15. Linear Inequations

Exercise 15.1

1. Question

Solve the following linear inequations in R

12x < 50, when

i. x ∈ R

ii. x ∈ Z

iii. x ∈ N

Answer

Given 12x < 50

$\Rightarrow \frac{12x}{12} < \frac{50}{12}$

$\therefore x < \frac{25}{6}$

i. x ∈ R

When x is a real number, the solution of the given inequation is $\left(-\infty, \frac{25}{6}\right)$.

ii. x ∈ Z

As $4 < \frac{25}{6} < 5$, when x is an integer, the maximum possible value of x is 4.

Thus, the solution of the given inequation is {…, –2, –1, 0, 1, 2, 3, 4}.

iii. x ∈ N

As $4 < \frac{25}{6} < 5$, when x is a natural number, the maximum possible value of x is 4 and we know the natural numbers start from 1.

Thus, the solution of the given inequation is {1, 2, 3, 4}.

2. Question

Solve the following linear inequations in R

–4x > 30, when

i. x ∈ R

ii. x ∈ Z

iii. x ∈ N

Answer

Given –4x > 30

$\Rightarrow -\frac{4x}{4} > \frac{30}{4}$

$\Rightarrow -x > \frac{15}{2}$

$\therefore x < -\frac{15}{2}$

i. x ∈ R

When x is a real number, the solution of the given inequation is $\left(-\infty, -\frac{15}{2}\right)$.

ii. x ∈ Z

As $-8 < -\frac{15}{2} < -7$, when x is an integer, the maximum possible value of x is –8.

Thus, the solution of the given inequation is {…, –11, –10, –9, –8}.

iii. x ∈ N

As natural numbers start from 1 and can never be negative, when x is a natural number, the solution of the given inequation is ∅.

3. Question

Solve the following linear inequations in R

4x – 2 < 8, when

i. x ∈ R

ii. x ∈ Z

iii. x ∈ N

Answer

Given 4x – 2 < 8

⇒ 4x – 2 + 2 < 8 + 2

⇒ 4x < 10

$\Rightarrow \frac{4x}{4} < \frac{10}{4}$

$\therefore x < \frac{5}{2}$

i. x ∈ R

When x is a real number, the solution of the given inequation is $\left(-\infty, \frac{5}{2}\right)$.

ii. x ∈ Z

As $2 < \frac{5}{2} < 3$, when x is an integer, the maximum possible value of x is 2.

Thus, the solution of the given inequation is {…, –2, –1, 0, 1, 2}.

iii. x ∈ N

As $2 < \frac{5}{2} < 3$, when x is a natural number, the maximum possible value of x is 2 and we know the natural numbers start from 1.

Thus, the solution of the given inequation is {1, 2}.

4. Question

Solve the following linear inequations in R

3x – 7 > x + 1

Answer

Given 3x – 7 > x + 1

⇒ 3x – 7 + 7 > x + 1 + 7

⇒ 3x > x + 8

$\Rightarrow 3x - x > x + 8 - x$

$\Rightarrow 2x > 8$

$\Rightarrow \frac{2x}{2} > \frac{8}{2}$

$\therefore x > 4$

Thus, the solution of the given inequation is $(4, \infty)$.

5. Question

Solve the following linear inequations in R

$x + 5 > 4x - 10$

Answer

Given $x + 5 > 4x - 10$

$\Rightarrow x + 5 - 5 > 4x - 10 - 5$

$\Rightarrow x > 4x - 15$

$\Rightarrow 4x - 15 < x$

$\Rightarrow 4x - 15 - x < x - x$

$\Rightarrow 3x - 15 < 0$

$\Rightarrow 3x - 15 + 15 < 0 + 15$

$\Rightarrow 3x < 15$

$\Rightarrow \frac{3x}{3} < \frac{15}{3}$

$\therefore x < 5$

Thus, the solution of the given inequation is $(-\infty, 5)$.

6. Question

Solve the following linear inequations in R

$3x + 9 \geq -x + 19$

Answer

Given $3x + 9 \geq -x + 19$

$\Rightarrow 3x + 9 - 9 \geq -x + 19 - 9$

$\Rightarrow 3x \geq -x + 10$

$\Rightarrow 3x + x \geq -x + 10 + x$

$\Rightarrow 4x \geq 10$

$\Rightarrow \frac{4x}{4} \geq \frac{10}{4}$

$\therefore x \geq \frac{5}{2}$

Thus, the solution of the given inequation is $\left[\frac{5}{2}, \infty\right)$.

7. Question

Solve the following linear inequations in R

$$2(3-x) \geq \frac{x}{5}+4$$

Answer

Given $2(3-x) \geq \frac{x}{5}+4$

$\Rightarrow 6-2x \geq \frac{x}{5}+4$

$\Rightarrow 6-2x \geq \frac{x+20}{5}$

$\Rightarrow (6-2x) \times 5 \geq \left(\frac{x+20}{5}\right) \times 5$

⇒ 30 – 10x ≥ x + 20

⇒ x + 20 ≤ 30 – 10x

⇒ x + 20 – 20 ≤ 30 – 10x – 20

⇒ x ≤ 10 – 10x

⇒ x + 10x ≤ 10 – 10x + 10x

⇒ 11x ≤ 10

$\Rightarrow \frac{11x}{11} \leq \frac{10}{11}$

$\therefore x \leq \frac{10}{11}$

Thus, the solution of the given inequation is $\left(-\infty, \frac{10}{11}\right]$.

8. Question

Solve the following linear inequations in R

$$\frac{3x-2}{5} \leq \frac{4x-3}{2}$$

Answer

Given $\frac{3x-2}{5} \leq \frac{4x-3}{2}$

$\Rightarrow \left(\frac{3x-2}{5}\right) \times 5 \leq \left(\frac{4x-3}{2}\right) \times 5$

$\Rightarrow 3x-2 \leq \frac{5(4x-3)}{2}$

$\Rightarrow 3x-2 \leq \frac{20x-15}{2}$

$\Rightarrow (3x-2) \times 2 \leq \left(\frac{20x-15}{2}\right) \times 2$

⇒ 6x – 4 ≤ 20x – 15

⇒ 20x – 15 ≥ 6x – 4

⇒ 20x – 15 + 15 ≥ 6x – 4 + 15

⇒ 20x ≥ 6x + 11

⇒ 20x – 6x ≥ 6x + 11 – 6x

$\Rightarrow 14x \geq 11$

$\Rightarrow \frac{14x}{14} \geq \frac{11}{14}$

$\therefore x \geq \frac{11}{14}$

Thus, the solution of the given inequation is $\left[\frac{11}{14}, \infty\right)$.

9. Question

Solve the following linear inequations in R

$-(x - 3) + 4 < 5 - 2x$

Answer

Given $-(x - 3) + 4 < 5 - 2x$

$\Rightarrow -x + 3 + 4 < 5 - 2x$

$\Rightarrow -x + 7 < 5 - 2x$

$\Rightarrow -x + 7 - 7 < 5 - 2x - 7$

$\Rightarrow -x < -2x - 2$

$\Rightarrow -x + 2x < -2x - 2 + 2x$

$\therefore x < -2$

Thus, the solution of the given inequation is $(-\infty, -2)$.

10. Question

Solve the following linear inequations in R

$\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}$

Answer

Given $\frac{x}{5} < \frac{3x-2}{4} - \frac{5x-3}{5}$

$\Rightarrow \frac{x}{5} < \frac{5(3x-2)-4(5x-3)}{4 \times 5}$

$\Rightarrow \frac{x}{5} < \frac{15x-10-20x+12}{20}$

$\Rightarrow \frac{x}{5} < \frac{2-5x}{20}$

$\Rightarrow \frac{x}{5} \times 20 < \left(\frac{2-5x}{20}\right) \times 20$

$\Rightarrow 4x < 2 - 5x$

$\Rightarrow 4x + 5x < 2 - 5x + 5x$

$\Rightarrow 9x < 2$

$\Rightarrow \frac{9x}{9} < \frac{2}{9}$

$\therefore x < \frac{2}{9}$

Thus, the solution of the given inequation is $\left(-\infty, \frac{2}{9}\right]$.

11. Question

Solve the following linear inequations in R

$$\frac{2(x-1)}{5} \le \frac{3(2+x)}{7}$$

Answer

Given $\frac{2(x-1)}{5} \le \frac{3(2+x)}{7}$

$\Rightarrow \frac{2x-2}{5} \le \frac{6+2x}{7}$

$\Rightarrow \left(\frac{2x-2}{5}\right) \times 5 \le \left(\frac{6+3x}{7}\right) \times 5$

$\Rightarrow 2x-2 \le \frac{5(6+2x)}{7}$

$\Rightarrow 2x-2 \le \frac{30+15x}{7}$

$\Rightarrow (2x-2) \times 7 \le \left(\frac{30+15x}{7}\right) \times 7$

⇒ 14x – 14 ≤ 30 + 15x

⇒ 14x – 14 + 14 ≤ 30 + 15x + 14

⇒ 14x ≤ 44 + 15x

⇒ 14x – 44 ≤ 44 + 15x – 44

⇒ 14x – 44 ≤ 15x

⇒ 15x ≥ 14x – 44

⇒ 15x – 14x ≥ 14x – 44 – 14x

∴ x ≥ –44

Thus, the solution of the given inequation is [–44, ∞).

12. Question

Solve the following linear inequations in R

$$\frac{5x}{2} + \frac{3x}{4} \ge \frac{39}{4}$$

Answer

Given $\frac{5x}{2} + \frac{3x}{4} \ge \frac{39}{4}$

$\Rightarrow \frac{2(5x)+3x}{4} \ge \frac{39}{4}$

$\Rightarrow \frac{13x}{4} \ge \frac{39}{4}$

$\Rightarrow \left(\frac{13x}{4}\right) \times 4 \ge \left(\frac{39}{4}\right) \times 4$

⇒ 13x ≥ 39

$\Rightarrow \frac{13x}{13} \geq \frac{39}{13}$

∴ $x \geq 3$

Thus, the solution of the given inequation is $[3, \infty)$.

13. Question

Solve the following linear inequations in R

$$\frac{x-1}{3} + 4 < \frac{x-5}{5} - 2$$

Answer

Given $\frac{x-1}{3} + 4 < \frac{x-5}{5} - 2$

$\Rightarrow \frac{x-1}{3} + 4 - 4 < \frac{x-5}{5} - 2 - 4$

$\Rightarrow \frac{x-1}{3} < \frac{x-5}{5} - 6$

$\Rightarrow \frac{x-1}{3} < \frac{x-5-30}{5}$

$\Rightarrow \frac{x-1}{3} < \frac{x-35}{5}$

$\Rightarrow \left(\frac{x-1}{3}\right) \times 3 \times 5 < \left(\frac{x-35}{5}\right) \times 3 \times 5$

$\Rightarrow 5(x - 1) < 3(x - 35)$

$\Rightarrow 5x - 5 < 3x - 105$

$\Rightarrow 5x - 5 + 5 < 3x - 105 + 5$

$\Rightarrow 5x < 3x - 100$

$\Rightarrow 5x - 3x < 3x - 100 - 3x$

$\Rightarrow 2x < -100$

$\Rightarrow \frac{2x}{2} < -\frac{100}{2}$

∴ $x < -50$

Thus, the solution of the given inequation is $(-\infty, -50)$.

14. Question

Solve the following linear inequations in R

$$\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$$

Answer

Given $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$

$\Rightarrow \frac{2x+3}{4} - 3 + 3 < \frac{x-4}{3} - 2 + 3$

$\Rightarrow \frac{2x+3}{4} < \frac{x-4}{3} + 1$

$\Rightarrow \frac{2x+3}{4} < \frac{x-4+3}{3}$

$\Rightarrow \frac{2x+3}{4} < \frac{x-1}{3}$

$\Rightarrow \left(\frac{2x+3}{4}\right) \times 3 \times 4 < \left(\frac{x-1}{3}\right) \times 3 \times 4$

⇒ 3(2x + 3) < 4(x – 1)

⇒ 6x + 9 < 4x – 4

⇒ 6x + 9 – 9 < 4x – 4 – 9

⇒ 6x < 4x – 13

⇒ 6x – 4x < 4x – 13 – 4x

⇒ 2x < –13

$\Rightarrow \frac{2x}{2} < -\frac{13}{2}$

$\therefore x < -\frac{13}{2}$

Thus, the solution of the given inequation is $\left(-\infty, -\frac{13}{2}\right)$.

15. Question

Solve the following linear inequations in R

$$\frac{5-2x}{3} < \frac{x}{6} - 5$$

Answer

Given $\frac{5-2x}{3} < \frac{x}{6} - 5$

$\Rightarrow \frac{5-2x}{3} < \frac{x-30}{6}$

$\Rightarrow \left(\frac{5-2x}{3}\right) \times 6 < \left(\frac{x-30}{6}\right) \times 6$

⇒ 2(5 – 2x) < x – 30

⇒ 10 – 4x < x – 30

⇒ 10 – 4x – 10 < x – 30 – 10

⇒ –4x < x – 40

⇒ x – 40 > –4x

⇒ x – 40 + 40 > –4x + 40

⇒ x > –4x + 40

⇒ x + 4x > –4x + 40 + 4x

⇒ 5x > 40

$\Rightarrow \frac{5x}{5} > \frac{40}{5}$

∴ x > 8

Thus, the solution of the given inequation is (8, ∞).

16. Question

Solve the following linear inequations in R

$$\frac{4+2x}{3} \geq \frac{x}{2} - 3$$

Answer

Given $\frac{4+2x}{3} \geq \frac{x}{2} - 3$

$\Rightarrow \frac{4+2x}{3} \geq \frac{x-6}{2}$

$\Rightarrow \left(\frac{4+2x}{3}\right) \times 3 \times 2 \geq \left(\frac{x-6}{2}\right) \times 3 \times 2$

⇒ 2(4 + 2x) ≥ 3(x – 6)

⇒ 8 + 4x ≥ 3x – 18

⇒ 8 + 4x – 8 ≥ 3x – 18 – 8

⇒ 4x ≥ 3x – 26

⇒ 4x – 3x ≥ 3x – 26 – 3x

∴ x ≥ –26

Thus, the solution of the given inequation is [–26, ∞).

17. Question

Solve the following linear inequations in R

$$\frac{2x+3}{5} - 2 < \frac{3(x-2)}{5}$$

Answer

Given $\frac{2x+3}{5} - 2 < \frac{3(x-2)}{5}$

$\Rightarrow \frac{2x+3-10}{5} < \frac{3x-6}{5}$

$\Rightarrow \frac{2x-7}{5} < \frac{3x-6}{5}$

$\Rightarrow \left(\frac{2x-7}{5}\right) \times 5 < \left(\frac{3x-6}{5}\right) \times 5$

⇒ 2x – 7 < 3x – 6

⇒ 2x – 7 + 7 < 3x – 6 + 7

⇒ 2x < 3x + 1

⇒ 3x + 1 > 2x

⇒ 3x + 1 – 1 > 2x – 1

⇒ 3x > 2x – 1

⇒ 3x – 2x > 2x – 1 – 2x

∴ x > –1

Thus, the solution of the given inequation is (–1, ∞).

18. Question

Solve the following linear inequations in R

$$x - 2 \leq \frac{5x + 8}{3}$$

Answer

Given $x - 2 \leq \frac{5x+8}{3}$

$\Rightarrow (x - 2) \times 3 \leq \left(\frac{5x + 8}{3}\right) \times 3$

⇒ 3(x – 2) ≤ 5x + 8

⇒ 3x – 6 ≤ 5x + 8

⇒ 3x – 6 + 6 ≤ 5x + 8 + 6

⇒ 3x ≤ 5x + 14

⇒ 5x + 14 ≥ 3x

⇒ 5x + 14 – 14 ≥ 3x – 14

⇒ 5x ≥ 3x – 14

⇒ 5x – 3x ≥ 3x – 14 – 3x

⇒ 2x ≥ –14

$\Rightarrow \frac{2x}{2} \geq -\frac{14}{2}$

∴ x ≥ –7

Thus, the solution of the given inequation is [–7, ∞).

19. Question

Solve the following linear inequations in R

$$\frac{6x - 5}{4x + 1} < 0$$

Answer

Given $\frac{6x-5}{4x+1} < 0$

For this inequation to be true, there are two possible cases.

i. 6x – 5 > 0 and 4x + 1 < 0

⇒ 6x – 5 + 5 > 0 + 5 and 4x + 1 – 1 < 0 – 1

⇒ 6x > 5 and 4x < –1

$\Rightarrow \frac{6x}{6} > \frac{5}{6} \text{ and } \frac{4x}{4} < -\frac{1}{4}$

$\Rightarrow x > \frac{5}{6} \text{ and } x < -\frac{1}{4}$

$\therefore x \in \left(\frac{5}{6}, \infty\right) \cap \left(-\infty, -\frac{1}{4}\right)$

However, $\left(\frac{5}{6}, \infty\right) \cap \left(-\infty, -\frac{1}{4}\right) = \emptyset$

Hence, this case is not possible.

ii. 6x – 5 < 0 and 4x + 1 > 0

⇒ 6x – 5 + 5 < 0 + 5 and 4x + 1 – 1 > 0 – 1

⇒ 6x < 5 and 4x > –1

$\Rightarrow \frac{6x}{6} < \frac{5}{6}$ and $\frac{4x}{4} > -\frac{1}{4}$

$\Rightarrow x < \frac{5}{6}$ and $x > -\frac{1}{4}$

$\therefore x \in \left(-\infty, \frac{5}{6}\right) \cap \left(-\frac{1}{4}, \infty\right)$

However, $\left(-\infty, \frac{5}{6}\right) \cap \left(-\frac{1}{4}, \infty\right) = \left(-\frac{1}{4}, \frac{5}{6}\right)$

Thus, the solution of the given inequation is $\left(-\frac{1}{4}, \frac{5}{6}\right)$.

20. Question

Solve the following linear inequations in R

$$\frac{2x-3}{3x-7} > 0$$

Answer

Given $\frac{2x-3}{3x-7} > 0$

For this inequation to be true, there are two possible cases.

i. 2x – 3 > 0 and 3x – 7 > 0

⇒ 2x – 3 + 3 > 0 + 3 and 3x – 7 + 7 > 0 + 7

⇒ 2x > 3 and 3x > 7

$\Rightarrow \frac{2x}{2} > \frac{3}{2}$ and $\frac{3x}{3} > \frac{7}{3}$

$\Rightarrow x > \frac{3}{2}$ and $x > \frac{7}{3}$

$\therefore x \in \left(\frac{3}{2}, \infty\right) \cap \left(\frac{7}{3}, \infty\right)$

However, $\left(\frac{3}{2}, \infty\right) \cap \left(\frac{7}{3}, \infty\right) = \left(\frac{7}{3}, \infty\right)$

Hence, $x \in \left(\frac{7}{3}, \infty\right)$

ii. 2x – 3 < 0 and 3x – 7 < 0

⇒ 2x – 3 + 3 < 0 + 3 and 3x – 7 + 7 < 0 + 7

⇒ 2x < 3 and 3x < 7

$\Rightarrow \frac{2x}{2} < \frac{3}{2}$ and $\frac{3x}{3} < \frac{7}{3}$

$\Rightarrow x < \frac{3}{2}$ and $x < \frac{7}{3}$

$\therefore x \in \left(-\infty, \frac{3}{2}\right) \cap \left(-\infty, \frac{7}{3}\right)$

However, $\left(-\infty, \frac{3}{2}\right) \cap \left(-\infty, \frac{7}{3}\right) = \left(-\infty, \frac{3}{2}\right)$

Hence, $x \in \left(-\infty, \frac{3}{2}\right)$

Thus, the solution of the given inequation is $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{7}{3}, \infty\right)$.

21. Question

Solve the following linear inequations in R

$$\frac{3}{x-2} < 1$$

Answer

Given $\frac{3}{x-2} < 1$

$\Rightarrow \frac{3}{x-2} - 1 < 1 - 1$

$\Rightarrow \frac{3}{x-2} - 1 < 0$

$\Rightarrow \frac{3-(x-2)}{x-2} < 0$

$\Rightarrow \frac{3-x+2}{x-2} < 0$

$\Rightarrow \frac{5-x}{x-2} < 0$

$\Rightarrow \frac{x-5}{x-2} > 0$

For this inequation to be true, there are two possible cases.

i. $x - 5 > 0$ and $x - 2 > 0$

$\Rightarrow x - 5 + 5 > 0 + 5$ and $x - 2 + 2 > 0 + 2$

$\Rightarrow x > 5$ and $x > 2$

$\therefore x \in (5, \infty) \cap (2, \infty)$

However, $(5, \infty) \cap (2, \infty) = (5, \infty)$

Hence, $x \in (5, \infty)$

ii. $x - 5 < 0$ and $x - 2 < 0$

$\Rightarrow x - 5 + 5 < 0 + 5$ and $x - 2 + 2 < 0 + 2$

$\Rightarrow x < 5$ and $x < 2$

$\therefore x \in (-\infty, 5) \cap (-\infty, 2)$

However, $(-\infty, 5) \cap (-\infty, 2) = (-\infty, 2)$

Hence, $x \in (-\infty, 2)$

Thus, the solution of the given inequation is $(-\infty, 2) \cup (5, \infty)$.

22. Question

Solve the following linear inequations in R

$$\frac{1}{x-1} \leq 2$$

Answer

Given $\frac{1}{x-1} \leq 2$

$$\Rightarrow \frac{1}{x-1} - 2 \leq 2 - 2$$

$$\Rightarrow \frac{1}{x-1} - 2 \leq 0$$

$$\Rightarrow \frac{1-2(x-1)}{x-1} \leq 0$$

$$\Rightarrow \frac{1-2x+2}{x-1} \leq 0$$

$$\Rightarrow \frac{3-2x}{x-1} \leq 0$$

$$\Rightarrow \frac{2x-3}{x-1} \geq 0$$

For this inequation to be true, there are two possible cases.

i. $2x - 3 \geq 0$ and $x - 1 > 0$

$\Rightarrow 2x - 3 + 3 \geq 0 + 3$ and $x - 1 + 1 > 0 + 1$

$\Rightarrow 2x \geq 3$ and $x > 1$

$$\Rightarrow x \geq \frac{3}{2} \text{ and } x > 1$$

$$\therefore x \in \left[\frac{3}{2}, \infty\right) \cap (1, \infty)$$

However, $\left[\frac{3}{2}, \infty\right) \cap (1, \infty) = \left[\frac{3}{2}, \infty\right)$

Hence, $x \in \left[\frac{3}{2}, \infty\right)$

ii. $2x - 3 \leq 0$ and $x - 1 < 0$

$\Rightarrow 2x - 3 + 3 \leq 0 + 3$ and $x - 1 + 1 < 0 + 1$

$\Rightarrow 2x \leq 3$ and $x < 1$

$$\Rightarrow x \leq \frac{3}{2} \text{ and } x < 1$$

$$\therefore x \in \left(-\infty, \frac{3}{2}\right] \cap (-\infty, 1)$$

However, $\left(-\infty, \frac{3}{2}\right] \cap (-\infty, 1) = (-\infty, 1)$

Hence, $x \in (-\infty, 1)$

Thus, the solution of the given inequation is $(-\infty, 1) \cup \left[\frac{3}{2}, \infty\right)$.

23. Question

Solve the following linear inequations in R

$$\frac{4x+3}{2x-5} < 6$$

Answer

Given $\frac{4x+3}{2x-5} < 6$

$\Rightarrow \frac{4x+3}{2x-5} - 6 < 6 - 6$

$\Rightarrow \frac{4x+3}{2x-5} - 6 < 0$

$\Rightarrow \frac{4x+3-6(2x-5)}{2x-5} < 0$

$\Rightarrow \frac{4x+3-12x+30}{2x-5} < 0$

$\Rightarrow \frac{-8x+33}{2x-5} < 0$

$\Rightarrow \frac{8x-33}{2x-5} > 0$

For this inequation to be true, there are two possible cases.

i. $8x - 33 > 0$ and $2x - 5 > 0$

$\Rightarrow 8x - 33 + 33 > 0 + 33$ and $2x - 5 + 5 > 0 + 5$

$\Rightarrow 8x > 33$ and $2x > 5$

$\Rightarrow x > \frac{33}{8}$ and $x > \frac{5}{2}$

$\therefore x \in \left(\frac{33}{8}, \infty\right) \cap \left(\frac{5}{2}, \infty\right)$

However, $\left(\frac{33}{8}, \infty\right) \cap \left(\frac{5}{2}, \infty\right) = \left(\frac{33}{8}, \infty\right)$

Hence, $x \in \left(\frac{33}{8}, \infty\right)$

ii. $8x - 33 < 0$ and $2x - 5 < 0$

$\Rightarrow 8x - 33 + 33 < 0 + 33$ and $2x - 5 + 5 < 0 + 5$

$\Rightarrow 8x < 33$ and $2x < 5$

$\Rightarrow x < \frac{33}{8}$ and $x < \frac{5}{2}$

$\therefore x \in \left(-\infty, \frac{33}{8}\right) \cap \left(-\infty, \frac{5}{2}\right)$

However, $\left(-\infty, \frac{33}{8}\right) \cap \left(-\infty, \frac{5}{2}\right) = \left(-\infty, \frac{5}{2}\right)$

Hence, $x \in \left(-\infty, \frac{5}{2}\right)$

Thus, the solution of the given inequation is $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{33}{8}, \infty\right)$.

24. Question

Solve the following linear inequations in R

$$\frac{5x-6}{x+6}<1$$

Answer

Given $\frac{5x-6}{x+6}<1$

$$\Rightarrow \frac{5x-6}{x+6}-1<1-1$$

$$\Rightarrow \frac{5x-6}{x+6}-1<0$$

$$\Rightarrow \frac{5x-6-(x+6)}{x+6}<0$$

$$\Rightarrow \frac{5x-6-x-6}{x+6}<0$$

$$\Rightarrow \frac{4x-12}{x+6}<0$$

$$\Rightarrow \frac{4(x-3)}{x+6}<0$$

$$\Rightarrow \frac{x-3}{x+6}<0$$

For this inequation to be true, there are two possible cases.

i. $x - 3 > 0$ and $x + 6 < 0$

$\Rightarrow x - 3 + 3 > 0 + 3$ and $x + 6 - 6 < 0 - 6$

$\Rightarrow x > 3$ and $x < -6$

$\therefore x \in (3, \infty) \cap (-\infty, -6)$

However, $(3, \infty) \cap (-\infty, -6) = \varnothing$

Hence, this case is not possible.

ii. $x - 3 < 0$ and $x + 6 > 0$

$\Rightarrow x - 3 + 3 < 0 + 3$ and $x + 6 - 6 > 0 - 6$

$\Rightarrow x < 3$ and $x > -6$

$\therefore x \in (-\infty, 3) \cap (-6, \infty)$

However, $(-\infty, 3) \cap (-6, \infty) = (-6, 3)$

Hence, $x \in (-6, 3)$

Thus, the solution of the given inequation is $(-6, 3)$.

25. Question

Solve the following linear inequations in R

$$\frac{5x+8}{4-x}<2$$

Answer

Given $\frac{5x+8}{4-x}<2$

$$\Rightarrow \frac{5x+8}{4-x}-2<2-2$$

$\Rightarrow \frac{5x+8}{4-x} - 2 < 0$

$\Rightarrow \frac{5x+8-2(4-x)}{4-x} < 0$

$\Rightarrow \frac{5x+8-8+2x}{4-x} < 0$

$\Rightarrow \frac{7x}{4-x} < 0$

$\Rightarrow \frac{x}{4-x} < 0$

For this inequation to be true, there are two possible cases.

i. x > 0 and 4 – x < 0

⇒ x > 0 and 4 – x – 4 < 0 – 4

⇒ x > 0 and –x < –4

⇒ x > 0 and x > 4

∴ x ∈ (0, ∞) ∩ (4, ∞)

However, (0, ∞) ∩ (4, ∞) = (4, ∞)

Hence, x ∈ (4, ∞)

ii. x < 0 and 4 – x > 0

⇒ x < 0 and 4 – x – 4 > 0 – 4

⇒ x < 0 and –x > –4

⇒ x < 0 and x < 4

∴ x ∈ (–∞, 0) ∩ (–∞, 4)

However, (–∞, 0) ∩ (–∞, 4) = (–∞, 0)

Hence, x ∈ (–∞, 0)

Thus, the solution of the given inequation is (–∞, 0) ∪ (4, ∞).

26. Question

Solve the following linear inequations in R

$$\frac{x-1}{x+3} > 2$$

Answer

Given $\frac{x-1}{x+3} > 2$

$\Rightarrow \frac{x-1}{x+3} - 2 > 2 - 2$

$\Rightarrow \frac{x-1}{x+3} - 2 > 0$

$\Rightarrow \frac{x-1-2(x+3)}{x+3} > 0$

$\Rightarrow \frac{x-1-2x-6}{x+3} > 0$

$\Rightarrow \frac{-x-7}{x+3} > 0$

$\Rightarrow \frac{-(x+7)}{x+3} > 0$

$\Rightarrow \frac{x+7}{x+3} < 0$

For this inequation to be true, there are two possible cases.

i. x + 7 > 0 and x + 3 < 0

⇒ x + 7 – 7 > 0 – 7 and x + 3 – 3 < 0 – 3

⇒ x > –7 and x < –3

∴ x ∈ (–7, ∞) ∩ (–∞, –3)

However, (–7, ∞) ∩ (–∞, –3) = (–7, –3)

Hence, x ∈ (–7, –3)

ii. x + 7 < 0 and x + 3 > 0

⇒ x + 7 – 7 < 0 – 7 and x + 3 – 3 > 0 – 3

⇒ x < –7 and x > –3

∴ x ∈ (–∞, –7) ∩ (–3, ∞)

However, (–∞, –7) ∩ (–3, ∞) = ∅

Hence, this case is not possible.

Thus, the solution of the given inequation is (–7, –3).

27. Question

Solve the following linear inequations in R

$$\frac{7x-5}{8x+3} > 4$$

Answer

Given $\frac{7x-5}{8x+3} > 4$

$\Rightarrow \frac{7x-5}{8x+3} - 4 > 4 - 4$

$\Rightarrow \frac{7x-5}{8x+3} - 4 > 0$

$\Rightarrow \frac{7x-5-4(8x+3)}{8x+3} > 0$

$\Rightarrow \frac{7x-5-32x-12}{8x+3} > 0$

$\Rightarrow \frac{-25x-17}{8x+3} > 0$

$\Rightarrow \frac{-(25x+17)}{8x+3} > 0$

$\Rightarrow \frac{25x+17}{8x+3} < 0$

For this inequation to be true, there are two possible cases.

i. 25x + 17 > 0 and 8x + 3 < 0

⇒ 25x + 17 – 17 > 0 – 17 and 8x + 3 – 3 < 0 – 3

⇒ 25x > –17 and 8x < –3

$\Rightarrow x > -\frac{17}{25}$ and $x < -\frac{3}{8}$

$\therefore x \in \left(-\frac{17}{25}, \infty\right) \cap \left(-\infty, -\frac{3}{8}\right)$

However, $\left(-\frac{17}{25}, \infty\right) \cap \left(-\infty, -\frac{3}{8}\right) = \left(-\frac{17}{25}, -\frac{3}{8}\right)$

Hence, $x \in \left(-\frac{17}{25}, -\frac{3}{8}\right)$

ii. 25x + 17 < 0 and 8x + 3 > 0

⇒ 25x + 17 – 17 < 0 – 17 and 8x + 3 – 3 > 0 – 3

⇒ 25x < –17 and 8x > –3

$\Rightarrow x < -\frac{17}{25}$ and $x > -\frac{3}{8}$

$\therefore x \in \left(-\infty, -\frac{17}{25}\right) \cap \left(-\frac{3}{8}, \infty\right)$

However, $\left(-\infty, -\frac{17}{25}\right) \cap \left(-\frac{3}{8}, \infty\right) = \emptyset$

Hence, this case is not possible.

Thus, the solution of the given inequation is $\left(-\frac{17}{25}, -\frac{3}{8}\right)$.

28. Question

Solve the following linear inequations in R

$$\frac{x}{x-5} > \frac{1}{2}$$

Answer

Given $\frac{x}{x-5} > \frac{1}{2}$

$\Rightarrow \left(\frac{x}{x-5}\right) \times 2 > \frac{1}{2} \times 2$

$\Rightarrow \frac{2x}{x-5} > 1$

$\Rightarrow \frac{2x}{x-5} - 1 > 1 - 1$

$\Rightarrow \frac{2x}{x-5} - 1 > 0$

$\Rightarrow \frac{2x-(x-5)}{x-5} > 0$

$\Rightarrow \frac{x+5}{x-5} > 0$

For this inequation to be true, there are two possible cases.

i. x + 5 > 0 and x – 5 > 0

⇒ x + 5 – 5 > 0 – 5 and x – 5 + 5 > 0 + 5

⇒ x > –5 and x > 5

∴ x ∈ (–5, ∞) ∩ (5, ∞)

However, (–5, ∞) ∩ (5, ∞) = (5, ∞)

Hence, x ∈ (5, ∞)

ii. x + 5 < 0 and x – 5 < 0

⇒ x + 5 – 5 < 0 – 5 and x – 5 + 5 < 0 + 5

⇒ x < –5 and x < 5

∴ x ∈ (–∞, –5) ∩ (–∞, 5)

However, (–∞, –5) ∩ (–∞, 5) = (–∞, –5)

Hence, x ∈ (–∞, –5)

Thus, the solution of the given inequation is (–∞, –5) ∪ (5, ∞).

Exercise 15.2

1. Question

Solve each of the following system of inequations in R

x + 3 > 0, 2x < 14

Answer

Given x + 3 < 0 and 2x < 14

Let us consider the first inequality.

x + 3 < 0

⇒ x + 3 – 3 < 0 – 3

⇒ x < –3

∴ x ∈ (–∞, –3) (1)

Now, let us consider the second inequality.

2x < 14

$\Rightarrow \frac{2x}{2} < \frac{14}{2}$

⇒ x < 7

∴ x ∈ (–∞, 7) (2)

From (1) and (2), we get

x ∈ (–∞, –3) ∩ (–∞, 7)

∴ x ∈ (–∞, –3)

Thus, the solution of the given system of inequations is (–∞, –3).

2. Question

Solve each of the following system of inequations in R

2x – 7 > 5 – x, 11 – 5x ≤ 1

Answer

Given 2x – 7 > 5 – x and 11 – 5x ≤ 1

Let us consider the first inequality.

2x – 7 > 5 – x

⇒ 2x – 7 + 7 > 5 – x + 7

⇒ 2x > 12 – x

⇒ 2x + x > 12 – x + x

⇒ 3x > 12

$\Rightarrow \frac{3x}{3} > \frac{12}{3}$

⇒ x > 4

∴ x ∈ (4, ∞) (1)

Now, let us consider the second inequality.

11 – 5x ≤ 1

⇒ 11 – 5x – 11 ≤ 1 – 11

⇒ –5x ≤ –10

$\Rightarrow \frac{-5x}{5} \leq \frac{-10}{5}$

⇒ –x ≤ –2

⇒ x ≥ 2

∴ x ∈ (2, ∞) (2)

From (1) and (2), we get

x ∈ (4, ∞) ∩ (2, ∞)

∴ x ∈ (4, ∞)

Thus, the solution of the given system of inequations is (4, ∞).

3. Question

Solve each of the following system of inequations in R

x – 2 > 0, 3x < 18

Answer

Given x – 2 > 0 and 3x < 18

Let us consider the first inequality.

x – 2 < 0

⇒ x – 2 + 2 < 0 + 2

⇒ x < 2

∴ x ∈ (2, ∞) (1)

Now, let us consider the second inequality.

3x < 18

$\Rightarrow \frac{3x}{3} < \frac{18}{3}$

⇒ x < 6

$\therefore x \in (-\infty, 6)$ (2)

From (1) and (2), we get

$x \in (2, \infty) \cap (-\infty, 6)$

$\therefore x \in (2, 6)$

Thus, the solution of the given system of inequations is (2, 6).

4. Question

Solve each of the following system of inequations in R

$2x + 6 \geq 0$, $4x - 7 < 0$

Answer

Given $2x + 6 \geq 0$ and $4x - 7 < 0$

Let us consider the first inequality.

$2x + 6 \geq 0$

$\Rightarrow 2x + 6 - 6 \geq 0 - 6$

$\Rightarrow 2x \geq -6$

$\Rightarrow \frac{2x}{2} \geq \frac{-6}{2}$

$\Rightarrow x \geq -3$

$\therefore x \in [-3, \infty)$ (1)

Now, let us consider the second inequality.

$4x - 7 < 0$

$\Rightarrow 4x - 7 + 7 < 0 + 7$

$\Rightarrow 4x < 7$

$\Rightarrow \frac{4x}{4} < \frac{7}{4}$

$\Rightarrow x < \frac{7}{4}$

$\therefore x \in \left(-\infty, \frac{7}{4}\right)$ (2)

From (1) and (2), we get

$x \in [-3, \infty) \cap \left(-\infty, \frac{7}{4}\right)$

$\therefore x \in \left[-3, \frac{7}{4}\right)$

Thus, the solution of the given system of inequations is $\left[-3, \frac{7}{4}\right)$

5. Question

Solve each of the following system of inequations in R

$3x - 6 > 0$, $2x - 5 > 0$

Answer

Given $3x - 6 > 0$ and $2x - 5 > 0$

Let us consider the first inequality.

3x – 6 > 0

⇒ 3x – 6 + 6 > 0 + 6

⇒ 3x > 6

⇒ $\frac{3x}{3} > \frac{6}{3}$

⇒ x > 2

∴ x ∈ (2, ∞) (1)

Now, let us consider the second inequality.

2x – 5 > 0

⇒ 2x – 5 + 5 > 0 + 5

⇒ 2x > 5

⇒ $\frac{2x}{2} > \frac{5}{2}$

⇒ $x > \frac{5}{2}$

∴ $x \in \left(\frac{5}{2}, \infty\right)$ (2)

From (1) and (2), we get

$x \in (2, \infty) \cap \left(\frac{5}{2}, \infty\right)$

∴ $x \in \left(\frac{5}{2}, \infty\right)$

Thus, the solution of the given system of inequations is $\left(\frac{5}{2}, \infty\right)$

6. Question

Solve each of the following system of inequations in R

2x – 3 < 7, 2x > –4

Answer

Given 2x – 3 < 7 and 2x > –4

Let us consider the first inequality.

2x – 3 < 7

⇒ 2x – 3 + 3 < 7 + 3

⇒ 2x < 10

⇒ $\frac{2x}{2} < \frac{10}{2}$

⇒ x < 5

∴ x ∈ (–∞, 5) (1)

Now, let us consider the second inequality.

2x > –4

⇒ $\frac{2x}{2} > \frac{-4}{2}$

$\Rightarrow x > -2$

$\therefore x \in (-2, \infty)$ (2)

From (1) and (2), we get

$x \in (-\infty, 5) \cap (-2, \infty)$

$\therefore x \in (-2, 5)$

Thus, the solution of the given system of inequations is $(-2, 5)$.

7. Question

Solve each of the following system of inequations in R

$2x + 5 \leq 0$, $x - 3 \leq 0$

Answer

Given $2x + 5 \leq 0$ and $x - 3 \leq 0$

Let us consider the first inequality.

$2x + 5 \leq 0$

$\Rightarrow 2x + 5 - 5 \leq 0 - 5$

$\Rightarrow 2x \leq -5$

$\Rightarrow \frac{2x}{2} \leq \frac{-5}{2}$

$\Rightarrow x \leq -\frac{5}{2}$

$\therefore x \in \left(-\infty, -\frac{5}{2}\right]$ (1)

Now, let us consider the second inequality.

$x - 3 \leq 0$

$\Rightarrow x - 3 + 3 \leq 0 + 3$

$\Rightarrow x \leq 3$

$\therefore x \in (-\infty, 3]$ (2)

From (1) and (2), we get

$x \in \left(-\infty, -\frac{5}{2}\right] \cap (-\infty, 3]$

$\therefore x \in \left(-\infty, -\frac{5}{2}\right]$

Thus, the solution of the given system of inequations is $\left(-\infty, -\frac{5}{2}\right]$

8. Question

Solve each of the following system of inequations in R

$5x - 1 < 24$, $5x + 1 > -24$

Answer

Given $5x - 1 < 24$ and $5x + 1 > -24$

Let us consider the first inequality.

$5x - 1 < 24$

⇒ 5x – 1 + 1 < 24 + 1

⇒ 5x < 25

⇒ $\frac{5x}{5} < \frac{25}{5}$

⇒ x < 5

∴ x ∈ (–∞, 5) (1)

Now, let us consider the second inequality.

5x + 1 > –24

⇒ 5x + 1 – 1 > –24 – 1

⇒ 5x > –25

⇒ $\frac{5x}{5} > \frac{-25}{5}$

⇒ x > –5

∴ x ∈ (–5, ∞) (2)

From (1) and (2), we get

x ∈ (–∞, 5) ∩ (–5, ∞)

∴ x ∈ (–5, 5)

Thus, the solution of the given system of inequations is (–5, 5).

9. Question

Solve each of the following system of inequations in R

3x – 1 ≥ 5, x + 2 > –1

Answer

Given 3x – 1 ≥ 5 and x + 2 > –1

Let us consider the first inequality.

3x – 1 ≥ 5

⇒ 3x – 1 + 1 ≥ 5 + 1

⇒ 3x ≥ 6

⇒ $\frac{3x}{3} \geq \frac{6}{3}$

⇒ x ≥ 2

∴ x ∈ (2, ∞) (1)

Now, let us consider the second inequality.

x + 2 > –1

⇒ x + 2 – 2 > –1 – 2

⇒ x > –3

∴ x ∈ (–3, ∞) (2)

From (1) and (2), we get

x ∈ (2, ∞) ∩ (–3, ∞)

∴ $x \in (2, \infty)$

Thus, the solution of the given system of inequations is $(2, \infty)$.

10. Question

Solve each of the following system of inequations in R

$11 - 5x > -4, 4x + 13 \leq -11$

Answer

Given $11 - 5x > -4$ and $4x + 13 \leq -11$

Let us consider the first inequality.

$11 - 5x > -4$

$\Rightarrow 11 - 5x - 11 > -4 - 11$

$\Rightarrow -5x > -15$

$\Rightarrow \frac{-5x}{5} > \frac{-15}{5}$

$\Rightarrow -x > -3$

$\Rightarrow x < 3$

∴ $x \in (-\infty, 3)$ (1)

Now, let us consider the second inequality.

$4x + 13 \leq -11$

$\Rightarrow 4x + 13 - 13 \leq -11 - 13$

$\Rightarrow 4x \leq -24$

$\Rightarrow \frac{4x}{4} \leq \frac{-24}{4}$

$\Rightarrow x \leq -6$

∴ $x \in (-\infty, -6]$ (2)

From (1) and (2), we get

$x \in (-\infty, 3) \cap (-\infty, -6]$

∴ $x \in (-\infty, -6]$

Thus, the solution of the given system of inequations is $(-\infty, -6]$.

11. Question

Solve each of the following system of inequations in R

$4x - 1 \leq 0, 3 - 4x < 0$

Answer

Given $4x - 1 \leq 0$ and $3 - 4x < 0$

Let us consider the first inequality.

$4x - 1 \leq 0$

$\Rightarrow 4x - 1 + 1 \leq 0 + 1$

$\Rightarrow 4x \leq 1$

$\Rightarrow \frac{4x}{4} \leq \frac{1}{4}$

$\Rightarrow x \leq \frac{1}{4}$

$\therefore x \in \left(-\infty, \frac{1}{4}\right]$ (1)

Now, let us consider the second inequality.

3 – 4x < 0

⇒ 3 – 4x – 3 < 0 – 3

⇒ –4x < –3

$\Rightarrow \frac{-4x}{4} < \frac{-3}{4}$

$\Rightarrow -x < -\frac{3}{4}$

$\Rightarrow x > \frac{3}{4}$

$\therefore x \in \left(\frac{3}{4}, \infty\right)$ (2)

From (1) and (2), we get

$x \in \left(-\infty, \frac{1}{4}\right] \cap \left(\frac{3}{4}, \infty\right)$

∴ x ∈ ∅

Thus, there is no solution of the given system of inequations.

12. Question

Solve each of the following system of inequations in R

x + 5 > 2(x + 1), 2 – x < 3(x + 2)

Answer

Given x + 5 > 2(x + 1) and 2 – x < 3(x + 2)

Let us consider the first inequality.

x + 5 > 2(x + 1)

⇒ x + 5 > 2x + 2

⇒ x + 5 – 5 > 2x + 2 – 5

⇒ x > 2x – 3

⇒ 2x – 3 < x

⇒ 2x – 3 + 3 < x + 3

⇒ 2x < x + 3

⇒ 2x – x < x + 3 – x

⇒ x < 3

∴ x ∈ (–∞, 3) (1)

Now, let us consider the second inequality.

2 – x < 3(x + 2)

⇒ 2 – x < 3x + 6

⇒ 2 – x – 2 < 3x + 6 – 2

⇒ $-x < 3x + 4$

⇒ $3x + 4 > -x$

⇒ $3x + 4 - 4 > -x - 4$

⇒ $3x > -x - 4$

⇒ $3x + x > -x + x - 4$

⇒ $4x > -4$

⇒ $\frac{4x}{4} > \frac{-4}{4}$

⇒ $x > -1$

∴ $x \in (-1, \infty)$ (2)

From (1) and (2), we get

$x \in (-\infty, 3) \cap (-1, \infty)$

∴ $x \in (-1, 3)$

Thus, the solution of the given system of inequations is $(-1, 3)$.

13. Question

Solve each of the following system of inequations in R

$2(x - 6) < 3x - 7$, $11 - 2x < 6 - x$

Answer

Given $2(x - 6) < 3x - 7$ and $11 - 2x < 6 - x$

Let us consider the first inequality.

$2(x - 6) < 3x - 7$

⇒ $2x - 12 < 3x - 7$

⇒ $2x - 12 + 12 < 3x - 7 + 12$

⇒ $2x < 3x + 5$

⇒ $3x + 5 > 2x$

⇒ $3x + 5 - 5 > 2x - 5$

⇒ $3x > 2x - 5$

⇒ $3x - 2x > 2x - 5 - 2x$

⇒ $x > -5$

∴ $x \in (-5, \infty)$ (1)

Now, let us consider the second inequality.

$11 - 2x < 6 - x$

⇒ $11 - 2x - 11 < 6 - x - 11$

⇒ $-2x < -x - 5$

⇒ $-x - 5 > -2x$

⇒ $-x - 5 + 5 > -2x + 5$

⇒ $-x > -2x + 5$

⇒ $-x + 2x > -2x + 5 + 2x$

⇒ x > 5

∴ x ∈ (5, ∞) (2)

From (1) and (2), we get

x ∈ (–5, ∞) ∩ (5, ∞)

∴ x ∈ (5, ∞)

Thus, the solution of the given system of inequations is (5, ∞).

14. Question

Solve each of the following system of inequations in R

5x – 7 < 3(x + 3), $1-\frac{3x}{2} \geq x-4$

Answer

Given 5x – 7 < 3(x + 3) and $1-\frac{3x}{2} \geq x-4$

Let us consider the first inequality.

5x – 7 < 3(x + 3)

⇒ 5x – 7 < 3x + 9

⇒ 5x – 7 + 7 < 3x + 9 + 7

⇒ 5x < 3x + 16

⇒ 5x – 3x < 3x + 16 – 3x

⇒ 2x < 16

⇒ $\frac{2x}{2} < \frac{16}{2}$

⇒ x < 8

∴ x ∈ (–∞, 8) (1)

Now, let us consider the second inequality.

$1-\frac{3x}{2} \geq x-4$

⇒ $\frac{2-3x}{2} \geq x-4$

⇒ $\left(\frac{2-3x}{2}\right) \times 2 \geq (x-4) \times 2$

⇒ 2 – 3x ≥ 2(x – 4)

⇒ 2 – 3x ≥ 2x – 8

⇒ 2 – 3x – 2 ≥ 2x – 8 – 2

⇒ –3x ≥ 2x – 10

⇒ 2x – 10 ≤ –3x

⇒ 2x – 10 + 10 ≤ –3x + 10

⇒ 2x ≤ –3x + 10

⇒ 2x + 3x ≤ –6x + 10 + 6x

⇒ 5x ≤ 10

$\Rightarrow \frac{5x}{5} \le \frac{10}{5}$

⇒ x ≤ 2

∴ x ∈ (–∞, 2] (2)

From (1) and (2), we get

x ∈ (–∞, 8) ∩ (–∞, 2]

∴ x ∈ (–∞, 2]

Thus, the solution of the given system of inequations is (–∞, 2].

15. Question

Solve each of the following system of inequations in R

$\frac{2x-3}{4} - 2 \ge \frac{4x}{3} - 6$, 2(2x + 3) < 6(x – 2) + 10

Answer

Given $\frac{2x-3}{4} - 2 \ge \frac{4x}{3} - 6$ and 2(2x + 3) < 6(x – 2) + 10

Let us consider the first inequality.

$\frac{2x-3}{4} - 2 \ge \frac{4x}{3} - 6$

$\Rightarrow \frac{2x-3-8}{4} \ge \frac{4x-18}{3}$

$\Rightarrow \frac{2x-11}{4} \ge \frac{4x-18}{3}$

$\Rightarrow \left(\frac{2x-11}{4}\right) \times 3 \times 4 \ge \left(\frac{4x-18}{3}\right) \times 3 \times 4$

⇒ 3(2x – 11) ≥ 4(4x – 18)

⇒ 6x – 33 ≥ 16x – 72

⇒ 6x – 33 + 33 ≥ 16x – 72 + 33

⇒ 6x ≥ 16x – 39

⇒ 16x – 39 ≤ 6x

⇒ 16x – 39 + 39 ≤ 6x + 39

⇒ 16x ≤ 6x + 39

⇒ 16x – 6x ≤ 6x + 39 – 6x

⇒ 10x ≤ 39

$\Rightarrow \frac{10x}{10} \le \frac{39}{10}$

$\Rightarrow x \le \frac{39}{10}$

$\therefore x \in \left(-\infty, \frac{39}{10}\right]$ (1)

Now, let us consider the second inequality.

2(2x + 3) < 6(x – 2) + 10

⇒ 4x + 6 < 6x – 12 + 10

⇒ 4x + 6 < 6x – 2

⇒ 4x + 6 – 6 < 6x – 2 – 6

⇒ 4x < 6x – 8

⇒ 6x – 8 > 4x

⇒ 6x – 8 + 8 > 4x + 8

⇒ 6x > 4x + 8

⇒ 6x – 4x > 4x + 8 – 4x

⇒ 2x > 8

$\Rightarrow \frac{2x}{2} > \frac{8}{2}$

⇒ x > 4

∴ x ∈ (4, ∞) (2)

From (1) and (2), we get

$x \in \left(-\infty, \frac{39}{10}\right] \cap (4, \infty)$

∴ x ∈ ∅

Thus, there is no solution of the given system of inequations.

16. Question

Solve each of the following system of inequations in R

$$\frac{7x-1}{2} < -3, \frac{3x+8}{5} + 11 < 0$$

Answer

Given $\frac{7x-1}{2} < -3$ and $\frac{3x+8}{5} + 11 < 0$

Let us consider the first inequality.

$\frac{7x-1}{2} < -3$

$\Rightarrow \left(\frac{7x-1}{2}\right) \times 2 < -3 \times 2$

⇒ 7x – 1 < -6

⇒ 7x – 1 + 1 < -6 + 1

⇒ 7x < -5

$\Rightarrow \frac{7x}{7} < \frac{-5}{7}$

$\Rightarrow x < -\frac{5}{7}$

$\therefore x \in \left(-\infty, -\frac{5}{7}\right)$ (1)

Now, let us consider the second inequality.

$\frac{3x+8}{5}+11<0$

$\Rightarrow \frac{3x+8+55}{5}<0$

$\Rightarrow \frac{3x+63}{5}<0$

$\Rightarrow \left(\frac{3x+63}{5}\right)\times 5<0\times 5$

⇒ 3x + 63 < 0

⇒ 3x + 63 – 63 < 0 – 63

⇒ 3x < -63

$\Rightarrow \frac{3x}{3}<\frac{-63}{3}$

⇒ x < -21

∴ x ∈ (–∞, –21) (2)

From (1) and (2), we get

$x \in \left(-\infty, -\frac{5}{7}\right) \cap (-\infty, -21)$

∴ x ∈ (–∞, –21)

Thus, the solution of the given system of inequations is (–∞, –21).

17. Question

Solve each of the following system of inequations in R

$\frac{2x+1}{7x-1}>5, \frac{x+7}{x-8}>2$

Answer

Given $\frac{2x+1}{7x-1}>5$ and $\frac{x+7}{x-8}>2$

Let us consider the first inequality.

$\frac{2x+1}{7x-1}>5$

$\Rightarrow \frac{2x+1}{7x-1}-5>5-5$

$\Rightarrow \frac{2x+1}{7x-1}-5>0$

$\Rightarrow \frac{2x+1-5(7x-1)}{7x-1}>0$

$\Rightarrow \frac{2x+1-35x+5}{7x-1}>0$

$\Rightarrow \frac{-33x+6}{7x-1}>0$

$\Rightarrow \frac{-3(11x-2)}{7x-1}>0$

$\Rightarrow \frac{11x-2}{7x-1} < 0$

For this inequation to be true, there are two possible cases.

i. 11x – 2 > 0 and 7x – 1 < 0

⇒ 11x – 2 + 2 > 0 + 2 and 7x – 1 + 1 < 0 + 1

⇒ 11x > 2 and 7x < 1

$\Rightarrow x > \frac{2}{11} \text{ and } x < \frac{1}{7}$

$\therefore x \in \left(\frac{2}{11}, \infty\right) \cap \left(-\infty, \frac{1}{7}\right)$

However, $\left(-\frac{17}{25}, \infty\right) \cap \left(-\infty, -\frac{3}{8}\right) = \emptyset$

Hence, this case is not possible.

ii. 11x – 2 < 0 and 7x – 1 > 0

⇒ 11x – 2 + 2 < 0 + 2 and 7x – 1 + 1 > 0 + 1

⇒ 11x < 2 and 7x > 1

$\Rightarrow x < \frac{2}{11} \text{ and } x > \frac{1}{7}$

$\therefore x \in \left(-\infty, \frac{2}{11}\right) \cap \left(\frac{1}{7}, \infty\right)$

However, $\left(-\infty, \frac{2}{11}\right) \cap \left(\frac{1}{7}, \infty\right) = \left(\frac{1}{7}, \frac{2}{11}\right)$

Hence, $x \in \left(\frac{1}{7}, \frac{2}{11}\right)$ (1)

Now, let us consider the second inequality.

$\frac{x+7}{x-8} > 2$

$\Rightarrow \frac{x+7}{x-8} - 2 > 0$

$\Rightarrow \frac{x+7-2(x-8)}{x-8} > 0$

$\Rightarrow \frac{x+7-2x+16}{x-8} > 0$

$\Rightarrow \frac{-x+23}{x-8} > 0$

$\Rightarrow \frac{-(x-23)}{x-8} > 0$

$\Rightarrow \frac{x-23}{x-8} < 0$

For this inequation to be true, there are two possible cases.

i. x – 23 > 0 and x – 8 < 0

⇒ x – 23 + 23 > 0 + 23 and x – 8 + 8 < 0 + 8

⇒ x > 23 and x < 8

∴ x ∈ (23, ∞) ∩ (–∞, 5)

However, (23, ∞) ∩ (–∞, 5) = ∅

Hence, this case is not possible.

ii. x – 23 < 0 and x – 8 > 0

⇒ x – 23 + 23 < 0 + 23 and x – 8 + 8 > 0 + 8

⇒ x < 23 and x > 8

∴ x ∈ (–∞, 23) ∩ (8, ∞)

However, (–∞, 23) ∩ (8, ∞) = (8, 23)

Hence, x ∈ (8, 23) (2)

From (1) and (2), we get

$x \in \left(\frac{1}{7}, \frac{2}{11}\right) \cap (8,23)$

∴ x ∈ ∅

Thus, there is no solution of the given system of inequations.

18. Question

Solve each of the following system of inequations in R

$0 < \frac{-x}{2} < 3$

Answer

Given $0 < \frac{-x}{2} < 3$

The above inequality can be split into two inequalities.

$0 < -\frac{x}{2}$ and $-\frac{x}{2} < 3$

Let us consider the first inequality.

$0 < -\frac{x}{2}$

$\Rightarrow 0 \times 2 < -\frac{x}{2} \times 2$

⇒ 0 < –x

⇒ –x > 0

⇒ x < 0

∴ x ∈ (–∞, 0) (1)

Now, let us consider the second inequality.

$-\frac{x}{2} < 3$

$\Rightarrow -\frac{x}{2} \times 2 < 3 \times 2$

⇒ –x < 6

⇒ x > –6

∴ x ∈ (–6, ∞) (2)

From (1) and (2), we get

$x \in (-\infty, 0) \cap (-6, \infty)$

$\therefore x \in (-6, 0)$

Thus, the solution of the given system of inequations is (–6, 0).

19. Question

Solve each of the following system of inequations in R

$10 \le -5(x - 2) < 20$

Answer

Given $10 \le -5(x - 2) < 20$

The above inequality can be split into two inequalities.

$10 \le -5(x - 2)$ and $-5(x - 2) < 20$

Let us consider the first inequality.

$10 \le -5(x - 2)$

$\Rightarrow 10 \le -5x + 10$

$\Rightarrow 10 - 10 \le -5x + 10 - 10$

$\Rightarrow 0 \le -5x$

$\Rightarrow 0 + 5x \le -5x + 5x$

$\Rightarrow 5x \le 0$

$\Rightarrow x \le 0$

$\therefore x \in (-\infty, 0]$ (1)

Now, let us consider the second inequality.

$-5(x - 2) < 20$

$\Rightarrow -5x + 10 < 20$

$\Rightarrow -5x + 10 - 10 < 20 - 10$

$\Rightarrow -5x < 10$

$\Rightarrow \frac{-5x}{5} < \frac{10}{5}$

$\Rightarrow -x < 2$

$\Rightarrow x > -2$

$\therefore x \in (-2, \infty)$ (2)

From (1) and (2), we get

$x \in (-\infty, 0] \cap (-2, \infty)$

$\therefore x \in (-2, 0]$

Thus, the solution of the given system of inequations is (–2, 0].

20. Question

Solve each of the following system of inequations in R

$-5 < 2x - 3 < 5$

Answer

Given $-5 < 2x - 3 < 5$

The above inequality can be split into two inequalities.

–5 < 2x – 3 and 2x – 3 < 5

Let us consider the first inequality.

–5 < 2x – 3

⇒ 2x – 3 > –5

⇒ 2x – 3 + 3 > –5 + 3

⇒ 2x > –2

$\Rightarrow \frac{2x}{2} > \frac{-2}{2}$

⇒ x > –1

∴ x ∈ (–1, ∞) (1)

Now, let us consider the second inequality.

2x – 3 < 5

⇒ 2x – 3 + 3 < 5 + 3

⇒ 2x < 8

$\Rightarrow \frac{2x}{2} < \frac{8}{2}$

⇒ x < 4

⇒ x > –2

∴ x ∈ (–∞, 4) (2)

From (1) and (2), we get

x ∈ (–1, ∞) ∩ (–∞, 4)

∴ x ∈ (–1, 4)

Thus, the solution of the given system of inequations is (–1, 4).

21. Question

Solve each of the following system of inequations in R

$$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$$

Answer

Given $\frac{4}{x+1} \leq 3 < \frac{6}{x+1}, x > 0$

The above inequality can be split into two inequalities.

$\frac{4}{x+1} \leq 3$ and $3 < \frac{6}{x+1}$

Let us consider the first inequality.

$$\frac{4}{x+1} \leq 3$$

As x > 0, we have x + 1 > 0.

$\Rightarrow \left(\frac{4}{x+1}\right) \times (x+1) \leq 3 \times (x+1)$

$\Rightarrow 4 \leq 3(x + 1)$

$\Rightarrow 4 \leq 3x + 3$

$\Rightarrow 3x + 3 \geq 4$

$\Rightarrow 3x + 3 - 3 \geq 4 - 3$

$\Rightarrow 3x \geq 1$

$\Rightarrow \frac{3x}{3} \geq \frac{1}{3}$

$\Rightarrow x \geq \frac{1}{3}$

$\therefore x \in \left[\frac{1}{3}, \infty\right)$ (1)

Now, let us consider the second inequality.

$3 < \frac{6}{x + 1}$

As $x > 0$, we have $x + 1 > 0$.

$\Rightarrow 3 \times (x + 1) \leq \left(\frac{6}{x + 1}\right) \times (x + 1)$

$\Rightarrow 3(x + 1) \leq 6$

$\Rightarrow 3x + 3 \leq 6$

$\Rightarrow 3x + 3 - 3 \leq 6 - 3$

$\Rightarrow 3x \leq 3$

$\Rightarrow \frac{3x}{3} \leq \frac{3}{3}$

$\Rightarrow x \leq 1$

$\therefore x \in (-\infty, 1]$ (2)

From (1) and (2), we get

$x \in \left[\frac{1}{3}, \infty\right) \cap (-\infty, 1]$

$\therefore x \in \left[\frac{1}{3}, 1\right]$

Thus, the solution of the given system of inequations is $\left[\frac{1}{3}, 1\right]$

Exercise 15.3

1. Question

Solve each of the following system of equations in R.

$\left|x + \frac{1}{3}\right| > \frac{8}{3}$

Answer

We know that,

$|x+a|>r \Longleftrightarrow x>r-a$ or $x<-(a+r)$

Here, $a = \frac{1}{3}$ and $r = \frac{8}{3}$

$\Rightarrow x > \frac{8}{3} - \frac{1}{3} \text{ or } x < -(\frac{8}{3} + \frac{1}{3})$

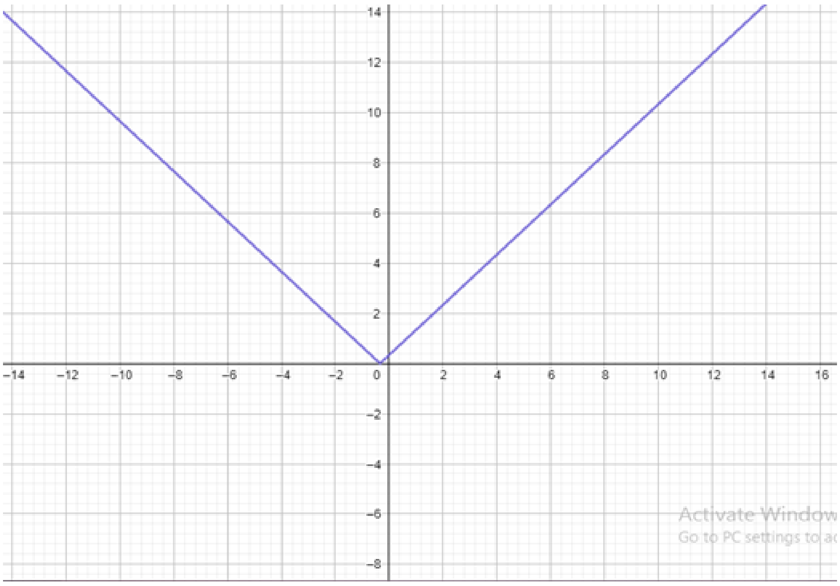
$\Rightarrow x > \frac{7}{3} \text{ or } x < -\frac{9}{3}$

$\Rightarrow x > \frac{7}{3} \text{ or } x < -3$

$\Rightarrow \ x \in \left(\frac{7}{3}, \infty\right) \text{ or } x \in (-\infty, -3)$

$\Rightarrow \ \mathbf{x} \, \boldsymbol{\epsilon}(-\infty, -\mathbf{3}) \cup \left(\frac{\mathbf{7}}{\mathbf{3}}, \infty\right)$

We can verify the answers using the graph as well.



2. Question

Solve each of the following system of equations in R.

$|4 - x| + 1 < 3$

Answer

|4–x|+1<3

Subtracting 1 from both the sides.

⇒ |4–x|+1–1<3–1

⇒ |4–x|<2

We know that,

|a–x|<r ⟺ a–r<x<a+r

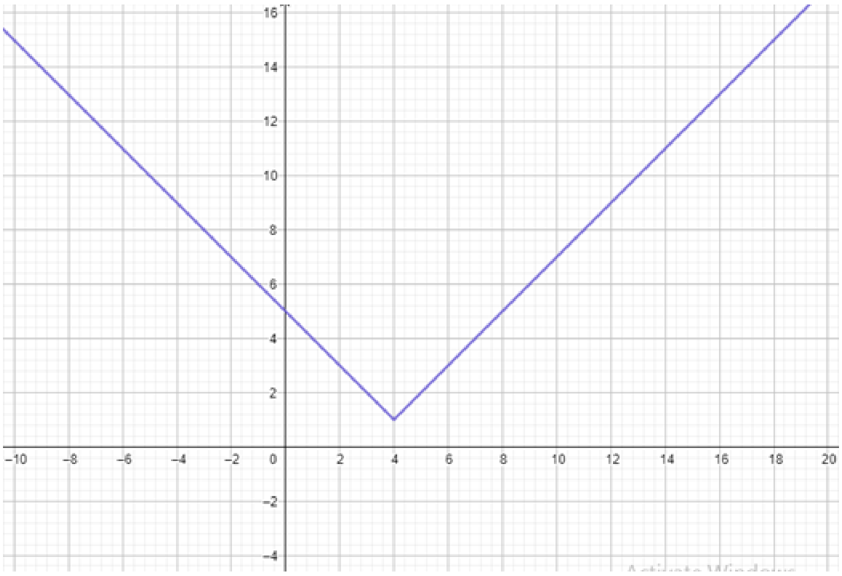
Here, a=4 and r=2

⇒ 4–2<x<4+2

⇒2<x<6

⇒ x ϵ(2, 6)

We can verify the answers using graph as well.



3. Question

Solve each of the following system of equations in R.

$$\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$$

Answer

The equation can be re–written as

$$\left|\frac{3x}{2}-\frac{4}{2}\right| \le \frac{5}{12}$$

$$\Rightarrow \left|\frac{3x}{2}-2\right| \le \frac{5}{12}$$

We know that,

|x–a|≤r ⟺ a–r ≤ x≤ a+r

Here, $a = 2 \text{ and } r = \frac{5}{12}$

$$\Rightarrow 2-\frac{5}{12} \le \frac{3x}{2} \le \ 2+\frac{5}{12}$$

$$\Rightarrow \frac{24-5}{12} \le \frac{3x}{2} \le \frac{24+5}{12}$$

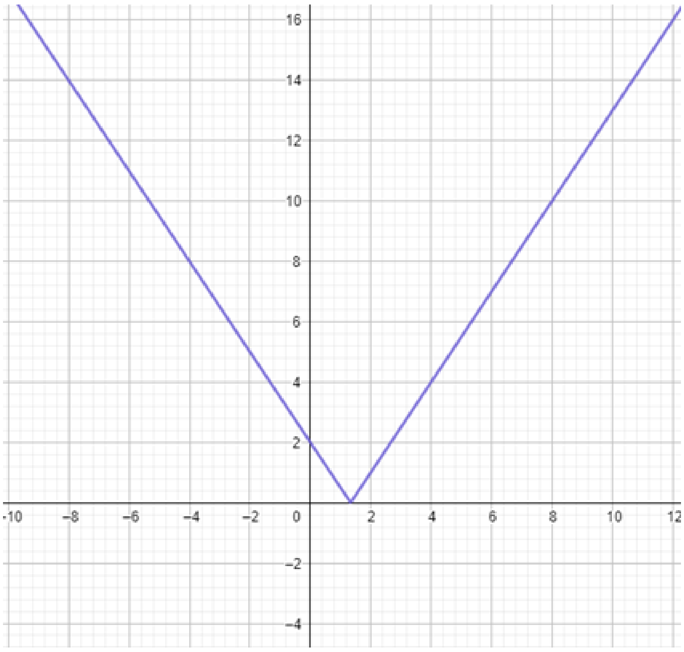
$$\Rightarrow \frac{19}{12} \le \frac{3x}{2} \le \frac{29}{12}$$

Now, multiplying the whole inequality by 2 and dividing by 3, we get

$$\Rightarrow \frac{19}{18} \le x \le \frac{29}{18}$$

$$\Rightarrow \ \mathbf{x} \in \left[\frac{\mathbf{19}}{\mathbf{18}}, \frac{\mathbf{29}}{\mathbf{18}}\right]$$

We can verify the answers using graph as well.



4. Question

Solve each of the following system of equations in R.

$$\frac{|x-2|}{x-2} > 0$$

Answer

$$\frac{|x-2|}{x-2} > 0$$

Clearly, x≠2, as it will lead equation unmeaningful.

Now, two case arise:

Case1: x–2>0

⇒ x>2

In this case |x–2|=x–2

⇒ x ϵ (2, ∞)….(1)

Case 2: x–2<0

⇒ x<2

In this case, |x–2|=–(x–2)

$$\frac{-(x-2)}{x-2} > 0$$

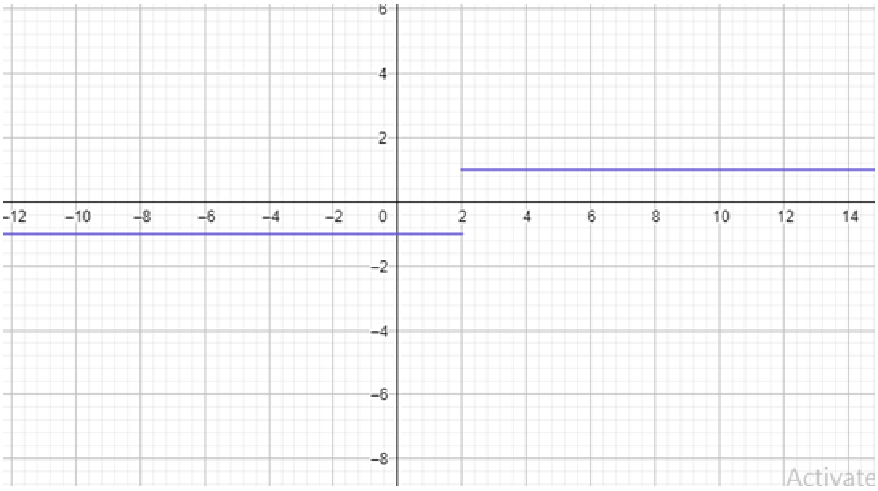
$\Rightarrow -1 > 0$

Inequality doesn’t get satisfy

∴, this case gets nullified.

⇒ $\mathbf{x \in (2, \infty)}$ **(from 1)**

We can verify the answers using graph as well.



5. Question

Solve each of the following system of equations in R.

$$\frac{1}{|x|-3}<\frac{1}{2}$$

Answer

We know that, if we take reciprocal of any inequality we need to change the inequality as well.

Also,

$|x|-3\neq 0$

$\Rightarrow |x|>3$ or $|x|<3$

For $|x|<3$

$\Rightarrow -3<x<3$

$\Rightarrow x\in(-3, 3)$ ....(1)

∴, The equation can be re–written as–

$$|x|-3>2$$

Adding 2 both the sides, we get–

$|x|-3+3> 2+3$

$\Rightarrow |x|>5$

We know that,

$|x|>a \Longleftrightarrow x<-a$ or $x>a$

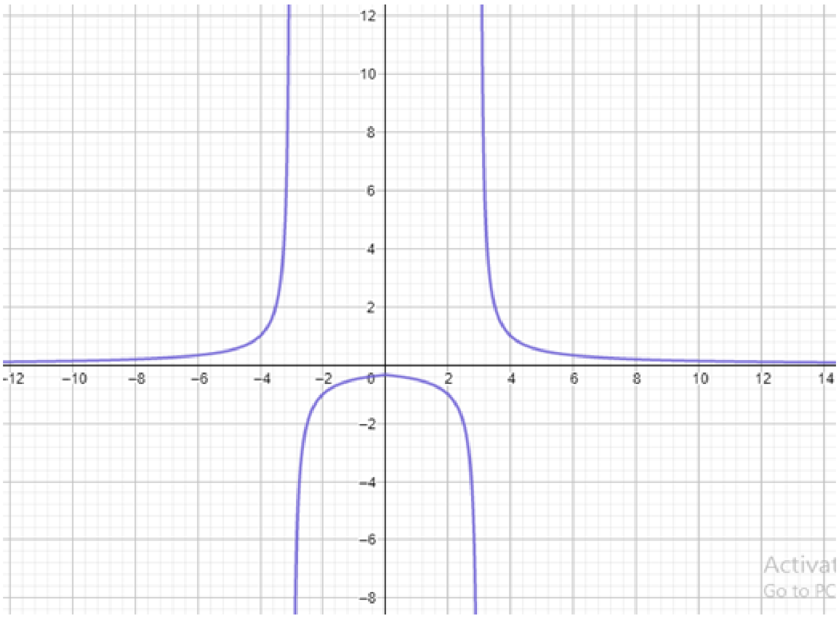
Here, $a=5$

$\Rightarrow x<-5$ or $x>5$ ....(2)

$\Rightarrow x\in(-\infty,-5)$ or $x\in(5, \infty)$

$\Rightarrow x \in(-\infty,-5) \cup (-3, 3) \cup (5, \infty)$ (from 1 and 2)

We can verify the answers using graph as well.



6. Question

Solve each of the following system of equations in R.

$$\frac{|x+2|-x}{x}<2$$

Answer

The equation can be re–written as

$$\frac{|x+2|}{x}-1<2$$

Adding 1 both the sides, we get,

$$\frac{|x+2|}{x}<3$$

Subtracting 3 both the sides

$$\Rightarrow \frac{|x+2|}{x}-3<0$$

Clearly, x≠0, as it will lead equation unmeaningful.

Now, two case arise:

Case1: x+2>0

⇒ x>–2

In this case |x+2|=x+2

$$\frac{x+2}{x}-3<0$$

$$\frac{x+2-3x}{x}<0$$

$$\frac{-(2x-2)}{x}<0$$

$$\frac{2x-2}{x}>0$$

Considering Numerator,

2x–2>0

⇒ x>1

⇒ x ϵ (1, ∞) ....(1)

Case 2: x+2<0

⇒ x<–2

In this case, |x+2|=–(x+2)

$\frac{-(x+2)}{x} - 3 < 0$

$\frac{-x-2-3x}{x} < 0$

$\frac{-(4x+2)}{x} < 0$

$\frac{4x+2}{x} > 0$

Considering Numerator,

4x+2>0

$\Rightarrow \; x > -\frac{1}{2}$

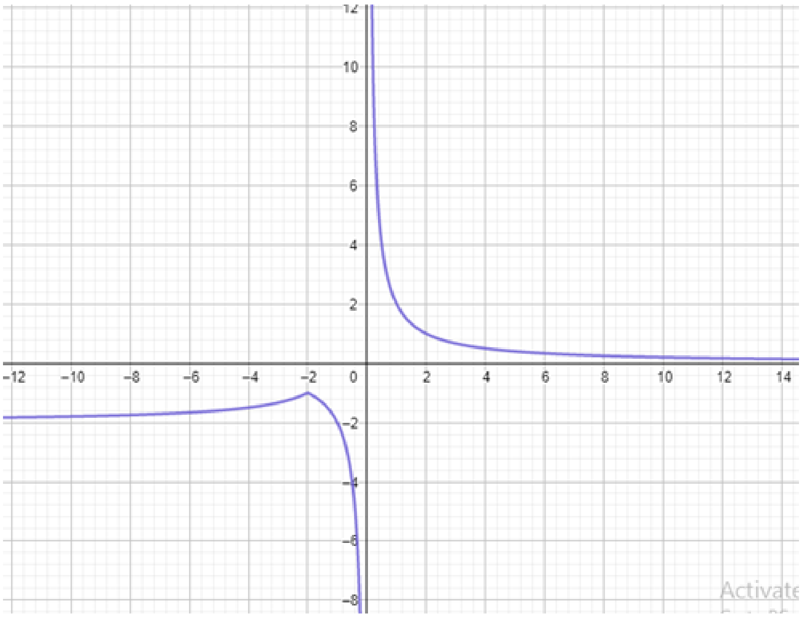
But x<–2

Now, from Denominator, we have–

⇒ x ϵ (–∞ , 0) ...(2)

$\Rightarrow \; x \in (-\infty, 0) \cup$ (1, ∞) (from 1 and 2)

We can verify the answers using graph as well.



7. Question

Solve each of the following system of equations in R.

$\left|\frac{2x-1}{x-1}\right| > 2$

Answer

x≠1, as it will lead equation unmeaningful.

Now, on subtracting 2 from both the sides, we get–

$$\left|\frac{2x-1}{x-1}\right|-2>0$$

Now, 3 case arises:

Case 1:1<x<∞

For this case, |2x–1|=2x–1 and |x–1|=x–1

$$\left|\frac{2x-1}{x-1}\right|-2>0$$

$$\Rightarrow \frac{2x-1}{x-1}-2>0$$

$$\Rightarrow \frac{2x-1-2x+2}{x-1}>0$$

$$\Rightarrow \frac{1}{x-1}>0$$

⇒ x ϵ (1, ∞) ...(1)

Case 2: $\frac{1}{2}<x<1$

For this case: |2x–1|=2x–1 and |x–1|=–(x–1)

$$\left|\frac{2x-1}{x-1}\right|-2>0$$

$$\Rightarrow \frac{2x-1}{-(x-1)}-2>0$$

$$\Rightarrow \frac{-(2x-1)-2x+2}{x-1}>0$$

$$\Rightarrow \frac{-4x+3}{x-1}>0$$

$$\Rightarrow \frac{4x-3}{x-1}<0$$

$\Rightarrow x \in (\frac{3}{4}, 1)$ ...(2)

Case 3: $-\infty < x < \frac{1}{2}$

For this case: |2x–1|=–(2x–1) and |x–1|=–(x–1)

$$\left|\frac{2x-1}{x-1}\right|-2>0$$

$$\Rightarrow \frac{-(2x-1)}{-(x-1)}-2>0$$

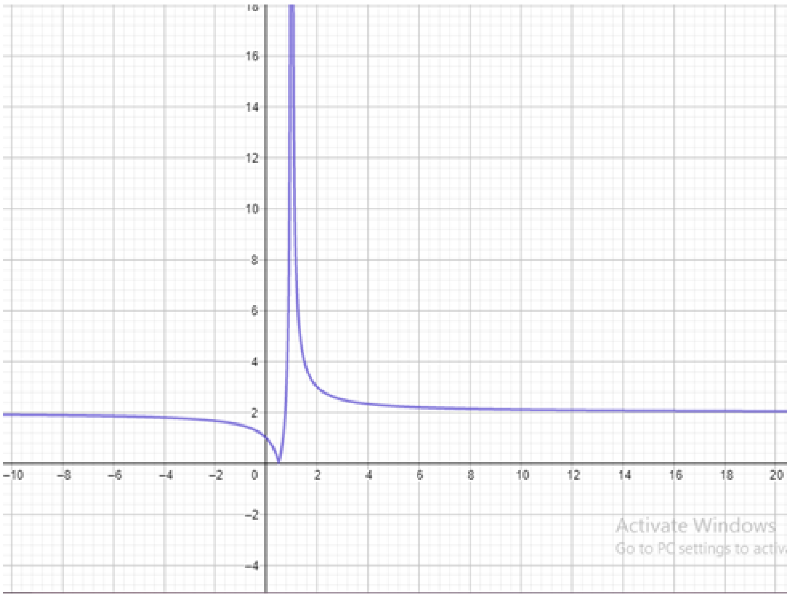
$$\Rightarrow \frac{2x-1-2x+2}{x-1}>0$$

$$\Rightarrow \frac{1}{x-1}>0$$

Which is not possible, hence, this will give no solution.

$\Rightarrow \mathbf{x \in \left(\frac{3}{4}, 1\right) \cup (1, \infty)}$ **(from 1 and 2)**

We can verify the answers using graph as well.



8. Question

Solve each of the following system of equations in R.

$$|x-1|+|x-2|+|x-3|\geq 6$$

Answer

Subtracting 6 from both the sides, we get–

$|x–1|+|x–2|+|x–3|–6\geq 0$

Here, we have 4 cases:

Case 1: $-\infty < x < 1$

For this case, $|x–1|=–(x–1)$, $|x–2|=–(x–2)$ and $|x–3|=–(x–3)$

$\Rightarrow –(x–1+x–2+x–3+6)\geq 0$

$\Rightarrow x–1+x–2+x–3+6<0$

$\Rightarrow 3x<0$

$\Rightarrow x<0$

$\Rightarrow x\in (-\infty , 0)$ ...(1)

Case 2:$1<x<2$

For this case, $|x–1|=x–1$, $|x–2|=–(x–2)$ and $|x–3|=–(x–3)$

$\Rightarrow x–1–x+2–x+3–6\geq 0$

$\Rightarrow –x–2\geq 0$

$\Rightarrow x+2<0$

$\Rightarrow x<–2$

Which doesn’t signify the interval

Case 3:$2<x<3$

For this case, $|x–1|=x–1$, $|x–2|=x–2$ and $x–3=–(x–3)$

$\Rightarrow x–1+x–2–x+3–6\geq 0$

$\Rightarrow x–6\geq 0$

$\Rightarrow x\geq 6$

Which doesn't signify the interval

Case 4:3<x<∞

For this case, |x–1|=x–1, |x–2|=x–2 and |x–3|=x–3

⇒ x–1+x–2+x–3–6≥0

⇒ x–1+x–2+x–3–6≥0

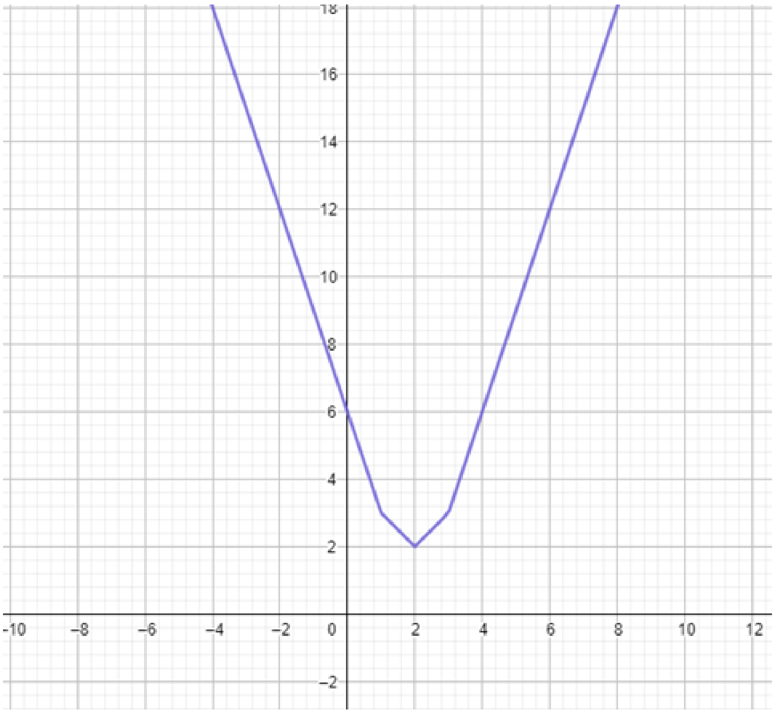
⇒ 3x–12>0

⇒ x>4

⇒ x∈ (4 , ∞) …(2)

⇒ x ∈ (–∞ , 0)⋃ (4 , ∞) (from 1 and 2)

We can verify the answers using graph as well.



9. Question

Solve each of the following system of equations in R.

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

Answer

$$\frac{|x-2|-1}{|x-2|-2} \leq 0$$

Clearly, |x–2|–2≠0

⇒ |x–2|≠2

⇒ x≠0 and x≠4

Now, 2 case arise:

Case 1:–∞ <x<2

For this, |x–2|=–(x–2)

$$\frac{-(x-2)-1}{-(x-2)-2} \leq 0$$

$$\frac{-x+1}{-x} \leq 0$$

$$\frac{x-1}{x} \leq 0$$

⇒ x ϵ (0,1] ...(1)

Case 2: 2<x<∞

For this, |x–2|=x–2

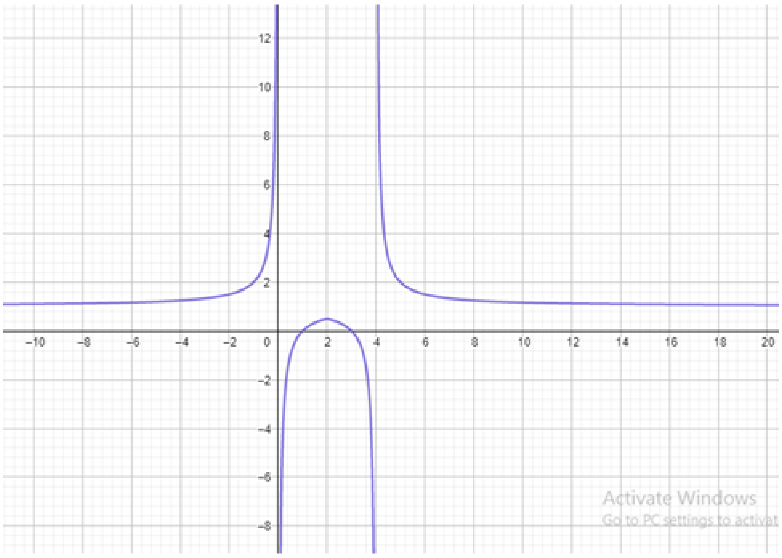
$$\frac{x-2-1}{x-2-2} \leq 0$$

$$\frac{x-3}{x-4} \leq 0$$

⇒ x ϵ [3,4) ...(2)

⇒ x ϵ (0,1] ⋃ [3, 4) (from 1 and 2)

We can verify the answers using graph as well.



10. Question

Solve each of the following system of equations in R.

$$\frac{1}{|x|-3} \leq \frac{1}{2}$$

Answer

We know that, if we take reciprocal of any inequality we need to change the inequality as well.

Also,

|x|–3≠0

⇒ |x|>3 or |x|<3

For |x|<3

⇒ –3<x<3

⇒ x ϵ (–3, 3) ...(1)

∴, The equation can be re–written as–

$|x| - 3 \geq 2$

Adding 2 both the sides, we get–

|x|–3+3≥ 2+3

⇒ |x|≥5

We know that,

|x |≥a ⟺ x≤–a or x≥a

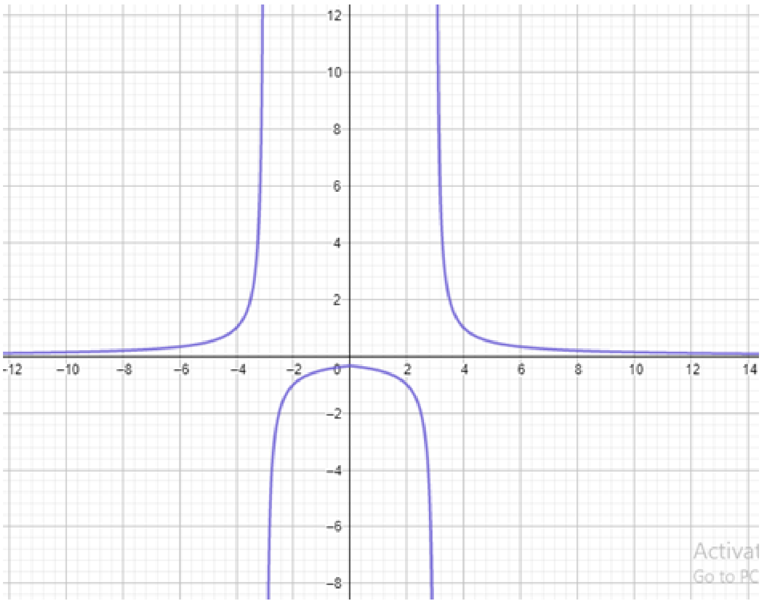
Here, a=5

⇒ x≤–5 or x≥5

⇒ x ϵ(–∞,–5] or x ϵ[5, ∞) …(2)

⇒ x ϵ(–∞,–5] ⋃ (–3, 3) ⋃ [5, ∞) (from 1 and 2)

We can verify the answers using graph as well.



11. Question

Solve each of the following system of equations in R.

$$|x + 1| + |x| > 3$$

Answer

Subtracting 3 from both sides, we get–

|x+1|+|x|–3>0

For this, we have 3 cases,

Case 1:–∞ <x<–1

For this, |x+1|=–(x+1) and |x|=–x

–(x+1)–x–3>0

–2x–1–3>0

2x+4<0

x<–2

⇒ x ϵ (–∞ , –2) …(1)

Case 2: –1<x<0

For this, $|x+1|=x+1$ and $|x|=-x$

$x+1-x-3>0$

$-2>0$

Which is absurd, thus it leads to no solution.

Case 3: $0<x<\infty$

For this, $|x+1|=x+1$ and $|x|=x$

$x+1+x-3>0$

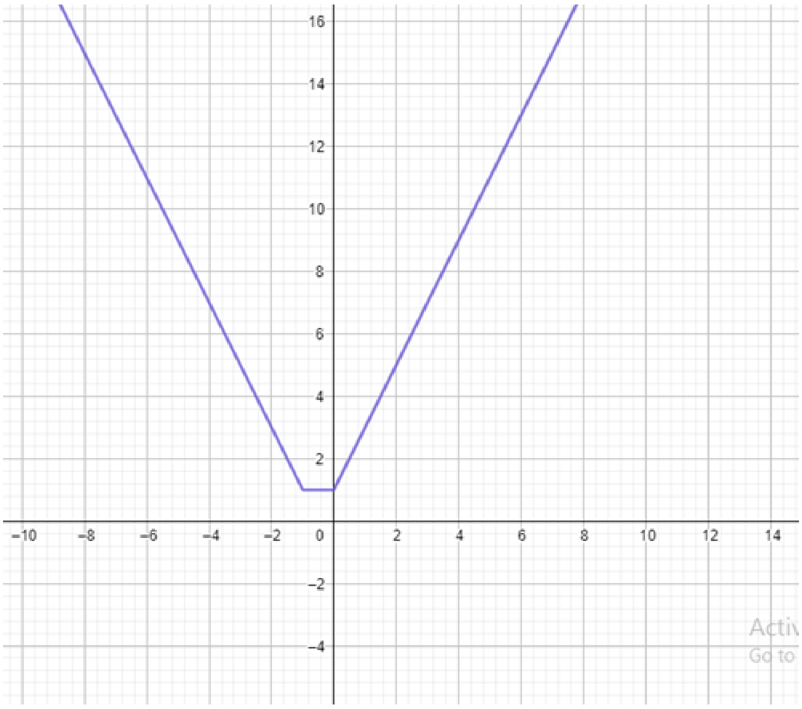
$2x-2>0$

$x>1$

$\Rightarrow x \in (1, \infty)$ ...(2)

$\Rightarrow x \in (-\infty, -2) \cup (1, \infty)$ (from 1 and 2)

We can verify the answers using graph as well.



12. Question

Solve each of the following system of equations in R.

$$1 \le |x-2| \le 3$$

Answer

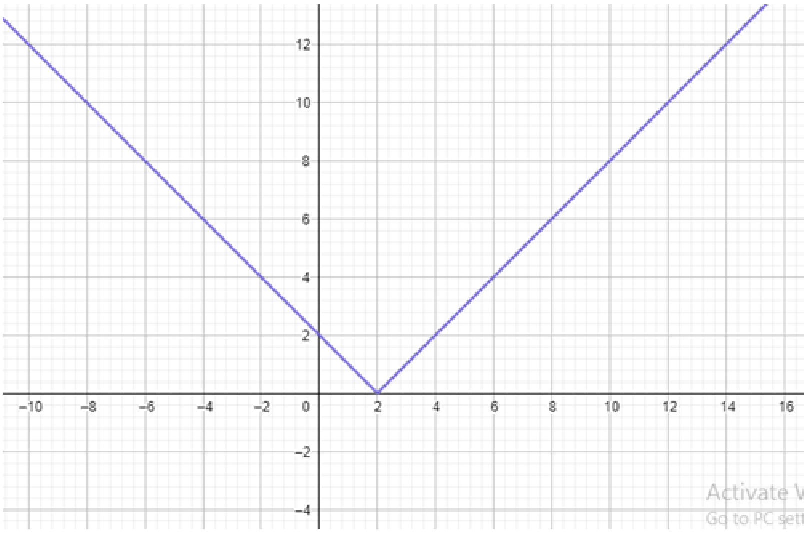
We know that,

$a \le |x-c| \le b \Longleftrightarrow x \in [-b+c, -a+c] \cup [a+c, b+c]$

$\therefore$, $1 \le |x-2| \le 3 \Longleftrightarrow x \in [-3+2, -1+2] \cup [1+2, 3+2]$

$\Longleftrightarrow x \in [-1, 1] \cup [3, 5]$

We can verify the answers using graph as well.



13. Question

Solve each of the following system of equations in R.

$|3-4x| \geq 9$

Answer

Subtracting 9 from both sides, we get–

|3–4x|–9≥0

For this, we have 2 cases,

Case 1:$-\infty < x < \frac{3}{4}$

For this, |3–4x|=3–4x

3–4x–9≥0

–4x–6≥0

4x≤–6

$x \leq -\frac{3}{2}$

$\Rightarrow x \in \left(-\infty, -\frac{3}{2}\right]$ ...(1)

Case 2: 0<x<∞

For this, |3–4x|=–(3–4x)

–3+4x–9≥0

4x–12≥0

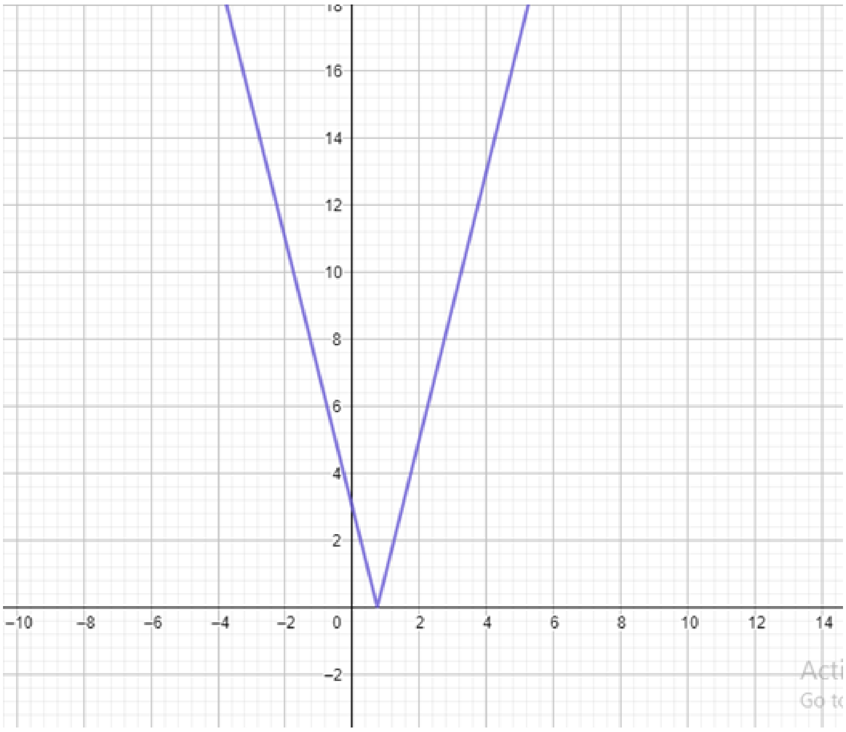
4x≥12

⇒ x≥3

⇒ x∈ [3, ∞) ...(2)

$\Rightarrow \mathbf{x \in \left(-\infty, -\frac{3}{2}\right] \cup [3, \infty)}$ **(from 1 and 2)**

We can verify the answers using graph as well.



Exercise 15.4

1. Question

Find all pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

Answer

We need to assume two consecutive odd positive integers.

So, let the smaller odd positive integer be x.

Then, the other odd positive integer will be (x + 2).

Given: Both these numbers are smaller than 10. …(i)

And their sum is more than 11. …(ii)

So,

From given statement (i),

$x < 10$ …(iii)

$x + 2 < 10$

$\Rightarrow x < 10 - 2$

$\Rightarrow x < 8$ …(iv)

From given statement (ii),

Sum of these two consecutive odd positive integers > 11

$\Rightarrow (x) + (x + 2) > 11$

$\Rightarrow x + x + 2 > 11$

$\Rightarrow 2x + 2 > 11$

$\Rightarrow 2(x + 1) > 11$

$\Rightarrow x + 1 > \frac{11}{2}$

$\Rightarrow x > \frac{11}{2} - 1$

$\Rightarrow x > \frac{11-2}{2}$

$\Rightarrow x > \frac{9}{2}$ …(v)

From inequalities (iv) & (v), we have

x < 8 & $x > \frac{9}{2}$

It can be merged and written as

$\frac{9}{2} < x < 8$

This means that x lies between 9/2 (or 4.5) and 8.

Note the odd positive integers lying between 4.5 and 8.

They are 5 and 7.

Also, consider inequality (iii), that is, x < 10.

So, from inequalities (iii) & (v), we have

x < 10 & $x > \frac{9}{2}$

It can be merged and written as

$\frac{9}{2} < x < 10$

Note that, the upper limit here has shifted from 8 to 10. Now, x is odd integer from 4.5 to 10.

So, the odd integers from 4.5 to 10 are 5, 7 and 9.

Now, let us find pairs of consecutive odd integers.

Let x = 5, then (x + 2) = (5 + 2) = 7.

Let x = 7, then (x + 2) = (7 + 2) = 9.

Let x = 9, then (x + 2) = (9 + 2) = 11. But, 11 is greater than 10.

Hence, all such pairs of odd consecutive positive integers required are (5, 7) and (7, 9).

2. Question

Find all pairs of consecutive odd natural number, both of which are larger than 10, such that their sum is less than 40.

Answer

We need to assume two consecutive odd natural numbers.

So, let the smaller odd natural number be x.

Then, the other odd natural number will be (x + 2).

Given: Both these numbers are larger than 10. …(i)

And their sum is less than 40. …(ii)

So,

From given statement (i),

x > 10 …(iii)

x + 2 > 10

⇒ x > 10 – 2

⇒ x > 8

Since, the number must be greater than 10, x > 8 can be ignored.

From given statement (ii),

Sum of these two consecutive odd natural numbers < 40

⇒ (x) + (x + 2) < 40

⇒ x + x + 2 < 40

⇒ 2x + 2 < 40

⇒ 2(x + 1) < 40

⇒ $x + 1 < \frac{40}{2}$

⇒ x + 1 < 20

⇒ x < 20 – 1

⇒ x < 19 …(iv)

From inequalities (iii) & (iv), we have

x > 10 & x < 19

It can be merged and written as

10 < x < 19

From this inequality, we can say that x lies between 10 and 19.

So, the odd natural numbers lying between 10 and 19 are 11, 13, 15 and 17. (Excluding 19 as x < 19)

Now, let us find pairs of consecutive odd natural numbers.

Let x = 11, then (x + 2) = (11 + 2) = 13

Let x = 13, then (x + 2) = (13 + 2) = 15

Let x = 15, then (x + 2) = (15 + 2) = 17

Let x = 17, then (x + 2) = (17 + 2) = 19.

Hence, all such pairs of consecutive odd natural numbers required are (11, 13), (13, 15), (15, 17) and (17, 19).

3. Question

Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

Answer

We need to assume two consecutive even positive integers.

So, let the smaller even positive integer be x.

Then, the other even positive integer will be (x + 2).

Given: Both these numbers are larger than 5. …(i)

And their sum is less than 23. …(ii)

So,

From given statement (i),

$x > 5$ ...(iii)

$x + 2 > 5$

$\Rightarrow x > 5 - 2$

$\Rightarrow x > 3$

Since, the number must be larger than 5, $x > 3$ can be ignored.

From given statement (ii),

Sum of these two consecutive even positive integers < 23

$\Rightarrow (x) + (x + 2) < 23$

$\Rightarrow x + x + 2 < 23$

$\Rightarrow 2x + 2 < 23$

$\Rightarrow 2(x + 1) < 23$

$\Rightarrow x + 1 < \frac{23}{2}$

$\Rightarrow x + 1 < 11.5$

$\Rightarrow x < 11.5 - 1$

$\Rightarrow x < 10.5$ ...(iv)

From inequalities (iii) & (iv), we have

$x > 5$ & $x < 10.5$

It can be merged and written as

$5 < x < 10.5$

From this inequality, we can say that x lies between 5 and 10.5.

So, the even positive integers lying between 5 and 10.5 are 6, 8 and 10.

Now, let us find pairs of consecutive even positive integers.

Let $x = 6$, then $(x + 2) = (6 + 2) = 8$

Let $x = 8$, then $(x + 2) = (8 + 2) = 10$

Let $x = 10$, then $(x + 2) = (10 + 2) = 12$.

Hence, all such pairs of consecutive even positive integers required are (6, 8), (8, 10) and (10, 12).

4. Question

The marks scored by Rohit in two tests were 65 and 70. Find the minimum marks he should score in the third test to have an average of at least 65 marks.

Answer

Given: Marks scored by Rohit in two tests are 65 and 70.

To find Minimum marks in the third test to make an average of at least 65 marks.

Let marks in the third test be x.

According to the question, we need to find minimum x for which the average of all three papers would be at least 65 marks.

That is,

Average marks in three papers ≥ 65 ...(i)

Now, we know that average is given as

$$\text{Average} = \frac{\text{Total sum of all numbers}}{\text{Total number of items}}$$

So, an average of the marks in these three tests is given by

$$\text{Average} = \frac{\text{Sum of marks in three tests}}{3}$$

$$\Rightarrow \text{Average} = \frac{\text{Marks in first two papers} + \text{Marks in third test}}{3}$$

$$\Rightarrow \text{Average} = \frac{65 + 70 + x}{3}$$

$$\Rightarrow \text{Average} = \frac{135 + x}{3}$$

Substituting this value of average in the inequality (i), we get

$$\frac{135 + x}{3} \geq 65$$

⇒ $(135 + x) \geq 65 \times 3$

⇒ $(135 + x) \geq 195$

⇒ $x \geq 195 - 135$

⇒ $x \geq 60$

This inequality means that Rohit should score at least 60 marks in his third test to have an average of at least 65 marks.

So, the minimum marks to get an average of 65 marks is 60.

Thus, the minimum marks required in the third test is 60.

5. Question

A solution is to be kept between 86^o and 95^oF. What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$.

Answer

We have been given that, a solution is kept between 86° F and 95° F.

Let $F_1 = 86°$ F

And let $F_2 = 95°$

The conversion formula is

$$F = \frac{9}{5}C + 32$$

We need to convert Fahrenheit into degree Celcius.

So, take $F_1 = 86°$ F

Using the formula, we have

$$F_1 = \frac{9}{5}C_1 + 32$$

$$\Rightarrow \frac{9}{5}C_1 = F_1 - 32$$

$\Rightarrow C_1 = \frac{5}{9}(F_1 - 32)$

$\Rightarrow C_1 = \frac{5}{9}(86 - 32)$

$\Rightarrow C_1 = \frac{5}{9} \times 54$

$\Rightarrow C_1 = 5 \times 6$

$\Rightarrow C_1 = 30° C$

Now, take $F_2 = 95° F$

Using the formula, we have

$F_2 = \frac{9}{5}C_2 + 32$

$\Rightarrow \frac{9}{5}C_2 = F_2 - 32$

$\Rightarrow C_2 = \frac{5}{9}(F_2 - 32)$

$\Rightarrow C_2 = \frac{5}{9}(95 - 32)$

$\Rightarrow C_2 = \frac{5}{9} \times 63$

$\Rightarrow C_2 = 5 \times 7$

$\Rightarrow C_2 = 35° C$

Therefore, the range of temperature of the solution in degree Celsius is 30° C and 35° C.

6. Question

A solution is to be kept between 30^oC and 35^oC. What is the range of temperature in degree Fahrenheit?

Answer

We have been given that, a solution is kept between 30° C and 35° C.

Let $C_1 = 30° C$

And let $C_2 = 35° C$

The conversion formula is given by

$F = \frac{9}{5}C + 32$

We need to convert degree Celsius to degree Fahrenheit.

So, take $C_1 = 30° C$

Using the formula, we have

$F_1 = \frac{9}{5}C_1 + 32$

$\Rightarrow F_1 = \frac{9}{5}(30) + 32$

$\Rightarrow F_1 = 9 \times 6 + 32$

$\Rightarrow F_1 = 54 + 32$

$\Rightarrow F_1 = 86° F$

Now, take $C_2 = 35° C$

Using the formula, we have

$$F_2 = \frac{9}{5}C_2 + 32$$

$$\Rightarrow F_2 = \frac{9}{5}(35) + 32$$

$\Rightarrow F_2 = 9 \times 7 + 32$

$\Rightarrow F_2 = 63 + 32$

$\Rightarrow F_2 = 95° F$

Therefore, the range of temperature of the solution in degree Fahrenheit is 86° F and 95° F.

7. Question

To receive grade 'A' in a course, one must obtain an average of 90 marks or more in five papers each of 100 marks. If Shikha scored 87, 95, 92 and 94 marks in first four papers, find the minimum marks that she must score in the last paper to get grade 'A' in the course.

Answer

Given that, there is total of five papers that Shikha has attended.

The score in the first four papers is 87, 95, 92 and 94.

To receive grade 'A,' the average marks in the five papers must be 90 or more.

Let marks in the fourth paper be x.

According to the question, we need to find minimum x for which the average of all five papers would be at least 90 marks.

That is,

Average marks in five papers ≥ 90 …(i)

Let us find the average marks in five papers. It is given by

$$\text{Average} = \frac{\text{Total sum of all numbers}}{\text{total number of items}}$$

So,

$$\text{Average} = \frac{\text{Sum of marks in five papers}}{5}$$

$$\Rightarrow \text{Average} = \frac{\text{Sum of marks in four papers} + x}{5}$$

$$\Rightarrow \text{Average} = \frac{87 + 95 + 92 + 94 + x}{5}$$

$$\Rightarrow \text{Average} = \frac{368 + x}{5}$$

Substituting this value of average in the inequality (i), we get

$$\frac{368 + x}{5} \geq 90$$

$\Rightarrow (368 + x) \geq 90 \times 5$

⇒ (368 + x) ≥ 450

⇒ x ≥ 450 – 368

⇒ x ≥ 82

This inequality means that Shikha should score at least 82 marks in her fifth test to have an average of at least 90 marks.

So, the minimum marks to get an average of 90 marks is 82.

Thus, the minimum marks required in the fifth test is 82.

8. Question

A company manufactures cassettes and its cost and revenue functions for a week are $C = 300 + \frac{3}{2}x$ and R = 2x respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold for the company to realize a profit?

Answer

We have been a week's data,

$\text{Cost of cassette}, C = 300 + \frac{3}{2}x$

Revenue, R = 2x

Where x = number of cassettes produced and sold in a week.

We know that profit is given by,

Profit = Revenue – Cost …(i)

Revenue is the income that a business has from its normal business activities, usually from the sale of goods and services to customers.

A cost is the value of money that has been used up to produce something or deliver a service and hence is not available for use anymore.

And Profit is the gain in the business.

So, it is justified that profit in any business would be measured by the difference in the capital generated by the business and the capital used up in the business.

Profit generated by the company manufacturing cassettes is given by,

Profit = R – C (from (i))

Where, R = Revenue

C = Cost of cassette

Here,

If R < C, then

Profit < 0

⇒ There is a loss.

If R = C, then

Profit = 0

⇒ There is no profit no loss.

If R > C, then

Profit > 0

⇒ There is a profit.

We need to find the number of cassettes sold to make a profit. That is, we need to find x.

So, R > C (to realize a profit)

Substituting values of R and C. We get

$2x > 300 + \frac{3}{2}x$

$\Rightarrow 2x - \frac{3}{2}x > 300$

$\Rightarrow \frac{4x - 3x}{2} > 300$

$\Rightarrow \frac{x}{2} > 300$

⇒ x > 300 × 2

⇒ x > 600

This means that x must be greater than 600.

Thus, the company must sell more than 600 cassettes to realize a profit.

9. Question

The longest side of a triangle is three times the shortest side, and the third side is 2 cm shorter than the longest side if the perimeter of the triangle at least 61 cm, Find the minimum length of the shortest-side.

Answer

We are given with a triangle,

The longest side of this triangle = 3 × Shortest side …(i)

The third side of this triangle = Longest side – 2 cm …(ii)

The perimeter of the triangle ≥ 61 cm …(iii)

Let

Shortest side of the triangle = a

The longest side of the triangle = b

The third side of the triangle = c

So

From (i),

b = 3 × a

⇒ b = 3a …(iv)

From (ii),

c = b – 2

⇒ c = 3a – 2 (∵ b = 3a) …(v)

Then, perimeter is given by

Perimeter of the triangle = a + b + c

Substituting the values of b and c from equation (iv) and (v) respectively, we get

Perimeter of the triangle = a + (3a) + (3a – 2)

⇒ Perimeter of the triangle = 7a – 2 …(vi)

Putting the value of perimeter of the triangle from (v) in inequality (iii), we get

$7a - 2 \geq 61$

$\Rightarrow 7a \geq 61 + 2$

$\Rightarrow 7a \geq 63$

$$\Rightarrow a \geq \frac{63}{7}$$

$\Rightarrow a \geq 9$

This means, ‘a’ which is the shortest side of the triangle is 9 or more than 9.

Thus, the minimum length of the shortest side of the triangle is 9 cm.

10. Question

How many liters of water will have to be added to 1125 liters of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Answer

Given: Volume of the existing solution = 1125 liters

Amount of acid in the existing solution = 45% of 1125 …(i)

And the rest 55% of 1125 liters is the amount of water in it, which need not be computed.

Let the water added (in liters) be x in 1125 liters of solution.

According to the question,

x liters of water has to be added to 1125 liters of the 45% solution.

We can say that, even if x liters of water is added to the 1125 liters of solution, acid content will not change. Only water content and the whole volume of the solution will get affected.

So, the resulted solution will have acid content as follows:

The acid content in the solution after adding x liters of water = 45% of 1125 …(ii)

[∵ we know that the amount of acid content will not change after adding water to the whole solution. So, from equation (i), we have this conclusion]

Also, according to the question,

This resulting mixture will contain more than 25% acid content.

So, we have

Acid content in the solution after adding x litres of water > 25% of new mixture

⇒ 45% of 1125 > 25% of (1125 + x) [∵ from equation (ii)]

$$\Rightarrow \frac{45}{100} \times 1125 > \frac{25}{100} \times (1125 + x)$$

$\Rightarrow 45 \times 1125 > 25(1125 + x)$

$\Rightarrow 9 \times 1125 > 5(1125 + x)$

$\Rightarrow 9 \times 225 > 1125 + x$

$\Rightarrow 2025 > 1125 + x$

$\Rightarrow x < 2025 - 1125$

$\Rightarrow x < 900$

Also,

This resulting mixture will contain less than 30% acid content.

So, we have

Acid content in the solution after adding x litres of water < 30% of new mixture

⇒ 45% of 1125 < 30% of (1125 + x) [∵ from equation (ii)]

$\Rightarrow \frac{45}{100} \times 1125 < \frac{30}{100} \times (1125 + x)$

⇒ 45 × 1125 < 30(1125 + x)

⇒ 9 × 1125 < 6(1125 + x)

⇒ 3 × 1125 < 2(1125 + x)

⇒ 3375 < 2250 + 2x

⇒ 2x + 2250 > 3375

⇒ 2x > 3375 – 2250

⇒ 2x > 1125

$\Rightarrow x > \frac{1125}{2}$

⇒ x > 562.5

Thus, we have got x < 900 and x > 562.5.

⇒ 562.5 < x < 900

Hence, the required liters of water to be added to 1125 liters of solution is between 562.5 liters and 900 liters.

11. Question

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If there are 640 liters of the 8% solution, how many liters of 2% solution will have to be added?

Answer

Volume of the 8% solution = 640 litres

Boric acid present in the 8% solution = 8% of 640 …(i)

And the rest 92% of 640 litres is water in the 8% solution.

Let volume of 2% solution added to 640 liters be x.

Boric acid present in 2% solution = 2% of x …(ii)

New volume of 8% solution = 640 + x …(iii)

Boric acid present in the new solution (that is, after adding x litres of 2% solution to 8% solution) = Boric acid present in the 8% solution + Boric acid present in the 2% solution [from (i) & (ii)]

⇒ Boric acid present in the new solution = 8% of 640 + 2% of x

⇒Boric acid present in the new solution = $\left(\frac{8}{100} \times 640\right) + \left(\frac{2}{100} \times x\right)$

⇒Boric acid present in the new solution = $\frac{2x}{100} + \left(\frac{8}{100} \times 640\right)$ …(iv)

According to the question,

The resulting mixture is to be more than 4% but less than 6% boric acid.

That is, the boric acid content in the resulting mixture must be more than 4% but less than 6% boric acid.

So, first let us take boric acid content in the resulting mixture to be more than 4%.

⇒ Boric acid present in the new solution > 4% of the new volume of 8% solution

$\Rightarrow \frac{2x}{100} + \left(\frac{8}{100} \times 640\right) > \frac{4}{100} \times (640 + x)$

[from (iii) & (iv)]

$\Rightarrow \frac{2x}{100} + \frac{8 \times 640}{100} > \frac{4(640 + x)}{100}$

$\Rightarrow \frac{2x + (8 \times 640)}{100} > \frac{4(640 + x)}{100}$

⇒ 2x + 5120 > 2560 + 4x

⇒ 5120 – 2560 > 4x – 2x

⇒ 2560 > 2x

⇒ 2x < 2560

$\Rightarrow x < \frac{2560}{2}$

⇒ x < 1280

Now, let us the take boric acid in the resulting mixture to be less than 6%.

⇒ Boric acid present in the new solution < 6% of the new volume of 8% solution

$\Rightarrow \frac{2x}{100} + \left(\frac{8}{100} \times 640\right) < \frac{6}{100} \times (640 + x)$

[from (iii) & (iv)]

$\Rightarrow \frac{2x}{100} + \frac{8 \times 640}{100} < \frac{6(640 + x)}{100}$

$\Rightarrow \frac{2x + (8 \times 640)}{100} < \frac{6(640 + x)}{100}$

⇒ 2x + 5120 < 3840 + 6x

⇒ 5120 – 3840 < 6x – 2x

⇒ 1280 < 4x

⇒ 4x > 1280

$\Rightarrow x > \frac{1280}{4}$

⇒ x > 320

We have

x < 1280 & x > 320

⇒ 320 < x < 1280

Hence, the required liters of 2% solution to be added to 8% of the solution is between 320 liters and 1280 liters.

12. Question

The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH reading is 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level is normal.

Answer

Given that,

pH value of the first reading = 7.48

pH value of the second reading = 7.85

We need to find the range of the pH value for the third reading so that the acidity level in the pool is normal.

But the acidity level in the pool is considered normal when the average pH reading of the three measurements is between 7.2 and 7.8.

That is, 7.2 < average pH reading of the three measurements < 7.8 ...(i)

Let us find the average pH reading of the three measurements.

For this, let the pH value of the third reading be x.

Then, the average is given by

$$\text{Average} = \frac{\text{Sum of pH value of all three readings}}{3}$$

$$\Rightarrow \text{Average} = \frac{\text{pH value of first reading} + \text{pH value of second reading} + \text{pH value of third reading}}{3}$$

$$\Rightarrow \text{Average} = \frac{7.48 + 7.85 + x}{3}$$

$$\Rightarrow \text{Average} = \frac{15.33 + x}{3}$$

Substituting this value of average in inequality (i), we get

$$7.2 < \frac{15.33 + x}{3} < 7.8$$

Multiply 3 throughout the inequality, we have

$$7.2 \times 3 < \frac{15.33 + x}{3} \times 3 < 7.8 \times 3$$

$\Rightarrow 22.6 < 15.33 + x < 23.4$

Now, subtract 15.33 throughout the inequality,

$\Rightarrow 22.6 - 15.33 < 15.33 + x - 15.33 < 23.4 - 15.33$

$\Rightarrow 7.27 < x < 8.07$

This means, x lies between values 7.27 and 8.07.

Thus, the pool's acidity level would be normal when the range of pH value in the third measurement would be between 7.27 and 8.07.

Exercise 15.5

1. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

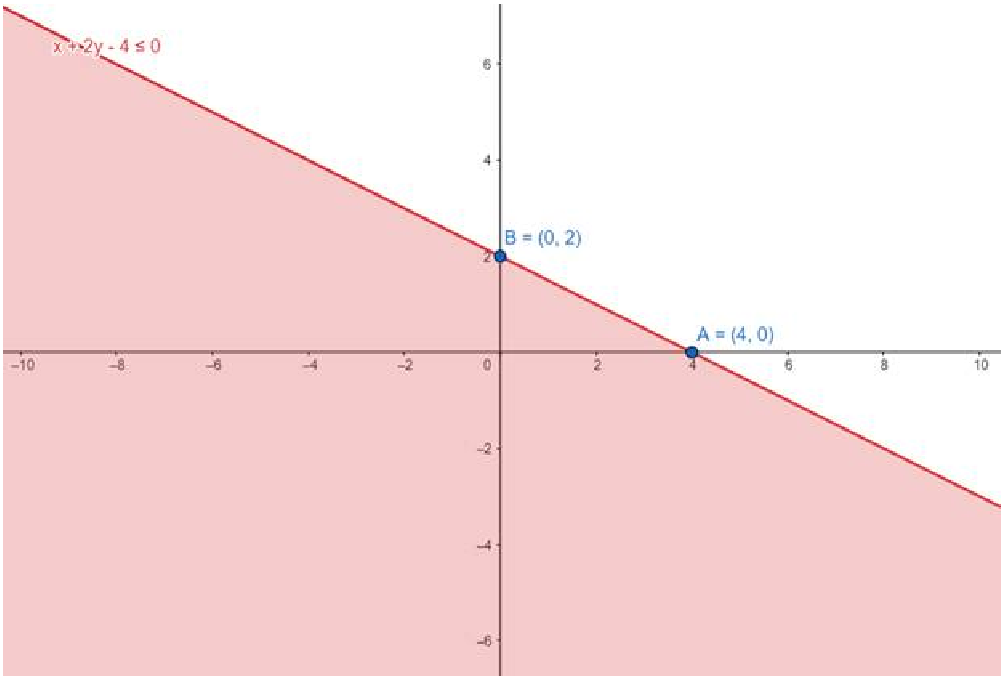
$x + 2y - 4 \le 0$

Answer

First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality, You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

x + 2y – 4 ≤ 0

x	0	2	4
y	2	1	0



2. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

x + 2y ≥ 6

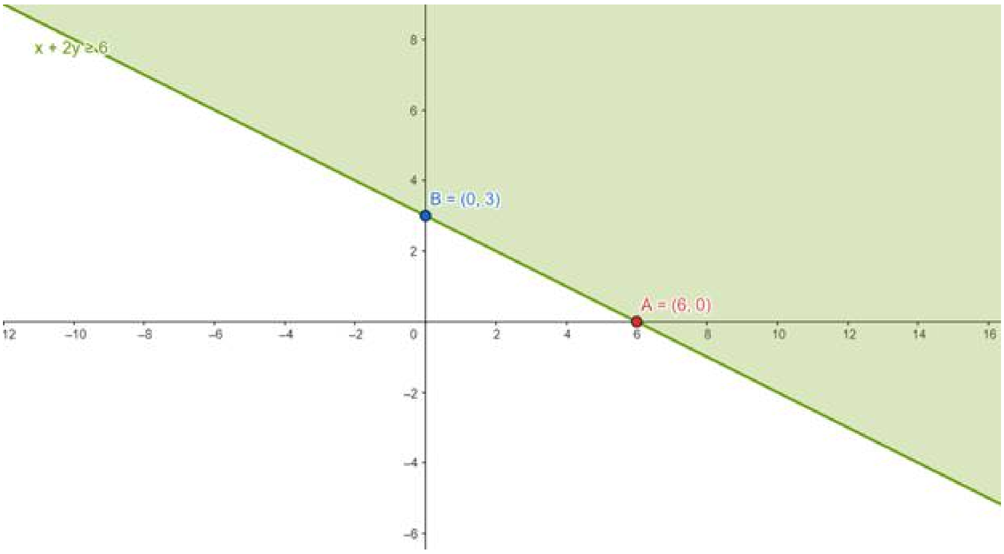
Answer

First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

x + 2y ≥ 6

x	0	2	6
y	3	2	0



3. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

x + 2 ≥ 0

Answer

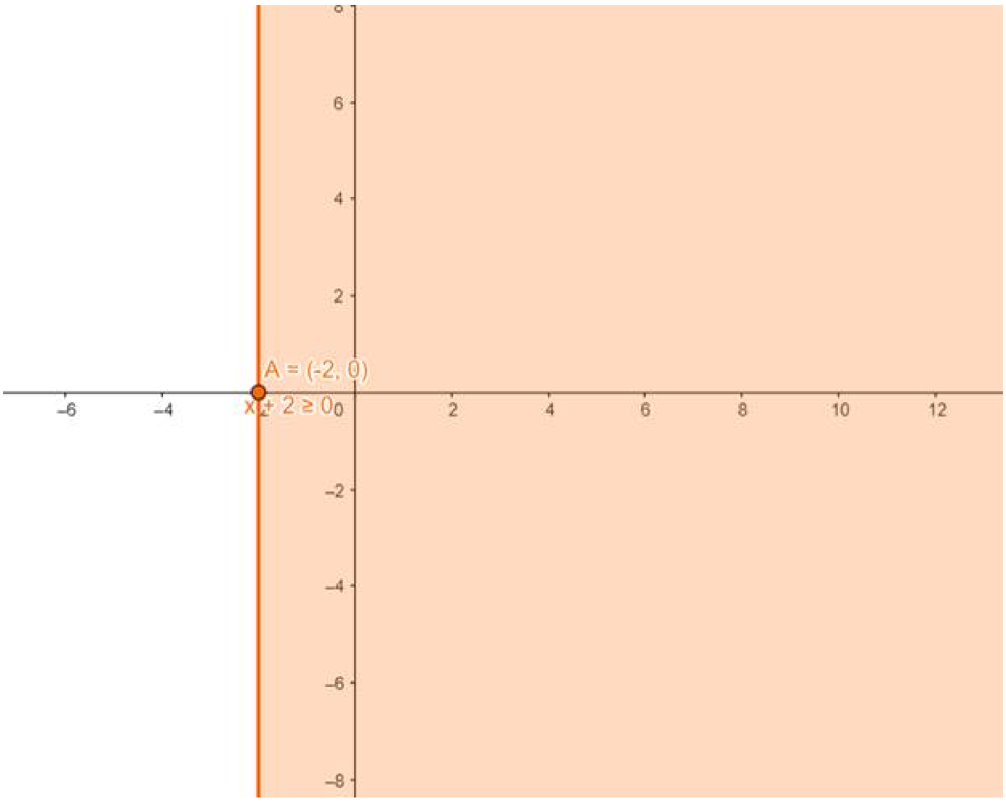
First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

x + 2 ≥ 0

x ≥ –2

As there is only one variable ‘x,’ and y = 0, which means that x has only one value when considered as an equation. Therefore no table is required in this problem.



4. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

x – 2y < 0

Answer

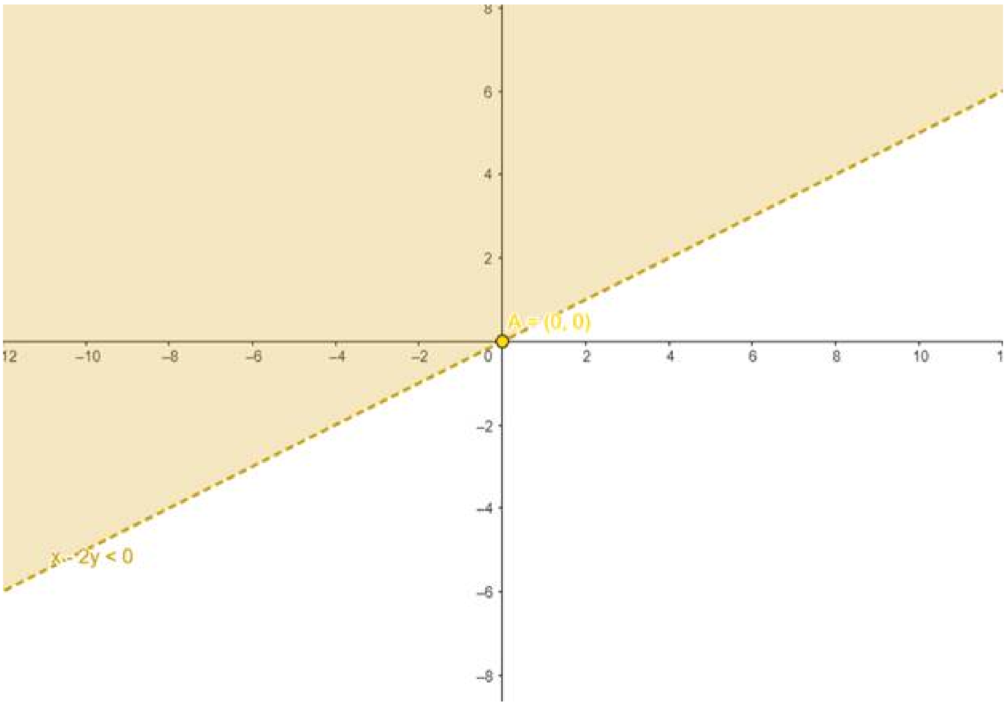
First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

x – 2y < 0

x < 2y

x	0	2	4
y	0	1	2



5. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

– 3x + 2y ≤ 6

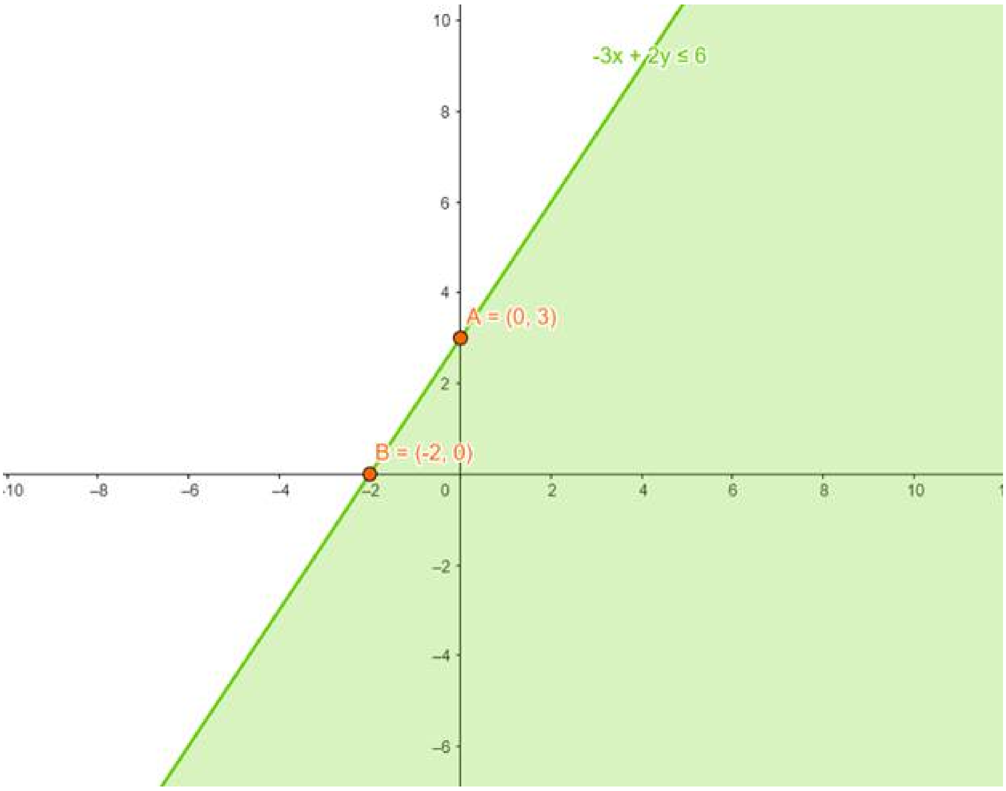
Answer

First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

– 3x + 2y ≤ 6

x	0	2	-2
y	3	6	0



6. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

$x \leq 8 - 4y$

Answer

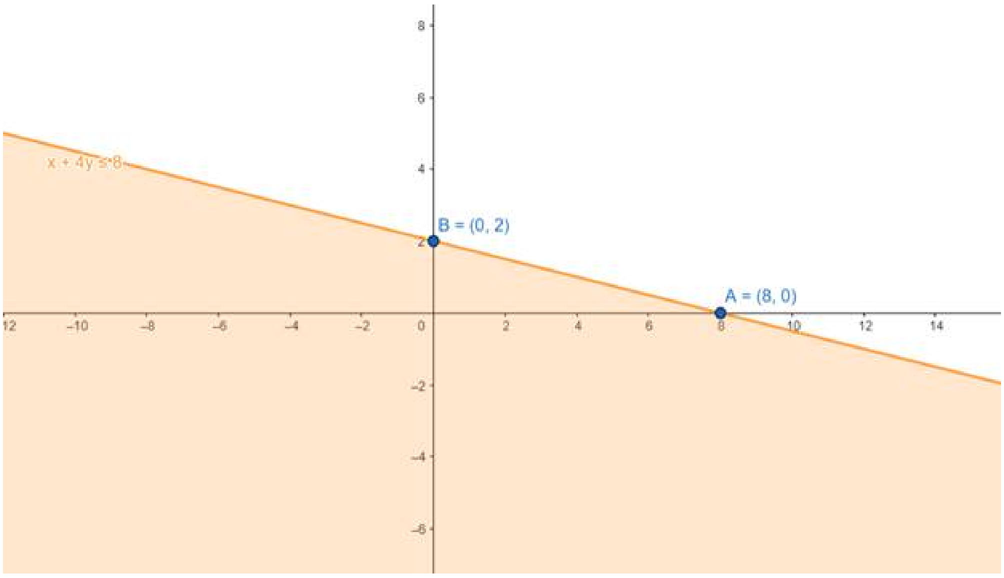
First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

$x \leq 8 - 4y$

$x + 4y \leq 8$

x	0	4	8
y	2	1	0



7. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

$0 \leq 2x - 5y + 10$

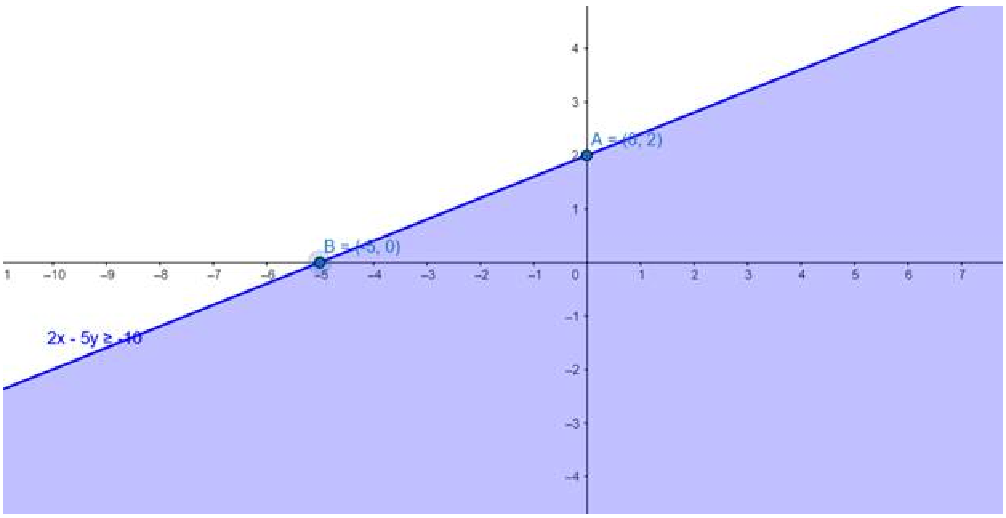
Answer

First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

$0 \leq 2x - 5y + 10$

x	0	5	–5
y	2	4	0



8. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

$3y > 6 - 2x$

Answer

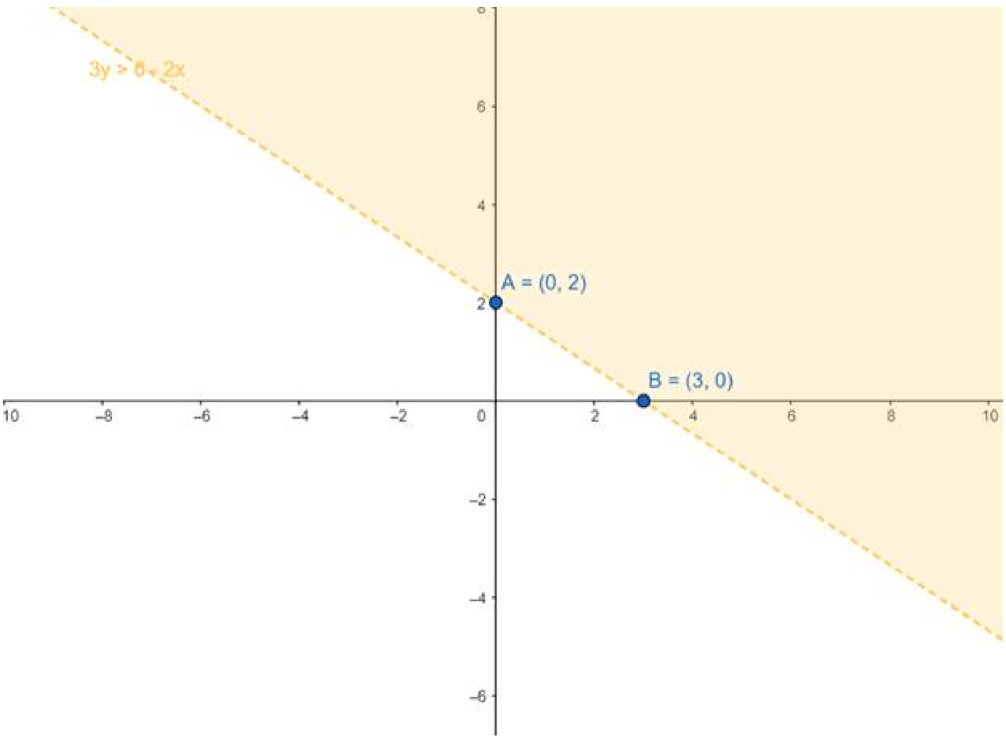
First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

3y > 6 – 2x

2x + 3y > 6

x	0	1	3
y	2	1.33	0



9. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

y> 2x – 8

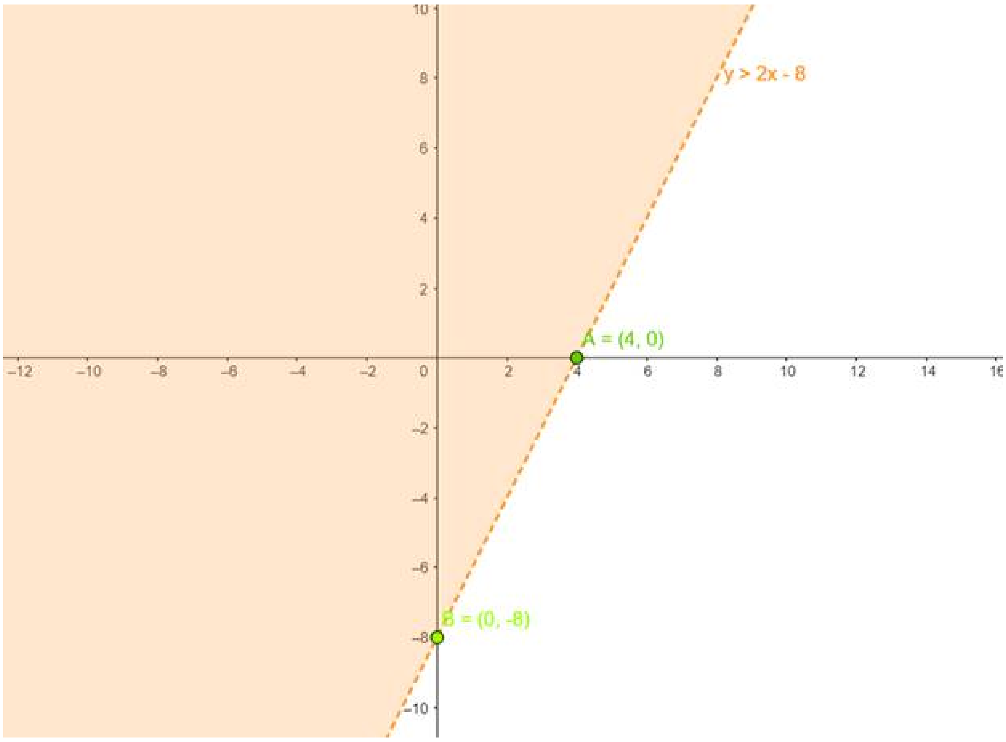
Answer

First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

y> 2x – 8

x	0	2	4
y	–8	–4	0



10. Question

Represent to solution set of the following inequations graphically in two dimensional plane:

3x – 2y ≤ x + y – 8

Answer

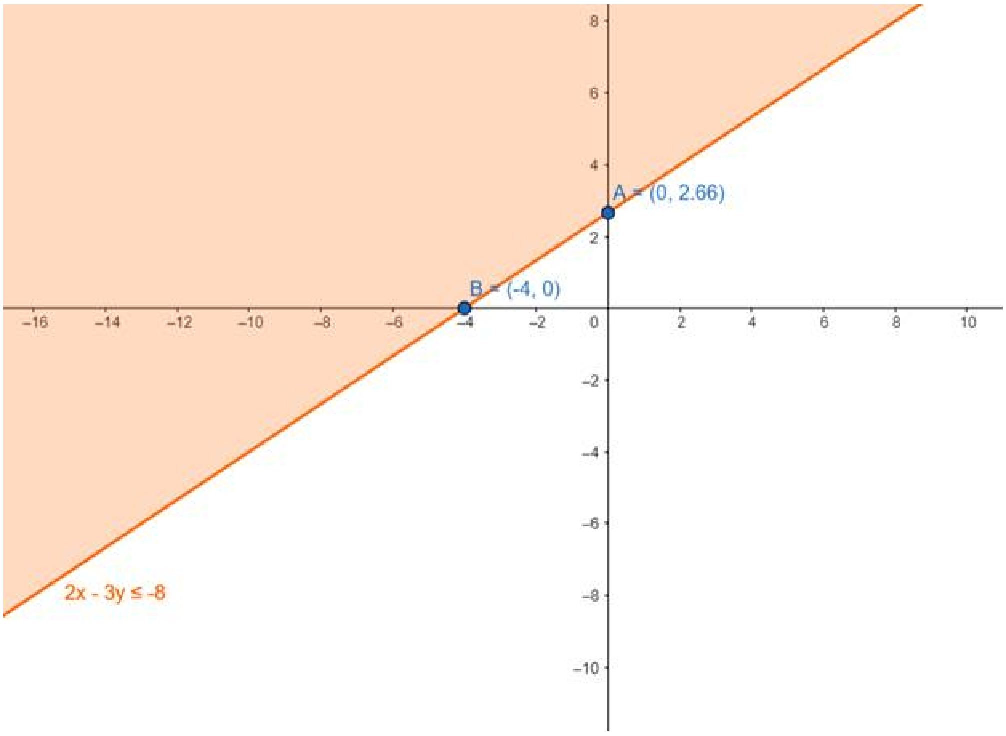
First, we will find the solutions of the given equation by hit and trial method and afterward we will plot the graph of the equation and shade the side containing solutions of the inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

3x – 2y ≤ x + y – 8

2x – 3y ≤ –8

x	0	2	–4
y	2.667	4	0



Exercise 15.6

1 A. Question

Solve the following systems of linear inequaitons graphically.

$2x + 3y \leq 6$, $3x + 2y \leq 6$, $x \geq 0$, $y \geq 0$

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

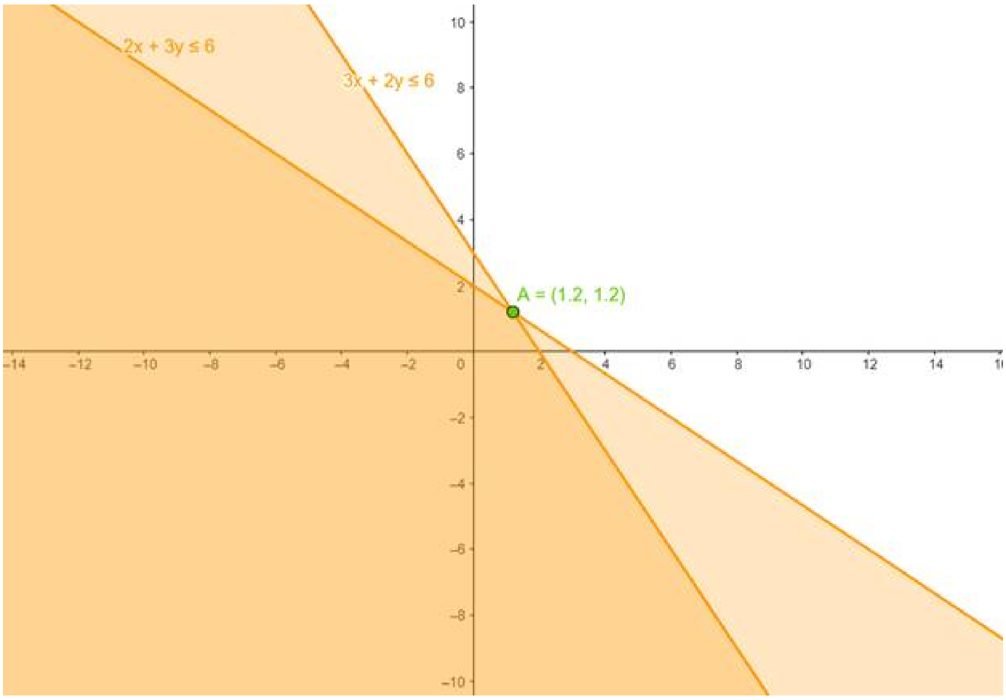
$2x + 3y \leq 6$

x	0	1	3
y	2	1.33	0

$3x + 2y \leq 6$

x	0	1	2
y	3	1.5	0

$x \geq 0$, $y \geq 0$



1 B. Question

Solve the following systems of linear inequaitons graphically.

2x + 3y≤ 6, x + 4y ≤ 4, x ≥ 0, y ≥ 0

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

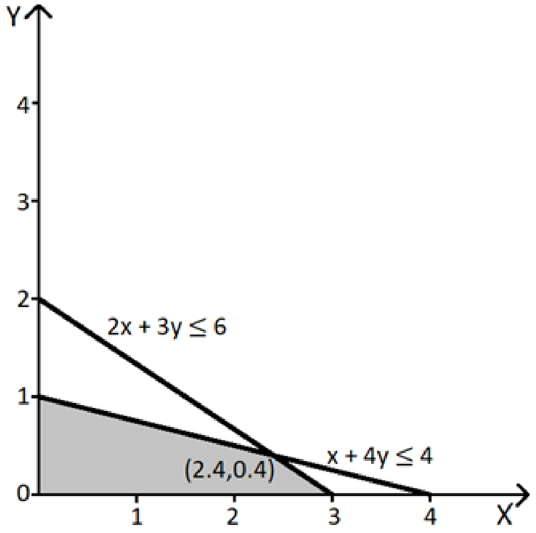
2x + 3y≤ 6

x	0	1	3
y	2	1.33	0

x + 4y ≤ 4

x	0	2	4
y	1	0.5	0

x ≥ 0, y ≥ 0



1 C. Question

Solve the following systems of linear inequations graphically.

x – y ≤ 1, x + 2y≤ 8, 2x + y ≥ 2, x ≥ 0, y ≥ 0

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

x – y ≤ 1

x	0	2	1
y	–1	1	0

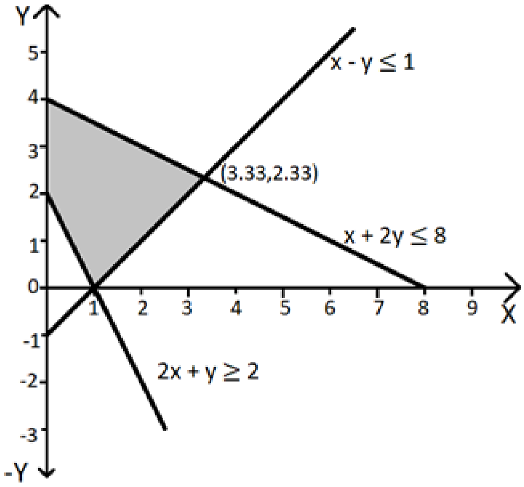
x + 2y≤ 8

x	0	4	8
y	4	2	0

2x + y ≥ 2

x	0	2	1
y	2	–2	0

x ≥ 0, y ≥ 0



1 D. Question

Solve the following systems of linear inequaitons graphically.

x + y ≥ 1, 7x + 9y ≤ 63, x ≤6, y ≤ 5, x ≥ 0, y ≥ 0

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

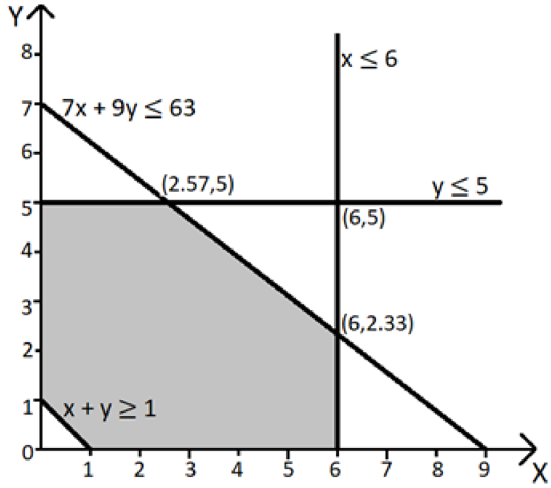
x + y ≥ 1

x	0	2	1
y	1	–1	0

7x + 9y ≤ 63

x	0	5	9
y	7	3.11	0

x ≤6, y ≤ 5, x ≥ 0, y ≥ 0



1 E. Question

Solve the following systems of linear inequations graphically.

2x + 3y≤35, y ≥ 3, x ≥ 2, x ≥ 0, y ≥ 0

Answer

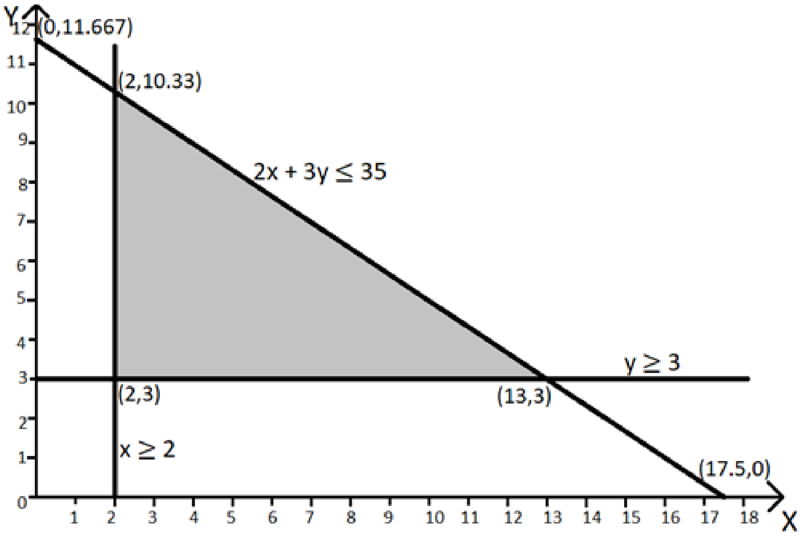
First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

2x + 3y≤35

x	0	5	17.5
y	11.667	8.33	0

y ≥ 3, x ≥ 2, x ≥ 0, y ≥ 0



2 A. Question

how that the solution set of the following linear inequations is empty set :

x – 2y ≥ 0, 2x – y ≤ –2, x ≥ 0, y ≥ 0

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality.

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

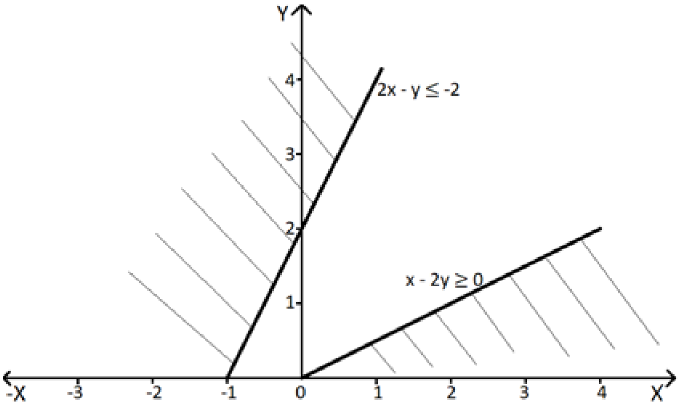
x – 2y ≥ 0

x	0	2	4
y	0	1	2

2x – y ≤ –2

x	0	1	–1
y	2	4	0

x ≥ 0, y ≥ 0



The lines do not intersect each other for x ≥ 0, y ≥ 0

Hence, there is no solution for the given inequations.

2 B. Question

how that the solution set of the following linear inequations is empty set :

x + 2y≤ 3, 3x + 4y≥ 12, y ≥ 1, x ≥ 0, y ≥ 0

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

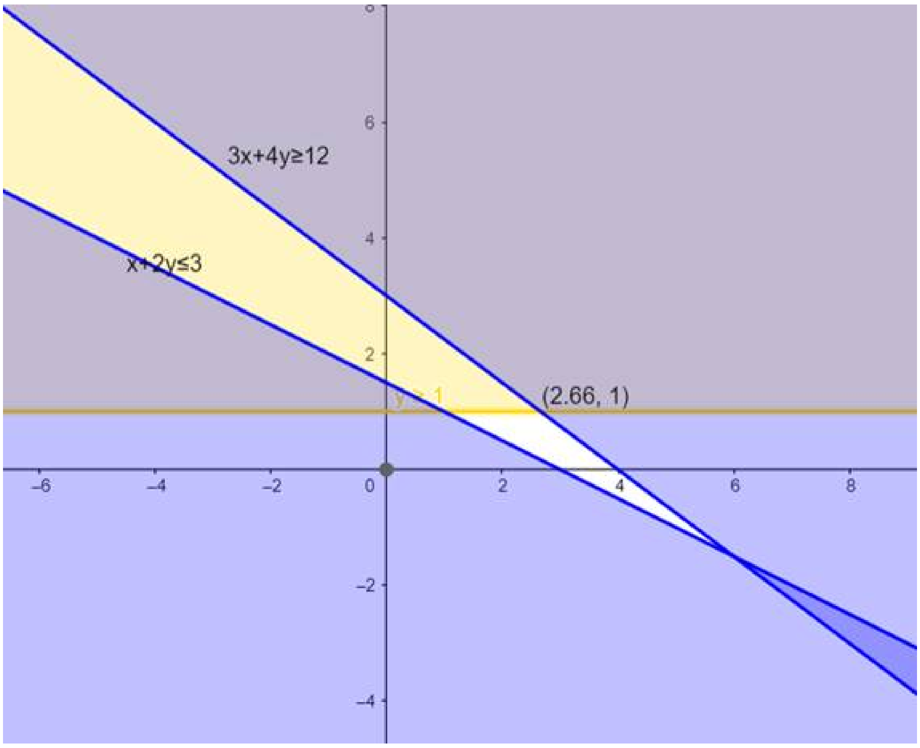
x + 2y≤ 3

x	0	1	3
y	1.5	1	0

3x + 4y≥ 12

x	0	2	4
y	3	1.5	0

y ≥ 1, x ≥ 0, y ≥ 0



3. Question

Find the linear inequations for which the shaded area in Fig. 15.41 is the solution set. Draw the diagram of the solution set of the linear inequations.

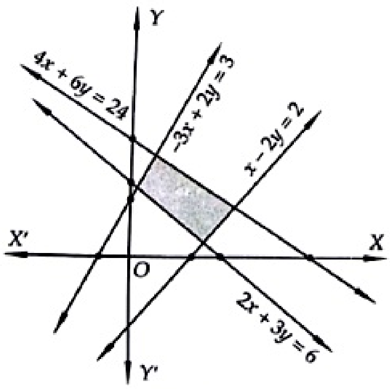


Fig. 15.41

Answer

In this question, we will apply the concept of a common solution area to find the signs of inequality by using their given equations and the given common solution area(shaded part).

If a line is in the form ax+by = c and **c is positive constant**(in case of negative c the rule becomes opposite), so there will be two cases to discuss, which are,

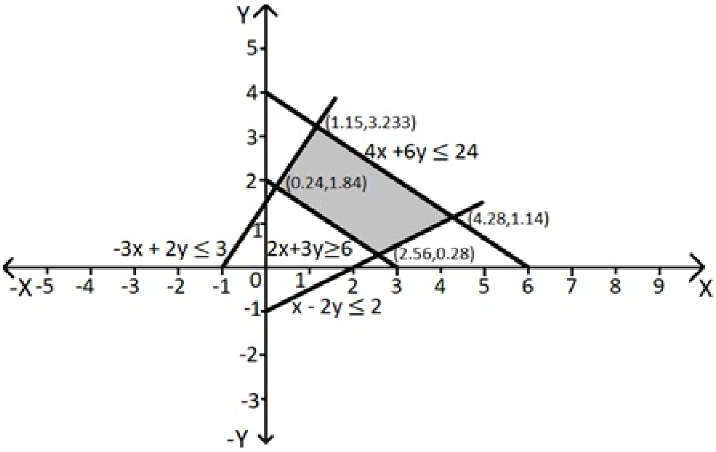
If a line is above origin :–

(i). If the shaded area is below the line then ***ax+by<c***

(ii). If the shaded area is above the line then ***ax+by>c***

If a line is below origin then the rule becomes opposite.

According to the rules the answer will be,



4. Question

Find the linear inequations for which the solution set is the shaded region given in Fig. 15.42.

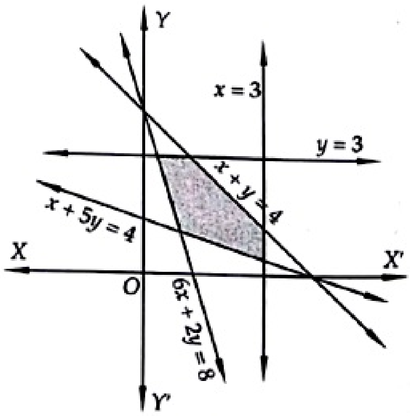


Fig. 15.42

Answer

In this question we will apply the concept of common solution area to find the signs of inequality by using their given equations and the given common solution area(shaded part).

If a line is in the form ax+by = c and **c is positive constant**(in case of negative c the rule becomes opposite), so there will be two cases to discuss, which are,

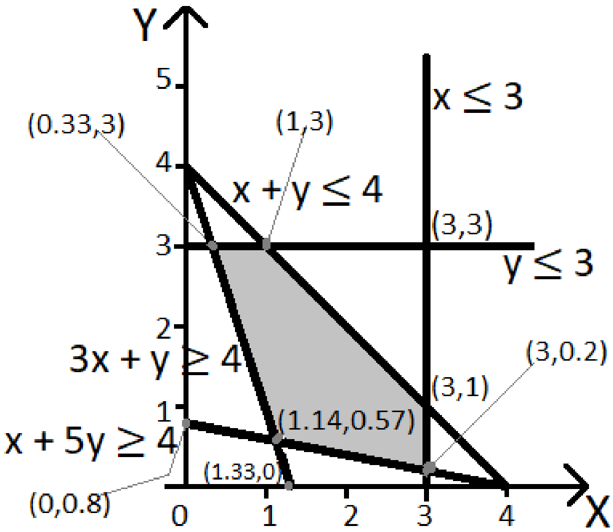
If a line is above origin :–

(i). If the shaded area is below the line then ***ax+by<c***

(ii). If the shaded area is above the line then ***ax+by>c***

If a line is below origin, then the rule becomes opposite.

According to the rules, the answer will be,



5. Question

Show that the solution set of the following linear inequations is an unbounded set :

$x + y \geq 9$, $3x + y \geq 12$, $x \geq 0$, $y \geq 0$.

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

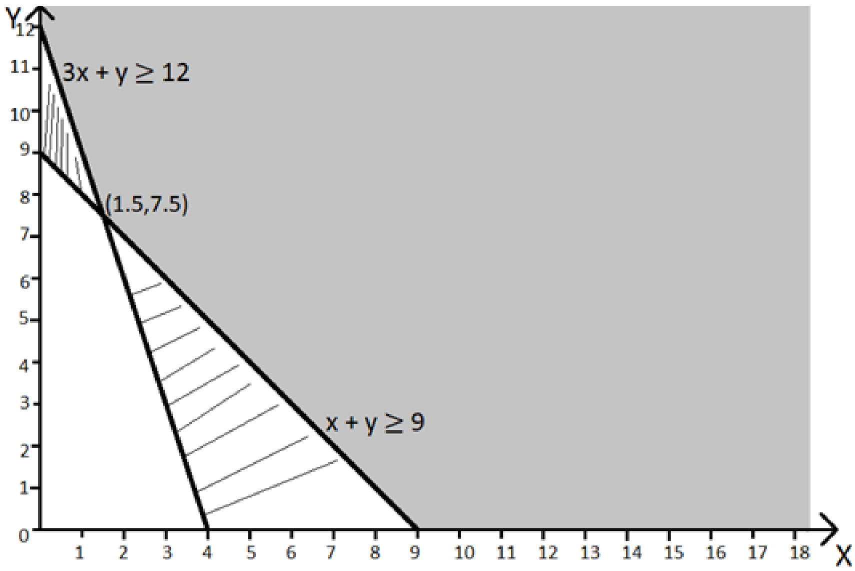
$x + y \geq 9$

x	0	5	9
y	9	4	0

$3x + y \geq 12$

x	0	2	4
y	12	6	0

$x \geq 0$, $y \geq 0$



6 A. Question

solve the following systems of inequations graphically :

$2x + y \geq 8$, $x + 2y \geq 8$, $x + y \leq 6$

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

$2x + y \geq 8$

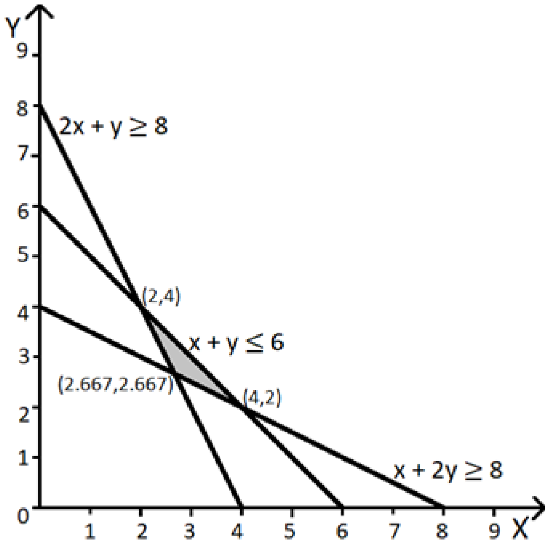
x	0	2	4
y	8	4	0

$x + 2y \geq 8$

x	0	4	8
y	4	2	0

$x + y \leq 6$

x	0	3	6
y	6	3	0



6 B. Question

solve the following systems of inequations graphically :

12 + 12y ≤ 840, 3x + 6y ≤ 300,8x + 4y ≤ 480, x ≥ 0, y ≥ 0

Answer

First we will find the solutions of the given equations by hit and trial method and afterwards we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

12 + 12y ≤ 840

x + y ≤ 70

x	0	35	70
y	70	35	0

3x + 6y ≤ 300

x + 2y ≤ 100

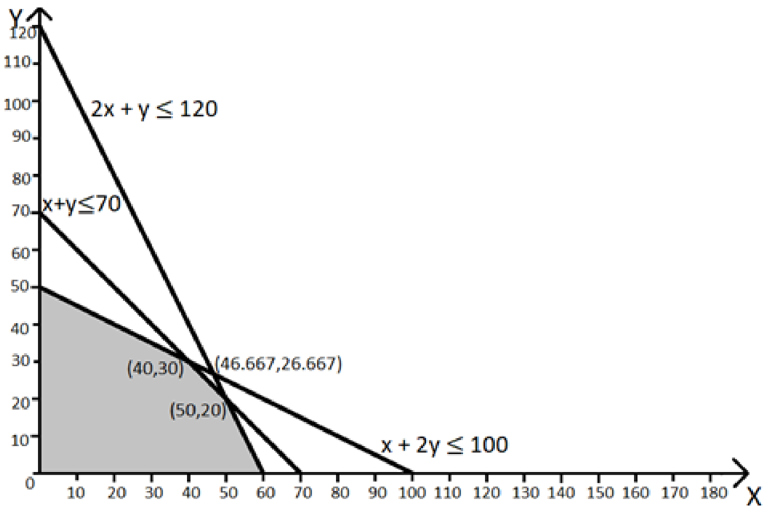
x	0	50	100
y	50	25	0

$8x + 4y \leq 480$

$2x + y \leq 120$

x	0	30	60
y	120	60	0

$x \geq 0, y \geq 0$



6 C. Question

solve the following systems of inequations graphically :

$x + 2y \leq 40, 3x + y \geq 30, 4x + 3y \geq 60, x \geq 0, y \geq 0$

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

$x + 2y \leq 40$

x	0	30	60
y	120	60	0

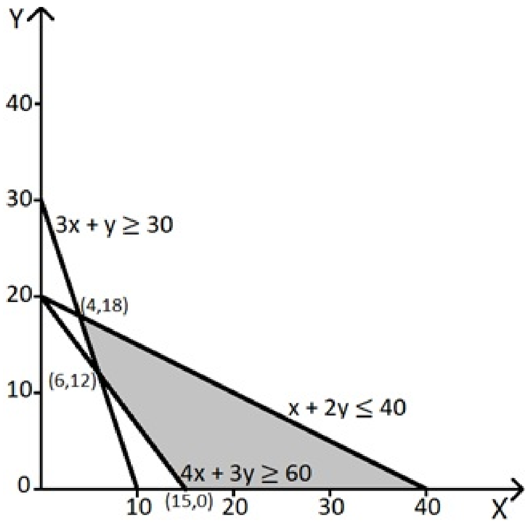
$3x + y \geq 30$

x	0	30	60
y	120	60	0

$4x + 3y \geq 60$

x	0	30	60
y	120	60	0

$x \geq 0, y \geq 0$



6 D. Question

solve the following systems of inequations graphically :

$5x + y \geq 10, 2x + 2y \geq 12, x + 4y \geq 12, x \geq 0, y \geq 0$

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality, You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

$5x + y \geq 10$

x	0	1	2
y	10	5	0

$2x + 2y \geq 12$

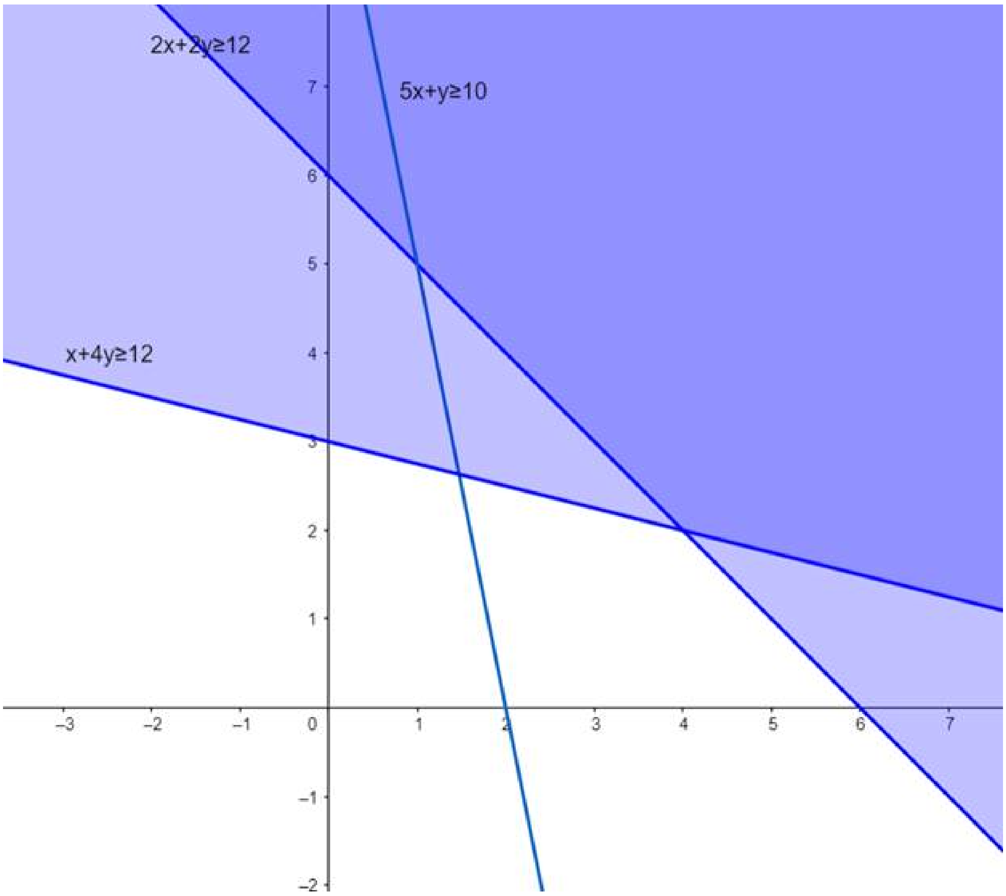
$x + y \geq 6$

x	0	3	6
y	6	3	0

$x + 4y \geq 12$

x	0	4	12
y	3	2	0

$x \geq 0, y \geq 0$



7. Question

Show that the following system of linear equations has no solution :

$x + 2y \geq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$.

Answer

First, we will find the solutions of the given equations by hit and trial method and afterward we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of the solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e., x and y–intercepts always,

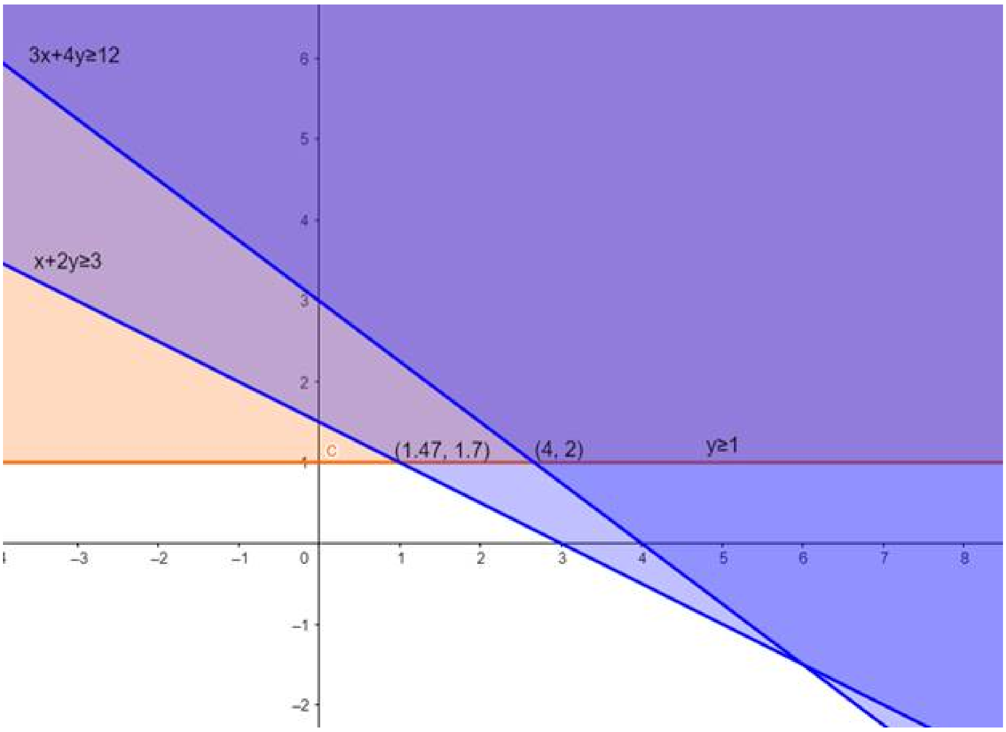
$x + 2y \geq 3$

x	0	1	3
y	1.5	1	0

$3x + 4y \geq 12$

x	0	2	4
y	3	1.5	0

x ≥ 0, y ≥ 1



8. Question

Show that the solution set of the following system of linear inequalities is an unbounded region 2x + y≥ 8, x + 2y≥ 10, x ≥ 0, y ≥ 0.

Answer

First we will find the solutions of the given equations by hit and trial method and afterwards we will plot the graph of the equations and shade the side with grey color containing common solutions or intersection of solution set of each inequality,

You can choose any value but find the two mandatory values which are at x = 0 and y = 0, i.e. x and y intercepts always,

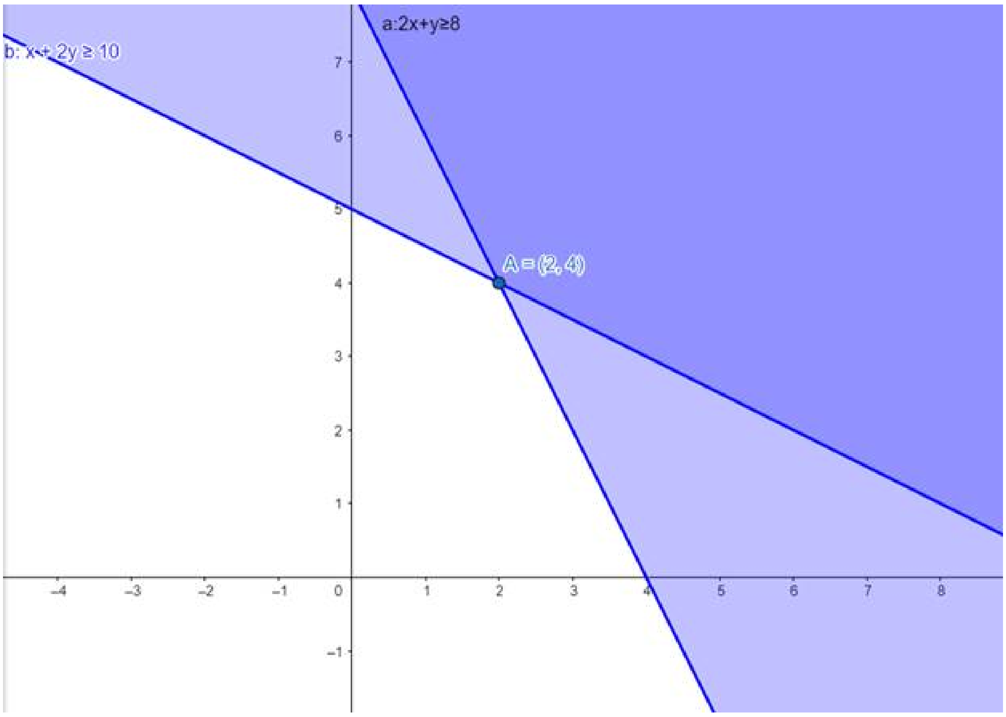
2x + y≥ 8

x	0	2	4
y	8	4	0

x + 2y≥ 10

x	0	2	10
y	5	4	0

x ≥ 0, y ≥ 0.



Very Short Answer

1. Question

Write the solution set of the inequation $\frac{x^2}{x-2} > 0$.

Answer

$\frac{x^2}{x-2} > 0$

⇒ $x^2 > 0$ and $x - 2 > 0$

⇒ $x > 0$ and $x > 2$

We have to take intersection of $x > 0$ and $x > 2$

So, answer should be $x \in (2, \infty)$

2. Question

Write the solution set of the inequation $x + \frac{1}{x} \geq 2$.

Answer

$x + \frac{1}{x} \geq 2$

⇒ $x + \frac{1}{x} - 2 \geq 0$

$\Rightarrow \frac{x^2-2x+1}{x} \geq 0$

Case I : $x^2 - 2x + 1 \geq 0$ and $x > 0$

$(x - 1)^2 \geq 0$ and $x > 0$

So, by taking intersection $x > 0$

Case II : : $x^2 - 2x + 1 \leq 0$ and $x < 0$

$(x - 1)^2 \leq 0$ and $x < 0$

Square term is always positive so case II is irrelevant.

Then, the final answer of question is $x \in (0, \infty)$

3. Question

Write the set of values of x satisfying the inequation $\left(x^2 - 2x + 1\right)\left(x - 4\right) \geq 0$.

Answer

$(x^2 - 2x + 1)(x - 4) \geq 0$

$(x - 1)^2(x - 4) \geq 0$

$\Rightarrow (x - 1)^2 \geq 0$ and $(x - 4) \geq 0$

$\Rightarrow$ Square term is always positive and $x \geq 4$

So, answer is $x \in [4, \infty)$

4. Question

Write the solution set of the inequation $|2 - x| = x - 2$.

Answer

$|2 - x| = x - 2$

$\Rightarrow |x - 2| = x - 2$

We know that mode is always positive or zero. So, $x - 2$ is also positive or zero.

$\Rightarrow x - 2 \geq 0$

$\Rightarrow x \geq 2$

So, final answer is $x \in [2, \infty)$

5. Question

Write the set of values of x satisfying $|x - 1| \leq 3$ and $|x - 1| \leq 1$.

Answer

$|x - 1| \leq 3$

$\Rightarrow -3 \leq x - 1 \leq 3$

$\Rightarrow -2 \leq x \leq 4$

$|x - 1| \leq 1$

$\Rightarrow -1 \leq x - 1 \leq 1$

$\Rightarrow 0 \leq x \leq 2$

We have to take intersection of $x \in [-2, 4]$ and $x \in [0, 2]$

So, final answer is x ∈ [0, 2]

6. Question

Write the solution set of the inequation $\left|\frac{1}{x}-2\right|<4$.

Answer

$\left|\frac{1}{x}-2\right|<4$

$-4<\left(\frac{1}{x}-2\right)<4$

$-2<\frac{1}{x}<6$

Part I : $\frac{1}{x}>-2$

$\Rightarrow x<\frac{-1}{2}$

Part II: $\frac{1}{x}<6$

$\Rightarrow x>\frac{1}{6}$

We have to take union of $x<\frac{-1}{2}$ and $x>\frac{1}{6}$

So, the final answer is $x\in\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{6},\infty\right)$.

7. Question

Write the number of integral solutions of $\frac{x+2}{x^2+1}>\frac{1}{2}$.

Answer

$\frac{x+2}{x^2+1}>\frac{1}{2}$

$\Rightarrow \frac{x+2}{x^2+1}-\frac{1}{2}>0$

$\Rightarrow \frac{2(x+2)-(x^2+1)}{2(x^2+1)}>0$

$\Rightarrow \frac{-x^2+2x+3}{2(x^2+1)}>0$

Here, denominator i.e., $x^2 + 1$ is always positive and not equal to zero. So, neglect it.

$\Rightarrow -x^2 + 2x + 3 > 0$

$\Rightarrow x^2 - 2x - 3 < 0$

$\Rightarrow (x - 3)(x + 1) < 0$

Case I : (x – 3) < 0 and (x + 1) > 0

⇒ x < 3 and x > -1

By takin intersection x ∈ (-1, 3)

Case II : (x – 3) > 0 and (x + 1) < 0

⇒ x > 3 and x < -1

By taking intersection x ∈ ∅. So, case II is irrelevant.

So, the complete solution is x ∈ (-1, 3)

The integral solution is 0, 1 and 2. So, number of integral

solution is 3.

8. Question

Write the set of values of x satisfying the inequations 5x + 2 < 3x + 8 and $\frac{x+2}{x-1} < 4$.

Answer

Part I : 5x + 2 < 3x + 8

⇒ 2x < 6

⇒ x < 3

Part II : $\frac{x+2}{x-1} < 4$

$\Rightarrow \frac{x+2}{x-1} - 4 < 0$

$\Rightarrow \frac{(x+2) - 4(x-1)}{x-1} < 0$

$\Rightarrow \frac{-3x+6}{x-1} < 0$

Case I : -3x + 6 < 0 and x – 1 > 0

⇒ x > 2 and x > 1

By taking intersection x ∈ (2, ∞)

Case II : -3x + 6 > 0 and x – 1 < 0

⇒ x < 2 and x < 1

By takin intersection x ∈ (-∞, 1)

Taking union of case I and case II, x ∈ (-∞, 1) (2, ∞)

We have to take intersection of part I and part II, we have

final answer i.e., x ∈ (-∞, 1) (2, 3)

9. Question

Write the solution set of $\left|x+\frac{1}{x}\right| > 2$.

Answer

$\left|x+\frac{1}{x}\right| > 2$

$\Rightarrow \; -2 < x + \frac{1}{x} < 2$

Part I : $x + \frac{1}{x} > -2$

$\Rightarrow \frac{x^2+2x+1}{x} > 0$

$\Rightarrow \dfrac{(x+1)^2}{x} > 0$

Square term is always positive and $(x + 1)^2 \neq 0$. So, x > 0 and x ≠ -1

Part II : $x + \frac{1}{x} < 2$

$\Rightarrow \dfrac{x^2 - 2x + 1}{x} < 0$

$\Rightarrow \dfrac{(x-1)^2}{x} < 0$

Square term is always positive and $(x - 1)^2 \neq 0$. So, x < 0 and x ≠ 1

So, by taking union of part I and part II we have final solution i.e., x ∈ R – {-1, 0, 1}

10. Question

Write the solution set of the inequation $|x - 1| \geq |x - 3|$.

Answer

|x – 1| ≥ |x – 3|

Squaring both sides

$|x - 1|^2 \geq |x - 3|^2$

$(x - 1)^2 \geq (x - 3)^2$

$x^2 - 2x + 1 \geq x^2 - 6x + 9$

4x – 8 ≥ 0

x ≥ 2

x ∈ [2, ∞)

MCQ

1. Question

Mark the Correct alternative in the following:

If x < 7, then

A. −x < −7

B. −x ≤ −7

C. −x > −7

D. −x ≥ −7

Answer

We know that when we change the sign of inequalities then greater tan changes to less than and vice versa also true.

So, -x > -7

2. Question

Mark the Correct alternative in the following:

If −3x + 17 < −13, then

A. x∈(10, ∞)

B. $x \in [10, \infty)$

C. $x \in (-\infty, 10]$

D. $x \in [-10, 10)$

Answer

$-3x + 17 < -13$

$-3x < -13 - 17$

$-3x < -30$

$3x > 30$

$x > 10$

$x \in (10, \infty)$

3. Question

Mark the Correct alternative in the following:

Given that x, y and b are real numbers and x < y, b > 0, then

A. $\frac{x}{b} < \frac{y}{b}$

B. $\frac{x}{b} \le \frac{y}{b}$

C. $\frac{x}{b} > \frac{y}{b}$

D. $\frac{x}{b} \ge \frac{y}{b}$

Answer

x < y and b > 0

$\frac{x}{b} < \frac{y}{b}$ because b is greater than zero.

4. Question

Mark the Correct alternative in the following:

If x is a real number and $|x| < 5$, then

A. $x \ge 5$

B. $-5 < x < 5$

C. $x \le -5$

D. $-5 \le x \le 5$

Answer

$|x| < 5$

$-5 < x < 5$

5. Question

Mark the Correct alternative in the following:

If x and a are real numbers such that a > 0 and $|x| > a$, then

A. x∈(−a, ∞)

B. x∈[−∞, a]

C. x∈(−a, a)

D. x∈(−∞, −a) (a, ∞)

Answer

|x| > a

x < -a and x > a

x ∈ (-∞, -a) (a, ∞)

6. Question

Mark the Correct alternative in the following:

If $|x - 1| > 5$, then

A. x∈(−4, 6)

B. x∈[−4, 6)

C. x∈(−∞, −4) (6, ∞)

Answer

|x – 1| > 5

x – 1 < -5 and x – 1 > 5

x < -4 and x > 6

x ∈ (-∞, -4) (6, ∞)

7. Question

Mark the Correct alternative in the following:

If $|x + 2| \leq 9$, then

A. x∈(−7, 11)

B. x∈[−11, 7]

C. x∈(−∞, −7) (11, ∞)

D. x∈(−∞, −7) [11, ∞)

Answer

|x + 2| ≤ 9

-9 ≤ x + 2 ≤ 9

-11 ≤ x ≤ 7

x ∈ [-11, 7]

8. Question

Mark the Correct alternative in the following:

The inequality representing the following graph is

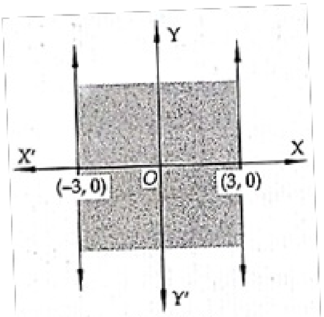


Fig. 15.43

A. $|x| < 3$

B. $|x| \leq 3$

C. $|x| > 3$

D. $|x| \geq 3$

Answer

The given figure is shaded between -3 to 3 on x-axis.

x ∈ [-3, 3]

-3 ≤ x ≤ 3

|x| ≤ 3

9. Question

Mark the Correct alternative in the following:

The linear inequality representing the solution set given in Fig. 15.44 is

Fig. 15.44

A. $|x| < 5$

B. $|x| > 5$

C. $|x| \geq 5$

D. $|x| \leq 5$

Answer

The given figure is highlighted between -∞ to 5 and 5 to ∞

So, x ∈ (-∞, 5] [5, ∞)

x ≤ -5 and x ≥ 5

|x| ≥ 5

10. Question

Mark the Correct alternative in the following:

The solution set of the inequation $|x + 2| \leq 5$ is

A. (−7, 5)

B. [−7, 3]

C. [−5, 5]

D. (−7, 3)

Answer

$|x + 2| \leq 5$

$-5 \leq x + 2 \leq 5$

$-7 \leq x \leq 3$

$x \in [-7, -3]$

11. Question

Mark the Correct alternative in the following:

If $\frac{|x-2|}{x-2} \geq 0$, then

A. $x \in [2, \infty)$

B. $x \in (2, \infty)$

C. $x \in (-8, 2)$

D. $x \in (-\infty, 2]$

Answer

$\frac{|x-2|}{x-2} \geq 0$

Case I : x > 2

$\frac{x-2}{x-2} \geq 0$

$1 \geq 0$

It is true that 1 is always greater than 0 so case I is also true x > 2

Case II : x < 2

$\frac{-(x-2)}{x-2} \geq 0$

$-1 \geq 0$

It is false that -1 is not greater than 0 so case II is also false.

So, the final solution is x > 2 i.e., $x \in (2, \infty)$

12. Question

Mark the Correct alternative in the following:

If $|x+3| \geq 10$, then

A. $x \in (-13, 7]$

B. $x \in (-13, 7)$

C. $x \in (-\infty, -13) \cup (7, \infty)$

D. $x \in (-\infty, -13] \cup [7, \infty)$

Answer

$|x + 3| \geq 10$

$x + 3 \leq -10$ and $x + 3 \geq 10$

$x \leq -13$ and $x \geq 7$

$x \in (-\infty, -13]\ [7, \infty)$