 cm and CQ = 3 cm, then PC is equal to



A. 11 cm

B. 5 cm

C. 24 cm

D. 38 cm

Answer

Given: PA = 8 cm and CQ = 3 cm

We know that the lengths of the two tangents from an exterior point to a circle are equal.

$$DA = DQ, PA = PB \text{ and } CQ = BC$$

$$PB = 8 \text{ cm } BC = 3 \text{ cm}$$

$$PC = PB - BC = 8 - 3 = 5 \text{ cm}$$

18. Question

$\triangle ABC$ is a right angled triangle where $\angle B = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm, $AD = 4$ cm, then CD is

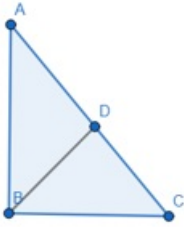
A. 24 cm

B. 16 cm

C. 32 cm

D. 8 cm

Answer



In $\triangle ABC$ and $\triangle ADB$,

$$\angle A = \angle A \text{ \{Common\}}$$

$$\angle B = \angle D \text{ \{90}^\circ \text{ each\}}$$

By the criterion of similar triangle AA which says that

if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

$$\triangle ABC \sim \triangle ADB \dots(1)$$

In $\triangle ABC$ and $\triangle BDC$,

$$\angle C = \angle C \text{ \{Common\}}$$

$$\angle B = \angle D \text{ \{90}^\circ \text{ each\}}$$

By the criterion of similar triangle AA which says that

if two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar

$$\triangle ABC \sim \triangle BDC \dots(2)$$

From (1) and (2),

$$\triangle BDC \sim \triangle ADB$$

$$\Rightarrow \frac{AD}{BD} = \frac{DB}{CD} \text{ \{Corresponding sides are in same ratio\}}$$

Putting the values,

$$\Rightarrow \frac{4}{8} = \frac{8}{CD}$$

$$\Rightarrow CD = 16 \text{ cm}$$

19. Question

The areas of two similar triangles are 16 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 3 cm, then the corresponding altitude of the other triangle is

- A. 6.5 cm
- B. 6 cm
- C. 4 cm
- D. 4.5 cm

Answer

Given: $\text{ar}(\triangle ABC) = 16 \text{ cm}^2$, $\text{ar}(\triangle DEF) = 36 \text{ cm}^2$ and altitude of first triangle = 3 cm

Areas of two similar triangles are in the ratio $16:36 = 4:9$

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides or altitudes

$$\text{Ratio of their corresponding altitudes} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Putting the values

$$\Rightarrow \frac{3}{\text{altitude of second triangle}} = \frac{2}{3}$$

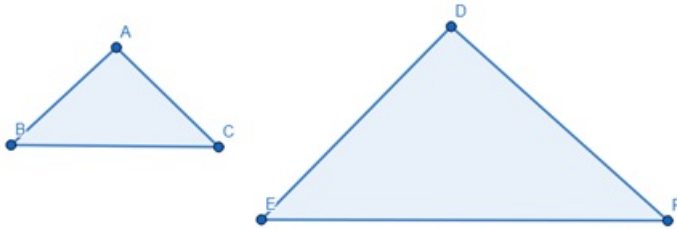
$$\Rightarrow \text{altitude of second triangle} = \frac{9}{2} = 4.5 \text{ cm}$$

20. Question

The perimeter of two similar triangles $\triangle ABC$ and $\triangle DEF$ are 36 cm and 24 cm respectively. If $DE = 10$ cm, then AB is

- A. 12 cm
- B. 20 cm
- C. 15 cm
- D. 18 cm

Answer



Given: Perimeter $(ABC) = 36 \text{ cm}^2$, Perimeter $(DEF) = 24 \text{ cm}^2$ and $DE = 10 \text{ cm}$

Perimeters of two similar triangles are in the ratio $36:24 = 3:2$

We know that if two triangles are similar, then the ratio of the corresponding sides is equal to the ratio of the corresponding perimeters.

$$\text{Ratio of their corresponding sides} \frac{AB}{DE} = \frac{3}{2}$$

Putting the values

$$\Rightarrow \frac{AB}{10} = \frac{3}{2}$$

$$\Rightarrow AB = 5 \times 3 = 15 \text{ cm}$$