Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If cot $\theta = \frac{1}{2}$ and sec $\phi = \frac{-5}{3}$, where θ lies in quadrant III and ϕ lies in quadrant II, then tan $(\theta + \phi)$ = ? **[1]**

   a) $\frac{-6}{11}$ b) $\frac{5}{11}$

   c) $\frac{2}{11}$ d) $\frac{10}{11}$

2. Let A and B be two sets containing four and two elements respectively.Then,the number of subsets of A×B,each having at least three elements is: **[1]**

   a) 256 b) 219

   c) 510 d) 275

3. Two dice each numbered from 1 to 6 are thrown together. Let A and B be two events given by **[1]**
   A: even number on the first die
   B: number on the second die is greater than 4
   What is the value of P(A ∪ B)?

   a) $\frac{1}{4}$ b) $\frac{1}{2}$

   c) $\frac{1}{6}$ d) $\frac{2}{3}$

4. $\lim\limits_{x \to 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$ is equal to **[1]**

   a) 1 b) 0

   c) 2 d) -1

5. The reflection of the point (4, -13) about the line 5x + y + 6 = 0 is **[1]**

   a) (1, 2) b) (0, 0)

c) (3, 4)

d) (-1, -14)

6. The set A = {x : x is a positive prime number less than 10} in the tabular form is **[1]**

a) {2, 3, 5, 7}

b) {1, 2, 3, 5, 7}

c) {3, 5, 7}

d) {1, 3, 5, 7, 9}

7. If z $= \frac{1}{(1-i)(2+3i)}$, then |z| = **[1]**

a) 1

b) $1/\sqrt{26}$

c) $4/\sqrt{26}$

d) $5/\sqrt{26}$

8. The minimum value of sin x + cos x is **[1]**

a) $-2\sqrt{2}$

b) $\sqrt{2}$

c) 0

d) $-\sqrt{2}$

9. Solve the system of inequalities: $-15 < \frac{3(x-2)}{5} \leq 0$ **[1]**

a) -13 < x < 13

b) -23 < x $\leq$ 2

c) -23 < x < 23

d) -13 < x < 2

10. cosec (-1110°) = ? **[1]**

a) -2

b) $\frac{-2}{\sqrt{3}}$

c) 2

d) $\frac{2}{\sqrt{3}}$

11. The number of subsets of a set containing n elements is **[1]**

a) 2^n-1

b) 2^n - 2

c) 2^n

d) n

12. If a, b, c are in GP and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ then x, y, z are in **[1]**

a) GP

b) AP

c) H.M.

d) HP

13. {C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)C_n} = ? **[1]**

a) (n - 2) . 2^{n-2}

b) (n - 1) (n + 2)

c) (n + 2) . 2^{n-1}

d) (n + 1)2^n

14. The solution set for (x + 3) + 4 > -2x + 5: **[1]**

a) (- ∞, 2)

b) $\left(\frac{-2}{3}, \infty\right)$

c) (- ∞, - 2)

d) (2, ∞)

15. If A and B are two given sets , then A ∩ (A ∩ B)^c is equal to **[1]**

a) B

b) A

c) A ∩ B^c

d) ϕ

16. $\cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta)$ is equal to **[1]**

a) 2 cos θ

b) 0

c) $2 \sin \theta$ d) 1

17. If z is any complex number, then $\frac{z-\bar{z}}{2i}$ is **[1]**

a) either 0 or purely imaginary b) purely imaginary

c) purely real d) either 0 or purely real

18. In how many ways can the letters of the word 'APPLE' be arranged? **[1]**

a) 90 b) 6

c) 120 d) 60

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**

Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** The proper measure of dispersion about the mean of a set of observations i.e. standard deviation is expressed as positive square root of the variance. **[1]**

Reason (R): The units of individual observations x_i and the unit of their mean are different that of variance. Since, variance involves sum of squares of $(x - \bar{x})$.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Find the domain of the relation, R = {(x, y): x, y ∈ z, y = 4} **[2]**

OR

Find the domain and the range of the real function: f(x) = $\frac{x^2-16}{x-4}$

22. Differentiate $\sin^3 x \cos^3 x$ w.r.t x **[2]**

23. Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases: Vertices at (±5, 0), Foci at (± 7, 0). **[2]**

OR

Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas $y^2 - 4y + 4x = 0$

24. Write E = {14, 21, 28, 35, 42, ..., 98} in set-builder form. **[2]**

25. The intercept cuts-off by a line from y-axis is twice than that from x-axis and the line passes through the point (1, 2). Find the equation of the line. **[2]**

Section C

26. Let A = {1, 2, 3, 4}, B = {1, 4, 9, 16, 25} and R be a relation defined from A to B as, R = {(x, y) : x ∈ A, y ∈ B and $y = x^2$} **[3]**

i. Depict this relation using arrow diagram.
ii. Find domain of R.
iii. Find range of R.
iv. Write co-domain of R.

27. Solve the following system of linear inequations: **[3]**

$3x - 6 \geq 0$

$4x - 10 \leq 6$

28. Show that the points A(4, 6, - 3), B(0, 2, 3) and C(-4, - 4, -1) form the vertices of an isosceles triangle. **[3]**

OR

Given that ,P(5, 4, - 2), Q(7, 6, -4) and R(11, 10, -8) are collinear points. Find the ratio in which Q divides PR.

29. Using binomial theorem, expand: $(\sqrt[3]{x} - \sqrt[3]{y})^6$ **[3]**

OR

Find the coefficient of x^5 in the product $(1 + 2x)^6 (1 - x)^7$ using binomial theorem.

30. Find the square roots: 7 - 24i. **[3]**

OR

If $(x + iy)^{1/3} = a + ib$, where x, y, a, b $\in$ R, then show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$.

31. Let A = {a, e, i, o, u}, B = {a, d, e, o, v) and C = {e, o, t, m] . Using Venn diagrams, verify that: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) **[3]**

Section D

32. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the card drawn is, **[5]**

i. A prime number
ii. An odd number
iii. A multiple of 5
iv. Not divisible by 3.

33. Solve: $\lim_{x \to 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}$ **[5]**

OR

Differentiate log sin x from first principles.

34. Find four numbers in GP, whose sum is 85 and product is 4096. **[5]**

35. Prove that $\cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} = \frac{1}{16}$ **[5]**

OR

If α, β are two different values of x lying between 0 and 2π which satisfy the equation 6 cos x + 8 sin x = 9, find the value of sin $(\alpha + \beta)$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.

i. Path traced by Arun represents which type of curve. Find the length of major axis? (1)
ii. Find the equation of the curve traced by Arun? (1)

iii. Find the eccentricity of path traced by Arun? (2)

OR

iv. Find the length of latus rectum for the path traced by Arun. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A teacher conducted a surprise test of Mathematics, Physics and Chemistry for class XI on Monday.

The mean and standard deviation of marks obtained by 50 students of the class in three subjects are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

i. Which of the three subjects shows the highest variability? (1)

ii. What is the coefficient of variation of marks obtained by the students in Chemistry? (1)

iii. What is the coefficient of variation of marks obtained by the students in Physics? (2)

OR

What is the coefficient of variation of marks obtained by the students in Mathematics? (2)

38. A permutation is **an act of arranging the objects or numbers in order**. Combinations are the way of selecting the objects or numbers from a group of objects or collections, in such a way that the order of the objects does not matter. **[4]**

ALLAHABAD

How many different words can be formed by using all the letters of the word ALLAHABAD?

i. In how many of them, vowels occupy the even position?

ii. In how many of them, both L do not come together?

Solution

Section A

1.

(c) $\frac{2}{11}$

Explanation: In quadrant III, $\sin\theta < 0, \cos\theta < 0$ and $\tan\theta > 0$

In quadrant II, $\sin\phi > 0, \cos\phi < 0$ and $\tan\theta < 0$

Now, $\cot\theta = \frac{1}{2} \Rightarrow \tan\theta = 2$

$\sec\phi = \frac{-5}{3} \Rightarrow \cos\phi = \frac{-3}{5}$

$\therefore \sin^2\theta = \left(1 - \cos^2\phi\right) = \left(1 - \frac{9}{25}\right) = \frac{16}{25} \Rightarrow \sin\phi = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan\phi = \left(\frac{4}{5} \times \frac{5}{-3}\right) = \frac{-4}{3}$

$\therefore \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{\left(2 - \frac{4}{3}\right)}{\left\{1 - \left(2 \times \frac{-4}{3}\right)\right\}} = \frac{\left(\frac{2}{3}\right)}{\left(1 + \frac{8}{3}\right)} = \left(\frac{2}{3} \times \frac{3}{11}\right) = \frac{2}{11}$

2.

(b) 219

Explanation: $n(A) = 4, n(B) = 2$

$n(A \times B) = 8$

$\therefore\ Number\ of\ subsets\ having\ at\ least\ 3\ elements$

$= 2^8 - (1 + {}^8C_1 + {}^8C_2) = 219$

3.

(d) $\frac{2}{3}$

Explanation: Let S be the sample space.

$\therefore$ n(S) = 36

A. even number on the first die

B. number on the second die is greater that 4

$\therefore$ n(A) = 18, n(B) = 12,

P(A) = $\frac{18}{36} = \frac{1}{2}$ and P(B) = $\frac{12}{36} = \frac{1}{3}$

Also, A ∩ B = {(2, 5), (2, 6), (4, 5), (4, 6), (6, 5), (6, 6)}

P(A ∩ B) = $\frac{6}{36} = \frac{1}{6}$

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

$= \frac{3+2-1}{6} = \frac{4}{6} = \frac{2}{3}$

4. **(a)** 1

Explanation: Given, $\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{(\sqrt{x+1} - \sqrt{1-x})(\sqrt{x+1} + \sqrt{1-x})}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{x+1-1+x}$

$= \lim\limits_{x\to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{2x} = \frac{1}{2} \cdot \lim\limits_{x\to 0} \frac{\sin x}{x}[\sqrt{x+1} + \sqrt{1-x}]$

Taking limits, we get

$= \frac{1}{2} \times 1 \times [\sqrt{0+1} + \sqrt{1-0}] = \frac{1}{2} \times 1 \times 2 = 1$

5.

(d) (-1, -14)

Explanation: Suppose (h, k) be the point of reflection of the given point (4, – 13) about the line 5x + y + 6 = 0.

The mid-point of the line segment joining points (h, k) and (4, – 13) is given by $\frac{h+4}{2}, \frac{k-13}{2}$

This point lies on the given line, thus we have $5\frac{h+4}{2} + \frac{k-13}{2} + 6 = 0$ or 5h + k + 19 = 0 ... (1)

Again the slope of the line joining points (h, k) and (4, –13) is given by $\frac{k+13}{h-4}$.

This line is perpendicular to the given line and therefore, $(\text{-}5)\ \frac{k+3}{h-4} = -1$

This gives 5k + 65 = h – 4 or h – 5k – 69 = 0 ... (2)

On solving (1) and (2), we obtain h = –1 and k = –14.

Therefore thethe point (-1, -14) is the reflection of the given point.

6. **(a)** {2, 3, 5, 7}

Explanation: Prime no. less then 10 is 2, 3, 5, 7 so

Set A = {2, 3, 5, 7}

7.

(b) $1/\sqrt{26}$

Explanation: $1/\sqrt{26}$

Let z $= \frac{1}{(1-i)(2+3i)}$

$\Rightarrow z = \frac{1}{2+i-3i^2}$

$\Rightarrow z = \frac{1}{2+i+3}$

$\Rightarrow z = \frac{1}{5+i} \times \frac{5-i}{5-i}$

$\Rightarrow z = \frac{5-i}{25-i^2}$

$\Rightarrow z = \frac{5-i}{25+1}$

$\Rightarrow z = \frac{5-i}{26}$

$\Rightarrow z = \frac{5}{26} - \frac{i}{26}$

$\Rightarrow |z| = \sqrt{\frac{25}{676} + \frac{1}{676}}$

$\Rightarrow z = \frac{1}{\sqrt{26}}$

8.

(d) $-\sqrt{2}$

Explanation: Let $f(x) = \sin x + cos\, x$

$\therefore f'(x) = \cos x - \sin x$

$\Rightarrow f''(x) = -\sin x - \cos x$

Now, $f'(x) = 0$

$\Rightarrow \cos x - \sin x = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow x = n\pi + \frac{\pi}{4}$, n $\in$ z

At x $= \pi + \frac{\pi}{4}$,

f'' (x) $= -\sin\left(\pi + \frac{\pi}{4}\right) - \cos\left(\pi + \frac{\pi}{4}\right)$

$= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$

$\therefore$ x $= \pi + \frac{\pi}{4}$ is point of minimum

Minimum value = $\sin\left(\pi + \frac{\pi}{4}\right) + \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

9.

(b) -23 < x $\leq$ 2

Explanation: $-15 < \frac{3(x-2)}{5} \leq 0$

$\Rightarrow -15 \cdot \frac{5}{3} < \frac{3(x-2)}{5} \cdot \frac{5}{3} \leq 0 \cdot \frac{5}{3}$

$\Rightarrow$ -25 < (x - 2) $\leq$ 0 + 2

$\Rightarrow$ -25 + 2 < x - 2 + 2 $\leq$ 2

$\Rightarrow$ -23 < x $\leq$ 2

10. **(a)** -2

Explanation: $180^\circ = \pi^c \Rightarrow 1110^\circ = \left(\frac{\pi}{180} \times 1110\right)^c = \left(\frac{37\pi}{6}\right)^c$

$\therefore$ cosec (-1110°) = -cosec 1110° = - cosec $\frac{37\pi}{6}$

$= -\operatorname{cosec}\left(6\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \;\; [\because \operatorname{cosec}(2n\pi + \theta) = \operatorname{cosec}\theta]$

11.

(c) 2^n

Explanation: 2^n

The total number of subsets of a finite set consisting of n elements is 2^n.

12.

(b) AP

Explanation: Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$ Then, a = k^x, b = k^y and c = k^z.

Since, a, b, c are in GP

$\Rightarrow b^2 = ac \Rightarrow (k^y)^2 = (k^x \times k^z)$

$\Rightarrow k^{2y} = k^{(x+z)} \Rightarrow 2y = x + z$

$\Rightarrow$ x, y, z are in AP.

13.

(d) $(n + 1)2^n$

Explanation: We have, $C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)C_n$

$= (C_0 + C_1 + C_2 + ... + C_n) + 2(C_1 + 2C_2 + ... + nC_n)$

$= 2^n + 2(n \cdot 2^{n-1}) = (n + 1) \cdot 2^n$

14.

(b) $\left(\frac{-2}{3}, \infty\right)$

Explanation: (x + 3) + 4 > -2x + 5

$\Rightarrow$ x + 7 > -2x + 5

$\Rightarrow$ x + 7 + 2x > -2x + 5 + 2x

$\Rightarrow$ 3x + 7 > 5

$\Rightarrow$ 3x + 7 - 7 > 5 - 7

$\Rightarrow$ 3x > -2

$\Rightarrow$ x > $\frac{-2}{3}$

$\Rightarrow x \in \left(\frac{-2}{3}, \infty\right)$

15.

(c) $A \cap B^c$

Explanation: $A \cap B^c$

A and B are two sets.

$A \cap B$ is the common region in both the sets.

$(A \cap B^c)$ is all the region in the universal set except $A \cap B$

Now, $A \cap (A \cap B)^c = A \cap B^c$

16.

(b) 0

Explanation: $\sin(270^\circ + \theta) = -\cos\theta$, $\sin(270^\circ - \theta) = -\cos\theta$, $\cos(180^\circ + \theta) = -\cos\theta$

$\therefore$ given exp $= \cos\theta - \cos\theta + \cos\theta - \cos\theta = 0$

17.

(c) purely real

Explanation: Let z = x + iy

Then $\bar{z}$ = x - iy

$\therefore$ z - $\bar{z}$ = (x + iy) - (x - iy) = 2iy

Now $\frac{z-\bar{z}}{2i}$ = y

Hence $\frac{z-\bar{z}}{2i}$ is purely real.

18.

(d) 60

Explanation: There are in all 5 letters out of which there are 2P, 1A, 1L and IE

$\therefore$ required number of ways $= \frac{5!}{(2!)(1!)(1!)(1!)}$ = 60.

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2$ +... + ${}^n c_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3}$ + ... $(-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: In the calculation of variance, we find that the units of individual observations x_i and the unit of their mean $\bar{x}$ are different from that of variance, since variance involves the sum of squares of $(x_i - \bar{x})$.

For this reason, the proper measure of dispersion about the mean of a set of observations is expressed as positive square-root of the variance and is called standard deviation.

Section B

21. Given, R = {(x, y) : x, y $\in$ z, xy = 4}

= {(-4, -1), (-2, -2), (-1, -4), (1, 4), (2, 2), (4, 1)}

$\therefore$ Domain of R = {-4, -2, -1, 1, 2, 4}

OR

Here we are given that, f(x) = $\frac{x^2-16}{x-4}$

Need to find: where the function is defined.

Let, f(x) = $\frac{x^2-16}{x-4}$ = y ....(i)

To find the domain of the function f(x) we need to equate the denominator of the function to 0

Therefore,

x - 4 = 0 or x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4

The domain of the function, $D_{\{f(x)\}} = (-\infty, 4) \cup (4, \infty)$

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 8

So, the range of the function is the set of all the real numbers except 8

The range of the function, $R_{f(x)} = (-\infty, 8) \cup (8, \infty)$

22. We have, $\frac{d}{dx}\left(\sin^3 x \cos^3 x\right) = \sin^3 x \cdot \frac{d}{dx}\cos^3 x + \cos^3 x \cdot \frac{d}{dx}\left(\sin^3 x\right)$ [Using Product Rule of differentiation]

$= \sin^3 x \cdot 3\cos^2 x(-\sin x) + \cos^3 x \cdot 3\sin^2 x \cdot \cos x$

$= -3 \sin^4 x \cos^2 x + 3 \cos^4 x \sin^2 x$

$= 3 \sin^2 x \cos^2 x (-\sin^2 x + \cos^2 x)$

$= 3\sin^2 x\cos^2 x \cdot \cos 2x$

$= \frac{3}{4} \cdot 4\sin^2 x\cos^2 x \cdot \cos 2x = \frac{3}{4}(2\sin x\cos x)^2\cos 2x$

$= \frac{3}{4}\sin^2 2x \cdot \cos 2x$

23. Since the vertices lie on the x-axis, so let the equation of the required hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ... (i)

The coordinates of its vertices and foci are ($\pm$ a, 0) and ($\pm$ ae, 0) respectively.

But, the coordinates of vertices and foci are given as ($\pm$ 5, 0) and ($\pm$ 7, 0) respectively.

$\therefore$ a = 5 and ae = 7 then 5e = 7 $\Rightarrow$ e $= \frac{7}{5}$

Now, $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 25\left(\frac{49}{25} - 1\right)$ = 24.

Substituting the values of a^2 and b^2 in (i), we obtain $\frac{x^2}{25} - \frac{y^2}{24} = 1$

Required equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{24} = 1$

OR

We are given:

$y^2 - 4y + 4x = 0$

$\Rightarrow (y - 2)^2 - 4 + 4x = 0$

$\Rightarrow (y - 2)^2 = - 4 (x - 1)$

Let Y = y - 2

X = x - 1

Then, we have:

$Y^2 = - 4X$

On comparing the given equation with $Y^2 = - 4aX$

$4a = 4 \Rightarrow a = 1$

$\therefore$ Vertex = (X = 0, Y = 0) = (x = 1, y = 2)

Focus = (X = -a, Y = 0) = (x - 1 = - 1, y - 2 = 0) = (x = 0, y = 2)

Equation of the directrix:

x = a

i.e. $x - 1 = 1 \Rightarrow x = 2$

Axis = Y = 0

i.e. $y - 2 = 0 \Rightarrow y = 2$

Therefore, length of the latus rectum = 4a = 4 units

24. Now,

$14 = 7 \times 2$

$21 = 7 \times 3$

$28 = 7 \times 4$

$35 = 7 \times 5$

$42 = 7 \times 6$

$98 = 7 \times 14$

Therefore,the given set can be write as

E = {x: x = 7n, n $\in$ N and $1 \leq n \leq 14$}

25. The equation of a line intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

Given, b = 2a

$\therefore (1) \Rightarrow \frac{x}{a} + \frac{y}{2a} = 1$

$\Rightarrow 2x + y = 2a$

since the line passes through the point (1, 2),

2 . 1 + 2 = 2a

$\Rightarrow a = 2$

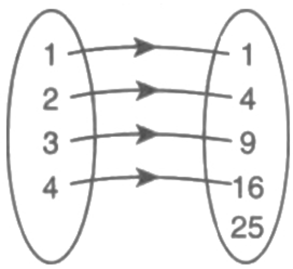
$\therefore$ Equation of the line is 2x + y - 4 = 0.

Section C

26. Given, A = {1,2,3,4 } and B = {1, 4, 9, 16, 25} and

R = {(x, y) : x $\in$ A, y $\in$ B and $y = x^2$}

i. Relation R = {(1, 1), (2, 4), (3, 9), (4, 16)}



ii. Domian of R = {1, 2, 3, 4}

iii. Range of R = {1, 4, 9, 16}

iv. Co-domain of R = {1, 4, 9, 16, 25}

27. The given system of inequations is 3x - 6 $\geq$ 0 ...(i)

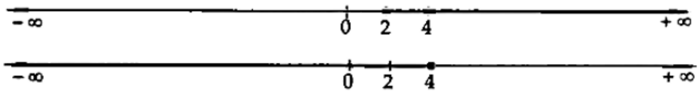
4x - 10 $\leq$ 6 .....(ii)

Now 3x - 6 $\geq$ 0 $\Rightarrow$ 3x $\geq$ 6 $\Rightarrow$ $\frac{3x}{3} \geq \frac{6}{3} \Rightarrow x \geq 2$

Solution set of inequation (i) is [2, ∞)

and, 4x -10 < 6 $\Rightarrow$ 4x $\leq$ 16 $\Rightarrow$ x < 4

$\therefore$ The solution set of inequation (ii) is (∞, 4]



The solution sets of inequations (i) and (ii) are represented graphically on the real line in the above figure.

Clearly, the intersection of these solution sets is the set [2,4].

Hence, the solution set of the given system of inequations is the interval [2,4].

28. To prove: Points A, B, C form an isosceles triangle.

Formula: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

D = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$

Here,

(x_1, y_1, z_1) = (4, 6, -3)

(x_2, y_2, z_2) = (0, 2, 3)

(x_3, y_3, z_3) = (-4, -4, -1)

Length AB = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$

$=\sqrt{(0-4)^2+(2-6)^2+(3-(-3))^2}$

$=\sqrt{(-4)^2+(-4)^2+(6)^2}$

$=\sqrt{16+16+36}$

Length AB $=\sqrt{68}=2\sqrt{17}$

Length BC = $\sqrt{\left(x_3-x_2\right)^2+\left(y_3-y_2\right)^2+\left(z_3-z_2\right)^2}$

$=\sqrt{(-4-0)^2+(-4-2)^2+(-1-3)^2}$

$=\sqrt{(-4)^2+(-6)^2+(-4)^2}$

$=\sqrt{16+36+16}$

Length BC $=\sqrt{68}=2\sqrt{17}$

Length AC = $\sqrt{\left(x_3-x_1\right)^2+\left(y_3-y_1\right)^2+\left(z_3-z_1\right)^2}$

$=\sqrt{(-4-4)^2+(-4-6)^2+(-1-(-5))^2}$

$=\sqrt{(-8)^2+(-10)^2+(2)^2}$

$=\sqrt{64+100+4}$

Length AC $=\sqrt{168}$

Here, AB = BC

$\therefore$ vertices A, B, C forms an isosceles triangle.

OR

Let Q divides PR in the ratio k : 1

Thus coordinates of Q are $\left[\frac{11k+5}{k+1}, \frac{10k+4}{k+1}, \frac{-8k-2}{k+1}\right]$

It is given that coordinates of Q are (7, 6, -4).

$\therefore \frac{11k+5}{k+1}=7,\ \frac{10k+4}{k+1}=6,\ \frac{-8k-2}{k+1}=-4$

Now solving these we get $k=\frac{1}{2}$

Thus Q divides PR in the ratio $\frac{1}{2}:1$ or 1 : 2

29. To find: Expension of $(\sqrt[3]{x}-\sqrt[3]{y})^6$ by means of binomial theorem..

Formula used: ${}^nC_r=\frac{n!}{(n-r)!(r)!}$

(a + b)^n = ${}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+\ldots\ldots+{}^nC_{n-1}ab^{n-1}+nC_nb^n$

We have, $(\sqrt[3]{x}-\sqrt[3]{y})^6$

We can write $\sqrt[3]{x}$, as $x^{\frac{1}{3}}$, and $\sqrt[3]{y}$, as $y^{\frac{1}{3}}$,

Now, we have to solve for $\left(x^{\frac{1}{3}} - y^{\frac{1}{3}}\right)^6$

$\Rightarrow \left[6C_0\left(x^{\frac{1}{3}}\right)^{6-0}\right] + \left[6C_1\left(x^{\frac{1}{3}}\right)^{6-1}\left(-y^{\frac{1}{3}}\right)^1\right] + \left[6C_2\left(x^{\frac{1}{3}}\right)^{6-2}\left(-y^{\frac{1}{3}}\right)^2\right] + \left[6C_3\left(x^{\frac{1}{3}}\right)^{6-3}\left(-y^{\frac{1}{3}}\right)^3\right]$

$+ \left[6C_4\left(x^{\frac{1}{3}}\right)^{6-4}\left(-y^{\frac{1}{3}}\right)^4\right] + \left[6C_5\left(x^{\frac{1}{3}}\right)^{6-5}\left(-y^{\frac{1}{3}}\right)^5\right] + \left[6C_6\left(-y^{\frac{1}{3}}\right)^6\right]$

$\Rightarrow \left[{}^6C_0\left(\frac{6}{x^3}\right)\right] - \left[{}^6C_1\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[6C_2\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - \left[6C_3\left(x^{\frac{3}{3}}\right)\left(y^{\frac{3}{3}}\right)\right]$

$+ \left[{}^6C_4\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[6C_5\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[6C_6\left(\frac{6}{y^3}\right)\right]$

$\Rightarrow \left[\frac{6!}{0!(6-0)!}\left(x^2\right)\right] - \left[\frac{6!}{1!(6-1)!}\left(x^{\frac{5}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(x^{\frac{4}{2}}\right)\left(x^{\frac{2}{3}}\right)\right]$

$- \left[\frac{6!}{3!(6-3)!}(x)(y)\right] + \left[\frac{6!}{4!(6-4)!}\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{5}}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[\frac{6!}{6!(6-6)!}\left(y^2\right)\right]$

$\Rightarrow \left[1\left(x^2\right)\right] - \left[6\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[15\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - [20(x)(y)] + \left[15\left(x^{\frac{2}{3}}\right)\left(\frac{4}{y^3}\right)\right]$

$- \left[6\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[1\left(y^2\right)\right]$

$\Rightarrow x^2 - 6x^{\frac{5}{2}}y^{\frac{1}{3}} + 15x^{\frac{4}{3}}y^{\frac{2}{3}} - 20xy + 15x^{\frac{2}{3}}y^{\frac{4}{3}} - 6x^{\frac{1}{3}}y^{\frac{5}{3}} + y^2$

Hence the result.

OR

Using binomial theorem

$(1 + 2x)^6 (1 - x)^7 = [{}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6]$
$[{}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7]$

$= [1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6]\ [1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7]$

$\therefore$ Coefficient of x^5 in the product

$= (1 \times -21) + (12 \times 35) + (60 \times -35) + (160 \times 21) + (240 \times -7) + (192 \times 1)$

= -21 + 420 - 2100 + 3360 - 1680 + 192 = 171

30. Let $\sqrt{7 - 24i}$ = x + iy. Then

$\sqrt{7 - 24i}$ = x + iy

$\Rightarrow 7 - 24i = (x + iy)^2$

$\Rightarrow 7 - 24i = (x^2 - y^2) + 2i\,xy$

$\Rightarrow x^2 - y^2 = 7$ ... (i)

and 2xy = -24 ... (ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$\Rightarrow (x^2 + y^2)^2 = 49 + 576 = 625$ [$\because x^2 + y^2 > 0$]

$\Rightarrow x^2 + y^2 = 25$... (iii)

add (i) and (iii), we get

$2x^2 = 32$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

put value of x in (I), we get

$y^2 = 9 \Rightarrow y = \pm 3$

From (ii) we observe that 2xy is negative. So, x and y are of opposite signs.

Hence, $\sqrt{7 - 24i} = \pm$ (4 - 3i)

OR

We have, $(x + iy)^{1/3} = a + ib$

$\Rightarrow x + iy = (a + ib)^3$ [cubing on both sides]

$\Rightarrow x + iy = a^3 + i^3b^3 + 3iab(a + ib)$

$\Rightarrow x + iy = a^3 - ib^3 + i\,3a^2 b - 3ab^2$

$\Rightarrow x + iy = a^3 - 3ab^2 + i\,(3a^2 b - b^3)$

On equating real and imaginary parts from both sides, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

Now, $\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

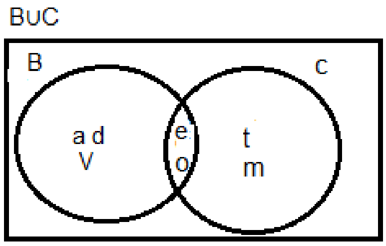
$= -2a^2 - 2b^2 = -2(a^2 + b^2)$

Hence proved.

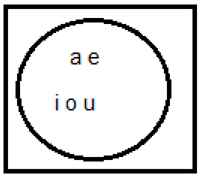
31. Here,it is given: A = {a, e, i, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

$B \cap C$ = {e, o} and $A \cup (B \cap C)$ = {a, e, i, o, u}

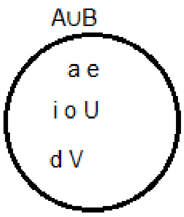
LHS

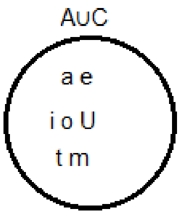


$A \cup (B \cap C)$



R.H.S: $A \cup B$ = {a, d, e, I, o, u, v} and $A \cup C$ = {a, e, I, o, u, t, m}





$(A \cup B) \cap (A \cup C)$ = {a, e, I, o, u}

L.H.S = R.H.S. [Verified]

Section D

32. Let S be the sample space

S = {1,2,3,4,5.....20}

Let E_1, E_2 and E_3, E_4 are the event of getting prime number, an odd number, multiple of 5 and not divisible by 3 respectively.

$P(E_1) = \frac{8}{20} = \frac{2}{5}$, $E_1 = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$P(E_2) = \frac{10}{20} = \frac{1}{2}$, $E_2 = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$P(E_3) = \frac{4}{20} = \frac{1}{5}$, $E_3 = \{5, 10, 15, 20\}$

$P(E_4) = \frac{14}{20} = \frac{7}{10}$, $E_4 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

33. Dividing $x^4 - 3x^3 + 2$ by $x^3 - 5x^2 + 3x + 1$

$$x^3 - 5x^3 + 3x + 1 \,\overline{)\, x^4 - 3x^3 + 2\,}\; x + 2$$
$$\pm x^4 \mp 5x^3 \pm 3x^2 \pm x$$
$$2x^3 - 3x^2 - x + 2$$
$$\pm 2x^3 \mp 10x^2 \pm 6x \pm 2$$
$$7x^2 - 7x$$

$\Rightarrow \lim\limits_{x\to1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1} = \lim\limits_{x\to1}(x + 2) + \lim\limits_{x\to1} \frac{7x^2 - 7x}{x^3 - 5x^3 + 3x + 1}$

$= \lim\limits_{x\to1} x + 2 + \lim\limits_{x\to1} \frac{7x(x-1)}{x^3 - 5x^3 + 3x + 1}$

$= \lim\limits_{x\to1} x + 2 + \lim\limits_{x\to1} \frac{7x(x-1)}{(x-1)(x^2 - 4x - 1)}$

$= \lim\limits_{x\to1} x + 2 + \lim\limits_{x\to1} \frac{7x}{(x^2 - 4x - 1)}$

$= 1 + 2 + \frac{7}{(1-4-1)}$

$= 3 - \frac{7}{4}$

$= \frac{12-7}{4}$

$= \frac{5}{4}$

OR

Let f (x) = log sin x. Then, f (x + h) = log sin (x + h)

$\therefore \frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log \sin(x+h) - \log \sin x}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{\frac{\sin(x+h)}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)}{\sin x}-1\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{\sin x}\right\}}{h}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{\sin x}\right\}}{h\left\{\frac{\sin(x+h)-\sin x}{\sin x}\right\}} \times \left\{\frac{\sin(x+h)-\sin x}{\sin x}\right\}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}} \times \frac{\sin(x+h)-\sin x}{h} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}} \times \lim_{h \to 0} \frac{2 \sin \frac{h}{2} \cos\left(x+\frac{h}{2}\right)}{h} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = $\lim_{h \to 0} \frac{\log\left\{1+\frac{\sin(x+h)-\sin x}{h}\right\}}{\left\{\frac{\sin(x+h)-\sin x}{h}\right\}} \times \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right) \cos\left(x+\frac{h}{2}\right)}{\frac{h}{2}} \times \frac{1}{\sin x}$

$\Rightarrow \frac{d}{dx}$ (f (x)) = 1 $\times$ cos x $\times$ $\frac{1}{\sin x}$ = cot x.

34. Let the four numbers in GP be

$\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3 ...(i)

Product of four numbers = 4096 [given]

$\Rightarrow \left(\frac{a}{r^3}\right)\left(\frac{a}{r}\right)$ (ar) (ar^3) = 4096

$\Rightarrow a^4 = 4096 \Rightarrow a^4 = 8^4$

On comparing the base of the power 4, we get

$\Rightarrow \frac{a}{r^3} + \frac{a}{r}$ + ar + ar^3 = 85

$\Rightarrow$ a $\left[\frac{1}{r^3} + \frac{1}{r} + r + r^3\right]$ = 85

$\Rightarrow$ 8 [$r^3 + \frac{1}{r^3}$] + 8 [r + $\frac{1}{r}$] = 85 [$\because$ a = 8]

$\Rightarrow$ 8 $\left[\left(r+\frac{1}{r}\right)^3 - 3\left(r+\frac{1}{r}\right)\right] + 8\left(r+\frac{1}{r}\right)$ = 85 [$\because a^3 + b^3 = (a+b)^3 - 3(a+b)$]

$\Rightarrow$ 8 $\left(r+\frac{1}{r}\right)^3$ - 16 $\left(r+\frac{1}{r}\right)$ - 85 = 0 ...(ii)

On putiing $\left(r+\frac{1}{r}\right)$ = x in Eq. (ii), we get

$8x^3 - 16x - 85 = 0$

$\Rightarrow (2x - 5)(4x^2 + 10x + 17) = 0$

$\Rightarrow$ 2x - 5 = 0 [$\because 4x^2 + 10x + 177 = 0$ has imaginary roots]

$\Rightarrow x = \frac{5}{2} \Rightarrow r + \frac{1}{r} = \frac{5}{2}$ [put x = r + $\frac{1}{r}$]

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow$ (r - 2) (2r - 1) = 0

$\Rightarrow$ r = 2 or r = $\frac{1}{2}$

On putting a = 8 and r = 2 or r = $\frac{1}{2}$ in Eq. (i), we obtain four numbers as

$\frac{8}{2^3}, \frac{8}{2}, 8 \times 2, 8 \times 2^3$

or $\frac{8}{(1/2)^3}, \frac{8}{(1/2)}, 8 \times \frac{1}{2}, 8 \times \left(\frac{1}{2}\right)^3$

⇒ 1, 4, 16, 64 or 64, 16, 4, 1.

35. **LHS =** cos $\frac{2\pi}{15}$ · cos $\frac{4\pi}{15}$ · cos $\frac{8\pi}{15}$ · cos $\frac{16\pi}{15}$

$= cos\ \frac{2\pi}{15}\ cos2\ \left(\frac{2\pi}{15}\right) cos4\ \left(\frac{2\pi}{15}\right)\ cos8\ \left(\frac{2\pi}{15}\right)$

Put $\frac{2\pi}{15} = \alpha$

⇒LHS = $\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha$

= $\frac{2\sin\alpha[\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha]}{2\sin\alpha}$ [multiplying numerator and denominator by $2sin\ \alpha$]

= $\frac{(2\sin\alpha\cdot\cos\alpha)\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha}{2\sin\alpha}$

= $\frac{2(\sin 2\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha\cdot\cos 8\alpha)}{2(2\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

= $\frac{(2\sin 2\alpha\cdot\cos 2\alpha)\cdot\cos 4\alpha\cdot\cos 8\alpha}{4\sin\alpha}$

= $\frac{2(\sin 4\alpha\cdot\cos 4\alpha)\cos 8\alpha}{2(4\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

= $\frac{2(\sin 8\alpha\cdot\cos 8\alpha)}{2(8\sin\alpha)}$

= $\frac{\sin 16\alpha}{16\sin\alpha} = \frac{\sin(15\alpha+\alpha)}{16\sin\alpha}$

Now, $15\alpha = 2\pi$,

= $\frac{\sin(2\pi+\alpha)}{16\sin\alpha} = \frac{\sin\alpha}{16\sin\alpha}$ = $\frac{1}{16}$ = RHS

∴ **LHS = RHS**

Hence proved.

OR

We have to find the value of sin $(\alpha + \beta)$

It is given that

6 cos x + 8 sin x = 9

⇒ 6 cosx = 9 - 8 sinx

⇒ 36 $\cos^2$x = (9 - 8 sinx)^2

⇒ 36(1 - $\sin^2$x) = 81 + 64 $\sin^2$x - 144 sinx

⇒ 100 $\sin^2$x - 144 sinx + 45 = 0

Now, α and β are the roots of the given equation;

therefore, $\cos\alpha$ and $\cos\beta$ are the roots of the above equation.

⇒ $\sin\alpha\sin\beta = \frac{45}{100}$

(Product of roots of a quadratic equation ax^2 + bx + c = 0 is $\frac{c}{a}$)

Again, 6 cosx + 8 sinx = 9

⇒ 8 sinx = 9 - 6 cosx

⇒ 64 $\sin^2$x = (9 - 6 cos x)^2

⇒ 64(1 - $\cos^2$x) = 81 + 36$\cos^2$x - 108 cosx

⇒ 100 $\cos^2$x - 108 cosx + 17 = 0

Now, α and β are the roots of the given equation;

therefore, sinα and sinβ are the roots of the above equation.

Therefore, $\cos\alpha\cos\beta = \frac{17}{100}$

Hence, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{17}{100} - \frac{45}{100}$

$= -\frac{28}{100}$

$= -\frac{7}{25}$

$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$

$= \sqrt{1-\left(\frac{-7}{25}\right)^2}$

$= \sqrt{\frac{576}{625}}$

$= \frac{24}{25}$

Section E

36. i. An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Hence path traced by Arun is ellipse.

Sum of the distances of the point moving point to the foci is equal to length of major axis =10m

ii. Given 2a = 10 & 2c = 8

$\Rightarrow$ a = 5 & c = 4

$c^2 = a^2 + b^2$

$\Rightarrow 16 = 25 + b^2$

$\Rightarrow b^2 = 25 - 16 = 9$

Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Required equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

iii. equation is of given curve is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

a = 5, b = 3 and given 2c = 8 hence c = 4

Eccentricity = $\frac{c}{a} = \frac{4}{5}$

OR

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

Hence a = 5 and b = 3

Length of latus rectum of ellipse is given by $\frac{2b^2}{a} = \frac{2\times9}{5} = \frac{18}{5}$

37. i. The subject with greater C.V. is more variable than others.

Therefore, the highest variability in marks is in Chemistry.

ii. Standard deviation of Chemistry = 20

C.V. (in Chemistry) = $\frac{20}{40.9} \times 100$ = 48.89

iii. Standard deviation of Physics = 15

The coefficient of variation, C.V. = $\frac{\text{Standard deviation}}{Mean} \times 100$

C.V. (in Physics) = $\frac{15}{32} \times 100$ = 46.87

OR

Standard deviation of Mathematics = 12

The coefficient of variation, C.V. = $\frac{\text{Standard deviation}}{Mean} \times 100$

C.V. (in Mathematics) = $\frac{12}{42} \times 100$ = 28.57

38. In a word ALLAHABAD, we have

Letters	A	L	H	B	D	Total
Number	4	2	1	1	1	9

So, the total number of words = $\frac{9!}{4!2!} = \frac{9\times8\times7\times6\times5}{2\times1}$ = 7560

i. There are 4 vowels and all are alike i.e., 4 A's.

Also, there are 4 even places which are 2nd, 4th, 6th and 8th. So, these 4 even places can be occupied by 4 vowels in $\frac{4!}{4!}$ = 1 way. Now, we are left with 5 places in which 5 letters, of which two are alike (2 L's) and other distinct, can be arranged in $\frac{5!}{2!}$ ways.

Hence, the total number of words in which vowels occupy the even places $= \frac{5!}{2!} \times \frac{4!}{4!} = \frac{5!}{2!}$ = 60

ii. Considering both L together and treating them as one letter. We have,

Letters	A	LL	H	B	D	Total
Number	4	1	1	1	1	8

Then, 8 letters can be arranged in $\frac{8!}{4!}$ ways.

So, the number of words in which both L come together = $\frac{8!}{4!}$ = 8 × 7 × 6 × 5 = 1680

Hence, the number of words in which both L do not come together

= Total number of words - Number of words in which both L come together

= 7560 - 1680 = 5880

Hence, the total number of words in which both L do not come together is 5880