Sample Question Paper - 31
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve for $x : \sqrt{2x+9}+x=13$

2. In a class test, 50 students obtained marks as follows:

Marks obtained	0–20	20–40	40–60	60–80	80–100
Number of students	8	6	15	12	9

Find the modal class and the median class.

3. If the first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$, then find the value of y.

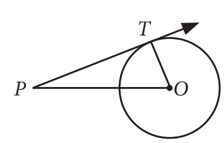
OR

Find the next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$.

4. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

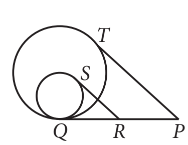
5. If two cubes, each of edge 4 cm are joined end to end, then find the surface area of the resulting cuboid.

6. In the given figure, point P is 13 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 12 cm. Then find the radius of the circle.



OR

In the following figure, PQ is the common tangent to both the circles. SR and PT are tangents. If $SR = 4$ cm, $PT = 7$ cm, then find the length of RP.



SECTION - B

7. Find 'p' if the mean of the given data is 15.45.

Class interval	0–6	6–12	12–18	18–24	24–30
Frequency	6	8	p	9	7

8. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

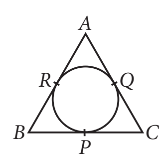
OR

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, then find the height of the building. (Use $\sqrt{3} = 1.73$)

9. Compare the modal ages of two groups of students appearing for an entrance test.

Age (in years)	16-18	18-20	20-22	22-24	24-26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

10. In the given figure, the incircle of ΔABC touches the sides BC, CA and AB at P, Q and R respectively. Prove that $(AR + BP + CQ) = (AQ + BR + CP) = \frac{1}{2}$ (Perimeter of ΔABC).



SECTION - C

11. Find the A.P. whose fourth term is 9 and the sum of its sixth term and thirteenth term is 40.

OR

The sum of the first seven terms of an A.P. is 182. If its 4^{th} and the 17^{th} terms are in the ratio 1 : 5, then find the A.P.

12. Draw a circle of radius 6 cm and draw a tangent to this circle making an angle of 30° with a line passing through the centre.

Case Study - 1

13. Anku and his friends went for a vacation in Manali. There they had a stay in tent for a night. Anku found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42 m and height of the cylinder is 22 m.

(i) How much canvas is needed to make the tent?

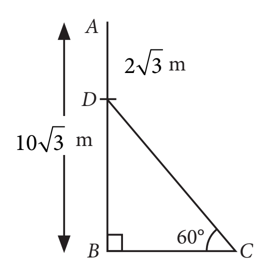
(ii) If each person needs 126 m^2 of floor, then how many persons can be accommodated in the tent?

Case Study - 2

14. Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distant from the foot of the tree.

(i) If the top of upper part of broken tree touches ground at a distance of 45 m (from the foot of the tree) and makes an angle of inclination 60°, then find the height of remaining part of the tree.

(ii) If $AB = 10\sqrt{3}$ m, $AD = 2\sqrt{3}$ m, then find the length of CD.



Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. We have, $\sqrt{2x+9}+x=13$

$\Rightarrow \sqrt{2x+9}=13-x$

Squaring both sides, we have $2x + 9 = (13 - x)^2$

$\Rightarrow 2x + 9 = 169 + x^2 - 26x$

$\Rightarrow x^2 - 28x + 160 = 0 \Rightarrow x^2 - 20x - 8x + 160 = 0$

$\Rightarrow x(x - 20) - 8(x - 20) = 0 \Rightarrow (x - 20)(x - 8) = 0$

$\therefore x = 20$ or 8

2. The cumulative frequency distribution table from the given data can be drawn as :

Marks obtained	Number of students	Cumulative frequency
0-20	8	8
20-40	6	14
40-60	15	29
60-80	12	41
80-100	9	50

The highest frequency is 15 and its corresponding class is 40 – 60. So, the modal class is 40 – 60.

Also, $n = 50 \Rightarrow n/2 = 25$. The cumulative frequency just greater than 25 is 29, which lies in the interval 40 – 60. So, the median class is 40 – 60.

3. Given, $3y - 1$, $3y + 5$ and $5y + 1$ are in A.P.

$\therefore 3y + 5 - (3y - 1) = 5y + 1 - (3y + 5)$

$\Rightarrow 3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$

$\Rightarrow 6 = 2y - 4 \Rightarrow y = \frac{10}{2} = 5$

OR

First term, $a = \sqrt{7}$ and common difference,

$d = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$

$\therefore$ Next term of the A.P. is $(a_4) = a + 3d$

$= \sqrt{7} + 3\sqrt{7} = 4\sqrt{7} = \sqrt{112}$

4. Given, $x^2 + 4x + k = 0$

For real roots, discriminant, $D \geq 0$

$\therefore b^2 - 4ac \geq 0 \Rightarrow 16 - 4(1)(k) \geq 0$

$\Rightarrow 16 - 4k \geq 0 \Rightarrow k \leq 4$

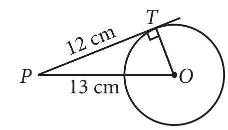
5. $\because$ Two cubes of edge 4 cm each are joined end to end to form a cuboid.

$\therefore$ For resulting cuboid, length $(l) = 4 + 4 = 8$ cm, breadth $(b) = 4$ cm and height $(h) = 4$ cm

$\therefore$ Surface area of cuboid

$= 2(lb + bh + hl) = 2(8 \times 4 + 4 \times 4 + 4 \times 8) = 160\text{ cm}^2$

6. In ΔPTO,



$OP^2 = PT^2 + OT^2$

$\Rightarrow 13^2 = 12^2 + OT^2$

$\Rightarrow 169 - 144 = OT^2$

$\Rightarrow 25 = OT^2 \Rightarrow OT = 5$ cm

OR

Since tangents drawn from an external point to a circle are equal in length.

$\therefore PQ = PT = 7$ cm and $RQ = RS = 4$ cm

Now, $RP = PQ - RQ = (7 - 4)$ cm $= 3$ cm

7. The frequency distribution table from the given data can be drawn as :

Class interval	x_i	f_i	$f_i x_i$
0–6	3	6	18
6–12	9	8	72
12–18	15	p	$15p$
18–24	21	9	189
24–30	27	7	189
Total		$\sum f_i = 30 + p$	$\sum f_i x_i = 468 + 15p$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 15.45 = \frac{468+15p}{30+p}$

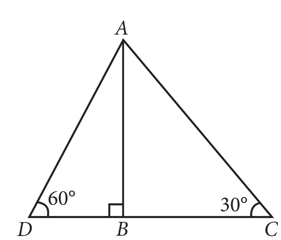
$\Rightarrow 463.5 + 15.45p = 468 + 15p$

$\Rightarrow 15.45p - 15p = 468 - 463.5$

$\Rightarrow 0.45p = 4.5 \Rightarrow p = 10$

8. Let $AB = 75$ m be the building and C, D be the positions of two men.

Now, in ΔABC,



$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC}$

$\Rightarrow BC = 75\sqrt{3}$ m

In ΔABD, $\tan 60° = \frac{AB}{BD}$

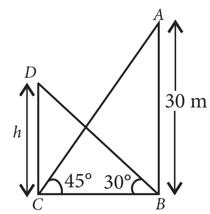
$\Rightarrow \sqrt{3} = \frac{75}{BD} \Rightarrow BD = \frac{75}{\sqrt{3}}$ m $= 25\sqrt{3}$ m

$\therefore$ Distance between the two men

$= BC + BD = 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3} = 173$ m

OR

Let AB be the tower of height 30 m and DC is the building of height h m.



In ΔABC, $\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{30}{BC} \Rightarrow BC = 30$ m

In ΔBDC, $\tan 30° = \frac{CD}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30} \Rightarrow \sqrt{3}h = 30$

$\Rightarrow h = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 17.32$

Thus, height of building is 17.32 m.

9. Maximum frequency in group A is 78 and its corresponding class is 18-20.

$\therefore$ Mode for group $A = 18 + \left(\frac{78-50}{2\times 78-50-46}\right)\times 2$

$= 18 + \frac{28}{30} = 18.9$ years.

Maximum frequency in group B is 89 and its corresponding class is 18-20.

$\therefore$ Mode for group $B = 18 + \left(\frac{89-54}{2\times 89-54-40}\right)\times 2$

$= 18 + \frac{70}{84} = 18.8$ years.

Since, 18.9 > 18.8

$\therefore$ Modal age of group A is greater than that of group B.

10. We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore AR = AQ$ ...(i)

$BP = BR$ ...(ii)

$CQ = CP$ ...(iii)

Adding (i), (ii) and (iii), we get

$(AR + BP + CQ) = (AQ + BR + CP) = k$(say)

Perimeter of $\Delta ABC = (AB + BC + CA)$

$= (AR + BR) + (BP + CP) + (CQ + AQ)$

$= (AR + BP + CQ) + (AQ + BR + CP) = k + k = 2k$

$\Rightarrow k = \frac{1}{2}$ (Perimeter of ΔABC)

$\therefore (AR + BP + CQ) = (AQ + BR + CP)$

$= \frac{1}{2}$ (Perimeter of ΔABC)

11. Given, $a_4 = 9$ and $a_6 + a_{13} = 40$

Now $a_4 = 9 \Rightarrow a + 3d = 9 \Rightarrow a = 9 - 3d$

Also, $a_6 + a_{13} = 40$

$\Rightarrow (a + 5d) + (a + 12d) = 40$

$\Rightarrow 2a + 17d = 40$

On substituting the value of a, we get

$2(9 - 3d) + 17d = 40$

$\Rightarrow 18 + 11d = 40 \Rightarrow 11d = 22$

$\Rightarrow d = 2 \therefore a = 9 - 3(2) = 3$

Thus, the A.P. is 3, 5, 7, 9 …

OR

Given, sum of first seven terms of an A.P., $S_7 = 182$

$\Rightarrow 182 = \frac{7}{2}[2a + (7-1)d]$

$\Rightarrow 364 = 14a + 42d \Rightarrow 26 = a + 3d$ ...(i)

Also, $\frac{a_4}{a_{17}} = \frac{1}{5} \Rightarrow \frac{a+3d}{a+16d} = \frac{1}{5}$

$\Rightarrow 5(a + 3d) = a + 16d$

$\Rightarrow 5a + 15d = a + 16d$

$\Rightarrow 4a - d = 0 \Rightarrow d = 4a$ ...(ii)

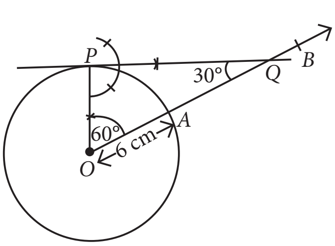
Substituting (ii) in (i), we get

$26 = a + 3(4a) \Rightarrow 13a = 26 \Rightarrow a = 2$

$\therefore d = 4(2) = 8$

Hence, the A.P. is formed as 2, 10, 18, ...

12. Steps of construction :



Step-I : Draw a circle with centre O and radius 6 cm.

Step-II : Draw a radius OA and produce it to B.

Step-III : Construct an $\angle AOP$ equal to the complement of 30° *i.e.*, 60°.

Step-IV : Draw a perpendicular to OP at P which intersects OB at Q.

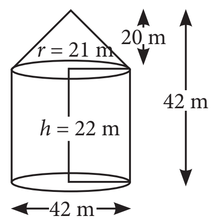
Hence, PQ is the required tangent such that $\angle OQP = 30°$.

13. (i) Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh = \pi r(l + 2h)$

$= \frac{22}{7} \times 21\ (29 + 44) = 4818\ \text{m}^2$

$\left[\because\ l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29\ \text{m}\right]$

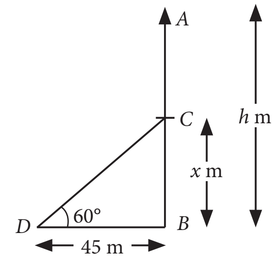


(ii) Area of floor $= \pi r^2$

$= \frac{22}{7} \times 21 \times 21 = 1386\ \text{m}^2$

$\therefore$ Number of persons that can be accommodated in the tent $= \frac{1386}{126} = 11$

14. (i) Let AB be the tree of height h m and let it broken at height of x m, as shown in figure.



Clearly $CD = AC = (h - x)$ m

Now, in ΔCBD, we have

$$\tan 60° = \frac{x}{45}$$

$\Rightarrow\ x = 45\sqrt{3}$ m

Thus, the height of remaining part of the tree is $45\sqrt{3}$ m.

(ii) Clearly, $BD = AB - AD$

$= \left(10\sqrt{3} - 2\sqrt{3}\right)\text{m} = 8\sqrt{3}\ \text{m}$

Now, in ΔBCD, we have

$$\sin 60° = \frac{BD}{DC}$$

$\Rightarrow\ \frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{DC}\ \Rightarrow\ DC = 16$ m