QUADRATIC AND OTHER EQUATIONS

15

We have already seen in the *Back to School* part of this block the key interrelationship between functions, equations and inequalities. In this chapter we are specifically looking at questions based on equations—with an emphasis on quadratic equations. Questions based on equations are an important component of the CAT and XAT exam and hence your ability to formulate and solve equations is a key skill in the development of your thought process for QA.

As you go through with this chapter, focus on understanding the core concepts and also look to create a framework in your mind which would account for the typical processes that are used for solving questions based on equations.

THEORY OF EQUATIONS

Equations in One Variable

An equation is any expression in the form $f(x) = 0$; the type of equation we are talking about depends on the expression that is represented by $f(x)$. The expression $f(x)$ can be linear, quadratic, and cubic or might have a higher power and accordingly the equation can be referred to as linear equations, quadratic equations, cubic equations, etc. Let us look at each of these cases one by one.

Linear Equations

$2x + 4$; we have the expression for $f(x)$ as a linear expression in x. Consequently, the equation $2x + 4 = 0$ would be characterised as a linear equation. This equation has exactly one root (solution) and can be seen by solving $2x + 4 = 0 \rightarrow x = -2$, which is the root of the equation. Note that the root or solution of the equation is the value of 'x' which would make the LHS of the equation equals the

RHS of the equation. In other words, the equation is satisfied when the value of x becomes equal to the root of the equation.

The linear function $f(x)$ when drawn would give us a straight line and this line will be intersecting with the x-axis at the point where the value of x equals the root (solution) of the equation.

Quadratic Equations

An equation of the form: $2x_2 - 5x + 4 = 0$; we have the expression for $f(x)$ as a quadratic expression in x. Consequently, the equation $2x_2 - 5x + 4 = 0$ would be characterised as a quadratic equation. This equation has exactly two roots (solutions) and leads to the following cases with respect to whether these roots are real/imaginary or equal/unequal.

Case 1: Both the roots are real and equal;

Case 2: Both the roots are real and unequal;

Case 3: Both the roots are imaginary.

A detailed discussion of quadratic equations and the analytical formula based approach to identify which of the above three cases prevails, follows later in this chapter.

The graph of a quadratic function is always U-shaped and will just touch the X-axis in the first case above; will cut the X-axis twice in the second case above and will not touch the X-axis at all in the third case above.

Note that the roots or solutions of the equation are the values of 'x' which would make the LHS of the equation equal the RHS of the equation. In other words, the equation is satisfied when the value of x becomes equal to the root of the equation.

Cubic Equations

An equation of the form: $x^3 + 2x^2 - 5x + 4 = 0$ where the expression $f(x)$ is a cubic expression in x. Consequently, the expression will have three roots or solutions.

Depending on whether the roots are real or imaginary, we can have the following two cases in this situation:

Case 1: All three roots are real; (Graph might touch/cut the x-axis once, twice or thrice.)

In this case, depending on the equality or inequality of the roots, we might have the following cases:

Case (i): All three roots are equal; (The graph of the function would intersect the X-axis only once as all the three roots of the equation coincide.)

Case (ii): Two roots are equal and one root is distinct; (In this case the graph cuts the X-axis at one point and touches the X-axis at another point where the other two roots coincide.)

Case (iii): All three roots are distinct from each other. (In this case the graph of the function cuts the x-axis at three distinct points.)

Case 2: One root is real and two roots are imaginary. (Graph would cut the X-axis only once.)

The shapes of the graph that a cubic function can take have already been discussed as a part of *Back to School* write up of this block.

> ***Note:*** For a cubic equation $ax^3 + bx^2 + cx + d = 0$ with roots as l, m and n:
>
> The product of its three roots viz: $l \times m \times n = -d/a$;
>
> The sum of its three roots viz: $l + m + n = -b/a$
>
> The pair wise sum of its roots taken two at a time viz: $lm + ln + mn = c/a$.

THEORY OF QUADRATIC EQUATIONS

An equation of the form

$$ax^2 + bx + c = 0 \qquad (1)$$

where a, b and c are all real and a is not equal to 0, is a quadratic equation. Then,

$D = (b^2 - 4ac)$ is the discriminant of the quadratic equation.

If $D < 0$ (i.e. the discriminant is negative) then the equation has no real roots.

If $D > 0$, (i.e. the discriminant is positive) then the equation has two distinct roots, namely,

$$x_1 = (-b + \sqrt{D})/2a, \quad \text{and} \quad x_2 = (-b - \sqrt{D})/2a$$

and then $$ax^2 + bx + c = a(x - x_1)(x - x_2) \qquad (2)$$

If $D = 0$, then the quadratic equation has equal roots given by

$$x_1 = x_2 = -b/2a$$

and then $$ax^2 + bx + c = a(x - x_1)^2 \qquad (3)$$

To represent the quadratic $ax^2 + bx + c$ in form (2) or (3) is to expand it into linear factors.

Properties of Quadratic Equations and their Roots

1. If D is a *perfect square* then the roots are *rational* and in case, it is not a perfect square then the roots are *irrational*.

2. In the case of imaginary roots ($D < 0$) and if $p + iq$ is one root of the quadratic equation, then the other must be the conjugate $p - iq$ and vice versa (where p and q are real and $i = \sqrt{-1}$).

3. If $p + \sqrt{q}$ is one root of a quadratic equation, then the other must be the conjugate $p - \sqrt{q}$ and vice versa, (where p is rational and $\sqrt{q}$ is a surd).

4. If $a = 1$, $b, c \in I$ and the roots are rational numbers, then the root must be an integer.

Sign of a Quadratic Expression

Let $f(x) = y = ax^2 + bx + c$ where a, b, c are real and $a \pi 0$, then $y = f(x)$ represents a parabola whose axis is parallel to y-axis. For some values of x, $f(x)$ may be positive, negative or zero. Also, if $a > 0$, the parabola opens upwards and for $a < 0$, the parabola opens downwards. This gives the following cases:

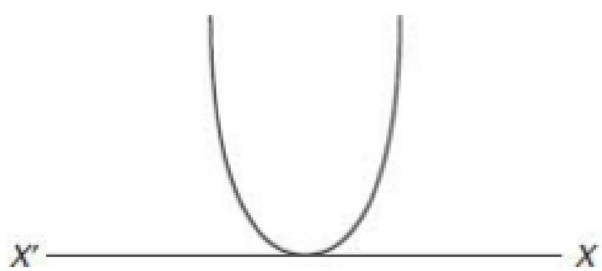
(i) $a > 0$ and $D < 0$ (The roots are imaginary)

The function $f(x)$ will always be positive for all real values of x. So $f(x) > 0\ \forall\ x \in R$. Naturally the graph as shown in the figure does not cut the X-axis.

(ii) When $a > 0$ and $D = 0$ (The roots are real and identical)

$f(x)$ will be positive for all values of x except at the vertex where $f(x) = 0$.

So, $f(x) \geq 0\ \forall\ x \in R$. Naturally, the graph touches the X-axis once.



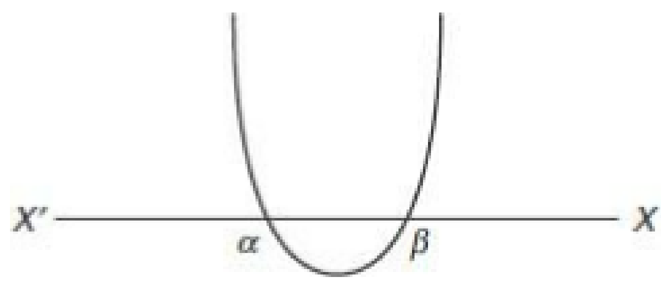
(iii) When $a > 0$ and $D > 0$ (The roots are real and distinct)

Let $f(x) = 0$ have two real roots α and β (α < β) then $f(x)$ will be positive for all real values of x which are lower than α or higher than β; $f(x)$ will be equal to zero when x is equal to either of α or β.

When x lies between a and b, then, $f(x)$ will be negative. Mathematically, this can be represented as

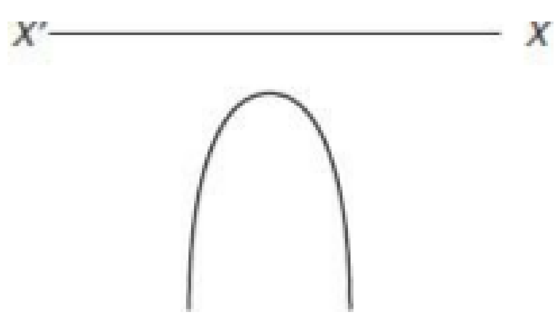
$f(x) > 0 \ \forall \ x \in (-\infty, a) \cup (\mathrm{b}, \infty)$

and $f(x) < 0 \ \forall \ x \in (\alpha, \beta)$ (Naturally, the graph cuts the X-axis twice)



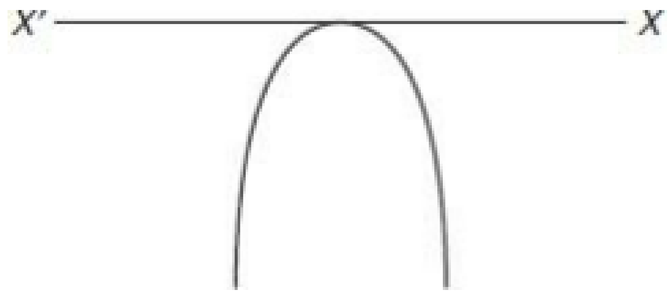
(iv) When $a < 0$ and $D < 0$ (Roots are imaginary)

$f(x)$ is negative for all values of x. Mathematically, we can write $f(x) < 0$ $\forall \ x \in R$. (The graph will not cut or touch the X-axis.)



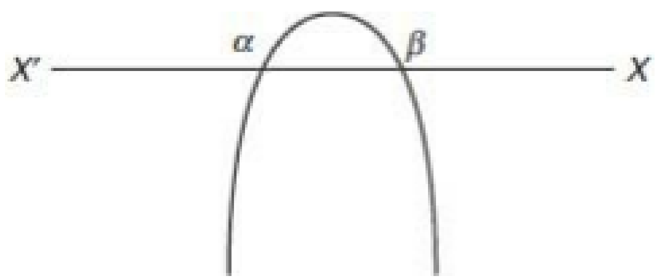
(v) When $a < 0$ and $D = 0$ (Roots are real and equal)

$f(x)$ is negative for all values of x except at the vertex where $f(x) = 0$, i.e. $f(x) \leq 0 \ \forall \ x \in R$ (The graph touches the X-axis once.)



(vi) When $a < 0$ and $D > 0$

Let $f(x) = 0$ have two roots α and β (α < β) then $f(x)$ will be negative for all real values of x that are lower than α or higher than β. $f(x)$ will be equal to zero when x is equal to either of α or β. The graph cuts the X-axis twice.



When x lies between α and β, then $f(x)$ will be positive.

Mathematically, this can be written as

$f(x) < 0\ \forall\ x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0\ \forall\ x \in (\alpha, \beta)$.

Sum of the roots of a quadratic

equation = $-b/a$.

Product of the roots of a quadratic equation = c/a

Equations in More than One Variable

Sometimes, an equation might contain not just one variable but more than one variable. In the context of an aptitude examination like the CAT, multiple variable equations may be limited to two or three variables. Consider this question from a past CAT examination which required the student to understand the interrelationship between the values of x and y.

The question was as follows:

$4x - 17y = 1$ where x and y are integers with $x,y > 0$ and $x, y < 1000$. How many pairs of values of (x, y) exist such that the equation is satisfied?

In order to solve this equation, you need to consider the fact that $4x$ in this equation should be looked upon as a multiple of 4 while $17y$ should be looked upon as a multiple of 17. A scan of values which exist such that a multiple of 4 is 1 more than a multiple of 17 starts from $52 - 51 = 1$, in which case the value of $x = 13$ and $y = 3$. This represents the first set of values for (x, y) that satisfies the equation.

The next pair of values in this case would happen if you increase x from 13 to 30 (increase by 17 which is the coefficient of y); at the same time increase y from 3 to 7 (increase by 4 which is the coefficient of x). The effect this has on $4x$ is to increase it by 68 while $17y$ would also increase by 68 keeping the value of $4x$ exactly 1 more than $17y$. In other words, at $x = 30$ and $y = 7$, the equation would give us $120–119 = 1$. Going further, you should realise that the same increases need to be repeated to again identify the pair of x, y values. ($x = 47$ and $y = 11$ gives us $188–187 = 1$)

Once you realise this, the next part of the visualisation in solving this question has to be on creating the series of values which would give us our desired outcomes every time.

This series can be viewed as

(13, 3); (30, 7); (47, 11); (64, 15)....and the series would basically be two arithmetic progressions running parallel to each other (viz: 13, 30, 47, 64, 81,....) and (3, 7, 11, 15, 19,...) and obviously the number of such pairs would depend on the number of terms in the first of these arithmetic progressions (since that AP would cross the upper limit of 1000 first).

You would need to identify the last term of the series below 1000. The series can be visualised as 13, 30, 47, 64,... 999 and the number of terms in this series is 986/17 + 1 = 59 terms. [Refer to the chapter on Arithmetic Progressions for developing the thinking that helps us do these last two steps.]

WORKED-OUT PROBLEMS

Problem 15.1 Find the value of $\sqrt{6+\sqrt{6+\sqrt{6+...}}}$

(a) 4

(b) 3

(c) 3.5

(d) 2.5

Solution: Let y = $\sqrt{6+\sqrt{6+\sqrt{6+...}}}$

Then, $y = \sqrt{6+y}$ ⇒ or $y^2 - y - 6 = 0$

Or, $(y + 2)(y - 3) = 0 \Rightarrow y = -2, 3$

$y = -2$ is not admissible

Hence $y = 3$

Alternative: Going through options:

option (a): for 4 to be the solution, value of the whole expression should be equal to 16. Looking into the expression, it cannot be equal to 16. So, option (a) cannot be the answer. option (b): for 3 to be the solution, the value of the expression should be 9.

So, the expression is = $\sqrt{6+\sqrt{6+\sqrt{6+...}}}$

But $\sqrt{6} \approx 2.4$, hence = $\sqrt{6+\sqrt{8.4}} \cong \sqrt{8.9+\cdots} \cong 3$

(Since, the remaining part is negligible in value)

Problem 15.2 One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly and got his roots as –6 and 1 respectively. The correct roots are

(a) 3, –2

(b) –3, 2

(c) – 6, –1

(d) 6, –1

Solution: Let a, b be the roots of the equation. Then $a + b = 5$ and $ab = -6$. So, a possible equation is $x^2 - 5x - 6 = 0$. The roots of the equation are 6 and –1.

Problem 15.3 If p and q are the roots of the equation $x^2 + px + q = 0$ then we can conclude that:

(a) $p = 1$

(b) $p = 1$ or 0 or $-\frac{1}{2}$

(c) $p = -2$

(d) $p = -2$ or 0

Solution: Since p and q are roots of the equation $x^2 + px + q = 0$,

$p^2 + p^2 + q = 0$ and $q^2 + pq + q = 0$

$\Rightarrow 2p^2 + q = 0$ and $q(q + p + 1) = 0$

$\Rightarrow 2p^2 + q = 0$ and $q = 0$ or $q = -p - 1$

When we use, $q = 0$ and $2p^2 + q = 0$

We get $p = 0$.

Or when we use $q = -p - 1$ and $2p^2 + q = 0$

We get $2p^2 - p - 1 = 0 \rightarrow$ which gives us

$p = 1$ or $p = -1/2$

Hence, there can be three values for p

i.e. $p = 1, p = 0$, or $p = -\frac{1}{2}$

Problem 15.4 If the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then a, b, c are in

(a) AP

(b) GP

(c) HP

(d) Cannot be determined

Solution: Since roots of the equation a $(b-c)x^2 + b\,(c-a)$x $+ c\,(a-b) = 0$ are equal

$\Rightarrow b^2\,(c-a)^2 - 4ac(b-c)(\text{a}-\text{b}) = 0$

$\Rightarrow b^2(c+a)^2 - 4abc(a+c) + 4a^2c^2 = 0$

$\Rightarrow [\text{b}(c+a) - 2ac]^2 = 0 \Rightarrow \text{b}(c+a) - 2ac = 0$

$\Rightarrow b = (2ac)/(a+c) \Rightarrow a, b, c,$ are in HP

Problem 15.5 The number of roots of the equation

$x - x - \frac{2}{(x-1)} = 1 - \frac{2}{(x-1)}$ is

(a) 0

(b) 1

(c) 2

(d) Infinite

Solution: The equation gives $x = 1$.

But $x = 1$ is not admissible because it gives $x - 1 = 0$ which, in turn, makes the whole expression like this: x–2/0 = 1–2/0. But 2/0 is not defined. Hence, no solution is possible.

Problem 15.6 If the roots of the equation $x^2 - bx + c = 0$ differ by 2, then which of the following is true?

(a) $c^2 = 4(c + 1)$

(b) $b^2 = 4c + 4$

(c) $c^2 = b + 4$

(d) $b^2 = 4(c + 2)$

Solution: Let the roots be α and α + 2.

Then $\alpha + \alpha + 2 = b \Rightarrow \alpha = (b - 2)/2$ (1)

and $\alpha(\alpha + 2) = c \Rightarrow \alpha^2 + 2\alpha = c$ (2)

Putting the value of a from (1) in (2).

$((b - 2)/2)^2 + 2(b - 2)/2) = c$

$\Rightarrow (b^2 + 4 - 4b)/4 + b - 2 = c$

$\Rightarrow b^2 + 4 - 8 = 4c$

$\Rightarrow b^2 = 4c + 4$

∴ Hence, option (b) is the correct answer.

Problem 15.7 What is the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other?

(a) $b^2 = 4ac$

(b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$

(d) $c^2 = 9a - b^2$

Solution:

$\therefore \alpha + 2\alpha = -(b/a)$ and $\alpha \times 2\alpha = c/a$

$\Rightarrow 3\alpha = -(b/a) \Rightarrow \alpha = -b/3a$

and $2\alpha^2 = c/a \Rightarrow 2(-b/3a)^2 = c/a$

$\Rightarrow 2b^2/9a^2 = c/a \Rightarrow 2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

Alternative: Assume any equation having two roots as 2 and 4 or any equation having two roots one of which is twice the other.

When roots are 2 and 4, then equation will be $(x-2)(x-4) = x^2 - 6x + 8 = 0$.

Now, check the options one by one and you will find only (b) as a suitable option.

Problem 15.8 Solve the system of equations

$$\begin{cases} 1/x + 1/y = 3/2, \\ 1/x^2 + 1/y^2 = 5/4 \end{cases}$$

Solution: Let $1/x = u$ and $1/y = v$. We then obtain

$$\begin{cases} u + v = 3/2, \\ u^2 + v^2 = 5/4 \end{cases}$$

From the first equation, we find $v = (3/2) - u$ and substitute this expression into the second equation

$$u^2 + ((3/2) - u)^2 = 5/4 \text{ or } 2u^2 - 3u + 1 = 0$$

when $u_1 = 1$ and $u_2 = 1/2$; consequently, $v_1 = 1/2$ and $v_2 = 1$. Therefore, $x_1 = 1$,

$$y_1 = 2 \text{ and } x_2 = 2, y_2 = 1$$

Answer: (1, 2) and (2, 1).

Problem 15.9 The product of the roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is –1. Then m is

(a) 1

(b) 1/3

(c) –1

(d) –1/3

Solution: We have $(2m - 1)/m = -1 \Rightarrow m = 1/3$

Problem 15.10 If $13x + 17y = 643$, where x and y are natural numbers, what is the value of two times the product of x and y? Assume x, y are both >10.

(a) 744

(b) 844

(c) 924

(d) 884

Solution: The solution of this question depends on your visualisation of the multiples of 13 and 17 which would satisfy this equation. (**Note:** Your reaction to $13x$ should be to look at it as a multiple of 13 while $17y$ should be looked at as a multiple of 17). A scan of multiples of 13 and 17 gives us the solution at 286 + 357 which would mean 13 × 22 and 17 × 21 giving us x as 22 and y as 21. The value of $2xy$ would be 2 × 22 × 21 = 2 × 462 = 924.

Problem 15.11 If $[x]$ denotes the greatest integer $\le x$, then

$$\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\cdots\left[\frac{1}{3}+\frac{198}{99}\right]=\boxed{}$$

(a) 99

(b) 66

(c) 132

(d) 167

Solution: When the value of the sum of the terms inside the function is less than 1, the value of its greatest integer function would be 0. In the above sequence of values, the value inside the bracket would become equal to 1 or more from the value $\left[\frac{1}{3}+\frac{66}{99}\right]$.

Further, this value would remain between 1 and 2 up till the term $\left[\frac{1}{3}+\frac{164}{99}\right]$.

There would be 99 terms each with a value of 1 between 66/99 to 164/99.

Hence, this part of the expression would each yield a value of 1 giving us:

$$\left[\frac{1}{3}+\frac{66}{99}\right]+\left[\frac{1}{3}+\frac{67}{99}\right]+\cdots\left[\frac{1}{3}+\frac{164}{99}\right]=99$$

Further, from $\left[\frac{1}{3}+\frac{165}{99}\right]+\left[\frac{1}{3}+\frac{167}{99}\right]+\cdots\left[\frac{1}{3}+\frac{198}{99}\right]$

= 2 × 34 = 68

This gives us a total value of 99 + 68 = 167 which means that option (*d*) is the correct answer.

LEVEL OF DIFFICULTY (I)

1. Find the maximum value of the expression $\frac{1}{x^2+5x+10}$.

   (a) $\frac{15}{2}$

   (b) 1

   (c) $\frac{4}{15}$

   (d) $\frac{1}{3}$

2. Find the maximum value of the expression $(x2 + 8x + 20)$.

   (a) 4

   (b) 2

   (c) 29

   (d) None of these

3. Find the minimum value of the expression $(p + 1/p)$; $p > 0$.

(a) 1

(b) 0

(c) 2

(d) Depends upon the value of p

4. If the product of roots of the equation $x^2 - 3(2a + 4)x + a^2 + 18a + 81 = 0$ is unity, then a can take the values as

(a) 3, – 6

(b) 10, – 8

(c) –10, –8

(d) –10, – 6

5. For the equation $2^{a+3} = 4^{a+2} - 48$, the value of a will be

(a) $\frac{-3}{2}$

(b) –3

(c) –2

(d) 1

6. The expression $a^2 + ab + b^2$ is _________for $a < 0, b < 0$

(a) $\neq 0$

(b) < 0

(c) > 0

(d) $= 0$

7. If the roots of the equation $x^2 + bx + c = 0$ differ by 2, then which of the following is true?

(a) $a^2c^2 = 4(1 + c)$

(b) $4b + c = 1$

(c) $c^2 = 4 + b$

(d) $b^2 = 4 (c + 1)$

8. If $f(x) = (x + 2)$ and $g(x) = (4x + 5)$, and $h(x)$ is defined as $h(x) = f(x) \cdot g(x)$, then sum of roots of $h(x)$ will be

(a) $\frac{3}{4}$

(b) $\frac{13}{4}$

(c) $\frac{-13}{4}$

(d) $\frac{-3}{4}$

9. If equation $x^2 + bx + 12 = 0$ gives 2 as its one of the roots and $x^2 + bx + q = 0$ gives equal roots then the value of b is

(a) $\frac{49}{4}$

(b) – 8

(c) 4

(d) $\frac{25}{2}$

10. If the roots of the equation $(a^2 + b^2)x^2 – 2(ac + bd)x + (c^2 + d^2) = 0$ are equal then which of the following is true?

(a) $ab = cd$

(b) $ad = bc$

(c) $ad = \sqrt{bc}$

(d) $ab = \sqrt{cd}$

11. For what value of c the quadratic equation $x^2 – (c + 6) x + 2(2c – 1) = 0$ has sum of the roots as half of their product?

(a) 5

(b) – 4

(c) 7

(d) 3

12. Two numbers a and b are such that the quadratic equation $ax^2 + 3x + 2b = 0$ has – 6 as the sum and the product of the roots. Find $a + b$.

(a) 2

(b) –1

(c) 1

(d) –2

13. If a and b are the roots of the quadratic equation $5y^2 – 7y + 1 = 0$. then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(a) $\frac{7}{25}$

(b) –7

(c) $\frac{-7}{25}$

(d) 7

14. Find the value of the expression $(\sqrt{x+(\sqrt{x+(\sqrt{x \cdots +}))}})$.

(a) $\frac{1}{2}\left[2\sqrt{(2x-1)}+1\right]$

(b) $\frac{1}{2}\left[\sqrt{(4x+1)}+1\right]$

(c) $\frac{1}{2}\left[2\sqrt{(2x-1)}-1\right]$

(d) $\frac{1}{2}\left[\sqrt{(4x-1)}-1\right]$

15. If $a = \sqrt{(7+4\sqrt{3})}$, what will be the value of $\left(a+\frac{1}{a}\right)$?

(a) 7

(b) 4

(c) 3

(d) 2

16. If the roots of the equation $(a^2 + b^2) x^2 - 2b(a + c) x + (b^2 + c^2) = 0$ are equal then a, b, c, are in

(a) AP

(b) GP

(c) HP

(d) Cannot be said

17. If a and b are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $a + \frac{1}{\beta}$ and $b + \frac{1}{\alpha}$ is

(a) $abx^2 + b(c + a) x + (c + a)^2 = 0$

(b) $(c + a) x^2 + b(c + a) x + ac = 0$

(c) $cax^2 + b(c + a)x + (c + a)^2 = 0$

(d) $cax^2 + b(c + a)x + c(c + a)^2 = 0$

18. If $x^2 + ax + b$ leaves the same remainder 5 when divided by $x - 1$ or $x + 1$, then the values of a and b are respectively

(a) 0 and 4

(b) 3 and 0

(c) 0 and 3

(d) 4 and 0

19. Find all the values of b for which the equation $x^2 - bx + 1 = 0$ does not possess real roots.

(a) $-1 < b < 1$

(b) $0 < b < 2$

(c) $-2 < b < 2$

(d) $-1.9 < b < 1.9$

Directions for Questions 20 to 24: Read the data given below and solve the questions that follow.

If a and b are roots of the equation $x^2 + x - 7 = 0$, then

20. Find $a^2 + b^2$.

(a) 10

(b) 15

(c) 5

(d) 18

21. Find $a^3 + b^3$.

(a) 22

(b) –22

(c) 44

(d) 36

22. For what values of c in the equation $2x^2 - (c^3 + 8c - 1)x + c^2 - 4c = 0$ the roots of the equation would be opposite in signs?

(a) $c \in (0, 4)$

(b) $c \in (-4, 0)$

(c) $c \in (0, 3)$

(d) $c \in (-4, 4)$

23. The set of real values of x for which the expression $x^2 - 9x + 20$ is negative is represented by

(a) $-4 < x < 4$

(b) $4 < x < 5$

(c) $x < 4$ or $x > 5$

(d) $-4 < x < 5$

24. For what values of k, the expression $x^2 + kx + 9$ becomes positive (given that x is real)?

(a) $k < 6$

(b) $k > 6$

(c) $|k| < 6$

(d) $|k| \leq 6$

25. If $9^{a-2} \div 3^{a+4} = 81^{a-11}$, then find the value of $3^{a-8} + 3^{a-6}$.

(a) 972

(b) 2916

(c) 810

(d) 2268

26. Find the number of solutions of $a^3 + 2^{a+1} = a^4$, given that n is a natural number less than 100.

(a) 0

(b) 1

(c) 2

(d) 3

27. The number of positive integral values of x that satisfies $x^3 - 32x - 5x^2 + 64 \leq 0$ is/are

(a) 4

(b) 5

(c) 6

(d) More than 6

28. Find the positive integral value of x that satisfies the equation $x^3 - 32x - 5x^2 + 64 = 0$.

(a) 5

(b) 6

(c) 7

(d) 8

29. If a, b, c are positive integers, such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{29}{72}$, how many sets of (a, b, c) exist?

(a) 3

(b) 4

(c) 5

(d) 6

30. The variables p, q, r and s are correlated with each other with the following relationship: $s^{0.5}/p = q/r^{2}$. The ranges of values for p, q and r are respectively: $-0.04 \le p \le -0.03, -0.25 \le q \le -0.09, 1 \le r \le 7$

Determine the difference between the maximum and minimum value of s.

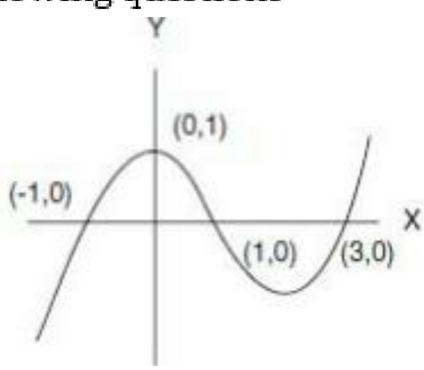
(a) – 0.2

(b) 0.02

(c) 0.1

(d) None of these

Directions for Questions 31 to 34: If graph of $y = f(x)$ is shown in the diagram below, then answer the following questions



31. If $f(x) = 0$ has 'n' real roots then n =

32. Sum of all roots of $f(x) = 0$ is

33. Total number of roots of $f(x) = 2$ is

34. Total number of roots of $f(|x|) = 0$ is

35. If m, n are the roots of $px^2 + qx + r = 0$ and $m + k, n + k$ are the roots of the equation $ax^2 + bx + c = 0$ then k = ?

(a) $\frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$

(b) $\frac{1}{2}\left[\frac{q}{p} - \frac{b}{a}\right]$

(c) $\frac{1}{2}\left[\frac{q}{p} + \frac{b}{a}\right]$

(d) None of these

36. If m, n are the roots of the equation $px^2 + qx + r = 0$ and km, kn are the roots of the equation $ax^2 + bx + c = 0$, then k is

(a) $\sqrt{\frac{cp}{ar}}$

(b) $\sqrt{\frac{cr}{ap}}$

(c) $\sqrt{\frac{ap}{cr}}$

(d) None of these

37. For how many real values of x, is $x^2 = |x|$?

38. For how many different values of p, does the equation $16x^2 + px + 9 = 0$ have equal roots?

39. If m, n are the roots of the equation $x^2 - 6x + 3 = 0$, then the equation whose roots are m^2, n^2 could be

(a) $x^2 - 36x + 9 = 0$

(b) $x^2 - 18x - 6 = 0$

(c) $x^2 - 42x + 9 = 0$

(d) None of these

40. If m, n are the roots of the equation $x^2 + ax + b = 0$ and m, n, a, b all are real numbers then which of the following options can never be a value for (a, b)?

(a) (1, 1)

(b) (3, 2)

(c) (4, 4)

(d) (7, 3)

41. If a and b are roots of the equation $x^2 + ax + b = 0$ then

(a) $a = 1, b = -2$

(b) $a = 0, b = -2$

(c) $a = 1, b = 0$

(d) None of these

42. Find the numbers of real roots of the equation $x^2 + 3|x| + 2 = 0$.

43. The number of real roots of the equation $x^2 + (x + 1)^2 + (x - 2)^2 = 0$

44. The numbers of real solutions of the equation $3^{2x^2+3x+1} = 0$

Directions for Questions 45 and 46: $\boldsymbol{f(x) = -x^2 + x - 4}$, then answer the following questions.

45. If $x \in$ R, then which of the following statements is true about $f(x)$ if $f(x) = 0$ has no real root?

    (a) The graph of $f(x) = -x^2 + x - 4$ opens upward and $f(x) = 0$ has no real root.

    (b) Graph of $f(x) = -x^2 + x - 4$ opens downward and $f(x) = 0$ has two real roots that are equal.

    (c) Graph of $f(x) = -x^2 + x - 4$ opens downward and $f(x) = 0$ has no real root.

    (d) None of these

46. If a, b are two positive integers and both a and b are greater than 1000 then $f(a).f(b)$ is

    (a) > 0

    (b) < 0

    (c) $= 0$

    (d) ≤ 0

47. Find the number of real roots of the equation $|x|^2 - 2|x| - 3 = 0$.

48. For how many real values of x is $3^{x^2-2x-1} = 9$?

49. For how many values of k, the absolute value of the difference between the roots of the equation $x^2 + kx + 15 = 0$ is 2?

51. In the above question for what value of x, does $f(x)$ attain its minimum value?

LEVEL OF DIFFICULTY (II)

1. The sum of roots of the equation $px^2 + qx + r = 0$ is equal to sum of squares of their reciprocals. If p, q and r are real numbers and $p \neq 0$, then pq^2, rp^2, qr^2 are in

   (a) A.P.

   (b) G.P.

   (c) H.P.

   (d) None of these

2. If one of the roots of the equation $ax^2 + bx + c = 0$ is less than –3 and other is greater than 3, where $a > 0$, then $4a - 2b + c$ is

   (a) > 0

   (b) < 0

   (c) $= 0$

   (d) Cannot be determined

3. In the previous question, $9a + 3|b| + c$ is

   (a) > 0

   (b) < 0

   (c) $= 0$

   (d) Cannot be determined

4. If $f(x) = x^3 - 3x^2 + 2x + 5$ and $g(x) = x^3 + x^2 + 7x + 6$ then what is the number of common roots of $f(x) = 0$ and $g(x) = 0$?

Directions for Questions 5 and 6:

5. Find the number of integral solutions of the equation, $x^2 - 4xy + 3y = 0$ where x and y are non-zero real numbers.

6. In the above question what would be the maximum possible value of $x + y$?

7. $f(x) = x^3 - (5 + a)\,x^2 + (6 + 5a)x - 6a$, where '$a$' is an even prime number.

   For which of the following value of 'x', is $f(x) > 0$?

   (a) –1

   (b) 1

   (c) 2

   (d) None of these

8. In the above question if 'a' is a double-digit odd prime number then, for which of the following range of x, $f(x)$ is greater than 0?

   (a) $3 < x < a$ or $x < 2$
   (b) $2 < x < 3$ or $x < a$

   (c) $3 < x < a$

   (d) $2 < x < 3$ or $x > a$

9. Let $f(x) = 4a^2 - 8 + 8\,(x - 1) - x^2$, where a is a constant. If the maximum value of $f(x)$ is 16, then the value of a = ?

(a) 2

(b) –2

(c) 3

(d) Either (a) or (b)

Directions for Questions 10 to 12: $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are natural numbers. When $f(x)$ is divided by x, the remainder is k^4, where 'k' is a prime number. The square root of the remainder when $f(x)$ is divided by $(x – k)$ is the perfect cube of a natural number. If a, b, c and d are in increasing geometric progression, then answer the following questions.

10. Find the value of 'k'.

11. What is the value of a?

12. $f(4)$ = ?

13. $f(x) = x^8 – 4$ and $f(k) = 0$. The value of $(k – 1)(k^{12} + k^{13} + k^{14} + k^{15} + ... + k^{51})$ = ?

14. $f(x) = px^3 – 2x + c$

    $f(1) = –5$ and $f(4) = 52$. For how many values of x, $f(x) = 0$?

15. $f(x) = x^3 – qx^2 – p^2 x + 288$. If p, q and –12 are the roots of the equation $f(x) = 0$, then find the value of $f(2)$.

16. Find the sum of values of x that satisfies the equation $\left(x^2-4x-4\right)^{x^2-16}-1=0$ is:

17. Find the product of the roots of the equation $x^2+10x+19=2\sqrt{x^2+10x+22}$.

18. How many common roots do the equations $1000x^2+1001x+1=0$ and $x^2+1001x+1000=0$ have?

Directions for Questions 19 and 20: The equation $x^4 - 594 = ax^3 + bx^2 + cx$ has exactly three distinct integral roots. If *a*, *b* and *c* are real numbers, then answer the following questions:

19. Find the largest value of '*b*'.

20. Find the minimum possible value of *a*.

Directions for Questions 21 and 22 If the roots of the equation $x^3 - 5x^2 + qx + r = 0$ represent the length of the sides of a triangle *PQR* and the length of the sides are perfect integers, then answer the following questions:

21. Find the maximum possible value of *r*.

22. Find the minimum possible value of the product of in radius and circumradius of the triangle.

23. Find the number of real roots of the equation $4^{-x} + 2x = 5$.

24. The sum of the reciprocals of the roots of the equation $px^2 + qx + r = 0$ is 7/23 and the product of the roots of the equation $rx^2 + qx + p = 0$ is 1/23, find the sum of the roots of the equation $qx^2 + px + r = 0$

    (a) 1/7

    (b) –7

    (c) –1/7

    (d) –7

25. If $3^{|2x+1|} = 9^{(4x-5)}$, then find the product of all possible values of *x*.

26. One day each of Neha's friends consumed some cold drink and some orange squash. Though the quantities of cold drink and orange squash varied for the friends, the total consumption of the two liquids was ex-

actly nine litres for each friend. If Neha had one-ninth of the total cold drink consumed and one-eleventh of the total orange squash consumed, find the ratio of the quantity of cold drink to that of orange squash consumed by Neha on that day?

(a) 3 : 2

(b) 5:4

(c) 2 : 1

(d) 1:1

27. $Q_1(x)$ and $Q_2(x)$ are quadratic functions such that $Q_1(10) = Q_2(8) = 0$. If the corresponding equations $Q_1(x) = 0$ and $Q_2(x) = 0$ have a common root and $Q_1(4) \times Q_2(5) = 36$, what is the value of the common root?

(a) 10

(b) 6

(c) Either 9 or 6

(d) Cannot be determined

28. For the above question, if it is known that the coefficient of x^2 for $Q_1(x) = 0$ is 1/15 and that of $Q_2(x) = 0$ is 1, then which of the following options is a possible value of the sum of the roots of $Q_2(x) = 0$?

(a) 18

(b) 36

(c) 25

(d) 11

29. Which of the following could be a possible value of 'x' for which, each of the fractions is in its simplest form, where [x] stands for the greatest integer less than or equal to 'x'?

$$\frac{[x]+7}{10}, \frac{[x]+18}{11}, \frac{[x]+31}{12}, \frac{[x]+46}{13}, \ldots, \frac{[x]+1489}{39} \text{ and } \frac{[x]+1567}{40}$$

(a) 95.71

(b) 93.71

(c) 94.71

(d) 92.71

30. $x - y = 6$ and $P = 7x2 - 12y2$, where $x, y > 0$. What is the maximum possible value of P?

(a) Infinite

(b) 352.8

(c) 957.6

(d) 604.8

31. If the equations $5x + 9y + 17z = a$, $4x + 8y + 12z = b$ and $2x + 3y + 8z = c$ have at least one solution for x, y and z and a, b, and c π 0, then which of the following is true?

(a) $4a - 3b - 3c = 0$

(b) $3a - 4b - 3c = 0$

(c) $4a - 3b - 4c = 0$

(d) Nothing can be said

32. If the roots of the equation $x^3 - ax^2 + bx - 1080 = 0$ are in the ratio 2 : 4 : 5, then find the value of the coefficient of x^2.

(a) 33

(b) 66

(c) –33

(d) 99

33. If the roots of the equation $px^3 - 20x^2 + 4x - 5 = 0$, where $p \neq 0$, are l, m and n, then what is the value of $\frac{1}{lm} + \frac{1}{mn} + \frac{1}{ln}$?

(a) 5

(b) –4

(c) 4

(d) 8

34. The cost of ten pears, eight grapes and six mangoes is ₹ 44. The cost of five pears, four grapes and three mangoes is ₹ 22. Find the cost of four mangoes and three grapes, if the cost of each of the items, in rupees, is a natural number and the cost of no two items is the same.

(a) ₹ 17

(b) ₹18

(c) ₹ 14

(d) Cannot be determined

35. The number of positive integral solutions to the system of equations $a_1 + a_2 + a_3 + a_4 + a_5 = 47$ and $a_1 + a_2 = 37$ is

(a) 2376

(b) 2246

(c) 2024

(d) 1296

36. If $f(x)$ is a quadratic polynomial, such that $f(5) = 75$ and $f(-5) = 55$, and $f(p) = f(q) = 0$, then find $p \times q$, given that the value of the constant term in the polynomial is 10.

(a) 2

(b) –3

(c) 5

(d) Cannot be determined

37. If $|a| + a + b = 75$ and $a + |b| - b = 150$, then what is the value of $|a| + |b|$?

(a) 105

(b) 60

(c) 90

(d) Cannot be determined

38. If $[x]$ represents the greatest integer less than or equal to x, then the value of $\left[315^{\frac{1}{3}}\right]+\left[316^{\frac{1}{3}}\right]+\left[317^{\frac{1}{3}}\right]+\ldots+\left[515^{\frac{1}{3}}\right]$ is

(a) 1383

(b) 1379

(c) 1183

(d) 1351

39. a, b, c, d and e are five consecutive integers $a < b < c < d < e$ and $a^2 + b^2 + c^2 = d^2 + e^2$. What is/are the possible value(s) of d?

(a) –2 and 10

(b) –1 and 11

(c) 0 and 12

(d) None of these

40. The 9-to-9 supermarket purchases x litres of fruit juice from the Fruit Garden Inc for a total price of \$ $4x^2$ and sells the entire x litres at a total price of \$ $10 \times (15 + 16x)$. Find the maximum amount of profit that the 9-to-9 supermarket makes in the process.

(a) 1700

(b) 1,000

(c) 1,750

(d) 1,300

41. If α, β are the roots of the quadratic equation $x^2 + mx + 1 = 0$ and γ, δ are the roots of the equation $x^2 + nx + 1 = 0$, then the value of $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ is equal to

(a) $n^2 - m^2$

(b) $m^2 - n^2$

(c) $2\ m^2 - n^2$

(d) None of these

42. Find the roots of the quadratic equation $bx^2 - 2ax + a = 0$.

(a) $\frac{\sqrt{b}}{\sqrt{b} \pm \sqrt{a-b}}$

(b) $\frac{\sqrt{a}}{\sqrt{b} \pm \sqrt{a-b}}$

(c) $\frac{\sqrt{a}}{\sqrt{a} \pm \sqrt{a-b}}$

(d) $\frac{\sqrt{a}}{\sqrt{a} \pm \sqrt{a+b}}$

43. If $x^2 + 3x - 10$ is a factor of $3x^4 + 2x^3 - ax^2 + bx - a + b - 4$, then the closest approximate values of a and b are

(a) 25, 43

(b) 52, 43

(c) 52, 67

(d) None of these

44. If x is real, the smallest value of the expression $3x^2 - 4x + 7$ is

(a) 2/3

(b) ¾

(c) 7/9

(d) None of these

45. If $0 < p < 1$ then the roots of the equation $(1 - p)x^2 + 4x + p = 0$ are ___?

(a) Real and of opposite sign

(b) Real and both negative

(c) Imaginary

(d) Real and both positive

46. The number of possible real solution(s) of y in the equation $y^2 - 2y \cos x + 1 = 0$ is____?

(a) 0

(b) 1

(c) 2

(d) 3

47. A polynomial $ax^3 + bx^2 + cx + d$ intersects the x–axis at 1 and –1, and y-axis at 2. The value of b is

(a) – 2

(b) 0

(c) 1

(d) 2

48. If $x^3 - mx^2 + nx - p = 0$ has three roots α, β, γ. If we add 3 in each of the roots of $x^3 - mx^2 + nx - p = 0$, then we get the roots of the equation $x^3 - ax^2 + bx - 27 = 0$. What is the value of $p + 9m + 3n$?

(a) 0

(b) 1

(c) –54

(d) 54

49. If the roots of the equation $x^3 + qx^2 + rx + s = 0$ are in GP, then which of the following is true?

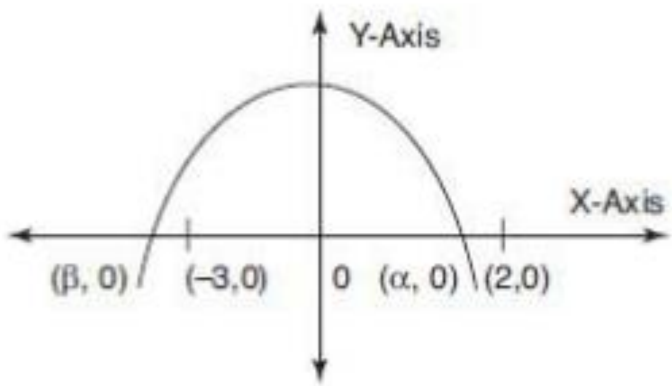
(a) $r^2 = q^2s$

(b) $r^3 = q^3s$

(c) $r = qs^2$

(d) $r = qs^3$

50. The graph of $ax^2 + bx + c$ is shown below. Then which of the following is true?



(a) $a < 0, b < 0, c > 0$

(b) $a < 0, b < 0, c < 0$

(c) $a < 0, b > 0, c > 0$

(d) $a < 0, b > 0, c < 0$

51. If 'x' is a real number then what is the number of solutions for the equation: $(x^8 + 12)^{1/2} = x^4 - 2$?

(a) 0

(b) 1

(c) 2

(d) Cannot be determined

52. How many integer values of 'p' are there such that the inequality $x^2 + 4px + (p + 3) > 0$ is true for all values of x?

53. If $g(x) = x^3 - px^2 - \frac{qx}{2} - r$ can be factorised as $(x - p)(x - q)(x - r)$, then $f(4) = ?$

54. The roots of $x^3 - px^2 + qx - r = 0$ are a, b, c while the roots of $x^3 + wx^2 + yz - 47 = 0$ are $a + 2, b + 2, c + 2$, then the value of $4p + 2q + r = ?$

Directions for Questions 55 and 56: One of the roots of the equation $x^2 + bx + 3b = 0$ ($b \in R$) is thrice the other root, then answer the following questions.

55. What is the value of $3b$?

56. Which of the following options correctly represents roots of the equation $x^2 - 33x + 17b = 0$?

(a) $b, b + 1$

(b) $b - 1, b$

(c) $b - 1, b + 1$

(d) None of these

57. The roots of $x^2 + 5x + a = 0$ are p and q while the roots of $x^2 + 23x + b = 0$ are q and r. If p, q, r are in an arithmetic progression then the value of $|a \times b| = ?$

58. $f(x) = (x - 2)(x^2 + 2x + 5)$

If a, b, c are the roots of $f(x) = 0$, then find the value of $a^3 + b^3 + c^3$.

59. If $f(x) = px2 + qx + r$ and $f(x)$ is exactly divisible by $(x + 2)$ and $(x + 3)$ but leaves remainder of 7 when divided by $(x - 1)$, find the approximate value of q.

60. If $f(x) = 4x - 7\sqrt{x}$, which of the following statements is true about the roots of the equation $f(x) = 2$?

(a) It has only one real root which is not an integer

(b) It has no real root

(c) It has one real root which is a positive integer

(d) It has two real roots

61. If the equation $ax2 + bx + a = 0 (a > 0)$ has real and positive roots then which of the following is always true?

(a) $b < 2a$

(b) $b < 0$

(c) $b \leq -2a$

(d) All the options are true

62. If p, q, r are real and $(x - p)(x - q)+(x - q)(x - r)+(x - r)(x - p)=0$ if $p \neq q \neq r$ then which of the following options is true?

(a) Roots of the given equations are imaginary.

(b) Roots of the given equation are real.

(c) Roots of the given equation are equal.

(d) None of these

63. If $f(x) = px^2 + qx + r$ and $g(x) = rx^2 + qx + p$. If one root of $f(x) = 0$ is 2 and one root of $g(x) = 0$ is ¼, then find the sum of the roots of $f(x)$ and $g(x)$.

64. The number of distinct points at which the curve $y^3 - 4y^2 + x^2 - 5x + 3y = 0$ intersects either the y-axis or the x-axis is.

65. If the sum of the roots of the quadratic equation $px^2 + qx + r = 0$ is equal to the sum of the square of their reciprocals, mark all the correct statements.

(a) r/p, p/q and q/r are in A. P.

(b) p/r, q/p and r/q are in G. P.

(c) p/r, q/p and r/q are in H. P.

(d) Both (a) and (c).

66. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes:

(i) One of them made a mistake in the constant term and got the roots as 5 and 9.

(ii) Another one committed an error in the coefficient of x and he got the roots as 12 and 4.

But in the meantime, they realised that they are wrong and they managed to get it right jointly. Which of the following could be the quadratic equation?

(a) $x^2 + 4x + 14 = 0$

(b) $2x^2 + 7x - 24 = 0$

(c) $x^2 - 14x + 48 = 0$

(d) $3x^2 - 17x + 52 = 0$

LEVEL OF DIFFICULTY (III)

1. $f(x) = ax^2 + bx + c$ & $a < 0$.

   The equation $f(x) = 0$ has two distinct roots which is from the set { −2, −1, 0, 1, 2}. How many different pairs of roots of $f(x)$ are possible such that $f(0)$ is greater than or equals to 0?.

   (a) 4

   (b) 6

   (c) 8

   (d) 10

2. $px^2 + qx + r = 0$ has one root greater than 3 and other root less than 1. Which of the following is necessarily true?

   (a) $p(4p + 2q + r) < 0$

   (b) $p(4p + 2q + r) > 0$

   (c) $p(4p - 2q + r) < 0$

   (d) $p(9p - 3q + r) < 0$

3. If $f(x, y) = 4^{x^y} + x^{4^y}$ then what is the total number of solutions for the equation $f(x, y) = 5$.

   (a) 0

   (b) 1

(c) 2

(d) More than 2.

Directions for Question 4 and 5: If 'd' is a root of the equation $ax^2 + bx + c = 0$ and 'c' is a root of the equation $ax^2 + bx + d = 0$ and if $c \neq d$ then answer the following questions:

4. Find the value of $c + d$

   (a) $a(b - 1)$

   (b) $\frac{b-1}{a}$

   (c) $\frac{1-b}{a}$

   (d) None of these

5. Which of the following equations has roots $-c$, $-d$?

   (a) $ax^2 + a(b - 1)x + b - 1=0$
   (b) $ax^2 - a(1 - b)x + 1 - b = 0$

   (c) $ax^2 + a(1 - b)x + 1 - b = 0$

   (d) None of these

6. The values of a quadratic function $f(x)$ is a positive for all values of x, except for $x = 4$. If $f(0) = 10$, find the value of $f(-4)$.

7. Find all values of 'a', such that 4 lies somewhere between the roots of the equation $3x^2 + 4ax + (a + 3) = 0$ for all values of x.

   (a) $a < -3$

   (b) $a < -2$

   (c) $a > 3$

(d) $a > 2$

8. $f(x)=x^3 - (5 + k)x^2 + (6 + 5k)x - 6k$, where '$k$' is an odd prime number and $k > 3$. What is the range of values of x for which $f(x) < 0$?

   (a) $2 < x < 3$ or $x > k$

   (b) $x < 2, 3 < x < k$

   (c) $x < 3, x > k$

   (d) None of these

9. $f(x, p) = a(x - p)^2 + b(x - p) + c$, where a, b, c are constants and $a < 0$ and 'p' is a natural number. It is given that the roots of the equation $ax^2 + bx + c = 0$ are 3, 4. Then,the value of x at which $f(x, 5)$ attains its maximum value is

   (a) 9

   (b) 8.5

   (c) 7.5

   (d) 5.5

10. $f(x) = [x]^2 - 11[x] + 30$, where $[x]$ represents the largest integer less than or equal to x, then what is the sum of all integer solutions of the equation $f(x) = 0$?

11. If all the three roots of the equation $x^3 - 12x^2 + px - 42 = 0$ are unequal and prime. Then find p.

Directions for Questions 12 and 13:

If $f(x) = x^3 - px^2 - 2qx + r$ and $g(x) = (x - p)(x - q)(x - r)$, answer the following questions.

12. Find the value of $p + q$ for which $f(x) = g(x)$,

13. Find the value of $f(4)$, if $f(x) = g(x)$,

14. If $f(x) = (b - 1)x^2 + (c + d)x + e$ and $a:b = 1:2$, $b:c = 2:3$, $c:d = 3:4$, $d:e = 4:5$, then find the square of difference of the roots of the equation $f(x) = 0$.

15. $\log_3 x \times \log_3 y + \log_3 z \times \log_3 xy = 11$, where x, y, z all are real numbers.

If $(\log_3 x)^2 + (\log_3 y)^2 = 14 - (\log_3 z)^2$ and $xyz = k_1^2 = k_2^3$ then $k_1 + k_2 =?$

Directions for Questions 16 to 18: $f(x) = ax^2 - 150x + 5b$ where a and b are two positive integers. Then answer the following questions

16. For how many ordered pairs (a, b) will $(x - 3)$ be a factor of $f(x)$?

17. Find the maximum possible value of $a + b$ for which $(x - 3)$ is a factor of $f(x)$.

18. If a and b are integers then for how many ordered pairs (a, b) will $(x - 3)$ be a factor of $f(x)$?

(a) 9

(b) 18

(c) 180

(d) None of these

Directions for Questions 19 and 20: If $f(x) = x^3 - 12x^2 + 47x = 60$. then answer the following questions:

19. How many quadratic equations of the form $x^2 + bx + c = 0$, can be formed such that both the roots of the quadratic equation are common with the roots of $f(x) = 0$?

(a) 3

(b) 5

(c) 6

(d) None of these

20. If the product of all the roots of the quadratic equations of the form $x^2 + bx + c = 0$, that can be formed such that both roots of the quadratic equation are common with the roots of $f(x) = 0$ is p then find $p^{1/4}$.

(a) 30

(b) 40

(c) 50

(d) 60

Directions for Questions 21 and 22: If all the roots of $(x - k)^3 (x - 7) - 27 = 0$ are integers then answer the following questions:

21. How many integer values can k have?
22. Find the difference between the maximum and minimum possible values of k.

23. Find the value of p for which the sum of the square of the roots of the equation $x^2 - (p - 3)x + (p - 4) = 0$ is minimum.

24. If $f(x) = x^2, g(x) = x^3 - 2^x$, where x is a positive integer then for how many values of x, is $f(x) = g(x)$?

Directions for Question 25 and 26: If $h(x)$ is a quadratic function which attains its maximum value of 10 at $x = 4$. If $t(x)$ is a function such that $3h(x) + 5t(x) = 0$, then answer the following questions:

25. Which of the following option is true?

(a) $h(x)$ and $t(x)$ have roots of opposite sign

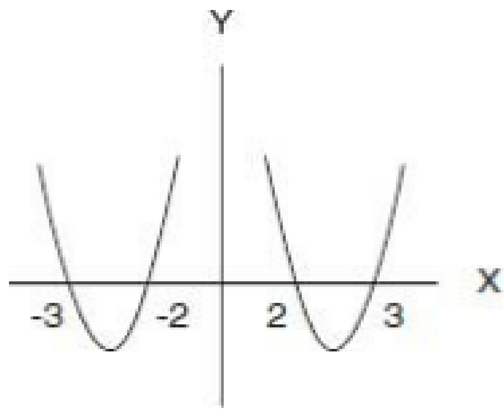
(b) Sum of roots of $h(x)$ and $t(x)$ is 0

(c) Sum of roots of $h(x)$ is equal to the sum of roots of $t(x)$

(d) None of these

26. Find the minimum value of $t(x)$.

27. The curve shown below can possibly represent which of the following equation (assume the curve does not exist at $x = 0$).



(a) $\frac{6}{|x|} = 5 + |x|$

(b) $-\frac{6}{|x|} = 5 - |x|$

(c) $\frac{6}{|x|} = 5 - |x|$

(d) None of these

28. If $f(x) = max(|x^2 - 4|, 2x + 4)$ then find the number of solutions of the equation $f(x) = \frac{5}{2}$.

29. Find the number of real solutions of the equation $||x| - 1| = e^x$.

30. Find the number of real solutions of equation $||x|-1|-2| = \left(\frac{1}{2}\right)^x$.

where 'p' is a positive integer then find q.

ANSWER KEY

Level of Difficulty (I)

1. (c)
2. (d)
3. (c)
4. (c)
5. (d)
6. (c)
7. (d)
8. (c)
9. (b)
10. (b)
11. (c)
12. (b)
13. (d)
14. (b)
15. (b)
16. (b)
17. (c)
18. (a)
19. (c)
20. (b)

21. (b)

22. (a)

23. (b)

24. (c)

25. (c)

26. (b)

27. (d)

28. (d)

29. (d)

30. (d)

31. 3

32. 3

33. 1

34. 4

35. (b)

36. (a)

37. 3

38. 2

39. (c)

40. (a)

41. (a)

42. 0

43. 0

44. 0

45. (c)

46. (a)

47. 2

48. 2

49. 2

50. 5

51. 4

Level of Difficulty (II)

1. (a)

2. (b)

3. (b)

4. 0

5. 2

6. 4

7. (d)

8. (d)

9. (d)

10. 2

11. 2

12. 240

13. 8184

14. 1

15. 0

16. 4

17. 13

18. 1

19. 1183

20. –591

21. –4

22. 0.4

23. 2

24. (a)

25. 1.65

26. (d)

27. (d)

28. (a)

29. (c)

30. (d)

31. (c)

32. (c)

33. (c)

34. (b)

35. (d)

36. (c)

37. (a)

38. (a)

39. (d)

40. (c)

41. (a)

42. (c)

43. (c)

44. (d)

45. (b)

46. (c)

47. (a)

48. (a)

49. (b)

50. (a)

51. (a)

52. 1

53. 31.5

54. 39

55. 48

56. (a)

57. 1568

58. 30

59. 2.92

60. (c)

61. (d)

62. (b)

63. 6.75

64. 4

65. (d)

66. (c)

Level of Difficulty (III)

1. (c)

2. (a)

3. (d)
4. (c)
5. (c)
6. 40
7. (a)
8. (b)
9. (b)
10. 11
11. 41
12. 3/2
13. 54
14. 29
15. 36
16. 9
17. 86
18. (d)
19. (c)
20. (d)
21. 4
22. 52
23. 4
24. 1
25. (c)
26. –6
27. (c)
28. 2

29. 3

30. 5

Solutions and Shortcuts

Level of Difficulty (I)

1. For the given expression to be a maximum, the denominator should be minimised. (Since, the function in the denominator has imaginary roots and is always positive). $x^2 + 5x + 10$ will be minimised at $x = -2.5$ and its minimum values at $x = -2.5$ is 3.75.

   Hence, required answer = 1/3.75 = 4/15.

2. Has no maximum.

3. The minimum value of $(p + 1/p)$ is at $p = 1$. The value is 2.

4. The product of the roots is given by: $(a^2 + 18a + 81)/1$.

   Since product is unity, we get: $a^2 + 18a + 81 = 1$.

   Thus, $a^2 + 18a + 80 = 0$

   Solving, we get: $a = -10$ and $a = -8$.

5. Solve through options. LHS = RHS for $a = 1$.

6. For a, b negative, the given expression will always be positive since, a^2, b^2 and ab are all positive.

7. To solve this, take any expression whose roots differ by 2.

   Thus, $(x - 3)(x - 5) = 0$

   $\Rightarrow x^2 - 8x + 15 = 0$

   In this case, $a = 1$, $b = -8$ and $c = 15$.

   We can see that $b^2 = 4(c + 1)$.

8. h $(x) = 4x^2 + 13x + 10$.

Sum of roots – 13/4.

9. $x^2 + bx + 12 = 0$ has 2 as a root.

Thus, $b = -8$.

10. Solve this by assuming each option to be true and then check whether the given expression has equal roots for the option under check.

Thus, if we check for option (b),

$ad = bc$,

We assume $a = 6$, $d = 4$, $b = 12$, $c = 2$ (any set of values that satisfies $ad = bc$).

Then $(a^2 + b^2)\,x^2 - 2(ac + bc)\,x + (c^2 + d^2) = 0$

$180\,x^2 - 120\,x + 20 = 0$

We can see that this has equal roots. Thus, option (b) is a possible answer. The same way if we check for a, c and d, we see that none of them gives us equal roots and can be rejected.

11. $(c + 6) = 1/2 \times 2(2c - 1) \Rightarrow c + 6 = 2c - 1 \Rightarrow c = 7$

12. $-3/a = -6 \Rightarrow a = ½$,

$2b/a = -6$ and $a = ½$

Gives us $b = -1.5$

$a + b = -1$

13. $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta$

$= (7/5)/(1/5) = 7$

14. $y = \sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}$

$\Rightarrow y = \sqrt{x+y}$

$\Rightarrow y^2 = x + y$

$y^2 - y - x = 0$

Solving quadratically, we have option (b) as the root of this equation.

15. The approximate value of $a = \sqrt{13.92} = 3.6$ (approx.) $a + 1/a = 3.6 + 1/3.6$ is closest to 4.

16. Solve by assuming values of *a*, *b*, and *c* in AP, GP and HP to check which satisfies the condition.

17. Assume any equation:

Say $x^2 - 5x + 6 = 0$

The roots are 2, and 3.

We are now looking for the equation, whose roots are:

(2 + 1/3) = 2.33 and (3 + 1/2) = 3.5

Also $a = 1$, $b = -5$ and $c = 6$.

Put these values in each option to see which gives 2.33 and 3.5 as its roots.

18. Remainder when $x^2 + ax + b$ is divided by $x - 1$ is obtained by putting $x = 1$ in the expression. Thus, we get.

$a + b + 1 = 5$ and

$b - a + 1 = 5$

$\Rightarrow b = 4$ and $a = 0$

19. $b^2 - 4 < 0 \Rightarrow -2 < b < 2$

20. $(a^2 + b^2) = (a + b)^2 - 2ab$

$= (-1)^2 - 2 \times (-7) = 15$

21. $(a^3 + b^3) = (a + b)^3 - 3ab(a + b)$

$= (-1)^3 - 3 \times (-7)(-1)$

$= -1 - 21 = -22$

22. For the roots to be opposite in sign, the product should be negative.

$(c^2 - 4c)/2 < 0 \Rightarrow 0 < c < 4$

23. The roots of the equation $x^2 - 9x + 20 = 0$ are 4 and 5. The expression would be negative for $4 < x < 5$.

24. The roots should be imaginary for the expression to be positive

i.e. $k^2 - 36 < 0$

thus, $-6 < k < 6$ or $|k| < 6$.

25. Simplifying the equation $9^{a-2} \div 3^{a+4} = 81^{a-11}$, we will get: $3^{2a-4} \div 3^{a+4} = 3^{4a-44}$. This gives us:

$2a - 4 - a - 4 = 4a - 44 \rightarrow a - 8 = 4a - 44 \rightarrow 3a = 36 \rightarrow a = 12$

Hence, we have to evaluate the value of $3^4 + 3^6 = 81 + 729 = 810$.

Hence, option (c) is correct.

26. In order to think of this situation, you need to think of the fact that "the cube of a number + a power of two" (LHS of the equation) should add up to the fourth power of the same number.

27. The values of x, where the above expression turns out to be negative or 0 are $x = 2, 3, 4, 5, 6, 7$ or 8.

Hence, option (d) is correct.

28. The value of the LHS will become $512 - 256 - 320 + 64 = 0$ when $x = 8$.

29. The above equation gets satisfied at $a = 9$, $b = 8$ and $c = 6$. (In order to visualise this, look for sets of 3 numbers with an LCM of 72). All different arrangements of (9, 8, and 6) will be possible values of (a, b, c). Possible arrangements of (a, b, c) are (9, 8, 6), (8, 9, 6), (6, 8, 9), (8, 6, 9), (6, 9, 8), (9, 6, 8). Thus, there are a total of six such sets possible. Hence, option (d) is correct.

30. The equation can be rewritten as $s^{0.5} = \frac{pq}{r^2}$ or $s = \frac{(pq)^2}{r^4}$

The maximum value of s will happen when pq is maximum and r is minimum. pq is maximum when $p = -0.04$ and $q = -0.25$ and $r = 1$ similarly s will be minimum when $p = -0.03$ and $q = -0.09$ and $r = 7$.

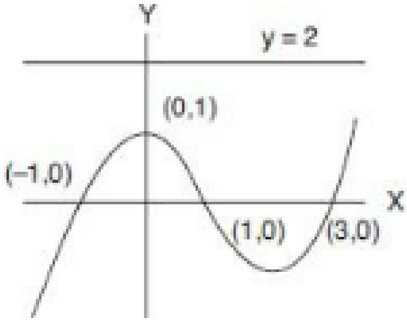
$$s_{max} = \frac{(-0.04 \times -0.25)^2}{1} \quad s_{min} = \frac{(-0.03 \times -0.09)^2}{7^4}$$

$$s_{max} - s_{min} = \frac{(-0.04 \times -0.25)^2}{1} - \frac{(-0.03 \times -0.09)^2}{7^4}$$

$$\approx 0.0001$$

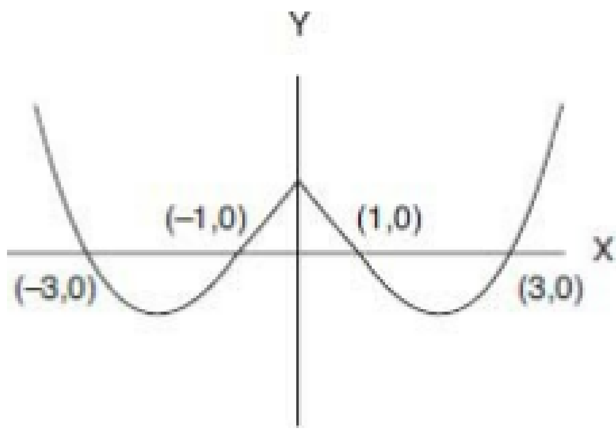
31. Graph of $f(x) = 0$ cuts x-axis at three distinct points. Therefore $f(x) = 0$ has three roots.

32. Sum of roots of $f(x) = 0$ is $-1 + 1 + 3 = 3$.

33. 

The $y = 2$ line intersects the curve $y = f(x)$ at only one point, hence total number of roots of $f(x) = 2$ is 1.

34. Process to draw graph $f(|x|)$, if graph of $f(x)$ is given: First erase the negative x portion of $f(x)$ then take a mirror image of positive X portion. Curve of $y = f(|x|)$ is shown below.



Here we can see that the curve of $y = f(|x|)$ cuts the x-axis at four distinct points, hence $y = f(|x|)$ has four real roots.

35. $m+n=-\frac{q}{p}$ (1)

$$(m+k)+(n+k)=-\frac{b}{a}$$

$$m+n+2k=-\frac{b}{a} \qquad (2)$$

Subtract equation (1) from equation (2):

$$2k=-\frac{b}{a}+\frac{q}{p}$$

$$k = \frac{1}{2}\left[\frac{q}{p} - \frac{b}{a}\right]$$

Option (b) is correct.

36. $mn = \frac{r}{p}$ (1)

$$(mk)(nk) = mnk^2 = \frac{c}{a} \quad (2)$$

Equation (2) ÷ equation (1)

$$k^2 = \frac{c}{a} \times \frac{p}{r}$$

$$k = \sqrt{\frac{cp}{ar}}$$

Thus, option (a) is correct.

37. For $x \geq 0 : x^2 = x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0, 1$

For $x < 0 : x^2 = -x$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0, -1$

Therefore, for three real values of x, we will get $x^2 = |x|$.

38. $16x^2 + px + 9 = 0$

For equal roots, the discriminant of the above equation must be zero.

$p^2 - 4 \times 16 \times 9 = 0$

$p^2 = 4 \times 16 \times 9$

$p = \pm (2 \times 4 \times 3) = \pm 24$

Therefore for two different values of p, $16x^2 + px + 9 = 0$, has equal roots.

39. m + $n = 6$, $mn = -3$

$(m + n)^2 = m^2 + n^2 + 2mn$

$36 = m^2 + n^2 - 6$

$m^2 + n^2 = 42$ (Sum of roots of the required equation)

Also, since $mn = -3$, the value of $m^2n^2 = 9$ (product of roots of the required equation)

Therefore the equation must be $x^2 - 42x + 9 = 0$..

Hence, option (c) is correct.

40. Checking from the options, if we put $a = 1$ and $b = 1$ (from option (a). then we get the equation, $x^2 + x + 1 = 0$, and this equation does not have any real roots.) So, this option is correct.

41. $a + b = -$a, $ab =$ b

$\Rightarrow ab =$ b

$\Rightarrow ab - b = 0$

$\Rightarrow$ (a $- 1)b = 0$

This gives us two possibilities: $b = 0$ or $a = 1$

For $b = 0$, $a + 0 = -a$, $\Rightarrow a = 0$

For $b = 0$, $a + 0 = -a$, $\Rightarrow a = 0$

For $a = 1$, $1 + b = -1 \Rightarrow b = -2$

Option (a) is correct.

42. For $x > 0$

$\Rightarrow x2 + 3x + 2 = 0$
$\Rightarrow x2 + 2x + x + 2 = 0$

$\Rightarrow (x + 2)(x + 1) = 0$

$\Rightarrow x = -1$ or -2

But here $x > 0$, so these values of x are not possible in this case.

For $x < 0$.

$\Rightarrow x2 - 3x + 2 = 0$

$\Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 1$ or 2

But here $x < 0$, so these values of x are not possible in this case.

Therefore, the given equation has no real root.

43. $x2 + (x + 1)2 + (x - 2)2 = 0$

It is possible when $x = 0$, $x + 1 = 0$, $x - 2 = 0$ (Since each of the terms within the brackets in the given expression is non-negative).

Thus, we get the required values of $x = 0$, $x = -1$, $x = 2$ all at the same time. It is not possible that a variable would have the same. Therefore, the given equation has no real root.

44. ax can never be equal to 0 for any real value of x. Therefore, 3^{2x^2+3x+1} can never be equal to 0 for any real value of x. So the given equation has no real solution.

45. $f(x) = -x^2 + x - 4$

The discriminant of the equation $f(x) = 0$ is less than zero. Therefore, $f(x) = 0$ has no real root and the coefficient of x^2 is less than 0. Therefore, $f(x) = -x^2 + x - 4$ opens downward. Hence, option (c) is correct.

46. $f(x)$ is less than 0 for all real values of x (As D < 0, a < 0).

Therefore, $f(a) < 0, f(b) < 0$

$f(a).f(b) > 0$

Hence, option (a) is correct.

47. $|x|^2 - 2|x| - 3 = 0$

$\Rightarrow (|x| - 3)(|x| + 1) = 0$

$\Rightarrow |x| = 3, -1$

$|x| = -1$ is not possible

$\Rightarrow |x| = 3$

$\Rightarrow x = \pm 3$

Therefore, for the given equation only two real roots are possible.

48. $3^{x^2-2x-1} = 9 = 3^2$

$\Rightarrow x^2 - 2x - 1 = 2$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3, -1$

The given equation has two solutions. Hence, there are two values of x at which the equation is satisfied.

49. Let the roots of the equation $x^2 + kx + 15 = 0$ be m, n.

$m + n = -k, mn = 15, m - n = 2$

$(m + n)^2 - 4mn = (m - n)^2$

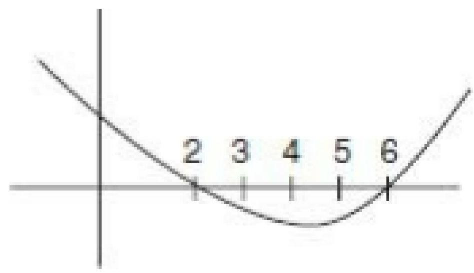
$k^2 - 60 = 4$

$k^2 = 64$

$k = +8, -8$

The correct answer is 2.

50. $f(x) = x^2 - 8x + 12 = (x - 2)(x - 6)$



$\therefore f(x) \leq 0$ for $x = 2, 3, 4, 5$ and 6

$\therefore$ the correct answer is 5.

51. $f(x) = 0$ has two roots 2, 6 and curve of $f(x)$ opens upwards. So $f(x)$ is minimum for $f'(x) = 0$. This gives us $2x - 8 = 0$. Thus, the function would attain its minimum at $x = 4$.

Therefore, $f(x)$ is minimum for $x = 4$.

Level of Difficulty (II)

1. Let the roots of the equation be a and b. According to the question:

$$a+b=\frac{1}{a^2}+\frac{1}{b^2}$$

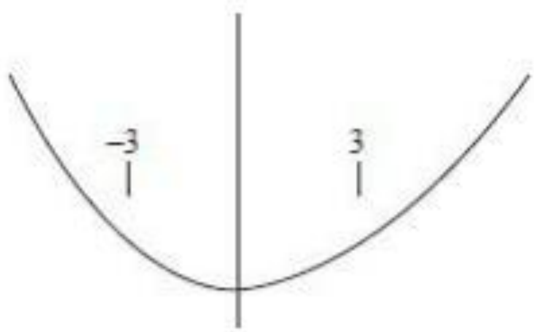
$$a+b=\frac{a^2+b^2}{a^2b^2}$$

$$a+b=\frac{(a+b)^2-2ab}{a^2b^2}$$

$$-\frac{q}{p}=\frac{\left(-\frac{q}{p}\right)^2-\frac{2r}{p}}{\left(\frac{r}{p}\right)^2}\Rightarrow 2rp^2=pq^2+qr^2$$

$pq2$, $rp2$, and $qr2$, are in $A.P.$

2. The graph of $ax^2+bx+c=0$ would look like:



So, at x = –3 and +3, the value of the given function must be negative for all values between (–3, 3). At x = –2, we have

$$a(-2)^2+b(-2)+c<0 \text{ or } 4a-2b+c<0$$

Hence, option (b) is correct.

3. $f(3)=9a+3b+c<0, f(-3)=9a-3b+c<0$

$$9a+3|b|+c<0$$

Hence, option (b) is correct.

4. $x^3 - 3x^2 + 2x + 5 = x^3 + x^2 + 7x + 6$

$4x^2 + 5x + 1 = 0$

$x = -1 \text{ or } -1/4$

None of the two values satisfies the given equations. Hence, the two equations do not have any common roots. Hence, correct answer is 0.

5. $x^2 - 4xy + 3y = 0$. This is a quadratic equation in the variable x. Using the formula for the roots of a quadratic equation, we have:

$x = 2y \pm \sqrt{4y^2 - 3y}$

For x to be an integer $4y^2 - 3y$ must be the perfect square of an integer.

$4y^2 - 3y = k^2$ or $y = \frac{[3 \pm \sqrt{9 + 16k^2}]}{8}$, where k is an integer. Again, for y to be an integer, $16k_2 + 9$ should be an integer.

Thus, where 'a' is an integer.

$(4k - a)(4k + a) = -9$

The only permissible values of k and a are 1 and 5 respectively.

Thus, the only possible value of k for which y is an integer is $k = 1$.

$\Rightarrow y = \frac{3+5}{8} = 1$ and $x = 2 \pm \sqrt{1} = 1$ and 3.

Hence, there are two possible integer pairs which are (1, 1) and (3, 1).

6. Maximum possible value of $x + y = 3 + 1 = 4$.

7. Since a is an even prime number, the expression becomes: $f(x) = x_3 - 7x_2 + 16x - 12 = (x - 2)_2 (x - 3)$.

For $f(x) > 0$, we have: $(x-2)^2(x-3) > 0$ we get $(x - 3) > 0$ or $x > 3$. Hence, for none of the given values of x, $f(x) > 0$.

Hence, option (d) is correct.

8. The expression is: $(x-a)(x-2)(x-3)>0$

As, a is a double-digit prime number so a = 11, 13, 17 or 19.

So, $(x-a)(x-2)(x-3)\ >0$ > 0

The above inequality holds true if all the three brackets are positive, which would occur for $x > a$, or if two of the brackets are negative and one bracket is positive. This occurs, if $2 < x < 3$. Hence, option (d) is correct.

9. $4a^2-8+8(x-1)-x^2=-(x^2-8x+16)+4a^2$

$=-(x-4)^2+4a^2$

$f(x)$ will be maximum at x = 4. Hence, at x = 4, $f(x) = 4a^2=16$ or $a = \pm 2$.

Hence, option (d) is correct.

10. When, $f(x)$ is divided by x, the remainder will be $f(0) = d = k4$.

Now, 'k' is prime number and a, b, c and d, form an increasing G.P.

$a = k$, $b = k2$, $c = k3$, $d = k4$, and subsequently the expression is: $f(x) = kx3 + k2x2 + k3x + k4$.

11. $a = 2$

12. $f(4)$ =

$2\times4^3+2^2\times4^2+2^3\times4+2^4=128+64+32+16=240$

13. $f(k)=k^8-4=0 or k^4=2$

$(k-1)(k^{12}+k^{13}+k^{14}+k^{15}+.....+k^{51})$

$=(k-1)\times k^{12}\times(1+k+k^2+...+k^{39})$

At this point, we can replace: $\left(1+k+k^2+...+k^{39}\right)$, with: $\frac{\left(k^{40}-1\right)}{k-1}$. Thus, we get:

$$\frac{(k-1)k^{12}\left(k^{40}-1\right)}{k-1}=k^{12}\left(k^{40}-1\right)=(8)\times\left(2^{10}-1\right)$$
$$=1023\times 8=8184$$

14. $f(1) = p - 2 + c = -5$

$p + c = -3$ (1)

$f(4) = 64p - 8 + c = 52$

$64p + c = 60$ (2)

Solving equations (1) and (2), we get: $p = 1$, $c = -4$.

$$f(x)=x^3-2x-4$$

There is only one real root for $f(x) = 0$. Hence, the correct answer is 1.

15. The sum of the roots of a cubic equation $ax3 + bx2 + cx + d = 0$ are given by: $-b/a$.

Hence, $p + q - 12 = q$ or $p = 12$

Also, using the product of roots, we get: $p \times q \times (-12) = -288$. Using $p = 12$, we get, $q = 2$.

Hence, $= f(x) = x^3-2x^2-144x+288$

$$f(2)=2^3-2\times 2^2-144\times 2+288=0$$

Hence, $f(2) = 0$.

16. $\left(x^2-4x-4\right)^{x^2-16}-1=0 \text{ or } \left(x^2-4x-4\right)^{x^2-16}=1$

This is possible when $x^2 - 4x - 4 = 1$ or $x^2 - 16 = 0$

For: $x^2 - 4x - 4 = 1$ → we get: x = 5 or –1

For: $x^2 - 16 = 0$ → we get x = –4 or +4.

Sum of all possible roots of the equation = –1 + 5 – 4 + 4 = 4.

17. Let: $\sqrt{x^2 + 10x + 22}$. Then, $x^2 + 10x + 19 = u^2 - 3$

Thus, the equation becomes: $u^2 - 3 = 2u$

$$u^2 - 2u - 3 = 0$$

$$(u - 3)(u + 1) = 0$$

u = 3 and by definition $u \neq -1$ (as a value inside a square root cannot be negative). The equation we have is:

$x^2 + 10x + 19 = 2\sqrt{x^2 + 10x + 22} \rightarrow$

$x^2 + 10x + 19 = 2 \times 3$

$x^2 + 10x + 13 = 0$

Product of the roots = c/a = 13.

18. Let's first find the intersection points of the two functions underlying the equations.

$1000x^2 + 1001x + 1 = x^2 + 1001x + 1000 \rightarrow$

$999x^2 = 999 \rightarrow x = \pm 1$

Of these two values of x, we can see that +1 is not a root of the equations, while –1 is a root for both the equations. Hence, the two equations have only 1 common root.

Solutions for Questions 19 and 20:

Let the roots of the equation $x^4 - 594 - ax^3 - bx^2 - cx = 0$ are p, p, q and r.

$pq + qr + pr + p.p + pq + pr = -b$ and $p \times p \times q \times r = -594$

b will be maximum when $pq + qr + pr + p.p + pq + pr$ is minimum. For this to be minimum, $p = 1$, $q = 2$ and $r = -297$.

Hence, the maximum possible value of $b =$

$-(1\times1+1\times2+1\times(-297)+1\times2+1\times(-297)+2\times(-297))$

$= 1183.$

For an equation with the highest power of x being the power 4, we have:

Generic Form: $ax4 + bx3 + cx2 + dx + e = 0$. Let the roots be: p, q, r, s.

Then: $p + q + r + s = -b/a$; $pq + pr + ps + qr + qs + rs = c/a$; $pqr + pqs + prs + qrs = -d/a$; and $pqrs = e/a$

20. $p + p + q + r = a$ or $2p + q + r = a$. a will be minimum when $2p + q + r$ is minimum. For this to be minimum, $p = -297$, $q = 1$ and $r = 2$.

    $a = 2p + q + r = -594 + 2 + 1 = -591$

Solutions for Questions 21 and 22:

21. Let the roots of the equation be a, b and c.

    $a + b + c = 5$ as a, b and c are the sides of a triangle. & $a + b > c$

    The only possible values of (a, b, c) are $(1, 2, 2)$. Product of the roots $= -r = 1\times2\times2 \rightarrow r = -4$

22. In-radius = $\frac{\Delta}{s}$

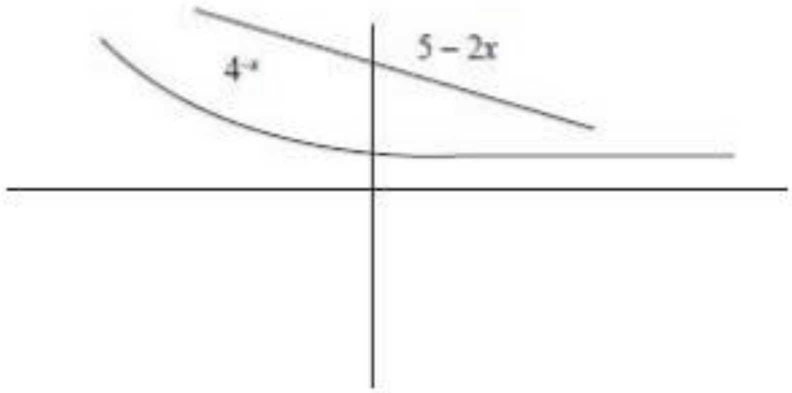
    Circum-radius = $abc/4\Delta$

    Product of in-radius and circum-radius =

$$\frac{\Delta}{s}\times\frac{abc}{4\Delta}=\frac{abc}{2(a+b+c)}=\frac{4}{10}$$

Hence, the product of in-radius and circum-radius = 0.4.

23. $4^{-x} = 5 - 2x$



The graphs of 4^{-x} and $5 - 2x$ are shown above. There are exactly two intersection points of 4^{-x} and $5 - 2x$. Hence, the correct answer is 2.

24. Let the roots of the equation $px^2 + qx + r = 0$ are a and b.

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{\left(-\frac{q}{p}\right)}{\frac{r}{p}} = -\frac{q}{r} = 7/23 \qquad (1)$$

$$\frac{p}{r} = \frac{1}{23} \qquad (2)$$

Equation 1/ equation 2:: $\frac{-\frac{q}{r}}{\frac{p}{r}} = \frac{\frac{7}{23}}{\frac{1}{23}} or -\frac{q}{p} = 7$

Sum of the roots of $qx^2 + px + r = 0$ is $-\frac{p}{q}$ which would be equal to 1/7 in this case.

Hence, option (a) is correct.

25. $f(x) = 0$

$3^{|2x+1|} = 9^{4x-5}$ or $3^{|2x+1|} = 3^{2(4x-5)}$

or $|2x+1| = 2\times(4x-5)$

This gives us two scenarios: $2x+1 = 2\times(4x-5)$ or

$2x+1 = -2\times(4x-5)$

When, $2x+1 = 2\times(4x-5)$

$2x+1 = 8x-10$ or $6x = 11$ or $x = 11/6$

When, we use: $2x+1 = -2\times(4x-5)$ or $10x = 9$ or $x = 9/10$

Product of the possible values of $x = \frac{11}{6}\times\frac{9}{10} = 1.65$.

26. Let there be 'n' friends of Neha which would mean that the amount of total liquids consumed by the group would be $9n$. Further let the total amount (in litres) of cold drink consumed by Neha and her friends be 'x' litres. Then, the amount of orange squash consumed by the friends will be $9n–x$. As per the given information, we know that Neha has consumed one-ninth of the total cold drink (i.e. $x/9$) and also that she has consumed one-eleventh of the total orange squash $(9n–x)/11$. Also, since Neha has consumed a total of 9 litres of the liquids, we will get:

$\frac{x}{9}+\frac{(9n-x)}{11} = 9 = 9$

$\Rightarrow 11x + 81n - 9x = 891$
$\rightarrow 2x + 81n = 891 \rightarrow x = (891 - 81n) \div 2$

In this equation, n is an integer and x should be less than $9n$. This gives rise to the inequality:

$0 < x < 9n \rightarrow 0 < (891–81n)/2 < 9n \rightarrow 0 < 891 - 81n < 18n$
The only value of n that satisfies this inequality is at $n = 10$.

This means that there were ten friends in the group and the amount of liquids consumed in total would have been 90 litres (9 litres each). Putting this value in the equation, we get: $2x + 810 = 891 \rightarrow x = 40.5$. This would mean that Neha consumes 40.5/9 = 4.5 litres of cold drink and hence, she would consume 4.5 litres of orange squash. The required ratio would be 1:1.

27. Let the common root be 'm'. Since we know that $Q1(10) = 0$, it means that 10 would be one of the roots of $Q1$. By the same logic it is given to us that 8 is one of the roots of $Q2$. Using this information, we have:

$Q_1 = c_1 \times (x - 10) \times (x - m)$

and $Q_2 = c_2 \times (x - 8) \times (x - m)$. (**Note:** c_1 and c_2 are constants (each not equal to 0)).

The only information we have beyond this is that the product $Q_1 (4) \times Q_2 (5) = 36$. However, it is evident that by replacing $x = 4$ and $x = 5$ in the expressions for Q_1 and Q_2 respectively, we would not get any conclusive value for m since the value of m will depend on the values of $c1$ and c_2. You can see this happening here:

$c_1 \times -6 \times (4 - m) \times c_2 \times -3 \times (5 - m) = 36$

$c_1 \times c_2 \times (20 - 9m + m^2) = 2$

In this equation, it can be clearly seen that the value of the common root 'm' would be dependent on the values of c_1 and c_2 and hence we cannot determine the answer to the question. Option (d) becomes the correct answer.

28. Since we got: $c_1 \times c_2 \times (20 - 9m + m^2) = 2$ as the equation in the previous solution, we can see that if we insert $c_1 = 1/15$ and $c_2 = 1$ in this equation we will get:

$m^2 - 9m + 20 = 30 \rightarrow m^2 - 9m - 10 = 0 \rightarrow (m-10)(m+1) = 0 \rightarrow m = 10$ or $m = -1$. i.e., the common roots for the two equations could either be 10 or –1 giving rise to two cases for the quadratic equation $Q_2 = 0$:

Case 1: When the common root is 10; Q_2 will become $\rightarrow (x-8)(x-10) = x^2 - 18x + 80$. The sum of roots for $Q2(x) = 0$ in this case will be 18.

Case 2: When the common root is –1; Q_2 will become $\rightarrow (x-8)(x+1) = x^2 - 7x - 8$. The sum of roots for $Q_1(x) = 0$ in this case will be 7.

Option (a) gives us a possible sum of roots as 18 and hence, is the correct answer.

29. In order to solve this question, the first thing we need to do is to identify the pattern of the numbers in the expression. The series 7, 18, 31, 46, etc., can be identified as 7, 7 + 11 × 1; 7 + 12 × 2; 7 + 13 × 3 and so on. Thus, the logic of the term when 39 is in the denominator is 7 + 39 × 38 = 1489 and the last term is 7 + 40 × 39 = 1567.

The series can be rewritten as:

$$\frac{[x]+7}{10}, 1+\frac{[x]+7}{11}, 2+\frac{[x]+7}{12}, 3+\frac{[x]+7}{13} \cdots 29+\frac{[x]+7}{39}$$

and $30 + \frac{[x]+7}{40}$

For each of these to be in their simplest forms, the value of [x] should be such that [x] + 7 is co-prime to each of the 31 denominators (from 10 to 40). From amongst the options, option (c) gives us a value such that [x] + 7 =101 which is a prime number and would automatically be co-prime with the other values.

30. $x - y = 6 \rightarrow x = 6 + y$. Substituting this value of x in the expression for the value of P, we get:

$P = 7(6 + y)^2 - 12y^2$

$P = 252 + 7y^2 + 84y - 12y^2 = 84y - 5y^2 + 252$

Differentiating P with respect to y and equating to zero we get:

$84 - 10y = 0 \rightarrow y = 8.4$

The maximum value of P will be obtained by inserting $y = 8.4$ in the expression. It gives us:

$84 \times 8.4 - 5 \times 8.42 + 252 = 705.6 - 5 \times 70.56 + 252 = 957.6 - 352.8 = 604.8$

31. Only in the case of option (c) do we get the LHS of the equation $4a - 3b - 4c = 0$ such that all the x, y and z cancel each other out. Hence, option (c) is the sole correct answer.

32. The equation can be thought of as $(x - 2m)(x - 4m)(x - 5m) = 0$. The value of the constant term would be given by $(-2m) \times (-4m) \times (-5m)$ which will give us an outcome of $-40m3$ which is equal to -1080. Solving $-40m3 = -1080 \rightarrow m = 3$. Hence, the roots of the equation being $2m$, $4m$ and $5m$ will be 6, 12 and 15 respectively. Hence, the equation would become $(x - 6)(x - 12)(x - 15) = 0$. The coefficient of x^2 will be $(-15x^2 - 6x^2 - 12x^2) = -33x^2$. Hence, the value of '$a$' will be +33. Hence, option (c) will be the correct answer.

33. The value of the expression $\frac{1}{lm} + \frac{1}{mn} + \frac{1}{ln} = l\,m\,n\,(l + m + n)/l2\,m^2\,n2 = (l + m + n)/l\,m\,n$. For any cubic equation of the form $ax^3 + bx^2 + cx + d = 0$, the sum of the roots is given by $-b/a$; while the product of the roots is given by $-d/a$. The ratio $(l + m + n)/l\,m\,n = b/d = -20/-5 = 4$.

Option (c) is correct.

34. When you look at this question, it seems that there are two equations with three unknowns. However, a closer look of the second equation shows us that the second equation is the same as the first equation.

i.e. $10p + 8g + 6m = 44$ and $5p + 4g + 3m = 22$ are nothing but one and the same equation. Hence, you have only one equation with three unknowns. However, before you jump to the 'cannot be determined' answer, consider this thought-process.

The cost of ten pears would be a multiple of 10 (since all costs are natural numbers). Similarly, the cost of eight grapes would be a multiple of 8 while the cost of six mangoes would be a multiple of 6.

Thus, the first equation can be numerically thought of as follows:

By fixing the cost of ten pears as a multiple of 10 and the cost of 8 grapes as a multiple of 8, we can see whether the cost of six mangoes turns out to be a multiple of 6.

Total cost	*Total cost of 10 pears*	*Total cost of 8 grapes*	*Total cost of 6 mangoes*	
44	10	16	18	Possible
44	10	24	10	Not possible
44	10	32	2	Not possible
44	20	8	16	Not possible
44	20	24	0	Not possible
44	30	8	6	Not possible, since both the cost of mangoes and grapes turns out to be ₹ 1 per unit (They have to be distinct.)

Thus, there is only one possibility that fits into the situation. The cost per pear = ₹ 1. The cost per grape = ₹ 2 per unit and the cost per mango = ₹ 3 per unit. Hence, the total cost of 4 mangoes + 3 grapes = 12 + 6 = ₹ 18.

Option (b) is correct.

35. There are 36 ways of distributing the sum of 37 between a_1 and a_2 such that both a_1 and a_2 are positive and integral. (From 1, 36; 2, 35; 3, 34; 4, 33...; 36, 1)

Similarly, there are $^9C_2$ (= 36) ways of distributing the residual value of 10 amongst a_3, a_4 and a_5. Thus, there are a total of 36 × 36 = 1296 ways of distributing the values amongst the five variables such that each of them is positive and integral. Option (d) is the correct answer.

36. Let the polynomial be $f(x) = ax^2 + bx + 10$.

The value of $f(5)$ in this case, would be:

$f(5) = 25a + 5b + 10 = 75$

$f(-5) = 25a - 5b + 10 = 45$

$f(5) - f(-5) = 10b = 30 \rightarrow b = 3$. The polynomial expression is: $ax^2 + 3x + 10$.

Further, if we put the value of $b = 3$ in the equation for $f(5)$, we would get: $25a + 15 + 10 = 75 \rightarrow 25a = 50 \rightarrow a = 2$.

Since $f(p)$ and $f(q)$ are both equal to zero, it means that p and q are the roots of the equation $2x^2 + 3x + 10 = 0$. Finding $p \times q$ would mean that we have to find the product of the roots of the equation. The product of the roots will be equal to 10/2 = 5. Option (c) is the correct answer.

37. The various possibilities for the values of a and b (in terms of their being positive or negative) will be as follows:

Possibility 1: a positive and b positive;

Possibility 2: a positive and b negative;

Possibility 3: a negative and b positive;

Possibility 4: a negative and b negative.

Let us look at each of these possibilities one-by-one and check out which one of them is possible.

Possibility 1: If a and b are both positive the second equation becomes $a = 150$ (since the value of $|b| - b$ would be equal to zero if b is positive). However, this value of 'a' does not fit the first equation since the value of the LHS would easily exceed 75 if we use $a = 150$ in the first equation. Hence, the possibility of both a and b being positive is not feasible and can be rejected.

Through similar thinking the possibilities 3 and 4 are also rejected. Consider this:

For possibility 3: a negative and b positive: the second equation will give us $a = 150$ which contradicts the presupposition that a is negative. Hence, this possibility can be eliminated.

For possibility 4: a negative and b negative: the first equation would give us $b = 75$ (since the value of $|a| + a$ will be equal to zero if a is negative) which contradicts the presupposition that b is negative. Hence, this possibility can be eliminated.

The only possibility that remains is possibility 2: a positive and b negative. In this case, the equations will transform as follows:

$|a| + a + b = 75$ will become $2a + b = 75$;

$a + |b| - b = 150$ will become $a - 2b = 150$.

Solving the two equations simultaneously, we will get the value of $a = 60$ and $b = -45$.

The sum of $|a| + |b| = 60 + 45 = 105$.

Option (a) is the correct answer.

38. The value of the expression will be dependent on the individual values of each of the terms in the expression. $[315^{1/3}]$ will give us a value of 6 and so would all the terms upto $[342^{1/3}]$. (as $6^3 = 216$ and $7^3 = 343$). Hence, the value of the expression from $[315^{1/3}] + [316^{1/3}] + \ldots + [342^{1/3}] = 28 \times 6 = 168$.

Similarly, the value of the expression from $[343^{1/3}] + [344^{1/3}] + \ldots + [511^{1/3}] = 169 \times 7 = 1183$.

Also, the value of the expression from $[512^{1/3}] + [513^{1/3}] + \ldots + [515^{1/3}] = 4 \times 8 = 32$.

Thus, the answer = 168 + 1183 + 32 = 1383. Option (a) is correct.

39. The five consecutive integers can be represented by: $(c - 2)$; $(c - 1)$; c; $(c + 1)$ and $(c + 2)$.

Then we have $a^2 + b^2 + c^2 = d^2 + e^2$, giving us:

$(c - 2)^2 + (c - 1)^2 + c^2 = (c + 1)^2 + (c + 2)^2 \rightarrow$

$3c^2 - 6c + 5 = 2c^2 + 6c + 5 \rightarrow$

$c^2 - 12c = 0 \rightarrow$

$c = 0$ or $c = 12$

Hence, the possible values of $d = c + 1$ would be 1 or 13.

Option (d) is correct.

40. The profit of the 9-to-9 supermarket would be:

Total sales price – Total cost price

$= 10 \times (15 + 16x) - 4x^2$

$= -4x^2 + 160x + 150$

The maximum value of this function can be traced by differentiating it with respect to x and equating to 0. We get:

$-8x + 160 = 0 \rightarrow x = 20$. The maximum value of profit will occur at $x = 20$.

The maximum profit will be $= -4 \times 20^2 + 160 \times 20 + 150 = -1600 + 3200 + 150 = 1750$.

Option (c) is the correct answer.

41 $\alpha + \beta = -m$ and $\alpha\beta = 1$

$\Rightarrow \gamma + \delta = -n$ and $\gamma\delta = 1$

$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = (\alpha - \gamma)(\beta + \delta)(\beta - \gamma)(\alpha + \delta)$

$= [\alpha\beta + \alpha\delta - \gamma\beta - \gamma\delta][\alpha\beta + \beta\delta - \alpha\gamma - \gamma\delta]$

$=[1 + \alpha\delta - \gamma\beta - 1][1 + \beta\delta - \gamma\alpha - 1]$

$=(\alpha\delta - \gamma\beta)(\beta\delta - \gamma\alpha)$

$= 1.\delta^2 - \alpha^2.1 - \beta^2.1 + \gamma^2.1 = (\delta^2 + \gamma^2) - (\alpha^2 + \beta^2)$

$= [(\delta + \gamma)^2 - 2\delta\gamma] - [(\alpha + \beta)^2 - 2\alpha\beta]$

$= [(-n)^2 - 2.1] - [(-m)^2 - 2.1] = n^2 - m^2$

Option (a) is correct.

42. Roots of the given equation $= \dfrac{2a \pm \sqrt{4a^2 - 4ab}}{2b}$

$$= \frac{a \pm \sqrt{a^2 - ab}}{b}$$

$$= \frac{\sqrt{a}\left(\sqrt{a} \pm \sqrt{a-b}\right)}{b} \times \frac{\sqrt{a} \mp \sqrt{a-b}}{\sqrt{a} \mp \sqrt{a-b}} = \frac{\sqrt{a}}{\sqrt{a} \pm \sqrt{a-b}}$$

Option (c) is correct.

43. $(x^2 + 3x - 10) = 0$

$(x - 2)(x + 5) = 0$ or $x = 2, -5$

Therefore, for $x = +2$ and $x = -5$

$3x^4 + 2x^3 - ax^2 + bx - a + b - 4 = 0$

For $x = 2 \rightarrow 3(2)^4 + 2(2)^3 - 4a + 2b - a + b - 4 = 0$

$48 + 16 - 4a + 2b - a + b - 4 = 0$

$5a - 3b = 60 \quad (1)$

For $x = -5$,

$3 \times (-5)^4 + 2 \times (-5)^3 - a \times (-5)^2 + b \times (-5) - a + b - 4 = 0$

$1875 - 250 - 25a - 5b - a + b - 4 = 0$

$26a + 4b = 1621 \quad (2)$

By solving equation (1) and (2) we get:

$a \approx 52, b \approx 67$

44. Let $f(x) = 3x^2 - 4x + 7$

$f(x) = 6x - 4 = 0$ or $x = 2/3$

The minimum value of $f(x)$ will then be given by: $3(2/3)^2 - 4(2/3) + 7 = 17/3$

45. Discriminant of the given equation is $16 - 4(1 - p)p$ or $16 - 4p(1 - p) > 0$ for $0 < p < 1$.

Sum of roots $\left(\frac{-4}{(1-p)}\right) < 0$ for $0 < p < 1$

Product of roots $\left(\frac{p}{1-p}\right) > 0$, for $0 < p < 1$

Therefore, roots of the given equation are real and negative.

46. $y^2 - 2y \cos x + 1 = 0$ (1)

For real y, $D \geq 0$, we get that $4 \cos^2 x \geq 4$

This will be true when $\cos x \geq 1$ and $\cos x \leq -1$.

As $-1 \leq \cos x \leq 1$: Hence, the only possible value of $\cos x$ for which $4(\cos^2 x - 1) \geq 0$ are 1 and -1.

Hence, number of possible real solutions are 2.

$\therefore$ Option (c) is the correct choice.

47. We are given that the expression $ax^3 + bx^2 + cx + d$ intersects the x-axis at 1 and -1. It means that at $x = 1$ and -1 the value of the given polynomial is equal to 0.

$\therefore a + b + c + d = 0$ and $-a + b - c + d = 0$

$\therefore 2(b + d) = 0$

$\therefore b + d = 0$ (1)

We are also given that; $ax^3 + bx^2 + cx + d$ intersects the y−axis at 2. This means that at $x = 0$, the value of the given polynomial is equal to 2.

$0 + d = 2$

$\therefore d = 2$ (2)

From equations (1) and (2), $b = -2$

48. $(\alpha + 3)(\beta + 3)(\gamma + 3) = 27$

$\alpha\beta\gamma + 9(\alpha + \beta + \gamma) + 3(\alpha\beta + \beta\gamma + \gamma\alpha) + 27 = 27$

$p + 9m + 3n = 0$ (**Note:** the product of the roots of the equation $x^3 - mx^2 + nx - p = 0$ will be equal to p; the sum of the roots would be equal to m; the pair-wise product of the roots will be equal to n.)

49. Using mathematical induction, let us assume the roots to be 1, 2, and 4.

The sum of the roots = $1 + 2 + 4 = -q \rightarrow q = -7$

The product of the roots = $1.2.4 = -s \rightarrow s = -8$

The pair-wise product of the roots, $r = 1 \times 2 + 2 \times 4 + 4 \times 1 \Rightarrow r = 14$

Only option (b) satisfies for $r = 14$, $q = -7$, $s = -8$

So option (b) is correct.

50. Graph opens downwards, so $a < 0$

As we can see that, $\alpha < 2$, $\beta < -3$

So sum of roots of $ax^2 + bx + c$ is less than 0 or

$$-\frac{b}{a} < 0$$

$\text{or}\ \frac{b}{a} > 0$

As a < 0, so $b < 0$

Product of the roots is also less than 0.

$\alpha\beta = \frac{c}{a} < 0$

As a < 0, so $c > 0$

Hence, option (a) is correct.

51. $x^8 + 12$ is always greater than x^8.

So $(x^8 + 12)^{1/2}$ will always be greater than $(x8)^{1/2} = x^4$

x^4 will always be greater than $x^4 - 2$

so $(x^8 - 12)^{1/2}$ will always be greater than $x^4 - 2$. Hence, the LHS and the RHS of the given equation can never be equal to each other. This means that there are no solutions for the given equation. Hence, option (a) is correct.

52. For the inequality to be true for all values of x, the quadratic expression $x^2 + 4px + (p + 3)$ should have imaginary roots. Using the discriminant < 0, we get:

$16p^2 - 4(p + 3) < 0$

$16p^2 - 4p - 12 < 0$

$4p^2 - p - 3 < 0$

$4p^2 - 4p + 3p - 3 < 0$

$4p\,(p - 1) + 3(p - 1) < 0$

$(4p + 3)(p - 1) < 0$

$p \in (-3/4, 1)$

Only possible integer value of p in this range would be at $p = 0$.

Hence, there is only one integer value of p that satisfies the condition.

53. $g(x) = x^3 - px^2 - \frac{q}{2}x - r = (x-p)(x-q)(x-r)$

$p + q + r = p$

$q + r = 0$ (1)

$$pq + qr + pr = -\frac{q}{2} \Rightarrow p(q+r) + qr = -\frac{q}{2} \Rightarrow r = -\frac{1}{2}$$

$$q = \frac{1}{2}$$

$pqr = r$

$pq = 1$

$p = 2$

$$g(4) = (4)^3 - 2(4)^2 - \frac{1}{4}(4) + \frac{1}{2}$$

$$= 64 - 32 - 1 + \frac{1}{2}$$

$= 31.5$

54. $(a + 2)(b + 2)(c + 2) = 47$

$abc + 4(a + b + c) + 2(ab + bc + ca) + 8 = 47$

$4p + 2q + r = 39$

55. Let the roots of $x^2 + bx + 3b = 0$ are a and $3a$.

$a + 3a = -b \Rightarrow 4a = -b$ (1)

$3a^2 = 3b \Rightarrow a^2 = b$ (2)

By solving equation (1) and (2), we get

$a = -4, b = 16$

Value of $3b = 3 \times 16 = 48$

56. The given equation is $x^2 - 33x + 17 \times 16 = 0$

$x^2 - 33x - 272 = 0$

Roots of the equation are 16, 17 or b, $b + 1$. Hence, option (a) is correct.

57. $p + q = -5$

$q + r = -23$

$p + 2q + r = -28$ or $(p + r) + 2q = -28$ (1).

Since, p, q and r are in arithmetic progression, the value of $(p + r) = 2q$. Using this in equation (1), we get:

$4q = -28$ or $q = -7$. Using the logic of the Arithmetic Progression, we can get the values of p and r respectively, as:

$p = 2, r = -16$
$|a \times b| = |pq \times qr| = |pq^2r| = |2 \times -7^2 \times -16| = 1568$

58. $f(x) = (x - 2)(x^2 + 2x + 5) = x^3 - 2x^2 + 2x^2 + 5x - 4x - 10$

$= x^3 + x - 10$

$a + b + c = 0$ abc = 10

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 = 3abc = 3 \times (10) = 30$

59. $f(x)$ is exactly divisible by $(x + 2), (x + 3)$

Therefore, $f(-2) = f(-3) = 0$

$4p - 2q + r = 0$ (1)

$9p - 3q + r = 0$ (2)

Also, since the remainder of $f(x)$ when divided by $(x - 1)$ is 7, we can use $f(1) = 7$, which in turn gives us that:

$p + q + r = 7(3)$

By solving equations (1), (2), (3) we get $q \approx 2.92$

60. $f(x) = 4x - 7\sqrt{x} = 2$

Let $t = \sqrt{x}$

$\Rightarrow 4t^2 - 7t = 2$

$\Rightarrow 4t^2 - 7t - 2 = 0$

$\Rightarrow 4t(t - 2) + 1(t - 2) = 0$

$\Rightarrow t = -\frac{1}{4}, 2$

$\Rightarrow \sqrt{x} = -\frac{1}{4}$ or 2. (However, the value of $\sqrt{x} = -\frac{1}{4}$ is not possible)

$\sqrt{x} = 2 \rightarrow x = 4$

∴ Only option (c) is correct.

61. $b^2 - 4a^2 \geq 0$ (roots are real)

$(b - 2a)(b + 2a) \geq 0$

Both the roots are positive, therefore, the sum of the roots must be greater than 0.

$-\frac{b}{a} > 0$

But a > 0, so b must be less than 0

$\therefore$ $b - 2a < 0$ or $b < 2a$ & $b + 2a \le 0$ or $b \le -2a$ Therefore, all the options are tr]ue.

62. $(x - p)(x - q) + (x - q)(x - r) + (x - r)(x - p) = 0$

$\Rightarrow x^2 - (p + q)x + pq + x^2 - (q + r)x + qr + x^2 - (p + r)x + pr = 0$

$3x^2 - 2(p + q + r)x + pq + qr + pr = 0$

Discriminant of the above equation $= 4(p + q + r)^2 - 12(pq + qr + pr)$

$= 4[p^2 + q^2 + r^2 + 2pq + 2pr + 2qr - 3pq - 3pr - 3qr]$

$= 4[p^2 + q^2 + r^2 - pr - qr - pr]$

$= = \frac{4}{2}\left[(p-q)^2 + (q-r)^2 + (p-r)^2\right]$

$= 2[(p - q)^2 + (q - r)^2 + (p - r)^2]$

As p, q, r are not equal to each other, so the discriminant of the given equation is always greater than 0, therefore the roots are real.

Thus, option (b) is true.

63. $f\left(\frac{1}{x}\right) = p\left(\frac{1}{x}\right)^2 + q\left(\frac{1}{x}\right) + r$

$= rx^2 + qx + p = g(x)$

Therefore, the roots of $g(x)$ and $f(x)$ are the inverse of each other.

$\therefore$ The roots of $f(x) = 0$ are 2 and 4.

The roots of $g(x) = 0$ are $\frac{1}{2}, \frac{1}{4}$

$\text{Required sum} = 2 + 4 + \frac{1}{2} + \frac{1}{4} = 6.75$

64. When the curve intersects the x-axis, then $y = 0$

$\Rightarrow x^2 - 5x = 0$

$\Rightarrow (x)(x - 5) = 0$ or $x = 0, 5$

When the curve intersects the y-axis then $x = 0$

$y^3 - 4y^2 + 3y = 0$

$y(y^2 - 4y + 3) = 0$

$y(y - 3)(y - 1) = 0$

$y = 0, 1, 3$

But the points $x = 0$ and $y = 0$ would coincide at the origin. So there are a total of 4 points.

These are (0, 0), (0, 1), (0, 3), (5, 0).

65. Let x and y be the roots of the given equation. Then,

$x + y = \frac{-q}{p}$ and $x \times y = \frac{r}{p}$

Now, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2 + y^2}{(xy)^2} = \frac{(x + y)^2 - 2xy}{(xy)^2}$

$$= \frac{\left(\frac{-q}{p}\right)^2 - 2\left(\frac{r}{p}\right)}{\left(\frac{r}{p}\right)^2}$$

As per the question:

$$\frac{-q}{p} = \frac{\left(\frac{-q}{p}\right)^2 - 2\left(\frac{r}{p}\right)}{\left(\frac{r}{p}\right)^2}$$

⇒ 2p2r = pq2 + qr2

$$\Rightarrow \frac{2p^2r}{pqr} = \frac{pq^2}{pqr} + \frac{qr^2}{pqr} \text{(dividing by pqr)}$$

$$\Rightarrow \frac{2p}{q} = \frac{q}{r} + \frac{r}{p} \quad (1)$$

From equation (1) it is clear that $\frac{r}{p}, \frac{p}{q}$ and $\frac{q}{r}$ are in arithmetic progression so option (a) is correct.

As $\frac{q^2}{p^2} \neq \frac{p}{r} \times \frac{r}{q} = \frac{p}{q}$, so option (b) is incorrect.

Option (c): As $\frac{r}{p}, \frac{p}{q}$ and $\frac{q}{r}$ are in arithmetic progression; their reciprocals are in harmonic progression. Therefore this option is also correct.

Hence option (d) is correct.

66. From (1), we have sum of roots = 14

and from (2) we have product of roots = 48. Checking the options, for option (c) we have that the sum of roots is 14 and the product of the roots is 48. Hence, option (c) is correct.

Level of Difficulty (III)

1. $f(0)$ is greater than or equals to 0 means that the value of $c \geq 0$. Also, since 'a' is negative as given in the question, the product of the roots given by $\frac{c}{a}$ will be negative or zero. So, the roots of $f(x) = 0$ are of opposite sign or

their product is zero.

So the possibilities for the two roots from the given values are: (0, 2), (0, 1), (1, −2), (0, −1), (0, −2), (−1, 2), (−2, 2), (−1, 1). Therefore a total of eight different sets are possible for the roots of $f(x) = 0$.

∴ option (c) is correct.

2. Let $f(x) = px^2 + qx + r$

$px^2 + qx + r = 0$. Here, two cases are possible:

Case 1: $p > 0$

When $p > 0$ then $px^2 + qx + r$ will open upwards and $p \times f(2) < 0$ will give us that $p(4p + 2q + r) < 0$. This is the same value as option (a). Hence, option (a) is correct. Option (b) is automatically rejected since it is the opposite of option (a).

(**Note:** Option (c) is rejected because In this case whether $p \times f(-2)$ [Which will give us $p(4p - 2q + r]$ is greater than or less than zero depends upon the location of the other root which is less than 1.Similarly, Option (d) is rejected as it is asking you to commit about the value of $p \times f(-3) < 0$. Whether this value is greater than or less than zero depends upon the location of the other root of $f(x) = 0$ (which is less than 1)).

So option (b), (c), and (d) are not necessarily true.

Case 2: $p < 0$

When $p < 0$ then $p\,f(2) < 0$ [because $f(x)$ will open downwards].

In this case whether $p\,f(-2)$, $p\,f(-3)$ will be greater than or less than zero depends upon the location of the other root of $f(x) = 0$. So only option (a) is correct in this case too.

3. $f(x, y) = 5$

$4^{x^y} + x^{4^y} = 5$

If $x = 1$, then the above equation satisfies for any integral value of y. So, option (d) is correct.

4. d is a root of the equation $ax^2 + bx + c = 0$

$\therefore ad^2 + bd + c = 0$ (1)

c is a root of the equation $ax^2 + bx + d = 0$

$\therefore ac^2 + bc + d = 0$ (2)

Equation (2) – equation (1):

$a(c^2 - d^2) + b(c - d) + (d - c) = 0$

$a(c - d)(c + d) + b(c - d) - (c - d) = 0$

$(c - d)[a(c + d) + b - 1] = 0$. In this expression, the value of $[a(c + d) + b - 1]$ has to be zero because it is given that $c \neq d$.

Solving: $[a(c+d)+b-1]=0$, we get $(c+d)=\frac{1-b}{a}$

$\therefore$ option (c) is correct.

5. $-(c+d)=\frac{b-1}{a}$

(iii) (From the solution of the previous question) (**Note:** This gives us the sum of the roots of the required equation, whose roots are $-c$ and $-d$.

Next in order to get the product of the roots of the required equation: Divide equation (1) by d and equation (2) by c and then by equating, we get:

$$\frac{ad^2+bd+c}{d}=\frac{ac^2+bc+d}{c}$$

$$\left(ad+b+\frac{c}{d}\right)=ac+b+\frac{d}{c}$$

$$a(c-d)=\frac{c}{d}-\frac{d}{c}=\frac{(c-d)(c+d)}{cd}$$

$$cd=\frac{c+d}{a}=\frac{1-b}{a^2}$$

(iv) (product of roots of the required equation)

Equation whose roots are $-c$, $-d$ is

x^2 – (sum of roots)x + product of roots = 0

$$x^2-\frac{b-1}{a}x+\frac{1-b}{a^2}=0$$

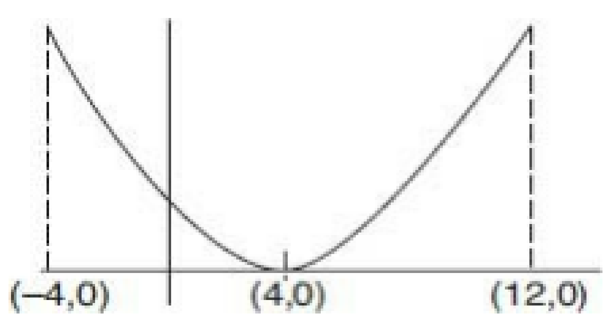
$ax^2 + a(1 - b)x + 1 - b = 0$

∴ option (c) is correct.

6. Let $f(x) = ax^2 + bx + c$

$f(4) = 16a + 4b + c = 0$ [At $x = 4$ $f(x) = 0$]

$f(0) = c = 10$



$f(x)$ is symmetric about $x = 4$.

$f(3) = f(5)$

$9a + 3b + c = 25a + 5b + c$

$16a + 2b = 0$

$8a + b = 0$

$b = -8a$

$f(4) = 0$

$16a + 4b + 10 = 0$

$16a - 32a + 10 = 0$

$$a = \frac{10}{16} = \frac{5}{8}$$

$f(-4) = 16a - 4b + 10$

$$= 16 \times \frac{5}{8} - 4 \times (-8) \times \frac{5}{8} + 10$$

$= 10 + 20 + 10 = 40$

Alternative method: the minimum value of $f(x)$ must be 0 and this minima occurs at $x = 4$.

Let $f(x) = a(x - 4)2$

$f(0) = 16a = 10$

$$a = \frac{5}{8}$$

Hence, $f(x) = \frac{5}{8}(x-4)^2$

$$f(-4) = \frac{5}{8} \times (-8)^2 = 40$$

7. 4 lies in between roots of the equations, so:

$3(4)_2 + 4a(4) + a + 3 < 0$

$48 + 16a + a + 3 < 0$

$17a + 51 < 0$

$a < -3$

∴ option (a) is correct.

8. $f(x) = x^3 - (5 + k)x^2 + (6 + 5k)x - 6k$

Let $k = 5$

$f(x) = x^3 - 10x^2 + 31x - 30$

By factorisation, we get:

$f(x) = (x - 2)(x - 3)(x - 5)$

$f(x) < 0$ for $x < 2, 3 < x < 5$

Similarly, we can put few other values of x and confirm that $f(x) < 0$ for $x < 2$ and for $3 < x < k$.

9. $f(x, 5) = a(x - 5)^2 + b(x - 5) + c$

$ax^2 + bx + c$ has roots 3, 4. Then roots of $f(x, 5)$ are $5 + 3, 5 + 4 = 8, 9$.

$f(x, 5)$ attains its maximum value at $\frac{8+9}{2} = \frac{17}{2} = 8.5$

∴ option (b) is correct.

10. $[x]^2 - 11[x] + 30 = 0$

 $[x]^2 - 5[x] - 6[x] + 30 = 0$

$([x] - 5)([x] - 6) = 0$

$[x] = 5, 6$

The integer solutions of x are at $x = 5$ and 6

So the required sum $= 5 + 6 = 11$.

11. If the three roots are x, y, z

$\Rightarrow x + y + z = 12$

Only one combination of $(x, y, z) = (2, 3, 7)$ is possible.

Product of the roots $= 2 \times 3 \times 7 = 42$.

So the chosen values are correct.

$p = 2 \times 3 + 3 \times 7 + 2 \times 7 = 6 + 21 + 14 = 41$

12. $f(x) = g(x)$

$x3 - px2 - 2qx + r = (x - p)(x - q)(x - r)$

$p + q + r = p \Rightarrow q + r = 0$ (1)

$pq + qr + pr = -2q$

$p(q + r) + qr = -2q$

As $q + r = 0$ from equation (1), therefore, $qr = -2q$

$r = -2$

$q = +2$

$pqr = -r$

$pq = -1$

$p = -1/2$

So the required value of $p + q = -\frac{1}{2} + 2 = \frac{3}{2}$

13. $f(x) = g(x)$

$f(x) = g(x) = (x - p)(x - q)(x - r)$

$$f(4) = \left(4 + \frac{1}{2}\right)(4-2)(4+2)$$

$$= \frac{9}{2} \times 2 \times 6$$

$= 54$

14. $a:b:c:d:e = 1:2:3:4:5$

$f(x) = x^2 + 7x + 5$

Let the roots of $f(x) = 0$ be A and B

$(A - B)^2 = (A + B)^2 - 4AB = (-7)^2 - 4 \times 5 = 29$

15. $\log_3 x \times \log_3 y + \log_3 z \times \log_3 xy = 11$

$\log_3 x \times \log_3 y + \log_3 z [\log_3 x + \log_3 y] = 11$

Let $\log_3 x = A, \log_3 y = B, \log_3 z = c$

$AB + C[A + B] = 11$

$AB + BC + AC = 11$

& $A^2 + B^2 = 14 - C^2$

$A^2 + B^2 + C^2 = 14$

This clearly identifies the values of A, B and C as 1,2, and 3. Thus, we get x, y and z as 3, 9 and 27 in no particular order. Also,

$xyz = 3^6 = (3^2)^3 = (3^3)^2$

Therefore $k_1 = 3^3$ and $k_2 = 3^2$

$k_1 + k_2 = 3^3 + 3^2 = 27 + 9 = 36$

16. $(x - 3)$ is a factor of $f(x)$ then $f(3) = 0$

$9a - 150 \times 3 + 5b = 0$

$9a - 450 + 5b = 0$

$a = 50 - \frac{5}{9}b$

a and b are positive integers, therefore, b must be a multiple of 9.

For $b = 9, a = 45$

$b = 18, a = 40$

$b = 27, a = 35$

$b = 36, a = 30$

$b = 45, a = 25$

$b = 54, a = 20$

$b = 63, a = 15$

$b = 72, a = 10$

$b = 81, a = 5$

Therefore, total possible ordered pairs of (a, b) is 9.

17. Maximum possible value of $a + b = 81 + 5 = 86$.

18. If a and b are integers then a and b both can be negative then there are infinite pairs of (a, b) for which $(x - 3)$ is a factor of $f(x)$.

Hence, option (d) is correct.

19. $f(x) = x^3 - 12x^2 + 47x - 60 = (x - 3)(x - 4)(x - 5)$

The possible equations are:

$(x-3)^2, (x-4)^2, (x-5)^2, (x-3)(x-4), (x-4)(x-5), (x-3)(x-5)$. We get a total of six such equations. Hence, option (c) is correct.

20. Required product = $3^2 \times 4^2 \times 5^2 \times 4 \times 5 \times 3 \times 5 \times 3 \times 4$

$p = 3^4 \times 4^4 \times 5^4$

$p^{1/4} = 3 \times 4 \times 5 = 60$

Option (d) is correct.

21. $(x-k)^3 (x-7) - 27 = 0$

$(x-k)^3 (x-7) = 27$

Roots of the equations are integers, which mean that x is an integer. Now the following four cases are possible:

Case 1: $(x-k)^3 = 27, x-7 = 1$

Case 2: $(x-k)^3 = 1, x-7 = 27$

Case 3: $(x-k)^3 = -27, x-7 = -1$

Case 4: $(x-k)^3 = -1, x-7 = -27$

Case 1: $x-7 = 1 \Rightarrow x = 8$

$(8-k)^3 = 27$

$8-k = 3$

$k = 5$

Case 2: $(x-7) = 27 \Rightarrow x = 34$

$(x-k)^3 = 1 \Rightarrow 34 - k = 1$

Case 3: $(x - k)^3 = -27$, $x - 7 = -1$

Therefore, $x = 6$

$x - k = -3$

$k = 9$

Case 4: $x - 7 = -27$, $x = -20$

$x - k = -1$ or $k = x + 1 = -20 + 1 = -19$

Therefore, four values are possible for k.

22. Required difference = 33 − (−19) =52

23. Let the roots of the quadratic equation be a, b.

$a + b = (p - 3)$

$ab = (p - 4)$

$a^2 + b^2 = (p - 3)^2 - 2(p - 4)$

$= (p - 3)^2 - 2(p - 4)$

$= (p - 3)^2 - 2(p - 3) + 1 + 1$

$= (p - 3 - 1)^2 + 1$

$= (p - 4)^2 + 1$

$(a^2 + b^2)$ is minimum for $p = 4$

24. $f(x) = g(x)$

$x^2 = x^3 - 2x$

$x^3 - x^2 = 2x$

$x^2(x - 1) = 2x$

for $x = 2$; $x^2(x - 1) = 2x$

For $x > 2$, either x^2 or $(x - 1)$ is odd and hence, $x^2(x - 1)$ cannot be equal to 2x. Therefore, $f(x) = g(x)$ only for one value of x.

25. $3h(x) + 5t(x) = 0$

$$t(x) = -\frac{3}{5}h(x)$$

If $h(x) = -(x - p)(x - q)$ then $t(x) = \frac{3}{5}(x - p)(x - q)$

Therefore, roots of $t(x)$ and $h(x)$ are same, so their sums are also equal. Option (c) is correct.

26. If $h(x)$ is maximum at $x = 4$ then $t(x)$ will be minimum at $t = 4$ and its minimum value will be $-\frac{3}{5}$ times of the maximum value of $h(x)$.

$$t(x)|_{min} = -\frac{3}{5}h(x)|_{max} = -\frac{3}{5} \times 10 = -6$$

27. If we see the graph carefully, then we get:

For $x < 0$ $f(x) = (x - (-3))(x - (-2)) = (x + 2)(x + 3)$

For $x > 0$ $f(x) = (x - 2)(x - 3)$

$\Rightarrow f(x) = (|x| - 2)(|x| - 3)$

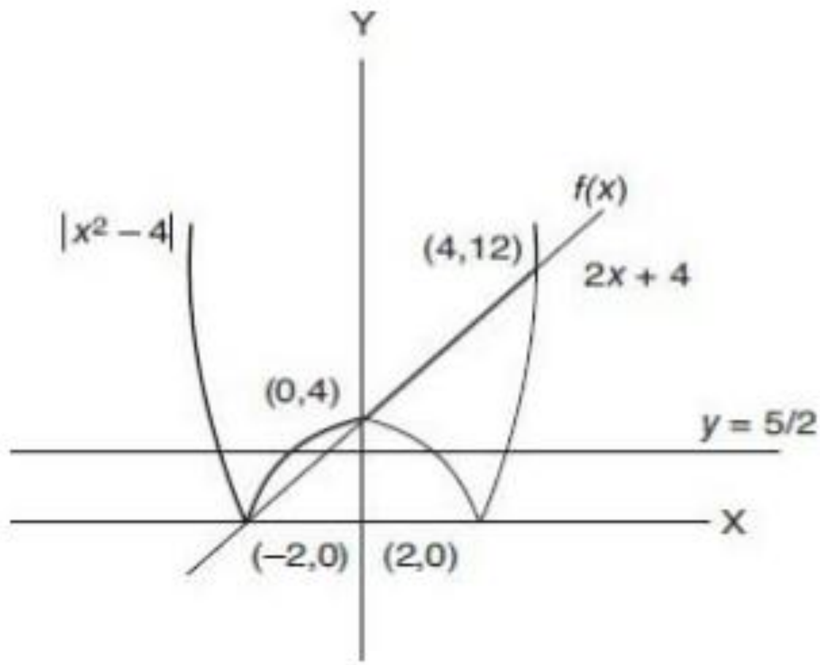
$|x|^2 - 5|x| + 6 = 0$

$$\frac{6}{|x|} = 5 - |x|$$

$\therefore$ Option (c) is correct.

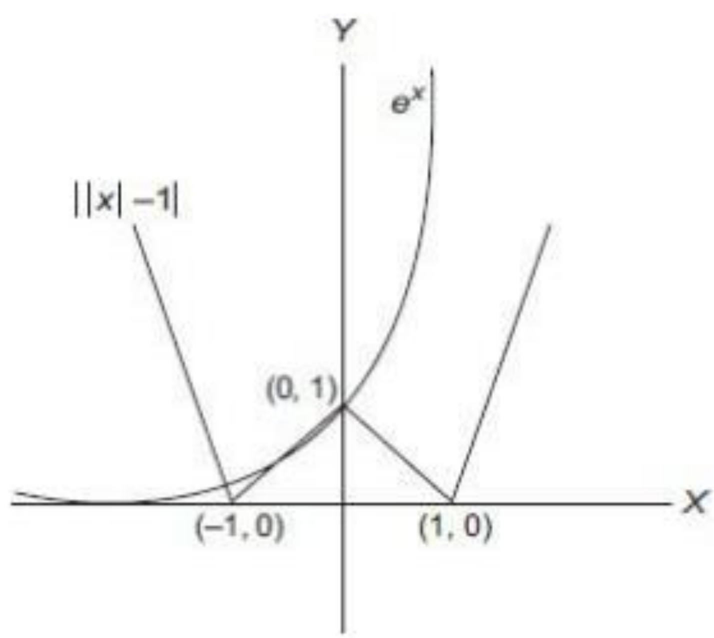
Alternately, you can try to put the values of x as −2, −3, 2 and 3 in the options to see which of the given options satisfies the conditions of the problem.

28. $f(x) = max(|x2 - 4|, 2x + 4)$



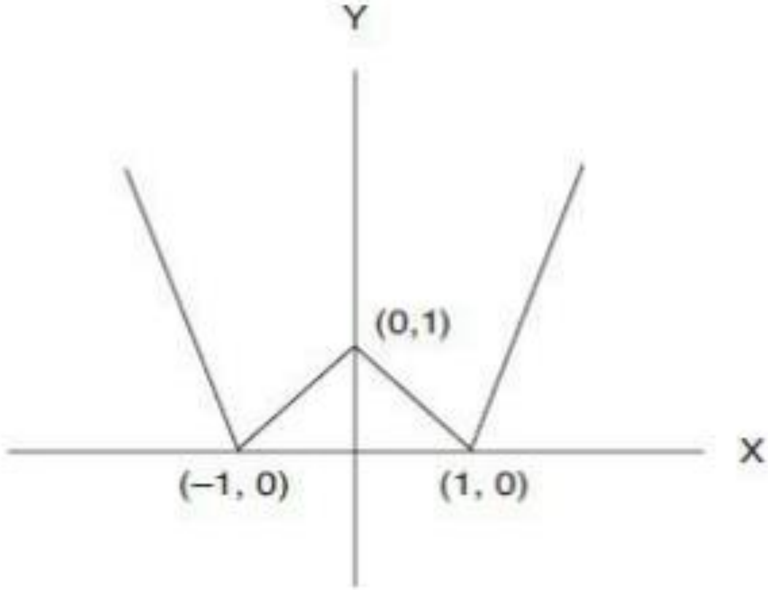
As we can see in the diagram shown above that $y = \frac{5}{2}$ intercepts $f(x)$ at two different points, therefore, the given equation has two solutions.

29. $||x| - 1| = ex$

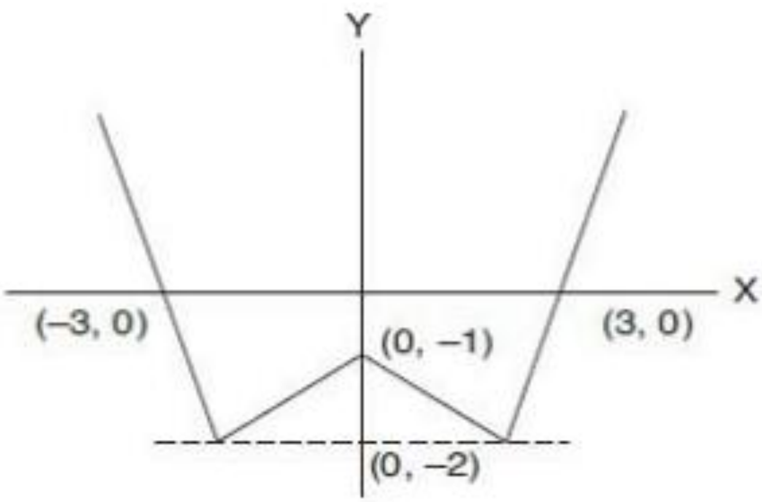


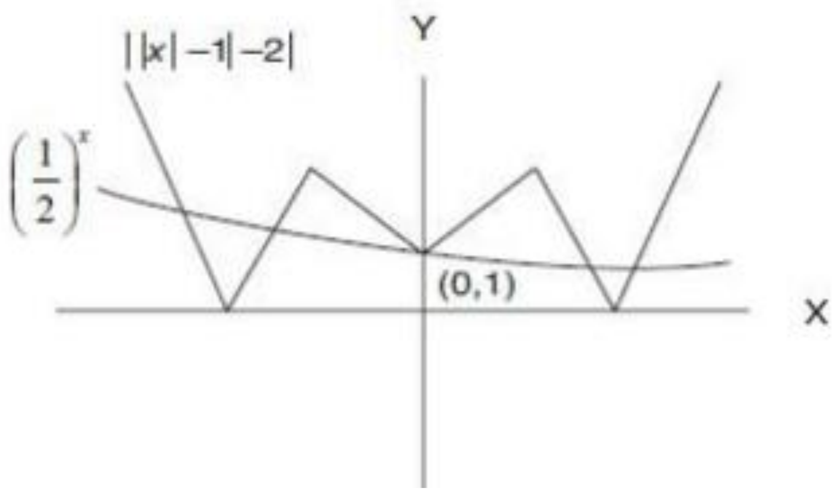
The curve of ex cuts the curve of $||x| - 1|$ at three points. Therefore, the given equation has three solutions.

30. $||x| - 1|$ would look like the figure shown below:



$||x| - 1| - 2$ will look like the figure shown below:





The graph of $(1/2)^x$ cuts the graph of $||x| - 1| - 2|$ at five distinct points. Therefore the given equation has five solutions.