Chapter

1

Rectangular Cartesian Co-ordinates

CONTENTS

Rene' Descartes

Geometry is one of the most ancient branch of mathematics. A Systematic study of geometry by the use of algebra was first carried out by celebrated French philosopher and mathematician Rene' Descartes (1596-1650), in his book 'La Geometrie' which was published in 1637.

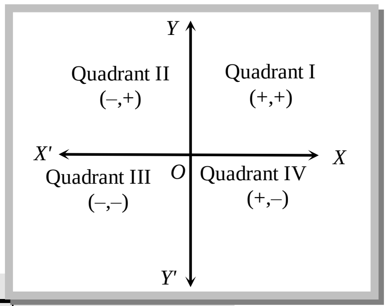
In order to relate algebra with geometry. Descartes established a relationship between the basic geometric concept of 'point' with basic algebraic entity 'number'. This relationship is called 'System of Co-ordinates'. Rene' Descartes related the position of a point with its distance from fixed line and its direction.

Leibnitz used the terms 'abscissa', ordinate and 'co-ordinate'. L' Hospital wrote (about 1700 A.D.) wrote an important text book on analytic geometry.

Rectangular Cartesian Co-ordinates

1.1 Introduction

Co-ordinates of a point are the real variables associated in an order to a point to describe its location in some space. Here the space is the two dimensional plane. The work of describing the position of a point in a plane by an ordered pair of real numbers can be done in different ways.



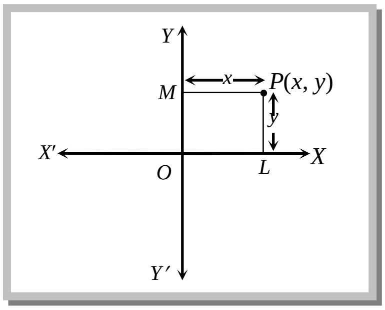
The two lines XOX' and YOY' divide the plane in four quadrants. XOY, YOX', $X'OY'$, $Y'OX$ are respectively called the first, the second, the third and the fourth quadrants. We assume the directions of OX, OY as positive while the directions of OX', OY' as negative.

Quadrant	***x*-coordinate**	***y*-coordinate**	**point**
First quadrant	+	+	(+,+)
Second quadrant	–	+	(–,+)
Third quadrant	–	–	(–,–)
Fourth quadrant	+	–	(+,–)

1.2 Cartesian Co-ordinates of a Point

This is the most popular co-ordinate system.

Let us consider two intersecting lines XOX' and YOY', which are perpendicular to each other. Let P be any point in the plane of lines. Draw the rectangle $OLPM$ with its adjacent sides OL,OM along the lines XOX', YOY' respectively. The position of the point P can be fixed in the plane provided the locations as well as the magnitudes of OL, OM are known.



Axis of *x* : The line XOX' is called axis of x.

Axis of *y* : The line YOY' is called axis of y.

Co-ordinate axes : x axis and y axis together are called axis of co-ordinates or axis of reference.

Origin : The point 'O' is called the origin of co-ordinates or the origin.

Oblique axes : If both the axes are not perpendicular then they are called as oblique axes.

Let $OL = x$ and $OM = y$ which are respectively called the abscissa (or x-coordinate) and the ordinate (or y-coordinate). The co-ordinate of P are (x, y).

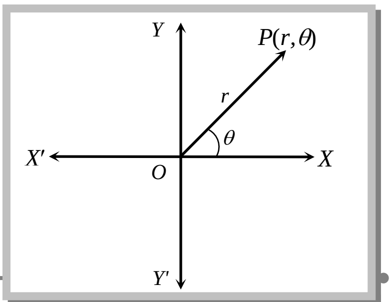
Note : ❑ Co-ordinates of the origin is (0, 0).

❑ The y co-ordinate of every point on x-axis is zero.

❑ The x co-ordinate of every point on y-axis is zero.

1.3 Polar Co-ordinates

Let OX be any fixed line which is usually called the initial line and O be a fixed point on it. If distance of any point P from the O is 'r' and $\angle XOP = \theta$, then (r, θ) are called the polar co-ordinates of a point P.

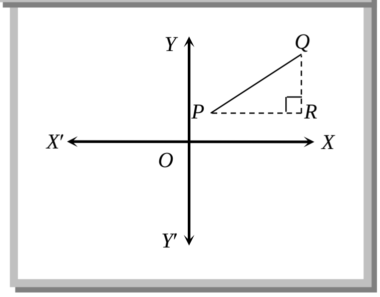


If (x, y) are the cartesian co-ordinates of a point P, then

$$x = r\cos\theta\,;\; y = r\sin\theta \;\text{ and }\; r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

1.4 Distance Formula



The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(PR)^2 + (QR)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note : ❑ The distance of a point $M(x_0, y_0)$ from origin $O\,(0, 0)$

$$OM = \sqrt{(x_0^2 + y_0^2)}.$$

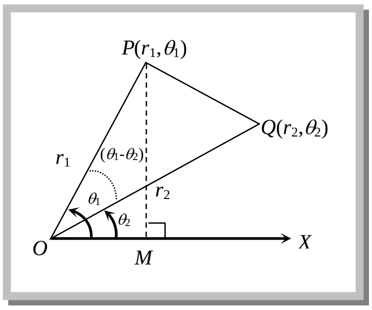
❑ If distance between two points is given then use $\pm$ sign.

❑ When the line PQ is parallel to the y-axis, the abscissa of point P and Q will be equal *i.e*, $x_1 = x_2$;

$\therefore \quad PQ = |y_2 - y_1|$

❑ When the segment PQ is parallel to the x-axis, the ordinate of the points P and Q will be equal *i.e.*, $y_1 = y_2$. Therefore $PQ = |x_2 - x_1|$

(1) **Distance between two points in polar co-ordinates :** Let O be the pole and OX be the initial line. Let P and Q be two given points whose polar co-ordinates are (r_1, θ_1) and (r_2, θ_2) respectively.



Then $OP = r_1, OQ = r_2$

$\angle POX = \theta_1$ and $\angle QOX = \theta_2$

then $\angle POQ = (\theta_1 - \theta_2)$

In ΔPOQ, from cosine rule $\cos(\theta_1 - \theta_2) = \dfrac{(OP)^2 + (OQ)^2 - (PQ)^2}{2OP.OQ}$

$$\therefore \quad (PQ)^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)$$

$$\therefore \quad PQ = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

Note : ❑ Always taking θ_1 and θ_2 in radians.

Example: 1 If the point (x, y) be equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, then **[MP PET 1983, 94]**

(a) $ax + by = 0$ (b) $ax - by = 0$ (c) $bx + ay = 0$ (d) $bx - ay = 0$

Solution: (d) Let points $P(x,y)$, $A(a+b, b-a)$, $B(a-b, a+b)$.

According to Question, $PA = PB$, *i.e.*, $PA^2 = PB^2$

$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow (a+b)^2 + x^2 - 2x(a+b) + (b-a)^2 + y^2 - 2y(b-a) = (a-b)^2 + x^2 - 2x(a-b) + (a+b)^2 + y^2 - 2y(a+b)$

$\Rightarrow 2x(a-b-a-b) = 2y(b-a-a-b) \Rightarrow -4bx = -4ay \Rightarrow bx - ay = 0$

Example: 2 If cartesian co-ordinates of any point are $(\sqrt{3}, 1)$, then its polar co-ordinates is

(a) $(2, \pi/3)$ (b) $(\sqrt{2}, \pi/6)$ (c) $(2, \pi/6)$ (d) None of these

Solution: (c) We know that $x = r\cos\theta$, $y = r\sin\theta$

$\therefore\ \sqrt{3} = r\cos\theta,\ \ 1 = r\sin\theta$

$r = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$, $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

Polar co-ordinates $= (2, \pi/6)$.

1.5 Geometrical Conditions

(1) **Properties of triangles**

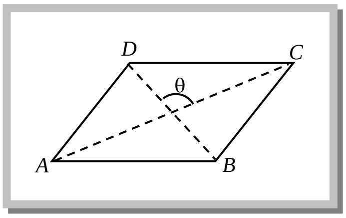
(i) In any triangle $ABC, AB + BC > AC$ and $|AB - BC| < AC$.

(ii) The ΔABC is equilateral $\Leftrightarrow AB = BC = CA$.

(iii) The ΔABC is a right angled triangle $\Leftrightarrow AB^2 = AC^2 + BC^2$ or $AC^2 = AB^2 + BC^2$ or $BC^2 = AB^2 + AC^2$.

(iv) The ΔABC is isosceles $\Leftrightarrow AB = BC$ or $BC = CA$ or $AB = AC$.

(2) **Properties of quadrilaterals**



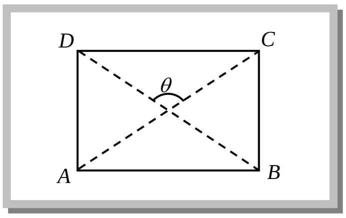
(i) The quadrilateral $ABCD$ is a parallelogram if and only if

(a) $AB = DC, AD = BC$, or (b) the middle points of BD and AC are the same,

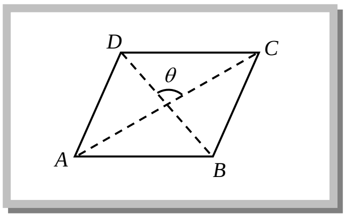
In a parallelogram diagonals AC and BD are not equal and $\theta \neq \frac{\pi}{2}$.

(ii) The quadrilateral $ABCD$ is a rectangle if and only if

(a) $AB = CD, AD = BC$ and $AC^2 = AB^2 + BC^2$ or, (b) $AB = CD, AD = BC, AC = BD$ or, (c) the middle points of AC and BD are the same and $AC=BD$. $(\theta \neq \pi/2)$

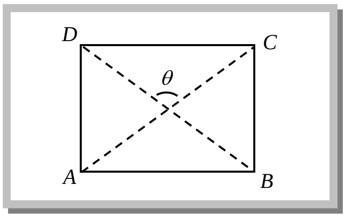


(iii) The quadrilateral $ABCD$ is a rhombus (but not a square) if and only if (a) $AB = BC = CD = DA$ and $AC \neq BD$ or, (b) the middle points of AC and BD are the same and $AB = AD$ but $AC \neq BD$. $(\theta = \pi/2)$



(iv) The quadrilateral $ABCD$ is a square if and only if

(a) $AB = BC = CD = DA$ and $AC = BD$ or (b) the middle points of AC and BD are the same and $AC = BD$, $(\theta = \pi/2)$, $AB = AD$.



Note :
- ❑ Diagonals of square, rhombus, rectangle and parallelogram always bisect each other.
- ❑ Diagonals of rhombus and square bisect each other at right angle.
- ❑ Four given points are collinear, if area of quadrilateral is zero.

Example: 3 ABC is an isosceles triangle. If the co-ordinates of the base are $B(1,3)$ and $C(-2,7)$ the co-ordinates of vertex A can be **[Orissa JEE 2002]**

(a) (1, 6) (b) $\left(-\frac{1}{2}, 5\right)$ (c) $\left(\frac{5}{6}, 6\right)$ (d) None of these

Solution: (c) Let the vertex of triangle be $A(x, y)$.

Then the vertex $A(x, y)$ is equidistant from *B* and *C* because *ABC* is an isosceles triangle, therefore

$(x-1)^2 + (y-3)^2 = (x+2)^2 + (y-7)^2 \Rightarrow 6x - 8y + 43 = 0$

Thus, any point lying on this line can be the vertex A except the mid point $\left(-\frac{1}{2}, 5\right)$ of *BC*. Hence vertex *A* is $\left(\frac{5}{6}, 6\right)$

Example: 4 The extremities of diagonal of parallelogram are the points (3, – 4) and (– 6,5) if third vertex is (– 2,1), then fourth vertex is **[Rajasthan PET 1987]**

(a) (1,0) (b) (–1,0) (c) (1,1) (d) None of these

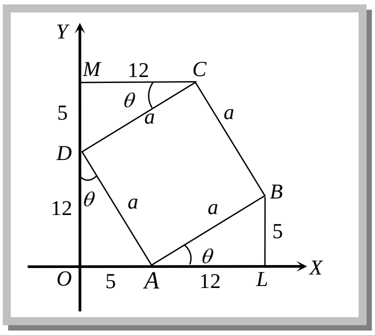
Solution: (b) Let $A(3,-4)$ and $C(-6,5)$ be the ends of diagonal of parallelogram *ABCD*. Let $B(-2,1)$ and *D* be (*x*, *y*), then mid points of diagonal *AC* and *BD* coincide. So, $\frac{x-2}{2} = \frac{-6+3}{2}$ and $\frac{y+1}{2} = \frac{5-4}{2}$

$x = -1,\ y = 0$. $\therefore$ Coordinates of *D* are (–1, 0)

Example: 5 The vertices *A* and *D* of square *ABCD* lie on positive side of *x* and *y*-axis respectively. If the vertex *C* is the point (12, 17), then the coordinate of vertex *B* are

(a) (14, 16) (b) (15, 3)

(c) (17, 5) (d) (17, 12)



Solution: (c) Let the co-ordinate of *B* be (*h*, *k*)

Draw *BL* and *CM* perpendicular to *x*-axis and *y*-axis.

$\therefore\ a\cos\theta = CM = OD = AL = 12$

and $a\sin\theta = DM = OA = BL = 5$

$\therefore\ k = BL = DM = OM - OD = 17 - 12 = 5$

$\therefore\ h = OL = OA + AL = 5 + 12 = 17$

Hence, Point *B* is (17, 5).

Example: 6 A triangle with vertices (4, 0); (–1, –1); (3, 5) is **[AIEEE 2002]**

(a) Isosceles and right angled (b) Isosceles but not right angled

(c) Right angled but not isosceles(d) Neither right angled nor isosceles

Solution: (a) Let A $(4,0)$; $B(-1,-1)$; $C(3,5)$ then

$AB = \sqrt{26},\ AC = \sqrt{26},\ BC = \sqrt{52}$; *i.e.* $AB = AC$

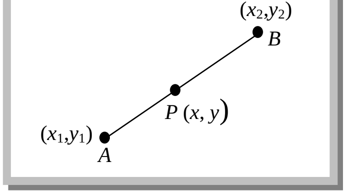
So triangle is isosceles and also $(BC)^2 = (AB)^2 + (AC)^2$. Hence ΔABC is right angled isosceles triangle.

1.6 Section Formulae

If $P(x,y)$ divides the join of $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio $m_1 : m_2 (m_1, m_2 > 0)$

(1) **Internal division :** If $P(x,y)$ divides the segment *AB* internally in the ratio of $m_1 : m_2$

$\Rightarrow \frac{PA}{PB} = \frac{m_1}{m_2}$

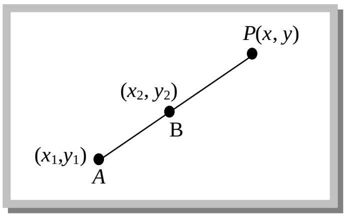


The co-ordinates of $P(x,y)$ are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

(2) **External division :** If $P(x,y)$ divides the segment *AB* externally in the ratio of $m_1 : m_2$

$\Rightarrow \quad \dfrac{PA}{PB} = \dfrac{m_1}{m_2}$



The co-ordinates of $P(x, y)$ are $x = \dfrac{m_1 x_2 - m_2 x_1}{m_1 - m_2}$ and $y = \dfrac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$

Note : ❑ If $P(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $\lambda : 1 (\lambda > 0)$, then $x = \dfrac{\lambda x_2 \pm x_1}{\lambda \pm 1}$; $y = \dfrac{\lambda y_2 \pm y_1}{\lambda \pm 1}$. Positive sign is taken for internal division and negative sign is taken for external division.

❑ The mid point of AB is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ [Here $m_1 : m_2 :: 1 : 1$]

❑ For finding ratio, use ratio $\lambda : 1$. If λ is positive, then divides internally and if λ is negative, then divides externally.

❑ Straight line $ax + by + c = 0$ divides the join of points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $\left(-\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$.

If ratio is –*ve* then divides externally and if ratio is +*ve* then divides internally.

Example: 7 The co-ordinate of the point dividing internally the line joining the points (4, –2) and (8, 6) in the ratio 7: 5 will be **[AMU 1979; MP PET 1984]**

(a) (16, 18) (b) (18, 16) (c) $\left(\dfrac{19}{3}, \dfrac{8}{3}\right)$ (d) $\left(\dfrac{8}{3}, \dfrac{19}{3}\right)$

Solution: (c) Let point (*x*, *y*) divides the line internally.

Then $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \dfrac{7(8) + 5(4)}{12} = \dfrac{19}{3}$, $y = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \dfrac{7(6) + 5(-2)}{12} = \dfrac{8}{3}$.

Example: 8 The line $x + y = 4$ divides the line joining the points (–1,1) and (5, 7) in the ratio **[IIT 1965, UPSEAT 1999]**

(a) 2 : 1 (b) 1 : 2 Internally (c) 1 : 2 Externally (d) None of these

Solution: (b) Required ratio = $-\left(\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right) = -\left(\dfrac{-1 + 1 - 4}{5 + 7 - 4}\right) = \dfrac{4}{8} = \dfrac{1}{2}$ (Internally)

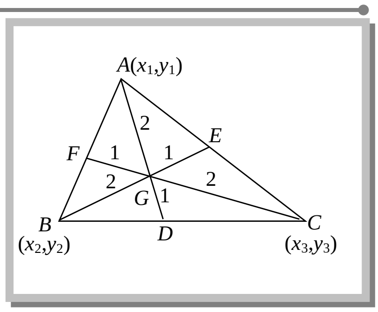
Example: 9 The line joining points (2, –3) and (–5, 6)is divided by *y*-axis in the ratio **[MP PET 1999]**

(a) 2 : 5 (b) 2 : 3 (c) 3 : 5 (d) 1 : 2

Solution: (a) Let ratio be k : 1 and coordinate of *y*-axis are (0, *b*). Therefore, $0 = \dfrac{k(-5) + 1(2)}{k + 1} \Rightarrow k = \dfrac{2}{5}$

1.7 Some points of a Triangle

(1) **Centroid of a triangle :** The centroid of a triangle is the point of intersection of its medians. The centroid divides the medians in the ratio $2 : 1$ (Vertex : base)



If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle. If *G* be the centroid upon one of the median (say) *AD*, then *AG* : *GD* = 2 : 1

$\Rightarrow$ Co-ordinate of *G* are $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

Example: 10 The centroid of a triangle is (2,7)and two of its vertices are (4, 8) and (–2, 6) the third vertex is **[Kerala (Engg.) 2002]**

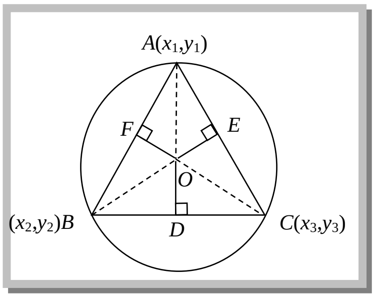
(a) (0, 0) (b) (4, 7) (c) (7, 4) (d) (7, 7)

Solution: (b) Let the third vertex (x, y)

$$2 = \frac{x+4-2}{3},\ 7 = \frac{y+8+6}{3},\ i.e.\ x = 4,\ \ y = 7$$

Hence third vertex is (4, 7).

(2) **Circumcentre :** The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of the sides of a triangle. It is the centre of the circle which passes through the vertices of the triangle and so its distance from the vertices of the triangle is the same and this distance is known as the circum-radius of the triangle.



Let vertices *A*, *B*, *C* of the triangle *ABC* be $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) and let circumcentre be $O(x, y)$ and then (x, y) can be found by solving

$$(OA)^2 = (OB)^2 = (OC)^2$$

i.e., $(x - x_1)^2 + (y - y_1)^2 = (x - x_2)^2 + (y - y_2)^2 = (x - x_3)^2 + (y - y_3)^2$

Note : ❑ If a triangle is right angle, then its circumcentre is the mid point of hypotenuse.

❑ If angles of triangle *i.e.*, *A*, *B*, *C* and vertices of triangle $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are given, then circumcentre of the triangle *ABC* is

$$\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$$

Example: 11 If the vertices of a triangle be (2, 1); (5, 2) and (3,4) then its circumcentre is **[IIT 1964]**

(a) $\left(\frac{13}{2}, \frac{9}{2}\right)$ (b) $\left(\frac{13}{4}, \frac{9}{4}\right)$ (c) $\left(\frac{9}{4}, \frac{13}{4}\right)$ (d) None of these

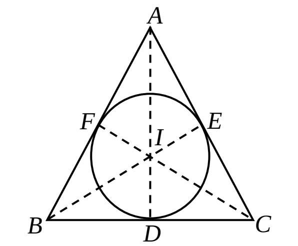
Solution: (b) Let circumcentre be $O(x, y)$ and given points are $A(2,1); B(5,2); C(3,4)$ and $OA^2 = OB^2 = OC^2$

$\therefore (x-2)^2 + (y-1)^2 = (x-5)^2 + (y-2)^2$ .....(i)

and $(x-2)^2 + (y-1)^2 = (x-3)^2 + (y-4)^2$ .....(ii)

On solving (i) and (ii), we get $x = \frac{13}{4}, y = \frac{9}{4}$

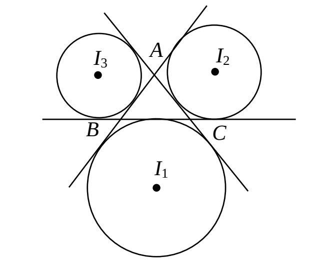
(3) **Incentre :** The incentre of a triangle is the point of intersection of internal bisector of the angles. Also it is a centre of a circle touching all the sides of a triangle.



Co-ordinates of incentre $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

Where *a*, *b*, *c* are the sides of triangle *ABC*.

(4) **Excircle :** A circle touches one side outside the triangle and other two extended sides then circle is known as excircle. Let ABC be a triangle then there are three excircles with three excentres. Let I_1, I_2, I_3 opposite to vertices *A*,*B* and *C* respectively. If vertices of triangle are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ then



$$I_1 \equiv \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

$$I_2 \equiv \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right),\ I_3 \equiv \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

Note : ❑ Angle bisector divides the opposite sides in the ratio of remaining sides *e.g.* $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

- Incentre divides the angle bisectors in the ratio $(b+c):a,(c+a):b$ and $(a+b):c$
- **Excentre :** Point of intersection of one internal angle bisector and other two external angle bisector is called as excentre. There are three excentres in a triangle. Co-ordinate of each can be obtained by changing the sign of *a,b,c* respectively in the formula of in-centre.

Example: 12 The incentre of the triangle with vertices $(1,\sqrt{3}),(0,0)$ and $(2,0)$ is **[IIT Screening 2000]**

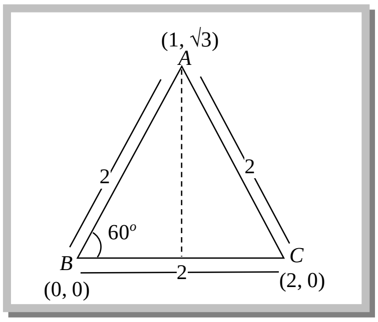
(a) $\left(1,\frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (d) $\left(1,\frac{1}{\sqrt{3}}\right)$

Solution: (d) $\because$ Here $AB = BC = CA$

$\therefore$ The triangle is equilateral .

So, the incentre is the same as the centroid.

$\therefore$ Incentre $=\left(\frac{1+0+2}{3},\frac{\sqrt{3}+0+0}{3}\right)=\left(1,\frac{1}{\sqrt{3}}\right)$.

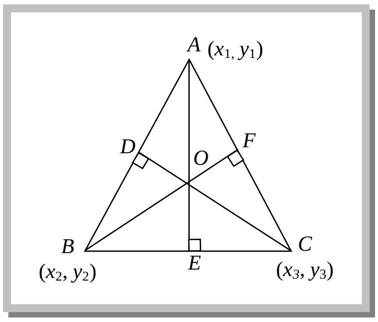


(5) **Orthocentre :** It is the point of intersection of perpendiculars drawn from vertices on opposite sides (called altitudes) of a triangle and can be obtained by solving the equation of any two altitudes.

Here *O* is the orthocentre since $AE \perp BC$, $BF \perp AC$ and $CD \perp AB$

then $OE \perp BC,\ OF \perp AC, OD \perp AB$

Solving any two we can get coordinate of *O*.



Note :
- If a triangle is right angled triangle, then orthocentre is the point where right angle is formed.
- If the triangle is equilateral then centroid, incentre, orthocentre, circum-centre coincides.
- Orthocentre, centroid and circum-centre are always collinear and centroid divides the line joining orthocentre and circum-centre in the ratio 2 : 1
- In an isosceles triangle centroid, orthocentre, incentre, circum-centre lie on the same line.

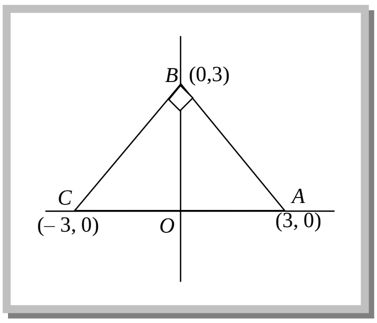
Example: 13 The vertices of triangle are (0, 3) (– 3, 0) and (3, 0). The co-ordinate of its orthocentre are **[AMU 1991; DCE 1994]**

(a) (0, – 2) (b) (0, 2) (c) (0, 3) (d) (0, –3)

Solution: (c) Here $AB \perp BC$.

In a right angled triangle, orthocentre is the point where right angle is formed.

$\therefore$ Orthocentre is (0, 3)



Example: 14 If the centroid and circumcentre of triangle are (3, 3); (6, 2), then the orthocentre is **[DCE 2000]**

(a) (9, 5) (b) (3, –1) (c) (– 3, 1) (d) (– 3, 5)

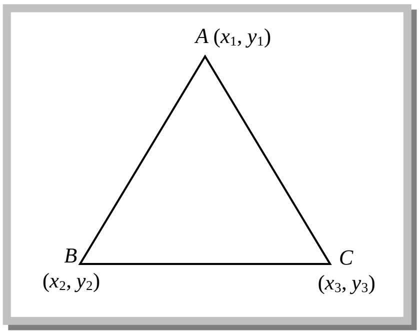
Solution: (d) Let orthocentre be (α, β). We know that centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1

$\therefore 3 = \frac{2(6)+1(\alpha)}{2+1} \Rightarrow \alpha = -3$, $3 = \frac{2(2)+1(\beta)}{2+1} \Rightarrow \beta = 5$

Hence orthocentre is (–3, 5).

1.8 Area of some Geometrical figures

(1) **Area of a triangle :** The area of a triangle *ABC* with vertices $A(x_1, y_1); B(x_2, y_2)$ and $C(x_3, y_3)$. The area of triangle *ABC* is denoted by 'Δ'and is given as



$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left|(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

In equilateral triangle

(i) Having sides *a*, area is $\frac{\sqrt{3}}{4}a^2$.

(ii) Having length of perpendicular as '*p*' area is $\frac{(p^2)}{\sqrt{3}}$.

Note : ❑ If a triangle has polar co-ordinates $(r_1, \theta_1), (r_2, \theta_2)$ and (r_3, θ_3) then its area

$$\Delta = \frac{1}{2}[r_1 r_2 \sin(\theta_2 - \theta_1) + r_2 r_3 \sin(\theta_3 - \theta_2) + r_3 r_1 \sin(\theta_1 - \theta_3)]$$

❑ If area is a rational number. Then the triangle cannot be equilateral.

(2) **Collinear points :** Three points $A(x_1, y_1); B(x_2, y_2); C(x_3, y_3)$ are collinear. If area of triangle is zero,

i.e., (i) $\Delta = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(ii) $AB + BC = AC$ or $AC + BC = AB$ or $AC + AB = BC$

(3) **Area of a quadrilateral :** If $(x_1, y_1); (x_2, y_2); (x_3, y_3)$ and (x_4, y_4) are vertices of a quadrilateral, then its

Area $= \frac{1}{2}[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_4 - x_4 y_3) + (x_4 y_1 - x_1 y_4)]$

Note : ❑ If two opposite vertex of rectangle are (x_1, y_1) and (x_2, y_2), then its area is $|(y_2 - y_1)(x_2 - x_1)|$.

❑ It two opposite vertex of a square are $A(x_1, y_1)$ and $C(x_2, y_2)$, then its area is

$$= \frac{1}{2}AC^2 = \frac{1}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$$

(4) **Area of polygon :** The area of polygon whose vertices are $(x_1, y_1); (x_2, y_2); (x_3, y_3); \ldots. (x_n, y_n)$ is

$= \frac{1}{2} | \{(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + ... + (x_n y_1 - x_1 y_n)\}|$

Or **Stair method :** Repeat first co-ordinates one time in last for down arrow use positive sign and for up arrow use negative sign.

$$\therefore \quad \text{Area of polygon} = \frac{1}{2}\left|\begin{array}{cc} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ : & : \\ : & : \\ x_n & y_n \\ x_1 & y_1 \end{array}\right| = \frac{1}{2}| \{(x_1 y_2 + x_2 y_3 + ... + x_n y_1) - (y_1 x_2 + y_2 x_3 + ... + y_n x_1)\}|$$

Example: 15 The area of the triangle formed by the points $(a, b+c), (b, c+a), (c, a+b)$ is

[IIT 1963; EAMCET 1982; Rajasthan PET 2003]

(a) abc (b) $a^2 + b^2 + c^2$ (c) $ab + bc + ca$ (d) 0

Solution: (d) Area $= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & b+c+a & 1 \\ c & c+a+b & 1 \end{vmatrix}$, (Applying $C_2 \to C_1 + C_2$) $= \frac{a+b+c}{2}\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$

Example: 16 Three points are $A(6, 3)$, $B(-3, 5)$, $C(4, -2)$ and $P(x, y)$ is a point, then the ratio of area of ΔPBC and ΔABC is

[IIT 1983]

(a) $\left|\frac{x+y-2}{7}\right|$ (b) $\left|\frac{x-y+2}{2}\right|$ (c) $\left|\frac{x-y-2}{7}\right|$ (d) None of these

Solution: (a) $\frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{\frac{1}{2}[-3(-2-y) + 4(y-5) + x(5+2)]}{\frac{1}{2}[6(5+2) - 3(-2-3) + 4(3-5)]} = \left|\frac{7x + 7y - 14}{49}\right| = \left|\frac{x+y-2}{7}\right|$

Example: 17 If the points $(2K, K)(K, 2K)$ and (K, K) with $K > 0$ enclose triangle of area 18 square units then the centroid of triangle is equal to

(a) (8, 8) (b) (4, 4) (c) (– 4, – 4) (d) $(4\sqrt{2}, 4\sqrt{2})$

Solution: (a) $\Delta = \frac{1}{2}\begin{vmatrix} 2K & K & 1 \\ K & 2K & 1 \\ K & K & 1 \end{vmatrix} = 18 \Rightarrow \frac{K^2}{2} = 18 \Rightarrow K = \pm 6$. Consider $K = +6$ because $K > 0$, then the points (12, 6) (6,12) and (6,6).

Hence, centroid $= \left(\frac{12+6+6}{3}, \frac{6+12+6}{3}\right) = (8, 8)$

Example: 18 If the points $(x+1, 2)$; $(1, x+2)$; $\left(\frac{1}{x+1}, \frac{2}{x+1}\right)$ are collinear, then x is **[Rajasthan PET 2002]**

(a) 4 (b) 0 (c) – 4 (d) None of these

Solution: (b,c) Let $A = (x+1, 2)$; $B = (1, x+2)$; $C = \left(\frac{1}{x+1}, \frac{2}{x+1}\right)$. A, B, C are collinear, if area of $\Delta ABC = 0$

$$\Rightarrow \begin{vmatrix} x+1 & 2 & 1 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & -x & 0 \\ 1 & x+2 & 1 \\ \frac{1}{x+1} & \frac{2}{x+1} & 1 \end{vmatrix} = 0 \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{vmatrix} x & 0 & 0 \\ 1 & x+3 & 1 \\ \frac{1}{x+1} & \frac{3}{x+1} & 1 \end{vmatrix} = 0 \quad (C_2 \to C_1 + C_2) \Rightarrow x^2(x+4) = 0 \Rightarrow x = 0, -4$$

Example: 19 The points (1, 1); $(0, \sec^2\theta)$; $(\text{cosec}^2\theta, 0)$ are collinear for **[Roorkee 1963]**

(a) $\theta = n\pi/2$ (b) $\theta \neq n\pi/2$ (c) $\theta = n\pi$ (d) None of these

Solution: (b) The given points are collinear, if Area of $\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sec^2\theta & 1 \\ \text{cosec}^2\theta & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1(\sec^2\theta) + 1(\text{cosec}^2\theta) + 1(-\text{cosec}^2\theta.\sec^2\theta) = 0$

$$\Rightarrow \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} - \frac{1}{\sin^2\theta.\cos^2\theta} = 0 \Rightarrow \frac{1}{\sin^2\theta.\cos^2\theta} - \frac{1}{\sin^2\theta.\cos^2\theta} = 0 \Rightarrow 0 = 0$$

Therefore the points are collinear for all value of θ, except only $\theta = \frac{n\pi}{2}$ because at $\theta = \frac{n\pi}{2}$, $\sec^2\theta = \infty$ (Not defined).

Example: 20 The points (0, 8/3) (1, 3) and (82, 30) are the vertices of **[IIT 1983; Rajasthan PET 1988]**

(a) An equilateral triangle (b) An isosceles triangle

(c) A right angled triangle (d) None of these

Solution: (d) Here $A = (0, 8/3)$, $B = (1,3)$ and $C = (82,30)$

$$AB = \sqrt{1+1/9} = \sqrt{10/9}\,,\ BC = \sqrt{(81)^2 + (27)^2} = 27\sqrt{10} = 81\sqrt{\frac{10}{9}}\,,\ AC = \sqrt{(82)^2 + (30-8/3)^2} = 82\sqrt{\frac{10}{9}}$$

Since $AB + BC = (1+81)\sqrt{\frac{10}{9}} = 82\sqrt{\frac{10}{9}} = AC$. $\therefore$ Points *A, B, C* are collinear.

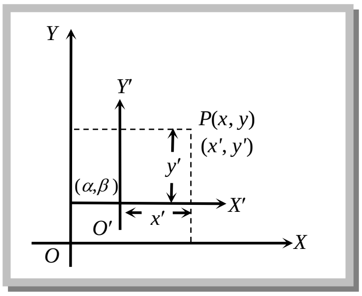
1.9 Transformation of Axes

(1) **Shifting of origin without rotation of axes :** Let $P \equiv (x, y)$ with respect to axes *OX* and *OY*.

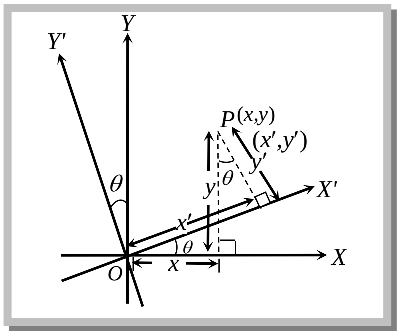
Let $O' \equiv (\alpha, \beta)$ with respect to axes *OX* and *OY* and let $P \equiv (x', y')$ with respect to axes *O'X'* and *O'Y'*, where *OX* and *O'X'* are parallel and *OY* and *O'Y'* are parallel.

Then $x = x' + \alpha, y = y' + \beta$ or $x' = x - \alpha, y' = y - \beta$

Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of *x* and $y + \beta$ in place of *y*.



(2) **Rotation of axes without changing the origin :** Let *O* be the origin. Let $P \equiv (x, y)$ with respect to axes *OX* and *OY* and let $P \equiv (x', y')$ with respect to axes *OX'* and *OY'* where $\angle X'OX = \angle YOY' = \theta$



then $x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

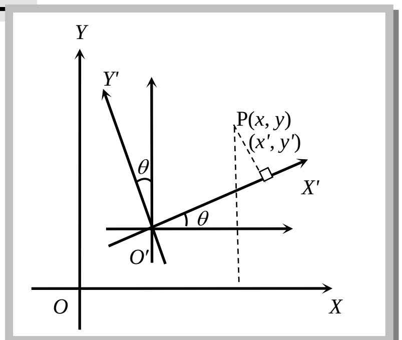
and $x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

	$x \downarrow$	$y \downarrow$
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

(3) **Change of origin and rotation of axes :** If origin is changed to $O'(\alpha, \beta)$ and axes are rotated about the new origin O' by an angle θ in the



anticlock-wise sense such that the new co-ordinates of $P(x,y)$ become (x',y') then the equations of transformation will be $x=\alpha+x'\cos\theta-y'\sin\theta$ and $y=\beta+x'\sin\theta+y'\cos\theta$

(4) **Reflection (Image of a point) :** Let (x,y) be any point, then its image with respect to

(i) x axis $\Rightarrow (x,-y)$ (ii) y-axis $\Rightarrow (-x,y)$ (iii) origin $\Rightarrow (-x,-y)$ (iv) line $y=x \Rightarrow (y,x)$

Example: 21 The point (2,3) undergoes the following three transformation successively,

(i) Reflection about the line $y=x$.

(ii) Transformation through a distance 2 units along the positive direction of y-axis.

(iii) Rotation through an angle of 45° about the origin in the anticlockwise direction.

The final coordinates of points are **[Roorkee 2000]**

(a) $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ (b) $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ (d) None of these

Solution: (b) (i) The new position after reflection is (3,2)

(ii) After transformation, it is (3, 2+ 2), *i.e*, (3, 4)

(iii) Rotation makes it $(3\cos 45^o-4\sin 45^o, 3\sin 45^o+4\cos 45^o)$, *i.e.* $\left(\frac{-1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$

Example: 22 Reflecting the point (2, –1) about y-axis, coordinate axes are rotated at 45^o angle in negative direction without shifting the origin. The new coordinates of the point are

(a) $\left(\frac{-1}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right)$ (b) $\left(\frac{-3}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ (d) None of these

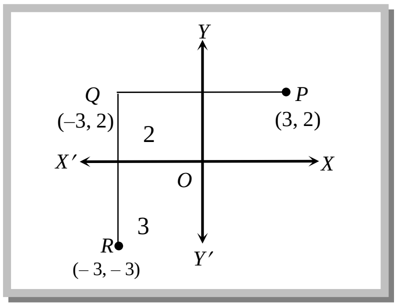
Solution: (a) The new position after reflection is (–2, –1)

Rotation makes it $[(-2)\cos(-45^o)+(-1)\sin(-45^o), -(-2)\sin(-45^o)+(-1)\cos(-45^o)]$, *i.e.*, $\left[\frac{-1}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right]$

Example: 23 The point (3, 2) is reflected in the y-axis and then moved a distance 5 units towards the negative side of y-axis. The co-ordinate of the point thus obtained are **[DCE 1997]**

(a) (3, –3) (b) (–3, 3) (c) (3, 3) (d) (–3, –3)

Solution: (d) Reflection in the y-axis of the point (3,2) is (–3, 2) when it moves towards the negative side of y- axis through 5 units, then the new position is (–3, 2– 5) =(– 3, – 3)



1.10 Locus

Locus : The curve described by a point which moves under given condition or conditions is called its locus.

Equation to the locus of a point : The equation to the locus of a point is the relation, which is satisfied by the coordinates of every point on the locus of the point.

Algorithm to find the locus of a point

Step I : Assume the coordinates of the point say (h, k) whose locus is to be found.

Step II : Write the given condition in mathematical form involving h , k.

Step III : Eliminate the variable (s), if any.

Step IV : Replace h by x and k by y in the result obtained in step III. The equation so obtained is the locus of the point which moves under some stated condition (s)

Note : ❑ Locus of a point P which is equidistant from the two point A and B is a straight line and is a perpendicular bisector of line AB.

❑ In above case if $PA = kPB$ where $k \neq 1$, then the locus of P is a circle.

❑ Locus of P if A and B is fixed.

(a) Circle, if $\angle APB$ = constant (b) Circle with diameter AB, if $\angle APB = \frac{\pi}{2}$

(c) Ellipse, if $PA + PB$ = constant (d) Hyperbola, if $PA - PB$ = constant

Example: 24 Let A (2, – 3) and B(– 2, 1) be vertices of triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **[AIEEE 2004]**

(a) $3x - 2y = 3$ (b) $2x - 3y = 7$ (c) $3x + 2y = 5$ (d) $2x + 3y = 9$

Solution: (d) Let third vertex C be (α, β)

$\therefore$ Centroid = $\left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$, *i.e.* $\left(\frac{\alpha}{3}, \frac{\beta-2}{3}\right)$

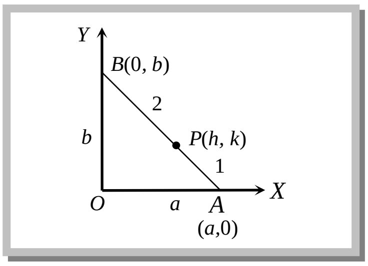
According to question, $2\left(\frac{\alpha}{3}\right) + 3\left(\frac{\beta-2}{3}\right) = 1 \Rightarrow 2\alpha + 3\beta - 6 = 3 \Rightarrow 2\alpha + 3\beta = 9$

Hence, locus of vertex C is $2x + 3y = 9$.

Example: 25 The ends of a rod of length l move on two mutually perpendicular lines. The locus of the point on the rod which divides it in the ratio 1 : 2 is **[IIT 1987; Rajasthan PET 1997]**

(a) $36x^2 + 9y^2 = 4l^2$ (b) $36x^2 + 9y^2 = l^2$ (c) $9x^2 + 36y^2 = 4l^2$ (d) None of these

Solution: (c) $AP : PB = 1 : 2$, then $h = \frac{1 \times 0 + 2 \times a}{1+2} = \frac{2a}{3}$ or $a = \frac{3h}{2}$, Similarly $b = 3k$



Now we have $OA^2 + OB^2 = AB^2 \Rightarrow \left(\frac{3h}{2}\right)^2 + (3k)^2 = l^2$

Hence locus of P (h, k) is given by $9x^2 + 36y^2 = 4l^2$

Example: 26 If A and B are two fixed points and P is a variable point such that $PA + PB = 4$, then the locus of P is a/an **[IIT 1989; UPSEAT 2001]**

(a) Parabola (b) Ellipse (c) Hyperbola (d) None of these

Solution: (b) We know that, $PA + PB$ = constant. Then locus of P is an ellipse.

***

Assignment

System of Co-ordinates

Basic Level

1. The distance between the points $(17, 105^o)$ and $(5\sqrt{2}, 60^o)$ is

(a) 13 (b) 12 (c) 11 (d) 10

2. In a plane, the co-ordinates (r, θ) of a point are equivalent

(a) $(r, -\theta)$ (b) $(-r, \theta)$ (c) $(-r, \pi+\theta)$ (d) $(r, \pi+\theta)$

3. The system of coordinates known as the cartesian system of coordinates was first introduced by

(a) Euclid (b) Euler (c) Descarte (d) Bhasker

4. Which of the following polar coordinates are associated to the same point

I : $(2, 30^o)$ II : $(3, 150^o)$

III : $(-2, 45^o)$ IV : $(-3, 330^o)$

V : $(3, -210^o)$ VI : $(-3, 30^o)$

(a) I, III and IV (b) II, IV and VI (c) II, IV, V and VI (d) IV and VI

Distance Formula

Basic Level

5. If the distance between the points (*a*, 2) and (3, 4) be 8, then *a* = [MNR 1978]

(a) $2+3\sqrt{15}$ (b) $2-3\sqrt{15}$ (c) $2\pm 3\sqrt{15}$ (d) $3\pm 2\sqrt{15}$

6. The distance between the points $(am_1^2, 2am_1)$ and $(am_2^2, 2am_2)$ is

(a) $a(m_1-m_2)\sqrt{(m_1+m_2)^2+4}$ (b) $(m_1-m_2)\sqrt{(m_1+m_2)^2+4}$

(c) $a(m_1-m_2)\sqrt{(m_1+m_2)^2-4}$ (d) $(m_1-m_2)\sqrt{(m_1+m_2)^2-4}$

7. The distance of the point $(b\cos\theta, b\sin\theta)$ from origin is [MP PET 1984]

(a) $b\cot\theta$ (b) b (c) $b\tan\theta$ (d) $b\sqrt{2}$

8. The distance between the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta,\ a\sin\beta)$ is

(a) $a\cos\frac{\alpha-\beta}{2}$ (b) $2a\cos\frac{\alpha-\beta}{2}$ (c) $a\sin\frac{\alpha-\beta}{2}$ (d) $2a\sin\frac{\alpha-\beta}{2}$

9. The point on y-axis equidistant from the points (3, 2) and (–1, 3) is

(a) (0, –3) (b) $(0,-3/2)$ (c) $(0,3/2)$ (d) (0, 3)

10. The point P is equidistant from A(1, 3), B(– 3, 5) and C(5, –1). Then PA = [EAMCET 2003]

(a) 5 (b) $5\sqrt{5}$ (c) 25 (d) $5\sqrt{10}$

11. The point whose abscissa is equal to its ordinate and which is equidistant from the points (1, 0) and (0, 3) is

(a) (1, 1) (b) (2, 2) (c) (3, 3) (d) (4, 4)

12. Mid-point of the sides AB and AC of a ΔABC are (3, 5) and (–3, –3) respectively, then the length of the side BC is

(a) 10 (b) 20 (c) 15 (d) 30

13. The distance of the middle point of the line joining the points $(a\sin\theta, 0)$ and $(0, a\cos\theta)$ from the origin

(a) $\frac{a}{2}$ (b) $\frac{1}{2}a(\sin\theta+\cos\theta)$ (c) $a(\sin\theta+\cos\theta)$ (d) a

14. A point on the line $y = x$ at a distance of 2 units from the origin is [MP PET 1984]

(a) $(0,\sqrt{2})$ (b) $(\sqrt{2},0)$ (c) $(2,2)$ (d) $(\sqrt{2},\sqrt{2})$

15. If the points (1, 1), (–1, –1) and $(-\sqrt{3},k)$ are vertices of an equilateral triangle then the value of k will be

(a) 1 (b) –1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

Advance Level

16. If O be the origin and if the coordinates of any two points Q_1 and Q_2 be (x_1,y_1) and (x_2,y_2) respectively, then $OQ_1.OQ_2\cos Q_1OQ_2 =$ [IIT 1961]

(a) $x_1x_2-y_1y_2$ (b) $x_1y_1-x_2y_2$ (c) $x_1x_2+y_1y_2$ (d) $x_1y_1+x_2y_2$

17. If the line segment joining the points $A(a,b)$ and $B(c, d)$ subtends an angle θ at the origin, then $\cos\theta$ is equal to [IIT 1961]

(a) $\frac{ab+cd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (b) $\frac{ac+bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (c) $\frac{ac-bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (d) None of these

18. The vertices of a triangle ABC are (0, 0), (2, –1) and (9, 2) respectively, then $\cos B =$ [AMU 1977]

(a) $\frac{11}{290}$ (b) $\frac{\sqrt{11}}{290}$ (c) $-\frac{11}{\sqrt{290}}$ (d) $-\sqrt{\frac{11}{290}}$

19. If $A(2,2), B(-4,-4), C(5,-8)$ are vertices of any triangle, then the length of median passes through C will be [Rajasthan PET 1988]

(a) $\sqrt{65}$ (b) $\sqrt{117}$ (c) $\sqrt{85}$ (d) $\sqrt{113}$

20. If a vertex of an equilateral triangle is on origin and second vertex is (4, 0), then its third vertex is

(a) $(2, \pm\sqrt{3})$ (b) $(3, \pm\sqrt{2})$ (c) $(2, \pm 2\sqrt{3})$ (d) $(3, \pm 2\sqrt{2})$

21. The locus of the point *P* equidistant from the points (x_1, y_1) and (x_2, y_2) is $(x_1 - x_2)x + (y_1 - y_2)y + c = 0$, then the value of *c* is

(a) $(x_1^2 - x_2^2) + (y_1^2 - y_2^2)$ (b) $\frac{1}{2}(x_1^2 + x_2^2 + y_1^2 + y_2^2)$ (c) $\frac{1}{2}(x_2^2 - x_1^2 + y_2^2 - y_1^2)$ (d) $\sqrt{x_1^2 - x_2^2 + y_1^2 - y_2^2}$

22. Let $S_1, S_2, ...$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 *cm*, then for which of the following values of *n* is the area of S_n less than 1 sq. *cm*.

(a) 7 (b) 8 (c) 9 (d) 10

Problems concerning to geometrical conditions

Basic Level

23. The three points (– 2, 2), (8, – 2) and (– 4, – 3) are the vertices of **[Rajasthan PET 1987]**

(a) An isosceles triangle (b) An equilateral triangle (c) A right angled triangle (d) None of these

24. The points $A(-4, -1); B(-2, -4); C(4, 0)$ and $D(2, 3)$ are the vertices of a

(a) Parallelogram (b) Rectangle (c) Rhombus (d) None of these

25. Two opposite vertices of a rectangle are (1,3) and (5,1). If the other two vertices of the rectangle lie on the line $y - x + \lambda = 0$, then $\lambda =$

(a) 1 (b) – 1 (c) 2 (d) None of these

26. Three vertices of a parallelogram are (1, 3) (2, 0) and (5, 1). Then its fourth vertex is **[Rajasthan PET 1988, 2001]**

(a) (3, 3) (b) (4, 4) (c) (4, 0) (d) (0, – 4)

27. The quadrilateral formed by the vertices (– 1, 1), (0, – 3), (5, 2) and (4, 6) will be **[Rajasthan PET 1986]**

(a) Square (b) Parallelogram (c) Rectangle (d) Rhombus

28. The triangle formed by the lines $x + y = 0$, $3x + y - 4 = 0$ and $x + 3y = 4$ is **[IIT 1983; MNR 1992; Rajasthan PET 1995; UPSEAT 2001]**

(a) Equilateral (b) Isosceles (c) Right angled (d) None of these

29. The following points $A(2a, 4a), B(2a, 6a)$ and $C(2a + \sqrt{3}a, 5a)$, $(a > 0)$ are the vertices of

(a) An acute angled triangle (b) An right angled triangle (c) An isosceles triangle (d)

30. The triangle joining the points $P(2,7), Q(4,-1), R(-2,6)$ is **[MP PET 1997]**

(a) Equilateral triangle (b) Right-angled triangle (c) Isosceles triangle (d) Scalene triangle

31. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of *c* will be **[IIT 1981]**

(a) 4 (b) – 4 (c) 2 (d) – 2

32. If the three vertices of a rectangle taken in order are the points (2, –2), (8, 4) and (5, 7). The coordinates of fourth vertex are

[Kurukshetra CEE 1993]

(a) (1, 1) (b) (1, –1) (c) (–1, 1) (d) None of these

33. If vertices of a quadrilateral are $A(0,0), B(3,4), C(7,7)$ and $D(4,3)$ then quadrilateral *ABCD* is a [Rajasthan PET 1986]

(a) Parallelogram (b) Rectangle (c) Square (d) Rhombus

34. The coordinates of the third vertex of an equilateral triangle whose two vertices are at (3, 4) and (–2, 3) are

(a) (1, 1) or (1, –1) (b) $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

(c) $(-\sqrt{3}, \sqrt{3})$ or $(\sqrt{3}, -\sqrt{3})$ (d) None of these

35. The quadrilateral joining the points (1, –2); (3, 0); (1, 2) and (–1, 0) is [Rajasthan PET 1999]

(a) Parallelogram (b) Rectangle (c) Square (d) Rhombus

36. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangle with vertices $(x_1, y_1); (x_2, y_2); (x_3, y_3)$ and $(a_1, b_1); (a_2, b_2); (a_3, b_3)$ must be

[IIT 1985]

(a) Similar (b) Congruent (c) Never congruent (d) None of these

37. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy [IIT 1986; Kurukshetra CEE 1998]

(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \leq 0$ (c) $2x - 3y - 12 \leq 0$ (d) All of these

38. The common property of points lying on *x*-axis, is [MP PET 1988]

(a) $x = 0$ (b) $y = 0$ (c) $a = 0, y = 0$ (d) $y = 0, b = 0$

39. Vertices of a figure are (– 2, 2); (– 2, – 1); (3, –1); (3, 2), it is a [Karnataka CET 1998]

(a) Square (b) Rhombus (c) Rectangle (d) Parallelogram

40. If *ABCD* is a quadrilateral, if the mid point of consecutive sides *AB, BC, CD* and *DA* are combined by straight lines, then the quadrilateral *PQRS* is always [Orissa JEE 2002]

(a) Square (b) Parallelogram (c) Rectangle (d) Rhombus

41. Three vertices of a parallelogram taken in order are $(-1, -6), (2, -5)$ and $(7, 2)$. The fourth vertex is

(a) (1, 4) (b) (4, 1) (c) (1, 1) (d) (4, 4)

42. If $P(1,2), Q(4,6) R(5,7)$ and $S(a,b)$ are the vertices of a parallelogram *PQRS*, then [IIT 1998]

(a) $a = 2, b = 4$ (b) $a = 3, b = 4$ (c) $a = 2, b = 3$ (d) $a = 3, b = 5$

Advance Level

43. The sides of a triangle are $3x + 4y, 4x + 3y$ and $5x + 5y$ where $x, y > 0$, then the triangle is [AIEEE 2002]

(a) Right angled (b) Obtuse angled (c) Equilateral (d) None of these

44. If the vertices of triangle have integral coordinates then the triangle is [IIT 1975; MP PET 1983]

(a) Equilateral (b) Never equilateral (c) Isosceles (d) None of these

45. The opposite angular points of a square are (3, 4) and (1, –1). Then the coordinates of other two vertices are [Roorkee 1985]

(a) $D\left(\frac{1}{2},\frac{9}{2}\right); B\left(-\frac{1}{2},\frac{5}{2}\right)$ (b) $D\left(-\frac{1}{2},\frac{9}{2}\right); B\left(\frac{1}{2},\frac{5}{2}\right)$ (c) $D\left(\frac{9}{2},\frac{1}{2}\right); B\left(-\frac{1}{2},\frac{5}{2}\right)$ (d) None of these

46. The quadrilateral formed by the lines $ax \pm by \pm c = 0$ is [Rajasthan PET 1998]

(a) Square (b) Rectangle (c) Rhombus (d) Parallelogram

Section Formulae

Basic Level

47. Point $\left(\frac{1}{2},\frac{-13}{4}\right)$ divides the line joining the points (3, – 5) and (– 7, 2) in the ratio of

(a) 1 : 3 internally (b) 3 : 1 internally (c) 1 : 3 externally (d) 3 : 1 externally

48. In what ratio does the *y*-axis divide the join of (–3, –4) and (1, –2) [Rajasthan PET 1995]

(a) 1 : 3 (b) 2 : 3 (c) 3 : 1 (d) None of these

49. The points which trisect the line segment joining the points (0, 0) and (9, 12) are [Rajasthan PET 1986]

(a) (3, 4), (6, 8) (b) (4, 3), (6, 8) (c) (4, 3), (8, 6) (d) (3, 4), (8, 6)

50. If the point dividing internally the line segment joining the points (a, b) and $(5, 7)$ in the ratio 2 : 1 be (4, 6) then

(a) $a = 1, b = 2$ (b) $a = 2, b = -4$ (c) $a = 2, b = 4$ (d) $a = -2, b = 4$

51. If *A* and *B* are the points $(-3, 4)$ and (2, 1). Then the co-ordinates of point *C* on *AB* produced such that *AC* = 2*BC* are

(a) (2, 4) (b) (3, 7) (c) (7, –2) (d) $\left(-\frac{1}{2},\frac{5}{2}\right)$

52. The line segment joining the points (1, 2) and (– 2, 1) is divided by the line $3x + 4y = 7$ in the ratio

(a) 3 : 4 (b) 4 : 3 (c) 9 : 4 (d) 4 : 9

Advance Level

53. If the points $P_1, P_2, P_3.....$ are the middle points of line segments $AB, P_1B, P_2B,.....$ respectively and particles of masses $m; \frac{m}{2}, \frac{m}{2^2},.....$ are placed respectively on these points. If *G* is the mass-centre of so placed infinite particles and $\overline{BG} = p\,\overline{BA}$, then *p* is [MP PET 1998]

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

54. If coordinates of the points *A* and *B* are (2, 4) and (4, 2) respectively and point *M* is such that *A*-*M*-*B* also *AB* = 3*AM*, then the coordinates of *M* are

(a) $\left(\frac{8}{3},\frac{10}{3}\right)$ (b) $\left(\frac{10}{3},\frac{14}{4}\right)$ (c) $\left(\frac{10}{3},\frac{6}{3}\right)$ (d) $\left(\frac{13}{4},\frac{10}{4}\right)$

55. The mid-points of sides of a triangle are (2, 1), (–1, –3) and (4, 5). Then the coordinates of its vertices are

(a) (7, 9), (– 3, – 7), (1, 1) (b) (– 3, – 7), (1, 1), (2, 3) (c) (1, 1), (2, 3), (– 5, 8) (d) None of these

56. The coordinates of the points *A, B, C* are (x_1, y_1), $(x_2, y_2), (x_3, y_3)$ and *D* divides the line *AB* in the ratio $l : k$. If *P* divides the line *DC* in the ratio $m : k + l,$ then the coordinates of *P* are

(a) $\left(\frac{kx_1 + lx_2 + mx_3}{k + l + m}, \frac{ky_1 + ly_2 + my_3}{k + l + m}\right)$

(b) $\left(\frac{lx_1 + mx_2 + kx_3}{l + m + k}, \frac{ly_1 + my_2 + ky_3}{l + m + k}\right)$

(c) $\left(\frac{mx_1 + kx_2 + lx_3}{m + k + l}, \frac{my_1 + ky_2 + ly_3}{m + k + l}\right)$

(d) None of these

Some points related to Triangle

Basic Level

57. If the coordinates of the vertices of a triangle be $(1, a), (2, b)$ and $(c^2, 3)$, then the centroid of the triangle

(a) Lies at the origin (b) Cannot lie on *x*-axis (c) Cannot lie on *y*-axis (d) None of these

58. If $A(4, -3), B(3, -2)$ and $C(2, 8)$ are the vertices of a triangle, then its centroid will be **[Rajasthan PET 1984, 1986]**

(a) (– 3, 3) (b) (3, 3) (c) (3, 1) (d) (1, 3)

59. Two vertices of a triangle are (5, 4) and (– 2, 4). If its centroid is (5, 6) then the third vertex has the coordinates **[MP PET 1993]**

(a) (12, 10) (b) (10, 12) (c) (–10, 12) (d) (12, –10)

60. The centroid of a triangle, whose vertices are (2, 1), (5, 2) and (3, 4) is **[IIT 1964]**

(a) $\left(\frac{8}{3},\frac{7}{3}\right)$ (b) $\left(\frac{10}{3},\frac{7}{3}\right)$ (c) $\left(-\frac{10}{3},\frac{7}{3}\right)$ (d) $\left(\frac{10}{3},-\frac{7}{3}\right)$

61. If the middle points of the sides of a triangle be (–2, 3), (4, –3) and (4, 5), then the centroid of the triangle is

(a) (5/3, 2) (b) (5/6, 1) (c) (2, 5/3) (d) (1, 5/6)

62. If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the excentre with respect to *B* is **[Rajasthan PET 2000]**

(a) $\left(\frac{ax_1 - bx_2 + cx_3}{a - b + c}, \frac{ay_1 - by_2 + cy_3}{a - b + c}\right)$

(b) $\left(\frac{ax_1 + bx_2 - cx_3}{a + b - c}, \frac{ay_1 + by_2 - cy_3}{a + b - c}\right)$

(c) $\left(\frac{ax_1 - bx_2 - cx_3}{a - b - c}, \frac{ay_1 - by_2 - cy_3}{a - b - c}\right)$

(d) None of these

63. If two vertices of an equilateral triangle have integral co-ordinates then the third vertex will have

(a) Integral co-ordinates (b) Co-ordinates which are rational

(c) At least one co-ordinate irrational (d) Co-ordinates which are irrational

64. If the orthocentre and centroid of triangle are (–3, 5), (3, 3), then the circumcentre is **[Kurukshetra CEE 1999]**

(a) (6, 2) (b) (0, 8) (c) (6, –2) (d) (0, 4)

65. The centroid and a vertex of an equilateral triangle are (1, 1) and (1, 2) respectively. Another vertex of the triangle can be

(a) $\left(\frac{2-\sqrt{3}}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{2+3\sqrt{3}}{2}, \frac{1}{2}\right)$ (c) $\left(\frac{2+\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) None of these

66. The incentre of triangle formed by lines $x = 0$, $y = 0$ and $3x + 4y = 12$ is **[Rajasthan PET 1990]**

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) (1, 1) (c) $\left(1, \frac{1}{2}\right)$ (d) $\left(\frac{11}{2}, 1\right)$

67. Orthocentre of triangle with vertices (0, 0), (3, 4), (4, 0) is **[IIT Screening 2003]**

(a) $\left(3, \frac{5}{4}\right)$ (b) (3, 12) (c) $\left(3, \frac{3}{4}\right)$ (d) (3, 9)

68. Orthocentre of the triangle whose vertices are (0, 0)), (2, – 1) and (1, 3) is **[ISM Dhanbad 1970; IIT 1967, 1974]**

(a) $\left(\frac{4}{7}, \frac{1}{7}\right)$ (b) $\left(-\frac{4}{7}, -\frac{1}{7}\right)$ (c) (– 4, – 1) (d) (4, 1)

69. The orthocentre of the triangle formed by the lines $4x - 7y + 10 = 0$, $x + y = 5$ and $7x + 4y = 15$ is **[IIT 1969, 1976]**

(a) (1, 2) (b) (1, –2) (c) (–1, – 2) (d) (–1, 2)

70. Coordinates of the orthocentre of the triangle whose sides are $x = 3$, $y = 4$ and $3x + 4y = 6$, will be **[MNR 1989]**

(a) (0, 0) (b) (3, 0) (c) (0, 4) (d) (3, 4)

71. The orthocentre of the triangle formed by (0, 0), (8, 0), (4, 6) is **[EAMCET 1991]**

(a) $\left(4, \frac{8}{3}\right)$ (b) (3, 4) (c) (4, 3) (d) (–3, 4)

72. If the line $3x + 4y - 24 = 0$ cuts the *x*-axis in *A* and *y*-axis in *B*, then incentre of ΔOAB (where *O* is the origin) is

(a) (1, 2) (b) (2, 2) (c) (12, 12) (d) (2, 12)

73. The distance between the orthocentre and circumcentre of the triangle with vertices (0, 0) , (0, *a*) and (*b*, 0) is

(a) $\frac{\sqrt{|a^2 - b^2|}}{2}$ (b) $a + b$ (c) $a - b$ (d) $\frac{\sqrt{a^2 + b^2}}{2}$

74. The incentre of the triangle formed by (0, 0); (5, 12); (16, 12) is **[EAMCET 1984]**

(a) (9, 7) (b) (7, 9) (c) (–9, 7) (d) (–7, 9)

75. If two vertices of a triangles are (6, 4); (2, 6) and its centroid is (4, 6), then the third vertex is **[Rajasthan PET 1996]**

(a) (4, 8) (b) (8, 4) (c) (6, 4) (d) None of these

76. If the vertices of a triangle be (*a*, 1); (*b*, 3) and (4, *c*), then the centroid of the triangle will lie on *x*-axis if

(a) $a + c = -4$ (b) $a + b = -4$ (c) $c = -4$ (d) $b + c = -4$

77. The vertices of a triangle are (0, 0), (3, 0) and (0, 4). Its orthocentre is at **[MNR 1982; Rajasthan PET 1997; DCE 1994]**

(a) (0, 0) (b) $\left(1, \frac{4}{3}\right)$ (c) $\left(\frac{3}{2}, 2\right)$ (d) None of these

Advance Level

78. The equations of the sides of a triangle are $x+y-5=0; x-y+1=0$ and $y-1=0,$ then the coordinates of the circumcentre are [MP PET 1996]

(a) (2, 1) (b) (1, 2) (c) (2, –2) (d) (1, – 2)

79. The mid points of the sides of a triangle are (5, 0); (5, 12) and (0, 12). The orthocentre of this triangle is

(a) (0, 0) (b) (10, 0) (c) (0, 24) (d) $\left(\frac{13}{3}, 8\right)$

80. The orthocentre of the triangle with vertices $\left(2, \frac{\sqrt{3}-1}{2}\right); \left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(2, -\frac{1}{2}\right)$ is [IIT 1993]

(a) $\left(\frac{3}{2}, \frac{\sqrt{3}-3}{6}\right)$ (b) $\left(2, -\frac{1}{2}\right)$ (c) $\left(\frac{5}{4}, \frac{\sqrt{3}-2}{5}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$

81. If the coordinates of the vertices of a triangle are rational numbers then which of the following points of the triangle will always have rational coordinates

(a) Centroid (b) Incentre (c) Circumcentre (d) Orthocentre

82. In the ΔABC , the coordinates of *B* are (0, 0), $AB = 2, \angle ABC = \frac{\pi}{3}$ and the middle point of *BC* has the coordinates (2, 0). The centroid of the triangle is

(a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{5}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{4+\sqrt{3}}{3}, \frac{1}{3}\right)$ (d) None of these

83. The vertices of triangle are (6, 0), (0, 6) and (6, 6). The distance between its circumcentre and centroid is

(a) $2\sqrt{2}$ (b) 2 (c) $\sqrt{2}$ (d) 1

84. Two vertices of a triangle are (5, –1) and (–2, 3). If orthocentre is the origin then co-ordinates of the third vertex are

(a) (7, 4) (b) (–4, 7) (c) (4, –7) (d) (– 4, – 7)

85. The orthocentre of the triangle formed by the lines $x+y=1,\ 2x+3y=6$ and $4x-y+4=0$ lies in quadrant [IIT 1985]

(a) First (b) Second (c) Third (d) Fourth

86. Two vertices of a triangle are (4, –3) and (–2, 5). If the orthocentre of the triangle is at (1, 2) , then the third vertex is [Roorkee 1987]

(a) (– 33, –26) (b) (33, 26) (c) (26, 33) (d) None of these

87. The equations to the sides of a triangle are $x-3y=0$, $4x+3y=5$ and $3x+y=0$. The line $3x-4y=0$ passes through [EAMCET 1994]

(a) The incentre (b) The centroid (c) The circumcentre (d) The orthocentre of the triangle

88. The vertices of a triangle are $|\, at_1t_2; a(t_1+t_2)|$, $|\, at_2t_3, a(t_2+t_3)|$, $|\, at_3t_1, a(t_3+t_1)|$, then the coordinates of its orthocentre are [IIT 1983]

(a) $|\, a, a(t_1+t_2+t_3+t_1t_2t_3)|$ (b) $[-a, a(t_1+t_2+t_3+t_1t_2t_3)]$

(c) $[-a, (t_1+t_2+t_3+t_1t_2t_3), a]$ (d) None of these

89. The equations of the three sides of a triangle are $x=2, y+1=0$ and $x+2y=4$. The coordinates of the circumcentre of the triangle are

(a) (4, 0) (b) (2, –1) (c) (0, 4) (d) None of these

Area of Some geometrical figures

Basic Level

90. The area of the triangle with vertices at (–4, 1), (1, 2), (4, – 3) is [EAMCET 1980]

(a) 14 (b) 16 (c) 15 (d) None of these

91. If the coordinates of the points *A, B, C* be (4, 4) (3, –2) and (3, – 16) respectively, then the area of the triangle *ABC* is [MP PET 1982]

(a) 27 (b) 15 (c) 18 (d) 7

92. If the vertices of a triangle are (5, 2), (2/3, 2) and (–4, 3), then the area of the triangle is [Kurukshetra CEE 2002]

(a) $\frac{28}{6}$ (b) $\frac{5}{2}$ (c) 43 (d) $\frac{13}{6}$

93. The area of a triangle whose vertices are (1, –1), (–1, 1) and (–1, –1) is given by [AMU 1981; Rajasthan PET 1989; MP PET 1993]

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 3

94. The vertices of a triangle *ABC* are $(\lambda, 2-2\lambda)$, $(-\lambda+1, 2\lambda)$ and $(-4-\lambda, 6-2\lambda)$. If its area be 70 units then number of integral values of λ is

(a) 1 (b) 2 (c) 4 (d) 0

95. The area of the pentagon whose vertices are (1, 2), (–3, 2), (4, 5), (–3, 3) and (–3, 0) is

(a) 15/2 unit^2 (b) 30 unit^2 (c) 45 unit^2 (d) None of these

Advance Level

96. If $A(6,3), B(-3,5), C(4,-2)$ and $D(x, 3x)$ are four points. If the ratio of area of ΔDBC and ΔABC is 1 : 2, then the value of *x* will be [IIT 1959]

(a) $\frac{11}{8}$ (b) $\frac{8}{11}$ (c) 3 (d) None of these

97. The point A divides the join of the points (– 5, 1) and (3, 5) in the ratio *k* : 1 and the coordinates of the points *B* and *C* are (1, 5) and (7, – 2) respectively. If the area of the triangle *ABC* be 2 units, then *k* = [IIT 1967; Kurukshetra CEE 1998]

(a) 6, 7 (b) 31/9, 9 (c) 7, 31/9 (d) 7, 9

98. The area of a triangle is 5. If two of its vertices are (2, 1), (3, –2) and the third vertex lies on the line $y = x + 3$, then the third vertex is

[IIT 1978; UPSEAT 1999]

(a) $\left(-\frac{7}{2}, -\frac{13}{2}\right)$ (b) $\left(-\frac{7}{2}, \frac{13}{2}\right)$ (c) $\left(\frac{7}{2}, -\frac{13}{2}\right)$ (d) $\left(\frac{7}{2}, \frac{13}{2}\right)$

99. The area of the triangle formed by the lines $7x - 2y + 10 = 0, 7x + 2y - 10 = 0$ and $y + 2 = 0$ is [IIT 1977]

(a) 8 sq. units (b) 12 sq. units (c) 14 sq. units (d) None of these

100. Area of the triangle with vertices (*a, b*), (x_1, y_1) and (x_2, y_2) where a, x_1, x_2 are in G.P. with common ratio '*r*' and b, y_1, y_2 are in G.P. with common ratio '*s*' is

(a) $ab(r-1)(s-1)(s-r)$ (b) $\frac{1}{2}ab(r+1)(s+1)(s-r)$ (c) $\frac{1}{2}ab(r-1)(s-1)(s-r)$ (d) $ab(r+1)(s+1)(r-s)$

101. If the area of the triangle whose vertices are $(b,c),(c,a)$ and (a,b) is Δ, then the area of triangle whose vertices are $(ac-b^2, ab-c^2)$ $(ba-c^2, bc-a^2)$ and $(cb-a^2, ca-b^2)$ is

(a) Δ^2 (b) $(a+b+c)^2\Delta$ (c) $a\Delta + b\Delta^2$ (d) None of these

102. $P(2,1), Q(4,-1), R(3,2)$ are the vertices of a triangle and if through *P* and *R* lines parallel to opposite sides are drawn to intersect in *S*, then the area of *PQRS* is

(a) 6 (b) 4 (c) 8 (d) 12

103. An equilateral triangle has each side equal to *a*. If the coordinates of its vertices are $(x_1, y_1);(x_2, y_2);(x_3, y_3)$, then the square of the determinant $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ equals

(a) $3a^4$ (b) $\frac{3a^4}{4}$ (c) $4a^4$ (d) None of these

104. Area of a $\Delta ABC = 20$ units and its vertices *A* and *B* are (–5, 0) and (3, 0) respectively. If its vertex *C* lies on the line $x - y = 2$, then *C* is **[IIT 1990]**

(a) (3, 5) (b) (– 3, – 5) (c) (– 5, 7) (d) None of these

105. Point *P* divides the line segment joining $A(-5,1)$ and $B(3,5)$ internally in the ratio $\lambda : 1$. If $Q \equiv (1,5), R \equiv (7,2)$ and area of $\Delta PQR = 2,$ then λ equals **[Kurukshetra CEE 1998]**

(a) 23 (b) 31/9 (c) 29/5 (d) None of these

Collinearity

Basic Level

106. Three points $(p+1,1),(2p+1,3)$ and $(2p+2,2p)$ are collinear if *p* = **[MP PET 1986]**

(a) – 1 (b) 1 (c) 2 (d) 0

107. If the points (*a*, 0), (0, *b*) and (1, 1) are collinear, then

(a) $\frac{1}{a^2}+\frac{1}{b^2}=1$ (b) $\frac{1}{a^2}-\frac{1}{b^2}=1$ (c) $\frac{1}{a}+\frac{1}{b}=1$ (d) $\frac{1}{a}-\frac{1}{b}=1$

108. If the points (*a, b*), (a', b') and $(a-a', b-b')$ are collinear, then **[Rajasthan PET 1999]**

(a) $ab' = a'b$ (b) $ab = a'b'$ (c) $aa' = bb'$ (d) $a^2 + b^2 = 1$

109. If the points $(k, 2-2k), (1-k, 2k)$ and $(-k-4, 6-2k)$ be collinear, then the possible values of *k* are

[AMU 1978; Rajasthan PET 1997]

(a) $\frac{1}{2}, -1$ (b) $1, -\frac{1}{2}$ (c) $1, -2$ (d) $2, -1$

110. If the points (–5, 1), (*p*, 5) and (10, 7) are collinear, then the value of *p* will be [MP PET 1984]

(a) 5 (b) 3 (c) 4 (d) 7

111. If the points $(-2, -5), (2, -2), (8, a)$ are collinear, then the value of *a* is [MP PET 2002]

(a) $-\frac{5}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

112. If the points (5, 5), (10, *K*) and (–5, 1) are collinear, then *K* = [MP PET 1994, 1999; Rajasthan PET 2003]

(a) 3 (b) 5 (c) 7 (d) 9

113. The points $(-a, -b), (a, b), (a^2, ab)$ are

(a) Vertices of an equilateral triangle (b) Vertices of a right angled triangle

(c) Vertices of an isosceles triangle (d) Collinear

114. The points $(3a, 0), (0, 3b)$ and $(a, 2b)$ are [MP PET 1982]

(a) Vertices of an equilateral triangle (b) Vertices of an isosceles triangle

(c) Vertices of a right angled isosceles triangle (d) Collinear

115. The points (*a*, *b*), (*c*, *d*) and $\left(\frac{kc+la}{k+l}, \frac{kd+lb}{k+l}\right)$ are

(a) Vertices of an equilateral triangle (b) Vertices of an isosceles triangle

(c) Vertices of a right angled triangle (d) Collinear

Advance Level

116. *A*, *B*, *C* are the points (*a*, *p*), (*b*, *q*) and (*c*, *r*) respectively such that *a*, *b*, *c* are in A.P. and *p*, *q*, *r* in G.P. If the points are collinear, then

(a) $p = q = r$ (b) $p^2 = q$ (c) $q^2 = r$ (d) $r^2 = p$

117. *A*, *B*, *C* are three collinear points such that *AB* = 2.5 and the co-ordinates of *A* and *C* are respectively (3, 4) and (11, 10), then the co-ordinates of the point *B* are

(a) $\left(5, \frac{11}{2}\right)$ (b) $\left(5, \frac{5}{2}\right)$ (c) $\left(1, \frac{11}{2}\right)$ (d) $\left(1, \frac{5}{2}\right)$

118. The points $(x, 2x), (2y, y)$ and $(3, 3)$ are collinear

(a) For all values of (x, y) (b) 2 is A.M. of *x*, *y* (c) 2 is G.M. of *x*, *y* (d) 2 is H.M. of *x*, *y*

119. If t_1, t_2 and t_3 are distinct, the points $(t_1, 2at_1 + at_1^3), (t_2, 2at_2 + at_2^3)$ and $(t_3, 2at_3 + at_3^3)$ are collinear if

(a) $t_1t_2t_3 = -1$ (b) $t_1 + t_2 + t_3 = t_1t_2t_3$ (c) $t_1 + t_2 + t_3 = 0$ (d) $t_1 + t_2 + t_3 = -1$

120. The points $(-a, -b), (0, 0), (a, b)$ and (a^2, ab) are [IIT 1979; Kurukshetra CEE 1993; Jamia Millia Entrance Exam. 2001]

(a) Collinear (b) Vertices of a rectangle (c) Vertices of a parallelogram (d) None of these

Transformation of Axes

Basic Level

121. The new coordinates of a point (4, 5), when the origin is shifted to the point (1, –2) are [MNR 1988; IIT 1989; UPSEAT 2000]

(a) (5, 3) (b) (3, 5) (c) (3, 7) (d) None of these

122. The co-ordinate axes are rotated through an angle $135°$. If the co-ordinates of a point *P* in the new system are known to be $(4, -3)$, then the co-ordinates of *P* in the original system are [EAMCET 2003]

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(\frac{1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$ (c) $\left(\frac{-1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$ (d) $\left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

123. If the axes be rotated through an angle of 60^o in the clockwise direction, the point (4, 2) in the new system was formally

(a) $(2-\sqrt{3}, 2\sqrt{3}+1)$ (b) $(2+\sqrt{3}, -2\sqrt{3}+1)$ (c) $(2-\sqrt{3}, 1-2\sqrt{3})$ (d) None of these

Advance Level

124. Without changing the direction of coordinate axes origin is transferred to (*h, k*), so that the linear (one degree) terms in the equation $x^2 + y^2 - 4x + 6y - 7 = 0$ are eliminated. Then the point (*h, k*) is

(a) (3, 2) (b) (– 3, 2) (c) (2, – 3) (d) None of these

125. The point (4, 1) undergoes the following two successive transformations

(i) reflection about the line $y = x$

(ii) rotation through a distance 2 units along the positive *x*-axis

Then the final coordinates of the point are

(a) (4, 3) (b) (3, 4) (c) (1, 4) (d) (7/2, 7/2)

Locus

Basic Level

126. Two points *A* and *B* have coordinates (1, 0) and (–1, 0) respectively and *Q* is a point which satisfies the relation $AQ - BQ = \pm 1$. The locus of *Q* is **[MP PET 1986]**

(a) $12x^2 + 4y^2 = 3$ (b) $12x^2 - 4y^2 = 3$ (c) $12x^2 - 4y^2 + 3 = 0$ (d) $12x^2 + 4y^2 + 3 = 0$

127. A point moves such that the sum of its distances from two fixed points (*ae*, 0) and (–*ae*, 0) is always 2*a*. Then equation of its locus is

[MNR 1981]

(a) $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{a^2(1-e^2)} = 1$ (c) $\frac{x^2}{a^2(1-e^2)} + \frac{y^2}{a^2} = 1$ (d) None of these

128. The locus of a point whose distance from the point $(-g, -f)$ is always '*a*', will be (where $k = g^2 + f^2 - a^2$)

(a) $x^2 + y^2 + 2gx + 2fy + k = 0$ (b) $x^2 - y^2 + 2gx + 2fy + k = 0$

(c) $x^2 + y^2 + 2xy + 2gx + 2fy + k = 0$ (d) None of these

129. The coordinates of the points A and B are $(a, 0)$ and $(-a, 0)$ respectively. If a point *P* moves so that $PA^2 - PB^2 = 2k^2$, when *k* is a constant, then the equation to the locus of the point *P* is

(a) $2ax - k^2 = 0$ (b) $2ax + k^2 = 0$ (c) $2ay - k^2 = 0$ (d) $2ay + k^2 = 0$

130. If the distance of any point *P* from the points $A(a+b, a-b)$ and $B(a-b, a+b)$ are equal, then the locus of *P* is

[Karnataka CET 2003]

(a) $x - y = 0$ (b) $ax + by = 0$ (c) $bx - ay = 0$ (d) $x + y = 0$

131. The locus of a point whose difference of distance from points (3, 0) and (–3, 0) is 4, is **[MP PET 2002]**

(a) $\frac{x^2}{4} - \frac{y^2}{5} = 1$ (b) $\frac{x^2}{5} - \frac{y^2}{4} = 1$ (c) $\frac{x^2}{2} - \frac{y^2}{3} = 1$ (d) $\frac{x^2}{3} - \frac{y^2}{2} = 1$

132. If *A* and *B* are two fixed points in a plane and $PA - PB$ = constant, then the locus of *P* is

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

133. If *A* and *B* are two points in a plane, so that *PA* + *PB* = constant, then the locus of *P* is **[MNR 1991]**

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

134. The equation of the locus of all points equidistant from the point (4, 2) and the *x*-axis, is **[Kurukshetra CEE 1993]**

(a) $x^2 + 8x + 4y - 20 = 0$ (b) $x^2 - 8x - 4y + 20 = 0$ (c) $y^2 - 4y - 8x + 20 = 0$ (d) None of these

135. The locus of a point which moves so that it is always equidistant from the points $A(a, 0)$ and $B(-a, 0)$ is

(a) A circle (b) Perpendicular bisector of the line segment *AB*

(c) A line parallel to *x*-axis (d) None of these

136. The locus of a point which moves so that its distance from *x*-axis is double of its distance from *y*-axis is **[AMU 1978; MP PET 1984]**

(a) $x = 2y$ (b) $y = 2x$ (c) $x = 5y + 1$ (d) $y = 2x + 3$

137. *O* is the origin and *A* is the point (3, 4). If a point *P* moves so that the line segment *OP* is always parallel to the line segment *OA*, then the equation to the locus of *P* is

(a) $4x-3y=0$ (b) $4x+3y=0$ (c) $3x+4y=0$ (d) $3x-4y=0$

138. If A and B are two fixed points in a plane and P is another variable point such that $PA^2+PB^2=$ constant, then the locus of the point P is

(a) Hyperbola (b) Circle (c) Parabola (d) Ellipse

139. If sum of distances of a point from the origin and line $x=2$ is 4, then its locus is [Rajasthan PET 1997]

(a) $x^2-12y=36$ (b) $y^2+12x=36$ (c) $y^2-12x=36$ (d) $x^2+12y=36$

140. The coordinates of the points A and B are $(ak, 0)$ and $\left(\frac{a}{k}, 0\right)$, $(k=\pm 1)$. If a point P moves so that $PA=k\,PB$, then the equation to the locus of P is

(a) $k^2(x^2+y^2)-a^2=0$ (b) $x^2+y^2-k^2a^2=0$ (c) $x^2+y^2+a^2=0$ (d) $x^2+y^2-a^2=0$

141. The equation of the locus of a point whose distance from (a, 0) is equal to its distance from y-axis, is

(a) $y^2-2ax=a^2$ (b) $y^2-2ax+a^2=0$ (c) $y^2+2ax+a^2=0$ (d) $y^2+2ax=a^2$

142. The locus of the point of intersection of lines $x\cos\alpha+y\sin\alpha=a$ and $x\sin\alpha-y\cos\alpha=b$ is (α is a variable)

(a) $2(x^2+y^2)=a^2+b^2$ (b) $x^2-y^2=a^2-b^2$ (c) $x^2+y^2=a^2+b^2$ (d) None of these

143. Two points A and B move on the x- axis and the y-axis respectively such that the distance between the two points is always the same. The locus of the middle point of AB is

(a) A straight line (b) A circle (c) A parabola (d) An ellipse

Advance Level

144. The locus of P such that area of $\Delta PAB=12\,sq.$ units, where $A(2,3)$ and $B(-4,5)$ is [EAMCET 1989]

(a) $(x+3y-1)\ (x+3y-23)=0$ (b) $(x+3y+1)(x+3y-23)=0$

(c) $(3x+y-1)(3x+y-23)=0$ (d) $(3x+y+1)(3x+y+23)=0$

145. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0), where t is a parameter is [AIEEE 2003]

(a) $(3x-1)^2+(3y)^2=a^2-b^2$ (b) $(3x-1)^2+(3y)^2=a^2+b^2$

(c) $(3x+1)^2+(3y)^2=a^2+b^2$ (d) $(3x+1)^2+(3y)^2=a^2-b^2$

146. If A is (2, 5), B is (4, –11) and C lies on $9x+7y+4=0$, then the locus of the centroid of the ΔABC is a straight line parallel to the straight line [MP PET 1986]

(a) $7x-9y+4=0$ (b) $9x-7y-4=0$ (c) $9x+7y+4=0$ (d) $7x+9y+4=0$

147. Two fixed points are $A(a,0)$ and $B(-a,0)$. If $\angle A-\angle B=\theta$, then the locus of point C of triangle ABC will be [Roorkee 1982]

(a) $x^2+y^2+2xy\tan\theta=a^2$ (b) $x^2-y^2+2xy\tan\theta=a^2$ (c) $x^2+y^2+2xy\cot\theta=a^2$ (d) $x^2-y^2+2xy\cot\theta=a^2$

148. If $A(-a,0)$ and $B(a,0)$ are two fixed points, then the locus of the point on which the line AB subtends the right angle, is

(a) $x^2+y^2=2a^2$ (b) $x^2-y^2=a^2$ (c) $x^2+y^2+a^2=0$ (d) $x^2+y^2=a^2$

149. The coordinates of the points *O*, *A* and *B* are (0, 0), (0, 4) and (6, 0) respectively. If a point *P* moves such that the area of ΔPOA is always twice the area of ΔPOB, then the equation to both parts of the locus of *P* is [IIT 1964]

(a) $(x-3y)(x+3y)=0$ (b) $(x-3y)(x+y)=0$ (c) $(3x-y)(3x+y)=0$ (d) None of these

150. A stick of length *l* rests against the floor and a wall of a room. If the stick begins to slide on the floor, then the locus of its middle point is

(a) A straight line (b) Circle (c) Parabola (d) Ellipse

151. Given the points $A(0,4)$ and $B(0,-4)$. Then the equation of the locus of the point $P(x,y)$ such that $|AP-BP|=6$, is

[IIT 1983; MP PET 1994]

(a) $\frac{x^2}{7}+\frac{y^2}{9}=1$ (b) $\frac{x^2}{9}+\frac{y^2}{7}=1$ (c) $\frac{x^2}{7}-\frac{y^2}{9}=1$ (d) $\frac{y^2}{9}-\frac{x^2}{7}=1$

152. If $P=(1,0), Q=(-1,0)$ and $R=(2,0)$ are three given points, then the locus of a point *S* satisfying the relation $SQ^2+SR^2=2SP^2$ is

[IIT 1988]

(a) A straight line parallel to *x*-axis (b) A circle through origin

(c) A circle with centre at the origin (d) A straight line parallel to *y*-axis

153. The locus of a point which moves in such a way that its distance from (0, 0) is three times its distance from the *x*-axis, as given by

[MP PET 1993]

(a) $x^2-8y^2=0$ (b) $x^2+8y^2=0$ (c) $4x^2-y^2=0$ (d) $x^2-4y^2=0$

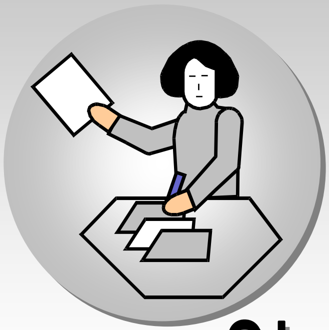
154. $A(a,0)$ and $B(-a,0)$ are two fixed points of triangle ABC. The vertex *C* moves in such a way that $\cot A+\cot B=\lambda$, where λ is a constant. Then the locus of the point *C* is [MP PET 1981]

(a) $y\lambda=2a$ (b) $ya=2\lambda$ (c) $y=\lambda a$ (d) None of these

155. A line of fixed length $(a+b)$ moves so that its ends are always on two fixed perpendicular lines. The locus of the point which divides this line into portions of lengths *a* and *b* is

(a) A circle (b) An ellipse (c) A hyperbola (d) None of these

***



Answer Sheet

Rectangular Cartesian Co-ordinates

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	c	c	c	d	a	b	d	b	d	b	b	a	d	c	c	b	c	c	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	c	c	b	a	b	b	b	a	b	b	c	d	b	c	d	d	b	c	b
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	c	b	b	c	c	a	c	a	c	c	d	c	a	a	a	c	c	a	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
c	a	c	a	a,c	b	c	b	a	d	a	b	b	b	a	c	a	a	a	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
a,c,d	b	c	d	a	b	d	b	a	a	d	d	a	a	a	a	c	d	c	c
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
b	b	b	b	a	c	c	a	a	a	b	c	d	d	d	a	a	d	c	a
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
c	d	b	c	b	b	a	a	b	a	a	a	d	b	b	b	a	b	b	d

141	142	143	144	145	146	147	148	149	150	151	152	153	154	155
b	c	b	b	b	c	d	d	a	b	d	d	a	a	b