Data Structures.

Books →

1) Sahni
2) Weiss
3) Kruse

} Level - 1

1) Coreman
2) Goodrich & Tamasia
3) Drozdek (interviews).

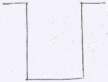
} Level - 2

1) Data structures - Forouzan.

what is a Data structure →

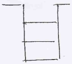
Data structure →
A mathematical or logical model.

Data structure.

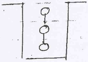


Stack

Physical structure →
Implementation of Data structure in physical memory.



Array stack:



linked list stack.

why do we need Data structures → ?!!

Solving a problem fastly by occupying optimum memory

time — Analysis. — space

Q.1. Calculate the locatn of an element A[0] in an array of [-5 to +5] where the starting locatn is 1000 and each element occupies two memory locations.

→ $\boxed{1010}$

Array is a contiguous and homogeneous.

a: array [lb .... ub] of elements.

lb → lower boundary , ub → upper boundary
c → count = ele. size.
Lo → starting location.

$$\therefore \boxed{loc\ A(i) = Lo + (i - lb) * c}$$

∴ applying this formula →

$$loc\ A(0) = 1000 + (0 + 5) * 2$$
$$= \boxed{1010}$$

Correlation with 'C' →

1000 1002 1004 1006 1008 1010

Correlation with 'C' →

$loc\ a[i] = L_0 + (i - lb) * C$

$= \&a_0 + (i-0) * C$

$\& a[i] = (a+i)$ (scalar arithmatic)

$\therefore a[i] = *(a+i)$

	10	12	14	16.
a →	a_0	a_1	a_2	a_3

a = name of the array.

= address of very first element.

$\therefore a = (\& a_0)$

char.	$10 + (1) \cdot 1$	→ 11
int	$10 + (1) \cdot 2$	→ 12
float	$10 + (1) \cdot 4$	→ 14
double	$10 + (1) \cdot 8$	→ 18
long double.	$10 + (1) \cdot 10$	→ 20.

Properties of planar graph →

1) In a planar graph, with 'n' vertices, sum of degrees of all vertices

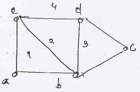
i.e. $\boxed{\sum_{i=1}^{n} \deg(v_i) = 2 \times}$ No. of edges in the graph.

$= 2|E|.$

2) Sum of degrees of regions theorem →

In a planar graph, with 'm' regions, sum of degrees of regions i.e.

$$\boxed{\sum_{i=1}^{m} (t_i) = 2 \cdot |E|}$$

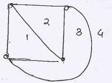


$3 + 3 + 3 + 5 = 14 = 7 \times 2$

Corollary 2.1. →

In a planar graph, if degree of each region is 'k' then sum of degrees of regions becomes $k \cdot |R|$.

$\therefore \boxed{k \cdot |R| = 2 \cdot |E|}$

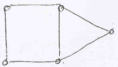


$\therefore 3 \times 4 = 2 \times 6.$

Corollary 2.2 →

In a planar graph, if degree of each region is at least 'k' (greater than or equal to k), then

$$\boxed{k \cdot |R| \leq 2 \cdot |E|}$$



$3 \cdot (3) \leq 2 \cdot (6).$

Corollary 2.3 →

Simple planar graph →

In a simple planar graph, with at least two edges degree of each region is greater than or equal to 3. (at least 3).

$\boxed{3 \cdot |R| \leq 2 \cdot |E|}$ ← For a simple planar graph.

Corollary 2.4 →

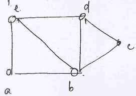
In a planar graph, if degree of each region is at most 'k' (i.e. $\leq k$), then

$$\boxed{k \cdot |R| \geq 2 \cdot |E|}$$

) Euler's Formula →

If 'G' is a connected planar graph, then

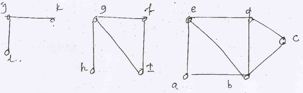
$$\boxed{|V| + |R| = |E| + 2}$$



Corollary 3.1 →

If G is a planar graph with 'k' components, then

$$\boxed{|V| + |R| = |E| + (k+1)}$$



$(2+5 = 13 + (3+1))$.

(4) Edge - Vertex Inequality →

If G is a connected planar graph, with degree of each region at least 'k' $(\geq k)$, then

No. of edges $\leq \frac{k}{k-2} \times ((\text{no. of vertices}) - 2)$

i.e. $\boxed{|E| \leq \frac{k}{k-2}(|V| - 2)}$

5)

If G is a simple connected planar graph, then

5.1) No. of edges i.e. $|E| \leq (3 \cdot |V| - 6)$

5.2) using Euler's formula,

$$\{|V| + |R| - 2\} \leq \{3|V| - 6\}$$

) In a complete bipartite graph, cycle of length odd not possible

$\therefore |E| \leq \{2 \cdot |V| - 4\}$.

5.8) There exists at least one vertex $v \in G$, such that

$deg \cdot (v) \leq 5$.

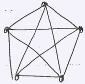
6) If G is a simple connected planar graph (with at least two edges) (and no triangles), then

$|E| \leq (2 \cdot |V| - 4)$

Note ⟹

1) $K_{3,3}$ is not a planar graph.

2) In K_5, triangle (i.e. cycle of odd length) exist.



So, K_5 is not a planar graph.

7) Kurtowski's thm ⟹

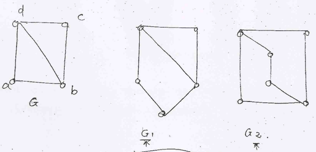
A graph G is non-planar iff. G has a subgraph which is homeomorphic to K_5 or $K_{3,3}$.

Note ⟹

Homeomorphic graphs ?

Two graphs 'G_1' and 'G_2' are said to be homeomorphic if each of these graphs can be obtained from the same graph G by dividing some edges of G with more vertices.

ex:



can be obtained from G by putting extra node

Note →

1) If any two graphs G1 and G2 are isomorphic, then they are homeomorphic also.

2) The converse of the above statement need not be true.

Corollary 7.1) → Any graph with 4 or fewer vertices is planar.

Corollary 7.2) → Any graph with 8 or fewer edges is planar.

Corollary 7.3) → The complete graph 'K_n' is planar iff. n is ≤ 4.

i.e. $n \leq 4$

Corollary 7.4) → The complete bipartite graph '$K_{m,n}$' is planar iff $m \leq 2$ (or) $n \leq 2$.

Corollary 7.5) → The simple nonplanar graph with min. no of vertices is the complete graph K_5.

Corollary 7.6) → The simple non-planar graph with min. no of vertices is $K_{3,3}$. (complete bipartite graph).

3) Polyhedral Graph →

A simple connected planar graph is called a "polyhedral graph" if

$\boxed{deg(v) \geq 3}$ $\forall v \in E$.

The following inequalities must hold good:-

i) $3 \cdot |V| \leq 2 \cdot |E|$.

ii) $3 \cdot |R| \leq 2 \cdot |E|$

Q.

1. Let G be a connected planar graph with 25 vertices, 60 edges. Find no. of bounded regions in G is?

→ By Euler's formula,

$|V| + |R| = |E| + 2$

$\therefore 25 + |R| = 60 + 2$ $\quad \therefore |R| = 37$.

Of these 37 regions, we have one unbounded region

$\therefore$ No. of bounded Regions = $\boxed{36}$

Q.2. Let G be a connected planar graph with 10 vertices with 10 vertices, 15 edges and 3 components. No. of regions in G is ?

→ By Euler's formula,

$$|V| + |R| = |E| + (K+1)$$

$$\therefore \; 10 + |R| = 15 + (3+1)$$

$$\therefore \boxed{|R| = 9}$$

Q.3. Let G be a connected planar graph with 20 vertices and degree of each vertex is 3. Find no. of regions in graph.

→ By sum of degrees of vertices thm,

$$\sum_{i=1}^{20} \deg(v_i) = 2 \cdot |E|$$

$$\therefore \; 20 \cdot (3) = 2 \cdot |E| \quad \therefore \boxed{|E| = 30}$$

$\therefore$ By Euler's formula,

$$|V| + |R| = |E| + 2$$

$$\therefore \; |R| = 32 - 20$$

$$\boxed{|R| = 12}$$

4. Let G be a connected planar graph with 35 regions, and degree of each region is 6. $|V| = ?$

by sum of degrees of regions rule,

$k \cdot |R| \leq 2|E|$ here $k \cdot |R| = 2 \cdot |E|$

$\therefore 6 \times 35 = 2 \times |E|$

$\therefore |E| = \underline{105}$

By Euler's formula,

No. of vertices $(|V|)$ + No. of regions $(|R|)$

$= 2 + |E|$

$\therefore |V| = 2 + |E| - |R| = 2 + 105 - 35$

$\therefore |V| = \boxed{72}$

5. Let G be a polyhedral graph with with 20 vertices, 30 edges and degree of each region is 'k'. then $\underline{k = ?}$

By Euler's formula,

$|V| + |R| = 2 + |E|$

$\therefore 20 + |R| = 2 + |E|$

$\therefore |R| = \underline{12}.$

By sum of degrees of regions,

if degree of each region is 'k', then

$k \cdot |R| = 2 \cdot |E|$.

$\therefore \quad k = \frac{2 \times 30}{\cancel{12}\, 6} = \boxed{5.}$

$\therefore \boxed{k = 5}$.

Q.6. Max. no. of edges possible in a simple connected planar graph with 8 vertices is ?

→ By theorem 5.1, no. of edges; $|E| \leq (3 \cdot |V| - 6)$

$\therefore \quad |E| \leq (3 \times 8) - 6 \quad \therefore \underline{|E| \leq 18}$.

$\therefore$ max. no. of edges possible is $\boxed{18}$.

Q.7. Min. no. of vertices necessary in a simple connected planar graph with 11 edges is ?

→ $11 \leq 3 \cdot |V| - 6$. → add 6 both sides.

$\therefore \quad 17 \leq 3 \cdot |V|$. → divide by 3.

$\therefore \quad 17/3 \leq |V|$.

$\therefore \quad |V| \geq 5.66$.

$\therefore \quad |V| \geq \boxed{6}$. $\therefore$ min. no. of vertices $= \boxed{6}$.

8. Min. no. of vertices necessory with a simple connected planar graph with 20 edges and degree of each region ≥ 5.

→ By edge-vertex inequality (thm 5)

$$|E| \leq \frac{k}{k-2}(|V|-2).$$

where, $k = 5$,

$$20 \leq \frac{5}{3}(|V|-2).$$

$\therefore$ $60 \leq 5(|V|-2)$.

$\therefore |V| \geq \frac{60}{5} + 2$

$\boxed{\therefore |V| \geq 14}$

8.9. Min. no. of vertices necessory in a simple connected planar graph with 15 regions is - ?

→ thm. 5.2.

$|R| \leq (2.|V| - 4)$

$\therefore$ $15 \leq 2.|V| - 4$.

$\therefore$ $|V| \geq \frac{19}{2}$ ≈ 9.5.

$\boxed{\therefore |V| \geq \cancel{\approx} 10.}$

Q.10. If G is a simple connected planar graph, then $\delta(G)$ (min. of the degrees of all degrees in G) cannot be

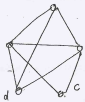
→ For any simple connected planar graph, there exists at least one vertex such that $\deg(v) \le 5$.

a) 3 b) 4 c) 5 ✓d) 6.

∴ $\boxed{\delta(G) \le 5}$ ∴ $\delta(G) \ne 6$.

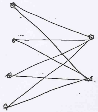
Q.11. Which of the following is (or) are not a planar graph?

a)



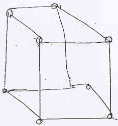
By Kurtosoki's thm, cor. 7.5, this graph is planar because the only simple nonplanar graph is K_5.

b)

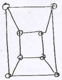


It is $K_{4,2}$. Corrolary 7.4. The complete bipartite graph is planar if any suffix ≤ 2.

c)

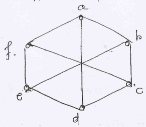


$\cong$

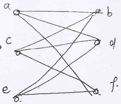


So, it's a planar graph.

d)

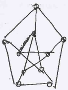


$\cong K_{3,3} \rightarrow$



It is not a planar graph.

e)



By edge-vertex inequality, we have

$\rightarrow \quad |E| \leq \frac{k}{k-2}(|V|-2).$

where, $k=5$.

$\therefore 15 \leq \frac{5}{5-2}(8).$

$\therefore 15 \nleq \frac{5}{3}\,8$

$\therefore$ edge-vertex inequality is not satisfied.

$\therefore$ G is not a planar graph.

8.12 which of the following is/are true ?

S1) A polyhedral graph with 30 edges and 11 regions does not exist.

By euler's formula,

Suppose a polyhedral graph with 30 edges and 11 regions exist. Then by euler's formula,

$|V| + |R| = |E| + 2$.

$\therefore |V| = 21$

By thm 8.1, for polyhedral graph, the foll. inequalities must hold good.

$3|V| \leq 2 \cdot |E|$ and $3 \cdot |R| \leq 2|E|$

$\therefore 3(21) \leq 2 \cdot (30)$

$\therefore 63 \nleq 60$

$\therefore$ Our assumption is not true.

$\therefore$ S1 is true.

S2) A. polyhedral graph with 7 edges does not exist.

→ Suppose, a polyhedral graph with 7 edges exist.

For polyhedral graph, the foll. inequalities must hold good

$3 \cdot |V| \leq 2 \cdot |E|$ and $3 \cdot |R| \leq 2 \cdot |E|$

$\therefore 3 \cdot |V| \leq 2 \times 7$ $\quad$ $3 \cdot |R| \leq 2 \times 7$

$3|V| \leq 14$ $\quad$ $\therefore |R| \leq 4.66.$

$\therefore |V| \leq 4.66.$ $\quad$ $\therefore |R| \leq 4.$

$\therefore |V| \leq 4$

By Euler's formula,

$|V| + |R| = |E| + 2.$

$\therefore 4 + 4 \geq 7 + 2$

$\therefore 8 \ngeq 9$

$\therefore$ it is not possible.

$\therefore$ our assumption is not true,

$\therefore$ such a graph does not exist.

$\therefore S_2$ is true. i.e. polyhedral graph with 7 edges does not exist.

13\. Let G be a simple nonplanar graph with min. no. of vertices, then G has

a) 5 vertices, 9 edges.

b) 5 vertices. 10 edges.

c) 6 vertices, 9 edges.

d) 6 vertices, 8 edges.

→ The simple nonplanar graph with min. no. of vertices is K_5 and K_5 has 5 vertices and 10 edges.

3.14. Let G be a simple nonplanar graph with min. no. edges, then G has

a) 5V, 9E

b) 5V, 10E

c) 6V, 9E → ($K_{3,3}$).

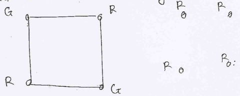
d) 6V, 8E.

→ The simple nonplanar graph with min no. of edges is $K_{3,3}$. $K_{3,3}$ has 6 vertices and 9 edges.

Colorings →

Vertex coloring →

An assignment of colors to the vertices of the graph G so that no two adjacent vertices have same color is called as "vertex coloring of G".



chromatic number of a graph →

The min. no. of colors reqd. for vertex coloring of graph 'G' is called "Chromatic number of G". and is denoted by "$\chi(G)$."

* chromatic no. of G = 1 iff G is a null graph.

* If G is not a null graph, then chromatic no. of G is ≥ 2.

) A graph G is said to be n-colorable if there is a vertex called coloring that uses at most n-colors. i.e.

$$\boxed{\chi(G) \leq n.}$$

* Any planar graph does not need more than 4 colors for vertex-coloring so that no two adjacent vertices have same color.

22/10/13

Four-color theorem →

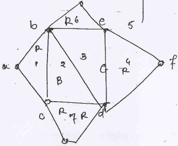
Every planar graph 'G' is "four-colorable".

i.e. $\chi(G) \leq 4$.

The above thm is valid for "map-coloring" / "Region-coloring" also.

(Assigning colors to the regions of a graph so that no two adjacent regions have same color.)

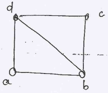
Two regions are said to be "adjacent" if they have a common edge.



Welch-Powel's Algorithm → (It gives upper limit of chromatic number)

* (This algorithm is used for "vertex-coloring".)

1) Arrange vertices in the descending order of their degrees.



$\{\underset{3}{b}, \underset{3}{d}, \underset{2}{a}, \underset{2}{c}\}$

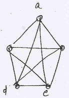
If two (ore) more vertices have same degree then arrange those vertices in alphabetical / numerical order.

2) Assign colors to the vertices in the above order, so that no two adjacent vertices have same color.

Q.1. The chromatic number of k_n = ?

✓a) n b) $n-1$ c) $\lfloor n/2 \rfloor$ d) $\lceil n/2 \rceil$

→ K_5



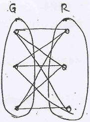
(every vertex is adjacent to every other vertex. in complete graph.)

∴ chromatic number is n for complete graph.

$\boxed{\chi(K_n) = n}$

Q.2. The chromatic no. of a complete bipartite graph $K_{m,n}$ is ?

→ $K_{3,3}$

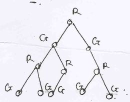


(No two vertices in the same group are adjacent. So, one color for each group).

$\therefore \chi(K_{m,n}) = 2$.

Q. 3. The chromatic number of a tree 'T' with 'n' vertices $(n \geq 2)$ is ?

→ Every tree can be represented as a bipartite graph and chromatic number of any bipartite graph is '2'.



$\therefore \chi(T) = 2.$

Q. 4. If G is a cyclic graph with 'n' vertices $(n \geq 3)$ and G has no cycles of odd length, then what is the chromatic no. of G. ?

⇒ G has no cycles of odd length. Thats why G is a "bipartite graph". (by thm)

$\therefore \chi(G) = 2.$

5. Chromatic number of a cycle graph C_n $(n \geq 3)$ is what ?

a) 2 b) 3 c) ✓ $n - 2\left\lfloor n/2 \right\rfloor + 2$ d). $n - 2 \cdot \left\lceil \frac{n}{2} \right\rceil + 1$

→ for even no. of vertices → 2 colors needed.

for odd no. of vertices → 3 colors needed.

$\therefore \chi(C_n) = n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2.$

Q.6. what is chromatic no. of wheel graph W_n $(n \geq 4)$.?

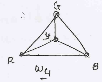
✓a) $n - 2 \cdot \left\lceil n/2 \right\rceil + 4$

b) $n - 2 \left\lfloor \frac{n}{2} \right\rfloor + 4$

c) 4

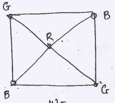
d) 3.

→



→ $\chi(G)$ $n=4$ i.e even $= \underline{4}.$

$n =$ odd $= 5$



$\chi(W_n)$ $(n = \text{odd})$ $= \underline{3}.$

$\therefore \chi(C_n) = n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2.$

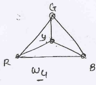
Q.6. what is chromatic no. of wheel graph W_n $(n \geq 4)$.?

✓a) $n - 2 \cdot \lceil n/2 \rceil + 4$

b) $n - 2 \left\lfloor \frac{n}{2} \right\rfloor + 4$

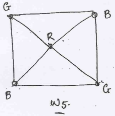
c) 4

d) 3.



$\chi(G)$ $n=4$ i.e even $= \underline{4}$.

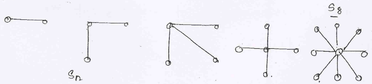
n = odd = 5



$\chi(W_n)$ (n=odd) $= \underline{3}$.

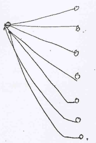
3.7. Chromatic no. of a star graph with n vertices $(n \geq 2)$

→



A star graph can be represented as a biparhite graph of $K_{1,n-1}$.

$S_8 \rightarrow K_{1,7}$ →

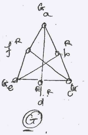


$\therefore \chi(G) = \underline{2}$

↑

star graph

3.8. Chromatic no. of the graph given below. =?



Applying welch-Powel's algom,

Vertex	a	b	c	d	e	f
color	C_1	C_2	C_1	C_2	C_1	C_2

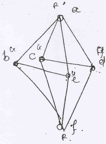
$\chi(G) \leq 2$ -①.

G is not a null graph.

$\therefore$ chromatic no. $\chi(G) \geq 2$. - ②.

From ① and ②, $\chi(G) = 2$

Q.9. chromatic no. of the graph shown below is ?



Applying Welch-Powel's algom.,

vertex	a	f	b	c	d	e
color	c_1	c_1	c_2	c_2	c_3	c_3

$\therefore$ $\chi(G) \leq 3$ - ①

Further, we have 3 mutually adjacent vertices $\{a, c, d\}$

And also they form a cycle of odd length.

$\therefore$ we require min 3 colors.

$\therefore$ $\chi(G) \geq 3$ - ②

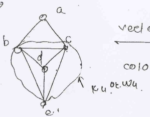
$\therefore$ From ① and ②,

$\chi(G) = 3$.

* Every 2-colorable graph is bipartite graph.

Q.10. Chromatic no. of the graph shown below,

→



vertex	b	c	d	e	a
color	C_1	C_2	C_3	C_4	C_3

$\therefore \chi(G) \leq 4. \quad -①$

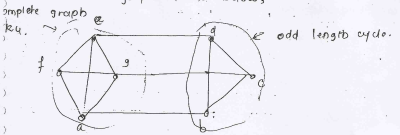
Further, we have four mutually adjacent vertices $\{b, c, d, e\}$

$\therefore \chi(G) \geq 4. \quad -②$

$\therefore$ from ① and ②,

$\chi(G) = 4.$

11. For the graph shown below,



G is a planar graph because we can draw w/o crossovers.

∴ So, by four-color thm,

$\chi(G) \leq 4.$ — ①

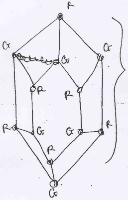
Further, we have four mutually adjacent vertices $\{a, e, f, g\}$ forming complete graph K_4.

∴ $\chi(G) \geq 4$ — ②.

∴ from ① and ②,

$\chi(G) = 4.$

12. Chromatic number of the graph shown below is ?

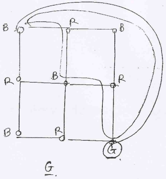


(G is a cyclic graph in which all the cycles are of even length.)

So, $\chi(G) = 2.$

∵ no odd length cycle.

Q.13. Chromatic number of the graph shown below is ?



G has a cycle of length 5.

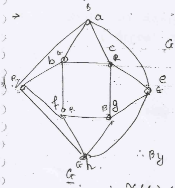
$\therefore \chi(G) \geq 3$. — (1)

By welch-powel's algom, 3 coloring is possible.

$\therefore \chi(G) = 3$

Q.14. What is chromatic no. of graph shown below. ?

→



G has cycles of length '3'.

$\therefore \chi(G) \geq 3$ — (1).

G is a planar graph.

$\therefore$ By 4-color thm,

$\therefore \chi(G) \leq 4$ — (2).

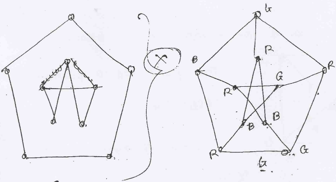
$\therefore \chi(G) = 4$

Applying welch-powel's algom,

Vertex	a	b	c	d	e	f	g	h
color	C_1	C_2	C_3	C_3	C_2	C_1	C_4	C_5

$\therefore \chi(G) \leq 5$ — ③.

15. chromatic number of the graph shown below?



G.

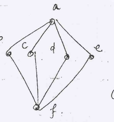
G has cycles of odd length.

$\therefore \chi(G) \geq 3$. — ①

3-coloring is possible

$\therefore \chi(G) = 3$.

Q.16. Chromatic no. of the graph shown below. ?



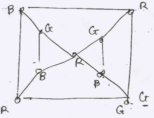
G.

The graph has no cycles of odd length

$\therefore \chi(G) = \underline{2}$

(It is a bipartite graph.)

Q.17. Chromatic no. of the graph shown below ?



The graph is a planar graph.

$\therefore \chi(G) \le 4.$

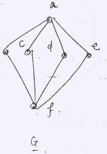
G has cycles of odd length.

$\therefore \chi(G) \ge 3.$

$\therefore$ 3-coloring is possible.

$\therefore \chi(G) = \underline{3}.$

Q.16. Chromatic no. of the graph shown below. ?

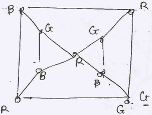


The graph has no cycles of odd length

$\therefore \chi(G) = \underline{2}$

(It is a bipartite graph.)

Q.17. Chromatic no. of the graph shown below ?



The graph is a planar graph.

$\therefore \chi(G) \le 4$.

$\underline{G}$ has cycles of odd length.

$\therefore \chi(G) \ge 3$.

$\therefore$ 3-coloring is possible.

$\therefore \chi(G) = \underline{3}$.