

CHAPTER – 6
OPERATION ON SETS VENN DIAGRAM

Exercise 6.1

1. If $A = \{0, 1, 2, 3, \dots, 8\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{0, 5, 10, 20\}$, find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $A \cap B$

(v) $A \cap C$

(vi) $B \cap C$

Also find the cardinal number of the sets $B \cup C$, $A \cup B$, $A \cap C$ and $B \cap C$.

Solution:-

From the question it is given that,

$$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

$$B = \{3, 5, 7, 9, 11\}$$

$$C = \{0, 5, 10, 20\}$$

(i) $A \cup B$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11\}$$

Cardinal number of set i.e. $n(A \cup B) = 11$

(ii) $A \cup C$

$$A \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 20\}$$

Cardinal number of set i.e. $n(A \cup C) = 11$

(iii) $B \cup C$

$$B \cup C = \{0, 3, 5, 7, 9, 10, 11, 20\}$$

Cardinal number of set i.e. $n(B \cup C) = 8$

(iv) $A \cap B$

$$A \cap B = \{3, 5, 7\}$$

Cardinal number of set i.e. $n(A \cap B) = 3$

(v) $A \cap C$

$$A \cap C = \{0, 5\}$$

Cardinal number of set i.e. $n(A \cap C) = 2$

(vi) $B \cap C$

$$B \cap C = \{5\}$$

Cardinal number of set i.e. $n(B \cap C) = 1$

2. Find A' when

(i) $A = \{0, 1, 4, 7\}$ and $E = \{x \mid x \in W, x \leq 10\}$

Solution:-

From the question,

$$A = \{0, 1, 4, 7\}$$

$$E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\text{Then, } A' = \{2, 3, 5, 6, 8, 9, 10\}$$

(ii) $A = \{\text{consonants}\}$ and $\xi = \{\text{alphabets of English}\}$

Solution:-

$$A = \{\text{consonants}\}$$

$\xi = \{\text{alphabets of English}\}$

$A' = \{\text{Vowels}\}$

(iii) $A = \{\text{boys in class VIII of all schools in Bengaluru}\}$ and $\xi = \{\text{students in class VIII of all schools in Bengaluru}\}$

Solution:-

$A' = \{\text{Girls in class VIII of all schools in Bengaluru}\}$

(iv) $A = \{\text{letters of KALKA}\}$ and $\xi = \{\text{letters of KOLKATA}\}$

Solution:-

From the question it is given that,

$A = \{K, A, L, K, A\}$

$\xi = \{K, O, L, K, A, T, A\}$

$A' = \{O, T\}$

(v) $A = \{\text{odd natural numbers}\}$ and $\xi = \{\text{whole numbers}\}$.

Solution:-

From the question it is given that,

$A = \{\text{odd natural numbers}\}$

$\xi = \{\text{whole numbers}\}$

$A' = \{0, \text{even whole numbers}\}$

3. If $A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 7\}$ and $B = \{x : x \in \mathbb{W} \text{ and } x \leq 4\}$, find

(i) $A \cup B$

(ii) $A \cap B$

(iii) $A - B$

(iv) $B - A$

Solution:-

From the question it is given that,

$$A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 7\}$$

$$A = \{4, 5, 6\}$$

$$B = \{x : x \in \mathbb{W} \text{ and } x \leq 4\}$$

$$B = \{0, 1, 2, 3, 4\}$$

Then,

(i) $A \cup B$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

(ii) $A \cap B$

$$A \cap B = \{4\}$$

(iii) $A - B$

$$A - B = \{5, 6\}$$

(iv) $B - A$

$$B - A = \{0, 1, 2, 3\}$$

4. If $P = \{x : x \in \mathbb{W} \text{ and } x < 6\}$ and $Q = \{x : x \in \mathbb{N} \text{ and } 4 \leq x \leq 9\}$, find

(i) $P \cup Q$

(ii) $P \cap Q$

(iii) $P - Q$

(iv) $Q - P$

Is $P \cup Q$ a proper superset of $P \cap Q$?

Solution:-

From the question it is given that,

$$P = \{x : x \in W \text{ and } x < 6\}$$

$$P = \{0, 1, 2, 3, 4, 5\}$$

$$Q = \{x : x \in N \text{ and } 4 \leq x \leq 9\}$$

$$Q = \{4, 5, 6, 7, 8, 9\}$$

(i) $P \cup Q$

$$P \cup Q = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(ii) $P \cap Q$

$$P \cap Q = \{4, 5\}$$

(iii) $P - Q$

$$P - Q = \{0, 1, 2, 3\}$$

(iv) $Q - P$

$$Q - P = \{6, 7, 8, 9\}$$

By observing the above sets, $P \cup Q$ is a proper superset of $P \cap Q$.

5. If A = (letters of word INTEGRITY) and B = (letters of word RECKONING), find

(i) $A \cup B$

(ii) $A \cap B$

(iii) $A - B$

(iv) $B - A$

Also verify that:

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b) $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$

$$(c) n(\mathbf{B} - \mathbf{A}) = n(\mathbf{A} \cup \mathbf{B}) - n(\mathbf{A}) = n(\mathbf{B}) - n(\mathbf{A} \cap \mathbf{B})$$

$$(d) n(\mathbf{A} \cup \mathbf{B}) = n(\mathbf{A} - \mathbf{B}) + n(\mathbf{B} - \mathbf{A}) + n(\mathbf{A} \cap \mathbf{B}).$$

Solution:

From the question it is given that,

$$\mathbf{A} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}\}$$

$$n(\mathbf{A}) = 7$$

$$\mathbf{B} = \{\mathbf{R}, \mathbf{E}, \mathbf{C}, \mathbf{K}, \mathbf{O}, \mathbf{N}, \mathbf{I}, \mathbf{G}\}$$

$$n(\mathbf{B}) = 8$$

Then,

$$(i) \mathbf{A} \cup \mathbf{B} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}\} \cup \{\mathbf{R}, \mathbf{E}, \mathbf{C}, \mathbf{K}, \mathbf{O}, \mathbf{N}, \mathbf{I}, \mathbf{G}\}$$

$$\mathbf{A} \cup \mathbf{B} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}, \mathbf{C}, \mathbf{K}, \mathbf{O}\}$$

$$n(\mathbf{A} \cup \mathbf{B}) = 10$$

$$(ii) \mathbf{A} \cap \mathbf{B} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}\} \cap \{\mathbf{R}, \mathbf{E}, \mathbf{C}, \mathbf{K}, \mathbf{O}, \mathbf{N}, \mathbf{I}, \mathbf{G}\}$$

$$\mathbf{A} \cap \mathbf{B} = \{\mathbf{I}, \mathbf{N}, \mathbf{E}, \mathbf{G}, \mathbf{R}\}$$

$$n(\mathbf{A} \cap \mathbf{B}) = 5$$

$$(iii) \mathbf{A} - \mathbf{B} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}\} - \{\mathbf{R}, \mathbf{E}, \mathbf{C}, \mathbf{K}, \mathbf{O}, \mathbf{N}, \mathbf{I}, \mathbf{G}\}$$

$$\mathbf{A} - \mathbf{B} = \{\mathbf{T}, \mathbf{Y}\}$$

$$n(\mathbf{A} - \mathbf{B}) = 2$$

$$(iv) \mathbf{B} - \mathbf{A} = \{\mathbf{I}, \mathbf{N}, \mathbf{T}, \mathbf{E}, \mathbf{G}, \mathbf{R}, \mathbf{Y}\} - \{\mathbf{R}, \mathbf{E}, \mathbf{C}, \mathbf{K}, \mathbf{O}, \mathbf{N}, \mathbf{I}, \mathbf{G}\}$$

$$\mathbf{B} - \mathbf{A} = \{\mathbf{C}, \mathbf{K}, \mathbf{O}\}$$

$$n(\mathbf{B} - \mathbf{A}) = 3$$

Now,

$$(a) n(\mathbf{A} \cup \mathbf{B}) = 10$$

$$n(\mathbf{A}) + n(\mathbf{B}) - n(\mathbf{A} \cap \mathbf{B}) = 7 + 8 - 5$$

$$= 15 - 5 = 10$$

Therefore, by comparing the above results

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$(b) n(A - B) = 2$$

$$n(A \cup B) - n(B) = 10 - 8 = 2$$

$$n(A) - n(A \cap B) = 7 - 5 = 2$$

Therefore, by comparing the above results

$$n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$$

$$(c) n(B - A) = 3$$

$$n(A \cup B) - n(A) = 10 - 7 = 3$$

$$n(B) - n(A \cap B) = 8 - 5 = 3$$

Therefore, by comparing the above results

$$n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$$

$$(d) n(A \cup B) = 10$$

$$n(A - B) + n(B - A) + n(A \cap B) = 2 + 3 + 5 = 10$$

Therefore, by comparing the above results

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

6. If $\xi = \{\text{natural numbers between 10 and 40}\}$

A = {multiples of 5} and

B = {multiples of 6}, then

(i) find $A \cup B$ and $A \cap B$

(ii) verify that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Solution:-

From the question it is given that,

$$\xi = \{\text{natural numbers between 10 and 40}\}$$

$$\xi = \{11, 12, 13, 14, 15, \dots, 39\}$$

ξ is a universal set and A and B are subsets of ξ .

Then, the elements of A and B are to be taken only from ξ .

$$A = \{\text{multiples of 5}\}$$

$$A = \{15, 20, 25, 30, 35\}$$

$$B = \{\text{multiples of 6}\}$$

$$B = \{12, 18, 24, 30, 36\}$$

$$(i) A \cup B = \{15, 20, 25, 30, 35, 40\} \cup \{12, 18, 24, 30, 36\}$$

$$A \cup B = \{15, 20, 25, 30, 35, 12, 18, 24, 36\}$$

$$A \cap B = \{30\}$$

$$(ii) n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 5$$

$$n(A) = 5$$

$$n(B) = 5$$

$$n(A \cap B) = 1$$

$$\text{Then, } n(A) + n(B) - n(A \cap B) = 5 + 5 - 1 = 9$$

By comparing the results,

$$9 = 9$$

$$\text{Therefore, } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

7. If $\xi = \{1, 2, 3, \dots, 9\}$, $A = \{1, 2, 3, 4, 6, 7, 8\}$ and $B = \{4, 6, 8\}$, then find.

(i) A'

(ii) B'

(iii) $A \cup B$

(iv) $A \cap B$

(v) $A - B$

(vi) $B - A$

(vii) $(A \cap B)'$

(viii) $A' \cup B'$

Also verify that:

(a) $(A \cap B)' = A' \cup B'$

(b) $n(A) + n(A') = n(\xi)$

(c) $n(A \cap B) + n((A \cap B)') = n(\xi)$

(d) $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$

Solution:-

From the question it is given that,

$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{1, 2, 3, 4, 6, 7, 8\}$$

$$B = \{4, 6, 8\}$$

$$(i) A' = \xi - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 7, 8\}$$

$$A' = \{5, 9\}$$

$$(ii) B' = \xi - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{4, 6, 8\}$$

$$B' = \{1, 2, 3, 5, 7, 9\}$$

$$(iii) A \cup B = \{1, 2, 3, 4, 6, 7, 8\} \cup \{4, 6, 8\}$$

$$A \cup B = \{1, 2, 3, 4, 6, 7, 8\}$$

$$(iv) A \cap B = \{1, 2, 3, 4, 6, 7, 8\} \cap \{4, 6, 8\}$$

$$A \cap B = \{4, 6, 8\}$$

$$(v) A - B = \{1, 2, 3, 4, 6, 7, 8\} - \{4, 6, 8\}$$

$$A - B = \{1, 2, 3, 7\}$$

$$(vi) B - A = \{4, 6, 8\} - \{1, 2, 3, 4, 6, 7, 8\}$$

$$B - A = \{ \}$$

$$(vii) (A \cap B)' = \xi - (A \cap B)$$

$$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{4, 6, 8\}$$

$$(A \cap B)' = \{1, 2, 3, 5, 7, 9\}$$

$$(viii) A' \cup B' = \{5, 9\} \cup \{1, 2, 3, 5, 7, 9\}$$

$$A' \cup B' = \{1, 2, 3, 5, 7, 9\}$$

Then,

$$n(\xi) = 9$$

$$n(A) = 7$$

$$n(A') = 2$$

$$n(B') = 6$$

$$n(A \cap B) = 3$$

$$n((A \cap B)') = 6$$

$$n(A' \cup B') = 6$$

$$n(A - B) = 4$$

$$n(B - A) = 0$$

$$n(A \cup B) = 7$$

$$(a) (A \cap B)' = A' \cup B'$$

$$(A \cap B)' = \{1, 2, 3, 5, 7, 9\}$$

$$A' \cup B' = \{1, 2, 3, 5, 7, 9\}$$

By comparing the results,

$$(A \cap B)' = A' \cup B'$$

$$(b) n(A) + n(A') = n(\xi)$$

$$7 + 2 = 9$$

$$9 = 9$$

Therefore, by comparing the results, $n(A) + n(A') = n(\xi)$

$$(c) n(A \cap B) + n((A \cap B)') = n(\xi)$$

$$3 + 6 = 9$$

$$9 = 9$$

Therefore, by comparing the results, $n(A \cap B) + n((A \cap B)') = n(\xi)$

$$(d) n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$$

$$4 + 0 + 3 = 7$$

$$7 = 7$$

Therefore, by comparing the results, $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$

8. If $\xi = \{x : x \in W, x \leq 10\}$, $A = \{x : x \geq 5\}$ and $B = \{x : 3 \leq x < 8\}$, then verify that:

$$(i) (A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

$$(iii) A - B = A \cap B'$$

$$(iv) B - A = B \cap A'$$

Solution:-

$$\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{5, 6, 7, 8, 9, 10\}$$

$$B = \{3, 4, 5, 6, 7\}$$

$$(i) (A \cup B)' = A' \cap B'$$

First consider the Left hand side (LHS) = $(A \cup B)'$

$$(A \cup B) = \{5, 6, 7, 8, 9, 10\} \cup \{3, 4, 5, 6, 7\}$$

$$(A \cup B) = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(A \cup B)' = \xi - A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$(A \cup B)' = \{0, 1, 2\}$$

Therefore, LHS $(A \cup B)' = \{0, 1, 2\}$

Then, Right hand side (RHS) = $A' \cap B'$

$$A' = \xi - A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{5, 6, 7, 8, 9, 10\}$$

$$A' = \{0, 1, 2, 3, 4\}$$

$$B' = \xi - B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 4, 5, 6, 7\}$$

$$B' = \{0, 1, 2, 8, 9, 10\}$$

$$A' \cap B' = \{0, 1, 2, 3, 4\} \cap \{0, 1, 2, 8, 9, 10\}$$

$$A' \cap B' = \{0, 1, 2\}$$

Therefore, RHS $A' \cap B' = \{0, 1, 2\}$

By comparing LHS and RHS

$$(A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

Consider LHS $(A \cap B)'$

$$(A \cap B) = \{5, 6, 7, 8, 9, 10\} \cap \{3, 4, 5, 6, 7\}$$

$$(A \cap B) = \{5, 6, 7\}$$

$$(A \cap B)' = \xi - (A \cap B) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{5, 6, 7\}$$

$$(A \cap B)' = \{0, 1, 2, 3, 4, 8, 9, 10\}$$

$$\text{Therefore, LHS} = (A \cap B)' = \{0, 1, 2, 3, 4, 8, 9, 10\}$$

$$\text{Now, consider RHS} = A' \cup B'$$

$$A' \cup B' = \{0, 1, 2, 3, 4\} \cup \{0, 1, 2, 8, 9, 10\}$$

$$A' \cup B' = \{0, 1, 2, 3, 4, 8, 9, 10\}$$

$$\text{Therefore, RHS} = A' \cup B' = \{0, 1, 2, 3, 4, 8, 9, 10\}$$

By comparing LHS and RHS

$$(A \cap B)' = A' \cup B'$$

$$\text{(iii) } A - B = A \cap B'$$

$$\text{Consider the LHS} = A - B$$

$$A - B = \{5, 6, 7, 8, 9, 10\} - \{3, 4, 5, 6, 7\}$$

$$A - B = \{8, 9, 10\}$$

$$\text{Therefore, LHS} = A - B = \{8, 9, 10\}$$

$$\text{Now, consider RHS} = A \cap B'$$

$$(A \cap B') = \{5, 6, 7, 8, 9, 10\} \cap \{0, 1, 2, 8, 9, 10\}$$

$$(A \cap B') = \{8, 9, 10\}$$

$$\text{Therefore, RHS} = (A \cap B') = \{8, 9, 10\}$$

By comparing LHS and RHS,

$$A - B = A \cap B'$$

$$\text{(iv) } B - A = B \cap A'$$

Consider the LHS = $B - A$

$$B - A = \{3, 4, 5, 6, 7\} - \{5, 6, 7, 8, 9, 10\}$$

$$B - A = \{3, 4\}$$

$$\text{Therefore, LHS} = B - A = \{3, 4\}$$

Now, consider RHS = $B \cap A'$

$$B \cap A' = \{3, 4, 5, 6, 7\} \cap \{5, 6, 7, 8, 9, 10\}$$

$$B \cap A' = \{3, 4\}$$

$$\text{Therefore, RHS} = B \cap A' = \{3, 4\}$$

By comparing the LHS and RHS, $B - A = B \cap A'$.

9. If $n(A) = 20$, $n(B) = 16$ and $n(A \cup B) = 30$, find $n(A \cap B)$.

Solution:

From the question it is given that,

$$n(A) = 20$$

$$n(B) = 16$$

$$n(A \cup B) = 30$$

As we know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$30 = 20 + 16 - n(A \cap B)$$

$$30 = 36 - n(A \cap B)$$

$$n(A \cap B) = 36 - 30$$

$$n(A \cap B) = 6$$

Therefore, $n(A \cap B) = 6$

10. If $n(\xi) = 20$ and $n(A') = 7$, then find $n(A)$.

Solution:

From the question it is given that,

$$n(\xi) = 20$$

$$n(A') = 7$$

We know that, $n(A') = n(\xi) - n(A)$

$$7 = 20 - n(A)$$

$$n(A) = 20 - 7$$

$$n(A) = 13$$

Therefore, $n(A) = 13$

11. If $n(\xi) = 40$, $n(A) = 20$, $n(B') = 16$ and $n(A \cup B) = 32$, then find $n(B)$ and $n(A \cap B)$.

Solution:

From the question it is given that,

$$n(\xi) = 40$$

$$n(A) = 20$$

$$n(B') = 16$$

$$n(A \cup B) = 32$$

We know that, $n(B') = n(\xi) - n(B)$

$$16 = 40 - n(B)$$

$$n(B) = 40 - 16$$

$$n(B) = 24$$

Then, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$32 = 20 + 24 - n(A \cap B)$$

$$32 = 44 - n(A \cap B)$$

$$n(A \cap B) = 44 - 32$$

$$n(A \cap B) = 12$$

Therefore, $n(A \cap B) = 12$

12. If $n(\xi) = 32$, $n(A) = 20$, $n(B) = 16$ and $n((A \cup B)') = 4$, find :

(i) $n(A \cup B)$

(ii) $n(A \cap B)$

(iii) $n(A - B)$

Solution:-

From the question it is given that,

$$n(\xi) = 32$$

$$n(A) = 20$$

$$n(B) = 16$$

$$n((A \cup B)') = 4$$

Then,

$$(i) n(A \cup B) = n(\xi) - n((A \cup B)')$$

$$= 32 - 4$$

$$n(A \cup B) = 28$$

(ii) $n(A \cap B)$

We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$28 = 20 + 16 - n(A \cap B)$$

$$28 = 36 - n(A \cap B)$$

$$n(A \cap B) = 36 - 28$$

$$n(A \cap B) = 8$$

$$(iii) n(A - B) = n(A) - n(A \cap B)$$

$$= 20 - 8$$

$$= 12$$

Therefore, $n(A - B) = 12$

13. If $n(\xi) = 40$, $n(A') = 15$, $n(B) = 12$ and $n((A \cap B)') = 32$, find :

(i) $n(A)$

(ii) $n(B')$

(iii) $n(A \cap B)$

(iv) $n(A \cup B)$

(v) $n(A - B)$

(vi) $n(B - A)$

Solution:-

From the question it is given that,

$$n(\xi) = 40$$

$$n(A') = 15$$

$$n(B) = 12$$

$$n((A \cap B)') = 32$$

(i) $n(A)$

We know that, $n(A) = n(\xi) - n(A')$

$$n(A) = 40 - 15$$

$$n(A) = 25$$

$$(ii) n(B')$$

$$\text{We know that, } n(B') = n(\xi) - n(B)$$

$$n(B') = 40 - 12$$

$$n(B') = 28$$

$$(iii) n(A \cap B) = n(\xi) - n((A \cap B)')$$

$$= 40 - 32$$

$$= 8$$

$$(iv) n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 25 + 12 - 8$$

$$= 37 - 8$$

$$= 29$$

$$(v) n(A - B) = n(A) - n(A \cap B)$$

$$= 25 - 8$$

$$= 17$$

$$(vi) n(B - A) = n(B) - n(A \cap B)$$

$$= 12 - 8$$

$$= 4$$

14. If $n(A - B) = 12$, $n(B - A) = 16$ and $n(A \cap B) = 5$, find:

(i) $n(A)$

(ii) $n(B)$

(iii) $n(A \cup B)$

Solution:-

From the question it is given that,

$$n(A - B) = 12$$

$$n(B - A) = 16$$

$$n(A \cap B) = 5$$

(i) $n(A)$

$$\text{We know that, } n(A) = n(A - B) + n(A \cap B)$$

$$= 12 + 5$$

$$= 17$$

(ii) $n(B)$

$$\text{We know that, } n(B) = n(B - A) + n(A \cap B)$$

$$= 16 + 5$$

$$n(B) = 21$$

(iii) $n(A \cup B)$

$$\text{We know that, } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 17 + 21 - 5$$

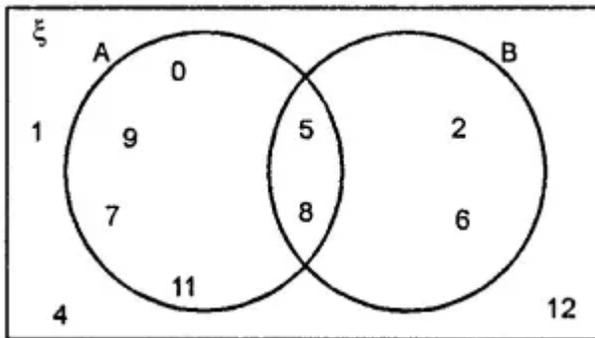
$$= 38 - 5$$

$$n(A \cup B) = 33$$

Exercise 6.2

1. From the adjoining Venn diagram, find the following sets :

- (i) A
- (ii) B
- (iii) ξ
- (iv) A'
- (v) B'
- (vi) $A \cup B$
- (vii) $A \cap B$
- (viii) $(A \cup B)'$
- (ix) $(A \cap B)'$



Solution:-

From the Venn diagram,

- (i) $A = \{0, 5, 7, 8, 9, 11\}$
- (ii) $B = \{2, 5, 6, 8\}$
- (iii) $\xi = \{0, 1, 2, 4, 5, 6, 7, 8, 9, 11, 12\}$
- (iv) A'

We know that $A' = \xi - A$

So, $A' = \{1, 2, 4, 6, 12\}$

(v) B'

We know that $B' = \xi - B$

So, $B' = \{0, 1, 4, 7, 9, 11, 12\}$

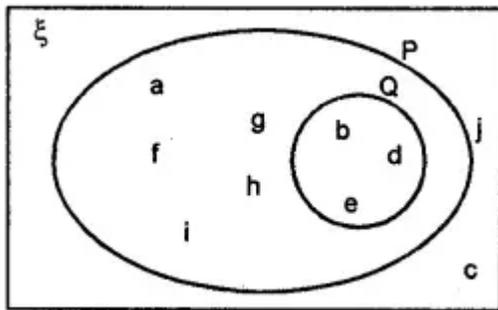
(vi) $A \cup B = \{0, 2, 5, 6, 7, 8, 9, 11\}$

(vii) $A \cap B = \{5, 8\}$

(viii) $(A \cup B)' = \{1, 4, 12\}$

(ix) $(A \cap B)' = \{0, 1, 2, 4, 6, 7, 9, 11, 12\}$

2. From the adjoining Venn diagram, find the following sets:



(i) P

(ii) Q

(iii) ξ

(iv) P'

(v) Q'

(vi) $P \cup Q$

(vii) $P \cap Q$

(viii) $(P \cup Q)'$

(ix) $(P \cap Q)'$

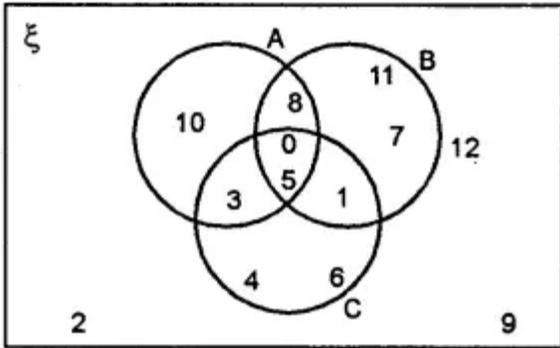
Solution:-

From the given Venn diagram, we have,

- (i) $P = \{a, b, d, e, f, g, h, i\}$
- (ii) $Q = \{b, d, e\}$
- (iii) $\xi = \{a, b, c, d, e, f, g, h, i, j\}$
- (iv) $P' = \{c, j\}$
- (v) $Q' = \{a, c, f, g, h, i, j\}$
- (vi) $P \cup Q = \{a, b, d, e, f, g, h, i\}$
- (vii) $P \cap Q = \{b, d, e\}$
- (viii) $(P \cup Q)' = \{c, j\}$
- (ix) $(P \cap Q)' = \{a, c, f, g, h, i, j\}$

3. From the adjoining Venn diagram, find the following sets :

- (i) ξ
- (ii) $A \cap B$
- (iii) $A \cap B \cap C$
- (iv) C'
- (v) $A - C$
- (vi) $B - C$
- (vii) $C - B$
- (viii) $(A \cup B)'$
- (ix) $(A \cup B \cup C)'$



Solution:-

From the Venn diagram,

$$A = \{0, 3, 5, 8, 10\}$$

$$B = \{0, 1, 5, 7, 8, 11\}$$

$$C = \{0, 1, 3, 4, 5, 6\}$$

$$(i) \xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$(ii) A \cap B = \{0, 5, 8\}$$

$$(iii) A \cap B \cap C = \{0, 5\}$$

$$(iv) C'$$

We know that $C' = \xi - C$

$$\text{So, } C' = \{2, 7, 8, 9, 10, 11, 12\}$$

$$(v) A - C = \{8, 10\}$$

$$(vi) B - C = \{7, 8, 11\}$$

$$(vii) C - B = \{3, 4, 6\}$$

$$(viii) (A \cup B)' = \{2, 4, 6, 9, 12\}$$

$$(ix) (A \cup B \cup C)' = \{2, 9, 12\}$$

4. Draw Venn diagrams to show the relationship between the following pairs of sets:

(i) $A = \{x \mid x \in \mathbb{N}, x = 2n, n \leq 5\}$ and $B = \{x \mid x \in \mathbb{W}, x = 4n, n < 5\}$

Solution:-

From the question it is given that,

$$A = \{x \mid x \in \mathbb{N}, x = 2n, n \leq 5\}$$

$$x = 2n = (2 \times 1) = 2 \dots (2 \times 5) = 10$$

$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{x \mid x \in \mathbb{W}, x = 4n, n < 5\}$$

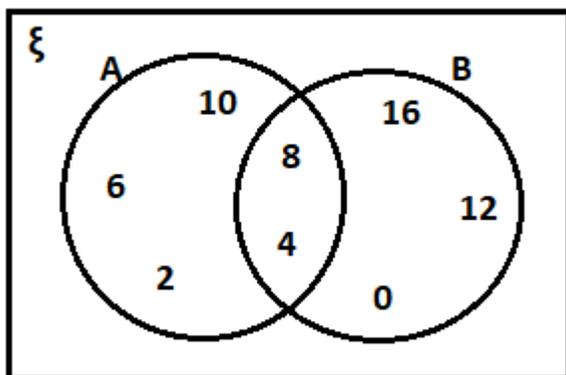
$$x = 4n = (4 \times 0) = 0 \dots (4 \times 4) = 16$$

$$B = \{4, 8, 12, 16\}$$

$$A \cap B = \{4, 8\}$$

Therefore, the sets are overlapping.

An adjoining Venn diagrams to show the relationship between the A and B sets,



(ii) $A = \{\text{prime factors of } 42\}$ and $B = \{\text{prime factors of } 60\}$

Solution:-

$$A = \{\text{prime factors of } 42\}$$

$$A = \{2, 3, 7\}$$

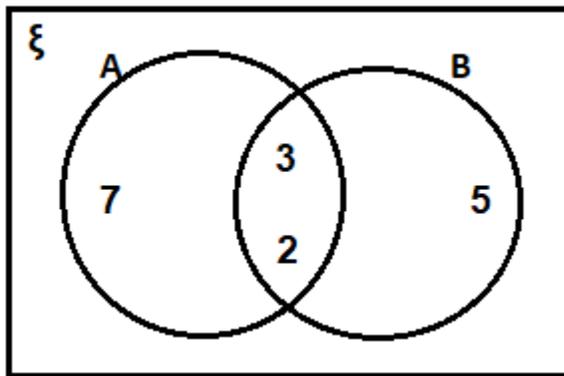
$$B = \{\text{prime factors of } 60\}$$

$$B = \{2, 3, 5\}$$

$$A \cap B = \{2, 3\}$$

Therefore, the sets are overlapping.

An adjoining Venn diagrams to show the relationship between the A and B sets,



(iii) $P = \{x \mid x \in \mathbb{W}, x < 10\}$ and $Q = \{\text{prime factors of } 210\}$

Solution:-

From the question it is given that,

$$P = \{x \mid x \in \mathbb{W}, x < 10\}$$

$$P = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

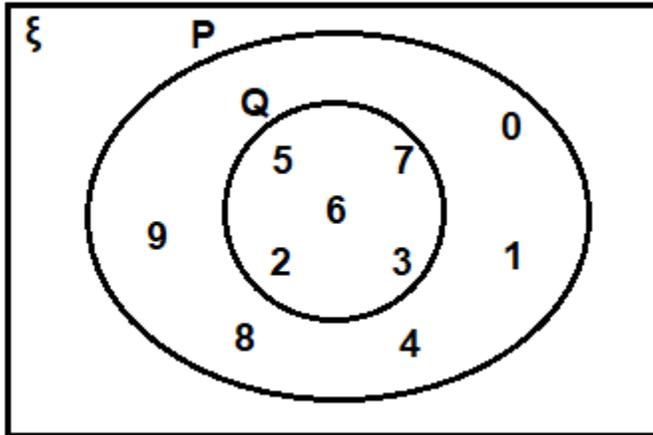
$$Q = \{\text{prime factors of } 210\}$$

$$Q = \{2, 3, 5, 6, 7\}$$

$P \cap Q = \{2, 3, 5, 6, 7\}$, Q is a subset of P.

Therefore, the sets are overlapping.

An adjoining Venn diagrams to show the relationship between the A and B sets,



**5. Draw a Venn diagram to illustrate the following information:
 $n(A) = 22$, $n(B) = 18$ and $n(A \cap B) = 5$.**

Hence find:

- (i) $n(A \cup B)$**
- (ii) $n(A - B)$**
- (iii) $n(B - A)$**

Solution:-

From the question it is given that,

$$n(A) = 22$$

$$n(B) = 18$$

$$n(A \cap B) = 5$$

Then, We know that $n(\text{only } A) = n(A) - n(A \cap B)$

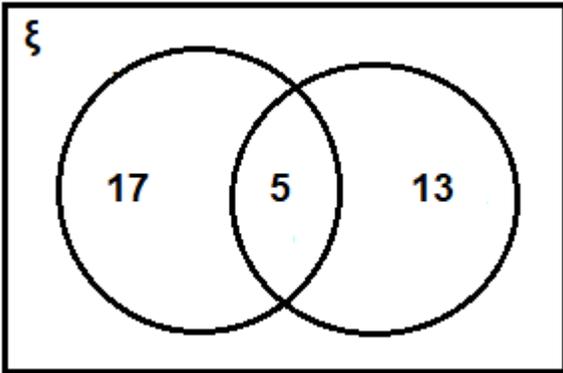
$$= 22 - 5$$

$$= 17$$

$$\text{So, } n(\text{only } B) = n(B) - n(A \cap B)$$

$$= 18 - 5$$

$$= 13$$



(i) $n(A \cup B)$

From the Venn diagram,

$$n(A \cup B) = 17 + 5 + 13 = 35$$

(ii) $n(A - B) = 17$

(iii) $n(B - A) = 13$

6. Draw a Venn diagram to illustrate the following information: $n(A) = 25$, $n(B) = 16$, $n(A \cap B) = 6$ and $n((A \cup B)') = 5$

Hence find:

(i) $n(A \cup B)$

(ii) $n(\xi)$

(iii) $n(A - B)$

(iv) $n(B - A)$

Solution:-

From the question it is given that,

$$n(A) = 25$$

$$n(B) = 16$$

$$n(A \cap B) = 6$$

$$n((A \cup B)') = 5$$

Then, We know that $n(\text{only } A) = n(A) - n(A \cap B)$

$$= 25 - 6$$

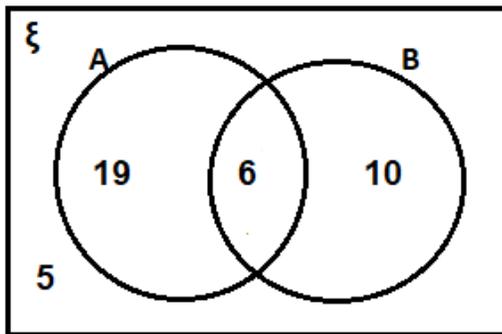
$$= 19$$

So, $n(\text{only } B) = n(B) - n(A \cap B)$

$$= 16 - 6$$

$$= 10$$

An adjoining Venn diagrams to show the relationship between the A and B sets,



From the Venn diagram,

$$(i) n(A \cup B) = 19 + 6 + 10 = 35$$

$$(ii) n(\xi) = 19 + 6 + 10 + 5 = 40$$

$$(iii) n(A - B) = 19$$

$$(iv) n(B - A) = 10$$

7. Given $n(\xi) = 25$, $n(A') = 7$, $n(B) = 10$ and $B \subset A$. Draw a Venn diagram to illustrate this information. Hence, find the cardinal number of the set $A - B$.

Solution:-

From the question it is given that,

$$n(\xi) = 25$$

$$n(A') = 7$$

$$n(B) = 10$$

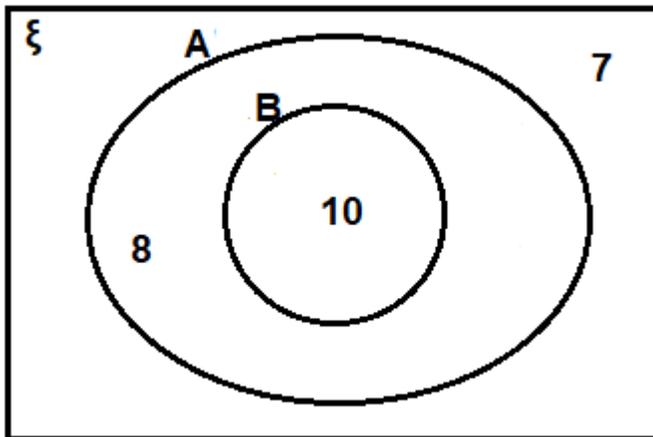
$$B \subset A$$

$$\text{Then, } n(A) = n(\xi) - n(A')$$

$$= 25 - 7$$

$$= 18$$

Venn diagrams to show the relationship between the A and B sets,



Therefore, the cardinal number of the set $A - B = 8$

8. In a group of 50 boys, 20 play only cricket, 12 play only football and 5 boys play both the games. Draw a Venn diagram and find the number of boys who play

- (i) At least one of the two games cricket or football.**
- (ii) Neither cricket nor football.**

Solution:-

Let us assume ξ be the all-boys

Let us assume P be the students who play cricket

And also assume that Q be the students who play football

From the question,

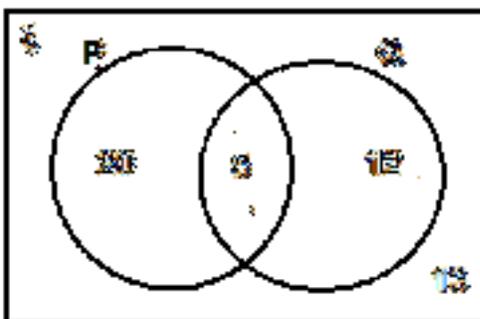
$$n(\xi) = 50$$

$$n(P) = 20$$

$$n(Q) = 12$$

$$n(P \cap Q) = 5$$

An adjoining Venn diagrams to show the relationship between the P and Q sets,



Then,

(i) at least one of the two games cricket or football, $n(P \cup Q) = 20 + 5 + 12 = 37$

(ii) neither cricket nor football = 13

9. In a group of 40 students, 26 students like orange but not banana, while 32 students like oranges. If all the students like at least one of the two fruits, find the number of students who like

(i) Both orange and banana

(ii) Only banana.

Draw a Venn diagram to represent the data.

Solution:-

Let us assume ξ be the total students

Let us assume P be the students who like orange

And also assume that Q be the students who like banana

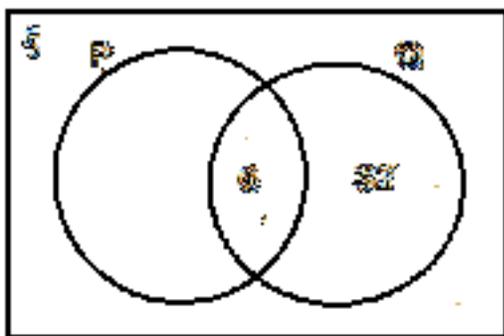
From the question it is given that,

$$n(\xi) = 40$$

$$n(P - Q) = 26$$

$$n(Q) = 32$$

$$n(P \cup Q) = 40$$



Then,

(i) Total number of students like both orange and banana. i.e. $n(P \cap Q) = 6$

(ii) Number of students like only banana = $40 - 32 = 8$

10. In a group of 60 persons, 45 speak Bengali, 28 speak English and all the persons speak at least one language. Find how many people speak both Bengali and English. Draw a Venn diagram.

Solution:-

Let us assume ξ be the group of persons

Let us assume P be the persons who speak Bengali

And also assume that Q be the persons who speak English

From the question,

$$n(\xi) = 60$$

$$n(P) = 45$$

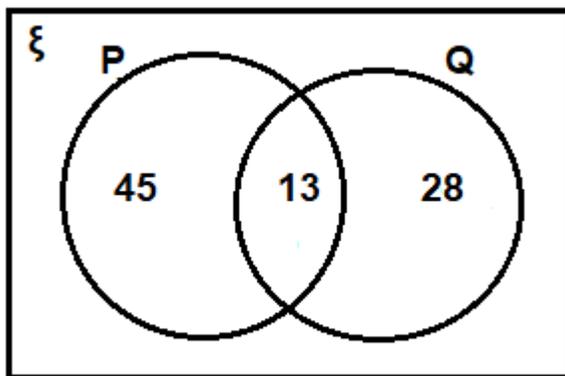
$$n(Q) = 28$$

$$n(P \cup Q) = 60$$

$$\text{Then, } n(P \cap Q) = n(P) + n(Q) - n(P \cup Q)$$

$$= 45 + 28 - 60$$

$$= 13$$



Mental Maths

Question 1: Fill in the blanks:

- (i) If A, B are two sets, then $A \cup B = \dots\dots\dots$
- (ii) If A, B are two sets, then $A \cap B = \dots\dots\dots$
- (iii) If A, B are two sets, then $A - B = \dots\dots\dots$
- (iv) A and B are disjoint sets if and only if $A \cap B = \dots\dots\dots$
- (v) A and B are overlapping sets if and only if $A \cap B = \dots\dots\dots$
- (vi) The set $\{x: x \in W, x < 3\}$ in the roster form = $\dots\dots\dots$
- (vii) If A is any set, then $A \cup \phi =$ and $A \cup \xi = \dots\dots\dots$
- (viii) If $\xi = \{\text{all digits in our number system}\}$ and $A = \{1, 2, 3, 4, 5\}$, then $A' = \dots\dots\dots$
- (ix) If A is any set and A' is its complement, then $A \cup A' =$ and $A \cap A' = \dots\dots\dots$

Solution:

- (i) If A, B are two sets, then $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- (ii) If A, B are two sets, then $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.
- (iii) If A, B are two sets, then $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.
- (iv) A and B are disjoint sets if and only if $A \cap B = \phi$
- (v) A and B are overlapping sets if and only if $A \cap B \neq \phi$.
- (vi) The set $\{x: x \in W, x < 3\}$ in the roster form = $\{0, 1, 2\}$.
- (vii) If A is any set, then $A \cup \phi = A$ and $A \cup \xi = \xi$.
- (viii) If $\xi = \{\text{all digits in our number system}\}$ and $A = \{1, 2, 3, 4, 5\}$, then $A' = \{0, 6, 7, 8, 9\}$.
- (ix) If A is any set and A' is its complement, then $A \cup A' = \xi$ and $A \cap A' = \phi$

Question 2: State whether the following statements are true (T) or false (F):

- (i) If ξ is the universal set and A is any set, then $A' = \{x; x \in \xi \text{ and } x \notin A\}$.
- (ii) If $A = \{0, 1, 2, 3, 4, 5\}$ and $B = \{0, 3, 5, 7\}$, then $A \cap B = B$.

(iii) If $A = \{0,1, 2,3,4, 5\}$ and $B = \{0,3,5,7\}$, then $A \cup B = A$.

(iv) If $\xi = \{\text{all digits in our number system}\}$, $A = \{\text{multiples of 2}\}$ and $B = \{\text{multiples of 3}\}$, then $A \cap B = \{6\}$.

Solution:

(i) If ξ is the universal set and A is any set,
then $A' = \{x ; x \in \xi, \text{ and } x \notin A\}$. True

(ii) If $A = \{0, 1, 2, 3, 4, 5\}$ and $B = \{0, 3, 5, 7\}$, then $A \cap B = B$. False

Correct:

As $A \cap B = \{0, 3, 5\}$

(iii) If $A = \{0, 1, 2, 3, 4, 5\}$ and $B = \{0, 3, 5, 7\}$, then $A \cup B = A$. False

Correct:

As $A \cup B = \{0, 1, 2, 3, 4, 5, 7\}$

(iv) If $\xi = \{\text{all digits in our number system}\}$, $A = \{\text{multiples of 2}\}$
and $B = \{\text{multiples of 3}\}$, then $A \cap B = \{6\}$. True

If $\xi = \{\text{all digits in our number system}\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{\text{multiples of 2}\} = \{2, 4, 6, 8\}$

$B = \{\text{multiples of 3}\} = \{3, 6, 9\}$

Then $A \cap B = \{6\}$

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 12):

Question 3.

If $A = \{x \mid x \text{ is a colour of rainbow}\}$ and $B = \{\text{white, red, green}\}$, then

$A \cap B$ is

(a) B

(b) $\{\text{green}\}$

(c) $\{\text{red}\}$

(d) $\{\text{green, red}\}$

Solution:

$A = \{x \mid x \text{ is a colour of rainbow}\}$

= {red, green, blue, violet, yellow, Indigo, orange}

$B = \{\text{white, red, green}\}$

$A \cap B = \{\text{red, green}\}$ (d)

Question 4: If $P = \{-1, 0, 1, 2, 5\}$ and $Q = \{3, 5, 7\}$, then $P \cup Q$ is

(a) $\{5\}$

(b) $\{-1, 0, 1, 2, 3, 7\}$

(c) $\{-1, 0, 1, 2, 3, 5, 7\}$

(d) none of these

Solution:

$P = \{-1, 0, 1, 2, 5\}$

$Q = \{3, 5, 7\}$

$\therefore P \cup Q = \{0, 1, 2, 3, 5, 7\}$ (c)

Question 5: If A and B are two sets, then $A - B$ is defined as

(a) $\{x \mid x \in A \text{ or } x \in B\}$

(b) $\{x \mid x \in A \text{ and } x \in B\}$

(c) $\{x \mid x \in A \text{ and } x \notin B\}$

(d) $\{x \mid x \in B \text{ and } x \notin A\}$

Solution:

A and B are two sets

$\therefore A - B = \{x \mid x \in A \text{ and } x \notin B\}$ (c)

Question 6: If A is any set, then $A \cup \phi$ is

(a) A

(b) ϕ

(c) 2,

(d) none of these

Solution:

$$A \cup \phi = A$$

Where A is any set. (a)

Question 7: $A \cap \xi$ is same as

(a) A

(b) ϕ

(c) A'

(d) ξ

Solution:

$$A \cap \xi = A \text{ (a)}$$

Question 8: If $\xi = W$ and $A = \{x \mid x \in W \text{ and } x \leq 10\}$, then A' is

(a) ϕ

(b) $\{x \mid x \in W \text{ and } 0 \leq x \leq 10\}$

(c) $\{x \mid x \in W \text{ and } x \leq 10\}$

(d) $\{x \mid x \in W \text{ and } x \geq 11\}$

Solution:

$$\xi = W, A = \{x \mid x \in W \text{ and } x \leq 10\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\therefore A' = \{x \mid x \in W, \text{ and } x \geq 11\} \text{ (d)}$$

Question 9: If $\xi = \{x \mid x \in W, x \leq 12\}$, $A = \{x \mid x \text{ is a multiple of } 3\}$ and

$B = \{x \mid x \text{ is a multiple of } 4\}$, then $A \cap B$ is

(a) ϕ

(b) $\{0\}$

(c) $\{12\}$

(d) $\{0, 12\}$

Solution:

$$A = \{x \mid x \text{ is a multiple of } 3\}$$

$$= \{0, 3, 6, 9, 12\}$$

$$\begin{aligned} B &= \{x \mid x \text{ is a multiple of } 4\} \\ &= \{0, 4, 8, 12\} \\ \therefore A \cap B &= \{0, 12\} \text{ (d)} \end{aligned}$$

Question 10: If $\xi = \{\text{all digits in our number system}\}$, $A = \{x \mid x \text{ is prime}\}$ and $B = \{x \mid x \text{ is even}\}$, then $B - A$ is

- (a) $\{4, 6, 8\}$
- (b) $\{0, 4, 6, 8\}$
- (c) $\{3, 5, 7\}$
- (d) $\{2\}$

Solution:

$$\begin{aligned} \xi &= \{\text{all digits in our number system}\} \\ &= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ A &= \{x \mid x \text{ is prime}\} \\ &= \{2, 3, 5, 7\} \\ B &= \{x \mid x \text{ is even}\} \\ &= \{0, 2, 4, 6, 8\} \\ B - A &= \{0, 4, 6, 8\} \text{ (b)} \end{aligned}$$

Question 11: If A and B are two sets such that $n(A) = 22$, $n(B) = 18$ and $n(A \cup B) = 35$, then $n(A \cap B)$

- (a) 4
- (b) 5
- (c) 15
- (d) 75

Solution:

$$\begin{aligned} n(A) &= 22, n(B) = 18 \\ n(A \cup B) &= 35 \\ n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 35 &= 22 + 18 - n(A \cap B) \\ n(A \cap B) &= 40 - 35 = 5 \text{ (b)} \end{aligned}$$

Question 12: In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of person who read neither is

(a) 180

(b) 210

(c) 260

(d) 290

Solution:

Total number of person = 840

Person who read Hindi = 450

$\Rightarrow n(A) = 450$

Person who read English = 300

$\Rightarrow n(B) = 300$

Person who read both = 200

$\Rightarrow n(A \cap B) = 200$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 450 + 300 - 200$

$= 750 - 200 = 550$

\therefore Person who read neither = $840 - 550 = 290$ (d)

Check Your Progress

Question 1: If $\xi = \{x: x \in \mathbb{N}, x < 25\}$ and $A = \{x: x \text{ is a composite number}\}$, then find A' in set builder form and also in roster form.

Solution:

$$\xi = \{x : x \in \mathbb{N}, x < 25\}$$

$$A = \{x : x \text{ is a composite number}\},$$

then find A' in set builder form and also in roster form

$$\xi = \{0, 2, 3, 4, \dots, 24\}$$

$$A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24\}$$

$$\therefore A' = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{x : x \in \mathbb{N}, x \text{ is a prime number less than } 25\}$$

Question 2: If $\xi = \{x \mid x \in \mathbb{I}, -5 \leq x \leq 5\}$, $A = \{-5, -3, 0, 3, 5\}$ and $B = \{-4, -2, 0, 2, 4\}$, then

(i) Find $A \cup B$ and $A \cap B$

(ii) Verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

(iii) Find A' and verify that $n(A) + n(A') = n(\xi)$.

(iv) Find B' and verify that $n(B') = n(\xi) - n(B)$

(v) Find $A \cup A'$, $A \cap A'$, $B \cup B'$ and $B \cap B'$.

(vi) Find $(A \cup B)'$ and $A' \cap B'$. Are they equal?

(vii) Find $(A \cap B)'$ and $A' \cup B'$. Are they equal?

Solution:

$$\xi = \{x \mid x \in \mathbb{I}, -5 \leq x \leq 5\}$$

$$= \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

$$n(\xi) = 11$$

$$A = \{-5, -3, 0, 3, 5\} \Rightarrow n(A) = 5$$

$$B = \{-4, -2, 0, 2, 4\} \Rightarrow n(B) = 5$$

$$(i) A \cup B = \{-5, -4, -3, -2, 0, 2, 3, 4, 5\}$$

$$\Rightarrow n(A \cup B) = 9$$

$$A \cap B = \{0\}$$

$$\Rightarrow n(A \cap B) = 1$$

$$(ii) n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{L.H.S.} = n(A \cup B) = 9$$

$$\text{R.H.S.} = n(A) + n(B) - n(A \cap B)$$

$$= 5 + 5 - 1 = 9$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(iii) A' = \xi - A = \{-4, -2, -1, 1, 2, 4\}$$

$$\Rightarrow n(A') = 6$$

$$\text{Now, } n(A) + n(A') = 5 + 6 = 11 = n(\xi)$$

Hence verified

$$(iv) B' = \xi - B = \{-5, -3, -1, 1, 3, 5\}$$

$$n(B') = 6$$

$$n(\xi) - n(B) = 11 - 5 = 6 = n(B')$$

$$(v) A \cup A' = \xi$$

$$A \cap A' = \phi$$

$$B \cup B' = \xi, B \cap B' = \phi$$

$$(vi) (A \cup B)' = \{-1, 1\} \text{ and } A' \cap B' = \{-1, 1\}$$

Yes both are equal.

$$(vii) (A \cap B)' = \{-5, -4, -3, -2, -1, 1, 2, 3, 4, 5\}$$

$$A' \cup B' = \{-5, -4, -3, -2, -1, 1, 2, 3, 4, 5\}$$

$$\text{Yes, } (A \cap B)' = A' \cup B'$$

Question 3: If $\xi = \{x \mid x \in \mathbb{N}, x \leq 12\}$, $A = \{\text{prime numbers}\}$ and $B = \{\text{odd numbers}\}$, then

(i) Find $A \cup B$ and $A \cap B$.

(ii) Verify that $n(A \cup B) + n(A \cap B) = n(A) + n(B)$.

(iii) Find A' and B' .

(iv) Find $(A \cup B)'$ and verify that $n(A \cup B) + n(A \cup B)' = n(\xi)$.

(v) Find $(A \cap B)'$ and $A' \cup B'$. Are they equal?

Solution:

Here,

$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{2, 3, 5, 7, 11\}$$

$$B = \{1, 3, 5, 7, 9, 11\}$$

$$(i) A \cup B = \{1, 2, 3, 5, 7, 9, 11\}$$

$$\text{and } A \cap B = \{3, 5, 7, 11\}$$

$$(ii) n(A \cup B) + n(A \cap B) = n(A) + n(B)$$

$$7 + 4 = 5 + 6$$

$$11 = 11$$

Hence, verified.

$$(iii) A' = \xi - A = \{1, 4, 6, 8, 9, 10, 12\}$$

$$\text{and } B' = \xi - B = \{2, 4, 6, 8, 10, 12\}$$

$$(iv) (A \cup B)' = \xi - (A \cup B) = \{4, 6, 8, 10, 12\}$$

To show,

$$n(A \cup B) + n(A \cup B)' = n(\xi)$$

$$7 + 5 = 12$$

$$12 = 12$$

Hence verified

$$(v) (A \cap B)' = \xi - (A \cap B)$$

$$= \{1, 2, 4, 6, 8, 9, 10, 12\}$$

$$A' \cup B' = \{1, 2, 4, 6, 8, 9, 10, 12\}$$

Yes, the given sets are equal

because they have the same elements.

Question 4: If $\xi = \{x : x \in \mathbb{N}, x \leq 12\}$, $A = \{x : x \geq 7\}$ and $B = \{x : 4 < x < 10\}$. Find :

(i) A'

(ii) B'

$$(iii) A \cup B$$

$$(iv) A \cap B$$

$$(v) A - B$$

$$(vi) B - A$$

$$(vii) (A \cup B)'$$

$$(viii) A' \cap B'$$

Also verify that:

$$(i) (A \cup B)' = A' \cap B'$$

$$(ii) A - B = A \cap B'$$

$$(iii) n(A \cup B) + n((A \cup B)') = n(\xi)$$

$$(iv) n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B).$$

Solution:

The given sets in the roster form are :

$$\xi = \{1, 2, 3, \dots, 12\}$$

$$A = \{7, 8, 9, 10, 11, 12\}$$

$$B = \{5, 6, 7, 8, 9\}$$

$$(i) A' = \xi - A$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} - \{7, 8, 9, 10, 11, 12\}$$

$$= \{1, 2, 3, 4, 5, 6\}$$

$$(ii) B' = \xi - B$$

$$= \{1, 2, 3, 12\} - \{5, 6, 7, 8, 9\}$$

$$= \{1, 2, 3, 4, 10, 11, 12\}$$

$$(iii) A \cup B = \{7, 8, 9, 10, 11, 12\} \cup \{5, 6, 7, 8, 9\} = \{5, 6, 7, 8, 9, 10, 11, 12\}$$

$$(iv) A \cap B = \{7, 8, 9, 10, 11, 12\} \cap \{5, 6, 7, 8, 9\} = \{7, 8, 9\}$$

$$(v) A - B = \{7, 8, 9, 10, 11, 12\} - \{5, 6, 7, 8, 9\} = \{10, 11, 12\}$$

$$(vi) B - A = \{5, 6, 7, 8, 9\} - \{7, 8, 9, 10, 11, 12\} = \{5, 6\}$$

$$(vii) (A \cup B)' = \xi - A \cup B$$

$$= \{1, 2, 3 \dots 12\} - \{5, 6, 7, 8, 9, 10, 11, 12\}$$

$$= \{1, 2, 3, 4\}$$

$$(viii) A' \cap B' = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 10, 11, 12\} = \{1, 2, 3, 4\}$$

Verification :

$$(i) (A \cup B)' = \{1, 2, 3, 4\} \text{ (By VII part)}$$

$$\text{and } A' \cap B' = \{1, 2, 3, 4\} \text{ (By VIII part)}$$

Hence $(A \cup B)' = A' \cap B'$ (Verified)

$$(ii) A - B = \{10, 11, 12\} \text{ (By (v) part)}$$

$$\text{and } A \cap B' = \{7, 8, 9, 10, 11, 12\} \cap \{1, 2, 3, 4, 10, 11, 12\} \text{ (By (ii) part)} \\ = \{10, 11, 12\}$$

Hence $A - B = A \cap B'$ (Verified)

$$(iii) n(A \cup B) + n((A \cup B)') = 8 + 4 = 12$$

$$n(\xi) = 12$$

Hence $n(A \cup B) + n((A \cup B)') = n(\xi)$ (Verified)

$$(iv) n(A \cup B) = 8$$

$$\text{and } n(A - B) + n(B - A) + n(A \cap B)$$

$$= 3 + 2 + 3 = 8$$

Hence $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$ (Verified)

Question 5: Given $A = \{\text{students who like cricket}\}$ and $B = \{\text{students who like tennis}\}$; $n(A) = 20$, $n(B) = 15$ and $n(A \cap B) = 5$.

Illustrate this through a Venn diagram. Hence find $n(A \cup B)$.

Solution:

$$\text{Here, } n(A) = 20$$

$$n(B) = 15$$

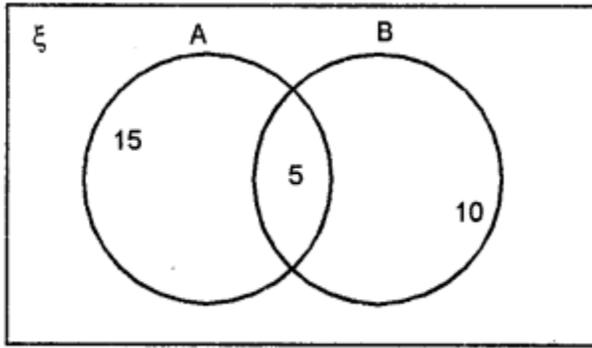
$$n(A \cap B) = 5$$

Now, only A i.e. students who like cricket only

$$= n(A) - n(A \cap B) = 20 - 5 = 15$$

Only B i.e. students who like tennis only

$$= n(B) - n(A \cap B) = 15 - 5 = 10$$



$n(A \cup B) = 15 + 5 + 10$ (from Venn diagram)

Using formula

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 20 + 15 - 5 = 35 - 5 = 30. \end{aligned}$$

Question 6: If $n(\xi) = 50$, $n(A) = 15$, $n(B) = 13$ and $n(A \cap B) = 10$. Find $n(A')$, $n(B')$ and $n(A \cup B)$.

Solution:

Here, $n(\xi) = 50$

$n(A) = 15$

$n(B) = 13$

$n(A \cap B) = 10$

Using formula,

$$n(A') = n(\xi) - n(A) = 50 - 15 = 35$$

$$n(B') = n(\xi) - n(B) = 50 - 13 = 37$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 15 + 13 - 10 = 18.$$

Question 7: If $n(\xi) = 60$, $n(A) = 35$, $n(B') = 36$ and $n((A \cap B)') = 51$, find :

(i) $n(B)$

(ii) $n(A \cap B)$

(iii) $n(A \cup B)$

(iv) $n(A - B)$

Solution:

Given $n(\xi) = 60$

$n(A) = 35$

$n(B') = 36$

and $n((A \cap B)') = 51$

(i) $n(B) = n(\xi) - n(B') = 60 - 36 = 24$

(ii) $n(A \cap B) = n(\xi) - n((A \cap B)') = 60 - 51 = 9$

(iii) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 35 + 24 - 9 = 59 - 9 = 50$

(iv) $n(A - B) = n(A) - n(A \cap B) = 35 - 9 = 26$

Question 8: In a city, 50% people read newspaper A, 45% read newspaper B, and 25% read neither A nor B. What percentage of people read both the newspapers A as well as B?

Solution:

Let A = Those people who read newspaper A

B = Those people who read newspaper B

Here $n(\xi) = 100\%$

$n(A) = 50\%$

$n(B) = 45\%$

$n(A \cup B)' = 25\%$

$\therefore n(A \cup B) = n(\xi) - n((A \cup B)') = 100\% - 25\% = 75\%$

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 75\% = 50\% + 45\% - n(A \cap B)$

$\Rightarrow n(A \cap B) = 95\% - 75\%$

$\Rightarrow n(A \cap B) = 20\%$