Triangles

Selected NCERT Questions

1. In the given Fig. 7.8, if *ABCD* is a trapezium in which *AB* ∥ *CD*, *E* and *F* are points on non-parallel sides *AD* and *BC* respectively such that *EF* is parallel to *AB*, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$. [*CBSE* 2020(30/1/1)]

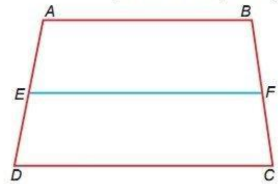


Fig. 7.8

Sol. We have *ABCD* as a trapezium.

Join *AC* which intersect *EF* at *G* (Fig. 7.9).

Now, in ΔCAB, we have

$GF \parallel AB$

$\Rightarrow \quad \frac{AG}{CG} = \frac{BF}{FC}$ (BPT) ..(*i*)

Also, in ΔADC, we have $EG \parallel DC$

$\Rightarrow \quad \frac{AE}{ED} = \frac{AG}{GC}$ (BPT) ..(*ii*)

From equations (*i*) and (*ii*), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

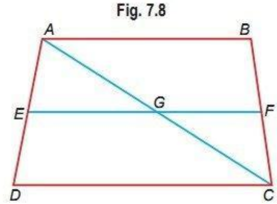


Fig. 7.9

2. *E* and *F* are points on the sides *PQ* and *PR* respectively of a ΔPQR. Show that $EF \parallel QR$ if *PQ* = 1.28 cm, *PR* = 2.56 cm, *PE* = 0.18 cm and *PF* = 0.36 cm.

Sol. We have, $PQ = 1.28$ cm, $PR = 2.56$ cm

$PE = 0.18$ cm, $PF = 0.36$ cm

Now, $EQ = PQ - PE = 1.28 - 0.18 = 1.10$ cm

and $FR = PR - PF = 2.56 - 0.36 = 2.20$ cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

and, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\quad \therefore \quad \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, $EF \parallel QR$ [By the converse of Basic Proportionality Theorem]

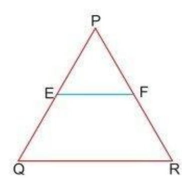


Fig. 7.10

3. In Fig. 7.11, if $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

Sol. Firstly, in ΔABC, we have

$LM \parallel CB$ (Given)

Therefore, by Basic Proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad ...(i)$$

Again, in ΔACD, we have

$LN \parallel CD$ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \quad ...(ii)$$

Now, from (*i*) and (*ii*), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

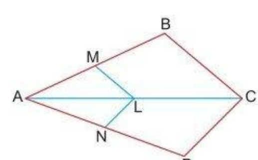


Fig. 7.11

4. **In Fig. 7.12, $DE \parallel OQ$ and $DF \parallel OR$, Show that $EF \parallel QR$.**

Sol. In ΔPOQ, we have

$DE \parallel OQ$ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \text{...}(i)$$

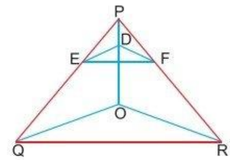


Fig. 7.12

Similarly, in ΔPOR, we have

$DF \parallel OR$ (Given)

$$\frac{PD}{DO} = \frac{PF}{FR} \quad \text{...}(ii)$$

Now, from (*i*) and (*ii*), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow \quad EF \parallel QR$$

[Applying the converse of Basic Proportionality Theorem in ΔPQR]

5. **The diagonals of a quadrilateral *ABCD* intersect each other at the point *O* such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that *ABCD* is a trapezium.**

Sol. $\frac{AO}{BO} = \frac{CO}{DO}$ (Given)

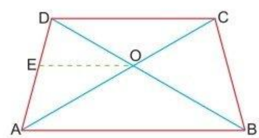


Fig. 7.13

$\Rightarrow$ $\frac{AO}{CO} = \frac{BO}{DO}$ ...(*i*)

In ΔABD, $EO \parallel AB$ (Construction)

$\therefore$ $\frac{AE}{ED} = \frac{BO}{DO}$ (By BPT) ...(*ii*)

From equations (*i*) and (*ii*)

$\frac{AE}{ED} = \frac{AO}{CO}$ $\Rightarrow$ $EO \parallel DC$ (Converse of BPT)

But $EO \parallel AB$ (Construction)

$\therefore$ $AB \parallel DC$

$\Rightarrow$ In quad *ABCD* since $AB \parallel DC$ $\Rightarrow$ *ABCD* is a trapezium.

6. ***E* is a point on the side *AD* produced of a parallelogram *ABCD* and *BE* intersects *CD* at *F*. Show that $\Delta ABE \sim \Delta CFB$.**

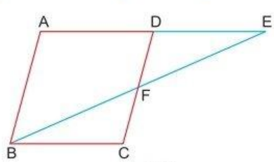


Fig. 7.14

Sol. In ΔABE and ΔCFB, we have

$\angle AEB = \angle CBF$ (Alternate angles)

$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\therefore$ $\Delta ABE \sim \Delta CFB$ (By *AA* criterion of similarity)

7. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG. If $\Delta ABC \sim \Delta FEG$, show that

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$ (iii) $\Delta DCA \sim \Delta HGF$

Sol. $\Delta ABC \sim \Delta FEG$, [Given]

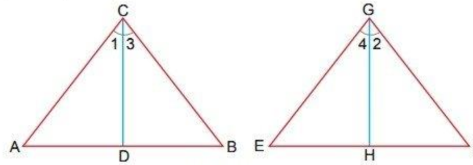


Fig. 7.15

$\Rightarrow$ $\angle A = \angle F$

$\angle B = \angle E$

$\angle C = \angle G$

and $\dfrac{AB}{FE} = \dfrac{BC}{EG} = \dfrac{AC}{FG}$ ...(a)

(i) In ΔACD and ΔFGH

$\angle A = \angle F$ [Given]

and $\angle 1 = \angle 2$ [$\frac{1}{2}\angle C = \frac{1}{2}\angle G$]

$\therefore$ $\Delta ACD \sim \Delta FGH$ [AA Similarity]

$\Rightarrow$ $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ [Corresponding sides of similar triangles]

(ii) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ [Proved above]

But $\dfrac{AC}{FG} = \dfrac{BC}{EG}$ [From (a)]

$\therefore$ $\dfrac{CD}{GH} = \dfrac{BC}{EG}$

In $\Delta DCB \sim \Delta HGE$

$\angle 3 = \angle 4$ [$\frac{1}{2}\angle C = \frac{1}{2}\angle G$]

and $\dfrac{CD}{GH} = \dfrac{BC}{EG}$ [Proved above]

$\therefore$ $\Delta DCB \sim \Delta HGE$ [SAS Similarity]

(iii) In ΔDCA and ΔHGF,

$\angle 1 = \angle 2$ [$\frac{1}{2}\angle C = \frac{1}{2}\angle G$]

and $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ [Proved above]

$\Rightarrow$ $\Delta DCA \sim \Delta HGF$ [SAS Similarity]

8. In the given Fig. 7.16, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔECF. [*CBSE* 2019 (30/5/1)]

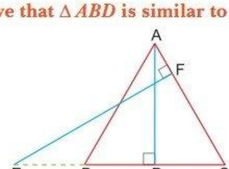


Fig. 7.16

Sol. $AB = AC \Rightarrow \angle C = \angle B$ ... (1) **1**

In ΔABD & ΔECF,

$\angle ADB = \angle EFC$ (each 90°) **1**

$\angle ABD = \angle ECF$ (by (1)) **1**

By AA similarity

$\Delta ABD \sim \Delta ECF$ **1**

[*CBSE Marking Scheme* 2019]

9. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. Let AB be a vertical pole of length 6 m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

Now, in ΔABC and ΔDEF, we have

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the Sun)

$\therefore$ $\Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\dfrac{AB}{DE} = \dfrac{BC}{EF}$

$\Rightarrow$ $\dfrac{6}{h} = \dfrac{4}{28}$ (Let $DE = h$)

$\Rightarrow$ $\dfrac{6}{h} = \dfrac{1}{7}$ $\Rightarrow$ $h = 42$

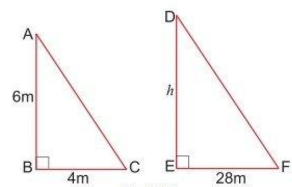


Fig. 7.17

Hence, height of tower, $DE = 42$ m

10. *ABCD* is a trapezium in which $AB \| DC$ and its diagonals intersect each other at the point O. Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

Sol. **Given:** $ABCD$ is a trapezium, in which $AB \| DC$ and its diagonals intersect each other at the point O.

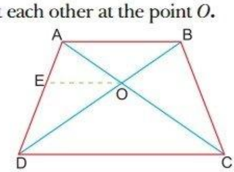


Fig. 7.18

To prove: $\dfrac{AO}{BO} = \dfrac{CO}{DO}$

Construction: Through O, draw $OE \| AB$ *i.e.*, $OE \| DC$.

Proof: In ΔADC, we have $OE \| DC$ (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \quad ...(i)$$

Now, in ΔABD, we have $OE \| AB$ (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \quad \Rightarrow \quad \frac{AE}{ED} = \frac{BO}{DO} \quad ...(ii)$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \quad \Rightarrow \quad \frac{AO}{BO} = \frac{CO}{DO}$$

11. *AD* and *PM* are median of triangles ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. [*CBSE* 2019(30/3/2)]

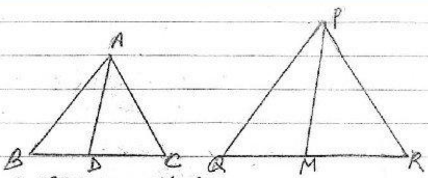
Sol.

17. Given,

$\Delta ABC \sim \Delta PQR$.

AD and PM

are medians of ΔABC and ΔPQR respectively.



Since, $\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}. \quad \text{——①}$$

D is the midpoint of BC (AD is median)

M is the midpoint of QR (PM is median)

$$\therefore BC = 2BD$$
$$QR = 2QM \quad \Big\}②$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} \quad [\text{from ①}]$$

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} \quad [\text{from ②}]$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \Rightarrow \quad \therefore \Delta ABD \sim \Delta PQM \quad \text{——ⓐ}$$

That is, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$$\frac{AB}{PQ} = \frac{AD}{PM}$$

Hence proved!

[*Topper's Answer* 2019]

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. *D* and *E* are respectively the points on the sides *AB* and *AC* of a triangle *ABC* such that $AD = 3$ cm, $BD = 5$ cm, $BC = 12.8$ cm and $DE \parallel BC$. Then length of *DE* (in cm) is

(*a*) 4.8 cm (*b*) 7.6 cm

(*c*) 19.2 cm (*d*) 2.5 cm

2. **Two similar figures are shown.**

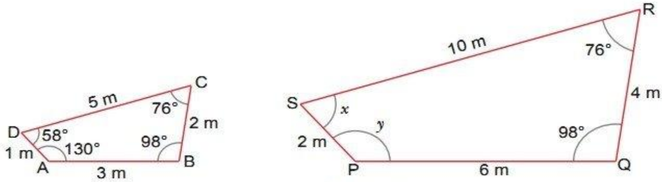


Fig. 7.19

What are the values of *x* and *y*?

(*a*) $x = 58°, y = 130°$ (*b*) $x = 98°, y = 76°$ (*c*) $x = 82°, y = 84°$ (*d*) $x = 130°, y = 84°$

3. **Consider the figure below.**

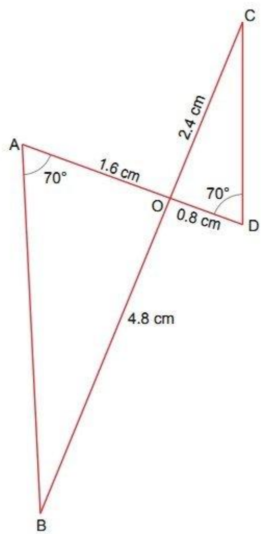


Fig. 7.20

Which of the following statement is correct about the triangles in the figure?

(*a*) $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$

(*b*) $\Delta AOB \sim \Delta DOC$ because $\angle AOB = \angle DOC$

(*c*) $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$ and $\angle BAO = \angle CDO$

(*d*) $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$ and $\angle AOB = \angle DOC$

4. **In Fig. 7.21, two line segments AC and BD intersect each other at the point P such that $PA = 6$ cm, $PB = 3$ cm, $PC = 2.5$ cm, $PD = 5$ cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to** [*NCERT Exemplar*]

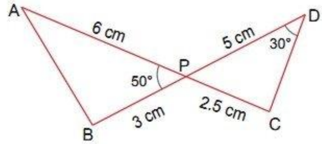


Fig. 7.21

(*a*) 50° (*b*) 30° (*c*) 60° (*d*) 100°

5. In the given figure, $QR \parallel AB$, $RP \parallel BD$, $CQ = x + 2$, $QA = x$, $CP = 5x + 4$, $PD = 3x$.

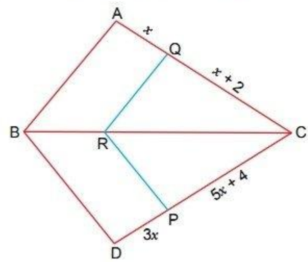


Fig. 7.22

The value of x is

(*a*) 1 (*b*) 6 (*c*) 3 (*d*) 9

6. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are [*NCERT Exemplar*]

(*a*) congruent but not similar (*b*) similar but not congruent
(*c*) neither congruent nor similar (*d*) congruent as well as similar

7. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, the following is true: [*NCERT Exemplar*]

(*a*) $DE = 12$ cm, $\angle F = 50°$ (*b*) $DE = 12$ cm, $\angle F = 100°$
(*c*) $EF = 12$ cm, $\angle D = 100°$ (*d*) $EF = 12$ cm, $\angle D = 30°$

8. Rohit is 6 feet tall. At an instant, his shadow is 5 feet long. At the same instant, the shadow of a pole is 30 feet long. How tall is the pole?

(*a*) 12 feet (*b*) 24 feet (*c*) 30 feet (*d*) 36 feet

9. In the following figure, Q is a point on PR and S is a point on TR. QS is drawn and $\angle RPT = \angle RQS$. [*CBSE Question Bank*]

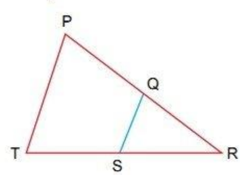


Fig. 7.23

Which of these criteria can be used to prove that ΔRSQ is similar to ΔRTP?

(*a*) AAA similarity criterion (*b*) SAS similarity criterion
(*c*) SSS similarity criterion (*d*) None of these

10. **Shown below are three triangles. The measures of two adjacent sides and included angle are given for each triangle. Which of these triangles are similar?** [*Competency Based Question*]

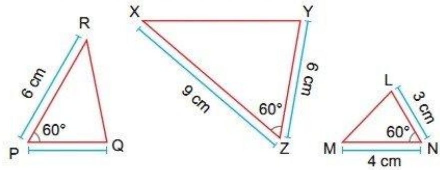


Fig. 7.24

(*a*) ΔRPQ and ΔXZY

(*b*) ΔRPQ and ΔMNL

(*c*) ΔXZY and ΔMNL

(*d*) ΔRPQ, ΔXZY and ΔMNL are similar to one another.

11. **In the figure below, $PQ \parallel CB$.**

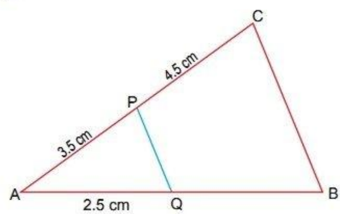


Fig. 7.25

To the nearest tenth, what is the length of QB?

(*a*) 1.4 cm (*b*) 1.7 cm

(*c*) 3.2 cm (*d*) 2.2 cm

12. **In the figure given below, $DE \parallel AC$ and $DF \parallel AE$. Which of these is equal to $\frac{BF}{FE}$?**

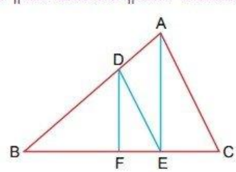


Fig. 7.26

(*a*) $\frac{DF}{AE}$ (*b*) $\frac{BE}{EC}$

(*c*) $\frac{BA}{AC}$ (*d*) $\frac{FE}{EC}$

Answers

1. (*a*)	2. (*a*)	3. (*d*)	4. (*d*)	5. (*a*)	6. (*b*)	7. (*b*)
8. (*d*)	9. (*a*)	10. (*a*)	11. (*c*)	12. (*b*)		

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC are parallel or not.

Sol. It is given that

$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

We have, $\frac{AY}{YC} = \frac{5}{9}$

Since, $\frac{AX}{XB} = \frac{3}{4} \neq \frac{5}{9} = \frac{AY}{YC}$

$\Rightarrow \quad \frac{AX}{XB} \neq \frac{AY}{YC}$

Hence XY is not parallel to BC.

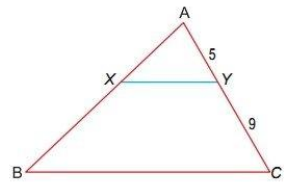


Fig. 7.27

2. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that $PQ = 12.5$ cm, $PA = 5$ cm, $BR = 6$ cm, and $PB = 4$ cm. Is $AB || QR$? Give reason.

Sol. Yes, $\frac{PA}{AQ} = \frac{5}{12.5-5} = \frac{5}{7.5} = \frac{2}{3}$

$$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$$

Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$

$\therefore \quad AB || QR$

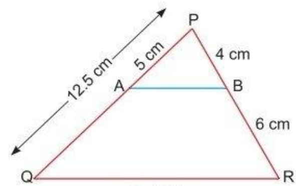


Fig. 7.28

3. In the figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB. [*CBSE Sample Paper* 2020]

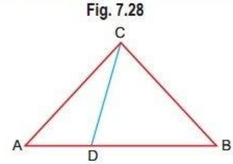


Fig. 7.29

Sol. $\Delta ACB \sim \Delta ADC$ (AA criterion)

$\Rightarrow \quad \frac{AC}{AD} = \frac{AB}{AC} \quad \Rightarrow \frac{6}{3} = \frac{AB}{6} \Rightarrow AB = 6 \times 2$

$\therefore \quad AB = 12$ cm

4. Observe the right triangle ABC, right angled at B as shown below.

What is the length of PC? [*Competency Based Question*]

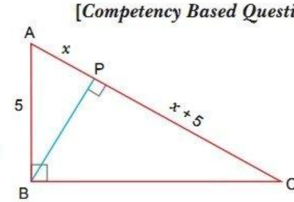


Fig. 7.30

Sol. We have

In ΔAPB and ΔABC

$\angle APB = \angle ABC = 90°$

$\angle BAP = \angle BAC$ (Common)

$\therefore \quad \Delta APB \sim \Delta ABC$ (By AA similarity criteria)

$\therefore \quad \frac{AB}{AC} = \frac{AP}{AB} \Rightarrow AB^2 = AC \cdot AP$

$\Rightarrow 25 = (2x + 5) . x$

$\Rightarrow 25 = 2x^2 + 5x \Rightarrow 2x^2 + 5x - 25 = 0$

$\Rightarrow 2x^2 + 10x - 5x - 25 = 0 \quad \Rightarrow 2x(x + 5) - 5(x + 5) = 0$

$\Rightarrow (x + 5)(2x - 5) = 0$

$\Rightarrow 2x - 5 = 0 \quad (\because x + 5 \neq 0 \Rightarrow x \neq -5$ length cannot be negative)

$\therefore x = \frac{5}{2} = 2.5$

Length of $PC = x + 5 = 2.5 + 5 = 7.5$ cm

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. In Fig. 7.31, $DE \parallel AC$ and $DC \parallel AP$, Prove that $\frac{BE}{EC} = \frac{BC}{CP}$. [*CBSE* 2020 (30/1/1)]

Sol. We have,

In ΔABC, $DE \parallel AC$

$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC}$ ...(*i*)

Also, in ΔABP, $DC \parallel AP$

$\therefore \quad \frac{BD}{DA} = \frac{BC}{CP}$ ...(*ii*)

From (*i*) and (*ii*), we have

$$\frac{BE}{EC} = \frac{BC}{CP}$$

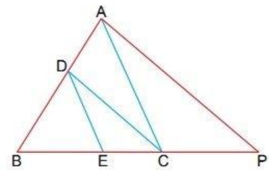


Fig. 7.31

2. In Fig. 7.32, $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x.

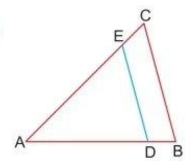


Fig. 7.32

Sol. In ΔABC, we have $\quad DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow \quad x(x-1) = (x-2)(x+2)$

$\Rightarrow \quad x^2 - x = x^2 - 4 \Rightarrow \quad x = 4$

3. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 9 cm long, find the length of the corresponding side of the second triangle.

[*CBSE* 2020 (30/4/1)]

Sol. Let the side of other triangle be x cm.

$\because$ Ratio of perimeters of two similar triangles is equal to ratio of their corresponding sides. ½

$\therefore \frac{9}{x} = \frac{30}{20}$ 1

$x = 6$ cm ½

[*CBSE Marking Scheme* 2020 (30/4/1)]

4. In Fig. 7.33, ΔPQR is right-angled at P. M is a point on QR such that PM is perpendicular to QR. Show that $PQ^2 = QM \times QR$. [*CBSE* 2020 (30/4/1)]

Sol. In ΔRPQ and ΔPMQ, we have

$\angle RPQ = \angle PMQ = 90°$

$\angle PQR = \angle MQP$ (Common angle)

$\therefore \quad \Delta RPQ \sim \Delta PMQ$ (By AA similarity criteria)

$\therefore \quad \frac{PQ}{QR} = \frac{QM}{PQ}$

$\Rightarrow \quad PQ^2 = QM \times QR$ Proved

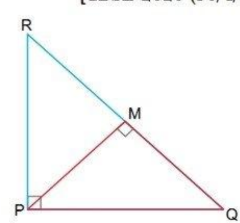


Fig. 7.33

5. In Fig. 7.34, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}.$$ [*CBSE* 2020 (30/2/1)]

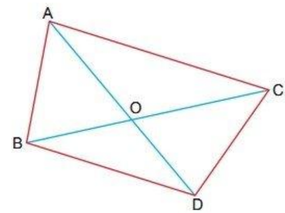


Fig. 7.34

Sol. We have,

ΔABC and ΔBCD both lie on the same base BC.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw $AE \perp BC$ and $DF \perp BC$

Proof: In ΔAEO and ΔDFO, we have

$\therefore \angle AOE = \angle DOF$ (Vertically opposite angles)

$\angle AEO = \angle DFO = 90°$

$\therefore \Delta AEO \sim \Delta DFO$ (By AA similarity criterion)

$\therefore \frac{AE}{DF} = \frac{AO}{DO}$ ... (*i*)

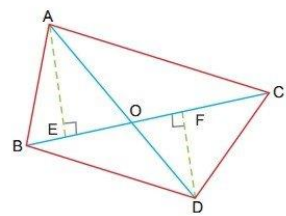


Fig. 7.35

Now, $\frac{ar(\Delta ABC)}{ar(\Delta BDC)} = \frac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AE}{DF}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$ (from (*i*)) Proved

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. In Fig. 7.36, if $\Delta ABC \sim \Delta DEF$ and their sides of length (in cm) are marked along them, then find the lengths of sides of each triangle. [*CBSE* 2020 (30/2/1)]

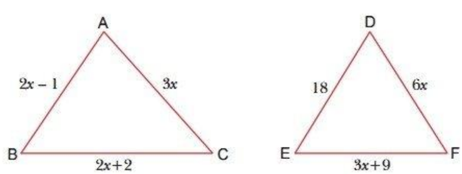


Fig. 7.36

Sol. Since, $\Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{1}{2}$

Now, we have

$$\frac{2x-1}{18} = \frac{1}{2} \text{ and } \frac{2x+2}{3x+9} = \frac{1}{2}$$

$\Rightarrow$	$4x - 2 = 18$	and,	$4x + 4 = 3x + 9$
$\Rightarrow$	$4x = 20$	$\Rightarrow$	$x = \frac{20}{4} = 5$
$\therefore$	$x = 5$	$\Rightarrow$	$4x - 3x = 9 - 4 = 5$
		$\Rightarrow$	$x = 5$

$\therefore$ Length of sides of ΔABC are

$AB = 2x - 1 = 2 \times 5 - 1 = 9$ cm

$BC = 2x + 2 = 2 \times 5 + 2 = 12$ cm

and, $AC = 3x = 3 \times 5 = 15$ cm

$\therefore$ Length of sides of ΔDEF are

$DE = 18$ cm, $EF = 3x + 9 = 3 \times 5 + 9 = 24$ cm

and $DF = 6x = 6 \times 5 = 30$ cm

2. In Fig. 7.37, $\Delta FEC \cong \Delta GDB$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$.

[*Competency Based Question*]

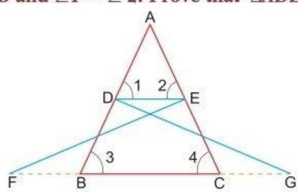


Fig. 7.37

Sol. Since, $\Delta FEC \cong \Delta GDB$

$\Rightarrow$ $EC = BD$ ...(*i*)

It is given that

$\angle 1 = \angle 2$

$\Rightarrow$ $AE = AD$ (Sides opposite to equal angles are equal) ...(*ii*)

Dividing (*ii*) by (*i*), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow$ $DE \parallel BC$ (By the converse of basic proportionality theorem)

$\Rightarrow$ $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ (Corresponding angles)

Thus, in Δ' s ADE and ABC, we have

$\angle A = \angle A$ (Common)

$\angle 2 = \angle 4$ (Proved above)

$\Rightarrow$ $\Delta ADE \sim \Delta ABC$ (By AA similarity)

3. In Fig. 7.38, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that BAC is an isosceles triangle.

[*CBSE* 2020 (30/1/1)]

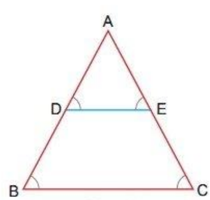


Fig. 7.38

Sol. Given, $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\because \quad \dfrac{AD}{DB} = \dfrac{AE}{EC} \quad \Rightarrow \quad DE \parallel BC$

$\therefore \quad \angle D = \angle B$ and $\angle E = \angle C$ (Corresponding angles)

But it is given that $\angle D = \angle E$

$\angle B = \angle C \quad \Rightarrow \quad AC = AB$ (Sides opposite to equal angles are equal)

$\therefore$ ΔBAC is an isosceles triangle. Proved.

4. In Fig. 7.39, $AB \parallel PQ \parallel CD$, $AB = x$ units, $CD = y$ units and $PQ = z$ units. Prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

[*Competency Based Question*]

Sol. In ΔADB and ΔPDQ,

Since $AB || PQ$

$\angle ABQ = \angle PQD$ (Corresponding angles)

$\angle ADB = \angle PDQ$ (Common)

$\Delta ADB \sim \Delta PDQ$ (By AA similarity)

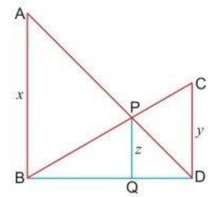


Fig. 7.39

$\therefore \quad \dfrac{DQ}{DB} = \dfrac{PQ}{AB} \quad \Rightarrow \quad \dfrac{DQ}{DB} = \dfrac{z}{x}$ ...(*i*)

Similarly, $\Delta PBQ \sim \Delta CBD$

and $\dfrac{BQ}{BD} = \dfrac{PQ}{CD} \quad \Rightarrow \quad \dfrac{BQ}{DB} = \dfrac{z}{y}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$\frac{z}{x} + \frac{z}{y} = \frac{DQ + BQ}{DB} = \frac{BD}{BD}$$

$$\frac{z}{x} + \frac{z}{y} = 1 \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

5. In Fig. 7.40, $ABCD$ is a trapezium with $AB||DC$. If ΔAED is similar to ΔBEC, prove that $AD = BC$.

Sol. In ΔEDC and ΔEBA we have

$\angle 1 = \angle 2$ (Alternate angles)

$\angle 3 = \angle 4$ (Alternate angles)

and $\angle CED = \angle AEB$ (Vertically opposite angles)

$\therefore \quad \Delta EDC \sim \Delta EBA$ (By AA criterion of similarity)

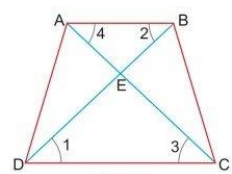


Fig. 7.40

$\Rightarrow \quad \dfrac{ED}{EB} = \dfrac{EC}{EA} \quad \Rightarrow \quad \dfrac{ED}{EC} = \dfrac{EB}{EA}$ ...(*i*)

It is given that $\Delta AED \sim \Delta BEC$.

$\therefore \quad \dfrac{ED}{EC} = \dfrac{EA}{EB} = \dfrac{AD}{BC}$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \quad \Rightarrow \quad (EB)^2 = (EA)^2 \quad \Rightarrow \quad EB = EA$$

Substituting $EB = EA$ in (*ii*), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC} = 1 \quad \Rightarrow \quad AD = BC$$

Long Answer Questions

Each of the following questions are of 5 marks.

1. **Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.** **[*CBSE* 2019 (30/2/1)]**

Sol. **Given:** A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$.

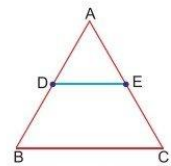


Fig. 7.41

Proof: Area of $\Delta ADE = \left(\frac{1}{2} \text{ base} \times \text{height}\right)$

So, $ar(\Delta ADE) = \frac{1}{2}(AD \times EN)$

and $ar(\Delta BDE) = \frac{1}{2}(DB \times EN)$

Similarly, $ar(\Delta ADE) = \frac{1}{2}(AE \times DM)$

and $ar(\Delta DEC) = \frac{1}{2}(EC \times DM)$

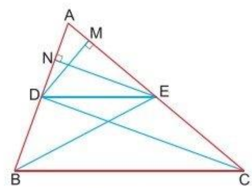


Fig. 7.42

Therefore, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ...(*i*)

and $\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ...(*ii*)

Now, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

So, $ar(\Delta BDE) = ar(\Delta DEC)$ ...(*iii*)

Therefore, from (*i*), (*ii*) and (*iii*) we have, $\frac{AD}{DB} = \frac{AE}{EC}$.

2. **In Fig. 7.43, P is the mid-point of BC and Q is the mid-point of AP. If BQ when produced meets AC at R, prove that $RA = \frac{1}{3} CA$.**

Sol. **Given:** In ΔABC, P is the mid-point of BC, Q is the mid-point of AP such that BQ produced meets AC at R.

To prove: $RA = \frac{1}{3} CA$

Construction: Draw $PS \parallel BR$, meeting AC at S.

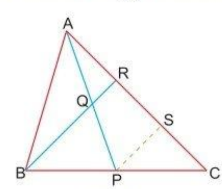


Fig. 7.43

Proof: In ΔBCR, P is the mid-point of BC and $PS \parallel BR$

$\therefore$ S is the mid-point of CR.

$\Rightarrow$ $CS = SR$ ...(*i*)

In ΔAPS, Q is the mid-point of AP and $QR \parallel PS$.

$\therefore$ R is the mid-point of AS.

$\Rightarrow$ $AR = RS$ ...(ii)

From (i) and (ii), we get

$$AR = RS = SC$$

$\Rightarrow$ $AC = AR + RS + SC = 3\,AR$ $\Rightarrow$ $AR = \frac{1}{3}AC = \frac{1}{3}CA$

Hence proved.

3. **Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC at L and AD produced to E. Prove that $EL = 2BL$.**

Sol. In ΔBMC and ΔEMD, we have

$MC = MD$ ($\because$ M is the mid-point of CD)

$\angle CMB = \angle DME$ (Vertically opposite angles)

and $\angle MBC = \angle MED$ (Alternate angles)

So, by AAS criterion of congruence, we have

$\Delta BMC \cong \Delta EMD$

$\Rightarrow$ $BC = DE$ (CPCT)

Also, $BC = AD$ ($\because$ $ABCD$ is a parallelogram)

Now, in ΔAEL and ΔCBL, we have

$\angle ALE = \angle CLB$ (Vertically opposite angles)

$\angle EAL = \angle BCL$ (Alternate angles)

$\therefore$ $\Delta AEL \sim \Delta CBL$ (By AA similarity)

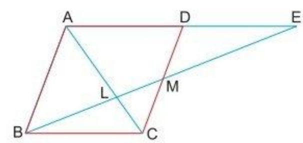


Fig. 7.44

$\Rightarrow$ $\frac{EL}{BL} = \frac{AE}{CB}$ $\Rightarrow$ $\frac{EL}{BL} = \frac{2BC}{BC}$ ($\because$ $AE = AD + DE = BC + BC = 2BC$)

$\Rightarrow$ $\frac{EL}{BL} = 2$ $\Rightarrow$ $EL = 2BL$

Case Study-based Questions

Each of the following questions are of 4 marks.

1. **Read the following and answer any four questions from (i) to (v).**

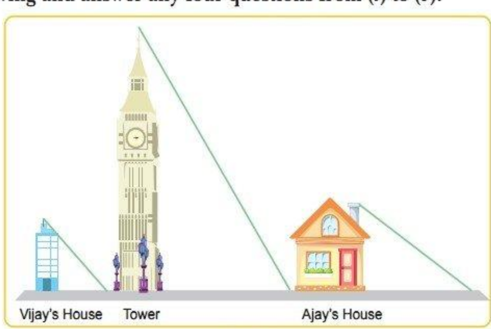


Fig. 7.45

Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground. ***[CBSE Question Bank]***

(i) What is the height of the tower?

(a) 20 m (b) 50 m (c) 100 m (d) 200 m

(ii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12m?

(a) 75 m (b) 50 m (c) 45 m (d) 60 m

(iii) What is the height of Ajay's house?

(a) 30 m (b) 40 m (c) 50 m (d) 20 m

(iv) When the tower casts a shadow of 40m, same time what will be the length of the shadow of Ajay's house?

(a) 16 m (b) 32 m (c) 20 m (d) 8 m

(v) When the tower casts a shadow of 40m, same time what will be the length of the shadow of Vijay's house?

(a) 15 m (b) 32 m (c) 16 m (d) 8 m

Sol. (i) Let h m be the height of tower, therefore using property of similar triangle between two triangle *i.e.* for Vijay's house and for tower with shadows.

We have,

$$\frac{20}{h}=\frac{10}{50} \Rightarrow h = 100 \text{ m}$$

$\therefore$ Height of tower is 100 m.

$\therefore$ Option (c) is correct.

(ii) When Vijay's house casts a shadow of 12 m, we have

$\frac{20}{h}=\frac{12}{x}$, where x is the length of shadow of tower.

$$\Rightarrow \frac{20}{100}=\frac{12}{x} \Rightarrow x = 60 \text{ m}$$

$\therefore$ Option (d) is correct.

(iii) Let H m be the height of Ajay's house.

$$\therefore \frac{20}{10}=\frac{H}{20} \Rightarrow H = 40 \text{ m}$$

$\therefore$ Option (b) is correct.

(iv) When the tower casts a shadow of 40 m.

$$\therefore \frac{100}{40}=\frac{\text{Height of Ajay's house}}{\text{Length of shadow of Ajay's house}}$$

$$\Rightarrow \frac{5}{2}=\frac{40}{\text{Shadow length}}$$

$\Rightarrow$ Length of shadow of Ajay house = 16 m.

$\therefore$ Option (a) is correct.

(v) We have,

$$\frac{100}{40}=\frac{\text{Height of Vijay's house}}{\text{Length of its shadow}}$$

$$\Rightarrow \frac{5}{2}=\frac{20}{\text{Length of its shadow}}$$

$\Rightarrow$ Length of shadow of Vijay house = 8 m

$\therefore$ Option (d) is correct.

2. The legs of an iron table form two triangles as shown in the picture.

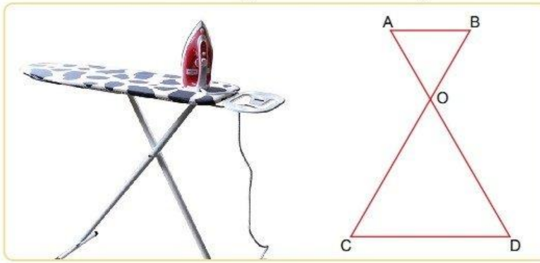


Fig. 7.46

Based on above information answer the following questions.

(i) Which similarity criteria is applicable to prove the two triangles similar?

(ii) If $AO = 30$ cm and $OD = 45$ cm, then find perimeter (ΔAOB) : perimeter (ΔCOD).

[*Competency Based Question*]

Sol. (i) Since $AB \parallel CD, \angle A = \angle D$ and $\angle B = \angle C$ (Alternate interior angles)

Also, $\angle AOB = \angle COD$ (Vertically opposite angles)

So, $\Delta AOB \sim \Delta DOC$ (By AAA similarity criteria)

Hence AAA similarity criteria is applicable.

(ii) $\frac{AO}{OD} = \frac{30}{45} = \frac{2}{3}$

Since $\Delta AOB \sim \Delta DOC$

$\Rightarrow \frac{\text{Perimeter}(\Delta AOB)}{\text{Perimeter}(\Delta DOC)} = \frac{AO}{OD} = \frac{2}{3}$ ($\because$ Ratio of perimeters of two similar triangles is equal to ratio of their corresponding sides)

Hence, the ratio is 2 : 3.

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. **Choose and write the correct option in each of the following questions.**

(i) In Fig. 7.47 below, $PQ \parallel CB$.

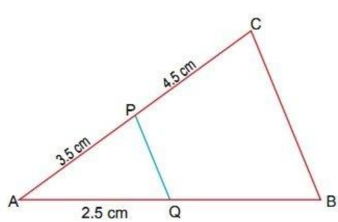


Fig. 7.47

To the nearest tenth, what is the length of QB?

(a) 1.4 cm (b) 1.7 cm (c) 1.8 cm (d) 2.2 cm (*ii*) Consider the Fig. 7.48 below.

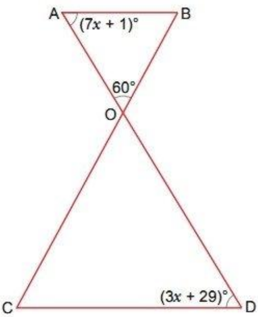


Fig. 7.48

Which of the following statement helps proving that triangle AOB is similar to triangle DOC? **[*Competency Based Question*]**

(*i*) $\angle B = 70°$, and (*ii*) $\angle C = 70°$

(*a*) Statement (*i*) alone is sufficient, but statement (*ii*) alone is not sufficient.

(*b*) Statement (*ii*) alone is sufficient, but statement (*i*) alone is not sufficient.

(*c*) Either (*i*) or (*ii*) statement alone is sufficient.

(*d*) Both statements together is sufficient, but neither statement alone is sufficient.

(*iii*) If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true? **[*NCERT Exemplar*]**

(*a*) $BC \cdot EF = AC \cdot FD$ (*b*) $AB \cdot EF = AC \cdot DE$

(*c*) $BC \cdot DE = AB \cdot EF$ (*d*) $BC \cdot DE = AB \cdot FD$

(*iv*) Ankit is 5 feet tall. He places a mirror on the ground and moves until he can see the top of a building. At the instant when Ankit is 2 feet from the mirror, the building is 48 feet from the mirror. How tall is the building?

(*a*) 96 feet (*b*) 120 feet

(*c*) 180 feet (*d*) 240 feet

■ Very Short Answer Questions: **[1 mark each]**

2. In Fig. 7.49, $GC \parallel BD$ and $GE \parallel BF$. If $AC = 3$ cm and $CD = 7$ cm, then find the value of $\frac{AE}{AF}$. **[*CBSE* 2019(*C*) (30/1/1)]**

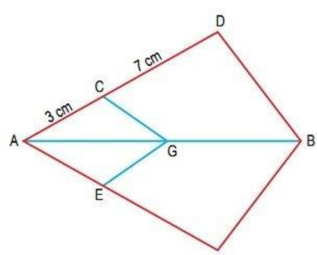


Fig. 7.49

3. In Fig. 7.50, $DE \| BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm. **[*CBSE* 2019 (30/2/1)]**

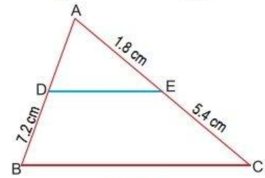


Fig. 7.50

4. A and B are respectively the points on the sides PQ and PR of a triangle PQR such that $PQ = 10.5$ cm, $PA = 4.5$ cm, $BR = 8$ cm and $PB = 6$ cm. Is $AB \parallel QR$?
5. If in two right triangles, one of the acute angle of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar?
6. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? **[*NCERT Exemplar*]**

■ Short Answer Questions-I: [2 marks each]

7. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$. **[*CBSE* 2018 (*C*) (30/1)]**
8. In Fig. 7.51, $\frac{OA}{OC} = \frac{OD}{OB}$. Prove that $\angle A = \angle C$ and $\angle B = \angle D$.

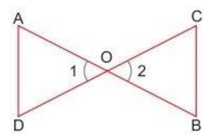


Fig. 7.51

9. Two poles of height 9 m and 15 m stand vertically upright on a plane ground. If the distance between their tops is 10m, then find the distance between their feet.
10. $\Delta ABC \sim \Delta DEF$. If $AB = 4$ cm, $BC = 3.5$ cm, $CA = 2.5$ cm and $DF = 7.5$ cm, then find perimeter of ΔDEF.
11. AD is the bisector of $\angle BAC$ in ΔABC. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then find BD.

■ Short Answer Questions-II: [3 marks each]

12. $ABCD$ is a trapezium with $AB \parallel DC$. E and F are points on non-parallel sides AD and BC respectively, such that $EF \parallel AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$. **[*CBSE* 2019 (*C*)(30/1/1)]**
13. In ΔABC, $DE \parallel BC$. If $AD = 4x - 3$, $AE = 8x - 7$, $BD = 3x - 1$ and $CE = 5x - 3$, find the value of x.
14. In Fig. 7.52, P is the mid-point of EF and Q is the mid-point of DP. If EQ when produced meets DF at R, prove that $RD = \frac{1}{3}DF$. **[*Competency Based Question*]**

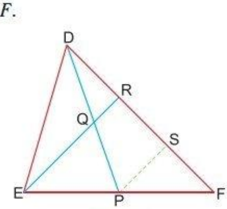


Fig. 7.52

15. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5m casts a shadow of 3 m, find how far she is away from the base of the pole.

16. In Fig. 7.53, $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$. **[CBSE 2019 (30/1/1)]**

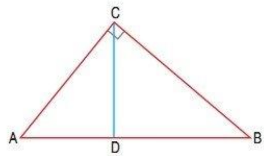


Fig. 7.53

17. In Fig. 7.54, $AB \parallel CD$. If $OA = 3x - 19$, $OB = x - 4$, $OC = x - 3$, and $OD = 4$, find x.

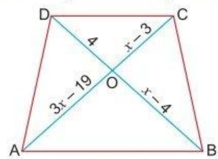


Fig. 7.54

■ Long Answer Questions: [5 marks each]

18. In Fig. 7.55, E is a point on side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Prove that $\Delta ABE \sim \Delta CFB$. ***[NCERT Exemplar]***

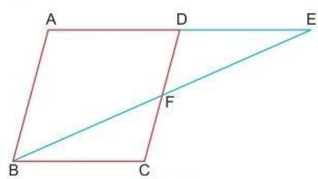


Fig. 7.55

19. In Fig. 7.56, $DEFG$ is a square and $\angle BAC = 90°$. Prove that:

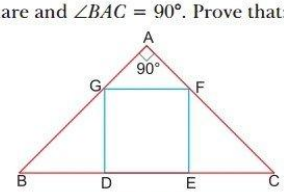


Fig. 7.56

(i) $\Delta AGF \sim \Delta DBG$
(ii) $\Delta AGF \sim \Delta EFC$
(iii) $\Delta DBG \sim \Delta EFC$
(iv) $DE^2 = BD \times EC$

20. In Fig. 7.57, OB is the perpendicular bisector of the line segment DE, $FA \perp OB$ and FE intersects OB at the point C. Prove that: $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$.

[Competency Based Question]

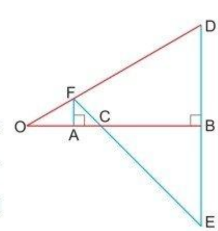


Fig. 7.57

Answers

1. (*i*) (*c*) (*ii*) (*c*) (*iii*) (*c*) (*iv*) (*b*)

2. 3 : 10 3. $AD = 2.4$ cm 4. Yes 5. Yes, by AA similarity

6. No 9. 8 m 10. 30 cm 11. $BD = 7.5$ cm 13. $x = 1$ or $x = \frac{1}{2}$

15. 9 m 17. 11 or 8