

CHAPTER 17

Triangles

17-1. Angles of a Triangle

Angle Sum Theorem

The angle sum of a triangle is 180° .

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Exterior Angle Theorem

The measure of an **exterior angle** of a triangle is equal to the sum of the measures of the two remote interior angles.

$$m\angle BCD = m\angle A + m\angle B$$

Isosceles Triangle Theorem

If two sides of a triangle are congruent, the angles opposite of those sides are congruent.

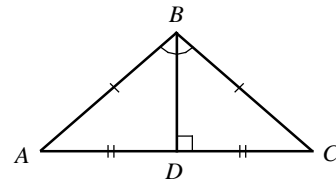
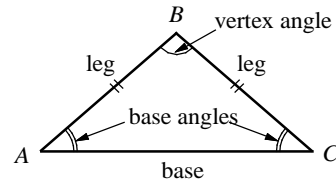
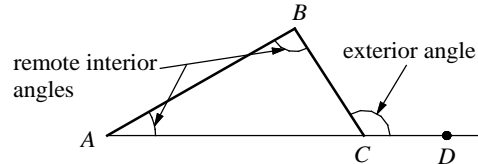
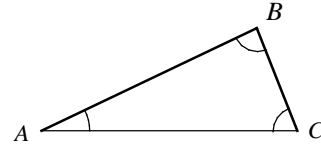
If $AB = BC$, then $m\angle C = m\angle A$.

The converse is also true.

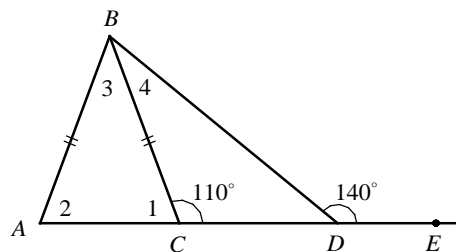
Isosceles Triangle Theorem - Corollary

If a line bisects the vertex angle of an isosceles triangle, the line is the perpendicular bisector of the base.

If $AB = BC$ and $m\angle ABD = m\angle CBD$,
then $\overline{BD} \perp \overline{AC}$ and $AD = CD$.



Example 1 □ a. In $\triangle ABC$ shown below, $AB = BC$, $m\angle BCD = 110$ and $m\angle BDE = 140$.
Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$.

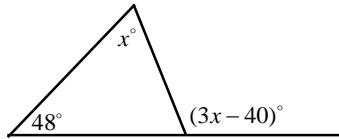


Solution □ $m\angle 1 + m\angle BCD = 180$
 $m\angle 1 + 110 = 180$
 $m\angle 1 = 180 - 110 = 70$
 $m\angle 2 = m\angle 1 = 70$
 $m\angle 3 + m\angle 2 = 110$
 $m\angle 3 + 70 = 110$
 $m\angle 3 = 40$
 $m\angle 4 + 110 = 140$
 $m\angle 4 = 30$

Straight angle equals 180° .
 Substitution
 Subtraction
 Isosceles Triangle Theorem
 Exterior Angle Theorem
 Substitution
 Subtraction
 Exterior Angle Theorem
 Subtraction

Exercises - Angles of a Triangle

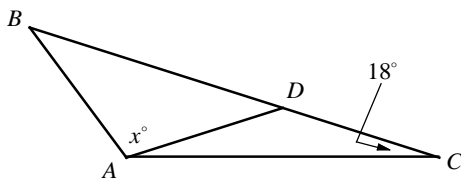
1



In the triangle above, what is the value of x ?

- A) 44
- B) 48
- C) 52
- D) 56

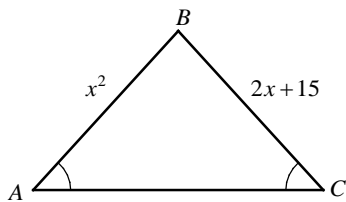
2



In $\triangle ABC$ above, if $AB = AD = DC$, what is the value of x ?

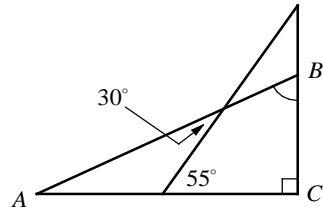
- A) 92
- B) 96
- C) 102
- D) 108

3



In $\triangle ABC$ above, $m\angle A = m\angle C$. If $x > 0$, what is the value of x ?

4

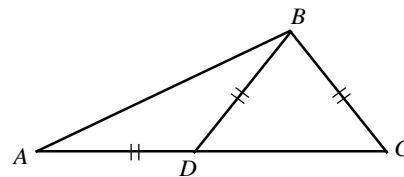


Note: Figure not drawn to scale.

In the figure above, $AC \perp BC$. What is the measure of $\angle ABC$?

- A) 50
- B) 55
- C) 60
- D) 65

5



In the figure above, $AD = BD = BC$. If $m\angle A = 26$, what is the measure of $m\angle DBC$?

- A) 68
- B) 72
- C) 76
- D) 82

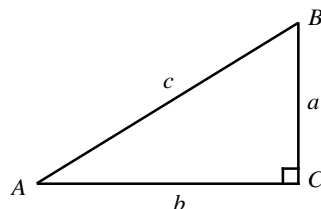
17-2. Pythagorean Theorem and Special Right Triangles

A triangle with a right angle is called a **right triangle**.

The side opposite to the right angle is called the **hypotenuse** and the other two sides are called **legs**.

In a right triangle the acute angles are complementary.

In triangle shown at right, $m\angle A + m\angle B = 90^\circ$.



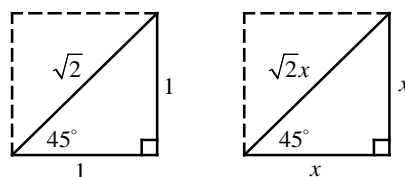
The **Pythagorean theorem** states that in a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

In right triangle ABC at the right, $a^2 + b^2 = c^2$.

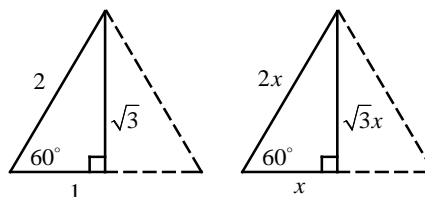
The converse is also true.

The Pythagorean theorem can be used to determine the ratios of the lengths of the sides of two special right triangles.

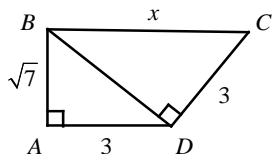
In a 45° - 45° - 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg. An isosceles right triangle is also called a 45° - 45° - 90° triangle.



In a 30° - 60° - 90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Example 1 □ In the figure below, find the value of x .



Solution □ $BD^2 = (\sqrt{7})^2 + 3^2 = 16$

$$x^2 = BD^2 + 3^2$$

$$x^2 = 16 + 9 = 25$$

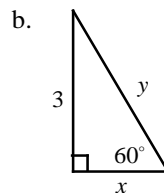
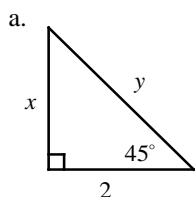
$$x = \sqrt{25} = 5$$

Pythagorean Theorem

Pythagorean Theorem

Substitution

Example 2 □ In the figures below, find the values of x and y .



Solution □ a. Since a 45° - 45° - 90° triangle is an isosceles right triangle, $x = 2$.

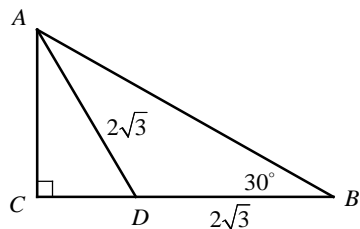
$$\text{In a } 45^\circ\text{-}45^\circ\text{-}90^\circ \text{ triangle, hypotenuse} = \sqrt{2} \cdot \text{leg} \Rightarrow y = 2\sqrt{2}$$

$$\text{b. In a } 30^\circ\text{-}60^\circ\text{-}90^\circ \text{ triangle, longer leg} = \sqrt{3} \cdot \text{shorter leg} \Rightarrow 3 = \sqrt{3}x \Rightarrow x = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\text{hypotenuse} = 2 \cdot \text{shorter leg} \Rightarrow y = 2\sqrt{3}$$

Exercises - Pythagorean Theorem and Special Right Triangles

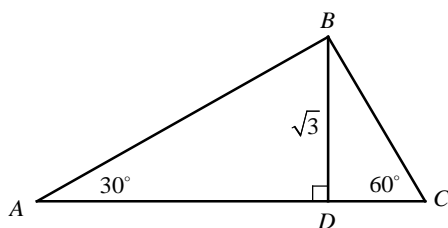
1



In the figure above, if $AD = BD = 2\sqrt{3}$, what is the length of AB ?

- A) $4\sqrt{3}$
- B) $3\sqrt{6}$
- C) 6
- D) $6\sqrt{2}$

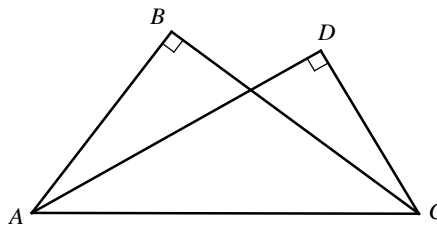
2



In $\triangle ABC$ above, $BD = \sqrt{3}$. What is the perimeter of $\triangle ABC$?

- A) $2\sqrt{2} + 6$
- B) $2\sqrt{3} + 6$
- C) $2\sqrt{6} + 6$
- D) $3\sqrt{2} + 6$

3

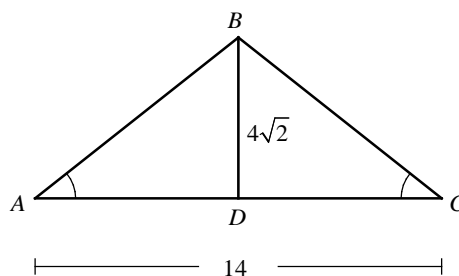


Note: Figure not drawn to scale.

In the figure above, $AB = 6$, $BC = 8$, and $CD = 5$. What is the length of AD ?

- A) $4\sqrt{3}$
- B) $5\sqrt{2}$
- C) $5\sqrt{3}$
- D) $6\sqrt{2}$

4



Note: Figure not drawn to scale.

In the figure above, $\angle A \cong \angle C$ and \overline{BD} bisects \overline{AC} . What is the perimeter of $\triangle ABC$?

- A) 32
- B) 36
- C) $14 + 10\sqrt{2}$
- D) $14 + 12\sqrt{2}$

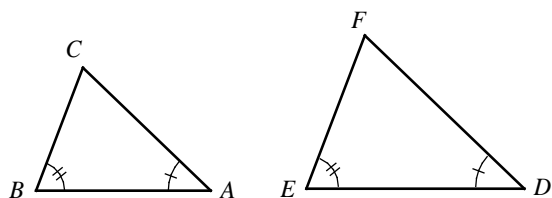
17-3. Similar Triangles and Proportional Parts

AA Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

If two triangles are similar, their corresponding angles are congruent and their corresponding sides are proportional.

If two triangles are similar, their perimeters are proportional to the measures of the corresponding sides.



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Therefore $\triangle ABC \sim \triangle DEF$, and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}.$$

Triangle Proportionality Theorem

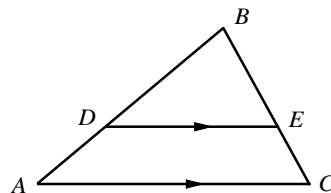
If a line parallel to one side of a triangle intersects the other two sides, it divides those sides proportionally.

In $\triangle ABC$, if $\overline{AC} \parallel \overline{DE}$ then $\triangle ABC \sim \triangle DBE$ by AA Similarity.

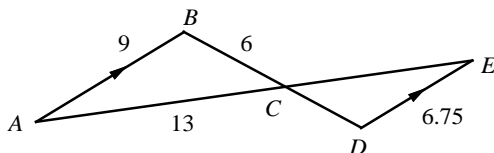
It follows that $\frac{AB}{DB} = \frac{CB}{EB} = \frac{AC}{DE}$. Also $\frac{BD}{DA} = \frac{BE}{EC}$, $\frac{BA}{DA} = \frac{BC}{EC}$,

$$\frac{BD}{DE} = \frac{BA}{AC}, \text{ and } \frac{BE}{DE} = \frac{BC}{AC}.$$

If D and E are the midpoints of \overline{AB} and \overline{BC} , $\overline{AC} \parallel \overline{DE}$ and $DE = \frac{1}{2} AC$.



Example 1 □ In the figure below, $\overline{AB} \parallel \overline{DE}$. Find CD and CE .



Solution □ $m\angle A = m\angle E$

$$m\angle BCA = m\angle DCE$$

$$\triangle ABC \sim \triangle EDC$$

Alternate Interior \angle s are \cong .

Vertical \angle s are \cong .

AA Similarity

$$\text{Therefore } \frac{AB}{ED} = \frac{BC}{DC} \Rightarrow \frac{9}{6.75} = \frac{6}{DC} \Rightarrow DC = 4.5$$

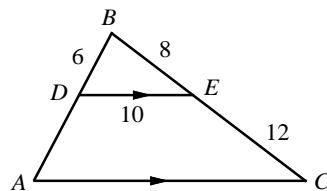
$$\frac{AB}{ED} = \frac{AC}{EC} \Rightarrow \frac{9}{6.75} = \frac{13}{EC} \Rightarrow EC = 9.75$$

Example 2 □ In the figure at right, $\overline{AC} \parallel \overline{DE}$.

Find the length of \overline{DA} and \overline{AC} .

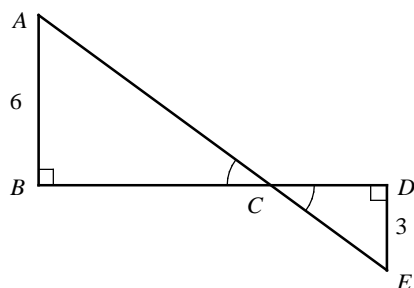
$$\text{Solution } \square \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{6}{DA} = \frac{8}{12} \Rightarrow DA = 9$$

$$\frac{BD}{DE} = \frac{BA}{AC} \Rightarrow \frac{6}{10} = \frac{6+9}{AC} \Rightarrow AC = 25$$



Exercises - Similar Triangles and Proportional Parts

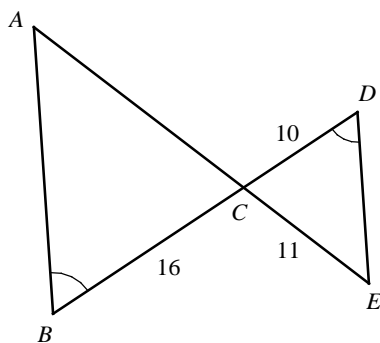
1



In the figure above, if $AB = 6$, $DE = 3$, and $BD = 12$, what is the length of AE ?

- A) 12
- B) $9\sqrt{2}$
- C) $8\sqrt{3}$
- D) 15

2

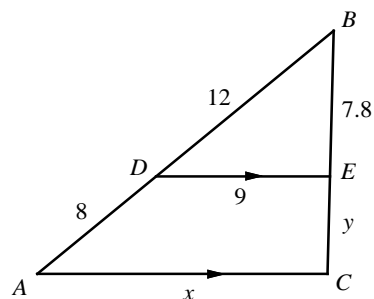


Note: Figure not drawn to scale.

In the figure above, $\angle B \cong \angle D$. If $BC = 16$, $CD = 10$, and $CE = 11$, what is the length of AE ?

- A) 16.8
- B) 17.2
- C) 17.6
- D) 18.4

Questions 3 and 4 refer to the following information.



In the figure above, $\overline{DE} \parallel \overline{AC}$.

3

What is the value of x ?

- A) 12.5
- B) 15
- C) 16.5
- D) 18

4

What is the value of y ?

- A) 5.2
- B) 5.6
- C) 6.0
- D) 6.4

17-4. Area of a Triangle

The **area** A of a triangle equals half the product of a base and the height to that base.

$$A = \frac{1}{2}b \cdot h$$

Area of equilateral triangle with side length of a .

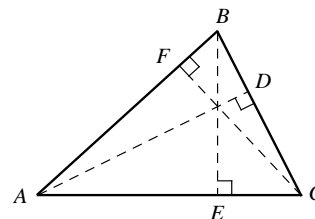
$$A = \frac{1}{2}(a)\left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$$

Any side of a triangle can be used as a base. The height that corresponds to the base is the perpendicular line segment from the opposite vertex to the base. The area of $\triangle ABC$ at the right

can be written in 3 different ways: area of $\triangle ABC = \frac{1}{2}BC \cdot AD = \frac{1}{2}AC \cdot BE = \frac{1}{2}AB \cdot CF$.

The **perimeter** P of a triangle is the sum of the lengths of all three sides.

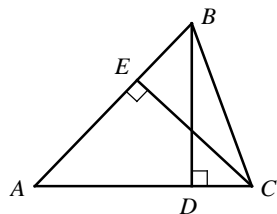
$$P = AB + BC + CA$$



Ratios of Areas of Two Triangles

1. If two triangles are similar with corresponding sides in a ratio of $a : b$, then the ratio of their areas equals $a^2 : b^2$.
2. If two triangles have equal heights, then the ratio of their areas equals the ratio of their bases.
3. If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.

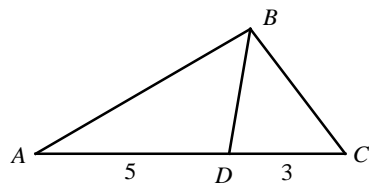
Example 1 □ In the figure below, if $AC = 6$, $BD = 4$, and $AB = 8$, what is the length of CE ?



Solution □ Area of $\triangle ABC = \frac{1}{2}AC \cdot BD = \frac{1}{2}AB \cdot CE$.

$$\Rightarrow \frac{1}{2}(6)(4) = \frac{1}{2}(8)(CE) \Rightarrow CE = 3$$

Example 2 □ In the figure below, $AD = 5$ and $DC = 3$. Find the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$.

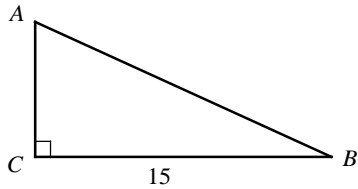


Solution □ The two triangles have the same height, so the ratio of the areas of the two triangles is equal to the ratio of their bases.

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle CBD} = \frac{AD}{CD} = \frac{5}{3}$$

Exercises - Area of a Triangle

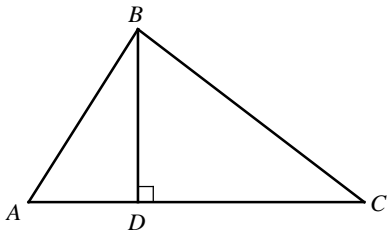
1



In the figure above, the area of right triangle ABC is 60. What is the perimeter of $\triangle ABC$?

- A) 34
- B) 36
- C) 38
- D) 40

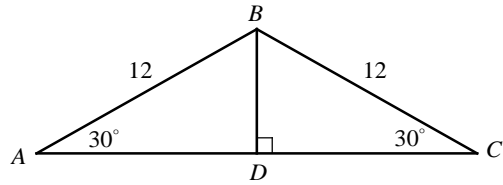
2



In triangle ABC above, if BD was increased by 50 percent and AC was reduced by 50 percent, how would the area of $\triangle ABC$ change?

- A) The area of $\triangle ABC$ would be decreased by 25 percent.
- B) The area of $\triangle ABC$ would be increased by 25 percent.
- C) The area of $\triangle ABC$ would not change.
- D) The area of $\triangle ABC$ would be decreased by 50 percent.

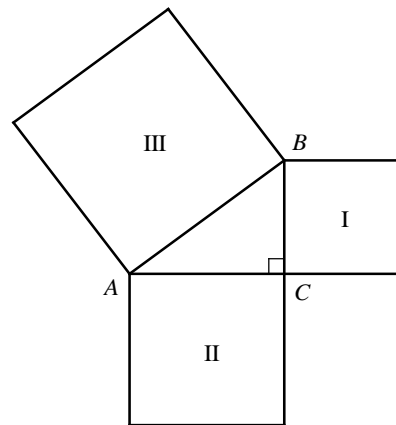
3



In the figure above, what is the area of $\triangle ABC$?

- A) $24\sqrt{3}$
- B) $30\sqrt{3}$
- C) $36\sqrt{3}$
- D) $48\sqrt{3}$

4

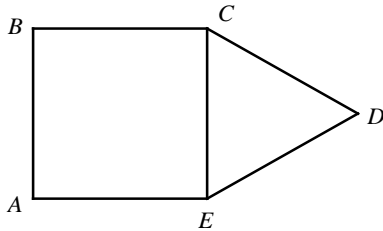


The figure above shows right triangle ABC and three squares. If the area of square region I is 80 square inches and the area of square region II is 150 square inches, which of the following is true about the area of square region III?

- A) Less than 230 square inches.
- B) More than 230 square inches.
- C) Equal to 230 square inches.
- D) It cannot be determined from the information given.

Chapter 17 Practice Test

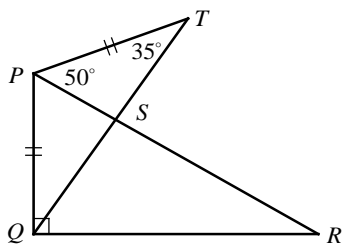
1



In the figure above, CDE is an equilateral triangle and $ABCD$ is a square with an area of $4x^2$. What is the area of triangle CDE in terms of x ?

- A) $\frac{\sqrt{3}}{2}x^2$
- B) $\sqrt{3}x^2$
- C) $\frac{3\sqrt{3}}{2}x^2$
- D) $2\sqrt{3}x^2$

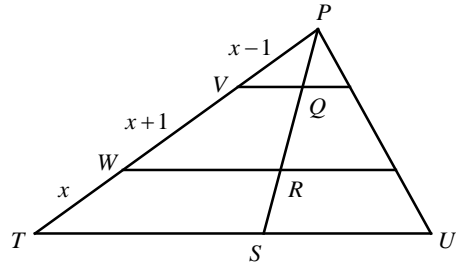
2



In the figure above, $\overline{PQ} \perp \overline{QR}$ and $\overline{PQ} \cong \overline{PT}$. What is the measure of $\angle R$?

- A) 30
- B) 35
- C) 40
- D) 45

3



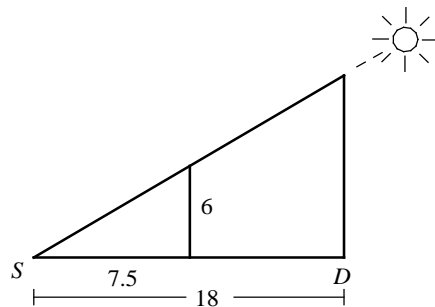
Note: Figure not drawn to scale.

In the figure above, $\overline{VQ} \parallel \overline{WR} \parallel \overline{TS}$.

If $PS = 15$, what is the length of \overline{RS} ?

- A) 4.5
- B) 5
- C) 6
- D) 6.5

4

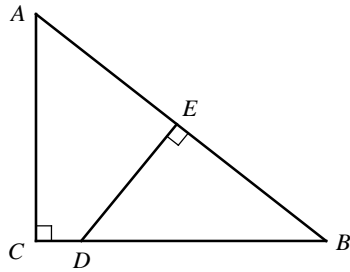


Note: Figure not drawn to scale.

A person 6 feet tall stands so that the ends of his shadow and the shadow of the pole coincide. The length of the person's shadow was measured 7.5 feet and the length of the pole's shadow, SD , was measured 18 feet. How tall is the pole?

- A) 12.8
- B) 13.6
- C) 14.4
- D) 15.2

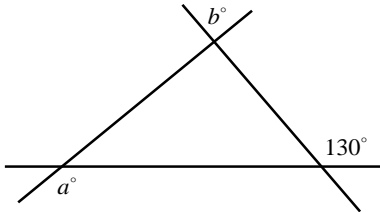
5



In the figure above, $\triangle ABC$ and $\triangle DBE$ are right triangles. If $AC = 12$, $BC = 15$, and $DE = 8$, what is the length of BE ?

- A) 8.5
- B) 9
- C) 9.5
- D) 10

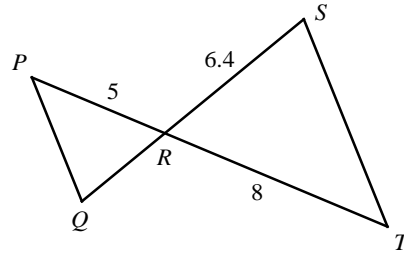
6



In the figure above, what is the value of $a - b$?

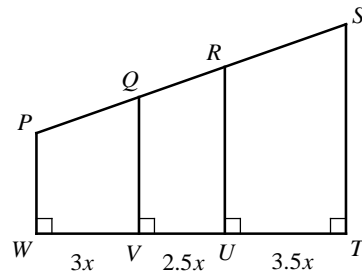
- A) 50
- B) 55
- C) 60
- D) 65

7



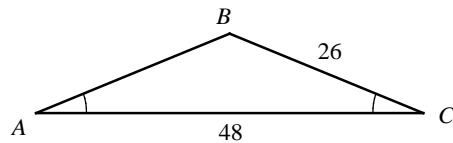
In the figure above, $\overline{PQ} \parallel \overline{ST}$ and segment PT intersects segment QS at R . What is the length of segment QS ?

8



In the figure above, if $PS = 162$, what is the length of segment QR ?

9



In the figure above, what is the area of the isosceles triangle ABC ?

Answer Key

Section 17-1

1. A 2. D 3. 5 4. D 5. C

Section 17-2

1. C 2. B 3. C 4. A

Section 17-3

1. D 2. C 3. B 4. A

Section 17-4

1. D 2. A 3. C 4. C

Chapter 17 Practice Test

1. B 2. A 3. B 4. C 5. D
6. A 7. 10.4 8. 45 9. 240

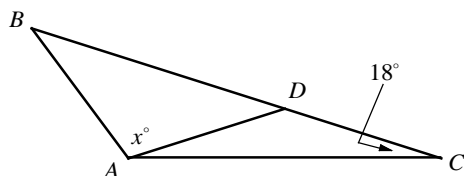
Answers and Explanations

Section 17-1

1. A

$$\begin{aligned} 3x - 40 &= x + 48 && \text{Exterior Angle Theorem} \\ 3x - 40 - x &= x + 48 - x && \text{Subtract } x \text{ from each side.} \\ 2x - 40 &= 48 && \text{Simplify.} \\ 2x &= 88 && \text{Add 40 to each side.} \\ x &= 44 \end{aligned}$$

2. D



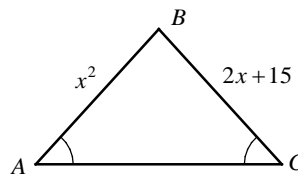
$$\begin{aligned} AD &= DC && \text{Given} \\ m\angle DAC &= m\angle DCA = 18 && \text{Isosceles } \Delta \text{ Theorem} \\ m\angle BDA &&& \text{Exterior } \angle \text{ Theorem} \\ &= m\angle DCA + m\angle DAC \\ m\angle BDA &= 18 + 18 && m\angle DAC = m\angle DCA = 18 \\ m\angle BDA &= 36 && \text{Simplify.} \\ AB &= AD && \text{Given} \\ m\angle DBA &= m\angle BDA = 36 && \text{Isosceles } \Delta \text{ Theorem} \end{aligned}$$

In triangle ABD , the angle sum is 180.

Thus, $x + 36 + 36 = 180$.

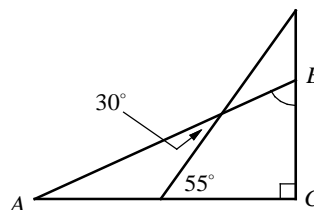
Solving the equation for x gives $x = 108$.

3. 5



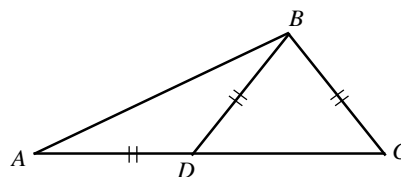
$$\begin{aligned} m\angle A &= m\angle C && \text{Given} \\ AB &= BC && \text{Isosceles } \Delta \text{ Theorem} \\ x^2 &= 2x + 15 && \text{Substitution} \\ x^2 - 2x - 15 &= 0 && \text{Make one side 0.} \\ (x + 3)(x - 5) &= 0 && \text{Factor.} \\ x + 3 = 0 &\text{ or } x - 5 = 0 && \text{Zero Product Property} \\ x = -3 &\text{ or } x = 5 && \\ \text{Since } x > 0, &\text{ the value of } x &\text{ is } 5. \end{aligned}$$

4. D



$$\begin{aligned} m\angle A + 30 &= 55 && \text{Exterior Angle Theorem} \\ m\angle A &= 25 \\ m\angle A + m\angle B &= 90 && \text{The acute } \angle\text{s of a right } \Delta \text{ are complementary.} \\ 25 + m\angle B &= 90 && m\angle A = 25. \\ m\angle B &= 65 \end{aligned}$$

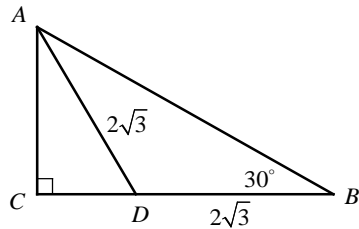
5. C



$$\begin{aligned} AD &= BD && \text{Given} \\ m\angle ABD &= m\angle A && \text{Isosceles } \Delta \text{ Theorem} \\ m\angle A &= 26 && \text{Given} \\ m\angle ABD &= 26 && m\angle A = 26 \\ m\angle BDC &&& \text{Exterior } \angle \text{ Theorem} \\ &= m\angle A + m\angle ABD \\ m\angle BDC &= 26 + 26 = 52 && m\angle A = m\angle ABD = 26 \\ BD &= BC && \text{Given} \\ m\angle C &= m\angle BDC && \text{Isosceles } \Delta \text{ Theorem} \\ m\angle C &= 52 && m\angle BDC = 52 \\ m\angle C + m\angle BDC + m\angle DBC &= 180 && \text{Angle Sum Theorem} \\ 52 + 52 + m\angle DBC &= 180 && m\angle C = m\angle BDC = 52 \\ m\angle DBC &= 76 \end{aligned}$$

Section 17-2

1. C



$AD = BD$ Given
 $m\angle BAD = m\angle B = 30$ Isosceles Δ Theorem
 $m\angle ADC = m\angle BAD + m\angle B$ Exterior \angle Theorem
 $m\angle ADC = 30 + 30 = 60$ $m\angle BAD = m\angle B = 30$
 ΔADC is a 30° - 60° - 90° triangle.

In a 30° - 60° - 90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$$AD = 2CD$$

$$2\sqrt{3} = 2CD$$

$$\sqrt{3} = CD.$$

$$BC = BD + CD = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

Triangle ABC is also a 30° - 60° - 90° triangle.

In a 30° - 60° - 90° triangle, the longer leg is $\sqrt{3}$ times as long as the shorter leg. Therefore,

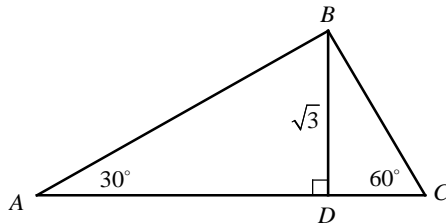
$$BC = \sqrt{3}AC$$

$$3\sqrt{3} = \sqrt{3}AC$$

$$3 = AC.$$

$$AB = 2AC = 2 \times 3 = 6$$

2. B



In the figure above, ΔABD and ΔBCD are 30° - 60° - 90° triangles.

In a 30° - 60° - 90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg. In ΔABD ,

$$AB = 2BD = 2\sqrt{3}$$

$$AD = \sqrt{3}BD = \sqrt{3} \cdot \sqrt{3} = 3.$$

In ΔBCD ,

$$BD = \sqrt{3}CD$$

$$\sqrt{3} = \sqrt{3}CD$$

$$1 = CD$$

$$BC = 2CD = 2 \cdot 1 = 2$$

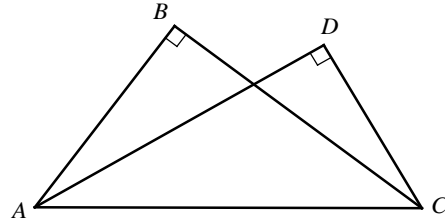
perimeter of ΔABC

$$= AB + BC + AC$$

$$= 2\sqrt{3} + 2 + (3+1)$$

$$= 2\sqrt{3} + 6$$

3. C



Note: Figure not drawn to scale.

$$AC^2 = AB^2 + BC^2 \quad \text{Pythagorean Theorem}$$

$$AC^2 = 6^2 + 8^2 = 100$$

$$AC^2 = AD^2 + CD^2 \quad \text{Pythagorean Theorem}$$

$$100 = AD^2 + 5^2$$

$$AC^2 = 100, CD = 5$$

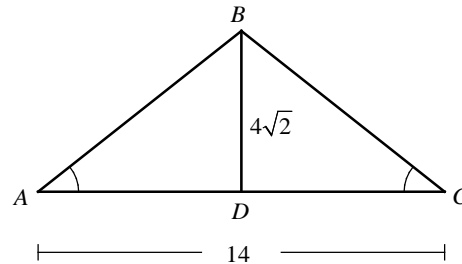
$$100 - 25 = AD^2$$

$$75 = AD^2$$

$$\sqrt{75} = AD$$

$$5\sqrt{3} = AD$$

4. A



Note: Figure not drawn to scale.

$$AD = CD = 7 \quad \text{Definition of segment bisector}$$

$$AB^2 = BD^2 + AD^2 \quad \text{Pythagorean Theorem}$$

$$AB^2 = (4\sqrt{2})^2 + 7^2 \quad \text{Substitution}$$

$$= 32 + 49 = 81$$

$$AB = \sqrt{81} = 9$$

Square root both sides.

$$AB = BC$$

Isosceles Triangle Theorem

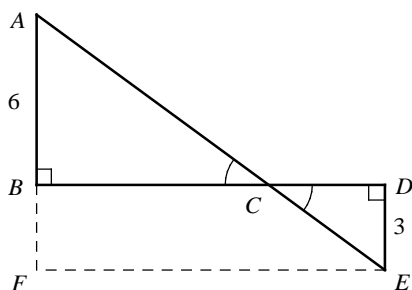
Perimeter of ΔABC

$$= AB + BC + AC$$

$$= 9 + 9 + 14 = 32$$

Section 17-3

1. D



Draw \overline{EF} , which is parallel and congruent to \overline{BD} .
Extend \overline{AB} to point F . Since $\overline{EF} \parallel \overline{BD}$, $\angle F$ is a right angle.

$$BD = EF = 12 \text{ and } DE = BF = 3$$

$$AF = AB + BF = 6 + 3 = 9$$

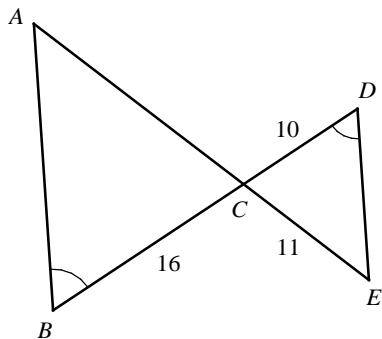
$$AE^2 = AF^2 + EF^2 \quad \text{Pythagorean Theorem}$$

$$= 9^2 + 12^2$$

$$= 225$$

$$AE = \sqrt{225} = 15$$

2. C



Note: Figure not drawn to scale.

$$\angle B \cong \angle D$$

Given

$$\angle ACB \cong \angle ECD$$

Vertical \angle s are \cong .

$$\triangle ACB \sim \triangle ECD$$

AA similarity

If two triangles are similar, their corresponding sides are proportional.

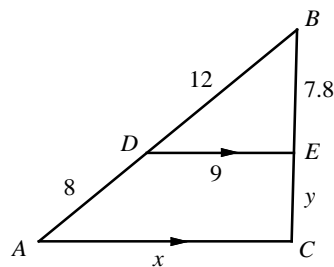
$$\frac{BC}{DC} = \frac{AC}{EC}$$

$$\frac{16}{10} = \frac{AC}{11}$$

$$10AC = 16 \times 11$$

$$AC = 17.6$$

3. B



$$\frac{BD}{DE} = \frac{BA}{AC} \Rightarrow \frac{12}{9} = \frac{20}{x} \Rightarrow 12x = 9 \cdot 20$$

$$\Rightarrow x = 15$$

4. A

$$\frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{12}{8} = \frac{7.8}{y} \Rightarrow 12y = 8 \times 7.8$$

$$\Rightarrow y = 5.2$$

Section 17-4

1. D

$$\text{Area of triangle } ABC = \frac{1}{2} BC \cdot AC$$

$$= \frac{1}{2} (15) AC = 60$$

$$\Rightarrow 7.5AC = 60 \Rightarrow AC = 8$$

$$AB^2 = AC^2 + BC^2 \quad \text{Pythagorean Theorem}$$

$$AB^2 = 8^2 + 15^2$$

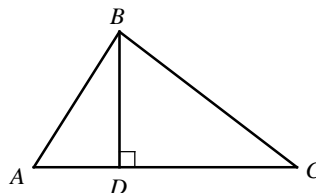
$$= 289$$

$$AB = \sqrt{289} = 17$$

$$\text{Perimeter of } \triangle ABC = AB + BC + AC$$

$$= 17 + 15 + 8 = 40$$

2. A



Let $BD = h$ and let $AC = b$.

If BD was increased by 50 percent, the new BD will be $h + 0.5h$, or $1.5h$.

If AC was reduced by 50 percent, the new AC will be $b - 0.5b$, or $0.5b$.

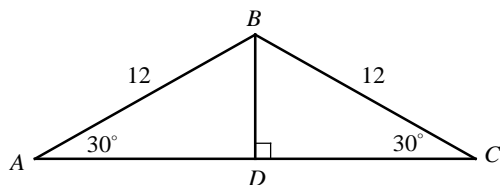
$$\text{The new area of } \triangle ABC = \frac{1}{2} (\text{new } AC) \times (\text{new } BD)$$

$$= \frac{1}{2}(0.5b)(1.5h) = \frac{1}{2}(0.75bh)$$

Because the area of the triangle before change was

$$\frac{1}{2}(bh), \text{ the area has decreased by 25 percent.}$$

3. C



$\triangle ABD$ and $\triangle CBD$ are 30° - 60° - 90° triangles.

In a 30° - 60° - 90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$$AB = 2BD$$

$$12 = 2BD$$

$$6 = BD$$

$$AD = \sqrt{3}BD$$

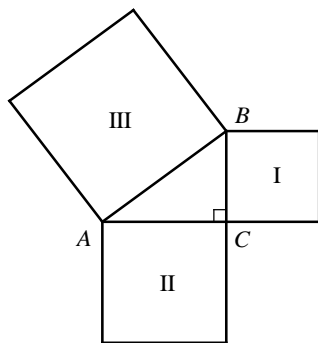
$$AD = \sqrt{3}(6) = 6\sqrt{3}$$

$$AC = 2AD = 12\sqrt{3}$$

$$\text{Area of } \triangle ABC = \frac{1}{2}AC \cdot BD = \frac{1}{2}(12\sqrt{3})(6)$$

$$= 36\sqrt{3}$$

4. C



The area of a square is the square of the length of any side.

The area of square region I = $BC^2 = 80$.

The area of square region II = $AC^2 = 150$.

The area of square region III = AB^2

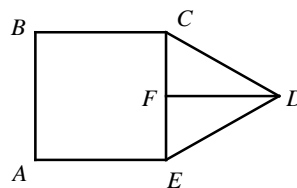
$$AB^2 = BC^2 + AC^2 \quad \text{Pythagorean Theorem}$$

$$= 80 + 150 = 230$$

Therefore, the area of square region III is 230.

Chapter 17 Practice Test

1. B



If the area of square $ABCD$ is $4x^2$, the length of the side of square $ABCD$ is $2x$.

Drawing \overline{DF} , a perpendicular bisector of \overline{CE} , makes two 30° - 60° - 90° triangles, $\triangle CDF$ and $\triangle EDF$.

$$CE = 2x$$

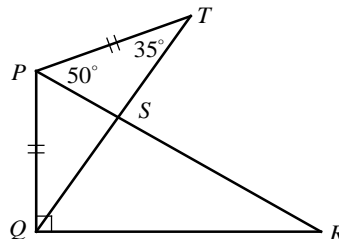
$$CF = \frac{1}{2}CE = \frac{1}{2}(2x) = x$$

$$DF = \sqrt{3}CF = \sqrt{3}x$$

$$\text{Area of } \triangle CDE = \frac{1}{2}CE \cdot DF = \frac{1}{2}(2x)(\sqrt{3}x)$$

$$= \sqrt{3}x^2$$

2. A



$$\overline{PQ} \cong \overline{PT}$$

Given

$$m\angle PQT = m\angle T = 35$$

Isosceles \triangle Theorem

$$m\angle PQT + m\angle T + m\angle QPT = 180$$

Angle Sum Theorem

$$35 + 35 + m\angle QPT = 180$$

Substitution

$$m\angle QPT = 110$$

$$m\angle QPT$$

Angle Addition Postulate

$$= m\angle QPR + m\angle RPT$$

$$110 = m\angle QPR + 50$$

Substitution

$$60 = m\angle QPR$$

$$\overline{PQ} \perp \overline{QR}$$

Given

$$m\angle PQR = 90$$

Definition of Right \angle

$$m\angle PQR + m\angle QPR + m\angle R = 180$$

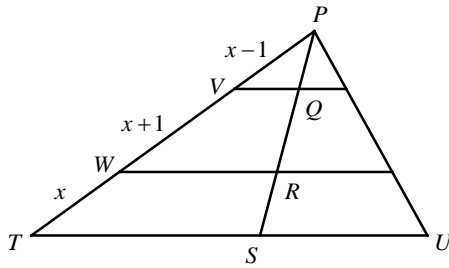
Angle Sum Theorem

$$90 + 60 + m\angle R = 180$$

Substitution

$$m\angle R = 30$$

3. B



Note: Figure not drawn to scale.

$$\text{Since } \overline{VQ} \parallel \overline{WR} \parallel \overline{TS}, \frac{PT}{PS} = \frac{x}{RS}.$$

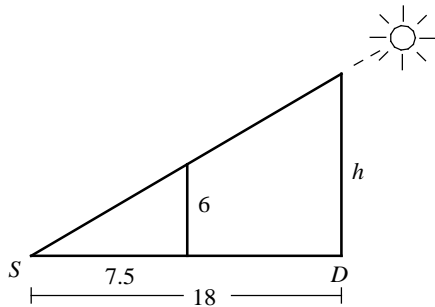
$$\frac{(x-1) + (x+1) + x}{15} = \frac{x}{RS} \quad \text{Substitution}$$

$$\frac{3x}{15} = \frac{x}{RS} \quad \text{Simplify.}$$

$$3x(RS) = 15x \quad \text{Cross Products}$$

$$RS = 5$$

4. C



Note: Figure not drawn to scale.

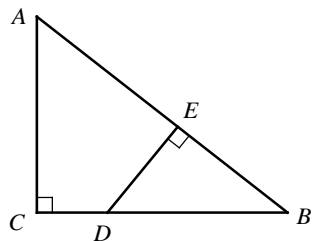
Let h = the length of the pole.

$$\frac{6}{7.5} = \frac{h}{18}$$

$$7.5h = 6 \times 18 \quad \text{Cross Products}$$

$$h = 14.4$$

5. D



$$m\angle C = m\angle BED$$

$$m\angle B = m\angle B$$

$$\triangle ABC \sim \triangle BDE$$

All right \angle s are equal.

Reflexive Property

AA Similarity Postulate

$$\frac{AC}{BC} = \frac{DE}{BE}$$

AA Similarity Postulate

$$\frac{12}{15} = \frac{8}{BE}$$

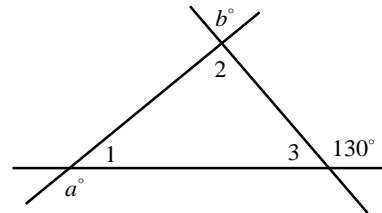
Substitution

$$12BE = 15 \times 8$$

$$BE = 10$$

Cross Products

6. A



$$m\angle 1 + m\angle 2 + m\angle 3 = 180 \quad \text{Angle Sum Theorem}$$

$$a + m\angle 1 = 180$$

Straight \angle measures 180.

$$m\angle 1 = 180 - a$$

$$m\angle 2 = b$$

Vertical \angle s are \cong .

$$130 + m\angle 3 = 180$$

Straight \angle measures 180.

$$m\angle 3 = 50$$

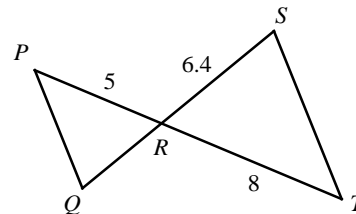
$$180 - a + b + 50 = 180 \quad \text{Substitution}$$

$$230 - a + b = 180$$

$$-a + b = -50$$

$$a - b = 50$$

7. 10.4



$$\overline{PQ} \parallel \overline{ST}$$

Given

$$m\angle P = m\angle T$$

If $\overline{PQ} \parallel \overline{ST}$, alternate

interior \angle s are \cong .

$$m\angle PRQ = m\angle TRS$$

Vertical \angle s are \cong .

$$\triangle PRQ \sim \triangle TRS$$

AA Similarity Postulate

$$\frac{PR}{TR} = \frac{RQ}{RS}$$

AA Similarity Postulate

$$\frac{5}{8} = \frac{RQ}{6.4}$$

Substitution

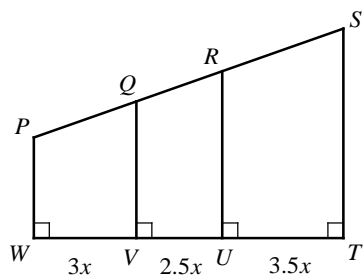
$$8RQ = 5 \times 6.4$$

Cross Products

$$RQ = 4$$

$$QS = SR + RQ = 6.4 + 4 = 10.4$$

8. 45



In the figure above, $\overline{PW} \parallel \overline{QV} \parallel \overline{RU} \parallel \overline{ST}$,
because they are all perpendicular to \overline{TW} .

Therefore, $\frac{PS}{WT} = \frac{QR}{VU}$.

$$\frac{162}{3x + 2.5x + 3.5x} = \frac{QR}{2.5x} \quad \text{Substitution}$$

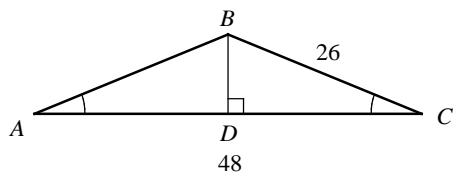
$$\frac{162}{9x} = \frac{QR}{2.5x} \quad \text{Simplify.}$$

$$9x(QR) = 162(2.5x) \quad \text{Cross Products}$$

$$9x(QR) = 405x \quad \text{Simplify.}$$

$$QR = 45$$

9. 240



Draw \overline{BD} perpendicular to \overline{AC} . Since $\triangle ABC$ is
an isosceles triangle, \overline{BD} bisects \overline{AC} .

Therefore, $AD = CD = \frac{1}{2}AC = \frac{1}{2}(48) = 24$.

$$CD^2 + BD^2 = BC^2 \quad \text{Pythagorean Theorem}$$

$$24^2 + BD^2 = 26^2$$

$$576 + BD^2 = 676$$

$$BD^2 = 100$$

$$BD = 10$$

$$\text{Area of } \triangle ABC = \frac{1}{2}(AC)(BD).$$

$$= \frac{1}{2}(48)(10)$$

$$= 240$$